Department of Electronic and Electrical Engineering

Reactive Components, Resonance and Filters

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Abstract

This report will be of interest to those involved in the study of analogue electronics. It is aimed at people looking to further their knowledge of filters and AC circuitry.

The document contains the theory behind the operation of high, low and band pass filters, as well as analysis of primary research data from the laboratory. The data in the report is displayed graphically and analysed using Python and the Jupyter Notebook.

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1. Introduction

Electronic signals can contain many different frequencies and sometimes it is desirable to remove a range of frequencies, allow a certain frequency to pass or remove a certain frequency. This is the job of the electronic filter. Different permutations of resistors, capacitors and inductors can produce circuits that act as different types of filter. The three types of filters focused on in this report are: high-pass, low-pass and band-pass. High-pass filters attenuate low frequencies thereby allowing higher frequencies to "pass", low-pass filters allow low frequencies and attenuate high frequencies and band-pass filters attenuate all frequencies other than a specific range or "band" or frequencies.

This report is designed to outline some of theory of these three filters, display some measured data and analyse said data.

- Section 2 contains circuit simulations made in AWR Design Environment and derivations of the transfer functions for each filter. For each transfer function the magnitude and phase angle functions are derived also.
- Section 3 presents the tabulated data, graphs and analysis of the data using Python to find cut-off and resonant frequencies, and explanations of the circuit's behaviour.

To obtain the tabulated data a simple apparatus was used: each filter was constructed on a small section of breadboard (this allows for fast, cheap and easy prototyping of circuits), a digital power supply was used to provide an AC input voltage and a digital oscilloscope was used to record the output voltage. The true values of each component were measured using the LCR bridge.

2. Theory

2.1 High-pass filter

The high pass filter consists of a resistor and a capacitor in series. The output voltage is high when the frequency of the input voltage is high. The output voltage decreases as the frequency of the input voltage decreases. High-pass filters are widely used in a variety of applications; they are used in the audio crossover of a speaker to allow only the higher frequencies to drive the tweeter and they are widely used in audio mixing equipment[1].

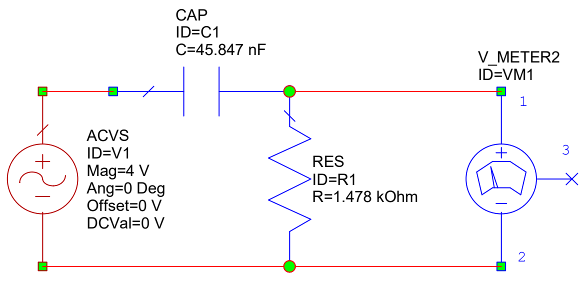

Figure 1a - High pass filter simulated on AWR

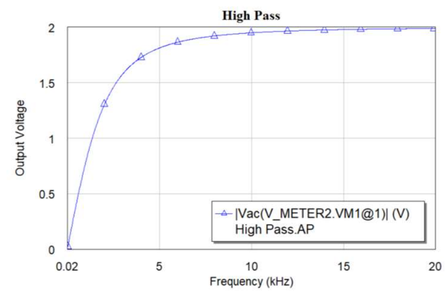

Figure 1b - Simulation result with 1999 data points

2.1.1 Derivations

From figure 1a V_1, C_1, R_1, and V_{M1} represent the input voltage V_{in}, capacitance C, resistance R, and output voltage V_{out} respectively.

The transfer function $H(\omega)$ is the ratio of the output voltage to the input voltage. It is a function in ω:

$$H(\omega) = \frac{V_{out}}{V_{in}} \quad (1)$$

The impedances of the resistor, capacitor, and circuit are calculated as follows:

$$Z_R = R \quad (2)$$

$$Z_C(\omega) = \frac{1}{j\omega C} \quad (3)$$

As the capacitor and resistor are in series the total amplitude is their sum:

$$Z_T(\omega) = R + \frac{1}{j\omega C} \quad (4)$$

Using the potential divider equation, the output voltage as a function of ω is:

$$V_{out}(\omega) = V_{in}(\omega) \cdot \frac{Z_R}{Z_C + Z_R} \quad (5)$$

$$V_{out}(\omega) = V_{in}(\omega) \cdot \frac{R}{\frac{1}{j\omega C} + R} \quad (6)$$

$$\frac{V_{out}(\omega)}{V_{in}(\omega)} = H(\omega) = \frac{j\omega CR}{1 + j\omega CR} \quad (7)$$

The amplitude of the transfer function $H(\omega)$ is the magnitude or modulus of the transfer function $|H(\omega)|$. Using equation (6):

$$|H(\omega)| = \frac{R}{\left|\frac{1}{j\omega C}+R\right|} \qquad (8)$$

$$|H(\omega)| = \frac{R}{\sqrt{R^2-\left(\frac{j}{\omega C}\right)^2}} \qquad (9)$$

$$|H(\omega)| = \frac{R}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}} \qquad (10)$$

$$|H(\omega)| = \frac{\omega CR}{\sqrt{1+(\omega CR)^2}} \qquad (11)$$

The phase angle θ of the transfer function $H(\omega)$ is the argument of the total impedance $Z_T(\omega)$ of the circuit:

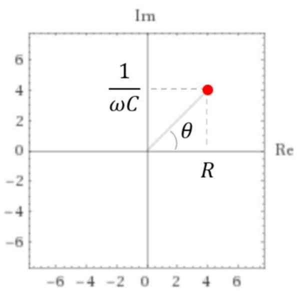

Figure 2 - Argand diagram

Figure 2 shows an Argand diagram representing the total impedance $Z_T(\omega)$. The phase angle θ is therefore:

$$\theta = \arg\big(Z_T(\omega)\big) = \arctan\left(\frac{1}{\omega CR}\right) \qquad (12)$$

2.2 Low-pass filter

The low-pass filter consists of a resistor and an inductor in series. The output voltage is high when the frequency of the input voltage is low. As the frequency of the input voltage increases the output voltage decreases and tends towards zero. As with the high-pass filter, the low-pass filter also features in the audio crossover device; it allows only low frequencies to drive the subwoofer[1].

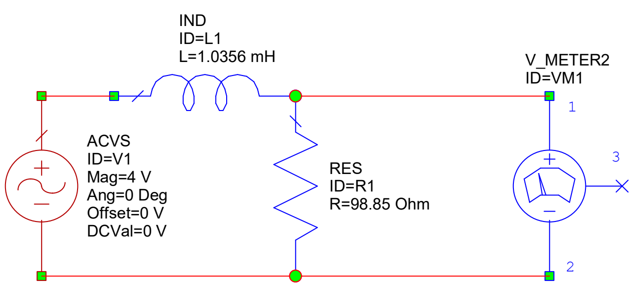

Figure 3a - Low-pass circuit built in AWR

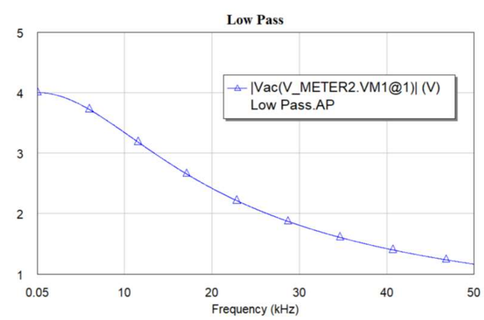

Figure 3b - Low-pass simulation in AWR

2.2.1 Derivations

From figure 6a V_1, L_1, R_1, and V_{M1} represent the input voltage V_{in}, inductance L, resistance R, and output voltage V_{out} respectively.

The transfer function $H(\omega)$ is the ratio of the output voltage to the input voltage. From equation (1):

$$H(\omega) = \frac{V_{out}}{V_{in}} \qquad (1)$$

The impedances of the resistor, inductor, and circuit are calculated as follows:

$$Z_R = R \qquad (2)$$

$$Z_L(\omega) = j\omega L \qquad (13)$$

As the inductor and resistor are in series the total impedance is their sum:

$$Z_T(\omega) = R + j\omega L \qquad (14)$$

Again, using the potential divider equation, the output voltage V_{out} as function of ω is:

$$V_{out}(\omega) = V_{in}(\omega) \cdot \frac{Z_R}{Z_L + Z_R} \qquad (15)$$

$$V_{out}(\omega) = V_{in}(\omega) \cdot \frac{R}{j\omega L + R} \qquad (16)$$

$$\frac{V_{out}(\omega)}{V_{in}(\omega)} = H(\omega) = \frac{R}{j\omega L + R} \qquad (17)$$

The amplitude of the transfer function $H(\omega)$ is the magnitude or modulus of the transfer function $|H(\omega)|$:

$$|H(\omega)| = \frac{R}{|j\omega L + R|} \qquad (18)$$

$$|H(\omega)| = \frac{R}{\sqrt{(\omega L)^2 + R^2}} \qquad (19)$$

The phase angle θ of the transfer function $H(\omega)$ is the argument of the total impedance $Z_T(\omega)$ of the circuit:

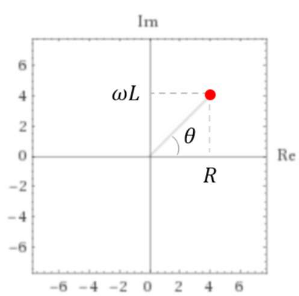

Figure 4 - Argand diagram

Figure 4 shows an Argand diagram representing the total impedance $Z_T(\omega)$. The phase angle θ is therefore:

$$\theta = \arg\big(Z_T(\omega)\big) = -\arctan\left(\frac{\omega L}{R}\right) \qquad (20)$$

The negative sign indicates the voltage lags the current.

2.3 Band-pass filter

The band-pass filter consists of a resistor, a capacitor and an inductor all in series. A band-pass filter will allow certain frequencies to pass whilst attenuating all other frequencies. The range of frequencies that are not attenuated and therefore allowed to pass is called the bandwidth or passband. Band-pass filters are widely used in wireless transmission, the common example being radio receivers. Band-pass filters are used to single out certain frequencies, this is called tuning.

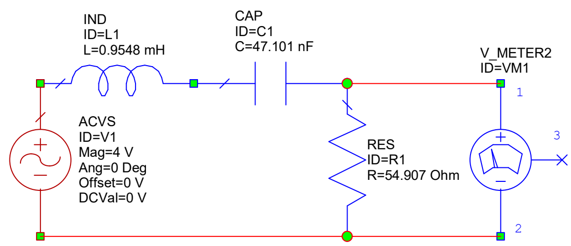

Figure 5a - Band-pass circuit built in AWR

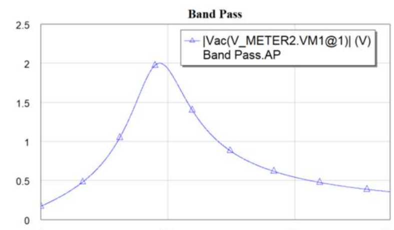

Figure 5b – Band-pass simulated in AWR

2.3.1 Derivations

From figure 5a V_1, L_1, C_1, R_1, and V_{M1} represent the input voltage V_{in}, inductance L, capacitance C, resistance R, and output voltage V_{out} respectively.

The transfer function $H(\omega)$ is the ratio of the output voltage to the input voltage. From equation (1):

$$H(\omega) = \frac{V_{out}}{V_{in}} \qquad (1)$$

The impedances of the resistor, inductor, and capacitor are calculated as follows:

$$Z_R = R \qquad (2)$$

$$Z_L(\omega) = j\omega L \qquad (13)$$

$$Z_C(\omega) = \frac{1}{j\omega C} \qquad (3)$$

Again, using the potential divider equation, the output voltage V_{out} as function of ω is:

$$V_{out}(\omega) = V_{in}(\omega) \cdot \frac{Z_R}{(Z_L + Z_C) + Z_R} \qquad (21)$$

$$V_{out}(\omega) = V_{in}(\omega) \cdot \frac{R}{j\omega L + \frac{1}{j\omega C} + R} \qquad (22)$$

$$\frac{V_{out}(\omega)}{V_{in}(\omega)} = H(\omega) = \frac{R}{j\omega L + \frac{1}{j\omega C} + R} \qquad (23)$$

The amplitude of the transfer function $H(\omega)$ is the magnitude or modulus of the transfer function $|H(\omega)|$:

$$|H(\omega)| = \frac{R}{\left|j\omega \;\; + \frac{1}{j\omega C} + R\right|} \qquad (24)$$

$$|H(\omega)| = \frac{R}{\left|j(\omega L - \frac{1}{\omega C}) + R\right|} \qquad (25)$$

$$|H(\omega)| = \frac{R}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} \qquad (26)$$

The phase angle θ of the transfer function $H(\omega)$ is the argument of the total impedance $Z_T(\omega)$ of the circuit:

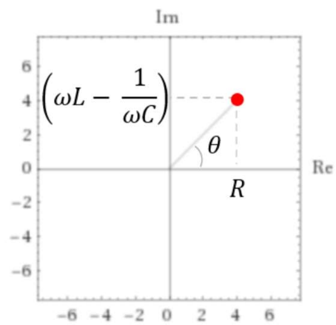

Figure 6 - Argand diagram

Figure 6 shows an Argand diagram representing the total impedance $Z_T(\omega)$. The phase angle θ is therefore:

$$\theta = \arg\left(Z_T(\omega)\right) = -\arctan\left(\frac{\left(\omega L - \frac{1}{\omega C}\right)}{R}\right) \qquad (25)$$

The negative sign indicates the voltage lags the current.

3. Experiments

3.1 Notes on Graphs and Results

Every graph in this section was produced using Python 3.6[2] with dependencies; numpy[3] and matplotlib[4] and the Jupyter Notebook[5] was used to display the graphs[1]. Each graph has two plots superimposed onto each other: one generated using the equations derived in section 2 and one generated using the results measured in the laboratory.

To obtain the results each filter was prototyped using breadboard. Then using digital oscilloscopes measurements for the input voltage, output voltage and phase angle were taken. For each filter the data is tabulated and then plotted.

3.2 High-pass filter

3.2.1 Results

The table below holds the results obtained from the laboratory for the high-pass filter:

$\boldsymbol{Frequency}\ (\boldsymbol{Hz})$	$\boldsymbol{V_{in}}(\boldsymbol{V})$	$\boldsymbol{V_{out}}\ (\boldsymbol{V})$	$\frac{\boldsymbol{V_{out}}}{\boldsymbol{V_{in}}}$	$\boldsymbol{Phase\ angle}\ (°)$
200	*4*	0.39	0.0975	80
1000	*4*	1.62	0.405	64
2000	*4*	2.61	0.6525	48
3000	*4*	3.14	0.785	38
4000	*4*	3.46	0.865	30
5000	*4.06*	3.65	0.899015	25
6000	*4.08*	3.79	0.928922	22
7000	*4.1*	3.89	0.94878	19
8000	*4.16*	3.96	0.951923	16
9000	*4.2*	4.04	0.961905	15
10000	*4.22*	4.09	0.969194	13
11000	*4.24*	4.13	0.974057	12
12000	*4.27*	4.16	0.974239	11
13000	*4.26*	4.18	0.981221	10
14000	*4.24*	4.17	0.983491	9
15000	*4.22*	4.13	0.978673	8
16000	*4.13*	4.06	0.983051	7
17000	*4.04*	3.96	0.980198	7
18000	*3.89*	3.83	0.984576	7
19000	*3.71*	3.68	0.991914	6
20000	*3.52*	3.48	0.988636	5

Table 1 - High pass filter

The frequency range used in the experiment was 20-20000Hz with increments of 1000Hz and the input voltage was 4V. As the frequency changed the input voltage fluctuated, this was not a large issue as it would not affect the outcome of the experiment because any variations in the input voltage would not affect the ratio between input and output voltages as the output voltage would also fluctuate as a result.

Figure 7a is the graph of the magnitude of the transfer function against the frequency of the input voltage. Figure 7b is graph of phase angle against frequency of the input voltage. The vertical and horizontal lines represent the cut-off frequency and corresponding magnitude or phase angle.

[1] To view the full Notebook, visit the GitHub repo: https://github.com/JamieWilliamsHackIT/linear-circuits-filters/blob/master/Filters.ipynb

In [82]:
```
# Plot the magnitude of the transfer function against frquency
%matplotlib inline
matplotlib.rcParams['figure.figsize'] = (16, 8)
plt.plot(f, np.absolute(H(f)))
plt.plot(measured_w, measured_H)
plt.axvline(x=cof, alpha=0.3)
plt.axhline(y=0.707, alpha=0.3)
plt.ylabel("Magnitude")
plt.xlabel("Frequency (Hz)")
keys = [
    mpatches.Patch(color='blue', label='Theoretical'),
    mpatches.Patch(color='orange', label='Measured'),
]
plt.legend(handles=keys)
plt.show()
```

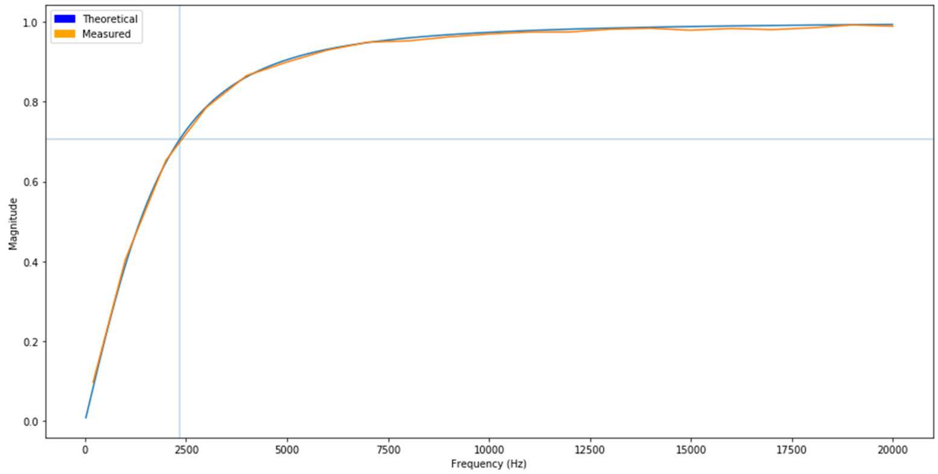

Figure 7a - High pass, magnitude against frequency

In [5]:
```
# Plot the phase angle against frquency
% matplotlib  inline
matplotlib.rcParams['figure.figsize'] = (16, 8)
theoretical = plt.plot(f, P(f))
measured = plt.plot(measured_f, measured_P)
plt.axvline(x=cof, alpha=0.3)
plt.axhline(y=45, alpha=0.3)
plt.ylabel("Phase angle (Degrees)")
plt.xlabel("Frequency (Hz)")
keys = [
    mpatches.Patch(color='blue', label='Theoretical'),
    mpatches.Patch(color='orange', label='Measured'),
]
plt.legend(handles=keys)
plt.show()
```

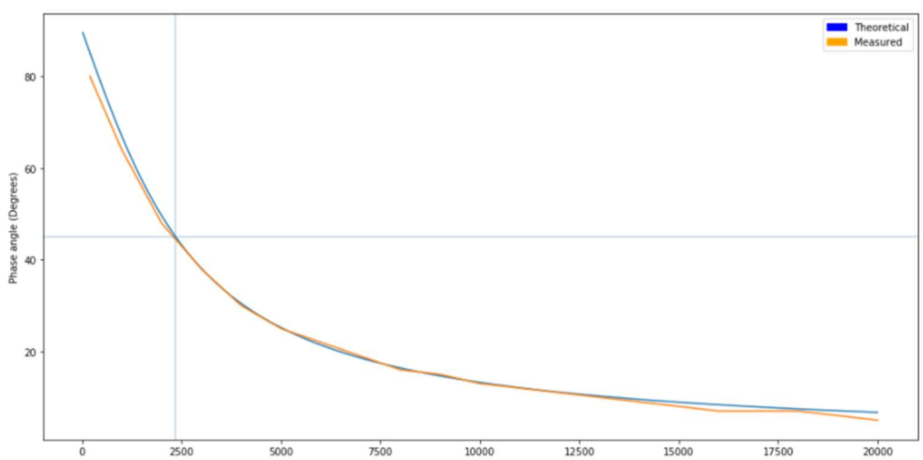

Figure 7b - High pass, phase angle against frequency

3.2.2 Cut-off Frequency and 45° Phase Angle Frequency

The cut-off frequency f_c for a high pass filter is calculated as follows:

$$f_c = \frac{1}{2\pi RC} \qquad (26)$$

Using the measured values:

$$R = 1.4748k\Omega \qquad (27)$$

$$C = 45.847nF \qquad (28)$$

$$f_c = \frac{1}{2\pi \cdot 1.4748\times10^3 \cdot 45.847\times10^{-9}} = 2353.835Hz \qquad (29)$$

$$f_c = 2.35kHz \qquad (30)$$

Figure x below shows the value for the cut-off frequency obtained from the magnitude versus frequency plot:

```
In [101]: # Get the frequency when the magnitude is one over root two (0.707...)
          np.interp((1/np.sqrt(2)), measured[0].get_ydata(), measured[0].get_xdata())

Out[101]: 2412.126650464509
```

Figure 8a - Code obtaining the cut-off frequency from graph

The difference between the theoretical cut-off frequency from equation (17) and cut-off frequency obtained from the plot (this was obtained by finding the frequency at which the magnitude was equal to $\frac{1}{\sqrt{2}}$) is only 63.388Hz.

Figure z below shows the value for the frequency when the phase angle is equal to 45°.

```
In [158]: # Get the frequency when the phase angle is equal to 45 degrees
          np.interp(45, measured_P[::-1], measured_f[::-1])

Out[158]: 2300.0
```

Figure 8b - Code obtaining the 45-degree frequency from graph

The `[::-1]` slice at the end of the `measured_P` and `measured_f` lists reverse the order of the data within them. This was necessary as the numpy interpolation function[6] would not work with decreasing functions. The resulting frequency was 2300Hz.

3.2.3 Circuit Behaviour

Looking at the impedances: the impedance of the capacitor is inversely proportional to the frequency:

$$Z_C(\omega) = \frac{1}{j\omega C} \quad (3)$$

$$Z_C(\omega) \propto \frac{1}{\omega} \quad (31)$$

From the total impedance equation (4) it is obvious that the total impedance will also decrease as the frequency increases.

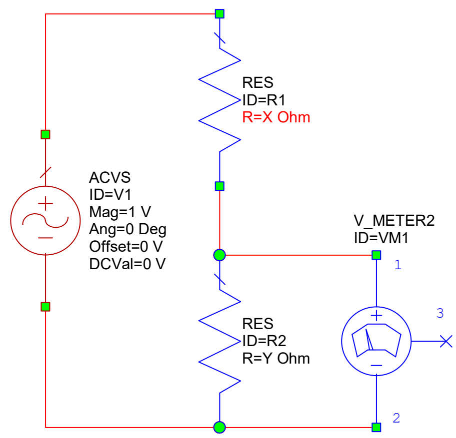

Figure 9 - Potential divider designed in AWR

Figure shows a simple potential divider. R_1 represents the impedance of the capacitor and R_2 represents the impedance of the resistor. From equation (31) we know that the impedance of the capacitor is inversely proportional to the frequency of the input voltage. Therefore, as the frequency increases the impedance of capacitor decreases, thus voltage across it will decrease (Ohm's law states the voltage across the capacitor is directly proportional to its impedance). If the voltage across the capacitor decreases the voltage across the resistor (which is the output voltage) will increase. This explains the shape of the curve in figure 7a.

3.3 Low-pass filter

3.3.1 Results

The table below holds the results obtained from the laboratory for the low-pass filter:

$\boldsymbol{Frequency}\ (\boldsymbol{Hz})$	$\boldsymbol{V_{in}}(\boldsymbol{V})$	$\boldsymbol{V_{out}}\ (\boldsymbol{V})$	$\frac{\boldsymbol{V_{out}}}{\boldsymbol{V_{in}}}$	$\boldsymbol{Phase\ angle}\ (°)$
50	4	3.88	0.97	0
2000	4	3.86	0.965	-7.4
4000	4.04	3.78	0.935644	-14
6000	4.08	3.7	0.906863	-21
8000	4.17	3.58	0.858513	-27
10000	4.02	3.29	0.818408	-33
12000	4.05	3.13	0.77284	-38
14000	4.03	2.95	0.73201	-42
16000	3.92	2.7	0.688776	-46
18000	4.01	2.62	0.653367	-50
20000	3.99	2.45	0.614035	-52
22000	4.03	2.32	0.575682	-56
24000	4	2.18	0.545	-59
26000	4.03	2.06	0.511166	-60
28000	4	1.93	0.4825	-61
30000	4	1.85	0.4625	-63
32000	4	1.77	0.4425	-65
34000	4.03	1.69	0.419355	-66
36000	4	1.6	0.4	-68
38000	3.98	1.53	0.384422	-69
40000	4	1.47	0.3675	-70
42000	4.02	1.44	0.358209	-71
44000	4	1.38	0.345	-72
46000	4	1.33	0.3325	-73
48000	3.97	1.28	0.322418	-74
50000	4	1.24	0.31	-75

Table 2 - Low pass filter

The frequency range used in this experiment was 50-50000Hz with increments of 2000Hz and, again, the input voltage was 4V. As the frequency was changed the input voltage fluctuated much less than during the high-pass filter tests.

Figure 10a is the graph of the magnitude of the transfer function against the frequency of the input voltage. Figure 10b is graph of phase angle against frequency of the input voltage. The vertical and horizontal lines represent the cut-off frequency and corresponding magnitude or phase angle.

In [175]:

```python
# Plot the magnitude of the transfer function against frequency
%matplotlib inline
matplotlib.rcParams['figure.figsize'] = (16, 8)
theoretical = plt.plot(f, np.absolute(H(f)))
measured = plt.plot(measured_f, measured_H)
plt.axvline(x=cof, alpha=0.3)
plt.axhline(y=(1/np.sqrt(2)), alpha=0.3)
plt.ylabel("Magnitude")
plt.xlabel("Frequency (Hz)")
keys = [
    mpatches.Patch(color='blue', label='Theoretical'),
    mpatches.Patch(color='orange', label='Measured'),
]
plt.legend(handles=keys)
plt.show()
```

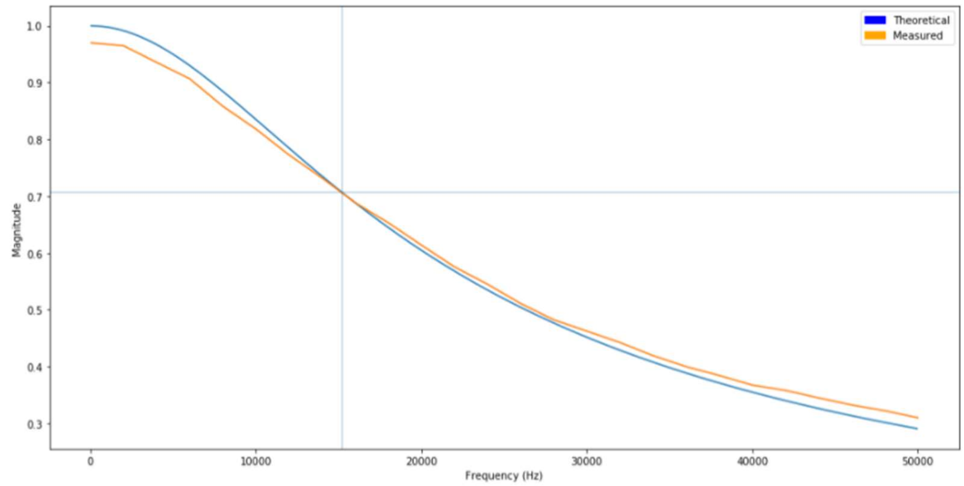

Figure 10a - Graph of magnitude against frequency

In [10]:

```python
# Plot the magnitude of the transfer function against frequency
% matplotlib  inline
matplotlib.rcParams['figure.figsize'] = (16, 8)
theoretical = plt.plot(f, P(f))
measured = plt.plot(measured_f, measured_P)
plt.axvline(x=cof, alpha=0.3)
plt.axhline(y=-45, alpha=0.3)
plt.ylabel("Phase angle (Degrees)")
plt.xlabel("Frequency (Hz)")
keys = [
    mpatches.Patch(color='blue', label='Theoretical'),
    mpatches.Patch(color='orange', label='Measured'),
]
plt.legend(handles=keys)
plt.show()
```

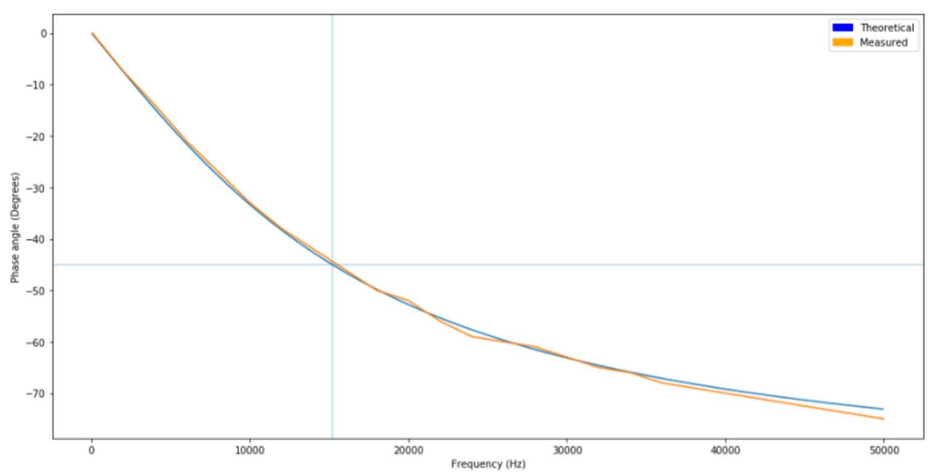

Figure 10b - Graph of phase angle against frequency

3.3.2 Cut-off Frequency and 45° Phase Angle Frequency

The cut-off frequency f_c for a low pass filter is calculated as follows:

$$f_c = \frac{R}{2\pi L} \tag{32}$$

Using the measured values:

$$R = 98.85\Omega \tag{33}$$

$$L = 1.0356mH \tag{34}$$

$$f_c = \frac{98.85}{2\pi \cdot 1.0356\times10^{-3}} = 15191.64361Hz \tag{35}$$

$$f_c = 15.19kHz \tag{36}$$

Figure 11a below shows the value for the cut-off frequency obtained from the magnitude versus frequency plot:

```
In [161]: # Get the frequency when the magnitude is one over root two (0.707...)
          np.interp((1/np.sqrt(2)), measured_H[::-1], measured_f[::-1])

Out[161]: 15152.005606526642
```

Figure 11a – Code to obtain the cut-off frequency from graph

The difference between the theoretical cut-off frequency from equation (36) and cut-off frequency obtained from the plot (this was obtained by finding the frequency at which the magnitude was equal to $\frac{1}{\sqrt{2}}$) is only 39.64Hz.

Figure 11b below shows the value for the frequency when the phase angle is equal to -45°.

```
In [164]: # Get the frequency when the phase angle is equal to -45 degrees
          np.interp(-45, measured_P[::-1], measured_f[::-1])

Out[164]: 15500.0
```

Figure 11b – Code to obtain the -45-degree frequency from phase vs frequency graph

Looking at equation (25):

$$\theta = \arctan\left(\frac{\omega L}{R}\right) \tag{37}$$

$$\tan(\theta) = \frac{2\pi f L}{R} \tag{38}$$

$$f(\theta) = \frac{\mathrm{R}\tan(\theta)}{2\pi L} \tag{39}$$

Setting $\theta = 45°$:

$$f(45°) = \frac{\mathrm{R}}{2\pi L} = f_c \tag{40}$$

This is also true for the high-pass filter and it explains why the results from figures 11a and 11b are so similar as they are approximations of the same value (the minus is omitted from the phase angle as it is only its magnitude that is important).

3.3.3 Circuit Behaviour

Again, looking at the impedances: the impedance of the inductor is directly proportional to the frequency:

$$Z_L(\omega) = j\omega L \quad (41)$$

$$Z_L(\omega) \propto \omega \quad (42)$$

From the total impedance equation (14) it is obvious that the total impedance will also increase as the frequency increases.

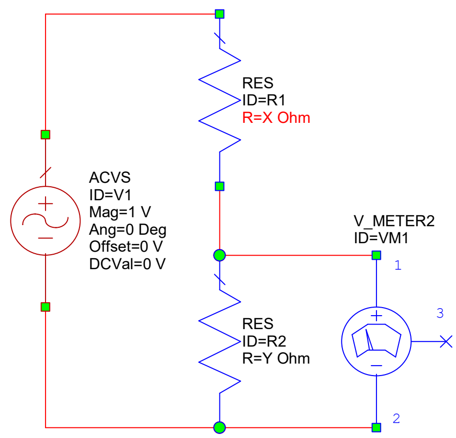

Figure 12 – Potential divider built using AWR

Using the potential divider example seen in section 3.2.3; R_1 represents the impedance of the inductor and R_2 represents the impedance of the resistor. As shown in equation (42) the impedance of the inductor is directly proportional to the frequency of the input voltage. Therefore, as the frequency increases (again due to Ohm's law) the voltage across the resistor R_1 increases, thus the voltage across resistor R_2 decreases which is the output voltage. This explains the shape of the graph in figure 10a.

3.4 Band-pass filter

3.4.1 Results

The table below holds the results obtained from the laboratory for the band-pass filter:

$\boldsymbol{Frequency}\ (\boldsymbol{Hz})$	$\boldsymbol{V_{in}}(\boldsymbol{V})$	$\boldsymbol{V_{out}}\ (\boldsymbol{V})$	$\frac{\boldsymbol{V_{out}}}{\boldsymbol{V_{in}}}$	$\boldsymbol{Phase\ angle}\ (°)$
5000	4.04	0.35	0.086634	82
8000	3.96	0.57	0.143939	80
11000	4.02	0.87	0.216418	76
14000	4.02	1.309	0.325622	68
17000	4.02	1.895	0.471393	57
20000	3.89	2.71	0.696658	39
20500	3.46	2.56	0.739884	35
21000	3.38	2.64	0.781065	31
21500	3.28	2.69	0.820122	25
22000	3.69	3.12	0.845528	20
22500	3.59	3.15	0.877437	14
23000	3.36	2.99	0.889881	9
23500	3.54	3.19	0.90113	3
24000	3.52	3.18	0.903409	-3
24500	3.54	3.17	0.89548	-8
25000	3.57	3.14	0.879552	-14
25500	3.63	3.11	0.856749	-18
26000	3.46	2.89	0.83526	-24
29000	3.93	2.62	0.666667	-44
32000	3.83	2.05	0.535248	-56
35000	4.12	1.86	0.451456	-63
38000	3.98	1.52	0.38191	-67
41000	4.14	1.39	0.335749	-71
44000	3.79	1.094	0.288654	-73
47000	3.87	1.016	0.262532	-75
50000	3.95	0.937	0.237215	-76
53000	4	0.898	0.2245	-78
56000	4.04	0.84	0.207921	-80
59000	4.1	0.791	0.192927	-81
60000	4.12	0.742	0.180097	-82

Table 3 - Band pass filter

The frequency range used in this experiment was 5000-60000Hz. There was more fluctuation of the input voltage during this test than there was with either the high-pass or low-pass filters. Initially increments of 3000Hz were used, using this data graph was plotted. There was an area of interest, near the resonant frequency, therefore near this frequency smaller increments of 500Hz were used to better understand and plot this area of the graph.

The two figures show the theoretical and measured plots for both the magnitude and phase angle versus frequency. The vertical and horizontal lines represent the resonant frequency and corresponding magnitude or phase angle.

In [14]:
```python
# Plot the magnitude of the transfer function against frequency
%matplotlib inline
matplotlib.rcParams['figure.figsize'] = (16, 8)
theoretical = plt.plot(f, np.absolute(H(f)))
measured = plt.plot(measured_f, measured_H)
plt.axvline(x=rf, alpha=0.3)
plt.axhline(y=np.amax(measured_H), alpha=0.3)
plt.ylabel("Magnitude")
plt.xlabel("Frequency (Hz)")
keys = [
    mpatches.Patch(color='blue', label='Theoretical'),
    mpatches.Patch(color='orange', label='Measured'),
]
plt.legend(handles=keys)
plt.show()
```

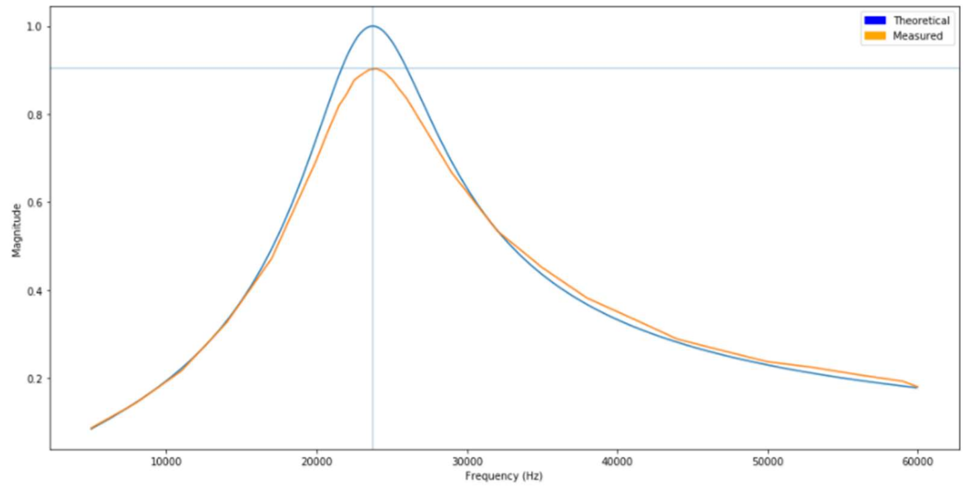

Figure 13a – Graph of magnitude against frequency

In [197]:
```python
# Plot the magnitude of the transfer function against frequency
%matplotlib inline
matplotlib.rcParams['figure.figsize'] = (16, 8)
theoretical = plt.plot(f, P(f))
measured = plt.plot(measured_f, measured_P)
plt.axvline(x=rf, alpha=0.3)
plt.axhline(y=0, alpha=0.3)
plt.ylabel("Phase angle (Degrees)")
plt.xlabel("Frequency (Hz)")
keys = [
    mpatches.Patch(color='blue', label='Theoretical'),
    mpatches.Patch(color='orange', label='Measured'),
]
plt.legend(handles=keys)
plt.show()
```

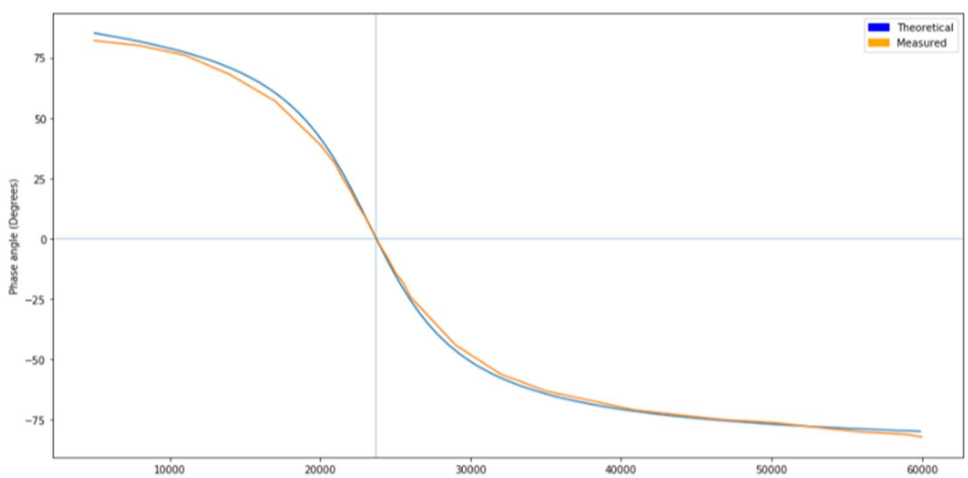

Figure 13b – Graph of phase angle against frequency

3.4.2 Resonant Frequency and 0° Phase Angle Frequency

The frequency at which the magnitude is maximum is called the resonant frequency and at this frequency the phase angle is 0°. Therefore, we can use the phase angle equation to determine the resonant frequency:

$$\theta = \arctan\left(\frac{\left(\omega L - \frac{1}{\omega C}\right)}{R}\right) \qquad (43)$$

$$\tan(\theta) = \frac{\left(\omega L - \frac{1}{\omega C}\right)}{R} \qquad (44)$$

Setting $\theta = 0°$:

$$\omega L - \frac{1}{\omega C} = 0 \qquad (45)$$

$$\omega L = \frac{1}{\omega C} \qquad (46)$$

$$\omega^2 = \frac{1}{LC} \qquad (47)$$

$$4\pi^2 f^2 = \frac{1}{LC} \qquad (48)$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \qquad (49)$$

Using the measured values:

$$R = 54.907\Omega \qquad (50)$$

$$L = 0.9548mH \qquad (51)$$

$$C = 47.101nF \qquad (52)$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{0.9548\times10^{-3} \cdot 47.101\times10^{-9}}} \qquad (53)$$

$$f_0 = 23732.79362Hz \qquad (54)$$

$$f_0 = 23.73kHz \qquad (55)$$

Figure 14a below shows the value for the resonant frequency obtained from the magnitude versus frequency plot:

```
In [44]: # Fit a polynomial to the curve and find the frequency where the magnitude is maximum
         max_y = np.amax(measured_H)
         poly_fit_graph = np.poly1d(np.polyfit(measured_H, measured_f, 15))
         poly_fit_graph(max_y)

Out[44]: 23992.341637790203
```

Figure 14a – Code to obtain resonant frequency from magnitude against frequency graph

To find the resonant frequency numpy's amax[7] function was used to obtain the maximum value for the magnitude, then using numpy's polyfit[8] function the frequency value pertaining to the maximum magnitude was found. The difference between the theoretical resonant frequency from equation (17) and resonant frequency obtained from the plot is only 259.6Hz.

```
In [33]: # Get the frequency when the phase angle is equal to 0 degrees
         np.interp(0, measured_P[::-1], measured_f[::-1])

Out[33]: 23750.0
```

Figure 14b – Code to obtain the resonant frequency from the phase angle against frequency graph

The code in figure 14b shows another linear interpolation to get the frequency at which the phase angle is equal to 0^0, this value is close to the theoretical value and the value obtained from figure y.

3.4.3 Circuit Behaviour

Looking at the transfer function from equation (23):

$$H(\omega) = \frac{R}{j\omega L + \frac{1}{j\omega C} + R} \tag{23}$$

This equation will be at a maximum when $j\omega L + \frac{1}{j\omega C} = 0$ as the transfer function will become $H(\omega) = \frac{R}{R} = 1$, at this point the frequency of the input voltage is called the **resonant frequency**. At high frequencies the denominator will become large as $\frac{1}{j\omega C}$ will tend to zero and $j\omega L$ will tend towards infinity, thus the transfer function will tend towards zero. At low frequencies the denominator will also become large as $j\omega L$ will tend towards zero and $\frac{1}{j\omega C}$ will tend towards infinity, thus the transfer function will, again, tend towards zero.

4. Conclusion

The series of experiments conducted in this laboratory have provided a deeper understanding of and appreciation for high, low and band pass filters. Also, it provided practical experience using industry standard equipment such as the digital oscilloscope.

In addition to practical experience this experiment helped develop understanding for simple data science techniques such as using the Jupyter Notebook with matplotlib to analyse data.

For the band-pass filter additional readings near the resonant frequency allowed for a more accurate analysis of the maximum output voltage. On repeating this experiment, it would be advisable to take more readings near the cut-off frequencies of both the high-pass and low-pass filters to reach a higher level of accuracy of the analysis at these frequencies.

Overall, the results for each filter match the theoretical plots closely, this indicates each circuit was correctly setup and the data were recorded correctly.

5. References

[1] https://en.wikipedia.org/wiki/High-pass_filter

[2] https://www.python.org/

[3] http://www.numpy.org/

[4] https://matplotlib.org/

[5] https://jupyter.org/

[6] https://docs.scipy.org/doc/numpy-1.13.0/reference/generated/numpy.interp.html

[7] https://docs.scipy.org/doc/numpy-1.13.0/reference/generated/numpy.amax.html

[8] https://docs.scipy.org/doc/numpy-1.13.0/reference/generated/numpy.polyfit.html

To view all the images, AWR files, and the full Jupyter Notebook visit the GitHub repository: https://github.com/JamieWilliamsHackIT/linear-circuits-filters