Math 114 - Fall 2016 - Assignment 2

Due: Friday, October 7 at 4:30PM in dropbox 7 outside of MC 4066

Dot Product

1. Find the angle between the vectors $\begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -\sqrt{3} \\ 1 \end{bmatrix}$.

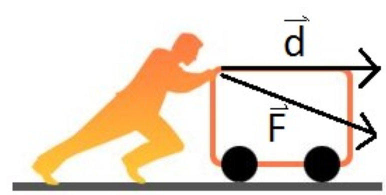
2. The Triangle Inequality states that any two vectors $\vec{x}$ and $\vec{y}$ in $\mathbb{R}^n$ satisfy

$$\|\vec{x}+\vec{y}\| \leq \|\vec{x}\| + \|\vec{y}\|. \tag{1}$$

   (a) Show that $\vec{x} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ and $\vec{y} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ satisfy the Triangle Inequality.

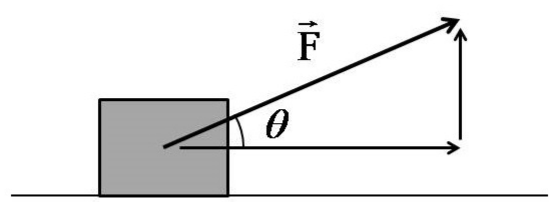
   (b) Show that the left and right sides of the inequality are the same for $\vec{x} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ and $\vec{y} = \begin{bmatrix} 3 \\ -6 \end{bmatrix}$. Why is this? (*Hint: Sketch the vectors and think about what the inequality is saying geometrically.)*

3. Alex pushes a car with a constant force of 100 N at an angle of 30° (or $\pi/6$ rad) to the horizontal. In doing so, Alex moves the car a distance of 10 m. The work (energy) that Alex does is given by the dot product of the force and displacement, $W = \vec{F} \cdot \vec{d}$. How much work does Alex do in total? (Note: Work is measured in units of Joules and 1 J is equal to 1 N·m.)



Projections

4. Elena pulls a box with force $\vec{F}$ at an angle θ relative to the horizontal. Compute the projection and perpendicular of $\vec{F}$ with respect to the horizontal. What do these represent physically?



5. Consider the line defined by $y = -3/2x + 2$ and the point $P = (5, 5)$.

   (a) Find a vector equation representing the line. Do vectors satisfying this equation form a subspace of $\mathbb{R}^2$? Why/Why not?

   (b) $Q = (4, -4)$ and $R = (0, 2)$ are two points falling on the line. Let $\vec{x}$ be the vector joining Q to P and $\vec{y}$ be the vector joining Q to R. Find all of $||\vec{x}||$, $proj_{\vec{y}}\vec{x}$, and $perp_{\vec{y}}\vec{x}$. Is one of the projections the same as before?

   (c) Let $\vec{x}$ be the vector joining R to P and $\vec{y}$ be the vector joining Q to R. Find all of $||\vec{x}||$, $proj_{\vec{y}}\vec{x}$, and $perp_{\vec{y}}\vec{x}$.

   (d) Based on the above, which vector could we use to describe the minimum distance between P and a point on the line?

   (e) Find the point on the line closest to P.

Cross Product

6. Determine the scalar equation of the plane that contains the points $P = (1, 5, 3)$, $Q = (2, 6, -1)$, and $R(1, 0, 1)$.

7. Let $\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$ and $\vec{y} = \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}$.

   (a) Compute $\vec{w} = \vec{x} \times \vec{y}$. Using the definition of length, find $||\vec{w}||$.

   (b) Compute the *product* of the following two scalars: $||\vec{x}||$ and $\|perp_{\vec{x}}\vec{y}\|$. You should get the same answer as part (a). Why?

   (c) Given new vectors $\vec{a} = \begin{bmatrix} 1 \\ 5 \\ 5 \end{bmatrix}$, $\vec{b}$ (components unknown) and the information that the angle between $\vec{a}$ and $\vec{b}$ is $\theta = \pi/4$, find $||proj_{\vec{b}}\vec{a}||$. Hint: You may find the diagram on p.53 helpful.

8. Suppose $\vec{w} = \vec{u} \times \vec{v}$, $\vec{u}, \vec{v} \in \mathbb{R}^3$.

   (a) If $\vec{u} \cdot \vec{v} = 0$, must $\vec{w} = \vec{0}$? Under what circumstances will $\vec{w} = \vec{0}$?

   (b) Find a vector $\vec{r}$ orthogonal to $\vec{u}, \vec{v}, \vec{w}$ as defined above when $\vec{u}, \vec{v}, \vec{w} \neq \vec{0}$. (*Hint: No calculation required.*)

Complex Numbers

9. Simplify the following in standard form:

   (a) $(1 + i\sqrt{3})(1 + i)$, (b) $\dfrac{\pi + i\pi}{1 - \sqrt{3}i}$

10. Given the provided root, find the remaining roots of the polynomial.

    (a) $y = x^3 - 2x^2 + 4x - 3, x = 1$

    (b) $y = x^3 - 4x^2 + 2x + 4, x = 2$