

MATH 114 - Fall 2016 - Assignment 3

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Problem 1. Complex Numbers - Euler's Formula

a)

$$\begin{aligned} &= e^{i\frac{\pi}{2}} \\ &= \cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \\ &= 0 + i(1) \\ &= i \end{aligned}$$

b)

$$\begin{aligned} &= e^{2\pi \frac{i}{3}} \\ &= \cos 2\pi + \frac{i}{3} \sin 2\pi \\ &= 1 + \frac{i}{3}(0) \\ &= 1 \end{aligned}$$

Problem 2. Complex Numbers - Euler's Formula

$$\begin{aligned} z &= \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \\ z &= e^{i\frac{\pi}{3}} \\ z^6 &= e^{6i2\pi} \\ &= \cos 2\pi + 6i \sin 2\pi \\ &= 1 + 6i(0) \\ &= 1 \end{aligned}$$

Problem 3. Systems of Equations

a)

$$\begin{aligned} &= \left[\begin{array}{cc|c} 3 & 2 & -9 \\ 1 & 4 & 7 \end{array} \right] R_2 - R_1 \\ &\sim \left[\begin{array}{cc|c} 3 & 2 & -9 \\ -2 & 2 & 16 \end{array} \right] R_1 + R_2 \\ &\sim \left[\begin{array}{cc|c} 1 & 4 & 7 \\ -2 & 2 & 16 \end{array} \right] \frac{1}{2}R_2 \\ &\sim \left[\begin{array}{cc|c} 1 & 4 & 7 \\ -1 & 1 & 8 \end{array} \right] R_2 + R_1 \\ &\sim \left[\begin{array}{cc|c} 1 & 4 & 7 \\ 0 & 5 & 15 \end{array} \right] \frac{1}{5}R_2 \\ &\sim \left[\begin{array}{cc|c} 1 & 4 & 7 \\ 0 & 1 & 3 \end{array} \right] R_1 - 4R_2 \\ &\sim \left[\begin{array}{cc|c} 1 & 0 & -5 \\ 0 & 1 & 3 \end{array} \right] \\ &\vec{x}_1 = -5, \vec{x}_2 = 3 \end{aligned}$$

b)

$$\begin{aligned} &= \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 1 & 1 & 1 & 2 \\ 1 & 3 & 4 & 0 \end{array} \right] R_3 - R_1 \\ &\sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & -1 \end{array} \right] R_2 - R_3 \\ &\sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 1 & 0 & 0 & 3 \\ 1 & 1 & 1 & -1 \end{array} \right] R_2 \Leftrightarrow R_1 \\ &\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 1 & 2 & 3 & 1 \\ 1 & 1 & 1 & -1 \end{array} \right] R_2 - 2R_3 - R_1 \\ &\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & -1 \end{array} \right] R_3 - R_2 \\ &\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \end{array} \right] R_3 \Leftrightarrow R_2 \end{aligned}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{array} \right]$$

Inconsistent, therefore no solutions.

c)

It is not possible to reduce to RREF fully, the closest that I could get is below. This is because it is inconsistent.

$$= \left[\begin{array}{ccc|c} 1 & 0 & \frac{7}{3} & \frac{5}{3} \\ 0 & 1 & \frac{1}{3} & \frac{-1}{3} \\ 0 & 0 & 0 & -3 \end{array} \right]$$

Problem 4. Solving two planes

$$3x_1 + 2x_2 - 4x_3 = 1$$

$$x_1 + x_2 - x_3 = 1$$

$$x_1 = 1 - x_2 + x_3$$

$$3(1 - x_2 + x_3) + 2x_2 - 4x_3 = 1$$

$$3 - 3x_2 + 3x_3 + 2x_2 - 4x_3 = 1$$

$$x_2 - x_3 = -2$$

$$\frac{-x_2 - x_3 = -2}{-1}$$

$$x_2 = 2 - x_3$$

$$x_1 = 1 - (2 - x_3) + x_3$$

$$x_1 = 1 - 2 + x_3 + x_3$$

$$x_1 = -1 + 2x_3$$

$$\vec{x} = (-1 + 2x_3) + (2 - x_3)$$

$$\vec{x} = x_3 + 1$$

The solution set represents a line.