Math 114 - Fall 2016 - Assignment 3

Due: Friday, October 21 at 4:30PM in dropbox 7 outside of MC 4066

Complex Numbers

1. Use Euler's Formula to convert the following complex numbers to standard form:

   (a) $e^{i\pi/2}$

   (b) $e^{2\pi i/3}$

2. Rewrite $z = \dfrac{1+\sqrt{3}i}{2}$ as an exponential using Euler's formula and then compute z^6.

Systems of Equations

3. Solve the following systems using an augmented matrix and performing elementary row operations to get to reduced row echelon form (RREF):

   a) $\begin{aligned} 3x_1 + 2x_2 &= -9 \\ x_1 + 4x_2 &= 7 \end{aligned}$

   b) $\begin{aligned} x_1 + 2x_2 + 3x_3 &= 1 \\ x_1 + x_2 + x_3 &= 2 \\ x_1 + 3x_2 + 4x_3 &= 0 \end{aligned}$

   c) $\begin{aligned} x_1 + 2x_2 + 3x_3 &= 1 \\ x_1 - x_2 + 2x_3 &= 2 \\ 2x_1 + x_2 + 5x_3 &= 0 \end{aligned}$

4. Consider two planes in $\mathbb{R}^3$ defined by $3x_1 + 2x_2 - 4x_3 = 1$ and $x_1 + x_2 - x_3 = 1$. Solve this system of equations to determine if the planes intersect. If so, find a vector equation for the solution set. What does this solution set represent geometrically?

5. Consider the vectors $\vec{a}_1 = (2, -3, 4)$, $\vec{a}_2 = (2, 6, 1)$, and $\vec{a}_3 = (-2, -12, 1)$. To test for linear independence, we are interested in the solution set of $x_1\vec{a}_1 + x_2\vec{a}_2 + x_3\vec{a}_3 = 0$ where x_1, x_2, and x_3 are unknowns (i.e., variables). Expand this out and show it is equivalent to the system of equations

$$\begin{aligned} 2x_1 + 2x_2 - 2x_3 &= 0 \\ -3x_1 + 6x_2 - 12x_3 &= 0 \\ 4x_1 + x_2 + x_3 &= 0. \end{aligned}$$

   Solve the system of equations by converting the augmented matrix to RREF and determine if the vectors are linearly independent. (Notice, we really didn't need the augmented matrix here since the right-hand sides stay zero. In other words, if we keep in mind that the right-hand sides are always zero, we can just row-reduce the coefficient matrix.)

6. Consider the vectors $\vec{a}_1 = (1, 0, 0)$, $\vec{a}_2 = (1, 1, 0)$, and $\vec{a}_3 = (0, 1, 1)$. To test for spanning, we need to show that we can write any vector $\vec{v} = (v_1, v_2, v_3)$ in $\mathbb{R}^3$ as a linear combination of $\vec{a}_1$, $\vec{a}_2$, and $\vec{a}_3$. In other words, we wish to show that we can solve $x_1\vec{a}_1 + x_2\vec{a}_2 + x_3\vec{a}_3 = \vec{v}$ where x_1, x_2, and x_3 are unknowns (i.e., variables) for any values of v_1, v_2, and v_3. Expand this out and show it is equivalent to the system of equations

$$\begin{aligned} x_1 + x_2 &= v_1 \\ x_2 + x_3 &= v_2 \\ x_3 &= v_3. \end{aligned}$$

   Solve the system of equations by converting the augmented matrix to RREF and determine if the vectors span $\mathbb{R}^3$. (Notice, the right-hand sides end up being some linear combinations of v_1, v_2, and v_3 which are, in general, some non-zero numbers. Therefore, we need the RREF to be the identity matrix otherwise the system is inconsistent.

7. Before each Fall semester, the university bookstore places massive orders for the textbooks that will be needs for the large first-year biology, chemistry, and physics courses. The following table gives for three different years the number of books bought for each course along with the total cost.

Year	Biology	Chemistry	Physics	Cost
2016	1400	1800	600	\$658,000
2011	1300	1700	500	\$587,000
2006	1200	1600	400	\$520,000

If we let x_b, x_c, and x_p be the price for a single biology, chemistry, or physics textbook respectively we can rewrite this information as a system of equations. What can you conclude about the price of each individual textbook from year to year? (*Tip: Start off your row reduction by scaling each row by* $1/100$ *to get easier numbers to work with.*)

8. While searching through your attic, you find a bag of gold coins (yes, it turns out some of your distant relatives were pirates, ARRR!). There are four types of gold coins and you'd like to know how much gold you've just found so you need to work out the mass of each type of coin. However, all you have is a balance scale and a few 10 g and 20 g weights. With some experimenting, you discover the following:

$$\begin{aligned} A+B+C+D &= 70\ g \\ B+C &= 30\ g \\ 2C+2D &= 90\ g \\ 4B+D &= 80\ g \end{aligned} \tag{1}$$

where A, B, C, and D are the unknown masses of each type of coin in grams. Determine the mass of each type of coin. (*Hint: Check your final values by plugging them back into the given system of equations.*)