MATH 114 - Fall 2016 - Assignment 3

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Problem 1. Complex Numbers - Euler's Formula

a)

$$
\begin{aligned}
&= e^{i\frac{\pi}{2}} \\
&= \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} \\
&= 0 + i(1) \\
&= i
\end{aligned}
$$

b)

$$
\begin{aligned}
&= e^{2\pi\frac{i}{3}} \\
&= \cos 2\pi + \frac{i}{3}\sin 2\pi \\
&= 1 + \frac{i}{3}(0) \\
&= 1
\end{aligned}
$$

Problem 2. Complex Numbers - Euler's Formula

$$
\begin{aligned}
&z = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3} \\
&z = e^{i\frac{\pi}{3}} \\
&z^6 = e^{6i2\pi} \\
&= \cos 2\pi + 6i\sin 2\pi \\
&= 1 + 6i(0) \\
&= 1
\end{aligned}
$$

Problem 3. Systems of Equations

a)

$$= \left[\begin{array}{cc|c} 3 & 2 & -9 \\ 1 & 4 & 7 \end{array}\right] R_2 - R_1$$

$$\sim \left[\begin{array}{cc|c} 3 & 2 & -9 \\ -2 & 2 & 16 \end{array}\right] R_1 + R_2$$

$$\sim \left[\begin{array}{cc|c} 1 & 4 & 7 \\ -2 & 2 & 16 \end{array}\right] \frac{1}{2} R_2$$

$$\sim \left[\begin{array}{cc|c} 1 & 4 & 7 \\ -1 & 1 & 8 \end{array}\right] R_2 + R_1$$

$$\sim \left[\begin{array}{cc|c} 1 & 4 & 7 \\ 0 & 5 & 15 \end{array}\right] \frac{1}{5} R_2$$

$$\sim \left[\begin{array}{cc|c} 1 & 4 & 7 \\ 0 & 1 & 3 \end{array}\right] R_1 - 4R_2$$

$$\sim \left[\begin{array}{cc|c} 1 & 0 & -5 \\ 0 & 1 & 3 \end{array}\right]$$

$\vec{x}_1 = -5, \vec{x}_2 = 3$

b)

$$= \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 1 & 1 & 1 & 2 \\ 1 & 3 & 4 & 0 \end{array}\right] R_3 - R_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & -1 \end{array}\right] R_2 - R_3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 1 & 0 & 0 & 3 \\ 1 & 1 & 1 & -1 \end{array}\right] R_2 \Leftrightarrow R_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 1 & 2 & 3 & 1 \\ 1 & 1 & 1 & -1 \end{array}\right] R_2 - 2R_3 - R_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & -1 \end{array}\right] R_3 - R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \end{array}\right] R_3 \Leftrightarrow R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

Inconsistent, therefore no solutions.

c)

It is not possible to reduce to RREF fully, the closest that I could get is below. This is because it is inconsistent.

$$= \left[\begin{array}{ccc|c} 1 & 0 & \frac{7}{3} & \frac{5}{3} \\ 0 & 1 & \frac{1}{3} & \frac{-1}{3} \\ 0 & 0 & 0 & -3 \end{array}\right]$$

Problem 4. Solving two planes

$$3x_1 + 2x_2 - 4x_3 = 1$$

$$x_1 + x_2 - x_3 = 1$$

$$x_1 = 1 - x_2 + x_3$$

$$3(1 - x_2 + x_3) + 2x_2 - 4x_3 = 1$$

$$3 - 3x_2 + 3x_3 + 2x_2 - 4x_3 = 1$$

$$x_2 - x_3 = -2$$

$$\frac{-x_2 - x_3 = -2}{-1}$$

$$x_2 = 2 - x_3$$

$$x_1 = 1 - (2 - x_3) + x_3$$

$$x_1 = 1 - 2 + x_3 + x_3$$

$$x_1 = -1 + 2x_3$$

$$\vec{x} = (-1 + 2x_3) + (2 - x_3)$$

$$\vec{x} = x_3 + 1$$

The solution set represents a line.

Problem 5. Solving System of Equations/Linear Independance

$$\begin{bmatrix} a_{1_1} & a_{2_1} & a_{3_1} \\ a_{1_2} & a_{2_2} & a_{3_2} \\ a_{1_3} & a_{2_3} & a_{3_3} \end{bmatrix} = \begin{bmatrix} 2 & 2 & 2 \\ -3 & 6 & -12 \\ 4 & 1 & 1 \end{bmatrix}$$

The System of Equations' coefficients map to the matrix above. This is the same as expanding the equation of $x_1\vec{a}_1 + x_2\vec{a}_2 + x_3\vec{a}_3$.
Therefore the vectors $\vec{a}_1, \vec{a}_2, \vec{a}_3$ are equivalent to the systems of equations.

$$= \left[\begin{array}{ccc|c} 2 & 2 & -2 & 0 \\ -3 & 6 & -12 & 0 \\ 4 & 1 & 1 & 0 \end{array}\right] \frac{1}{2}R_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ -3 & 6 & -12 & 0 \\ 4 & 1 & 1 & 0 \end{array}\right] R_2 + 3R_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 9 & -15 & 0 \\ 4 & 1 & 1 & 0 \end{array}\right] R3_{-}4R_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 9 & -15 & 0 \\ 0 & -3 & 5 & 0 \end{array}\right] \frac{1}{9}R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & \frac{-5}{3} & 0 \\ 0 & -3 & 5 & 0 \end{array}\right] R_3 + 3R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & \frac{-5}{3} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] R_3 + 3R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & \frac{-5}{3} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] R_1 - R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & \frac{2}{3} & 0 \\ 0 & 1 & \frac{-5}{3} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

The system of equations is linearly independant.

Problem 6. Solving System of Equations/Spanning

$$\begin{bmatrix} a_{1_1} & a_{2_1} & a_{3_1} \\ a_{1_2} & a_{2_2} & a_{3_2} \\ a_{1_3} & a_{2_3} & a_{3_3} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

The System of Equations' coefficients map to the matrix above. This is the same as expanding the equation of $x_1\vec{a}_1 + x_2\vec{a}_2 + x_3\vec{a}_3$.

Therefore the vectors $\vec{a}_1, \vec{a}_2, \vec{a}_3$ are equivalent to the systems of equations.

$$= \left[\begin{array}{ccc|c} 1 & 1 & 0 & v_1 \\ 0 & 1 & 1 & v_2 \\ 0 & 0 & 1 & v_3 \end{array}\right] R_1 - R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -1 & v_1 - v_2 \\ 0 & 1 & 1 & v_2 \\ 0 & 0 & 1 & v_3 \end{array}\right] R_1 + R_3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & v_1 - v_2 + v_3 \\ 0 & 1 & 1 & v_2 \\ 0 & 0 & 1 & v_3 \end{array}\right] R_2 - R_3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & v_1 - v_2 + v_3 \\ 0 & 1 & 0 & v_2 - v_3 \\ 0 & 0 & 1 & v_3 \end{array}\right]$$

System of Equations is consistent. Therefore the vectors $\vec{a}_1, \vec{a}_2, \vec{a}_3$ span $\mathbb{R}^3$.

Problem 7. Textbooks

$$1400x_b + 1800x_c + 600x_p = 658,000$$
$$1300x_b + 1700x_c + 500x_p = 587,000$$
$$1200x_b + 1600x_c + 400x_p = 520,000$$

$$= \left[\begin{array}{ccc|c} 1400 & 1800 & 600 & 658000 \\ 1300 & 1700 & 500 & 587000 \\ 1200 & 1600 & 400 & 520000 \end{array}\right] \frac{1}{100} \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 14 & 18 & 6 & 6580 \\ 13 & 17 & 5 & 5870 \\ 12 & 16 & 4 & 5200 \end{array}\right] \tfrac{1}{14} R_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & \frac{9}{7} & \frac{3}{7} & 470 \\ 13 & 17 & 5 & 5870 \\ 12 & 16 & 4 & 5200 \end{array}\right] R_2 - 13R_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & \frac{9}{7} & \frac{3}{7} & 470 \\ 0 & \frac{2}{7} & \frac{-4}{7} & -240 \\ 12 & 16 & 4 & 5200 \end{array}\right] R_3 - 12R_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & \frac{9}{7} & \frac{3}{7} & 470 \\ 0 & \frac{2}{7} & \frac{-4}{7} & -240 \\ 0 & \frac{4}{7} & \frac{-8}{7} & -440 \end{array}\right] \frac{7}{2} R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & \frac{9}{7} & \frac{3}{7} & 470 \\ 0 & 1 & -2 & -840 \\ 0 & \frac{4}{7} & \frac{-8}{7} & -440 \end{array}\right] R_3 + \frac{-4}{7} R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & \frac{9}{7} & \frac{3}{7} & 470 \\ 0 & 1 & -2 & -840 \\ 0 & 0 & 0 & 40 \end{array}\right] R_1 + \frac{-9}{7} R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 3 & 1550 \\ 0 & 1 & -2 & -840 \\ 0 & 0 & 0 & 40 \end{array}\right]$$

Because the RREF is inconsistent, there is no solution. This means the prices of each textbook have not changed as the equations are parallel to eachother. This could have been resolved from the coefficients of the equations as they are multiples of eachother.

Problem 8. PIRATE - YARR

Putting the System of Equations into an augmented coefficient matrix.

$$= \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 70 \\ 0 & 1 & 1 & 0 & 30 \\ 0 & 0 & 2 & 2 & 90 \\ 0 & 4 & 0 & 1 & 80 \end{array}\right] \frac{1}{2} R_3$$

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 70 \\ 0 & 1 & 1 & 0 & 30 \\ 0 & 0 & 1 & 1 & 45 \\ 0 & 4 & 0 & 1 & 80 \end{array}\right] R_1 - R_2$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 40 \\ 0 & 1 & 1 & 0 & 30 \\ 0 & 0 & 1 & 1 & 45 \\ 0 & 0 & -4 & 1 & -40 \end{array}\right] R_4 + 4R_3$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 40 \\ 0 & 1 & 1 & 0 & 30 \\ 0 & 0 & 1 & 1 & 45 \\ 0 & 0 & 0 & 5 & 140 \end{array}\right] \frac{1}{5} R_4$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 40 \\ 0 & 1 & 1 & 0 & 30 \\ 0 & 0 & 1 & 1 & 45 \\ 0 & 0 & 0 & 1 & 28 \end{array}\right] R_3 - R_4$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 40 \\ 0 & 1 & 1 & 0 & 30 \\ 0 & 0 & 1 & 0 & 17 \\ 0 & 0 & 0 & 1 & 28 \end{array}\right] R_2 - R_3$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 40 \\ 0 & 1 & 0 & 0 & 13 \\ 0 & 0 & 1 & 0 & 17 \\ 0 & 0 & 0 & 1 & 28 \end{array}\right] R_1 - R_4$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 12 \\ 0 & 1 & 0 & 0 & 13 \\ 0 & 0 & 1 & 0 & 17 \\ 0 & 0 & 0 & 1 & 28 \end{array}\right]$$

$A = 12, B = 13, C = 17, D = 28$

$12 + 13 + 17 + 28 = 70$

$13 + 17 = 30$

$2(13) + 2(28) = 90$

$4(13) + 28 = 80$