MATH 114 - Fall 2016 - Assignment 3

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Problem 1. Complex Numbers - Euler's Formula

a)

$$\begin{aligned}
&= e^{i\frac{\pi}{2}} \\
&= \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} \\
&= 0 + i(1) \\
&= i
\end{aligned}$$

b)

$$\begin{aligned}
&= e^{2\pi\frac{i}{3}} \\
&= \cos 2\pi + \frac{i}{3}\sin 2\pi \\
&= 1 + \frac{i}{3}(0) \\
&= 1
\end{aligned}$$

Problem 2. Complex Numbers - Euler's Formula

$$\begin{aligned}
&z = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3} \\
&z = e^{i\frac{\pi}{3}} \\
&z^6 = e^{6i2\pi} \\
&= \cos 2\pi + 6i\sin 2\pi \\
&= 1 + 6i(0) \\
&= 1
\end{aligned}$$

Problem 3. Systems of Equations

a)

$$= \left[\begin{array}{cc|c} 3 & 2 & -9 \\ 1 & 4 & 7 \end{array}\right] R_2 - R_1$$

$$\sim \left[\begin{array}{cc|c} 3 & 2 & -9 \\ -2 & 2 & 16 \end{array}\right] R_1 + R_2$$

$$\sim \left[\begin{array}{cc|c} 1 & 4 & 7 \\ -2 & 2 & 16 \end{array}\right] \frac{1}{2}R_2$$

$$\sim \left[\begin{array}{cc|c} 1 & 4 & 7 \\ -1 & 1 & 8 \end{array}\right] R_2 + R_1$$

$$\sim \left[\begin{array}{cc|c} 1 & 4 & 7 \\ 0 & 5 & 15 \end{array}\right] \frac{1}{5}R_2$$

$$\sim \left[\begin{array}{cc|c} 1 & 4 & 7 \\ 0 & 1 & 3 \end{array}\right] R_1 - 4R_2$$

$$\sim \left[\begin{array}{cc|c} 1 & 0 & -5 \\ 0 & 1 & 3 \end{array}\right]$$

$\vec{x}_1 = -5, \vec{x}_2 = 3$

b)

$$= \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 1 & 1 & 1 & 2 \\ 1 & 3 & 4 & 0 \end{array}\right] R_3 - R_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & -1 \end{array}\right] R_2 - R_3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 1 & 0 & 0 & 3 \\ 1 & 1 & 1 & -1 \end{array}\right] R_2 \Leftrightarrow R_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 1 & 2 & 3 & 1 \\ 1 & 1 & 1 & -1 \end{array}\right] R_2 - 2R_3 - R_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & -1 \end{array}\right] R_3 - R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \end{array}\right] R_3 \Leftrightarrow R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

Inconsistent, therefore no solutions.

c)

It is not possible to reduce to RREF fully, the closest that I could get is below. This is because it is inconsistent.

$$= \left[\begin{array}{ccc|c} 1 & 0 & \frac{7}{3} & \frac{5}{3} \\ 0 & 1 & \frac{1}{3} & \frac{-1}{3} \\ 0 & 0 & 0 & -3 \end{array}\right]$$

Problem 4. Solving two planes

$$3x_1 + 2x_2 - 4x_3 = 1$$

$$x_1 + x_2 - x_3 = 1$$

$$x_1 = 1 - x_2 + x_3$$

$$3(1 - x_2 + x_3) + 2x_2 - 4x_3 = 1$$

$$3 - 3x_2 + 3x_3 + 2x_2 - 4x_3 = 1$$

$$x_2 - x_3 = -2$$

$$\frac{-x_2 - x_3 = -2}{-1}$$

$$x_2 = 2 - x_3$$

$$x_1 = 1 - (2 - x_3) + x_3$$

$$x_1 = 1 - 2 + x_3 + x_3$$

$$x_1 = -1 + 2x_3$$

$$\vec{x} = (-1 + 2x_3) + (2 - x_3)$$

$$\vec{x} = x_3 + 1$$

The solution set represents a line.

Problem 5. Solving System of Equations/Linear Independance

$$\begin{bmatrix} a_{1_1} & a_{2_1} & a_{3_1} \\ a_{1_2} & a_{2_2} & a_{3_2} \\ a_{1_3} & a_{2_3} & a_{3_3} \end{bmatrix} = \begin{bmatrix} 2 & 2 & 2 \\ -3 & 6 & -12 \\ 4 & 1 & 1 \end{bmatrix}$$

The System of Equations' coefficients map to the matrix above. This is the same as expanding the equation of $x_1\vec{a}_1 + x_2\vec{a}_2 + x_3\vec{a}_3$.
Therefore the vectors $\vec{a}_1, \vec{a}_2, \vec{a}_3$ are equivalent to the systems of equations.

$$= \left[\begin{array}{ccc|c} 2 & 2 & -2 & 0 \\ -3 & 6 & -12 & 0 \\ 4 & 1 & 1 & 0 \end{array}\right] \frac{1}{2}R_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ -3 & 6 & -12 & 0 \\ 4 & 1 & 1 & 0 \end{array}\right] R_2 + 3R_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 9 & -15 & 0 \\ 4 & 1 & 1 & 0 \end{array}\right] R3_{-}4R_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 9 & -15 & 0 \\ 0 & -3 & 5 & 0 \end{array}\right] \frac{1}{9}R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & \frac{-5}{3} & 0 \\ 0 & -3 & 5 & 0 \end{array}\right] R_3 + 3R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & \frac{-5}{3} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] R_3 + 3R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & \frac{-5}{3} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] R_1 - R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & \frac{2}{3} & 0 \\ 0 & 1 & \frac{-5}{3} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

The system of equations is linearly independant.

Problem 6. Solving System of Equations/Spanning

$$\begin{bmatrix} a_{1_1} & a_{2_1} & a_{3_1} \\ a_{1_2} & a_{2_2} & a_{3_2} \\ a_{1_3} & a_{2_3} & a_{3_3} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

The System of Equations' coefficients map to the matrix above. This is the same as expanding the equation of $x_1\vec{a}_1 + x_2\vec{a}_2 + x_3\vec{a}_3$.

Therefore the vectors $\vec{a}_1, \vec{a}_2, \vec{a}_3$ are equivalent to the systems of equations.

$$= \left[\begin{array}{ccc|c} 1 & 1 & 0 & v_1 \\ 0 & 1 & 1 & v_2 \\ 0 & 0 & 1 & v_3 \end{array}\right] R_1 - R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -1 & v_1 - v_2 \\ 0 & 1 & 1 & v_2 \\ 0 & 0 & 1 & v_3 \end{array}\right] R_1 + R_3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & v_1 - v_2 + v_3 \\ 0 & 1 & 1 & v_2 \\ 0 & 0 & 1 & v_3 \end{array}\right] R_2 - R_3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & v_1 - v_2 + v_3 \\ 0 & 1 & 0 & v_2 - v_3 \\ 0 & 0 & 1 & v_3 \end{array}\right]$$

System of Equations is consistent. Therefore the vectors $\vec{a}_1, \vec{a}_2, \vec{a}_3$ span $\mathbb{R}^2$