MATH 114 - Fall 2016 - Assignment 6

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November 28, 2016

Problem 1. Eigenvectors and Eigenvalues

$$A = \begin{bmatrix} -4 & 0 & 0 \\ 2 & -8 & 4 \\ -4 & 5 & 0 \end{bmatrix}$$

$$(A - \lambda I) = \begin{bmatrix} -4-\lambda & 0 & 0 \\ 2 & -8-\lambda & 4 \\ -4 & 5 & 0-\lambda \end{bmatrix} =$$

$$C(A) = -(\lambda - 2)(\lambda + 4)(\lambda + 10)$$

$$\lambda_1 = 2, \lambda_2 = -4, \lambda_3 = -10$$

$$\vec{x_1} = t \begin{bmatrix} 0 \\ \frac{2}{5} \\ 1 \end{bmatrix}$$

$$\vec{x_2} = t \begin{bmatrix} 6 \\ 4 \\ 1 \end{bmatrix}$$

$$\vec{x_3} = t \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$$

Eigenspace:

Span of $\vec{x_1}$ is $\left\{ \begin{bmatrix} 0 \\ \frac{2}{5} \\ 1 \end{bmatrix} \right\}$ and the dimension of it is 1.

Span of $\vec{x_2}$ is $\left\{ \begin{bmatrix} 6 \\ 4 \\ 1 \end{bmatrix} \right\}$ and the dimension of it is 1.

Span of $\vec{x_3}$ is $\left\{ \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix} \right\}$ and the dimension of it is 1.

Geometric Multiplicity of all Eigenvalues and vectors is 1.
Algebraic Multiplicity of Eigenvalues and vectors is also 1.

Problem 2.

By utilizing synthetic division, as we are given a factor of the characteristic polynomial, we can get the quadratic that can then be factored.

$$(\lambda - 1)(2\lambda^2 + 3\lambda - 5) = (\lambda - 1)(\lambda - 1)(\lambda + \frac{5}{2})$$

Algebraic Multiplicity of λ_1 is 2, as it appears twice. Algebraic Multiplicity of λ_3 is 1.

Problem 3. Diagonalization