MATH 114, Fall 2016. Assignment 2

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Problem 1. Angle between two vectors

$$\cos\theta = \frac{-2}{2*2}$$

$$\cos\theta = \frac{-1}{2}$$

$$\arccos\frac{-1}{2}$$

$$90^\circ or \frac{\pi}{2}$$

Problem 2. Triangle Inequality

$$\|\vec{x}\| = \sqrt{5}$$

$$\|\vec{y}\| = \sqrt{5}$$

$$\|\vec{x}\| + \|\vec{y}\| = 2\sqrt{5}$$

$$\|\vec{x} + \vec{y}\| = 3\sqrt{5}$$

$$3\sqrt{5} \geq 2\sqrt{5}$$

$\vec{x}$ and $\vec{y}$ both have the same direction vector.

$\vec{y}$ is $\vec{x}$ scaled by a factor of 3. Because of this the Triangle Inequality is satisfied.

$$\|\vec{x}\| = \sqrt{5}$$

$$\|\vec{y}\| = 3\sqrt{5}$$

$$\|\vec{x} + \vec{y}\| = 4\sqrt{5}$$

$$4\sqrt{5} \leq \sqrt{5} + 3\sqrt{5}$$

Problem 3. Work - Dot Product

$$100\cos\frac{\pi}{6} = \vec{F}\vec{d}$$

$$100\frac{\sqrt{3}}{2} = 50\sqrt{3}$$

$$F = 50\sqrt{3}J$$

Problem 4. Projections

The projection on the horizontal is the horizontal component of F and the perpendicular of the horizontal is they vertical component of F. To calculate the projection, the formula of $\frac{\vec{F} \cdot \vec{H}}{\|\vec{H}\|}$ can be used to obtain the horizontal component.

Problem 5. Lines and Projections

a)

The vector equation of $y = \frac{-3}{2}x + 2$ is $\vec{v} = \begin{bmatrix} 2 \\ -3 \end{bmatrix} t + \begin{bmatrix} 0 \\ 2 \end{bmatrix}$ This does not satisfy the requirements of a subspace of $\mathbb{R}^2$ because it does not intersect the origin. And thus is not a subspace.

b)

$$\vec{x} = \begin{bmatrix} 1 \\ 9 \end{bmatrix} \vec{y} = \begin{bmatrix} -4 \\ 6 \end{bmatrix}$$

$$\|\vec{x}\| = \sqrt{82}$$

$$\text{proj}_{\vec{x}}\vec{y} = \frac{\vec{x} \cdot \vec{y}}{\|\vec{y}\|}$$

$$\text{proj}_{\vec{x}}\vec{y} = \frac{20}{2\sqrt{13}}$$

$$\text{proj}_{\vec{x}}\vec{y} = \frac{10}{\sqrt{13}}$$

c)

$$\vec{x} = \begin{bmatrix} 5 \\ 3 \end{bmatrix} \vec{y} = \begin{bmatrix} -4 \\ 6 \end{bmatrix}$$

$$\text{proj}_{\vec{x}}\vec{y} = \frac{\vec{x} \cdot \vec{y}}{\|\vec{y}\|}$$

$$\text{proj}_{\vec{x}}\vec{y} = \frac{-2}{2\sqrt{13}}$$

$$\text{proj}_{\vec{x}}\vec{y} = \frac{-1}{\sqrt{13}}$$

d)

e)

Problem 6. Scalar Equation of a Plane

$$\vec{PQ} = \begin{bmatrix} 1 \\ 1 \\ -4 \end{bmatrix}$$

$$\vec{PR} = \begin{bmatrix} 0 \\ -5 \\ -2 \end{bmatrix}$$

$$\vec{n} = \begin{bmatrix} (1)(-2) - (-5)(-4) \\ (-4)(0) - (1)(-2) \\ (1)(-5) - (1)(0) \end{bmatrix}$$

$$\vec{n} = \begin{bmatrix} -22 \\ 2 \\ -5 \end{bmatrix}$$

$$\vec{P} = -22x + 2y - 5z$$

$$-22(x-1) + 2(y-5) - 5(z-3) = 0$$

$$-22x + 2y - 5z = 27$$

Problem 7. Cross Product

a)

$$\vec{w} = \begin{bmatrix} (2)(3) - (2)(-1) \\ (2)(-2) - (1)(1) \\ (1)(-1) - (2)(-2) \end{bmatrix}$$

$$\vec{w} = \begin{bmatrix} 8 \\ -4 \\ 5 \end{bmatrix}$$

$$\|\vec{w}\| = \sqrt{105}$$

b)

$$\|\vec{x}\| = 3$$

The answer will be the same because $\vec{y}$ is $\vec{x}$ flipped. This is the definition of the perpendicular vector.

c)

$$\vec{a} = \begin{bmatrix} 1 \\ 5 \\ 5 \end{bmatrix}$$

$$\text{proj}_{\vec{a}}\vec{b} = \frac{\vec{a} \cdot \vec{b}}{\left\|\vec{b}\right\|}$$

$$\vec{a} \cdot \vec{b} = \|\vec{a}\| \left\|\vec{b}\right\| \cos \frac{pi}{4}$$

$$\vec{a} \cdot \vec{b} = \sqrt{51} \left\|\vec{b}\right\| (-\frac{1}{4})$$

Problem 8. Cross Product Theory

a)
When $\vec{u} \cdot \vec{v}$ equals 0, the two vectors are orthagonol. The case of when the cross product of the two is equal to 0 is when either $\vec{u}$ or $\vec{v}$ is equal to the $\vec{0}$ vector. Alternatively, but not applicable to this case is when the two vectors are parallel or antiparallel.
b)
$\vec{w}$ is the third and final vector that can be orthagonal to both $\vec{u}$ and $\vec{v}$. Unless the dimension is changed, there cannot be another vector that meets this.
The space would need to be $\mathbb{R}^4$ or greater in order to solve this question.

Problem 9. Complex Numbers

a)

$$(1 + i\sqrt{3})(1 + i)$$

$$(1 + i) - (1 - i)\sqrt{3}$$

$$1 - \sqrt{3} + i(1 + \sqrt{3})$$

b)

$$\frac{\pi + i\pi}{1 - \sqrt{3i}}$$

$$\frac{\pi + i\pi}{1 - \sqrt{3i}} \times \frac{1 + \sqrt{3i}}{1 + \sqrt{3i}}$$

$$\frac{(1 - i)\pi}{\sqrt{3} + i}$$

Problem 10. Solving Polynomials

As we are given a factor, we can use synthetic division to get the quadratic to solve. The quadratics specified below are the result of it.
a)

$$x^2 - x + 3 = 0$$

$$x = \frac{-1 \pm \sqrt{1 - 4(1)(3)}}{2}$$

$$x = \frac{-1 \pm \sqrt{-11}}{2}$$

Root 1 $x = \frac{1}{2}(-1 + 11i)$
Root 2 $x = \frac{1}{2}(-1 - 11i)$
b)

$$x^2 - 2x - 2 = 0$$
$$x = \frac{2 \pm \sqrt{4 - 4(1)(-2)}}{2}$$
$$x = \frac{1}{2}(2 \pm 2\sqrt{3})$$

Root 2 $x = \frac{1}{2}(2 + 2\sqrt{3})$
Root 3 $x = \frac{1}{2}(2 - 2\sqrt{3})$