

MATH 114, Fall 2016. Assignment 2

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Problem 1. Angle between two vectors

$$\cos \theta = \frac{-2}{2 * 2}$$

$$\cos \theta = \frac{-1}{2}$$

$$\arccos \frac{-1}{2}$$

$$90^\circ \text{ or } \frac{\pi}{2}$$

Problem 2. Triangle Inequality

$$\|\vec{x}\| = \sqrt{5}$$

$$\|\vec{y}\| = \sqrt{5}$$

$$\|\vec{x}\| + \|\vec{y}\| = 2\sqrt{5}$$

$$\|\vec{x} + \vec{y}\| = 3\sqrt{5}$$

$$3\sqrt{5} \geq 2\sqrt{5}$$

\vec{x} and \vec{y} both have the same direction vector.

\vec{y} is \vec{x} scaled by a factor of 3. Because of this the Triangle Inequality is satisfied.

$$\|\vec{x}\| = \sqrt{5}$$

$$\|\vec{y}\| = 3\sqrt{5}$$

$$\|\vec{x} + \vec{y}\| = 4\sqrt{5}$$

$$4\sqrt{5} \leq \sqrt{5} + 3\sqrt{5}$$

Problem 3. Work - Dot Product

$$100 \cos \frac{\pi}{6} = \vec{F}d$$

$$100 \frac{\sqrt{3}}{2} = 50\sqrt{3}$$

$$F = 50\sqrt{3}J$$

Problem 4. Projections

The projection on the horizontal is the horizontal component of F and the perpendicular of the horizontal is the vertical component of F
To calculate the projection, the formula of

$$\frac{\vec{F} \cdot \vec{H}}{\|\vec{H}\|}$$

can be used to obtain the horizontal component.

Problem 5. Lines and Projections

a

The vector equation of $y = \frac{-3}{2}x + 2$ is

$$\vec{v} = \begin{bmatrix} 2 \\ -3 \end{bmatrix} t + \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

This does not satisfy the requirements of a subspace of \mathbb{R}^2 because it does not intersect the origin.

b

$$\vec{x} = \begin{bmatrix} 1 \\ 9 \end{bmatrix} \vec{y} = \begin{bmatrix} -4 \\ 6 \end{bmatrix}$$

$$\|\vec{x}\| = \sqrt{82}$$

$$\text{proj}_{\vec{x}} \vec{y} = \frac{\vec{x} \cdot \vec{y}}{\|\vec{y}\|}$$

$$\text{proj}_{\vec{x}} \vec{y} = \frac{20}{2\sqrt{13}}$$

$$\text{proj}_{\vec{x}}\vec{y} = \frac{10}{\sqrt{13}}$$

c

$$\vec{x} = \begin{bmatrix} 5 \\ 3 \end{bmatrix} \quad \vec{y} = \begin{bmatrix} -4 \\ 6 \end{bmatrix}$$

$$\text{proj}_{\vec{x}}\vec{y} = \frac{\vec{x} \cdot \vec{y}}{\|\vec{y}\|}$$

$$\text{proj}_{\vec{x}}\vec{y} = \frac{-2}{2\sqrt{13}}$$

$$\text{proj}_{\vec{x}}\vec{y} = \frac{-1}{\sqrt{13}}$$

d

I dislike projections

Problem 6. Scalar Equation of a Plane

$$\vec{PQ} = \begin{bmatrix} 1 \\ 1 \\ -4 \end{bmatrix}$$

$$\vec{PR} = \begin{bmatrix} 0 \\ -5 \\ -2 \end{bmatrix}$$

$$\vec{n} = \begin{bmatrix} (1)(-2) - (-5)(-4) \\ (-4)(0) - (1)(-2) \\ (1)(-5) - (1)(0) \end{bmatrix}$$

$$\vec{n} = \begin{bmatrix} -22 \\ 2 \\ -5 \end{bmatrix}$$

$$\vec{P} = -22x + 2y - 5z$$

$$-22(x - 1) + 2(y - 5) - 5(z - 3) = 0$$

$$-22x + 2y - 5z = 27$$

Problem 7. Cross Product

a

$$\vec{w} = \begin{bmatrix} (2)(3) - (2)(-1) \\ (2)(-2) - (1)(1) \\ (1)(-1) - (2)(-2) \end{bmatrix}$$

$$\vec{w} = \begin{bmatrix} 8 \\ -4 \\ 5 \end{bmatrix}$$

$$\|\vec{w}\| = \sqrt{105}$$

b

$$\|\vec{x}\| = 3$$

c

$$\vec{a} = \begin{bmatrix} 1 \\ 5 \\ 5 \end{bmatrix}$$

$$\text{proj}_{\vec{a}} \vec{b} = \frac{\vec{a} \cdot \vec{b}}{\|\vec{b}\|}$$

$$\vec{a} \cdot \vec{b} = \|\vec{a}\| \|\vec{b}\| \cos \frac{\pi}{4}$$

$$\vec{a} \cdot \vec{b} = \sqrt{51} \|\vec{b}\| \left(-\frac{1}{4}\right)$$

Problem 8. Cross Product Theory

a

When $\vec{u} \cdot \vec{v}$ equals 0, the two vectors are orthogonal.

The case of when the cross product of the two is equal to 0 is when either \vec{u} or \vec{v} is equal to the $\vec{0}$ vector.

Alternatively, but not applicable is when the two vectors are parallel or antiparallel.

b

Problem 9. Complex Numbers

a

$$\begin{aligned} & (1 + i\sqrt{3})(1 + i) \\ & (1 + i) - (1 - i)\sqrt{3} \\ & 1 - \sqrt{3} + i(1 + \sqrt{3}) \end{aligned}$$

b

$$\begin{aligned} & \frac{\pi + i\pi}{1 - \sqrt{3}i} \\ & \frac{\pi + i\pi}{1 - \sqrt{3}i} \times \frac{1 + \sqrt{3}i}{1 + \sqrt{3}i} \\ & \frac{(1 - i)\pi}{\sqrt{3} + i} \end{aligned}$$

Problem 10. Solving Polynomials

As we are given a factor, we can use synthetic division to get the quadratic to solve.

The quadratics specified below are the result of it.

a

$$\begin{aligned} & x^2 - x + 3 = 0 \\ & x = \frac{-1 \pm \sqrt{1 - 4(1)(3)}}{2} \\ & x = \frac{-1 \pm \sqrt{-11}}{2} \\ & x = \frac{1}{2}(-1 + 11i) \\ & x = \frac{1}{2}(-1 - 11i) \end{aligned}$$

b

$$\begin{aligned} & x^2 - 2x - 2 = 0 \\ & x = \frac{2 \pm \sqrt{4 - 4(1)(-2)}}{2} \\ & x = \frac{1}{2}(2 \pm 2\sqrt{3}) \\ & x = \frac{1}{2}(2 + 2\sqrt{3}) \\ & x = \frac{1}{2}(2 - 2\sqrt{3}) \end{aligned}$$