TZ1

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Higher level
Paper 2

Monday 9 May 2022 (morning)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

93/110 = 84.6%

15 pages

2222–7107

Please **do not** write on this page.

Answers written on this page will not be marked.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

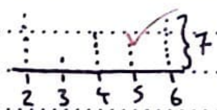
The number of hours spent exercising each week by a group of students is shown in the following table.

Exercising time (in hours)	Number of students
2	5
3	1
4	4
5	3
6	x

The median is 4.5 hours.

(a) Find the value of x. [2]

(b) Find the standard deviation. [2]

(a) 

$\boxed{x=7}$ ✓

2

(b) GDC : $\sigma x = 1.5843$

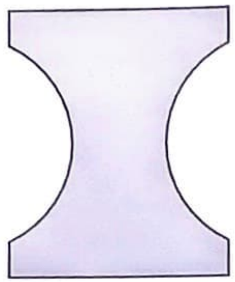
≈ 1.58 ✓✓ 2

4

2. [Maximum mark: 6]

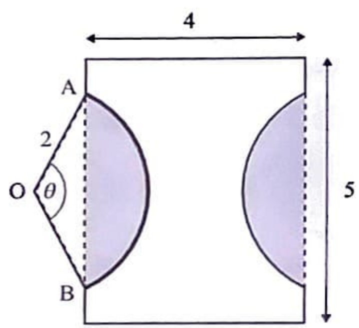
A company is designing a new logo. The logo is created by removing two equal segments from a rectangle, as shown in the following diagram.

diagram not to scale



The rectangle measures 5 cm by 4 cm. The points A and B lie on a circle, with centre O and radius 2 cm, such that $\text{A}\hat{\text{O}}\text{B} = \theta$, where $0 < \theta < \pi$. This information is shown in the following diagram.

diagram not to scale



(a) Find the area of one of the shaded segments in terms of θ. [3]

(b) Given that the area of the logo is $13.4\,\text{cm}^2$, find the value of θ. [3]

(This question continues on the following page)

(Question 2 continued)

(a) $A = A_{\text{SECTOR}} - A_{\text{TRIANGLE}}$ ✓

$= \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$

$= 2\theta - 2\sin\theta$ ✓

$= 2(\theta - \sin\theta)$ ✓ 3

(b) $A = 13.4 = (4)(5) - (2)(2)(\theta - \sin\theta)$ ✓

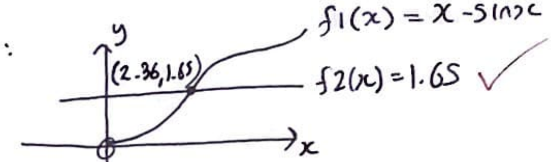
$\therefore 13.4 = 20 - 4\theta + 4\sin\theta$

$\therefore 4\theta - \sin\theta = 20 - 13.4$

$= 6.6$

$\therefore \theta - \sin\theta = 1.65$ ✓

[GDC]:



$\therefore \boxed{\theta = 2.36^c}$ ✓

3

6

3. [Maximum mark: 6]

A discrete random variable, X, has the following probability distribution:

x	0	1	2	3
$\mathrm{P}(X=x)$	0.41	$k-0.28$	0.46	$0.29-2k^2$

(a) Show that $2k^2-k+0.12=0$. [1]

(b) Find the value of k, giving a reason for your answer. [3]

(c) Hence, find $\mathrm{E}(X)$. [2]

(a) $0.41+(k-0.28)+0.46+(0.29-2k^2)=1$

$\therefore -2k^2+k+(0.41-0.28+0.46+0.29-1)=0$

$\therefore -2k^2+k-0.12=0$

$\therefore 2k^2-k+0.12=0$ 1

(b) polyRoots$(2x^2-x+0.12,x)$ $\{0.2, 0.3\}$

$\boxed{\cancel{\therefore k = 0.2, 0.3}}$

~~$\Rightarrow$ P(X=3) has a squared term. Hence~~

~~(c) $E(X) = 0.2-0.28$~~

$\therefore k = 0.3$ $\{k-0.28 \geq 0\}$ 3

M1

(c) $E(X) = 0\cdot 0.41 + (0.3-0.28) + 2(0.46) + 3(0.29-2(0.3^2))$

$\boxed{=1.68}$ 1

5

4. [Maximum mark: 7]

A particle moves along a straight line so that its velocity, $v\,\mathrm{m\,s^{-1}}$, after t seconds is given by $v(t)=e^{\sin t}+4\sin t$ for $0\le t\le 6$.

(a) Find the value of t when the particle is at rest. [2]

(b) Find the acceleration of the particle when it changes direction. [3]

(c) Find the total distance travelled by the particle. [2]

(a) Using (Graph 1): $t = 3.35$ s 2

(b) Using (Graph 2): at $t=3.35$ s,

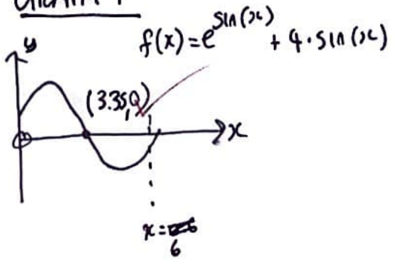
$\frac{d}{dt}(v(t)) = a(t) = -4.71\ \mathrm{ms^{-2}}$ 3

(c) Distance $= \int_0^6 |v(t)|\,dt$

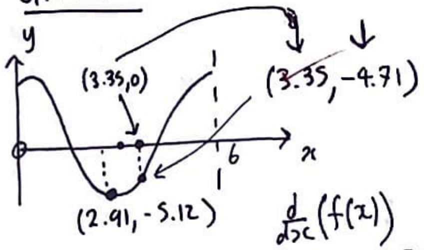
$= 20.7534$ m

≈ 20.8 m 2

GDC:
GRAPH 1



GDC:
GRAPH 2



7

5. [Maximum mark: 6]

Let A and B be two independent events such that $\mathrm{P}(A \cap B') = 0.16$ and $\mathrm{P}(A' \cap B) = 0.36$.

(a) Given that $\mathrm{P}(A \cap B) = x$, find the value of x. [4]

(b) Find $\mathrm{P}(A' \mid B')$. [2]

(a) $P(B) = P(A' \cap B) + \underset{\uparrow}{\overset{x}{P(A \cap B)}} = P(A' \cap B) + x$

$P(A) = P(A \cap B') + P(A \cap B) = P(A \cap B') + x$

$P(A \cap B) = (x + P(A' \cap B))(x + P(A \cap B'))$

$\therefore x = (x + 0.36)(x + 0.16)$

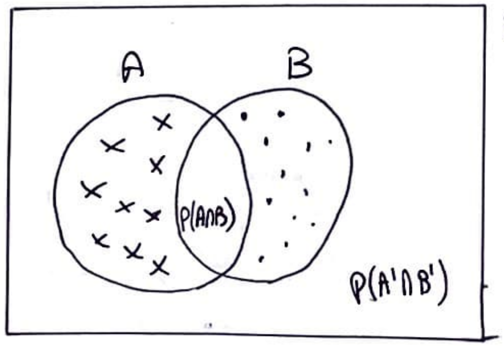
$\therefore x = x^2 + 0.52x + 0.0576$

$\therefore 0 = x^2 - 0.48x + 0.0576$

$\therefore \boxed{x = 0.24}$ {GDC polySolve}

(b) $P(A' \mid B') = \dfrac{P(A' \cap B')}{P(B')} = \dfrac{1 - 0.24 - 0.16 - 0.36}{1 - (0.36 + 0.24)}$

$\boxed{\therefore P(A' \mid B') = 0.4}$



x: $P(A \cap B')$

•: $P(A' \cap B)$

4

6. [Maximum mark: 6]

Consider the function $f(x) = 2^x - \dfrac{1}{2^x}$, $x \in \mathbb{R}$.

(a) Show that f is an odd function. [2]

The function g is given by $g(x) = \dfrac{x-1}{x^2 - 2x - 3}$, where $x \in \mathbb{R}$, $x \neq -1$, $x \neq 3$.

(b) Solve the inequality $f(x) \geq g(x)$. [4]

(a) $f(-x) = 2^{-x} - 1/2^{-x}$ M1

$= 1/2^x - 2^x$

$= -(2^x - 1/2^x)$ A1

$\therefore f(-x) = -f(x)$

$\therefore f$ is an odd function

2

(b) Using (Graph 1): $f(x) \geq g(x)$ when:

$-1.27 \leq x < -1$ A1

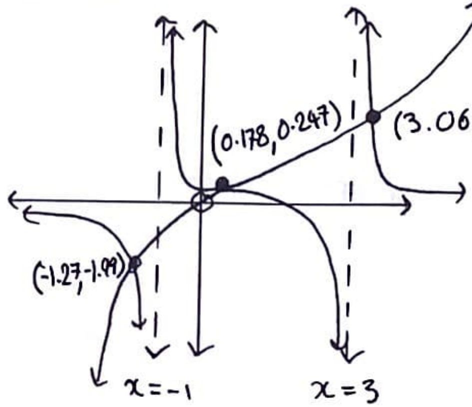
$0.178 \leq x < 3$ A1

$3.06 \leq x$ A1

4

GDC GRAPH 1:

M1



$f1(x) = 2^x - \dfrac{1}{2^x}$

$f2(x) = \dfrac{x-1}{x^2 - 2x - 3}$

6

7. [Maximum mark: 9]

Consider the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ such that $\boldsymbol{a} = \begin{pmatrix} 12 \\ -5 \end{pmatrix}$ and $|\boldsymbol{b}| = 15$.

(a) Find the possible range of values for $|\boldsymbol{a} + \boldsymbol{b}|$. [2]

Consider the vector $\boldsymbol{p}$ such that $\boldsymbol{p} = \boldsymbol{a} + \boldsymbol{b}$.

(b) Given that $|\boldsymbol{a} + \boldsymbol{b}|$ is a minimum, find $\boldsymbol{p}$. [2]

Consider the vector $\boldsymbol{q}$ such that $\boldsymbol{q} = \begin{pmatrix} x \\ y \end{pmatrix}$, where $x, y \in \mathbb{R}^+$.

(c) Find $\boldsymbol{q}$ such that $|\boldsymbol{q}| = |\boldsymbol{b}|$ and $\boldsymbol{q}$ is perpendicular to $\boldsymbol{a}$. [5]

~~(a)~~

$\cancel{\vec{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \rightarrow \sqrt{b_1^2 + b_2^2} = 15}$

$\cancel{|a+b| = \left|\begin{pmatrix} 12+b_1 \\ -5+b_2 \end{pmatrix}\right| = \sqrt{(12+b_1)^2 + (-5+b_2)^2}}$

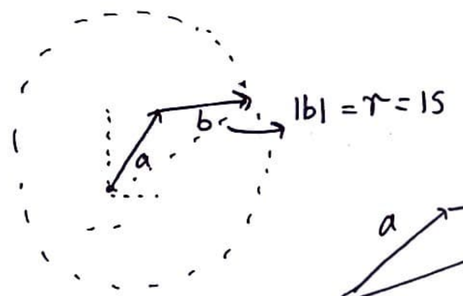
$\cancel{= \sqrt{144 + 24b_1 + b_1^2 + 25 - 10b_2 + b_2^2}}$

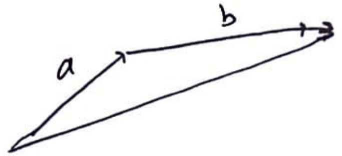
$\cancel{= \sqrt{169 + 24b_1 - 10b_2 + 15^2}}$

$\cancel{= \sqrt{394 + 24b_1 - 10b_2}}$

~~COMPLET~~

completed in answer booklet





7

8. [Maximum mark: 4]

Consider the equation $kx^2 - (k+3)x + 2k + 9 = 0$, where $k \in \mathbb{R}$.

(a) Write down an expression for the product of the roots, in terms of k. [1]

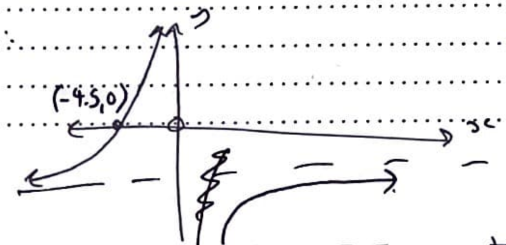
(b) Hence or otherwise, determine the values of k such that the equation has one positive and one negative real root. [3]

(a) $kx^2 - (k+3)x + (2k+9) = 0$

Product $= -\dfrac{2k+9}{k}$ ✗

$= -2 - 9/k$

(b) one + and 1 neg occurs when product < 0 ✓ 1



Hence, $\boxed{k < -4.5}$ or $\boxed{k > 0}$ ✗

$\Delta = (-k-3)^2 - 4k(2k+9)$

$= k^2 + 6k + 9 - 8k - 36k$

$= k^2 - 22k + 9 > 0$

$\therefore 0.416995 \leq k \leq 21.583.$

Hence, $\boxed{0 < k < 21.583}$

1

Turn over

9. [Maximum mark: 7]

Mary, three female friends, and her brother, Peter, attend the theatre. In the theatre there is a row of 10 empty seats. For the first half of the show, they decide to sit next to each other in this row.

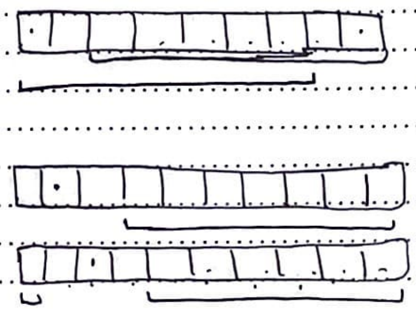
(a) Find the number of ways these five people can be seated in this row. [3]

For the second half of the show, they return to the same row of 10 empty seats. The four girls decide to sit at least one seat apart from Peter. The four girls do not have to sit next to each other.

(b) Find the number of ways these five people can now be seated in this row. [4]

(a) $\cancel{^{10}P_5 = 30240}$ $5! \times 6 = 6! = 720$ 3

(b)



$\therefore$ edge seat: $1\times1\times{}^8P_4 + {}^8P_4\times1\times1$

$= \cancel{13440}$ $1680\times2 = 3360$

inner seat: ${}^7P_4 \times 8 = \cancel{17640}$ $\cancel{5880}$ 6720

$\therefore$ ~~total = 17640 + 13440~~

~~= 31080~~

total = 3360 + 6720

= 10080

A1A1A1

3

6

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 20]

Consider the function $f(x)=\sqrt{x^2-1}$, where $1 \le x \le 2$.

(a) Sketch the curve $y=f(x)$, clearly indicating the coordinates of the endpoints. [2]

(b) (i) Show that the inverse function of f is given by $f^{-1}(x)=\sqrt{x^2+1}$.

(ii) State the domain and range of f^{-1}. [5]

The curve $y=f(x)$ is rotated 2π about the y-axis to form a solid of revolution that is used to model a water container.

(c) (i) Show that the volume, $V\,\text{m}^3$, of water in the container when it is filled to a height of h metres is given by $V=\pi\left(\frac{1}{3}h^3+h\right)$.

(ii) Hence, determine the maximum volume of the container. [5]

At $t=0$, the container is empty. Water is then added to the container at a constant rate of $0.4\,\text{m}^3\,\text{s}^{-1}$.

(d) Find the time it takes to fill the container to its maximum volume. [2]

(e) Find the rate of change of the height of the water when the container is filled to half its maximum volume. [6]

Do **not** write solutions on this page.

11. [Maximum mark: 16]

A bakery makes two types of muffins: chocolate muffins and banana muffins.

The weights, C grams, of the chocolate muffins are normally distributed with a mean of 62 g and standard deviation of 2.9 g.

(a) Find the probability that a randomly selected chocolate muffin weighs less than 61 g. [2]

(b) In a random selection of 12 chocolate muffins, find the probability that exactly 5 weigh less than 61 g. [2]

The weights, B grams, of the banana muffins are normally distributed with a mean of 68 g and standard deviation of 3.4 g.

Each day 60% of the muffins made are chocolate.

On a particular day, a muffin is randomly selected from all those made at the bakery.

(c) (i) Find the probability that the randomly selected muffin weighs less than 61 g.

(ii) Given that a randomly selected muffin weighs less than 61 g, find the probability that it is chocolate. [7]

The machine that makes the chocolate muffins is adjusted so that the mean weight of the chocolate muffins remains the same but their standard deviation changes to σ g. The machine that makes the banana muffins is not adjusted. The probability that the weight of a randomly selected muffin from these machines is less than 61 g is now 0.157.

(d) Find the value of σ. [5]

Do **not** write solutions on this page

12. [Maximum mark: 19]

Consider the differential equation $x^2\frac{dy}{dx} = y^2 - 2x^2$ for $x > 0$ and $y > 2x$. It is given that $y = 3$ when $x = 1$.

(a) Use Euler's method, with a step length of 0.1, to find an approximate value of y when $x = 1.5$. [4]

(b) Use the substitution $y = vx$ to show that $x\frac{dv}{dx} = v^2 - v - 2$. [3]

(c) (i) By solving the differential equation, show that $y = \frac{8x + x^4}{4 - x^3}$.

(ii) Find the actual value of y when $x = 1.5$.

(iii) Using the graph of $y = \frac{8x + x^4}{4 - x^3}$, suggest a reason why the approximation given by Euler's method in part (a) is not a good estimate to the actual value of y at $x = 1.5$. [12]

References:

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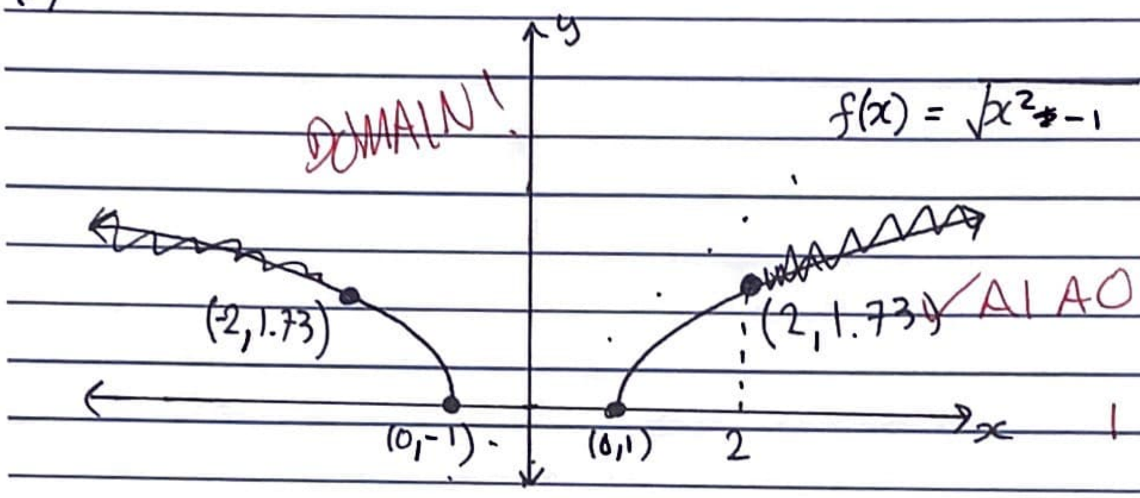
Example / Ejemplo 27 [2][7] Example / Ejemplo 3 [ ][3]

[1][0]

(a)



(b)(i) $x = \sqrt{y^2-1}$ ✓

$\therefore y^2-1 = x^2$

$\therefore y^2 = x^2+1$ ✓

$\therefore y = \sqrt{x^2+1}$

$\therefore f^{-1}(x) = \sqrt{x^2+1}$ ✓ 3

(ii) ~~Domain : $\{x \mid x \geq 1\}$~~

~~Range : $\{y \mid y \leq -1, y \geq 1\}$~~

Domain : $\{x \mid 0 \leq x \leq 1.73\}$ ✓

Range : $\{y \mid -2 \leq y \leq -1, 1 \leq y \leq 2\}$ ✗ 1

04AX01

(c)(i)

$$V = \pi \int_0^h (y^2+1)\,dy$$

$$= \pi\left[\tfrac{1}{3}y^3 + y\right]_0^h$$

3

$$= \pi\left(\tfrac{1}{3}h^3 + h - 0\right)$$

$$= \pi\left(\tfrac{1}{3}h^3 + h\right)$$

(ii)

$$\cancel{V = \pi\left(\tfrac{1}{3}h^3 + h\right)}$$

$$\cancel{\therefore\ dV/dh = \pi h^2 + \pi}$$

$$\cancel{V_{MAX}} = \pi \lim_{h\to\infty}\left(\tfrac{1}{3}h^3 + h\right)$$

$$= \pi \lim_{h\to\infty}\left(\frac{h^3+3h}{3}\right)$$

$$= \pi \lim_{h\to\infty}\left(\frac{1+3/h^2}{3/h^3}\right)$$

$$\cancel{\therefore V_{MAX} = \infty}$$

VMAX occurs at $h = 1.73$

$$\therefore V_{MAX} = \pi\left(\tfrac{1}{3}(1.73)^3 + 1.73\right)$$

$$= 10.857$$

$$\therefore V_{MAX} \approx 10.9\ m^3$$

2

(d) $dV/dt = 0.4$

at $t=0$, $V=0$

If $dV/dt = 0.4$, then $V = 0.4t$

$$\therefore 0.4t = 10.857$$

$$\therefore t = 27.1425\ s$$

2

$$\therefore t = 27.1\ s$$

(e) $0.4t = \dfrac{10.857}{2}$

$$\therefore t = 13.57125$$

$$V = \pi\left(\tfrac{1}{3}h^3 + h\right)$$

$$\therefore \frac{dV}{dt} = \frac{\pi}{3}(3)h^2\, dh/dt + \pi\, dh/dt$$ M1

$$= \frac{dh}{dt}\left(\pi h^2 + \pi\right)$$ A1

$$\therefore 0.4 = dh/dt\left(\pi h^2 + \pi\right)$$

M1

when $V = 5.4285$, $5.4285 = \pi\left(\tfrac{1}{3}h^3 + h\right)$

$\therefore h = 1.18011\ m$ {nSolve}

A1

$$\therefore 0.4 = \frac{dh}{dt}\left(\pi(1.18011)^2 + \pi\right)$$ M1

6

$$\therefore dh/dt = 0.0532\ ms^{-1}$$ A1

18

11

(a) $C \sim N(62, 2.9^2)$

$P(C \leq 61) =$

normalcdf$(-9E999, 61, 62, 2.9) = 0.365112$ ✓M1

$\therefore P(C \leq 61) \approx 0.365$ ✓A1 2

(b) $X \sim B(12, 0.365112)$

$\therefore P(X=5) = $ binomPdf$(12, 0.365112, 5)$

$= 0.213666$ ✓M1

$\therefore P(X=5) \approx 0.214$ ✓A1 2

(c) $B \sim N(68, 3.4^2)$

(i) $P(B \leq 61) = $ Normalcdf$(-9E999, 61, 68, 3.4)$

$= 0.019756$ ✓A1

✓(M1) A1

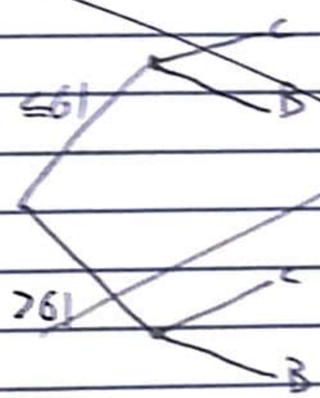
$\therefore P(\text{muffin} \leq 61) = (0.6)(0.365112) + (0.4)(0.019756)$

$= 0.22697$

$\therefore P(\text{muffin} \leq 61) \approx 0.227$ ✓A1 4

~~(ii) $P(\text{choc} \mid \text{muff} \leq 61) = \dfrac{P(\text{choc} \cap \text{muff} \leq 61)}{P(\text{muff} \leq 61)}$~~

~~$= \dfrac{0.22697 \times 0.6}{0.22697}$~~



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Example Ejemplo 27 | 2 7 | Example Ejemplo 3 | 3 |

11

(c)(ii) $P(M=B) = 0.4$ $P(C \le 61) = 0.365112$

$P(M=C) = 0.6$ $P(C \ge 61) = 0.634888$

$P(M \le 61) = 0.22697$ $P(M \ge 61) = 0.77303$

$$\therefore P(M=C \mid M \le 61) = \frac{P(M \le 61)P(C \le 61)}{P(M \le 61)P(C \le 61) + P(M \ge 61)P(C \ge 61)}$$

$$= \frac{(0.22697)(0.365112)}{(0.22697)(0.365112) + (0.77303)(0.634888)}$$

$$= 0.144$$

(d) $P(M \le 61) = (0.4)(0.019756) + (0.6)P(C \le 0.61)$ (M1)

$\therefore 0.157 = 0.0079024 + 0.6P(C \le 0.61)$

$\therefore P(C \le 0.61) = 0.248496$ A1

$\therefore P\left(C \le \frac{61-62}{\sigma}\right) = 0.248496$ M1

$\therefore -1/\sigma = \text{invNorm}(0.248496$

$= -0.67923$

$\therefore \sigma = 1.47225$

$\therefore \sigma \approx 1.47$ A1A1   5

13

12 (a) $x^2 \, dy/dx = y^2 - 2x^2$ $\quad x > 0 \quad y > 2x$

~~at (1,3):~~

~~$dy/dx = 3^2 - 2$~~

~~$= 9 - 2$~~

~~$\therefore dy/dx = 7$~~

n	x	y	
0	1	3	M1 $y_{n+1} = y_n + 0.1 \times f(x_n, y_n)$
1	1.1	3.7	$f(x_n, y_n) = \frac{y^2 - 2x^2}{x^2}$
2	1.2	4.63	A1
3	1.3	5.92	A1
4	1.4	7.780	4
5	1.5	10.7	A1 => give y = 10.7 as the final answer eg. y = 10.7

(b) $(vx)^2 (v + x\,dv/dx) = (vx)^2 - 2x^2$ M1 $\quad \begin{cases} y = vx \\ dy/dx = v + x\,dv/dx \end{cases}$ A1

$\therefore vx^2 + x^3 \, dv/dx = v^2x^2 - 2x^2$

$\therefore v + x \, dv/dx = v^2 - 2$

$\therefore x \, dv/dx = v^2 - v - 2$ A1 $\quad$ 3

(c)(i) $\frac{1}{v^2 - v - 2} \frac{dv}{dx} = \frac{1}{x}$ M1A1

Partial: $\frac{1}{(v-2)(v+1)} = \frac{A}{v-2} + \frac{B}{v+1}$ M1

$\therefore 1 = (v+1)A + (v-2)B$

when $v = -1$, $B = -1/3$

when $v = 2$, $A = 1/3$ $\quad$ M0M0A0

$\therefore \int \left(\frac{1}{3(v-2)} - \frac{1}{3(v+1)} \right) dv = \int \frac{1}{x} dx$ A1

$\therefore \frac{1}{3} \ln|v-2| - \frac{1}{3} \ln|v+1| = \ln|x| + C$ A1

$\therefore \ln \left| \frac{v-2}{v+1} \right| = \ln|x^3| + C$

$\therefore \frac{v-2}{v+1} = x^3 + C$

as $y = vx$, $v = y/x$ M1 M1

$\therefore \frac{\frac{y}{x} - 2}{\frac{y}{x} + 1} = x^3$

$\therefore \frac{y - 2x}{y + x} = x^3$

$\therefore y - 2x = x^4 + x^3 y$

$\therefore y(1 - x^3) = x^4 + 2x$ $\quad$ 7

$\therefore y = \frac{x^4 + 2x}{1 - x^3}$

$y = 3, x = 1, v = 3$

$\therefore \frac{1}{3}\ln|1| - \frac{1}{3}\ln|4| = \ln|1| + C$

$0 - \frac{1}{3}\ln|4| = C$

$\therefore C = \frac{1}{3} \ln \frac{1}{4}$

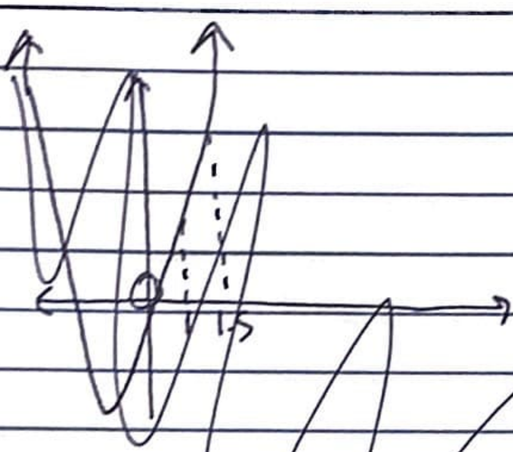
$\ln \left| \frac{v-2}{v+1} \right| = \ln|x^3| + \ln\left|\frac{1}{4}\right|$

$\therefore \frac{v-2}{v+1} = \frac{x^3}{4}$

(ii) $$y = \frac{8(1.5)+(1.5)^4}{4-(1.5)^3}$$

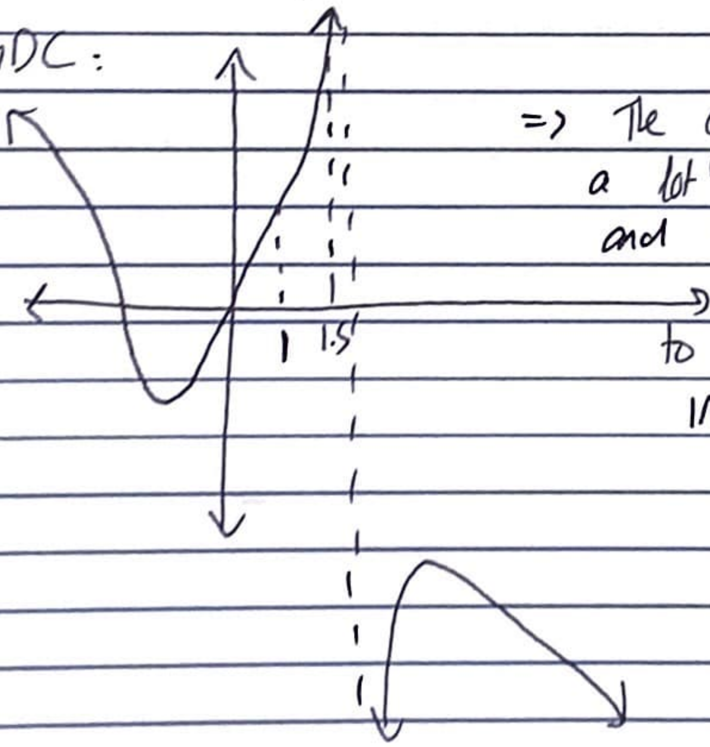
$= 27.3$ ✓ A1 1

(iii)



~~The gradient changes a lot in between steps of 0.1. This causes~~

GDC:



=> The gradient changes a lot ~~at~~ between $x=1$ and $x=1.5$, causing the estimate to ~~be~~ be very inaccurate R1

1

1

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ANSWER BOOKLET
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③

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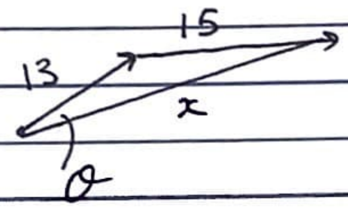
Example Ejemplo 27 | 27 | Example Ejemplo 3 | 3 |

| 7 |

(a)



$\Rightarrow 15^2 = 13^2 + x^2 - 2(13)(x)\cos\theta, \quad 0 \le \theta \le 2\pi$

$\therefore\ 56 = x^2 - 26x\cos\theta$

$\therefore\ \cancel{x^2}$

$\therefore\ 56 = x^2 - 26x\lambda, \quad \lambda = \cos\theta, \quad -1 \le \lambda \le 1$

$\Rightarrow$ when $\lambda = -1, \quad 56 = x^2 + 26x$

$\therefore x = -14.8815, 1.88153.$

$\Rightarrow$ when $\lambda = 1, \quad 56 = x^2 - 26x$

$\therefore x = -2, 28.$

~~As x~~

$\therefore\ \cancel{1.88153 \le x \le 28}$

$\therefore\ 2 \le x \le 28$

$2 \le |a+b| \le 28$ A1 A0

04AX01

(b) $p = a + b$

~~When~~ $\cancel{|a+b|}$ ~~is a minimum,~~ $\cancel{|a+b| = 2}$

$\cancel{\therefore p = \begin{pmatrix} 12 \\ -5 \end{pmatrix} - k\begin{pmatrix} 12 \\ 5 \end{pmatrix}}$

~~as~~ $\cancel{\vec{b} \parallel \vec{a}}$

$\cancel{\therefore 2 = \sqrt{(12-12k)^2 + (-5-5k)^2}}$

unit vector $\hat{a} = \frac{1}{13}\begin{pmatrix} 12 \\ -5 \end{pmatrix}$

$= \begin{pmatrix} 12/13 \\ -5/13 \end{pmatrix}$

$\therefore p = -2 \times \begin{pmatrix} 12/13 \\ -5/13 \end{pmatrix}$ ✓ M1

$= \begin{pmatrix} -24/13 \\ 10/13 \end{pmatrix}$

$= \frac{1}{13}\begin{pmatrix} -24 \\ 10 \end{pmatrix}$

$= \frac{2}{13}\begin{pmatrix} -12 \\ 5 \end{pmatrix}$ ✓ A1

2

(c) $|q| = |b|$

$q \cdot a = 0$

$\therefore \begin{pmatrix} x \\ y \end{pmatrix} \cdot \begin{pmatrix} 12 \\ -5 \end{pmatrix} = 0$ ✓ M1

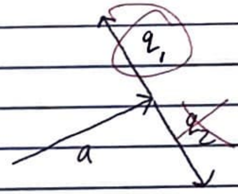
$\therefore 12x - 5y = 0 \quad \ldots (1)$

$\therefore x = 5y/12$ ✓ ← A1

$|q| = |b|$

$\therefore \sqrt{x^2+y^2} = 15$

$\therefore x^2+y^2 = 225. \quad \ldots (2)$



(1) → (2) ✓ M1

$\therefore \frac{25y^2}{144} + y^2 = 225$

$\therefore 25y^2 + 144y^2 = 32400$

$\therefore y^2 = 32400/169$

$\therefore y = \pm 180/13 \quad \ldots (3)$ ✗

~~(3) → (2):~~

$\cancel{x^2 = 225 - \left(\frac{180}{3}\right)^2}$

$\cancel{= -3375}$

(3) → (1):

$x = \pm 75/13$ ✗

Hence, $q = \pm\begin{pmatrix} 75 \\ 180/3 \end{pmatrix}$ ✗ ~~A1~~

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