Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Higher level
Paper 1

Monday 1 November 2021 (afternoon)

2 hours

Candidate session number

21	N	TZ	0	P	1	M	A	H	L

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

$\frac{65}{88} = 73\%$

Time taken : 1:37
Time remaining: 0:00

14 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

Given that $\frac{dy}{dx} = \cos\left(x - \frac{\pi}{4}\right)$ and $y = 2$ when $x = \frac{3\pi}{4}$, find y in terms of x.

$dy/dx = \cos(x - \pi/4)$

$\therefore y = \int \cos(x - \pi/4)\, \cancel{dy}\, dx$

$\therefore y = \sin(x - \pi/4) + c$

$\rightarrow 2 = \sin\left(\frac{3\pi - \pi}{4}\right) + c$

$\therefore 2 = \sin \pi/2 + c$

$\therefore c = \cancel{2}\ 1$

hence, $y = \sin(x - \pi/4) + 1$

4

2. [Maximum mark: 9]

The function f is defined by $f(x)=\dfrac{2x+4}{3-x}$, where $x \in \mathbb{R}$, $x \neq 3$.

(a) Write down the equation of

(i) the vertical asymptote of the graph of f;

(ii) the horizontal asymptote of the graph of f. [2]

(b) Find the coordinates where the graph of f crosses

(i) the x-axis;

(ii) the y-axis. [2]

ai) $3 - x = 0$

$\therefore x = 3$ {vertical asymptote}

aii) $y = \lim_{x \to \infty} \frac{2x+4}{3-x}$

$\therefore y = -2$ {horizontal asymptote}

bi) x-intercept at $y = 0$:

$2x + 4 = 0$

$\therefore$ ~~$x = -1/2$~~

$\therefore x = -2$

$\therefore$ ~~$y = 2(-2)+4$~~

$\therefore$ coordinate of x-int is $(-2, 0)$

bii) y-intercept at $x = 0$:

$y = \frac{4}{3}$

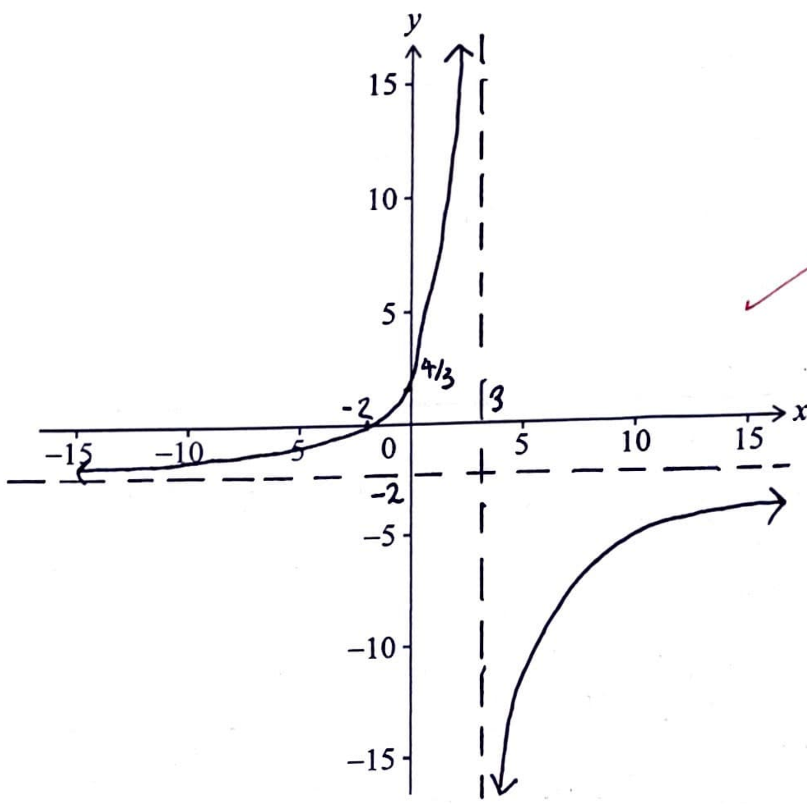
$\therefore$ coordinate of y-int is $(0, \frac{4}{3})$

4

(This question continues on the following page)

(Question 2 continued)

(c) Sketch the graph of f on the axes below. **[1]**



The function g is defined by $g(x)=\dfrac{ax+4}{3-x}$, where $x \in \mathbb{R}$, $x \neq 3$ and $a \in \mathbb{R}$.

(d) Given that $g(x)=g^{-1}(x)$, determine the value of a. **[4]**

$g^{-1}(x)$ is at $x = ay+4/3-y$

$\therefore (3-y)x = ay+4$

$\therefore 3x - yx = ay + 4$

$\therefore y(-x-a) = 4-3x$

$\therefore y = \dfrac{4-3x}{-x-a}$

$\therefore y = \dfrac{3x-4}{x+a} = g^{-1}(x)$

If $g^{-1}(x) = g(x)$, then

$(3x-4)(3-x) = (ax+4)(x+a)$

$\therefore 9x - 3x + 4x - 12 - (ax^2 + a^2x + 4x + 4a) = 0$

$\therefore -ax^2 - a^2x - 4x + x - 12 - 4a = 0$

3. [Maximum mark: 5]

Solve the equation $\log_3 \sqrt{x} = \dfrac{1}{2\log_2 3} + \log_3(4x^3)$, where $x > 0$.

$\log_3 (x^{1/2}) = \frac{1}{2\log_2 3} + \log_3(4x^3)$

$\therefore \log_3\left(\frac{\sqrt{x}}{4x^3}\right) = \frac{1}{2\log_2 3}$

$\therefore \log_3\left(\frac{1}{4} \cdot x^{\frac{1}{2}-3}\right) = \frac{1}{2\left(\frac{\log_3 3}{\log_3 2}\right)}$

$\therefore \log_3\left(\frac{1}{4} x^{-5/2}\right) = \frac{1}{2}\log_3 2$

$\cancel{\frac{1}{4} x^{2}\sqrt{x} = \sqrt{2}}$

$\therefore \cancel{x^{-5/2}}$

$\therefore x^{-5/2} = 4\sqrt{2}$

$\therefore x^{5/2} = \frac{1}{4\sqrt{2}}$

$\therefore x = \sqrt[5]{\frac{1}{16 \cdot 2}}$

$\therefore x = \sqrt[5]{\frac{1}{32}}$

$\therefore x = 1/2$

5

4. [Maximum mark: 5]

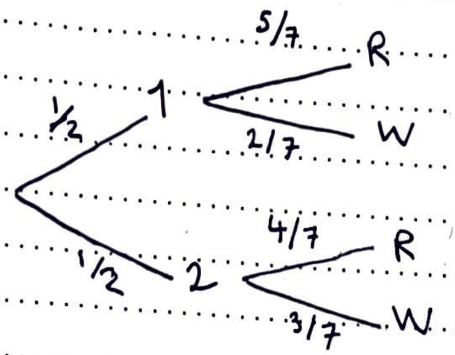
Box 1 contains 5 red balls and 2 white balls.
Box 2 contains 4 red balls and 3 white balls.

(a) A box is chosen at random and a ball is drawn. Find the probability that the ball is red. [3]

Let A be the event that "box 1 is chosen" and let R be the event that "a red ball is drawn".

(b) Determine whether events A and R are independent. [2]

a)



$\therefore P(R) = \frac{1}{2} \times \frac{5}{7} + \frac{1}{2} \times \frac{4}{7}$

$= 5/14 + 4/14$

$\therefore P(R) = 9/14$

b) $P(R) = 9/14$ $\quad P(A) = \cancel{14} \ 1/2$

$\therefore P(A) + P(R) = (9+7)/14$

$= 16/14$

$= 8/7$

$\therefore \cancel{B} \ P(A \cap R) = |1 - 8/7|$

$= |-1/7|$

$= 1/7$

$\therefore P(\text{only } A) = \frac{1}{2} - \frac{1}{7}$

$= \frac{7-2}{14}$

$= 5/14$

$\therefore$

If independent: $P(A \cap \cancel{B}) = P(A) \cancel{\times} P(R)$

$= \frac{9}{14} \times \frac{1}{2}$

$= 9/\cancel{28} = RHS$

$LHS = 1/7 \neq RHS$

4

5. [Maximum mark: 7]

The function f is defined for all $x \in \mathbb{R}$. The line with equation $y = 6x - 1$ is the tangent to the graph of f at $x = 4$.

(a) Write down the value of $f'(4)$. [1]

(b) Find $f(4)$. [1]

The function g is defined for all $x \in \mathbb{R}$ where $g(x) = x^2 - 3x$ and $h(x) = f(g(x))$.

(c) Find $h(4)$. [2]

(d) Hence find the equation of the tangent to the graph of h at $x = 4$. [3]

a) $f'(4) = 6$

b) $f(4)$ is ~~toute~~ touching $y = 6x - 1$ at $x = 4$:

$\therefore f(4) = 6(4) - 1$

$= 23$

c) $g(x) = x^2 - 3x$

$\therefore h(x) = f(x^2 - 3x)$

$\therefore h(4) = f(16 - 12)$

$= f(4)$

$= 23$

d) ~~$y = 6x + 1$~~ ~~$h'(x) = f'(2x - 3)$~~

$\therefore$ ~~$h'(4) = f'(8-3)$~~

$=$ ~~f'~~(5)

$h'(x) = f'(x^2 - 3x) \times (2x - 3)$

$\therefore h'(4) = f'(4) \times (8 - 3)$

$= 6 \times 5$

$= 30$

$\therefore m = 30$

$\rightarrow y = 30x + b$

when $x = 4$, $y = 23$

$\therefore 23 = 120 + b$

$\therefore b = -97$

$\therefore y = 30x - 97$

7

6. [Maximum mark: 7]

(a) Show that $2x-3-\dfrac{6}{x-1}=\dfrac{2x^2-5x-3}{x-1}$, $x\in\mathbb{R}$, $x\neq 1$. [2]

(b) Hence or otherwise, solve the equation $2\sin 2\theta-3-\dfrac{6}{\sin 2\theta-1}=0$ for $0\le\theta\le\pi$, $\theta\neq\dfrac{\pi}{4}$. [5]

a) $2x-3-\dfrac{6}{x-1} = \dfrac{(2x-3)(x-1)-6}{x-1}$ ✓

$= \dfrac{2x^2-2x-3x+3-6}{x-1}$

$= \dfrac{2x^2-5x-3}{x-1}$ ✓

b) $\dfrac{2x^2-5x-3}{x-1} = 0$

$\therefore 2x^2-5x-3=0$

$\therefore 2x^2-6x+x-3=0$

$\therefore 2x(x-3)+(x-3)=0$

$\therefore (2x+1)(x-3)=0$ ✓

$\rightarrow$ substitute $x=\sin 2\theta$:

$2\sin 2\theta+1=0$

$\therefore \sin 2\theta = -1/2$

$\therefore 2\theta = \pi/6$ {acute}

$\therefore \theta = \pi/12$

$\therefore 2\sin 2\theta+1=0$ DNE within domain

$\sin 2\theta = 3 \rightarrow$ does not exist as $-1<\sin 2\theta<1$

$\sin 2\theta - 3 = 0$

$\therefore \sin 2\theta = 3$ ✓

$\therefore 2\sin\theta\cos\theta = 3$

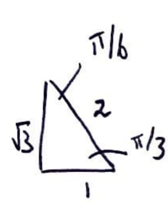
$\therefore \sin\theta\cos\theta = 3/2$

$\therefore \sin\theta\sqrt{1-\sin^2\theta} = 3/2$

$\therefore \sin^2\theta(1-\sin^2\theta)=9/4$

$\therefore \sin^2\theta-\sin^4\theta = 9/4$

$\therefore -4\sin^4\theta+4\sin^2\theta-9=0$



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7. [Maximum mark: 7]

The equation $3px^2 + 2px + 1 = p$ has two real, distinct roots.

(a) Find the possible values for p. [5]

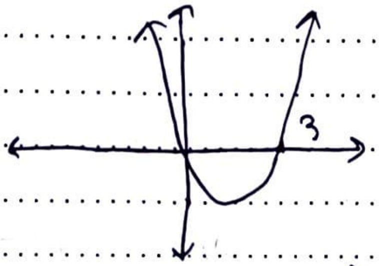
(b) Consider the case when $p = 4$. The roots of the equation can be expressed in the form $x = \dfrac{a \pm \sqrt{13}}{6}$, where $a \in \mathbb{Z}$. Find the value of a. [2]

a) $3px^2 + 2px + 1 = p$

$\therefore \Delta = (2p)^2 - 4(3p)(1) > 0$ ✗ { two distinct real roots }

$\therefore 4p^2 - 12p > 0$

$\therefore p^2 - 3p > 0$ ✓



x-int: $p^2 - 3p = 0$

$\therefore p(p-3) = 0$

$\therefore p = 0, \ p = 3$ ✓

$\Rightarrow p < 0, \quad p > 3$ ✓ ✗ ECF

b) When $p = 4$, $12x^2 + 8x + 1 = 0$

$\therefore \cancel{x = \frac{-8 \pm \sqrt{64 - 4(12)(1)}}{24}}$

$\cancel{= \frac{-8 \pm \sqrt{}}{}}$

$\therefore 12x^2 + 8x + 1 - 4 = 0$

$\therefore 12x^2 + 8x - 3 = 0$

$\therefore x = \dfrac{-8 \pm \sqrt{64 - 4(12)(-3)}}{24}$

$= \dfrac{-8 \pm \sqrt{208}}{24}$ ✓

$= \dfrac{-8 \pm 2\sqrt{13}}{24}$

$\cancel{\therefore a = 8/24} = -8/24 = -2/6$

$\therefore a = -2$ ✓

$64 - 48 = 14$

$144 + 64 = 208$

$48 \times 3 = 144$

$64 \pm \sqrt{208}$

$208 \div 8 = 26$

$8 \times 26 = 208$

$\therefore 4 \times 13 = 208$

6

8. [Maximum mark: 7]

Solve the differential equation $\frac{dy}{dx} = \frac{\ln 2x}{x^2} - \frac{2y}{x}$, $x > 0$, given that $y = 4$ at $x = \frac{1}{2}$.

Give your answer in the form $y = f(x)$.

$dy/dx = \frac{\ln 2x}{x^2} - \frac{2y}{x}$

let $y = ux$

$\therefore dy/dx = v + x\,du/dx$

$\therefore v + x\frac{du}{dx} = \frac{\ln 2x}{x^2} - \frac{2ux}{x}$

$= \frac{\ln 2x}{x^2} - 2u$

$\therefore \frac{du}{dx} = \frac{\frac{\ln 2x}{x^2} - 3u}{x}$

$= \frac{\ln 2x - 3x^2 u}{x^3}$

$\therefore u = \int \frac{\ln 2x}{x^3}dx - \int \frac{3x^2 v}{x^3}dx$

$u = \ln 2x \quad du = 2/2x = 1/x$

$= -\frac{1}{2}\ln 2x . x^{-2} - \int -\frac{1}{2}x^{-2} \cdot 1/x\,dx - \int \frac{3x^2}{x^3}dx$

$dv = x^{-3} \quad v = -\frac{1}{2}x^{-2}$

$= -\frac{\ln 2x}{x^2} + \frac{1}{2}\int \frac{1}{x^3}dx - \int \frac{1}{u}du$

let $u = x^3$

$\therefore du = 3x^2 \cancel{dx\ du}$

$du = 3x^2\,dx$

$= -\frac{\ln 2x}{x^2} + \frac{1}{2} \cdot -\frac{1}{2}x^{-2} - \ln|u| + c$

$= -\frac{\ln 2x}{x^2} - \frac{1}{4x^2} - \ln x^3 + c$

$= -\frac{4\ln 2x + 1}{4x^2} - 3\ln x + c$

$\therefore \frac{y}{x} = -\frac{4\ln 2x + 1}{4x^2} - 3\ln x + c$

$\therefore y = -\frac{4\ln 2x + 1}{4x} - 3x\ln x + c$

X

when $y = 4$, $x = 1/2$:

$4 = -\frac{4\ln(1) + 1}{2} - \frac{3}{2}\ln(1/2) + c$

$= -1/2 - 3/2 . 1/2 \ln(2) + c$

$\therefore c = 4 + 1/2 + 3/4 \ln 2$

$= \frac{8+2}{4} + \frac{3}{4}\ln 2$

$= \frac{10}{4} + \frac{3}{4}\ln 2$

$\rightarrow y = -\frac{4\ln 2x + 1}{4x} - 3x\ln x + \frac{10}{4} + \frac{3}{4}\ln 2$

2

9. [Maximum mark: 7]

Consider the expression $\dfrac{1}{\sqrt{1+ax}} - \sqrt{1-x}$ where $a \in \mathbb{Q}$, $a \neq 0$.

The binomial expansion of this expression, in ascending powers of x, as far as the term in x^2 is $4bx + bx^2$, where $b \in \mathbb{Q}$.

(a) Find the value of a and the value of b. [6]

(b) State the restriction which must be placed on x for this expansion to be valid. [1]

a) $(1+ax)^{-\frac{1}{2}} - (1-x)^{\frac{1}{2}}$

$= 1^{-1/2}\left(1 - \frac{1}{2}\left(\frac{ax}{1}\right) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2}\left(\frac{ax^2}{1}\right)\right) - 1\left(1 + \frac{1}{2}\left(\frac{-x}{1}\right) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2}\left(\frac{-x}{1}\right)^2\right)$

$= 1 - \frac{ax}{2} + \frac{\frac{3}{4}ax^2}{2} - \frac{1}{2}\left(1 - \frac{x}{2} - \frac{\frac{1}{4}}{2}(x^2)\right)$ ✗

$= ~~1 - \frac{ax}{2} + \frac{3ax^2}{8} - \frac{1}{2} + \frac{x}{4} + \frac{x^2}{16}~~$

$= 1 - \frac{ax}{2} + \frac{3ax^2}{8} - 1 + \frac{x}{2} + \frac{x^2}{8}$ ✓

$= x\left(\frac{1}{2} - \frac{a}{2}\right) + x^2\left(\frac{3a+1}{8}\right) = 4bx + bx^2$ ✓

(1) $4b = -8/2$
$\therefore b = -1$ ✓

(2) $b = \frac{3a+1}{8}$
$\therefore -8 = 3a+1$ ✓
$\therefore a = -3$

ECF

b) $|ax| < 1$
$\therefore -1 < ax < 1$ ✗
$\therefore -1 < -3x < -3$
$\therefore x < 1/3$
$x > 1$

$|x| < 1$

5

⑤

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 16]

A particle P moves along the x-axis. The velocity of P is $v\,\mathrm{m\,s}^{-1}$ at time t seconds, where $v(t) = 4 + 4t - 3t^2$ for $0 \le t \le 3$. When $t = 0$, P is at the origin O.

(a) (i) Find the value of t when P reaches its maximum velocity.

(ii) Show that the distance of P from O at this time is $\frac{88}{27}$ metres. [7]

(b) Sketch a graph of v against t, clearly showing any points of intersection with the axes. [4]

(c) Find the total distance travelled by P. [5]

11. [Maximum mark: 14]

(a) Prove by mathematical induction that $\frac{\mathrm{d}^n}{\mathrm{d}x^n}\left(x^2\mathrm{e}^x\right) = \left[x^2 + 2nx + n(n-1)\right]\mathrm{e}^x$ for $n \in \mathbb{Z}^+$. [7]

(b) Hence or otherwise, determine the Maclaurin series of $f(x) = x^2\mathrm{e}^x$ in ascending powers of x, up to and including the term in x^4. [3]

(c) Hence or otherwise, determine the value of $\lim_{x\to 0}\left[\frac{\left(x^2\mathrm{e}^x - x^2\right)^3}{x^9}\right]$. [4]

Do **not** write solutions on this page.

12. [Maximum mark: 22]

Consider the equation $(z-1)^3 = \mathrm{i}$, $z \in \mathbb{C}$. The roots of this equation are ω_1, ω_2 and ω_3, where $\mathrm{Im}(\omega_2) > 0$ and $\mathrm{Im}(\omega_3) < 0$.

(a) (i) Verify that $\omega_1 = 1 + \mathrm{e}^{\mathrm{i}\frac{\pi}{6}}$ is a root of this equation.

(ii) Find ω_2 and ω_3, expressing these in the form $a + \mathrm{e}^{\mathrm{i}\theta}$, where $a \in \mathbb{R}$ and $\theta > 0$. [6]

The roots ω_1, ω_2 and ω_3 are represented by the points A, B and C respectively on an Argand diagram.

(b) Plot the points A, B and C on an Argand diagram. [4]

(c) Find AC. [3]

Consider the equation $(z-1)^3 = \mathrm{i}z^3$, $z \in \mathbb{C}$.

(d) By using de Moivre's theorem, show that $\alpha = \dfrac{1}{1 - \mathrm{e}^{\mathrm{i}\frac{\pi}{6}}}$ is a root of this equation. [3]

(e) Determine the value of $\mathrm{Re}(\alpha)$. [6]

References:

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Candidate session number: / Numéro de session du candidat: / Número de convocatoria del alumno: 21 N TZ 0 P 1 - M A H L

Candidate name: / Nom du candidat: / Nombre del alumno: JAMIE SULLIVAN

Please write question numbers in the following format: / Veuillez numéroter les questions en utilisant la présentation suivante: / Sírvase escribir los números de las preguntas en el siguiente formato:

1 2 3 4 5 6 7 8 9 10

ai) $v(t) = 4+4t-3t^2$

$\therefore a(t) = 4-6t$

when $a(t) = 0 = 4-6t$, max velocity

$\therefore 6t = 4$

$\therefore t = 4/6$

$\therefore t = \cancel{3}\,2/3$ seconds

aii) $s(t) = 4t + \frac{4}{2}t^2 - \frac{3}{3}t^3 + c$

$= 4t + 2t^2 - t^3 + c$

when $t=0$, $s(0)=0$

$\therefore 0 = c$

$\therefore c = 0$

$8\times 4 = 56$

$8\times 9 = 80-2$

$= 72$

hence, at $t = 2/3$ seconds:

$s(2/3) = \cancel{\frac{8}{37}}\ 4(\frac{2}{3}) + 2(\frac{4}{9}) - (\frac{2}{3})^3$

$= 8/3 + 8/9 - 8/27$

$= \cancel{(56+24-8)}/27$

$= \cancel{82/27}$

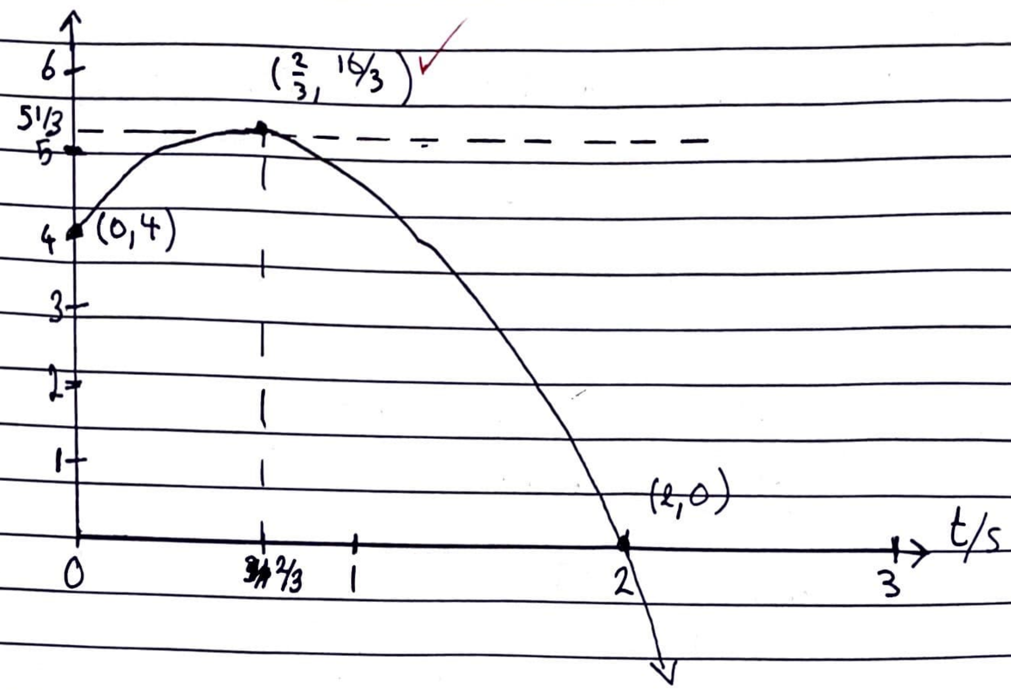
$\frac{72}{27} + \frac{24}{27} - \frac{8}{27}$

$= 88/27$ m

7

10

b)



maximum velocity $= v(2/3) = 4 + 4(2/3) - 3(2/3)^2$

$= 4 + 8/3 - 3(4/9)$

$= 4 + 8/3 - 4/3$

$= 4 + 4/3$

$= 16/3 \ ms^{-1}$

at $t = 0,\ v(0) = 4$

at $t = 3,\ v(3) = 4 + 12 - 3 \times 9$

$= 16 - 27$

$= -11$

x-int: $4 + 4t - 3t^2 = 0$

$\therefore -3t^2 + 6t - 2t + 4 = 0$

$\therefore -3t(t-2) - (t-2) = 0$

$\therefore (-3t-1)(t-2) = 0$

$\therefore t = 2,\ \cancel{t = -1/3}$

3

c) 1 $S_{total} = \int_0^3 s(t)\,dt$

$= \int_0^3 (4t + 2t^2 - t^3)\,dt$

$= \left[\frac{4}{2}t^2 + \frac{2}{3}t^3 - \frac{1}{4}t^4\right]_0^3$ X

$= \left[2t^2 + \frac{2}{3}t^3 - \frac{1}{4}t^4\right]_0^3$

$= 2(3^2) + \frac{2}{3}(3^3) - \frac{1}{4}(3^4)$

$= 2(9) + \frac{2}{3}(\cancel{27}) - \frac{1}{4}(81)$ 0

$= 18 + \cancel{2/9}\ 2\times9 - 81/4$

$= 18 + 18 - 81/4$

$= 36 - 81/4$

$= (144 - 81)/4$

$= 63/4$ m

$36 \times 4 = 144$

$144 - 81 = 63$

did not account for area under the x-axis properly

1

a) $\frac{d^n}{dx^n}(x^2e^x) = [x^2+2nx+n(n-1)]e^x$

Step 1: prove for $n=1$:

$LHS = \frac{d}{dx}(x^2e^x)$

$= x^2e^x + e^x.2x$

$= [x^2+2x]e^x$

$RHS = [x^2+2(1)x+1(1-1)]e^x$

$= [x^2+2x]e^x$

$= LHS$

$\therefore$ P_1 is true

Step 2: assume $n=k$:

$\therefore$ ~~$\frac{d^k}{dx^k}$~~ $\frac{d^k}{dx^k}(x^2e^x) = [x^2+2kx+k(k-1)]e^x$

Step 3: prove $n=k+1$:

$\frac{d^{k+1}}{dx^{k+1}} = \cancel{} [x^2+2(k+1)x+(k+1)k]e^x$

$\therefore$ $LHS = \frac{d}{dx}\left(\frac{d^k}{dx^k}\right)$

$= \frac{d}{dx}[x^2+2kx+k^2-k]e^x$

$= e^x(2x+2k) + [x^2+2kx+k^2-k]e^x$

$= (2x+2k+x^2+2kx+k^2-k)e^x$

$= (x^2+2x(k+1)+k^2+2k-k)e^x$

$= (x^2+2(k+1)x+k(k+2-1))e^x$

$= [x^2+2(k+1)x+k(k+1)]e^x$

$= LHS$

$\therefore P_{k+1}$ ~~P_k~~ is true when P_k is true

Step 4: because P_1 is true, and P_{k+1} is true whenever P_k is true, then P_n is true for all $n \in Z^+$ by mathematical induction.

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1 2 3 4 5 6 7 8 9 10

11

b) $f(x) = x^2 e^x$ ✓ $f(0) = 0$

$f'(x) = \cancel{2}[x^2 + 2x]e^x$ $f'(0) = 0$

$f''(x) = [x^2 + 4x + 2(1)]e^x$ $f''(\cancel{2}\,0) = 2e^2 \rightarrow e^0 = 1$

$f'''(x) = [x^2 + 2(3)x + 3(2)]e^x$

$= [x^2 + 6x + 6]e^x$ $f'''(\cancel{3}\,0) = 6e^3 \rightarrow e^0 = 1$

$f^{(4)}(x) = [x^2 + 2(4)x + 4(3)]e^x$

$= [x^2 + 8x + 12]e^x$ $f^{(4)}(0) = 12e^4 \rightarrow e^0 = 1$

$\therefore$ maclaurin series, $f(x) \approx f(0) + xf'(0) + \frac{x^2}{2}f''(0) + \frac{x^3}{3!}f'''(0) + \frac{x^4}{4!}f^{(4)}(0)$

$\therefore f(x) \approx 0 + x(0) + \frac{x^2}{2}(2e^2) + \frac{x^3}{6}(6e^3) + \frac{x^4}{24}(12e^4)$

$\therefore f(x) \approx x^2e^2 + x^3e^3 + \frac{1}{2}x^4e^4$ ✓ ECF

(2)

c)

maclaurin series $f(x) \approx x^2e^x + x^3e^3 + \cancel{\tfrac{1}{4}}\ \tfrac{1}{2}x^4e^4$

$\therefore \lim_{x\to 0}\left(\frac{(x^2e^x + x^3e^3 + \frac{1}{2}x^4e^4 - x^2)^3}{x^4}\right) = \lim_{x\to 0}\Big($

X

12 ai)