TZ1

Diploma Programme
Programme du diplôme
Programa del Diploma

# Mathematics: analysis and approaches
# Higher level
# Paper 3

Thursday 12 May 2022 (morning)

1 hour

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**Instructions to candidates**

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[55 marks]**.

60 min 70 min

Q1 ( 19/27 24/27) = 65.5%.

Q2 ( 17/28 25/28) = 89.1%.

19/10/22

5 pages

2222–7108

ANSWER BOOKLET
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1 1: $\frac{19}{27}$ 2: $\frac{17}{28}$ = 65.5%

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4 PAGES / PÁGINAS = w/ ext. time: 1: $\frac{24}{27}$ 2: $\frac{25}{28}$ = 89.1%

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Example Ejemplo 27 [2][7] Example Ejemplo 3 [ ][3]

1

(a)(i) $P_r(n) = \frac{(r-2)n^2-(r-4)n}{2}$

$\therefore P_3(n) = \frac{(3-2)n^2-(3-4)n}{2}$ ✓ A1

$= \frac{n^2+n}{2}$

$= \frac{n(n+1)}{2}$ ✓ A1 2

(ii) $351 = \frac{n(n+1)}{2}$ ✓ M1

$\therefore 702 = n^2+n$

$\therefore n^2+n-702=0$

$\therefore n = -27, 26$ 2

$\therefore n = 26 \quad \{n>0\}$ ✓ A1

✓ M1

(b)(i) $P_3(n)+P_3(n+1) = \frac{n(n+1)}{2} + \frac{(n+1)(n+2)}{2}$

$= \frac{n(n+1)+(n+1)(n+2)}{2}$

$= \frac{(n+1)(n+n+2)}{2}$ ✓ A1

$= \frac{(n+1)(2n+2)}{2}$ ✓ A1 3

$= (n+1)^2$.

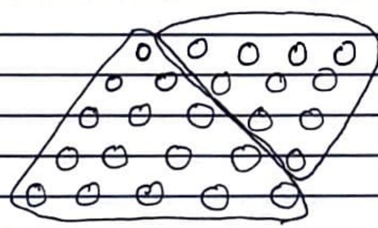
~~(b)(ii) The sum of two consecutive numbers equals the square of the second number n+1~~

~~in other words ⇒ The triangular number n+1 equals the integer number (n+1), squared minus the previous triangular number~~

~~(b)(iii) $P_3(4) + P_3(5) = 25$.~~

(b)(ii) The sum of two consecutive triangular numbers is a square number. A1 ✓ 1

(b)(iii) $P_3(4)$:



$P_3(4)$

$P_3(5)$ ✓A1

width = 5 height = 5

∴ square number. 1





(c)

$8P_3(n)+1 = 8\frac{n(n+1)}{2}+1$ ✓ A1

$= \cancel{4n(n+1)+1}$

$= 4n^2+4n+1$ ✓M1

$= 4n^2+2n+2n+1$

$= \cancel{4n(n+1)+2(n}$

$= 2n(2n+1)+(2n+1)$

$= (2n+1)(2n+1)$

$= (2n+1)^2$ ✓A1 3

As $(2n+1)$ is odd for all $n \in \mathbb{Z}^+$ then $8P_3(n)+1 = (2n+1)^2$ which is an odd number, squared

(d) $u_1 = 1$ $d = 3$ ✓A1 $(7-4=4-1=3)$

$\therefore P_5(n) = S_n$

$= \frac{n}{2}(2u_1+(n-1)d)$ ✓M1

$= \frac{n}{2}(2+(n-1)(3))$

$= \frac{n}{2}(2-3+3n)$

$= \frac{n}{2}(-1+3n)$ 3

$= \frac{3n^2}{2}-\frac{n}{2}$

$= \frac{3n^2-n}{2}$ ✓A1

$= \frac{n(3n-1)}{2}$, $n \in \mathbb{Z}^+$

(e)

$$P_3(n) = P_5(n)$$

$$\therefore \frac{n(n+1)}{2} = \frac{n(3n-1)}{2}$$

$$\therefore n(n+1) = n(3n-1)$$

$$\therefore n+1 = 3n-1$$

$$\therefore 0 = 2n-2$$

$$\therefore n = 1$$

$$\frac{k(k+1)}{2} = \frac{n(3n-1)}{2}$$ M1 1

$$\therefore k^2+k = 3n^2-n$$

$$\therefore (20^2)+20 = 3(12)^2-12$$

$$\Rightarrow P_3(20) = 210 = P_5(12) = 210$$

$$P_5(n)+P_5(n+1) = \frac{n(3n-1)}{2} + \frac{(n+1)(3(n+1)-1)}{2}$$

$$= \frac{n(3n-1) + (n+1)(3n+2)}{2}$$

$$= \frac{n(3n-1) + (n+1)(3n+2)}{2}$$

$$= \frac{3n^2 - \cancel{n} + 3n^2 + \cancel{n} + 3n + 2}{2}$$

$$= \frac{6n^2+3n+2}{2}$$

$$=$$

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Example Ejemplo 27 | 2 7 |    Example Ejemplo 3 | 3 |

| 1 |

(f) $P_r(n) = 1 + (1+r-2) + (1+2r-4) + \cdots + (1+(n-1)(r-2))$

~~Prove~~ for $n=1$ :

$\sum_{m=1}^{1} (1+(m-1)(r-2)) = 1$

$P_r(1) = \frac{(r-2)-(r-4)}{2}$

$= \frac{-2+4}{2}$

$= 1$ ✓ A1

$\therefore$ true for $n=1$.

Assume true for $n=k$.

$\therefore \sum_{m=1}^{k} (1+(m-1)(r-2)) = \frac{(r-2)k^2-(r-4)k}{2}$ ✓ M1

~~Prove~~ Consider $n=k+1$

$\therefore \sum_{m=1}^{k+1} (1+(m-1)(r-2)) = \frac{(r-2)(k+1)^2-(r-4)(k+1)}{2}$ ✓ M1

3

~~$\therefore LHS = \frac{(r-2)k^2-(r-4)k}{2} + 1+(k+1-1)(r-2)$~~

~~$= \frac{(r-2)(k^2)-(r-4)(k)+2+2k(r-2)}{2}$~~

~~$= \frac{(r-2)k^2+(r-2)(k)-(r-4)(k)+2+k(r-2)}{2}$~~

~~$= (r-2)($~~

19

full 8 marks for subsequent working in following booklet.

04AX01

2 (a)(i) $(z-1)(z^2-8z+17)=0$ $\quad z \in C$

$\boxed{4-i}$ ✓A1 1

(ii) $\mu = \frac{4+i+(4-i)}{2}$

$= \frac{8}{2}$

$= 4$ ✓A1 1

(b)(i) $f(x) = (x-1)(x^2-8x+17)$ ✓M1

$= x^3-8x^2+17x$

$\quad -x^2+8x-17$

$= x^3-9x^2+25x-17$ ✓A1

$\therefore f'(x) = 3x^2-18x+25$

at $x=4$, $f'(4) = 1 = m$ ✓A1

$\therefore y-3 = 1(x-4)$ ✓A1

$y = x-4+3$

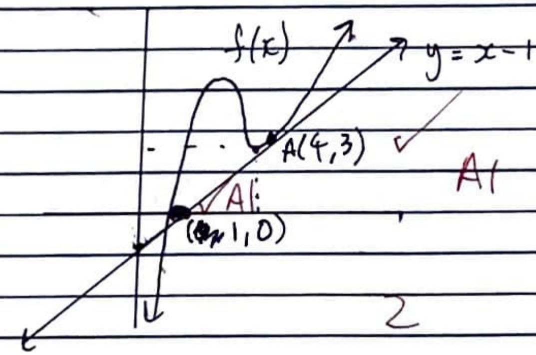
$\therefore y = x-1$ which is tangent to $f(x)$ @ $A(4,3)$

4





(c)




✓A1

2

(d)(i) $g(x) = x^3-2ax^2+a^2x+b^2x$

$\quad -rx^2+2arx-a^2r-b^2r$

$\therefore g'(x) = 3x^2-4ax+a^2+b^2-2rx+2ar$ ✓M1

$= \cancel{3x^2}$

$= x^2-2ax+a^2+b^2+2x^2-2ax-2rx+2ar$

$= x^2-2ax+a^2+b^2+2(x^2-ax-rx+ar)$

$= x^2-2ax+a^2+b^2+2(x(x-r)-a(x-r))$

$= 2(x-\cancel{a}r)(x-a)) + x^2-2ax+a^2+b^2$ ✓A1

~~(ii)~~

2

(d)(ii) ~~$g'(x) =$~~

~~m at g~~

$$\cancel{m \text{ at } g'(a) = g'(a)}$$
$$\cancel{= 2(x-r)}$$
$$\cancel{= 2(a-r)(a-a) + x^2 - 2a^2 + a^2 + b^2}$$
$$\cancel{= a^2 + b^2 - 2a^2 + x^2}$$
$$\cancel{= b^2 - a^2 + x^2}$$

$$\cancel{\therefore y - g(a) = (b^2 - a^2 + x^2)(x - a)}$$

$\therefore$

$$\cancel{\therefore \text{ at } y = 0, \quad -g(a) = (x-a)(b^2 - a^2 + x^2)}$$

$$m \text{ at } g(a) = g'(a)$$
$$= 2(a-r)(a-a) + a^2 - 2a^2 + a^2 + b^2$$
$$= b^2 \quad \checkmark \text{A1}$$

$$\therefore y - g(a) = b^2(x-a)$$
$$\therefore y = b^2 x - ab^2 + g(a)$$
$$= b^2(x-a) + g(a) \quad \checkmark \text{M1}$$

At $(r, 0)$:
$$0 = b^2(r-a) + g(a)$$
$$= b^2(r-a) + (a-r)(a^2 - 2a^2 + a^2 + b^2)$$
$$= b^2(r-a) + (a-r)(b^2) \quad \checkmark \text{M1}$$
$$= b^2(r-a+a-r) \quad \checkmark \text{A1} \checkmark \text{A1}$$
$$= 0.$$

$\checkmark$ R1.

6

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Example Ejemplo 27 | 2 7 |   Example Ejemplo 3 | 3 |

2

(e) $g'(z) = 0$

$\therefore 2(z-r)(z^2-2az+a^2+b^2) = 0$

$\therefore z = r$

$z^2-2az+a^2+b^2 = 0$

$\therefore z = \dfrac{2a \pm \sqrt{4a^2-4a^2-4b^2}}{2}$

$= \dfrac{2a \pm \sqrt{-4b^2}}{2}$

$= \dfrac{2a \pm 2bi}{2}$

$= a \pm bi$

As $\sqrt{g'(a)} = b$, R1 then the roots can be expressed as : $a \pm i\sqrt{g'(a)}$. R1 (f)(i)

roots are $a \pm \sqrt{g'(a)}\, i$

$= a \pm \sqrt{16}\ i$

$= a \pm 4i$

$0 = (x-r)($

(ii) $0 = (-2-r)(4+4a+a^2+16)$

$\therefore a^2+4a+16 = 0$

$80 = (a-r)(a^2-2a^2+a^2+16)$

$= (a-r)(16)$

$\therefore a-r = 5 \quad \ldots (1)$

$\therefore r = a-5$

$0 = (-2-r)(4+4a+a^2+16)$

$= (-2-r)(a^2+4a+20)$

$= -2a^2-8a-40-a^2r-4ar-20r$

$= (-2-r)a^2+(-8-4r)a-40-20r$

$= (2+r)a^2+(8+4r)a+40+20r$

sub (1):

$0 = (2+a-5)a^2+(8+4a-20)a+40+20(a-5)$

$= (a-3)a^2+(4a-20)a+40+20a-100$

$= a^3-3a^2+4a^2-20a+2a-60$

$\therefore a = 5.$

$\Rightarrow$ roots are $5 \pm \sqrt{16}\ i$

$= 5 \pm 4i$





(ii) $C_2: (5, 4)$

17

(f)(i) $r = -2$ ✓A1

✓M1

$g(a) = 80 = (x-r)(a^2-2a^2+a^2+16)$

$= (a+2)(16)$

$= 16a+32$

$\therefore 16a = 48$

$\therefore a = 3.$ ✓A1

$\therefore$ Roots are $z = r = -2$

$z = 3 \pm \sqrt{g'(3)}\ i$

$= 3 \pm \sqrt{16}\ i$

$= 3 \pm 4i$ ✓A1

3

(ii) ~~$C_2: (5, -4)$~~ $C_2: (3, -4)$ 1

$\square$ = extra time.

(9)(i) $g'(x) = 2(x^3 - 2ax^2 + a^2x + b^2x$
$- rx^2 + 2arx - a^2r - b^2r)$

$= 2(x^3$

$= 2x^3 - 4ax^2 + 2a^2x + 2b^2x - 2rx^2$
$+ 4arx - 2a^2r - 2b^2r$

$\therefore g''(x) = 6x^2 - 8ax + 2a^2 + 2b^2 - 4rx$
$+ 4ar$

$= 6x^2 - 8ax - 4rx + 2a^2 + 2b^2 + 4ar$

$\therefore 6x^2 - 8ax - 4rx + 2a^2 + 2b^2 + 4ar = 0$

$g'(x) = 2(x-r)(x-a) + x^2 - 2ax + a^2 + b^2$ M1
$= 2(x^2 - ax - rx + ar) + x^2 - 2ax + a^2 + b^2$
$= 2x^2 - 2ax - 2rx + 2ar + x^2 - 2ax + a^2 + b^2$
$= 3x^2 - 4ax - 2rx + 2ar + a^2 + b^2$

$\therefore g''(x) = 6x - 4a - 2r = 0$

$\therefore 6x = 4a + 2r$

$\therefore x = \frac{2a+r}{3}$

A1

$\therefore x = \frac{1}{3}(2a + r)$

$\boxed{2}$

(ii) r a

Halfway between the two. a & r.

X

r a

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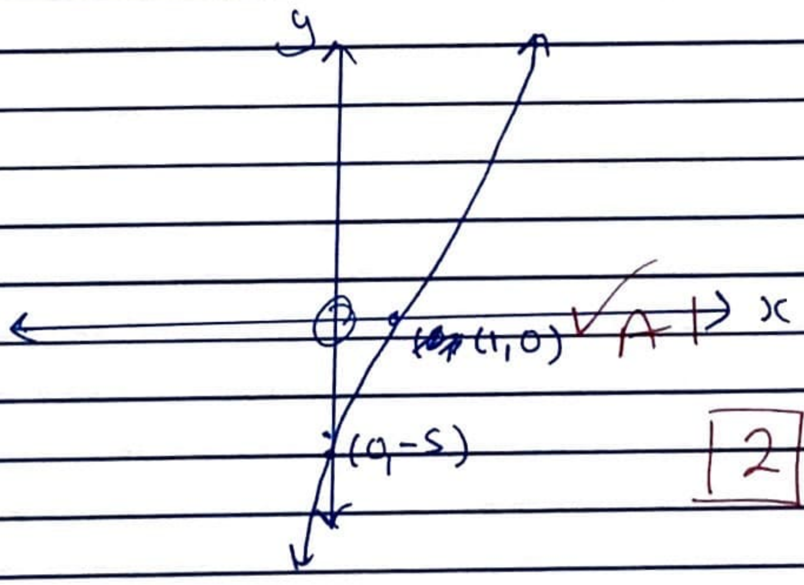
Example / Ejemplo 27 | 2 | 7 |

Example / Ejemplo 3 | | 3 |

(h)(i) $y = (x-1)(x^2-2x+1+4)$

$\therefore y = (x-1)(x^2-2x+5)$ A1




(ii) $x$ coord of P is $\frac{1}{3}(3) = 1$

$\therefore y$ coord of P is 0

$x$ coord of A is $a=1$

$y$ coord of A is $g(a)=0$

$\therefore$ P is $(1,0) =$ A $(1,0)$ A1

1



04AX01

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Example / Ejemplo 27 [27] Example / Ejemplo 3 [3]

1

(f) contd.

$$LHS = \frac{(r-2)k^2 - (r-4)k}{2} + 1 + (k+1-1)(r-2)$$

$$= \frac{(r-2)k^2 - (r-4)k + 2 + 2k(r-2)}{2}$$

$$= \frac{(r-2)k^2 - (r-4)k - (r-4) + r - 4 + 2 + 2k(r-2)}{2}$$

$$= \frac{(r-2)k^2 - (r-4)(k+1) + r - 2 + 2k(r-2)}{2}$$

$$= \frac{k^2r - 2k^2 + r - 2 + 2kr - 4k - (r-4)(k+1)}{2}$$

$$= \frac{k^2(r-2) + (r-2) + 2k(r-2) - (r-4)(k+1)}{2}$$ M1

$$= \frac{(r-2)(k^2+1+2k) - (r-4)(k+1)}{2}$$

$$= \frac{(r-2)(k+1)^2 - (r-4)(k+1)}{2}$$ A1 A1

$= RHS$

$\therefore$ Step 4: as true for $n=1$ and true for $n=k+1$ whenever $n=k$ is assumed to be true, true for all $n \in \mathbb{Z}^+$ by mathematical induction. R1