N17/5/MATHL/HP2/ENG/TZ0/XX/M

Diploma Programme
Programme du diplôme
Programa del Diploma

# Markscheme

## November 2017

## Mathematics

## Higher level

## Paper 2

17 pages

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## Instructions to Examiners

### Abbreviations

***M*** Marks awarded for attempting to use a valid **Method**; working must be seen.

***(M)*** Marks awarded for **Method**; may be implied by **correct** subsequent working.

***A*** Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

***(A)*** Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

***R*** Marks awarded for clear **Reasoning**.

***N*** Marks awarded for **correct** answers if **no** working shown.

***AG*** Answer given in the question and so no marks are awarded.

### Using the markscheme

**1 General**

Mark according to RM™ Assessor instructions and the document **“Mathematics HL: Guidance for e-marking November 2017”**. It is essential that you read this document before you start marking.
In particular, please note the following.

- Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.
- If a part is **completely correct**, (and gains all the “must be seen” marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.
- All the marks will be added and recorded by RM™ Assessor.

**2 Method and Answer/Accuracy marks**

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, for example, ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (for example, substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*, do **not** split the marks.

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- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct ***FT*** working shown, award ***FT*** marks as appropriate but do not award the final ***A1*** in that part.

  Examples

| | **Correct answer seen** | **Further working seen** | **Action** |
|---|---|---|---|
| **1.** | $8\sqrt{2}$ | $5.65685...$ *(incorrect decimal value)* | Award the final ***A1*** *(ignore the further working)* |
| **2.** | $\frac{1}{4}\sin 4x$ | $\sin x$ | Do not award the final ***A1*** |
| **3.** | $\log a - \log b$ | $\log(a-b)$ | Do not award the final ***A1*** |

**3 *N* marks**

*Award* ***N*** *marks for* ***correct*** *answers where there is* ***no*** *working*.

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

**4 Implied marks**

*Implied marks appear in* ***brackets****, for example,* ***(M1)****, and can only be awarded if* ***correct*** *work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

**5 Follow through marks**

*Follow through (****FT****) marks are awarded where an incorrect answer from one* ***part*** *of a question is used correctly in* ***subsequent*** *part(s). To award* ***FT*** *marks,* ***there must be working present*** *and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (for example, $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

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**6 Misread**

*If a candidate incorrectly copies information from the question, this is a misread (**MR**).*
*A candidate should be penalized only once for a particular misread. Use the **MR** stamp to indicate that this has been a misread. Then deduct the first of the marks to be awarded, even if this is an **M** mark, but award all others so that the candidate only loses **[1 mark]**.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (for example, $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

**7 Discretionary marks *(d)***

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

**8 Alternative methods**

*Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.*

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

**9 Alternative forms**

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

**Example**: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives:

$$f'(x) = \big(2\cos(5x-3)\big)5 \quad \big(=10\cos(5x-3)\big)$$ ***A1***

Award ***A1*** for $\big(2\cos(5x-3)\big)5$, even if $10\cos(5x-3)$ is not seen.

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## 10 Accuracy of Answers

*Candidates should **NO LONGER** be penalized for an accuracy error **(AP)**.*

*If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures. Please check work carefully for **FT**.*

## 11 Crossed out work

*If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.*

## 12 Calculators

*A GDC is required for paper 2, but calculators with symbolic manipulation features (for example, TI-89) are not allowed.*

**Calculator notation**
The Mathematics HL guide says:

*Students must always use correct mathematical notation, not calculator notation.*

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

## 13 More than one solution

*Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.*

## 14. Candidate work

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. This work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

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## Section A

**1.** let $b$ be the cost of one banana, $k$ the cost of one kiwifruit, and $m$ the cost of one melon

attempt to set up three linear equations ***(M1)***

$2b+3k+4m=658$

$5b+2k+8m=1232$

$5b+4k=300$ ***(A1)***

attempt to solve three simultaneous equations ***(M1)***

$b=36,\ k=30,\ m=124$

banana costs (\$)0.36, kiwifruit costs (\$)0.30, melon costs (\$)1.24 ***A1***

***[4 marks]***

**2.** (a) $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

$\Rightarrow 0.75 = \dfrac{0.6}{\mathrm{P}(B)}$ ***(M1)***

$\Rightarrow \mathrm{P}(B)\left(=\dfrac{0.6}{0.75}\right)=0.8$ ***A1***

***[2 marks]***

(b) $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\Rightarrow 0.95 = \mathrm{P}(A)+0.8-0.6$ ***(M1)***

$\Rightarrow \mathrm{P}(A) = 0.75$ ***A1***

***[2 marks]***

(c) **METHOD 1**

$\mathrm{P}(A' \mid B) = \dfrac{\mathrm{P}(A' \cap B)}{\mathrm{P}(B)} = \dfrac{0.2}{0.8} = 0.25$ ***A1***

$\mathrm{P}(A' \mid B) = \mathrm{P}(A')$ ***R1***

hence $A'$ and $B$ are independent ***AG***

| | |
|---|---|
| **Note:** | If there is evidence that the student has calculated $\mathrm{P}(A' \cap B) = 0.2$ by assuming independence in the first place, award ***A0R0***. |

*continued…*

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*Question 2 continued*

**METHOD 2**

**EITHER**

$\mathrm{P}(A) = \mathrm{P}(A|B)$ ***A1***

**OR**

$\mathrm{P}(A) \times \mathrm{P}(B) = 0.75 \times 0.80 = 0.6 = \mathrm{P}(A \cap B)$ ***A1***

**THEN**

$A$ and $B$ are independent ***R1***

hence $A'$ and $B$ are independent ***AG***

**METHOD 3**

$\mathrm{P}(A') \times \mathrm{P}(B) = 0.25 \times 0.80 = 0.2$ ***A1***

$\mathrm{P}(A') \times \mathrm{P}(B) = \mathrm{P}(A' \cap B)$ ***R1***

hence $A'$ and $B$ are independent ***AG***

***[2 marks]***

***Total [6 marks]***

**3.** **METHOD 1**

area = (four sector areas radius 9) + (four sector areas radius 3) ***(M1)***

$= 4\left(\frac{1}{2}9^2\frac{\pi}{9}\right) + 4\left(\frac{1}{2}3^2\frac{7\pi}{18}\right)$ ***(A1)(A1)***

$= 18\pi + 7\pi$

$= 25\pi\left(= 78.5\text{cm}^2\right)$ ***A1***

**METHOD 2**

area =

(area of circle radius 3) + (four sector areas radius 9) – (four sector areas radius 3) ***(M1)***

$\pi 3^2 + 4\left(\frac{1}{2}9^2\frac{\pi}{9}\right) - 4\left(\frac{1}{2}3^2\frac{\pi}{9}\right)$ ***(A1)(A1)***

**Note:** Award ***A1*** for the second term and ***A1*** for the third term.

$= 9\pi + 18\pi - 2\pi$

$= 25\pi\left(= 78.5\text{cm}^2\right)$ ***A1***

**Note:** Accept working in degrees.

***[4 marks]***

**4.** let $X$ be the random variable "amount of caffeine content in coffee"

$\mathrm{P}(X > 120) = 0.2$, $\mathrm{P}(X > 110) = 0.6$ ***(M1)***

$(\Rightarrow \mathrm{P}(X < 120) = 0.8$, $\mathrm{P}(X < 110) = 0.4)$

**Note:** Award ***M1*** for at least one correct probability statement.

$$\frac{120-\mu}{\sigma} = 0.84162\ldots, \ \frac{110-\mu}{\sigma} = -0.253347\ldots$$ ***(M1)(A1)(A1)***

**Note:** Award ***M1*** for attempt to find at least one appropriate $z$-value.

$120 - \mu = 0.84162\sigma$, $110 - \mu = -0.253347\sigma$

attempt to solve simultaneous equations ***(M1)***

$\mu = 112$, $\sigma = 9.13$ ***A1***

***[6 marks]***

**5.** attempt to use tan, or sine rule, in triangle BXN or BXS ***(M1)***

$$\mathrm{NX} = 80\tan 55^{\circ}\left(=\frac{80}{\tan 35^{\circ}} = 114.25\right)$$ ***(A1)***

$$\mathrm{SX} = 80\tan 65^{\circ}\left(=\frac{80}{\tan 25^{\circ}} = 171.56\right)$$ ***(A1)***

Attempt to use cosine rule ***M1***

$\mathrm{SN}^2 = 171.56^2 + 114.25^2 - 2 \times 171.56 \times 114.25\cos 70^{\circ}$ ***(A1)***

$\mathrm{SN} = 171(\mathrm{m})$ ***A1***

**Note:** Award final ***A1*** only if the correct answer has been given to 3 significant figures.

***[6 marks]***

**6.** (a) let $X$ be the number of bananas eaten in one day

$X \sim \mathrm{Po}(0.2)$

$\mathrm{P}(X \geq 1) = 1 - \mathrm{P}(X = 0)$ ***(M1)***

$= 0.181\left(=1-\mathrm{e}^{-0.2}\right)$ ***A1***

***[2 marks]***

(b) **EITHER**

let $Y$ be the number of bananas eaten in one week

$Y \sim \mathrm{Po}(1.4)$ ***(A1)***

$\mathrm{P}(Y = 0) = 0.246596\ldots\left(=\mathrm{e}^{-1.4}\right)$ ***(A1)***

**OR**

let $Z$ be the number of days in one week at least one banana is eaten

$Z \sim \mathrm{B}(7, 0.181\ldots)$ ***(A1)***

$\mathrm{P}(Z = 0) = 0.246596\ldots$ ***(A1)***

*continued…*

*Question 6 continued*

**THEN**

$52 \times 0.246596\ldots$ ***(M1)***

$= 12.8\left(= 52\mathrm{e}^{-1.4}\right)$ ***A1***

***[4 marks]***

***Total [6 marks]***

**7. METHOD 1**

let roots be $\alpha$ and $3\alpha$ ***(M1)***

sum of roots $(4\alpha) = \frac{8}{7}$ ***M1***

$\Rightarrow \alpha = \frac{2}{7}$ ***A1***

**EITHER**

product of roots $\left(3\alpha^2\right) = \frac{p}{7}$ ***M1***

$p = 21\alpha^2 = 21 \times \frac{4}{49}$

**OR**

$7\left(\frac{2}{7}\right)^2 - 8\left(\frac{2}{7}\right) + p = 0$ ***M1***

$\frac{4}{7} - \frac{16}{7} + p = 0$

**THEN**

$\Rightarrow p = \frac{12}{7}(=1.71)$ ***A1***

**METHOD 2**

$x = \frac{8 \pm \sqrt{64-28p}}{14}$ ***(M1)***

$\frac{8+\sqrt{64-28p}}{14} = 3\left(\frac{8-\sqrt{64-28p}}{14}\right)$ ***M1A1***

$8+\sqrt{64-28p} = 24 - 3\sqrt{64-28p} \Rightarrow \sqrt{64-28p} = 4$ ***(M1)***

$p = \frac{12}{7}(=1.71)$ ***A1***

***[5 marks]***

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**8.** **EITHER**

$x^2 = 2\sec\theta$

$2x\dfrac{\mathrm{d}x}{\mathrm{d}\theta} = 2\sec\theta\tan\theta$ ***M1A1***

$\displaystyle\int \frac{\mathrm{d}x}{x\sqrt{x^4-4}}$

$\displaystyle = \int \frac{\sec\theta\tan\theta\,\mathrm{d}\theta}{2\sec\theta\sqrt{4\sec^2\theta-4}}$ ***M1A1***

**OR**

$x = \sqrt{2}\left(\sec\theta\right)^{\frac{1}{2}}\left(=\sqrt{2}\left(\cos\theta\right)^{-\frac{1}{2}}\right)$

$\dfrac{\mathrm{d}x}{\mathrm{d}\theta} = \dfrac{\sqrt{2}}{2}\left(\sec\theta\right)^{\frac{1}{2}}\tan\theta\left(=\dfrac{\sqrt{2}}{2}\left(\cos\theta\right)^{-\frac{3}{2}}\sin\theta\right)$ ***M1A1***

$\displaystyle\int \frac{\mathrm{d}x}{x\sqrt{x^4-4}}$

$\displaystyle = \int \frac{\sqrt{2}\left(\sec\theta\right)^{\frac{1}{2}}\tan\theta\,\mathrm{d}\theta}{2\sqrt{2}\left(\sec\theta\right)^{\frac{1}{2}}\sqrt{4\sec^2\theta-4}}\left(=\int \frac{\sqrt{2}\left(\cos\theta\right)^{-\frac{3}{2}}\sin\theta\,\mathrm{d}\theta}{2\sqrt{2}\left(\cos\theta\right)^{-\frac{1}{2}}\sqrt{4\sec^2\theta-4}}\right)$ ***M1A1***

**THEN**

$\displaystyle = \frac{1}{2}\int \frac{\tan\theta\,\mathrm{d}\theta}{2\tan\theta}$ ***(M1)***

$\displaystyle = \frac{1}{4}\int \mathrm{d}\theta$

$= \dfrac{\theta}{4} + c$ ***A1***

$x^2 = 2\sec\theta \Rightarrow \cos\theta = \dfrac{2}{x^2}$ ***M1***

| **Note:** This ***M1*** may be seen anywhere, including a sketch of an appropriate triangle. |
|---|

so $\dfrac{\theta}{4} + c = \dfrac{1}{4}\arccos\left(\dfrac{2}{x^2}\right) + c$ ***AG***

***[7 marks]***

**9.** (a) $12!(=479001600)$ ***A1***

***[1 mark]***

(b) **METHOD 1**

$8 \times 2 = 16$ ways of sitting Helen and Nicky, $10!$ ways of sitting everyone else ***(A1)***

$16 \times 10!$

$= 58060800$ ***A1***

**METHOD 2**

$8 \times 1 \times 10!(=29030400)$ ways if Helen sits in the front or back row

$4 \times 2 \times 10!(=29030400)$ ways if Helen sits in the middle row ***(A1)***

| **Note:** Award ***A1*** for one correct value. |
|---|

$2 \times 29030400$

$= 58060800$ ***A1***

***[2 marks]***

(c) **METHOD 1**

$9 \times 2 \times 10!(=65318400)$ ways if Helen and Nicky sit next to each other ***(A1)***

attempt to subtract from total number of ways ***(M1)***

$12! - 9 \times 2 \times 10!$

$= 413683200$ ***A1***

**METHOD 2**

$6 \times 10 \times 10!(=217728000)$ ways if Helen sits in column 1 or 4 ***(A1)***

$6 \times 9 \times 10!(=195955200)$ ways if Helen sits in column 2 or 3 ***(A1)***

$217728000 + 195955200$

$= 413683200$ ***A1***

***[3 marks]***

***Total [6 marks]***

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## Section B

**10.** (a) (i) attempt to use quotient rule or product rule **_M1_**

$$f'(x) = \frac{\sin x\left(\frac{1}{2}x^{-\frac{1}{2}}\right) - \sqrt{x}\cos x}{\sin^2 x}\left(= \frac{1}{2\sqrt{x}\sin x} - \frac{\sqrt{x}\cos x}{\sin^2 x}\right)$$ **_A1A1_**

> **Note:** Award **_A1_** for $\frac{1}{2\sqrt{x}\sin x}$ or equivalent and **_A1_** for $-\frac{\sqrt{x}\cos x}{\sin^2 x}$ or equivalent.

setting $f'(x) = 0$ **_M1_**

$$\frac{\sin x}{2\sqrt{x}} - \sqrt{x}\cos x = 0$$

$\frac{\sin x}{2\sqrt{x}} = \sqrt{x}\cos x$ or equivalent **_A1_**

$\tan x = 2x$ **_AG_**

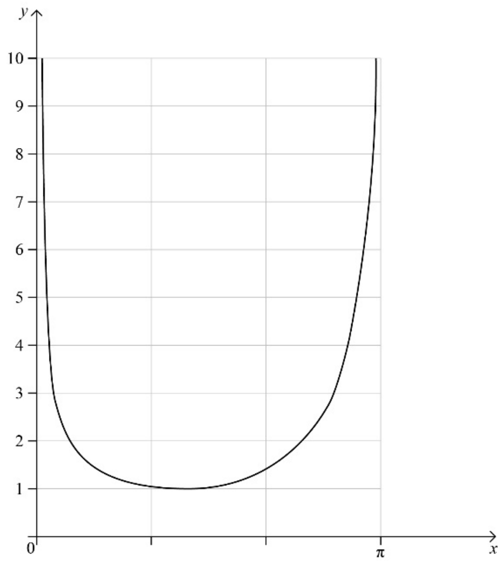
(ii) $x = 1.17$

$0 < x \le 1.17$ **_A1A1_**

> **Note:** Award **_A1_** for $0 < x$ and **_A1_** for $x \le 1.17$. Accept $x < 1.17$.

***[7 marks]***

(b)




concave up curve over correct domain with one minimum point above the *x*-axis. **_A1_**

approaches $x = 0$ asymptotically **_A1_**

approaches $x = \pi$ asymptotically **_A1_**

> **Note:** For the final **_A1_** an asymptote must be seen, and $\pi$ must be seen on the x-axis or in an equation.

***[3 marks]***

*continued…*

*Question 10 continued*

(c) $f'(x)\left(=\dfrac{\sin x\left(\frac{1}{2}x^{-\frac{1}{2}}\right)-\sqrt{x}\cos x}{\sin^2 x}\right)=1$ ***(A1)***

attempt to solve for $x$ ***(M1)***

$x=1.96$ ***A1***

$y=f(1.96...)$

$=1.51$ ***A1***

***[4 marks]***

(d) $V=\pi\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\dfrac{x\,\mathrm{d}x}{\sin^2 x}$ ***(M1)(A1)***

**Note:** ***M1*** is for an integral of the correct squared function (with or without limits and/or $\pi$).

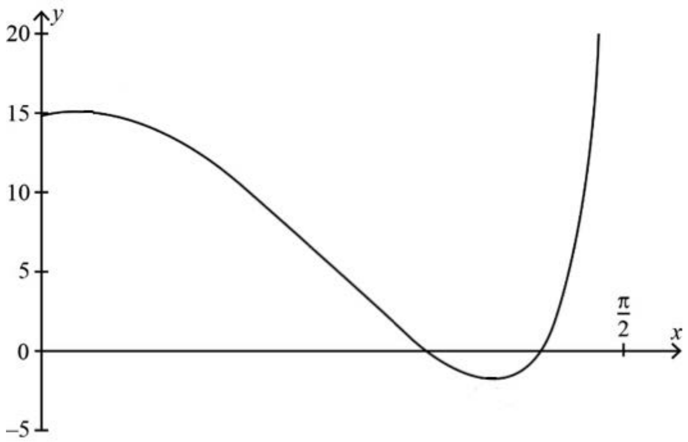
$=2.68(=0.852\pi)$ ***A1***

***[3 marks]***

***Total [17 marks]***

**11.** (a) (i) $f'(x)=4\sin x\cos x+14\cos 2x+\sec^2 x$ (or equivalent) ***(M1)A1***

(ii)




***A1A1A1A1***

**Note:** Award ***A1*** for correct behaviour at $x=0$, ***A1*** for correct domain and correct behaviour for $x\to\dfrac{\pi}{2}$, ***A1*** for two clear intersections with $x$-axis and minimum point, ***A1*** for clear maximum point.

*continued…*

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*Question 11 continued*

(iii) $x = 0.0736$ ***A1***
$x = 1.13$ ***A1***

***[8 marks]***

(b) (i) attempt to write $\sin x$ in terms of $u$ only ***(M1)***

$\sin x = \dfrac{u}{\sqrt{1+u^2}}$ ***A1***

(ii) $\cos x = \dfrac{1}{\sqrt{1+u^2}}$ ***(A1)***

attempt to use $\sin 2x = 2\sin x\cos x \left(= 2\dfrac{u}{\sqrt{1+u^2}}\dfrac{1}{\sqrt{1+u^2}}\right)$ ***(M1)***

$\sin 2x = \dfrac{2u}{1+u^2}$ ***A1***

(iii) $2\sin^2 x + 7\sin 2x + \tan x - 9 = 0$

$\dfrac{2u^2}{1+u^2} + \dfrac{14u}{1+u^2} + u - 9 \ (= 0)$ ***M1***

$\dfrac{2u^2 + 14u + u\left(1+u^2\right) - 9\left(1+u^2\right)}{1+u^2} = 0$ (or equivalent) ***A1***

$u^3 - 7u^2 + 15u - 9 = 0$ ***AG***

***[7 marks]***

(c) $u = 1$ or $u = 3$ ***(M1)***
$x = \arctan(1)$ ***A1***
$x = \arctan(3)$ ***A1***

| **Note:** Only accept answers given the required form. |
|---|

***[3 marks]***

***Total [18 marks]***

**12.** (a) $150000 \times 1.035^{20}$ ***(M1)(A1)***

$= \$298468$ ***A1***

**Note:** Only accept answers to the nearest dollar. Accept \$298469.

***[3 marks]***

(b) attempt to look for a pattern by considering 1 year, 2 years *etc* ***(M1)***
recognising a geometric series with first term $P$ and common ratio 1.02 ***(M1)***

**EITHER**

$P + 1.02P + \ldots + 1.02^{19}P \left(= P\left(1 + 1.02 + \ldots + 1.02^{19}\right)\right)$ ***A1***

**OR**

explicitly identify $u_1 = P$, $r = 1.02$ and $n = 20$ (may be seen as $S_{20}$). ***A1***

**THEN**

$$S_{20} = \frac{\left(1.02^{20} - 1\right)P}{\left(1.02 - 1\right)}$$ ***AG***

***[3 marks]***

(c) $24.297\ldots P = 298468$ ***(M1)(A1)***
$P = 12284$ ***A1***

**Note:** Accept answers which round to 12284.

***[3 marks]***

(d) (i) **METHOD 1**

$Q\left(1.028^n\right) = 5000\left(1 + 1.028 + 1.028^2 + 1.028^3 + \ldots + 1.028^{n-1}\right)$ ***M1A1***

$$Q = \frac{5000\left(1 + 1.028 + 1.028^2 + 1.028^3 + \ldots + 1.028^{n-1}\right)}{1.028^n}$$ ***A1***

$$= \frac{5000}{1.028} + \frac{5000}{1.028^2} + \ldots + \frac{5000}{1.028^n}$$ ***AG***

*continued…*

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*Question 12 continued*

**METHOD 2**

the initial value of the first withdrawal is $\frac{5000}{1.028}$ ***A1***

the initial value of the second withdrawal is $\frac{5000}{1.028^2}$ ***R1***

the investment required for these two withdrawals is $\frac{5000}{1.028}+\frac{5000}{1.028^2}$ ***R1***

$Q=\frac{5000}{1.028}+\frac{5000}{1.028^2}+\ldots+\frac{5000}{1.028^n}$ ***AG***

(ii) sum to infinity is $\dfrac{\frac{5000}{1.028}}{1-\frac{1}{1.028}}$ ***(M1)(A1)***

$=178571.428\ldots$

so minimum amount is $\$178572$ ***A1***

| **Note:** Accept answers which round to $\$178571$ or $\$178572$. |
|---|

***[6 marks]***

***Total [15 marks]***