Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 1

Monday 13 November 2017 (afternoon)

Candidate session number

17	N	TZ	0	P	1	M	A	H	L

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.

$\frac{78}{100} = 78\%$ 28/9/22

14 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Solve the equation $\log_2(x+3)+\log_2(x-3)=4$.

$\log_2((x+3)(x-3)) = \log_2 2^4$

$\therefore (x+3)(x-3) = 16$

$\therefore x^2 - 9 - 16 = 0$

$\therefore x^2 - 25 = 0$

$\therefore (x+5)(x-5) = 0$

$\therefore x = 5, \quad x = -5$

$x = 5$.

4

2. [Maximum mark: 6]

The points A and B are given by $A(0, 3, -6)$ and $B(6, -5, 11)$.

The plane Π is defined by the equation $4x - 3y + 2z = 20$.

(a) Find a vector equation of the line L passing through the points A and B. [3]

(b) Find the coordinates of the point of intersection of the line L with the plane Π. [3]

(a) $A(0,3,-6)$ $B(6,-5,11)$

$\therefore \overrightarrow{AB} = \begin{pmatrix} 6-0 \\ -5-3 \\ 11+6 \end{pmatrix} = \begin{pmatrix} 6 \\ -8 \\ 17 \end{pmatrix}$

$\therefore$ $L: \begin{pmatrix} 0 \\ 3 \\ -6 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ -8 \\ 17 \end{pmatrix}$

3

(b) $L: \; x = 6\lambda \quad y = 3 - 8\lambda \quad z = -6 + 17\lambda$

$\therefore 4(6\lambda) - 3(3-8\lambda) + 2(-6+17\lambda) = 20$

$\therefore 24\lambda - 9 + 24\lambda - 12 + 34\lambda = 20$

$\therefore 82\lambda = 41$

$\therefore \lambda = 1/2$

$48 + 34 = 82$

$\therefore$ Point of intersection:

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ -6 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 6 \\ -8 \\ 17 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ 5/2 \end{pmatrix}$

$-6 + \frac{17}{2} = \frac{-12+17}{2} = 5/2$

$\therefore$ Point of intersection is $(3, -1, \frac{5}{2})$

3

6

3. [Maximum mark: 6]

Consider the polynomial $q(x) = 3x^3 - 11x^2 + kx + 8$.

(a) Given that $q(x)$ has a factor $(x - 4)$, find the value of k. [3]

(b) Hence or otherwise, factorize $q(x)$ as a product of linear factors. [3]

(a) $q(x) = 3x^3 - 11x^2 + kx + 8$

$q(4) = 3(4^3) - 11(4^2) + 4k + 8 = 0$

$\therefore 3(64) - 11(16) + 4k + 8 = 0$

$\therefore 4(3\cdot 16 - 11\cdot 4 + k + 2) = 0$

$\therefore 48 - 44 + 2 + k = 0$

$\therefore k = -6$

3

(b) $q(x) = 3x^3 - 11x^2 - 6x + 8$

$= (x-4)(ax^2 + bx + c)$

$= ax^3 + bx^2 + cx - 4ax^2 - 4bx - 4c$

$= ax^3 + (b-4a)x^2 + (c-4b)x - 4c$

$\therefore$ Equating coefficients: $\boxed{a = 3}$

$-4c = 8 \rightarrow \boxed{c = -2}$

$c - 4b = -6$

$\therefore 4b = -2 + 6$

$\therefore 4b = 4$

$\therefore \boxed{b = 1}$

Hence, $q(x) = (x-4)(3x^2 + x - 2)$

$= \cancel{(x-4)(x+3)(3x-2)}$

$= (x-4)(3x^2 - 2x + 3x - 2)$

$= (x-4)(3x-2)(x+1)$

3

6

4. [Maximum mark: 4]

Find the coefficient of x^8 in the expansion of $\left(x^2 - \dfrac{2}{x}\right)^7$.

~~$\left(x^2 - \frac{2}{x}\right)^7 = x^{14} + 7x^{12}\left(\frac{2}{x}\right) + \binom{7}{2}x^{10}\left(\frac{4}{x^2}\right) + \dots$~~

~~Hence, for x^7, $x^{2(7-n)}\left(\frac{1}{x^n}\right) = x^7$~~

~~$\therefore x^{14-2n-n} = x^7$~~

~~$\therefore 14 - 3n = 7$~~

~~$\therefore 3n = 7$~~

~~$\therefore n =$~~

$$T_{r+1} = \binom{n}{r}(x^2)^{n-r}\left(-\frac{2}{x}\right)^r$$ ✓

$$= \binom{7}{r}(x^2)^{7-r}\left(-\frac{2}{x}\right)^r = ax^8$$

$\therefore$ Finding r: $(x^2)^{7-r}\left(\frac{1}{x}\right)^r = x^8$

$\therefore (x^{14-2r})(x^{-r}) = x^8$

$\therefore x^{14-3r} = x^8$

$\therefore 3r = 6$

$\therefore r = 2$ ✓

$$\therefore \binom{7}{3}(x^2)^5\left(-\frac{2}{x}\right)^2 = \frac{7!}{3!\,4!}x^{10} \times \frac{4}{x^2}$$ ✗

$$= \frac{7\times 6\times 5\times 4}{3\times 2} \times x^8$$

$$= 7\times 5\times 4\times x^8$$

$\therefore$ coefficient $= 140$ ✓

ECF

3

5. [Maximum mark: 5]

A particle moves in a straight line such that at time t seconds $(t \geq 0)$, its velocity v, in ms^{-1}, is given by $v = 10te^{-2t}$. Find the exact distance travelled by the particle in the first half-second.

$V = 10te^{-2t}$, where $V = \frac{ds}{dt}$

$\therefore \frac{ds}{dt} = 10te^{-2t}$

$\therefore s = 10\int te^{-2t}\,dt$ $\left[\begin{array}{ll} u = t & du = 1\,dt \\ dv = e^{-2t} & v = -\frac{1}{2}e^{-2t} \end{array}\right]$

$= -\frac{1}{2}te^{-2t} - \int -\frac{1}{2}e^{-2t}\,dt$

$= -\frac{1}{2}te^{-2t} + \frac{1}{2}(-\frac{1}{2})e^{-2t} + C$

$= -\frac{1}{2}te^{-2t} - \frac{1}{4}e^{-2t} + C$

$V = 10te^{-2t}$

$\therefore S = 10\int_0^{1/2} te^{-2t}\,dt$

$= 10\left(\left[-\frac{1}{2}te^{-2t}\right]_0^{1/2} - \int_0^{1/2} -\frac{1}{2}e^{-2t}\,dt\right)$

$= 10\left(\left(-\frac{1}{2}(\frac{1}{2})e^{-1}\right) - 0 + \frac{1}{2}\left[-\frac{1}{2}e^{-2t}\right]_0^{1/2}\right)$

$= -\frac{10}{4e} + 5\left(-\frac{1}{2}e^{-1} - \frac{1}{2}e^{0}\right)$ ECF

$= -\frac{10}{4e} - \frac{5}{2e} - \frac{5}{2}$ Metres

$= -\frac{20}{4e} - \frac{5}{2}$

$= -\frac{5}{e} - \frac{5}{2}$ Metres

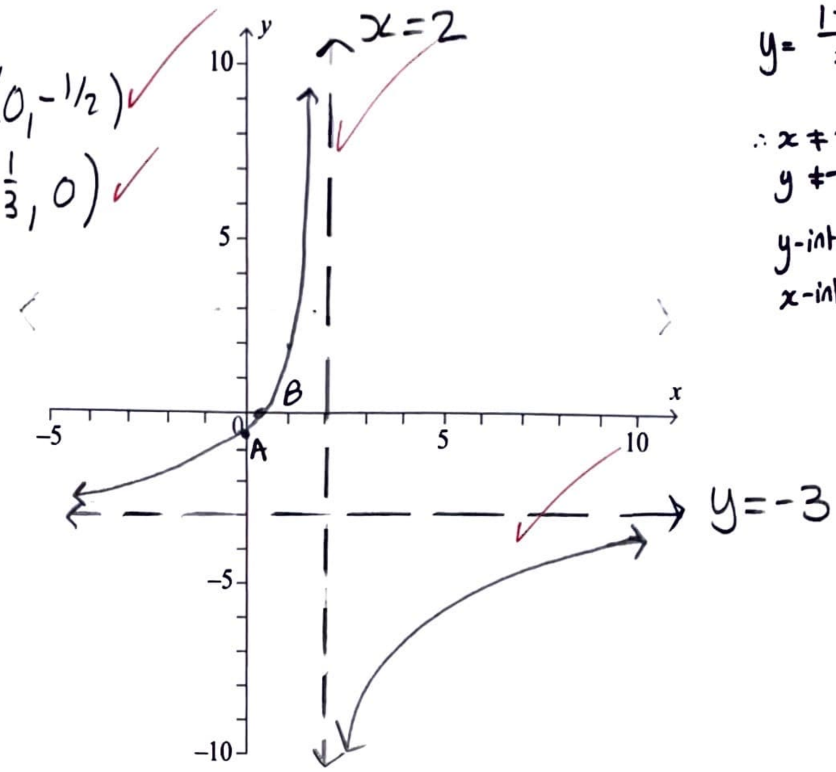
4

6. [Maximum mark: 9]

(a) Sketch the graph of $y = \dfrac{1-3x}{x-2}$, showing clearly any asymptotes and stating the coordinates of any points of intersection with the axes. [4]

A $(0, -\tfrac{1}{2})$

B $(\tfrac{1}{3}, 0)$



$y = \dfrac{1-3x}{x-2}$

$\therefore x \neq 2$

$y \neq -3$

y-int : $-\tfrac{1}{2}$

x-int : $x = \tfrac{1}{3}$

4

(b) Hence or otherwise, solve the inequality $\left|\dfrac{1-3x}{x-2}\right| < 2$. [5]

(b) $\left|\dfrac{1-3x}{x-2}\right| < 2$

~~Step 1: for $x < 2$:~~

~~$\dfrac{1-3x}{x-2} < 2$~~

~~$\therefore 1-3x > 2x-4$~~

~~$\therefore 5 > 5x$~~

~~$\therefore x < 5$~~

for $\tfrac{1}{3} < x < 2$:

$\dfrac{1-3x}{x-2} < 2$

$\therefore 1-3x > 2x-4$

$\therefore -5x > -5$

$\therefore x < 1$

for $x < \tfrac{1}{3}$

$\dfrac{1-3x}{x-2} > -2$

$\therefore 1-3x < -2x+4$

$\therefore -x < 3$

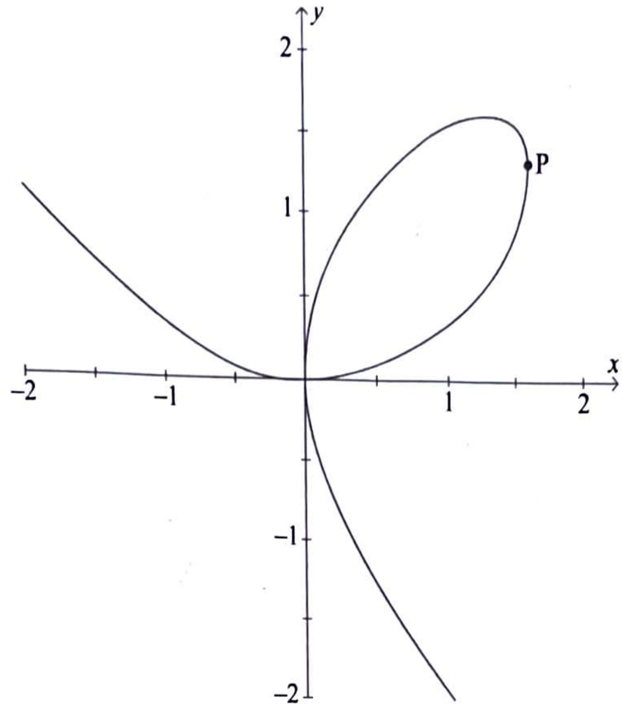
$\therefore x > -3$

$\Rightarrow -3 < x < \cancel{4/3}\ 1$ 5

9

7. [Maximum mark: 8]

The folium of Descartes is a curve defined by the equation $x^3 + y^3 - 3xy = 0$, shown in the following diagram.



Determine the exact coordinates of the point P on the curve where the tangent line is parallel to the y-axis.

$x^3 + y^3 - 3xy = 0$

$\therefore 3x^2 + 3y^2 \, dy/dx - 3(x \, dy/dx + y) = 0$

$\therefore 3x^2 + 3y^2 \, dy/dx - 3x \, dy/dx - 3y = 0$

$\therefore dy/dx \, (3y^2 - 3x) - 3y + 3x^2 = 0$

$\therefore \frac{dy}{dx} = \frac{3y - 3x^2}{3y^2 - 3x}$

When tangent is vertical, $\frac{\text{rise}}{\text{run}}$ = undefined.

$\therefore 3y^2 - 3x = 0$

$\therefore x = y^2 \quad \ldots (1)$

Substitute (1) into original equation

$\therefore x^3 + y^3 - 3xy = 0$, $x = y^2$

$\therefore (y^2)^3 + y^3 - 3y^2y = 0$

$\therefore y^6 + y^3 - 3y^3 = 0$

$\therefore y^6 - 2y^3 = 0$

$\therefore y^3(y^3 - 2) = 0$

$\cancel{\therefore y^3 = 0 \quad y = 0}$ OR $y^2 - 2 = 0$

$x = 0$ $\therefore y = \sqrt[3]{2}$

$\therefore x^3 + (\sqrt{2})^3 - 3\sqrt{2}x = 0$

$\therefore x^3 - 3\sqrt{2}x + 2\sqrt{2} = 0$

$\cancel{\therefore x^3 + \sqrt[3]{2}x + \sqrt[3]{2}}$

$\cancel{\therefore y^3 + (\sqrt[3]{2})^3 - 3\sqrt[3]{2}y = 0}$

$\cancel{\therefore y^3 - 3\sqrt[3]{2}y + 2 = 0}$

$\therefore x = (\sqrt[3]{2})^2$

$= \sqrt[3]{4}$

$\therefore$ coordinate $(\sqrt[3]{4}, \sqrt[3]{2})$

6

8. [Maximum mark: 7]

Determine the roots of the equation $(z+2\mathrm{i})^3 = 216\mathrm{i}$, $z \in \mathbb{C}$, giving the answers in the form $z = a\sqrt{3} + b\mathrm{i}$ where $a, b \in \mathbb{Z}$.

$(z+2i)^3 = 216i \quad , z \in \mathbb{C}$

$\therefore (z+2i)^3 = 216 \operatorname{cis} \pi/2$

$\therefore z+2i = \sqrt[3]{216} \operatorname{cis}\left(\frac{\pi+2k\pi}{6}\right)$

$\therefore z+2i = \sqrt[3]{8\times 27} \operatorname{cis}\left(\frac{\pi+2k\pi}{6}\right)$

$= 2\times 3 \operatorname{cis}\left(\frac{\pi+2k\pi}{6}\right)$

$= 6\operatorname{cis}\left(\frac{\pi+2k\pi}{6}\right)$

$\therefore z+2i = 6\operatorname{cis}(-\pi/6), \quad 6\operatorname{cis}\pi/6, \quad 6\operatorname{cis}(\pi/2)$

$= 6\cos\pi/6 - 6\sin\pi/6\, i, \quad 6\cos\pi/6 + 6\sin\pi/6\, i, \quad 6i$

$= \frac{6\sqrt{3}}{2} - \frac{6}{2}, \quad 6\frac{\sqrt{3}}{2} + \frac{6}{2}, \quad 6i$

$= 3\sqrt{3} - 3i, \quad 3\sqrt{3}+3i, \quad 6i$

$\therefore$ ~~$z = 3\sqrt{3}$~~

$z_1 = 3\sqrt{3} + i$

$z_2 =$

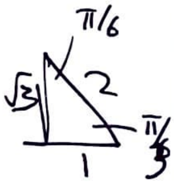
81 / 3 / 27·3

64 / 8 / 2

81 / 9 / 9

49 / 7 / 3

36 / 6 / 216



$\pi - \frac{5\pi}{6} = \frac{\pi}{6}$

$(z+2i)^3 = 216i$

$= 216 \operatorname{cis}\left(\frac{\pi}{2} + 2k\pi\right)$

$= 216 \operatorname{cis}\left(\frac{\pi + 4k\pi}{2}\right)$

$\therefore \quad z + 2i = 6 \operatorname{cis}\left(\frac{\pi + 4k\pi}{6}\right)$

$\therefore \quad z = 6\left(\cos\left(\frac{\pi+4k\pi}{6}\right) + i\sin\left(\frac{\pi+4k\pi}{6}\right)\right) - 2i$

$\cancel{\Rightarrow z_1 = 6\cos(-\pi) + i\sin(-\pi)(i) - 2i}$

$\cancel{= -6 - 2i}$

$\Rightarrow z_2 = 6\cos\frac{\pi}{3} + 6i\sin\frac{\pi}{3} - 2i$

$\cancel{= \frac{6\sqrt{3}}{2} + \frac{6}{2}i - 2i}$

$\cancel{= 3\sqrt{3} + i}$

$\pi/6$, $\sqrt{3}$, 2, 1, $\pi/3$

$\Rightarrow z_3 = 6\cos\frac{5\pi}{3}$

$z_1 =$

ANSWER BOOKLET

6

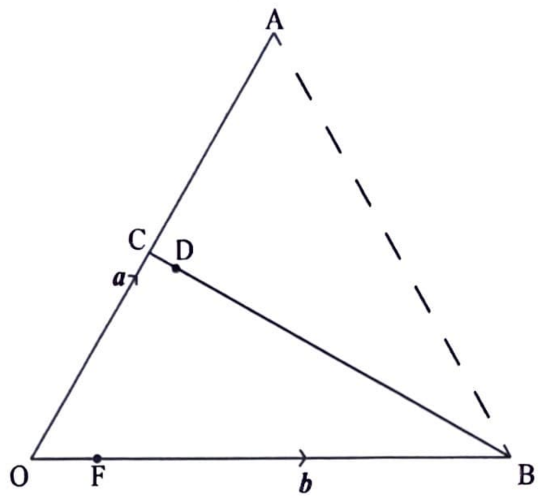
Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 18]

In the following diagram, $\vec{OA} = \boldsymbol{a}$, $\vec{OB} = \boldsymbol{b}$. C is the midpoint of $[OA]$ and $\vec{OF} = \frac{1}{6}\vec{FB}$.



(a) Find, in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$

(i) $\vec{OF}$;

(ii) $\vec{AF}$. [3]

It is given also that $\vec{AD} = \lambda\vec{AF}$ and $\vec{CD} = \mu\vec{CB}$, where $\lambda, \mu \in \mathbb{R}$.

(b) Find an expression for

(i) $\vec{OD}$ in terms of $\boldsymbol{a}, \boldsymbol{b}$ and λ;

(ii) $\vec{OD}$ in terms of $\boldsymbol{a}, \boldsymbol{b}$ and μ. [4]

(c) Show that $\mu = \frac{1}{13}$, and find the value of λ. [4]

(d) Deduce an expression for $\vec{CD}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$ only. [2]

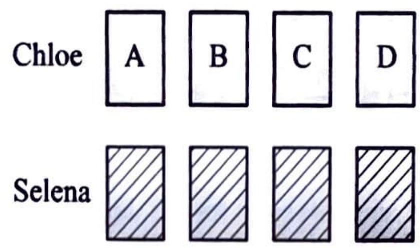
(e) Given that area $\Delta OAB = k(\text{area } \Delta CAD)$, find the value of k. [5]

Do **not** write solutions on this page.

10. [Maximum mark: 11]

Chloe and Selena play a game where each have four cards showing capital letters A, B, C and D.

Chloe lays her cards face up on the table in order A, B, C, D as shown in the following diagram.



Selena shuffles her cards and lays them face down on the table. She then turns them over one by one to see if her card matches with Chloe's card directly above.

Chloe wins if **no** matches occur; otherwise Selena wins.

(a) Show that the probability that Chloe wins the game is $\frac{3}{8}$. [6]

Chloe and Selena repeat their game so that they play a total of 50 times.

Suppose the discrete random variable X represents the number of times Chloe wins.

(b) Determine

(i) the mean of X;

(ii) the variance of X. [5]

Do **not** write solutions on this page.

11. [Maximum mark: 21]

Consider the function $f_n(x) = (\cos 2x)(\cos 4x)\ldots(\cos 2^n x)$, $n \in \mathbb{Z}^+$.

(a) Determine whether f_n is an odd or even function, justifying your answer. [2]

(b) By using mathematical induction, prove that

$$f_n(x) = \frac{\sin 2^{n+1} x}{2^n \sin 2x}, \; x \neq \frac{m\pi}{2} \text{ where } m \in \mathbb{Z}.$$ [8]

(c) Hence or otherwise, find an expression for the derivative of $f_n(x)$ with respect to x. [3]

(d) Show that, for $n > 1$, the equation of the tangent to the curve $y = f_n(x)$ at $x = \frac{\pi}{4}$ is $4x - 2y - \pi = 0$. [8]

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Example / Ejemplo 27 | 27 — Example / Ejemplo 3 | 3

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$\vec{OA} = a \qquad \vec{OB} = b \qquad C = \text{mid}[OA]$

$\vec{OF} = \frac{1}{6}\vec{FB}$

(a) (i) $OF + FB = b$

$\therefore \boxed{OB = b}$

$\therefore \vec{OF} = \frac{1}{7}b$

(ii) $\vec{AF} = AB + BF$

$= AO + OB + BO + OF$

$= -a + b - b + \frac{1}{7}b$

$= \frac{1}{7}b - a$

3

(b) $\vec{AD} = \lambda\vec{AF} \qquad \vec{CD} = \mu\vec{CB} \qquad \lambda, \mu \in \mathbb{R}$

(ii) $\vec{OD} = OC + CD$

$= a/2 + \mu CB$

$= a/2 + \mu(CO + OB)$

$= a/2 + \mu(-a/2 + b)$

$= a/2 - (a/2 - b)\mu$

(i) $\vec{OD} = \vec{OA} + \vec{AD}$

$= a + \lambda(AO + OF)$

$= a + \lambda(-a + \frac{1}{7}b)$

4

(c) $\frac{a}{2} - \mu(\frac{a}{2} - b) = a + \lambda(-a + \frac{1}{7}b)$

$\therefore \frac{a}{2} - \frac{\mu a}{2} + \mu b = a - \lambda a + \frac{\lambda b}{7}$

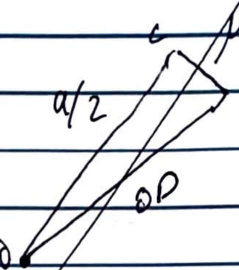
$\therefore \frac{1}{2} - \mu/2 + \mu = -\lambda + \lambda/7$

$\therefore 1 - \mu + 2\mu = -2\lambda + 2\lambda/7$

$\therefore 1 + \mu = -2\lambda + 2\lambda/7$

$\therefore 7 + 7\mu = -14\lambda + 2\lambda$

$\therefore 7 + 7\mu = -12\lambda$



$\mu \vec{CB} = \mu(\vec{CO} + \vec{OB})$

$= \mu(b - a)$

$\therefore \vec{OD} = \frac{a}{2} + \mu(b-a)$

$\therefore \frac{a}{2} + \mu(b-a) = \frac{a}{2} - \mu(\frac{a}{2} - b)$

$\therefore a + 2\mu b - 2\mu a = a - \mu a + 2\mu b$

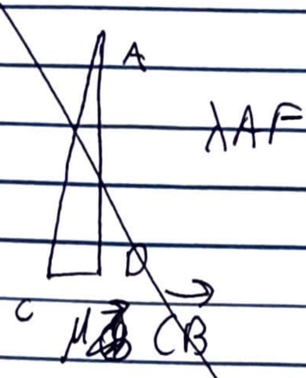
$\therefore -2\mu a = -\mu a$

$\therefore -\mu a = 0$

$\therefore$

7

$\cancel{\mu = \frac{1}{13}}$



$$a + \lambda\left(\frac{1}{7}b - a\right) = \frac{a}{2} - \left(\frac{a}{2} - b\right)\mu$$

$$\therefore a + \frac{\lambda b}{7} - \lambda a = \frac{a}{2} - \frac{\mu a}{2} - \mu b$$

$$\therefore 14a + 2\lambda b - 14\lambda a = 7a - 7\mu a - 14\mu b$$

$$\therefore 7a + (2\lambda - 14\mu)b = (-7\mu + 14\lambda)a$$

$$\mu = \frac{1}{13} \qquad \lambda = 7/13$$

(d) $\vec{CD} = \mu \vec{CB}$

$= \mu(\vec{CO} + \vec{OB})$

$= \mu\left(-\frac{a}{2} + b\right)$

$= \frac{1}{13}(b - a/2)$

(e)

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Example / Ejemplo 27 | 27 | Example / Ejemplo 3 | 3 |

A B C D
B A D C
B C D A

10

(a) Total combinations = 24

~~P(win)~~ = P(MATCH) = $1\times3! + 1\times1\times2! + 1\times1\times1\times1! + 1$

$= 6+2+1+1$

$= 10$

P(MATCH) = $1\times3!$ -

A B C D
A D B C
A C D B

(b) (i) $X \sim B(50, 3/8)$

$\therefore \mu = E(X) = np$

$= 50 \times 3/8$

$= 150/8$

$= 75/4$

2+2
+2+2
+1+1+1

(ii) $Var(X) = np(1-p)$

$= 75/4\,(1 - 3/8)$

$= 75/4\,(5/8)$

$= \dfrac{375}{32}$

$75 \times 5 = 375$

5

6

04AX01

11

(a) $f_n(x) = (\cos 2x)(\cos 4x)\dots(\cos 2^n x)$

2

$f_n(x)$ is an even funcion because

$\cos\theta = \cancel{4}\ \cos(-\theta)$ for all $\theta \in \mathbb{R}$

$\therefore$ will be symmetrical around y-axis.

(b) $f_n(x) = \dfrac{\sin(2^{n+1}x)}{2^n \sin 2x}$ $\quad x \neq \dfrac{m\pi}{2}$

Step 1: Prove for $n=1$

$\therefore$ LHS $= \cos 2x$

RHS $= \dfrac{\sin 4x}{2\sin 2x}$

$= \dfrac{2\sin 2x \cos 2x}{2\sin 2x}$

$= \cos 2x$

$=$ LHS

Step 2: Assume : inductive hypothesis.

~~cos 2x x co~~

$f_k(x) = \dfrac{\sin(2^{k+1}x)}{2^k \sin 2x}$

Step 3: Prove true for $n=k+1$.

$$f_{k+1}(x) = \frac{\sin(2^{k+2}x)}{2^{k+1}\sin 2x}$$

$$\therefore f_{k+1}(x) = (\cos 2x)(\cos 4x)\ldots(\cos 2^k x)(\cos 2^{k+1}x)$$

$$= \frac{\sin(2^{k+1}x)}{2^k \sin 2x}\cos(2^{k+1}x) \quad [\text{by IH}]$$

$$= \frac{2\sin(2^{k+1}x)\cos(2^{k+1}x)}{2\cdot 2^k \sin 2x}$$

$$= \frac{\cancel{2}\sin(2\cdot 2^{k+1}x)}{2^{k+1}\sin 2x}$$

$$= \frac{\cancel{2}\sin(2^{k+2}x)}{2^{k+1}\sin 2x}$$

$\therefore$ True for $n=k+1$ whenever $n=k$ is true

Step 4: Because ~~&~~ true for $n=1$, and true for $n=k+1$ whenever $n=k$ is assumed to be true, it is true for all $n \in \mathbb{Z}^+$ by mathematical induction.

~~(c) $f_n(x) = \frac{\sin 2^{n+1}x}{2^n \sin 2x}$~~

~~$\therefore \cancel{f'_n(x)} = \frac{1}{2^n} \times \frac{\cancel{2}\sin(2\times 2^n x)}{\sin(2x)}$~~

~~$= \frac{1}{2^n} \times \frac{2\sin(2^n x)\cos(2^n x)}{2\sin x \cos x}$~~

8

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3

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Example Ejemplo 27 | 27 — Example Ejemplo 3 | 3

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(c) $f_n(x) = \dfrac{\sin 2^{n+1}x}{2^n \sin 2x}$

$\therefore f_n'(x) = \dfrac{(2^n\sin 2x)(\cancel{2^{n+1}}\cos 2^{n+1}x \times 2^{n+1}) - (\sin 2^{n+1}x)(2^{n+1}\cancel{\sin}\cos 2x)}{(2^n \sin 2x)^2}$

3

$= \dfrac{2^n 2^{n+1} \sin 2x \cos 2^{n+1}x - 2^{n+1}\sin 2^{n+1}x\cos 2x}{(2^n\sin 2x)^2}$

(d) $f_n'\left(\frac{\pi}{4}\right) = \dfrac{2^n 2^{n+1} \sin\frac{\pi}{2}\cos\left(2^{n+1}\left(\frac{\pi}{4}\right)\right) - 2^{n+1}\sin\left(2^{n+1}\left(\frac{\pi}{4}\right)\right)\cos\left(\frac{\pi}{2}\right)}{\left(2^n \sin \pi/2\right)^2}$

$= \dfrac{2^{2n+1}(1)\cos(2^n \times \pi/2) - 2^{n+1}\sin(2^n\times\pi/2)(0)}{2^{2n}}$

$= \dfrac{2\cos(2^n \times \pi/2)}{\cancel{2^n}}$ ~~$- 2\sin(2^n \times \pi/$~~

$\therefore f_n'\left(\frac{\pi}{4}\right) = 2\cos\left(2^{n+1}\pi/4\right)$

$= 2$

8

$f_n(\pi/4) = 0$

4

$\therefore y - 0 = 2(x - \pi/4)$

$\therefore y = 2x - \pi/2 \rightarrow 2y - 2x - \pi = 0$

04AX01

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Example Ejemplo 27 | 27
Example Ejemplo 3 | 3

8

$(z+2i)^3 = 216i$

$= 216 \operatorname{cis}\left(\frac{\pi}{2}+2k\pi\right)$

$= 216 \operatorname{cis}\left(\frac{\pi+4k\pi}{2}\right)$ ✓

$\therefore z+2i = 6 \operatorname{cis}\left(\frac{\pi+4k\pi}{6}\right), \quad k=-1,0,1$

$\therefore z+2i = 6\operatorname{cis}\left(-\frac{3\pi}{6}\right), 6\operatorname{cis}\left(\pi/6\right), 6\operatorname{cis}\left(\frac{5\pi}{6}\right)$ ✓

$= 6\operatorname{cis}\left(-\pi/2\right), 6\operatorname{cis}\left(\pi/6\right), 6\operatorname{cis} 5\pi/6$

$= -6i, 6\left(\frac{\sqrt{3}}{2}+i\times\frac{1}{2}\right), 6\left(-\frac{\sqrt{3}}{2}+i\times\frac{1}{2}\right)$

$= -6i, 3\sqrt{3}+6i/2, -3\sqrt{3}+\frac{6}{2}i$ ✗ ECF

$\therefore z = -8i, 3\sqrt{3}+\left(\frac{3}{2}-2\right)i, -3\sqrt{3}+\left(\frac{3}{2}-2\right)i$

$\therefore z = -8i, 3\sqrt{3}-\frac{3}{2}i, -3\sqrt{3}-\frac{3}{2}i$ ✓

$\Rightarrow z_1 = 0\sqrt{3}-8i$ ✓
$z_2 = 3\sqrt{3}-\frac{3}{2}i$ ✓
$z_3 = -3\sqrt{3}-\frac{3}{2}i$ ✓

04AX01