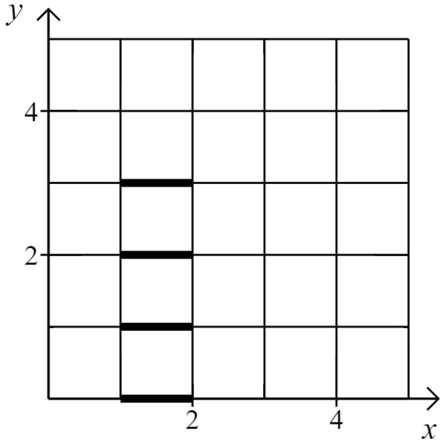
Markscheme

ID: 3013

Mathematics: analysis and approaches

Higher level

1. (a) The segments are shown in bold below. A1A1



(b)

(i) 1 A1

(ii) ${}^{2}C_{1} = 2$ M1A1

(iii) ${}^{3}C_{1} = 3$ M1A1

(iv) ${}^{4}C_{1} = 4$ M1A1

(c) ENNN, NENN, NNEN, NNNE A1A1

(d) The probability of travelling NNNE is 0.5^3 since after travelling north three times she has no choice but to go east. R1 A1

All of the other probabilities in part (c) are 0.5^4. A1

(e) The probability of Bob reaching points (2,0), (2,1), (2,2) and (2,3) is equal to the probability of Alic reaching points (1,0), (1,1), (1,2) and (1,3). A1

The probability of meeting on the segment connecting (1,0) to (2,0) is

$$0.5^4 = 0.0625$$ A1

The probability of meeting on the segment connecting (1,1) to (2,1) is

$$\left(2 \times 0.5^2\right)^2 \times 0.5^2 = 0.0625$$ A1

The probability of meeting on the segment connecting (1,2) and (2,2) is

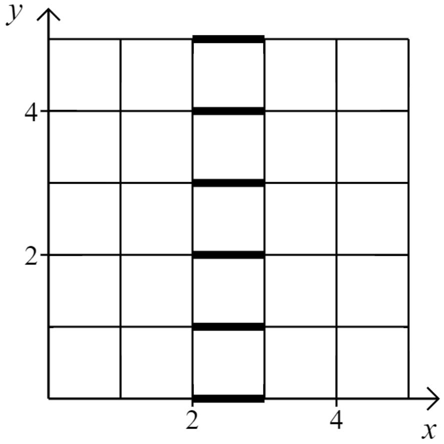
$$\left(3 \times 0.5^3\right)^2 \times 0.5^2 = 0.035156$$ A1

The probability of meeting on the segment connecting (1,3) to (3,3) is

$$\left(0.5^3 + 3 \times 0.5^4\right)^2 = 0.097656$$

The overall probability is 0.258. A1

(f) They can meet on the segments shown in bold below. A1



The probability of meeting on the segment connecting (2,0) to (3,0) is

$$(0.5^3)^2 \qquad \text{A1}$$

The probability of meeting on the segment connecting (2,1) to (3,1) is

$$\left({}^3C_2 \times 0.5^4\right)^2 \qquad \text{A1}$$

The probability of meeting on the segment connecting (2,2) to (3,2) is

$$\left({}^4C_2 \times 0.5^5\right)^2 \qquad \text{A1}$$

The probability of meeting on the segment connecting (2,3) to (3,3) is

$$\left({}^5C_2 \times 0.5^6\right)^2 \qquad \text{A1}$$

The probability of meeting on the segment connecting (2,4) to (3,4) is

$$\left({}^6C_2 \times 0.5^7\right)^2 \qquad \text{A1}$$

The probability of Alice reaching (2,5) is

$$0.5^5 + {}^5C_1 \times 0.5^6 + {}^6C_2 \times 0.5^7 \qquad \text{M1}$$

The probability of meeting on the segment conencting (2,5) to (3,5) is therefore

$$\left(0.5^5 + {}^5C_1 \times 0.5^6 + {}^6C_2 \times 0.5^7\right)^2 \qquad \text{A1}$$

The overall probability is therefore 0.175 A1

2. (a) Since PQ is perpendicular to the line $y = mx$ the gradient is $-\dfrac{1}{m}$ R1A1

(b)

(i) Let the coordinates of point Q be (a,b). Use the gradient formula to determine an expression for the gradient of PQ and set this equal to the answer from part (a). M1

$$\frac{b-y_1}{a-x_1} = -\frac{1}{m} \quad \text{A1}$$

Since point Q lies on the line $y = mx$ we must have $b = ma$. R1

So

$$\frac{ma-y_1}{a-x_1} = -\frac{1}{m} \quad \text{M1}$$

Rearranging gives

$$a = \frac{x_1 + y_1 m}{m^2+1} \quad \text{A1}$$

(ii) Since $b = ma$ we have R1

$$b = \frac{m(x_1 + y_1 m)}{m^2+1} \quad \text{A1}$$

(c) Use the distance formula M1

$$d^2 = \left(x_1 - \frac{x_1 + y_1 m}{m^2+1}\right)^2 + \left(y_1 - \frac{m(x_1 + y_1 m)}{m^2+1}\right)^2 \quad \text{A1}$$

Simplify

$$d^2 = \left(\frac{x_1 m^2 - y_1 m}{m^2+1}\right)^2 + \left(\frac{y_1 - x_1 m}{m^2+1}\right)^2 \quad \text{A1}$$

Write as one fraction and factorise the numerator.

$$d^2 = \frac{\left(m(x_1 m - y_1)\right)^2 + (y_1 - x_1 m)^2}{(m^2+1)^1} \quad \text{M1}$$

so

$$d^2 = \frac{(y_1 - x_1 m)^2 (m^2+1)}{(m^2+1)^2} \quad \text{A1}$$

which simplifies to

$$d^2 = \frac{(y_1 - x_1 m)^2}{m^2+1} \quad \text{A1}$$

(d) The coordinates of the opposing players are (4,1.2), (8,-1.8), (13,-3) and (14,2.4). A1A1

The sum of the square of the distances is therefore

$$\frac{(1.4-4m)^2+(-1.8-8m)^2+(-3-13m)^2+(2.4-14m)^2}{m^2+1}$$ M1

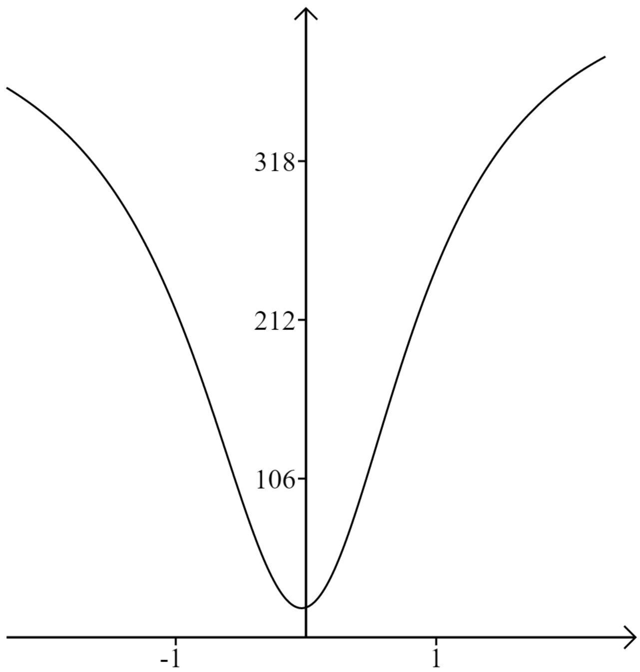
which simplifies to

$$\frac{445m^2+28.4m+19.96}{m^2+1}$$ A1

(e) To meet the goal the value of m has to be between $\pm\frac{3.2}{14}=\pm 0.22857$. M1A1

Let D represent the sum fo the squared distances. Use a GDC to graph the function M1

$$D=\frac{445m^2+28.4m+19.96}{m^2+1}$$



The maximum value occurs when $m=0.229$. A1

(f) The value for m in part (e) is not the best value to maximise he chance for scoring a goal. The diagram suggests a value close to zero would be better. A1A1

The reason why this model produces such a result is because we square the distances. So if we aim to the edge of the goal then the square of two of the distances will be very large. R1 They will be larger than squaring four shorter distances which is what happens when we aim in the middle of the goal. R1