Mathematics: Analysis and Approaches
Higher level
2022 Semester 2 Examinations
Paper 2

ST ANDREW'S CATHEDRAL SCHOOL
FOUNDED 1885

Monday, August 29th (morning)

2 hours

Candidate number

JAMES SULLIVAN

0	0	3	3	7	6	0	0	4	1

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all the questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

$50 + 34 = 84.$

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you could sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Consider a geometric sequence with a first term of 4 and a fourth term of −2.916.

(a) Find the common ratio of this sequence. [3]

(b) Find the sum to infinity of this sequence. [2]

a) $u_1 = 4$ $\quad u_4 = -2.916 = \cancel{u_1 + 3d}\ u_1 r^3$

$\therefore -2.916 = 4r^3$

$\therefore r^3 = \cancel{-2.96}\ \frac{-2.916}{4}$

$\therefore r = -9/10$

3

b) Check convergence, $|r| = 9/10 < 1$

$\therefore S_\infty = \frac{u_1}{1-r}$

$= \frac{4}{1-(-9/10)}$

$= 40/19$

≈ 2.11

2

5

2. [Maximum mark: 10]

It is given that $f(x) = 3x^4 + ax^3 + bx^2 - 7x - 4$ where a and b are positive integers.

(a) Given that $x^2 - 1$ is a factor of $f(x)$ find the value of a and the value of b. [4]

(b) Factorize $f(x)$ into a product of linear factors. [3]

(c) Using your graph state the range of values of c for which $f(x) = c$ has exactly two distinct real roots. [3]

a) $x^2 - 1 = (x+1)(x-1)$ {factors}

$\cancel{\therefore (x+1)(x-1)(}$

$\therefore f(-1) = 3 - a + b + 7 - 4 = 0$

$\therefore 6 - a + b = 0$ 4

$f(1) = 3 + a + b - 7 - 4 = 0$

$\therefore a + b - 8 = 0$

$\therefore a = 7, \; b = 1$ {G.D.C linSolve}

b) $f(x) = (x+1)(x-1)(ax^2 + bx + c)$

$= (x^2-1)(ax^2 + bx + c)$

$= ax^4 + bx^3 + cx^2 - ax^2 - bx - c$

$= ax^4 + bx^3 + (c-a)x^2 - bx - c$

$\therefore a = 3, \; b = 7, \; c = 4$ {equating coefficients}

$\therefore \; 3x^2 + 7x + 4 = 3x^2 + 3x + 4x + 4$

$= 3x(x+1) + 4(x+1)$

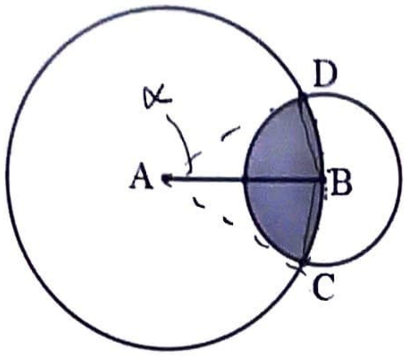
$= (3x+4)(x+1)$ 3

$\therefore f(x) = (x+1)(x-1)(3x+4)(x+1)$ 10

c) IN ANSWER BOOKLET 3

3. [Maximum mark: 8]

The diagram shows two circles with centres at the points A and B and radii $2r$ and r, respectively. The point B lies on the circle with centre A. The circles intersect at the points C and D.



Let α be the measure of the angle CAD and θ be the measure of the angle CBD in radians.

(a) Find an expression for the shaded area in terms of α, θ and r. [3]

(b) Show that $\alpha = 4\arcsin\frac{1}{4}$. [2]

(c) Hence find the value of r given that the shaded area is equal to 4. [3]

a) ~~$A = \frac{1}{2}r^2$~~

$A = \left(\frac{1}{2}(2r)^2\alpha - \frac{1}{2}(2r)^2\sin\alpha\right)$

$+ \left(\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta\right)$

$= (2r^2\alpha - 2r^2\sin\alpha) + \left(\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta\right)$ 2

$= 2r^2(2\cancel{\alpha} - \sin\alpha) + \frac{1}{2}r^2(\theta - \sin\theta)$

b) UNFINISHED

Writing space continued on next page

Question 3 (writing space continued)

c) $\alpha = 4\arcsin \frac{1}{4}$

$\therefore$ ~~$\frac{\alpha}{4} = \arcsin(1/4)$~~

$\therefore A = r^2(2\arcsin 1/4 - 1/4) + \frac{1}{2}r^2(\theta - \sin\theta)$

UNFINISHED

X

2

4. [Maximum mark: 6]

(a) Express the binomial coefficient $\binom{3n+1}{3n-2}$ as a polynomial in terms of n. [3]

(b) Hence find the least value of n for which $\binom{3n+1}{3n-2} > 10^6$. [3]

a) $\binom{3n+1}{3n-2} = \dfrac{(3n+1)!}{(3n-2)!\,(3n+1-3n+2)!}$

$= \dfrac{(3n+1)!}{(3n-2)!\,(3!)}$

$= \dfrac{(3n+1)(3n)(3n-1)\cancel{(3n-2)!}}{\cancel{(3n-2)!}\times 6}$

$= \frac{1}{6}(3n+1)(3n-1)(3n)$ 3

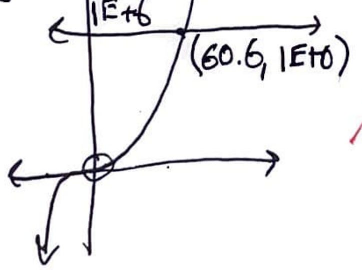
$= \frac{1}{6}(9n^2-1)(3n)$

$= \frac{1}{6}(27n^3-3n)$

$= \frac{9}{2}n^3 - \frac{1}{2}n$

b) $\frac{9}{2}n^3 - \frac{1}{2}n > 10^6$

G.D.C



$\therefore$ ~~n = 61~~

$n=61$ is the least value for the condition 3

6

5. [Maximum mark: 7]

The curve C has equation $e^{2y} = x^3 + y$.

(a) Show that $\frac{dy}{dx} = \frac{3x^2}{2e^{2y} - 1}$. [3]

(b) The tangent to C at the point P is parallel to the y-axis.

Find the x-coordinate of P. [4]

a) $e^{2y} = x^3 + y$

$\therefore 2\frac{dy}{dx} e^{2y} = 3x^2 + dy/dx$ {implicit}

$\therefore \frac{dy}{dx}(2e^{2y} - 1) = 3x^2$

$\therefore dy/dx = \frac{3x^2}{2e^{2y} - 1}$

b) Parallel to y-axis : $\frac{dy}{dx}$ = undefined

$\therefore 2e^{2y} - 1 = 0$

$e^{2y} = 1/2$

$\therefore 2y = \ln(1/2)$

$\therefore 2y = -\ln 2$

$\therefore y = -\frac{1}{2}\ln 2$

at $y = -\frac{1}{2}\ln 2$, $e^{2(-\frac{1}{2}\ln 2)} = x^3 + (-\frac{1}{2}\ln 2)$

$\therefore x^3 = e^{-\ln 2} + \frac{1}{2}\ln 2$

$= \frac{1}{2} + \frac{1}{2}\ln 2$

$= \frac{1}{2}(1 + \ln 2)$

≈ 0.846574

$\therefore x = \sqrt[3]{\frac{1}{2}(1+\ln 2)}$

≈ 0.945994

$\therefore x \approx 0.946$

7

6. [Maximum mark: 7]

By using the substitution $x^2 = 2\sec\theta$, show that $\int \frac{dx}{x\sqrt{x^4-4}} = \frac{1}{4}\arccos\left(\frac{2}{x^2}\right) + c$.

$x^2 = 2\sec\theta$

$$\int \frac{1}{x\sqrt{x^4-4}}\,dx = \int \frac{1}{\sqrt{2\sec\theta}\sqrt{(2\sec\theta)^2-4}}$$

If $x^2 = 2\sec\theta$

$dx\,2x = 2\sec\theta\tan\theta\,d\theta$

$\therefore dx = \frac{\cancel{2}\sec\theta\tan\theta}{\cancel{2}x}\,d\theta$

$$\therefore \int \frac{1}{x\sqrt{x^4-4}}\,dx = \int \frac{1}{x\sqrt{(2\sec\theta)^2-4}}\frac{\sec\theta\tan\theta}{x}\,d\theta$$

$$= \int \frac{1}{x^2\sqrt{(2\sec\theta)^2-4}}\sec\theta\tan\theta\,d\theta$$

$$= \int \frac{1}{2\sec\theta\sqrt{4\sec^2\theta-4}}\sec\theta\tan\theta\,d\theta$$

$$= \int \frac{1}{2\sqrt{4(1+\tan^2\theta)-4}}\tan\theta\,d\theta$$

$$= \frac{1}{2}\int \frac{1}{\sqrt{\tan^2\theta}}\tan\theta\,d\theta$$

$$= \frac{1}{2}\int d\theta$$

$$= \frac{1}{2}\theta + C$$

If $x^2 = 2\sec\theta$, then $\sec\theta = \frac{x^2}{2}$

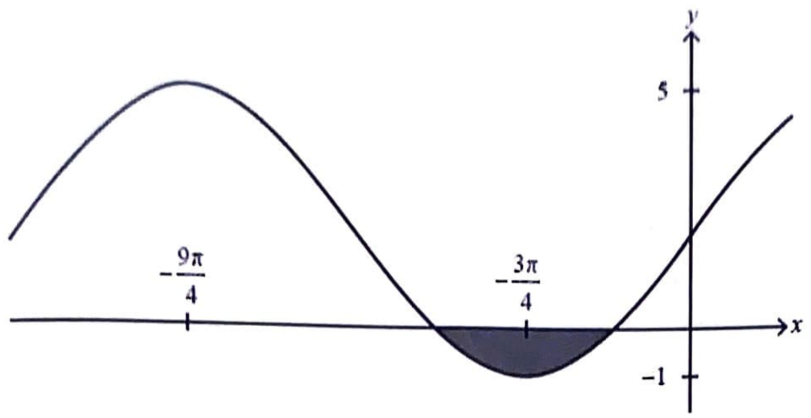
$\therefore \cos\theta = 2/x^2$

$\therefore \theta = \arccos(2/x^2)$

7

7. [Maximum mark: 8]

The following diagram shows part of the graph of $y = p + q\sin(rx)$. The graph has a local maximum point at $\left(-\frac{9\pi}{4}, 5\right)$ and a local minimum point at $\left(-\frac{3\pi}{4}, -1\right)$.



(a) Determine the values of p, q and r. [4]

(b) Hence find the area of the shaded region. [4]

a) $q = \frac{\text{MAX} - \text{MIN}}{2} = \frac{5+1}{2} = 3$

$p = \frac{\text{MAX} + \text{MIN}}{2} = \frac{5-1}{2} = 2$

$r = 2\pi / T$

$\hookrightarrow$ [$T = 2 \times$ distance between $-\frac{9\pi}{4}$ & $-\frac{3\pi}{4}$

$= 2 \times \left(-\frac{3\pi}{4} + \frac{9\pi}{4}\right)$

$= 2 \times (6\pi/4$

$= 3\pi$]

$\therefore r = \frac{2\pi}{3\pi} = \frac{2}{3}$

$\therefore q = 3 \quad p = 2, \quad r = 2/3$

b) CONTINUED IN ANSWER BOOKLET

8

8. [Maximum mark: 7]

There are eight boys and five girls who attend the Mathematics Club. How many ways can the teacher select a group of 6 students form the club to represent the school in a Mathematics competition if:

(a) There are no gender restrictions [2]

(b) The team is to be made up of three girls and three boys [2]

(c) At least two of each gender are included in the team [3]

a) Total = 8 + 5 = 13

$\therefore$ Possibilities = ${}^{13}C_6$ ~~6!~~

= 1716

b) Possibilities = ${}^{8}C_3 \times {}^{5}C_3$

= 560

remaining

c) Possibilities = ${}^{8}C_2 \times {}^{5}C_2 \times {}^{9}C_2$

= 10080

1

Possibilities = ${}^{8}C_2 \times {}^{5}C_4 + {}^{8}C_3 \times {}^{5}C_3 + {}^{8}C_4 \times {}^{5}C_2$

= 1400

(2b 4g) + (3b 3g) + (4b 2g)

5

1

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 20]

A particle P moves in a straight line such that after time t seconds, its velocity, v in $\mathrm{m\,s^{-1}}$, is given by $v = e^{-3t}\sin 6t$, where $0 < t < \frac{\pi}{2}$.

(a) Find the times when P comes to instantaneous rest. [2]

At time t, P has displacement $s(t)$; at time $t = 0$, $s(0) = 0$.

(b) Use the method of integration by parts to show that

$$s = \frac{2}{15} - \frac{e^{-3t}\sin 6t + 2e^{-3t}\cos 6t}{15} \qquad [7]$$

(c) Find the maximum displacement of P, in metres, from its initial position. [2]

(d) Find the total distance travelled by P in the first 1.5 seconds of its motion. [2] ?? why $\int v(t)\,dt$

At successive times when the acceleration of P is $0\ \mathrm{m\,s^{-2}}$, the velocities of P form a geometric sequence. The acceleration of P is zero at times t_1, t_2, t_3 where $t_1 < t_2 < t_3$ and the respective velocities are v_1, v_2, v_3.

(e) Show that, at these times, $\tan 6t = 2$. [2]

(f) Hence show that $\frac{v_2}{v_1} = \frac{v_3}{v_2} = -e^{-\frac{\pi}{2}}$. ?? [5]

10. [Maximum mark: 14] ??

(a) (i) Explain why it is appropriate to use L'Hopital's rule to find $\lim_{x\to\infty} x^3 e^{-x}$ [2]

(ii) Hence find $\lim_{x\to\infty} x^3 e^{-x}$ [3]

(b) Show that the improper integral $\int_0^\infty x^3 e^{-x}\,dx$ converges and state its value. [9]

$$-\frac{1}{6}e^{-3t}\cos 6t - \frac{1}{2}\left(\frac{1}{6}e^{-3t}\sin 6t - \int -\frac{1}{2}e^{-3t}\sin 6t\,dt\right)$$

$$-\frac{1}{6}e^{-3t}\cos 6t - \frac{1}{12}e^{-3t}\sin 6t - \frac{1}{4}\int e^{-3t}\sin 6t\,dt$$

Do **not** write solutions on this page.

11.[Maximum mark: 18]

The plane Π_1 contains the points P(1, 6, −7), Q(0, 1, 1) and R(2, 0, −4).

(a) Find the Cartesian equation of the plane containing P, Q and R. [6]

(b) The Cartesian equation of the plane Π_2 is given by $x - 3y - z = 3$.

Given that Π_1 and Π_2 meet in a line L, verify that the vector equation of L can be given by $\boldsymbol{r} = \begin{pmatrix} \frac{5}{4} \\ 0 \\ -\frac{7}{4} \end{pmatrix} + \lambda \begin{pmatrix} \frac{1}{2} \\ 1 \\ -\frac{5}{2} \end{pmatrix}$. [3]

The Cartesian equation of the plane Π_3 is given by $ax + by + cz = 1$.

(c) Given that Π_3 is parallel to the line L, show that $a + 2b - 5c = 0$. [1]

(d) Consider the case that Π_3 contains L.

Show that $5a - 7c = 4$. [2]

(e) Given that Π_3 is equally inclined to both Π_1 and Π_2, determine two distinct possible Cartesian equations for Π_3. [6]

End of paper 2

IB practice exams 2022

ANSWER BOOKLET (1)

4 PAGES

ST ANDREW'S CATHEDRAL SCHOOL
FOUNDED 1885

Candidate session number: 003376-0041

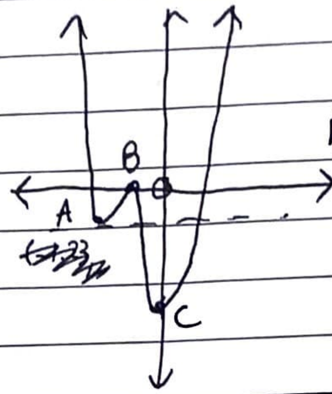
Candidate Name: JAMES SULLIVAN

At the start of each answer to a question, write the question number in the box using your normal handwriting

Example 27 | 2 | 7 | Example 3 | | 3 |

| | 2 |

c) G.D.C



A $(-1.23, -0.037)$
B $(-1, 0)$
C $(0.476, -6.2)$

$\therefore \; -6.2 < C < -0.037$ ✓

AND

$C > 0$ ✓

3

| | 7 |

~~b) Intercepts:~~ $y = \cancel{2 + 3\sin\left(\tfrac{2}{3}x\right) = 0}$

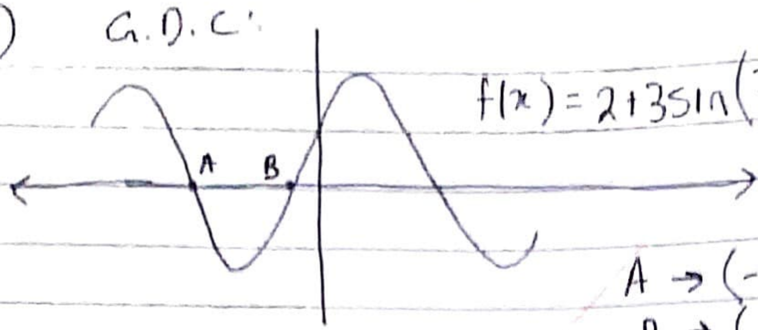
$\cancel{\therefore \; \sin\left(\tfrac{2}{3}x\right) = -2/3}$

$\cancel{\therefore \; x = \tfrac{3}{2}\arcsin(-2/3)}$

heart
mind
life

7

b) G.D.C:



$f(x) = 2+3\sin\left(\frac{2}{3}x\right)$

$A \rightarrow (-3.62, 0)$
$B \rightarrow (-1.09, 0)$

$$\therefore \text{Area} = \left| \int_{-3.62}^{-1.09} \left(2+3\sin\left(\tfrac{2}{3}x\right)\right) dx \right|$$

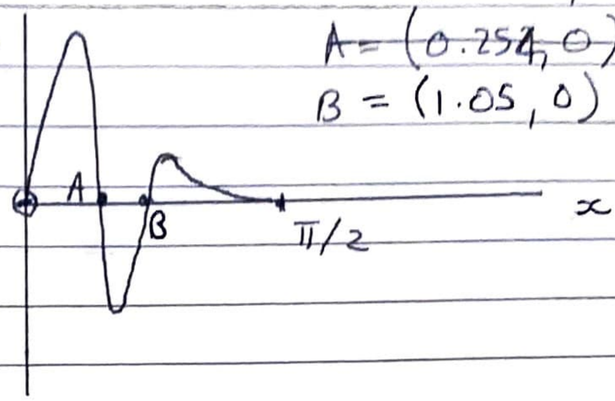
$$= 1.66177 \qquad \{G.D.C\}$$

$$\approx 1.66 \text{ units}^2$$

19

a) $v = e^{-3t} \sin 6t$

G.D.C:



A = (0.524, 0)

~~A = (0.254, 0)~~

B = (1.05, 0)

$\therefore$ when $v = e^{-3t} \sin 6t = 0$,

$t = 0.524 = \pi/6$ 2.

~~OR~~ AND

$t = 1.05 = \pi/3$

b) $v(t) = e^{-3t} \sin 6t$

$\therefore s(t) = \int e^{-3t} \sin 6t \, dt$

$\left[\begin{array}{ll} \text{let } u = e^{-3t} & \therefore du = -3e^{-3t} \\ \text{let } dv = \sin 6t & \therefore v = -\frac{1}{6}\cos 6t \end{array} \right]$

$\therefore \int e^{-3t} \sin 6t \, dt = -\frac{1}{6} e^{-3t} \cos 6t - \int -\frac{1}{6} \cos 6t \times -3e^{-3t} \, dt$

$= -\frac{1}{6} e^{-3t} \cos 6t - \int \frac{1}{2} e^{-3t} \cos 6t \, dt$

$= -\frac{1}{6} e^{-3t} \cos 6t - \frac{1}{2} \int e^{-3t} \cos 6t \, dt$

$\left[\begin{array}{ll} \text{let } u = e^{-3t} & \therefore du = -3e^{-3t} \\ \text{let } dv = \cos 6t & \therefore v = \frac{1}{6} \sin 6t \end{array} \right]$

$= -\frac{1}{6} e^{-3t} \cos 6t - \frac{1}{2} \left[\frac{1}{6} e^{-3t} \sin 6t - \int \frac{1}{6} \sin 6t \times -3e^{-3t} \, dt \right.$

~~$\therefore \int e^{-3t} \sin 6t \, dt = -\frac{1}{6} e^{-3t} \cos 6t - \frac{1}{12} e^{-3t} \sin 6t + \frac{1}{2} \int e^{-3t} \sin 6t \, dt$~~

~~$\therefore \frac{1}{2} \int e^{-3t} \sin 6t \, dt = -\frac{1}{12} \left(2e^{-3t} \cos 6t + e^{-3t} \sin 6t \right)$~~

~~$\therefore \int e^{-3t} \sin 6t \, dt = -\frac{1}{6} \left(2e^{-3t} \cos 6t + e^{-3t} \sin 6t \right) + c$~~

$= -\frac{1}{6} e^{-3t} \cos 6t - \frac{1}{12} e^{-3t} \sin 6t - \frac{1}{4} \int e^{-3t} \sin 6t \, dt$

$\therefore \int e^{-3t}\sin 6t\,dt = -\frac{1}{6}e^{-3t}\cos 6t - \frac{1}{12}e^{-3t}\sin 6t - \frac{1}{4}\int e^{-3t}\sin 6t\,dt$

$\therefore \frac{5}{4}\int e^{-3t}\sin 6t\,dt = -\frac{1}{6}\left(e^{-3t}\cos 6t + \frac{1}{2}e^{-3t}\sin 6t\right) + c$

$\therefore \int e^{-3t}\sin 6t\,dt = -\frac{2}{15}\left(e^{-3t}\cos 6t + \frac{1}{2}e^{-3t}\sin 6t\right) + c$

$= \cancel{e^{-3}}$

$$= \frac{-(e^{-3t}\sin 6t + 2e^{-3t}\cos 6t)}{15} + c$$

$$\therefore s(t) = -\frac{e^{-3t}\sin 6t + 2e^{-3t}\cos 6t}{15} + c$$

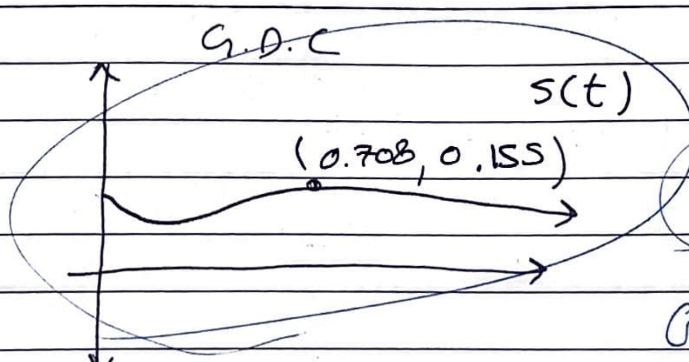
Sub $s(0) = 0$

$$\therefore 0 = -\frac{e^{0}\sin(0) + 2e^{0}\cos(0)}{15} + c$$

$\therefore c = 2/15$

$$\therefore s(t) = \frac{2}{15} - \frac{e^{-3t}\sin 6t + 2e^{-3t}\cos 6t}{15}$$

c)



→ wrong graph

GDC does not graph integrals.

$\therefore$ maximum displacement $= 1.55$ m ✗.
at $t = 0.708$ s

d) $$\int_0^{1.5} s(t)\,dt = \int_0^{1.5}\left(\frac{2}{15} - \frac{e^{-3t}\sin 6t + 2e^{-3t}\cos 6t}{15}\right)dt$$

$\approx \cancel{1.72}\ 0.173124$ m [G.D.C]

≈ 0.173 m

IB practice exams 2022

ANSWER BOOKLET 2

4 PAGES

ST ANDREW'S CATHEDRAL SCHOOL

FOUNDED 1885

Candidate session number: 003376-0041

Candidate Name: JAMES SULLIVAN

At the start of each answer to a question, write the question number in the box using your normal handwriting

Example 27 | 2 | 7 | Example 3 | | 3 |

e)

$$v = e^{-3t}\sin 6t$$

$$\therefore \frac{dv}{dt} = \sin 6t \cdot -3e^{-3t} + e^{-3t} \cdot 6\cos 6t$$

$$= \cancel{-3}$$

$$= -3e^{-3t}\sin 6t + 6e^{-3t}\cos 6t$$

$$\therefore \frac{dv}{dt} = 0 = -3\sin 6t + 6\cos 6t$$

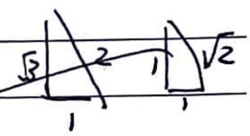
$$\therefore 3\sin 6t = 6\cos 6t$$

$$\therefore \cancel{\tan 6t = 2}$$

$$\therefore \tan 6t = 2$$

2

f)



$$\tan 6t = 2$$

$$\therefore 6t = \arctan(2)$$

$$= \pi/4 + k\pi$$

$$= (\pi + 4k\pi)/4$$

$$\therefore t = (\pi + 4k\pi)/24$$

heart
mind
life

d) $\tan 6t = 2$

$\therefore 6t = \arctan(2)$

$\therefore t = \frac{1}{6}\arctan(2)$

$\therefore v_1 =$

UNFINISHED

4

12

(e)

(f)

1 0

a) i) $\lim_{x\to\infty} x^3 e^{-x} = \lim_{x\to\infty} \left(x^3 / e^{-x} \right)$

$= \lim_{x\to\infty} \left(\frac{f(x)}{g(x)} \right)$

$= \lim_{x\to\infty} \left(\frac{f'(x)}{g'(x)} \right) \quad \{LH's\}$

~~ii) ∴ $\lim_{x\to\infty} \left(\frac{x^3}{e^{-x}} \right) = \lim_{x\to\infty} \left(\frac{3x^2}{-e^{-x}} \right) \quad \{LH's\}$~~

~~$= \lim_{x\to\infty} \left(\frac{6x}{e^{-x}} \right) \quad \{LH's \text{ again}\}$~~

~~$= \lim_{x\to\infty} \left(\frac{6}{-e^{-x}} \right) \quad \{LH's \text{ again}\}$~~

~~$= 6 / -e^{-\infty}$~~

~~$=$~~

$\lim_{x\to\infty} \left(\frac{x^3}{e^x} \right) = \lim_{x\to\infty} \left(\frac{3x^2}{e^x} \right) \quad \{LH's\}$

$= \lim_{x\to\infty} \left(\frac{6x}{e^x} \right) \quad \{LH's\}$

$= \lim_{x\to\infty} \left(\frac{6}{e^x} \right) \quad \{LH's\}$

$= 6 / e^{\infty}$

$= 0$

3

IB practice exams 2022

ANSWER BOOKLET ③

4 PAGES

ST ANDREW'S CATHEDRAL SCHOOL

FOUNDED 1885

Candidate session number

0	0	3	3	7	6	-	0	0	4	1

Candidate Name

JAMES SULLIVAN

At the start of each answer to a question, write the question number in the box using your normal handwriting

Example 27 [2][7] Example 3 [ ][3]

b) as $x \to \infty$, $\lim_{x\to\infty}(x^3e^{-x}) \to 0$ (from part a)

$\therefore$ converges

$\int_0^\infty x^3 e^{-x}\,dx = \left[-x^3e^{-x}\right]_0^\infty - \int_0^\infty -3x^2e^{-x}\,dx$

$\begin{bmatrix} u = x^3 & du = 3x^2 \\ dv = e^{-x} & v = -e^{-x} \end{bmatrix}$

Learn improper integrals.

$= 0 + 3\int_0^\infty x^2e^{-x}\,dx$

$= 3\left(\left[-x^2e^{-x}\right]_0^\infty - \int_0^\infty -e^{-x}\cdot 2x\,dx\right.$

$\begin{bmatrix} u = x^2 & du = 2x \\ dv = e^{-x} & v = -e^{-x} \end{bmatrix}$

$= 3\left(0 + 2\int_0^\infty xe^{-x}\,dx\right)$

$= 6\int_0^\infty xe^{-x}\,dx$

$\begin{bmatrix} u = x & du = 1 \\ dv = e^{-x} & v = -e^{-x} \end{bmatrix}$

$= 6\left(\left[-xe^{-x}\right]_0^\infty - \int_0^\infty -e^{-x}\,dx\right)$

$= 6\left(0 + \int_0^\infty e^{-x}\,dx\right)$

$= \left[-6e^{-x}\right]_0^\infty$

$= \left(6e^{-\infty} - 6e^0\right)$ X.

$= 0 - 6$

$= -6$

4

heart mind life

1 1

a) Π_1 : $P(1,6,-7)$
$Q(0,1,1)$
$R(2,0,-4)$

$$\therefore \overrightarrow{PQ} = \begin{pmatrix} 0-1 \\ 1-6 \\ 1-(-7) \end{pmatrix} = \begin{pmatrix} -1 \\ -5 \\ 8 \end{pmatrix}$$

$$\overrightarrow{PR} = \begin{pmatrix} 2-1 \\ 0-6 \\ -4-(-7) \end{pmatrix} = \begin{pmatrix} 1 \\ -6 \\ 3 \end{pmatrix}$$

$$\therefore \vec{n} = \begin{pmatrix} -1 \\ -5 \\ 8 \end{pmatrix} \times \begin{pmatrix} 1 \\ -6 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} -15-(-6)(8) \\ 8-(-3) \\ 6-(-5) \end{pmatrix}$$

$$= \begin{pmatrix} 33 \\ 11 \\ 11 \end{pmatrix}$$

$$= 11\begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$$

$\therefore \quad 3x + y + z = d$

6

Substitute $Q(0,1,1)$:
$d = 1+1$
$= 2$

$\therefore \underline{\underline{3x + y + z = 2}}$

heart
mind
life

b) $x = \frac{5}{4} + \frac{1}{2}\lambda$
$y = 0 + \lambda$
$z = -\frac{7}{4} - \frac{5}{2}\lambda$

test in $3x + y + z = 2$:

$3\left(\frac{5}{4} + \frac{1}{2}\lambda\right) + \lambda - \frac{7}{4} - \frac{5}{2}\lambda = \frac{15}{4} + \frac{3}{2}\lambda + \lambda - \frac{7}{4} - \frac{5}{2}\lambda$
$= 2 + 0\lambda$
$= 2$ ✓

test in $x - 3y - z = 3$

$\therefore \frac{5}{4} + \frac{1}{2}\lambda - 3(\lambda) - \left(-\frac{7}{4} - \frac{5}{2}\lambda\right)$
$= \frac{5}{4} + \frac{1}{2}\lambda - 3\lambda + \frac{7}{4} + \frac{5}{2}\lambda$

3

$= 3 + 0\lambda$
$= 3$ ✓

$\therefore L_1$ fits Π_1 and Π_2

c) Π_3 : $ax + by + cz = 1$

$\therefore \vec{n}_3 = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \rightarrow \begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} 1/2 \\ 1 \\ -5/2 \end{pmatrix} = 0$

$\therefore \frac{1}{2}a + b - \frac{5}{2}c = 0$

$\therefore a + 2b - 5c = 0$

1

d) $\Pi_3 : ax + by + cz = 1$

$L: \quad x = \frac{5}{4} + \frac{1}{2}\lambda$

$y = \lambda$

$z = -\frac{7}{4} - \frac{5}{2}\lambda$

$\therefore L \rightarrow \Pi_3 :$

$\left(\frac{5}{4} + \frac{1}{2}\lambda\right)a + (\lambda) b$

$\Pi_3 : \quad ax + by + cz = 1$

$\therefore L$ position vector: $\begin{pmatrix} 5/4 \\ 0 \\ -7/4 \end{pmatrix}$

2

$L \rightarrow \Pi_3 :$

$a\left(5/4\right) + 0y - 7/4\, c = 1$

$\therefore \quad 5a - 7c = 4$

ST ANDREW'S CATHEDRAL SCHOOL
FOUNDED 1885

Candidate session number
003376-0041

Candidate Name
JAMES SULLIVAN

At the start of each answer to a question, write the question number in the box using your normal handwriting

Example 27 [2][7] Example 3 [ ][3]

e) If Π_3 is equally inclined to Π_1 and Π_2

$\cos\theta_1 = \cos\theta_2$ and $\cos\theta_1 = -\cos\theta_2$

$\therefore n_1 = \begin{pmatrix}3\\1\\1\end{pmatrix} \quad n_2 = \begin{pmatrix}1\\-3\\-1\end{pmatrix} \quad n_3 = \begin{pmatrix}a\\b\\c\end{pmatrix}$

Case 1:

$\therefore \dfrac{n_1 \cdot n_3}{|n_1||n_3|} = \dfrac{n_1 \cdot n_2}{|n_1||n_2|}$

$\therefore \dfrac{3a+b+c}{\sqrt{11}\sqrt{a^2+b^2+c^2}} = \dfrac{a-3b-c}{\sqrt{11}\sqrt{a^2+b^2+c^2}}$

$\therefore 3a+b+c = a-3b-c$

$\therefore 2a+4b+2c = 0$

$\therefore$ linsolve with $\begin{cases} 2a+4b+2c=0 \\ 5a-7c=4 \\ \cancel{2a-5b}\ a+2b-5c=0 \end{cases}$

$a = \frac{4}{5},\ b = -\frac{2}{5},\ c = 0$

~~$\therefore$ coordinates are~~ $\begin{pmatrix}4/5\\-2/5\\0\end{pmatrix}$

$\therefore \frac{4}{5}x - \frac{2}{5}y = 1$

case 2 :

$$3a+b+c = -(a-3b-c)$$

$\therefore$ $3a+b+c = -a+3b+c$

$\therefore$ $4a-2b = 0$

linsolve $\rightarrow$ $4a-2b=0$, $5a-7c=4$, $a+2b-5c=0$

$\therefore$ $a=-2$, $b=-4$, $c=-2$

$\therefore$ $\underline{-2x-4y-2z=1}$ 6 ☺

18 ☺