# Practice Set A: Paper 2 Mark scheme

## SECTION A

**1** **a** $= \frac{4}{3}\pi(3^3) \times 1.45$ (M1)

$= 164\,\text{g}$ A1

**b** Each volume [mass] is $\frac{1}{8}$ the previous one. A1

Sum to infinity $= \frac{164}{1-\frac{1}{8}} = 187\,\text{g}$ M1A1

Hence the mass is always smaller than 200 g. A1

*[6 marks]*

**2** $\text{E}(X) = \int_{2\pi}^{3\pi} 0.4106x \sin x\sqrt{x-2\pi}\,\text{d}x [= 8.018]$ M1

A1 for correct limits A1

$\text{E}(X^2) = \int_{2\pi}^{3\pi} 0.4106x^2 \sin x\sqrt{x-2\pi}\,\text{d}x [= 64.71]$ M1

$\text{Var}(X) = 64.71 - 8.018^2$ M1

$= 0.425$ (A1)

$\sqrt{0.425} = 0.652$ (A1)

*[6 marks]*

**3** Attempt sine rule:

$\frac{\sin\theta}{x-1} = \frac{\sin 2\theta}{x+2}$ A1

Use double angle formula:

$= \frac{2\sin\theta\cos\theta}{x+2}$ M1

$2\cos\theta(x-1) = x+2$ A1

Rearrange for $x$:

$x(2\,\text{Cos}\theta - 1) = 2 + 2\cos x$ M1

$x = \frac{2(1+\cos\theta)}{2\cos\theta - 1}$ A1

Use $\cos\theta = 2\cos^2\left(\frac{\theta}{2}\right) - 1$:

$= \frac{4\cos^2\left(\frac{\theta}{2}\right)}{4\cos^2\left(\frac{\theta}{2}\right) - 3}$ M1

Divide by $\cos^2\left(\frac{\theta}{2}\right)$, clearly using $= \frac{1}{\cos\left(\frac{\theta}{2}\right)} = \sec\left(\frac{\theta}{2}\right)$:

$= \frac{4}{4 - 3\sec^2\left(\frac{\theta}{2}\right)}$ A1AG

*[7 marks]*

**4** **a** A A1

Gradient is zero and changing from positive to negative R1

**b** B, D A1

and E A1

Second derivative is zero and changes sign R1

*[5 marks]*

**5** $(4-x)^{-\frac{1}{2}}$ M1

$= 4^{-\frac{1}{2}}\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}$ M1

$\approx \frac{1}{2}(1+\frac{x}{8}+\ldots$ A1

$\ldots+\frac{3}{8}\left(-\frac{x}{4}\right)^2-\frac{5}{16}\left(-\frac{x}{4}\right)^3)$ M1

$= \frac{1}{2}+\frac{x}{16}+\frac{3x^2}{256}+\frac{5x^3}{2048}$ A1

Valid for $|x|<4$ A1

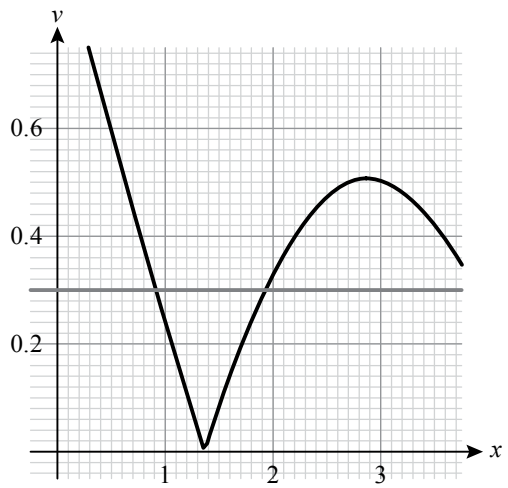
*[6 marks]*

**6** **a** integrate $|v|$ (M1)

With limits 0 and 5 (M1)

Distance = 1.8 m A1

**b** Sketch $\left|\frac{dv}{dt}\right|$ [or $\frac{dv}{dt}$] (M1)




Intersect with $y = 0.3$ [or with both 0.3 and $-0.3$] (M1)

$t = 0.902$ and 1.93 seconds A1

*[6 marks]*

**7** Consider f(1):

$1^2 - 1 = -(1)^2 + b(1) + c$ M1

$\Rightarrow b + c = +1$ A1

Consider f□(1):

$+2(1) = -2(1) + b$ M1A1

$\Rightarrow b = 4$ A1

$c = -3$ A1

*[6 marks]*

**8** $|\mathbf{a}||\mathbf{b}|\cos\theta = 17$ M1

$|\mathbf{a}||\mathbf{b}|\sin\theta = \sqrt{4+1+25}$ [$= \sqrt{30}$] M1

$\tan\theta = \frac{\sqrt{30}}{17}$ M1A1

$\theta = 17.9°$ A1

*[5 marks]*

**9 a** 8! seen A1
8!2!2! = 161280 (M1)A1

**b** Pair 1 stands together: 9!2! [= 725760] M1
2 × 725760 –(their **a**) [=1 290 240] M1
$\dfrac{10! - 1\,290\,240}{10!}$ M1
= 0.644 (3 s.f.) A1
*[7 marks]*

## SECTION B

**10 a** Paper 1: mean = 78.9, SD = 17.4 A1
Paper 2: mean = 74.0, SD = 15.1 A1
Paper 1 has higher marks on average. A1
Paper 2 has more consistent marks. A1
*[4 marks]*

**b** $r = 0.868$ A1
$> 0.532$ M1
There is evidence of positive correlation between the two sets of marks. A1
*[3 marks]*

**c i** Find regression line $x$ on $y$ M1
$x = 0.997y + 5.16$ A1
$0.997 \times 86 + 5.16 \approx 91$ marks A1
**ii** Can't be used. A1
Mark is outside of the range of available data (interpolation) R1
*[5 marks]*

**d i** Boundary for 7: inverse normal of 0.88 M1
Boundary = 81 A1
5 students A1
**ii** Use $B(12, 0.12)$ (M1)
P(>5) = 1 – P($\leq$ 5) (M1)
= 0.001 44 A1
*[6 marks]*

**e** Scaled mark = $\dfrac{80}{110}$ × original mark (M1)
Mean = 57.4 A1
SD = 12.7 A1
*[3 marks]*
*Total [21 marks]*

**11 a** Separate variables and attempt integration M1
$\displaystyle\int \frac{dy}{y} = \int \tan x \, dx$ A1
$\ln y = -\ln|\cos x| + c$ A1
$y = Ae^{-\ln(\cos x)}$ M1
$= \dfrac{A}{\cos x}$ A1
*[5 marks]*

**b i** $\displaystyle\int -\tan x \, dx = \int \frac{-\sin x}{\cos x} dx = \ln(\cos x)$ M1A1
$I = e^{\ln(\cos x)} = \cos x$ M1(AG)
**ii** $y\cos x = \displaystyle\int \cos^2 x \, dx$ M1
$= \displaystyle\int \frac{\cos 2x + 1}{2} dx$ M1
$= \dfrac{1}{4}\sin 2x + \dfrac{1}{2}x + c$ A1
$y = \dfrac{\sin 2x}{4\cos x} + \dfrac{x}{2\cos x} + \dfrac{c}{\cos x} \left( = \dfrac{\sin x}{2} + \dfrac{x \sec x}{2} + c\sec x \right)$ A1
*[7 marks]*

**c** Use $y_{n+1} = y_n + 0.1(y_n^2 \tan x_n + \cos x_n)$ M1A1

Table of values – at least the first two rows correct M1

| $x$ | $y'$ | $y$ |
|---|---|---|
| 0 | 1.000 | 2.000 |
| 0.1 | 1.437 | 2.100 |
| 0.2 | 2.001 | 2.244 |
| 0.3 | 2.803 | 2.444 |
| 0.4 | 4.058 | 2.724 |
| 0.5 | 6.229 | 3.130 |

$y(0.5) = 3.13$ A1

*[4 marks]*

*Total [16 marks]*

**12 a i** Equate $x$, $y$, $z$ components:

$$\begin{cases} 5 + 7\lambda = 1 - \mu\,(1) \\ 3 + 2\lambda = -8 + 3\mu\,(2) \\ 1 - 3\lambda = -2 + 2\mu\,(3) \end{cases}$$ M1A1

From, e.g. (1) and (2): $\lambda = -1,\ \mu = 3$ A1A1

Check in (3):

$1 - 3(-1) = 4$

$-2 + 2(3) = 4$

So lines intersect. M1AG

**ii** Substitute their values of $\lambda$ and $\mu$ into either equation

$$\mathbf{r} = \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix} - \begin{pmatrix} 7 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$$ (M1)

So coordinates $(-2,\ 1,\ 4)$ A1

*[7 marks]*

**b** Attempt to find cross product of direction vectors:

$$\begin{pmatrix} 7 \\ 2 \\ -3 \end{pmatrix} \times \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}$$ (M1)

$$= \begin{pmatrix} 13 \\ -11 \\ 23 \end{pmatrix}$$ A1

*[2 marks]*

**c** $\mathbf{r}$ · their $\mathbf{n}$ = their $\mathbf{p}$ · their $\mathbf{n}$

$$\mathbf{r} \cdot \begin{pmatrix} 13 \\ -11 \\ 23 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 13 \\ -11 \\ 23 \end{pmatrix}$$ (M1)

$$\mathbf{r} \cdot \begin{pmatrix} 13 \\ -11 \\ 23 \end{pmatrix} = 55$$ A1

*[2 marks]*

**d** $\overrightarrow{QP} = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix} - \begin{pmatrix} -11 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 9 \\ 1 \\ 3 \end{pmatrix}$ (M1)

$$\cos\phi = \frac{\begin{pmatrix} 9 \\ 1 \\ 3 \end{pmatrix} . \begin{pmatrix} 13 \\ -11 \\ 23 \end{pmatrix}}{\sqrt{9^2+1^2+3^2}\sqrt{13^2+11^2+23^2}}$$ M1A1

$$= \frac{25}{39}$$ A1

$\sin\theta = \sin(90-\phi) = \cos\phi$ M1

So, $\sin\theta = \frac{25}{39}$ A1

*[6 marks]*

**e** $d = \left|\overrightarrow{QP}\right| \sin\theta$ (M1)

$$= \frac{25\sqrt{91}}{39}$$ A1

*[2 marks]*

*Total [19 marks]*