

Markscheme

May 2016

Mathematics

Higher level

Paper 1

18 pages

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Instructions to Examiners

Abbreviations

- M** Marks awarded for attempting to use a valid **Method**; working must be seen.
- (M)** Marks awarded for **Method**; may be implied by **correct** subsequent working.
- A** Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding **M** marks.
- (A)** Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.
- R** Marks awarded for clear **Reasoning**.
- N** Marks awarded for **correct** answers if **no** working shown.
- AG** Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM™ Assessor instructions and the document “**Mathematics HL: Guidance for e-marking May 2016**”. It is essential that you read this document before you start marking. In particular, please note the following:

- Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.
- If a part is **completely correct**, (and gains all the “must be seen” marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp **A0** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.
- All the marks will be added and recorded by RM™ Assessor.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award **M0** followed by **A1**, as **A** mark(s) depend on the preceding **M** mark(s), if any.
- Where **M** and **A** marks are noted on the same line, eg **M1A1**, this usually means **M1** for an **attempt** to use an appropriate method (eg substitution into a formula) and **A1** for using the **correct** values.
- Where the markscheme specifies **(M2)**, **N3**, etc., do **not** split the marks.

- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final **A1**. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct **FT** working shown, award **FT** marks as appropriate but do not award the final **A1** in that part.

Examples

| | Correct answer seen | Further working seen | Action |
|----|----------------------|---|---|
| 1. | $8\sqrt{2}$ | 5.65685... (incorrect decimal value) | Award the final A1 (ignore the further working) |
| 2. | $\frac{1}{4}\sin 4x$ | $\sin x$ | Do not award the final A1 |
| 3. | $\log a - \log b$ | $\log(a - b)$ | Do not award the final A1 |

3 N marks

Award **N** marks for **correct** answers where there is **no** working.

- Do **not** award a mixture of **N** and other marks.
- There may be fewer **N** marks available than the total of **M**, **A** and **R** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

Implied marks appear in **brackets eg (M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s). To award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part.

- If the question becomes much simpler because of an error then use discretion to award fewer **FT** marks.
- If the error leads to an inappropriate value (eg $\sin \theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent A** marks can be awarded, but **M** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Misread

If a candidate incorrectly copies information from the question, this is a misread (**MR**). A candidate should be penalized only once for a particular misread. Use the **MR** stamp to indicate that this has been a misread. Then deduct the first of the marks to be awarded, even if this is an **M** mark, but award all others so that the candidate only loses one mark.

- If the question becomes much simpler because of the **MR**, then use discretion to award fewer marks.
- If the **MR** leads to an inappropriate value (eg $\sin \theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks (d)

An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, etc.
- Alternative solutions for part-questions are indicated by **EITHER . . . OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2 \sin(5x - 3)$, the markscheme gives

$$f'(x) = (2 \cos(5x - 3)) 5 \quad (= 10 \cos(5x - 3)) \quad \mathbf{A1}$$

Award **A1** for $(2 \cos(5x - 3)) 5$, even if $10 \cos(5x - 3)$ is not seen.

10 Accuracy of Answers

Candidates should **NO LONGER** be penalized for an accuracy error (**AP**).

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures. Please check work carefully for **FT**.

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

13 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

14. Candidate work

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. This work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

Section A

1. EITHER

eliminating a variable, x , for example to obtain $y + 3z = -16$ and $-5y - 3z = 8$ **M1A1**

attempting to find the value of one variable **M1**

point of intersection is $(-1, 2, -6)$ **A1A1A1**

OR

attempting row reduction of relevant matrix, eg. $\begin{pmatrix} 2 & 1 & -1 & 6 \\ 1 & 3 & 1 & -1 \\ 1 & 2 & -2 & 15 \end{pmatrix}$ **M1**

correct matrix with two zeroes in a column, eg. $\begin{pmatrix} 2 & 1 & -1 & 6 \\ 0 & 5 & 3 & -8 \\ 0 & 1 & 3 & -16 \end{pmatrix}$ **A1**

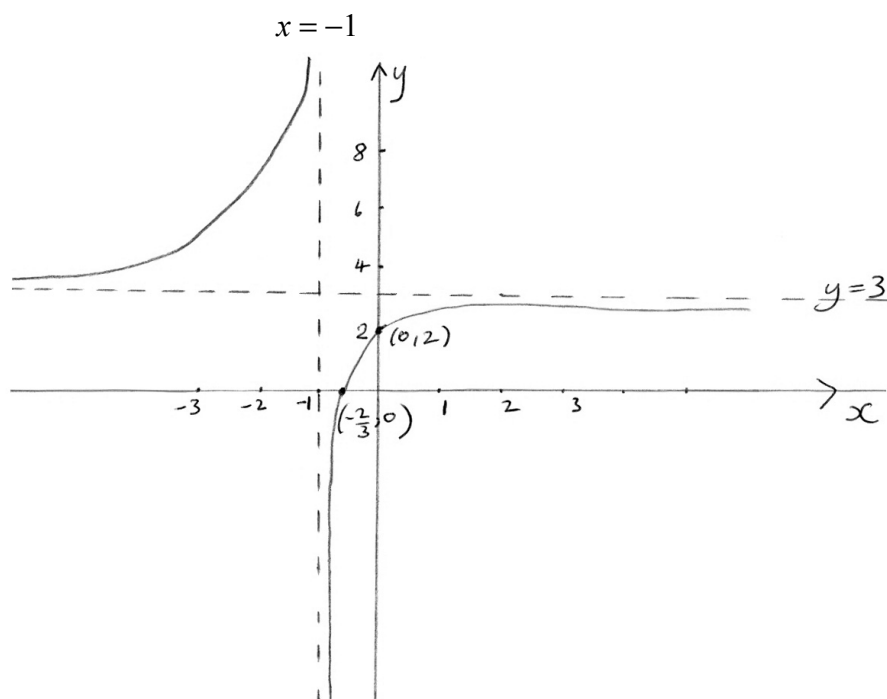
further attempt at reduction **M1**

point of intersection is $(-1, 2, -6)$ **A1A1A1**

Note: Allow solution expressed as $x = -1$, $y = 2$, $z = -6$ for final **A** marks.

[6 marks]

2.



A1A1A1A1A1

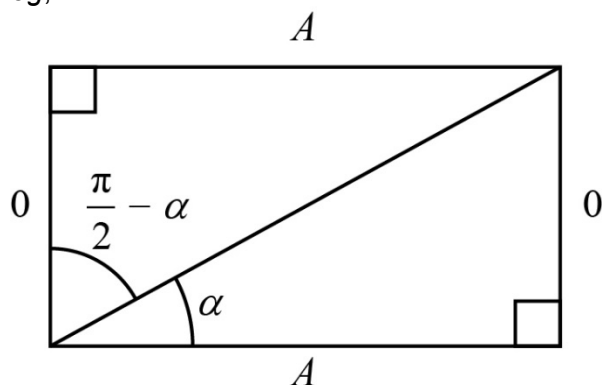
Note: Award **A1** for correct shape, **A1** for $x = -1$ clearly stated and asymptote shown,

A1 for $y = 3$ clearly stated and asymptote shown, **A1** for $\left(-\frac{2}{3}, 0\right)$ and **A1** for $(0, 2)$.

[5 marks]

3. (a) **EITHER**

use of a diagram and trig ratios
eg,



$$\tan \alpha = \frac{O}{A} \Rightarrow \cot \alpha = \frac{A}{O}$$

$$\text{from diagram, } \tan\left(\frac{\pi}{2} - \alpha\right) = \frac{A}{O}$$

R1

OR

$$\text{use of } \tan\left(\frac{\pi}{2} - \alpha\right) = \frac{\sin\left(\frac{\pi}{2} - \alpha\right)}{\cos\left(\frac{\pi}{2} - \alpha\right)} = \frac{\cos \alpha}{\sin \alpha}$$

R1

THEN

$$\cot \alpha = \tan\left(\frac{\pi}{2} - \alpha\right)$$

AG

[1 mark]

$$(b) \int_{\tan \alpha}^{\cot \alpha} \frac{1}{1+x^2} dx = [\arctan x]_{\tan \alpha}^{\cot \alpha}$$

(A1)

Note: Limits (or absence of such) may be ignored at this stage.

$$= \arctan(\cot \alpha) - \arctan(\tan \alpha)$$

(M1)

$$= \frac{\pi}{2} - \alpha - \alpha$$

(A1)

$$= \frac{\pi}{2} - 2\alpha$$

A1

[4 marks]

Total [5 marks]

4.
$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{ax_2^2 + bx_2 + c - (ax_1^2 + bx_1 + c)}{x_2 - x_1} \quad (M1)$$

$$= \frac{a(x_2^2 - x_1^2) + b(x_2 - x_1)}{x_2 - x_1} \quad A1$$

$$= \frac{a(x_2 - x_1)(x_2 + x_1) + b(x_2 - x_1)}{x_2 - x_1} \quad (A1)$$

$$= a(x_2 + x_1) + b \quad (x_1 \neq x_2) \quad A1$$

$$\frac{f'(x_2) + f'(x_1)}{2} = \frac{(2ax_2 + b) + (2ax_1 + b)}{2} \quad M1$$

$$= \frac{2a(x_2 + x_1) + 2b}{2}$$

$$= a(x_2 + x_1) + b \quad A1$$

so Hayley's conjecture is correct AG

[6 marks]

5. (a) $X \sim B(5, p)$ (M1)

$$P(X = 4) = \binom{5}{4} p^4 (1 - p) \text{ (or equivalent)} \quad A1$$

[2 marks]

(b) (i) $\frac{d}{dp}(5p^4 - 5p^5) = 20p^3 - 25p^4$ M1A1

$$5p^3(4 - 5p) = 0 \Rightarrow p = \frac{4}{5} \quad M1A1$$

Note: Do not award the final **A1** if $p = 0$ is included in the answer.

(ii) $E(X) = np = 5\left(\frac{4}{5}\right)$ (M1)

$$= 4 \quad A1$$

[6 marks]

Total [8 marks]

6. (a) $1, nx, \frac{n(n-1)}{2}x^2, \frac{n(n-1)(n-2)}{6}x^3$

A1A1

Note: Award **A1** for the first two terms and **A1** for the next two terms.

Note: Accept $\binom{n}{r}$ notation.

Note: Allow the terms seen in the context of an arithmetic sum.

Note: Allow unsimplified terms, eg, those including powers of 1 if seen.

[2 marks]

(b) (i) **EITHER**

using $u_3 - u_2 = u_4 - u_3$ **(M1)**

$$\frac{n(n-1)}{2} - n = \frac{n(n-1)(n-2)}{6} - \frac{n(n-1)}{2} \quad \mathbf{A1}$$

attempting to remove denominators and expanding (or vice versa) **M1**

$$3n^2 - 9n = n^3 - 6n^2 + 5n \text{ (or equivalent, eg, } 6n^2 - 12n = n^3 - 3n^2 + 2n) \quad \mathbf{A1}$$

OR

using $u_2 + u_4 = 2u_3$ **(M1)**

$$n + \frac{n(n-1)(n-2)}{6} = n(n-1) \quad \mathbf{(A1)}$$

attempting to remove denominators and expanding (or vice versa) **M1**

$$6n + n^3 - 3n^2 + 2n = 6n^2 - 6n \text{ (or equivalent)} \quad \mathbf{(A1)}$$

THEN

$$n^3 - 9n^2 + 14n = 0 \quad \mathbf{AG}$$

(ii) $n(n-2)(n-7) = 0$ or $(n-2)(n-7) = 0$ **(A1)**

$$n = 7 \text{ only (as } n \geq 3) \quad \mathbf{A1}$$

[6 marks]

Total [8 marks]

7. (a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
 $= P(A) + P(B) - P(A)P(B)$ (M1)
 $= p + p - p^2$ A1
 $= 2p - p^2$ AG
 [2 marks]

(b) $P(A|A \cup B) = \frac{P(A \cap (A \cup B))}{P(A \cup B)}$ (M1)

Note: Allow $P(A \cap A \cup B)$ if seen on the numerator.

$$= \frac{P(A)}{P(A \cup B)} \quad (A1)$$

$$= \frac{p}{2p - p^2} \quad A1$$

$$= \frac{1}{2 - p} \quad A1$$

[4 marks]

Total [6 marks]

8. let $P(n)$ be the proposition that $n(n^2 + 5)$ is divisible by 6 for $n \in \mathbb{Z}^+$
 consider $P(1)$:
 when $n = 1$, $n(n^2 + 5) = 1 \times (1^2 + 5) = 6$ and so $P(1)$ is true R1
 assume $P(k)$ is true ie, $k(k^2 + 5) = 6m$ where $k, m \in \mathbb{Z}^+$ M1

Note: Do not award M1 for statements such as “let $n = k$ ”.

consider $P(k + 1)$:

$$(k + 1)((k + 1)^2 + 5) \quad M1$$

$$= (k + 1)(k^2 + 2k + 6)$$

$$= k^3 + 3k^2 + 8k + 6 \quad (A1)$$

$$= (k^3 + 5k) + (3k^2 + 3k + 6) \quad A1$$

$$= k(k^2 + 5) + 3k(k + 1) + 6 \quad A1$$

$k(k + 1)$ is even hence all three terms are divisible by 6 R1

$P(k + 1)$ is true whenever $P(k)$ is true and $P(1)$ is true, so $P(n)$ is true for $n \in \mathbb{Z}^+$ R1

Note: To obtain the final R1, four of the previous marks must have been awarded.

[8 marks]

9. (a) **EITHER**

$$\text{LHS} = \frac{\sqrt{3}-1}{\frac{\sqrt{6}-\sqrt{2}}{4}} + \frac{\sqrt{3}+1}{\frac{\sqrt{6}+\sqrt{2}}{4}} \quad \text{M1}$$

$$= \frac{\sqrt{3}-1}{\frac{\sqrt{3}-1}{2\sqrt{2}}} + \frac{\sqrt{3}+1}{\frac{\sqrt{3}+1}{2\sqrt{2}}} \quad \text{A1}$$

$$= 2\sqrt{2} + 2\sqrt{2} \quad \text{A1}$$

$$\text{LHS} = 4\sqrt{2} \Rightarrow x = \frac{\pi}{12} \text{ is a solution} \quad \text{AG}$$

OR

$$\text{LHS} = \frac{\sqrt{3}-1}{\frac{\sqrt{6}-\sqrt{2}}{4}} + \frac{\sqrt{3}+1}{\frac{\sqrt{6}+\sqrt{2}}{4}} \quad \text{M1}$$

$$= \frac{(\sqrt{3}-1)\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right) + (\sqrt{3}+1)\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)}{\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)} \quad \text{A1}$$

$$= 2\sqrt{18} - 2\sqrt{2} \text{ (or equivalent)} \quad \text{A1}$$

$$\text{LHS} = 4\sqrt{2} \Rightarrow x = \frac{\pi}{12} \text{ is a solution} \quad \text{AG}$$

[3 marks]

$$(b) \quad \frac{\sqrt{2}}{4} \left(\frac{\sqrt{3}-1}{\sin x} + \frac{\sqrt{3}+1}{\cos x} \right) = 2 \Rightarrow \frac{\sin \frac{\pi}{12}}{\sin x} + \frac{\cos \frac{\pi}{12}}{\cos x} = 2 \quad \text{M1}$$

$$\frac{\sin \frac{\pi}{12} \cos x + \cos \frac{\pi}{12} \sin x}{\sin x \cos x} = 2 \quad \text{M1}$$

$$\sin \frac{\pi}{12} \cos x + \cos \frac{\pi}{12} \sin x = 2 \sin x \cos x$$

$$\sin \left(\frac{\pi}{12} + x \right) = \sin 2x \quad \text{A1}$$

$$\frac{\pi}{12} + x = \pi - 2x \text{ or } \pi - \left(\frac{\pi}{12} + x \right) = 2x \quad \text{(M1)}$$

$$x = \frac{11\pi}{36} \quad \text{A1}$$

[5 marks]

Total [8 marks]

Section B

10. (a) **EITHER**

$$\mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} \text{ and } \mathbf{d} = \begin{pmatrix} p \\ 2 \\ 1 \end{pmatrix}$$

A1A1

and $\mathbf{n} \neq k\mathbf{d}$

R1

OR

$$\mathbf{n} \times \mathbf{d} = \begin{pmatrix} -5 \\ 3p - 1 \\ 2 - p \end{pmatrix}$$

M1A1

the vector product is non-zero for $p \in \mathbb{R}$

R1

THEN

L is not perpendicular to Π

AG

[3 marks]

(b) **METHOD 1**

$$(2 + p\lambda) + (q + 2\lambda) + 3(1 + \lambda) = 9$$

M1

$$(q + 5) + (p + 5)\lambda = 9$$

(A1)

$$p = -5 \text{ and } q = 4$$

A1A1

METHOD 2

direction vector of line is perpendicular to plane, so

$$\begin{pmatrix} p \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} = 0$$

M1

$$p = -5$$

A1

$(2, q, 1)$ is common to both L and Π

$$\text{either } \begin{pmatrix} 2 \\ q \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} = 9 \text{ or by substituting into } x + y + 3z = 9$$

M1

$$q = 4$$

A1

[4 marks]

continued...

Question 10 continued

(c) (i) **METHOD 1**

α is the acute angle between \mathbf{n} and L

$$\text{if } \sin \theta = \frac{1}{\sqrt{11}} \text{ then } \cos \alpha = \frac{1}{\sqrt{11}} \quad (M1)(A1)$$

$$\text{attempting to use } \cos \alpha = \frac{\mathbf{n} \cdot \mathbf{d}}{|\mathbf{n}||\mathbf{d}|} \text{ or } \sin \theta = \frac{\mathbf{n} \cdot \mathbf{d}}{|\mathbf{n}||\mathbf{d}|} \quad M1$$

$$\frac{p+5}{\sqrt{11} \times \sqrt{p^2+5}} = \frac{1}{\sqrt{11}} \quad A1A1$$

$$(p+5)^2 = p^2 + 5 \quad M1$$

$$10p = -20 \text{ (or equivalent)} \quad A1$$

$$p = -2 \quad AG$$

METHOD 2

α is the angle between \mathbf{n} and L

$$\text{if } \sin \theta = \frac{1}{\sqrt{11}} \text{ then } \sin \alpha = \frac{\sqrt{10}}{\sqrt{11}} \quad (M1)A1$$

$$\text{attempting to use } \sin \alpha = \frac{|\mathbf{n} \times \mathbf{d}|}{|\mathbf{n}||\mathbf{d}|} \quad M1$$

$$\frac{\sqrt{(-5)^2 + (3p-1)^2 + (2-p)^2}}{\sqrt{11} \times \sqrt{p^2+5}} = \frac{\sqrt{10}}{\sqrt{11}} \quad A1A1$$

$$p^2 - p + 3 = p^2 + 5 \quad M1$$

$$-p + 3 = 5 \text{ (or equivalent)} \quad A1$$

$$p = -2 \quad AG$$

$$(ii) \quad p = -2 \text{ and } z = -1 \Rightarrow \frac{x-2}{-2} = \frac{y-q}{2} = -2 \quad (A1)$$

$$x = 6 \text{ and } y = q - 4 \quad (A1)$$

$$\text{this satisfies } II \text{ so } 6 + q - 4 - 3 = 9 \quad M1$$

$$q = 10 \quad A1$$

[11 marks]

Total [18 marks]

11. (a) use of $\pi \int_a^b x^2 dy$ (M1)

Note: Condone any or missing limits.

$$V = \pi \int_0^{\pi} (3 \cos 2y + 4)^2 dy \quad (A1)$$

$$= \pi \int_0^{\pi} (9 \cos^2 2y + 24 \cos 2y + 16) dy \quad A1$$

$$9 \cos^2 2y = \frac{9}{2} (1 + \cos 4y) \quad (M1)$$

$$= \pi \left[\frac{9y}{2} + \frac{9}{8} \sin 4y + 12 \sin 2y + 16y \right]_0^{\pi} \quad M1A1$$

$$= \pi \left(\frac{9\pi}{2} + 16\pi \right) \quad (A1)$$

$$= \frac{41\pi^2}{2} (\text{cm}^3) \quad A1$$

Note: If the coefficient “ π ” is absent, or eg, “ 2π ” is used, only **M** marks are available.

[8 marks]

- (b) (i) attempting to use $\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dV}{dh}$ with $\frac{dV}{dt} = 2$ (M1)

$$\frac{dh}{dt} = \frac{2}{\pi(3 \cos 2h + 4)^2} \quad A1$$

- (ii) substituting $h = \frac{\pi}{4}$ into $\frac{dh}{dt}$ (M1)

$$\frac{dh}{dt} = \frac{1}{8\pi} (\text{cm min}^{-1}) \quad A1$$

Note: Do not allow FT marks for (b)(ii).

[4 marks]

- (c) (i) $\frac{d^2h}{dt^2} = \frac{d}{dt} \left(\frac{dh}{dt} \right) = \frac{dh}{dt} \times \frac{d}{dh} \left(\frac{dh}{dt} \right)$ (M1)

$$= \frac{2}{\pi(3 \cos 2h + 4)^2} \times \frac{24 \sin 2h}{\pi(3 \cos 2h + 4)^3} \quad M1A1$$

Note: Award **M1** for attempting to find $\frac{d}{dh} \left(\frac{dh}{dt} \right)$.

$$= \frac{48 \sin 2h}{\pi^2 (3 \cos 2h + 4)^5} \quad A1$$

continued...

Question 11 continued

(ii) $\sin 2h = 0 \Rightarrow h = 0, \frac{\pi}{2}, \pi$ **A1**

Note: Award **A1** for $\sin 2h = 0 \Rightarrow h = 0, \frac{\pi}{2}, \pi$ from an incorrect $\frac{d^2h}{dt^2}$.

(iii) **METHOD 1**

$\frac{dh}{dt}$ is a minimum at $h = 0, \pi$ and the container is widest at these values

R1

$\frac{dh}{dt}$ is a maximum at $h = \frac{\pi}{2}$ and the container is narrowest at this value

R1

[7 marks]

Total [19 marks]

12. (a) **EITHER**

$$w^7 = \left(\cos \frac{2\pi}{7} + i \sin \frac{2\pi}{7} \right)^7 \quad \text{(M1)}$$

$$= \cos 2\pi + i \sin 2\pi \quad \text{A1}$$

$$= 1 \quad \text{A1}$$

so w is a root **AG**

OR

$$z^7 = 1 = \cos(2\pi k) + i \sin(2\pi k) \quad \text{(M1)}$$

$$z = \cos\left(\frac{2\pi k}{7}\right) + i \sin\left(\frac{2\pi k}{7}\right) \quad \text{A1}$$

$$k = 1 \Rightarrow z = \cos\left(\frac{2\pi}{7}\right) + i \sin\left(\frac{2\pi}{7}\right) \quad \text{A1}$$

so w is a root **AG**

[3 marks]

(b) (i) $(w - 1)(1 + w + w^2 + w^3 + w^4 + w^5 + w^6)$

$$= w + w^2 + w^3 + w^4 + w^5 + w^6 + w^7 - 1 - w - w^2 - w^3 - w^4 - w^5 - w^6 \quad \text{M1}$$

$$= w^7 - 1 (= 0) \quad \text{A1}$$

(ii) $w^7 - 1 = 0$ and $w - 1 \neq 0$ **R1**

so $1 + w + w^2 + w^3 + w^4 + w^5 + w^6 = 0$ **AG**

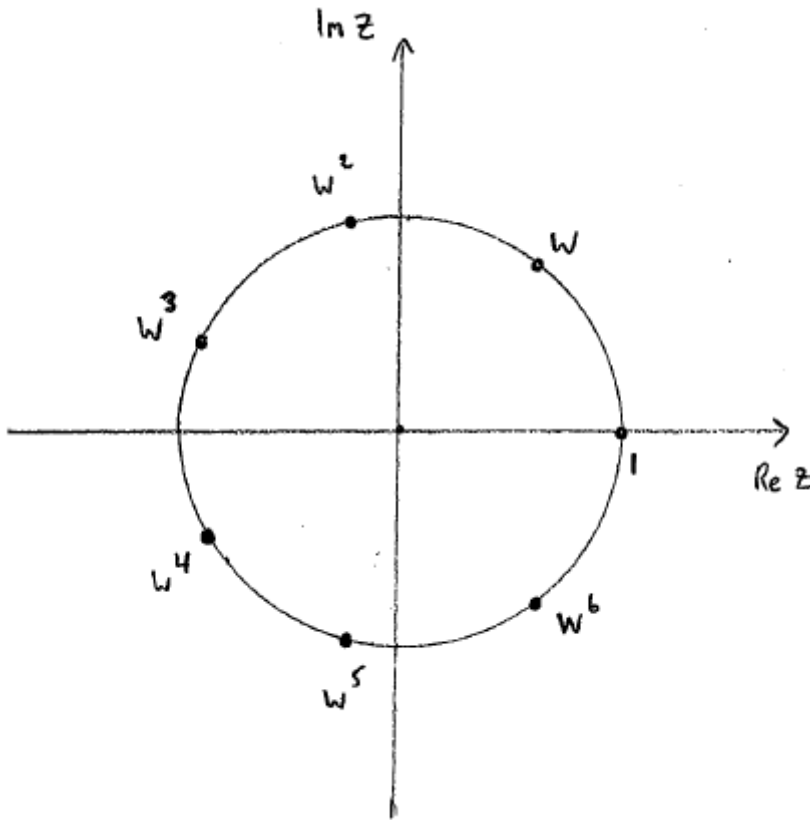
[3 marks]

continued...

Question 12 continued

- (c) the roots are $1, w, w^2, w^3, w^4, w^5$ and w^6

A1



7 points equidistant from the origin

A1

approximately correct angular positions for $1, w, w^2, w^3, w^4, w^5$ and w^6

A1

Note: Condone use of *cis* notation for the final two **A** marks.

Note: For the final **A** mark there should be one root in the first quadrant, two in the second, two in the third, one in the fourth, and one on the real axis.

[3 marks]

(d) (i) $\alpha^* = (w + w^2 + w^4)^*$
 $= w^* + (w^2)^* + (w^4)^*$
 since $w^* = w^6, (w^2)^* = w^5$ and $(w^4)^* = w^3$
 $\Rightarrow \alpha^* = w^6 + w^5 + w^3$

A1

R1

AG

continued...

Question 12 continued

$$(ii) \quad b = -(\alpha + \alpha^*) \quad (\text{using sum of roots (or otherwise)}) \quad (M1)$$

$$b = -(w + w^2 + w^3 + w^4 + w^5 + w^6) \quad (A1)$$

$$= -(-1)$$

$$= 1$$

A1

$$c = \alpha\alpha^* \quad (\text{using product of roots (or otherwise)}) \quad (M1)$$

$$c = (w + w^2 + w^4)(w^6 + w^5 + w^3)$$

EITHER

$$= w^{10} + w^9 + w^8 + 3w^7 + w^6 + w^5 + w^4 \quad (A1)$$

$$= (w^6 + w^5 + w^4 + w^3 + w^2 + w) + 3 \quad (M1)$$

$$= 3 - 1 \quad (A1)$$

OR

$$= w^{10} + w^9 + w^8 + 3w^7 + w^6 + w^5 + w^4 \quad (= w^4(1 + w + w^3)(w^3 + w^2 + 1)) \quad (A1)$$

$$= w^4(w^6 + w^5 + w^4 + w^2 + w + 1 + 3w^3) \quad (M1)$$

$$= w^4(w^6 + w^5 + w^4 + w^3 + w^2 + w + 1 + 2w^3)$$

$$= w^4(2w^3) \quad (A1)$$

THEN

$$= 2$$

A1

[10 marks]

$$(e) \quad z^2 + z + 2 = 0 \Rightarrow z = \frac{-1 \pm i\sqrt{7}}{2} \quad (M1A1)$$

$$\operatorname{Im}(w + w^2 + w^4) > 0 \quad (R1)$$

$$\operatorname{Im} \alpha = \frac{\sqrt{7}}{2} \quad (A1)$$

Note: Final **A** mark is independent of previous **R** mark.

[4 marks]

Total [23 marks]