Mathematics: analysis and approaches

Higher level

Paper 3

ID: 3009

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[55 marks]**.

Attempt 1: $\frac{35}{55} = 64\%$.

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ANSWER BOOKLET
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$\frac{35}{56} = 66\%$.

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Example / Ejemplo 27 | 27

Example / Ejemplo 3 | 3

1

$g(x) = 2x - x^2$ $g'(x) = 2 - 2x$

(a)(i) $h = 2$

x_n	$g'(x)$	y_n
0	2	0
2	-2	4
4	-6	0

(ii) $h = 1$

x_n	$g'(x_n)$	y_n
0	2	0
1	0	2
2	-2	2
3	-4	0
4	-6	-4

04AX01

(iii) $h = 0.5$

x_n	$g'(x_n)$	y_n
0	2	0
0.5	1	0.5 ✗
1	0	0.5
1.5	-1	0
2	-2	-0.5
2.5	-3	-1.5
3	-4	-3
3.5	-5	-5 ✓
4	-6	-7.5

ECF

10

(b) $g(4) = 2x - x^2$
$= 8 - 16$
$= -8$ ✓

2

(c) As $h \to 0$, $y_n \to g(x_n)$

English: as step length decreases, the answer from Eulers method becomes more accurate. ✓

2

(d) $f(x) = g'(x)$

$F(x) =$ antiderivative of $\boxed{f(x) = g'(x)}$

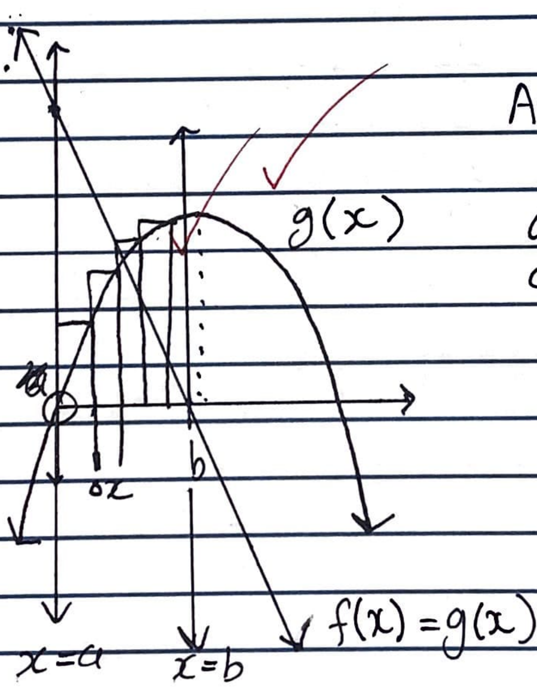
(i) $g(b) = g(a) + \lim_{n \to \infty} \sum_{i=0}^{n} g'(x_i) \Delta x$

$\therefore F(b) = F(a) + \lim_{n \to \infty} \sum_{i=0}^{n} f(x_i) \Delta x$ 2

(ii) $F(b) - F(a) = \lim_{n \to \infty} \sum_{i=0}^{n} f(x_i) \Delta x$ {part (di)}

$\therefore \int_a^b f(x)\,dx = \lim_{n \to \infty} \sum_{i=0}^{n} f(x_i) \Delta x$ 2

(e)



As $\Delta x \to 0$ due to $n \to \infty$, the area of each box on the graph tends to the area underneath the curve.

As $A = \sum_{i=0}^{n} f(x_i) \Delta x$, then $\int_a^b f(x)\,dx = \lim_{n \to \infty} A$

5

23

2

(a) $y = 1/2x$

$\therefore \int_1^{k+1} \frac{1}{2x}\,dx = \left[\ln x \times 1/2\right]_1^{k+1}$

$= \frac{1}{2} \times (\ln(k+1) - \ln(1))$

$= \ln(k+1)/2$

3

(b) $\sum_{n=1}^{k} \frac{1}{2n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8}$

$\therefore u_1 = 1/2 \qquad d = 1/2$

$\therefore S_k = \frac{k}{2}(2(\frac{1}{2}) + (k-1)(1/2))$

$= \frac{k}{2}(1 + \frac{1}{2}(k-1))$

$= \frac{k}{2}(\frac{1}{2} + \frac{1}{2}k)$

$= \frac{k}{4}(1+k)$

$= (k+k^2)/4 \qquad (1)$

upper rectangles: $A = \sum_{n=1}^{k} \frac{1}{2n}$

lower ~~upper~~ rectangles: $A = \sum_{n=2}^{k+1} \frac{1}{2n} < \ln(k+1)$

$\therefore \frac{1}{2(k+1)} + \sum_{n=2}^{k} < \ln(k+1)$

$\therefore 1/2 + 1/(2(k+1)) + \sum_{n=1}^{k} \frac{1}{2n} < \ln(k+1)$

$\therefore \sum_{n=1}^{k} \frac{1}{2n} < \frac{(k+1)(1+\ln(k+1)) - 1}{2(k+1)}$

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Example Ejemplo 27 | 2 | 7 | Example Ejemplo 3 | | 3 |

| 2 |

(c) $\lim_{k\to\infty} \sum_{n=1}^{\infty} \frac{1}{2n} = \lim_{k\to\infty}\left(\frac{k}{2}(2u_1+(k-1)d)\right)$

×

$u_1 = 1/2 \qquad d = 1/2$

$= \lim_{k\to\infty}\left(\frac{k}{2}\left(1+(k-1)/2\right)\right)$

$= \lim_{k\to\infty}\left(\frac{k}{2}\left(1-1/2+k/2\right)\right)$

$= \lim_{k\to\infty}\left(\frac{k}{4}+\frac{k^2}{4}\right)$

$\Rightarrow$ cannot use L'H's due to denominator

$\Rightarrow \lim_{k\to\infty}\left(\frac{k}{4}+\frac{k^2}{4}\right) = \infty$

$\Rightarrow$ diverges.

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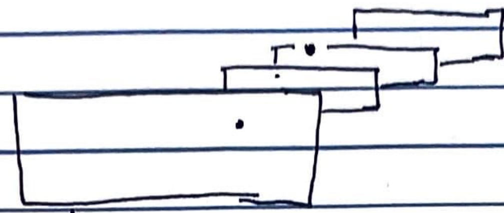
(e)



table

x - coord of centre of mass of hypothetical card ON the table is

$x = -1/2$

$\therefore x_1 = -1/2 + 1/8 = -3/8$

$x_2 = -1/2 + 1/8 + 1/6 = -5/24$

$x_3 = -1/2 + 1/8 + 1/6 + 1/4 = 1/24$

$x_4 = -1/2 + 1/8 + 1/6 + 1/4 + 1/2 = 13/24$

$\therefore (x_1 + x_2 + x_3 + x_4)/4 = 0$

(f) $\cancel{x_n = -1/2 + \sum_{r=n}^{n} \frac{1}{2r}}$

let $x = 0 \rightarrow x_R = 1/2$

$$\left.\begin{aligned} x_1 &= 1/8 \\ x_2 &= 1/8 + 1/6 \\ x_3 &= 1/8 + 1/6 + 1/4 \\ x_4 &= 1/8 + 1/6 + 1/4 + 1/2 \end{aligned}\right\} \bar{X} = 1/2$$

$\therefore LHS = \dfrac{x_1 + x_2 + x_3 + \cdots + x_n}{n}$

$= \dfrac{\frac{n}{2n} + \frac{n-1}{2(n-1)} + \frac{n-2}{2(n-2)} + \cdots + \frac{2}{4} + \frac{1}{2}}{n}$

$= \left(\sum_{r=1}^{n} \frac{r}{2r}\right) \Big/ n = 1/2$

Step 1: prove $n=1$

$$LHS = \frac{\sum_{r=1}^{1} \frac{r}{2r}}{1} = \frac{1/2}{1} = 1/2 = RHS$$

$\therefore$ true for $n=1$

Step 2: ~~prov~~ assume $n=k$ is true

$$\therefore \frac{\sum_{r=1}^{k} \frac{r}{2r}}{k} = 1/2 \quad \text{(inductive hypothesis)}$$

Step 3: consider $n=k+1$:

$$\frac{\sum_{r=1}^{k+1} \frac{r}{2r}}{k+1} = \frac{1}{2}$$

$$LHS = \frac{\sum_{r=1}^{k} \frac{r}{2r} + \frac{k+1}{2(k+1)}}{k+1}$$

$$= \frac{1/2 + \cancel{(k+1)}/2(\cancel{k+1})}{k+1} \quad \text{(by inductive hypothesis)}$$

$$= \frac{1/2}{k+1} + \frac{1/2}{k+1}$$

$$= \frac{1}{k+1}$$

3

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(g) As $\lim_{k\to\infty} \sum_{n=1}^{k} \frac{1}{2n}$ diverges, this is possible, and the induction was proven for any $k \in \mathbb{Z}^+$

(h) $d = \sum_{n=1}^{k} \frac{1}{2n} = 5$

$\therefore \frac{k}{2}(1+(k-1)(\frac{1}{2})) = 5$

$\therefore k(1+\frac{1}{2}k-\frac{1}{2}) = 10$

$\therefore k+\frac{1}{2}k^2-\frac{1}{2}k = 10$

$\therefore 2k+k^2-k = 20$

$\therefore k^2+k-20 = 0$

$\therefore k=-5,\ 4$

$\therefore k=4$.

$\frac{1}{6} - \frac{1}{4}$

$(\frac{1}{6})/(\frac{1}{4}) = \frac{3}{2}$

$\frac{8}{6} = \frac{4}{3}$

04AX01

(f) $x_1 = 1/8$

$x_2 = 1/8 + 1/6$

$x_3 = 1/8 + 1/6 + 1/4$

$x_4 = 1/8 + 1/6 + 1/4 + 1/2$.

$$\bar{x}_n = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{}$$

$$= \frac{\frac{1}{2} + \left(\frac{1}{4}\right)(2) + \left(\frac{1}{6}\right)(3) + \cdots + \left(\frac{1}{2(n-1)}\right)(n-1) + \left(\frac{1}{2n}\right)(n)}{n}$$

$\cancel{x_1 = \sum}$

$$= \frac{\frac{1}{2} + \frac{2}{4} + \frac{3}{6} + \cdots + \left(\frac{1}{2(n-1)}\right)(n-1) + \left(\frac{1}{2n}\right)(n)}{n}$$

$$= \frac{\sum_{n=1}^{K} \frac{n}{2n}}{K}.$$

↓