1 Problem

Mathematics: Analysis and Approaches
Higher level
2022 Semester 2 Examinations
Paper 1

ST ANDREW'S CATHEDRAL SCHOOL
FOUNDED 1885

Friday, August 26rd (afternoon).

2 hours

Candidate number

0	0	3	3	7	6	0	0	4	1

JAMES SULLIVAN

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Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is **not allowed** for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all the questions in the answer booklet provided. Fill in your session number on the front of the answer booklet and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

$43 + \cancel{48} = \cancel{91}$

$49 = 92$

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

Consider the functions f, g, defined for $x \in \mathbb{R}$, given by

$f(x) = e^{-x} \sin x$ and $g(x) = e^{-x} \cos x$.

(a) Find an expression for $f'(x) + g'(x)$ [2]

(b) Hence, or otherwise, find $\int_0^{\pi} e^{-x} \sin x \, dx$. [2]

a) $f'(x) = e^{-x}\cos x - e^{-x}\sin x$

$= e^{-x}(\cos x - \sin x)$

$g'(x) = -e^{-x}\cos x - e^{-x}\sin x$

$= e^{-x}(-\cos x - \sin x)$

$\therefore f'(x) + g'(x) = e^{-x}(\cos x - \sin x - \sin x - \cos x)$

$= -2e^{-x}\sin x$

b) $\int_0^{\pi} e^{-x}\sin x\,dx = \frac{1}{-2}\int -2e^{-x}\sin x\,dx$

$= \left[-\frac{1}{2}(f(x) + g(x))\right]_0^{\pi}$

$= \left[-\frac{1}{2}(e^{-x}\sin x + e^{-x}\cos x)\right]_0^{\pi}$

$= \left[-\frac{1}{2}e^{-x}\sin x - \frac{1}{2}e^{-x}\cos x\right]_0^{\pi}$

$= -\frac{1}{2}e^{-\pi}(0) - \frac{1}{2}e^{-\pi} - (0 - \frac{1}{2}e^{0})$

$= -\frac{1}{2}(e^{-\pi} + 1)$

2

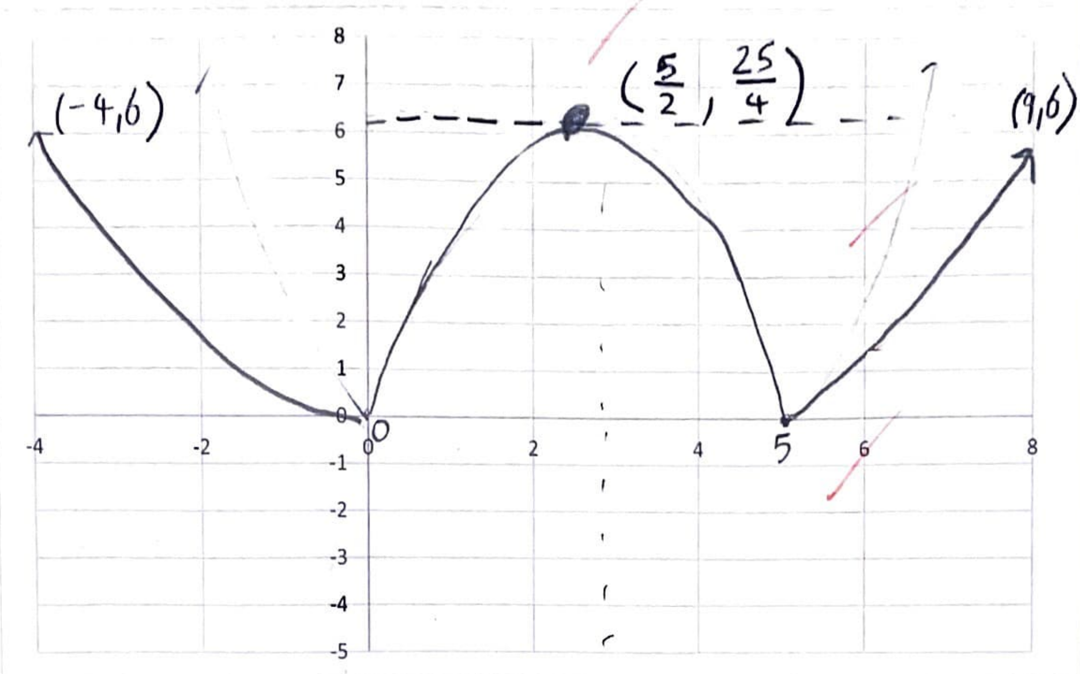
2

4.

2. [Maximum mark: 7]

(a) Sketch the function $y = |x^2 - 5x|$, clearly indicating all key points. [3]

(b) Hence or otherwise, solve $|x^2 - 5x| < 6$ [4]



3

a) $x^2 - 5x = 0 \Rightarrow x = 0,\ x = 5$

$\therefore x(x-5) = 0$

~~AOS = $-b/2a$ = $5/2 \rightarrow y = \frac{25}{4} - \frac{25}{2}$~~

~~vertex~~ $= \cancel{-25/4}$

AOS = $5/2 \rightarrow y = \left(\frac{5}{2}\right)^2 - 5\left(5/2\right)$

$= \frac{25}{4} - \frac{25}{2}$

$= -25/4$

b) $x^2 - 5x < 36$ why 36? → 6

$\therefore x^2 - 5x - 36 < 0$

$\therefore x = \frac{5 \pm \sqrt{25 + 144}}{2}$

$= \frac{5 \pm 13}{2} = 9, -\cancel{8}\ -4$

continued in answer book

3

3

6

3. [Maximum mark: 7]

Solve the simultaneous equations

$$\log_2 6x = 1 + 2\log_2 y$$

$$1 + \log_6 x = \log_6(15y - 25).$$

$\log_2 6x = 1 + 2\log_2 y = \dfrac{\log_6 6x}{\log_6 2}$ ... (1)

$1 + \log_6 x = \log_6(15y-25)$ ... (2)

$\therefore$ (1) $\cancel{\to (2)}$: $1 + 2\log_2 y = \dfrac{\log_6 6 + \log_6 x}{\log_6 2}$

$= \dfrac{1 + \log_6 x}{\log_6 2}$

$\therefore \log_6 x = (\log_6 2)(1 + 2\log_2 y)$

$\therefore 1 + (\log_6 2)(1 + 2\log_2 y) = \log_6(15y - 25)$

1) $\log_2 6x = \log_2 2 + \log_2 y^2 \to 6x = 2y^2$

2) $\log_6 6 + \log_6 x = \log_6(15y - 25)$

$\therefore 6x = 15y - 25$

$\therefore$ ~~$x = 15y - 31$~~

~~② → ①~~

~~$6(15y-31) = 2y^2$~~

not finished

$\therefore 2y^2 = 15y - 25$

$\therefore y = 5/2, \quad x = 25/12$

OR $y = 5, \quad x = 25/3$

4. [Maximum mark: 6]

A and B are independent events such that $\mathrm{P}(A) = \mathrm{P}(B) = p$, $p \neq 0$.

(a) Show that $\mathrm{P}(A \cup B) = 2p - p^2$. [2]

(b) Find $\mathrm{P}(A|A \cup B)$ in simplest form. [4]

a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A)P(B)$

$= p + p - p^2$

$= 2p - p^2$

2

b) $P(A|A \cup B) = \dfrac{P(A \cap (A \cup B))}{P(A \cup B)}$

$= \dfrac{P(A)}{P(A \cup B)}$

$= \dfrac{p}{2p - p^2}$

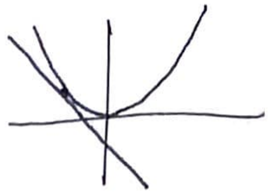
$= \dfrac{p}{p(2-p)}$

$= \dfrac{1}{2-p}$

4

6

=> Why is $x=-1$ a solution ?



5. [Maximum mark: 7]

Find the x-coordinates of all the points on the curve

$y = 2x^4 + 6x^3 + \frac{7}{2}x^2 - 5x + \frac{3}{2}$ at which the tangent to the curve is parallel to the tangent at $(-1,\ 6)$.

$y = 2x^4 + 6x^3 + \frac{7}{2}x^2 - 5x + \frac{3}{2}$

$\therefore \frac{dy}{dx} = 8x^3 + 18x^2 + 7x - 5$

at $x = -1$, $\frac{dy}{dx} = -8 + 18 - 7 - 5$

$= 3 - 5$

$= -2$ at $x = -1$

$\therefore -2 = 8x^3 + 18x^2 + 7x - 5$

$\therefore 8x^3 + 18x^2 + 7x - 3 = 0$?

$= (x+2)(ax^2 + bx + c)$

$$
\begin{array}{r l}
 & 8x^2 + 2x + 3 \\
x + 2 \,\big) & 8x^3 + 18x^2 + 7x - 3 \\
 & -(8x^3 + 16x^2) \\
 & 2x^2 + 7x \\
 & -(2x^2 + 4x) \\
 & 3x - 3 \\
 & 3x + 6 \\
\end{array}
$$

ran out of time :c

$\therefore 8x^3 + 18x^2 + 7x - 3 = (x+2)(8x^2 + 2x + 3)$

4

6. [Maximum mark: 7]

A and B are acute angles such that $\cos A = \frac{2}{3}$ and $\sin B = \frac{1}{3}$.

Show that $\cos(2A + B) = -\frac{2\sqrt{2}}{27} - \frac{4\sqrt{5}}{27}$.

$$\cos(2A+B) = \cos 2A \cos B - \sin 2A \sin B$$

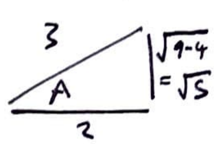
$$= (2\cos^2 A - 1)\cos B - 2\sin A \cos A \sin B$$

$$= \cancel{\left(2\left(\tfrac{4}{9}\right) - 1\right)}$$

$$= \left(2\left(\tfrac{4}{9}\right)-1\right)\left(\tfrac{2\sqrt{2}}{3}\right) - 2\left(\tfrac{\sqrt{5}}{3}\right)\left(\tfrac{2}{3}\right)\left(\tfrac{1}{3}\right)$$

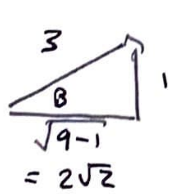
$$= \frac{8-9}{9} \times \frac{2\sqrt{2}}{3} - \frac{4\sqrt{5}}{27}$$

$$= -\frac{2\sqrt{2}}{27} - \frac{4\sqrt{5}}{27}$$



$\cos A = \frac{2}{3}$

$\sin A = \frac{\sqrt{5}}{3}$



$\sin B = 1/3$

$\cos B = \frac{2\sqrt{2}}{3}$

7

7. [Maximum mark: 6]

The quadratic equation $x^2 - 2kx + (k-1) = 0$ has roots α and β such that $\alpha^2 + \beta^2 = 4$.

Without solving the equation, find the possible values of the real number k.

$x^2 - 2kx + (k-1) = 0 \quad \ldots(1)$

~~$(\alpha+\beta)$~~ $\therefore \alpha\beta = k-1$

$\alpha+\beta = -\frac{(-2k)}{1} = 2k$

A1K M1

If $(\alpha+\beta)^2 = \alpha^2+\beta^2 - 2\alpha\beta$, then

$(2k)^2 = 4 - 2(k-1)$

$\therefore 4k^2 + 2(k-1) - 4 = 0$ M1C

$\therefore 4k^2 + 2k - 2 - 4 = 0$

$\therefore 4k^2 + 2k - 6 = 0$

$\therefore 2k^2 + k - 3 = 0$

$\therefore 2k^2 + 3k - 2k - 3 = 0$

$\therefore 2k(k-1) + 3(k-1) = 0$

$\therefore (2k+3)(k-1) = 0$

$\therefore$

$2k+3 = 0$

$\therefore k = -3/2$

$k = 1$

5.

8. [Maximum mark: 10]

Consider the differential equation $\frac{dy}{dx} + \frac{x}{x^2+1}y = x$ where $y = 1$ when $x = 0$.

(a) Show that $\sqrt{x^2+1}$ is an integrating factor for this differential equation. [4]

(b) Solve the differential equation giving your answer in the form $y = f(x)$. [6]

a) $I(x) = e^{\int \frac{x}{x^2+1}dx}$ let $u = x^2+1$, $du = 2x\,dx$

$= e^{\frac{1}{2}\int \frac{2x}{u}dx}$

$= e^{\frac{1}{2}\int \frac{1}{u}du}$

$= e^{\frac{1}{2}\ln|u|}$

$= e^{\frac{1}{2}\ln|x^2+1|}$

$= e^{\ln|\sqrt{x^2+1}|}$

$= \sqrt{x^2+1}$

b) $\sqrt{x^2+1}\frac{dy}{dx} + \sqrt{x^2+1} \times \frac{x}{x^2+1}y = x\sqrt{x^2+1}$

$\therefore \frac{d}{dx}\left[\sqrt{x^2+1}\,y\right] = x\sqrt{x^2+1}$

$\therefore y\sqrt{x^2+1} = \int x\sqrt{x^2+1}\,dx$

[let $u = x^2+1$, $du = 2x\,dx$

$= \frac{1}{2}\int 2x\sqrt{u}\,dx$

$= \frac{1}{2}\int \sqrt{u}\,du$

$= \frac{1}{2}\int u^{1/2}\,du$

$= \frac{2}{3} \times \frac{1}{2} \times u^{3/2} + c$

$= \frac{1}{3}u^{3/2} + c$

hence, $y = \frac{\frac{1}{3}(x^2+1)^{3/2} + c}{\sqrt{x^2+1}}$

when $x = 0$, $y = 1$:

$1 = \frac{\frac{1}{3}(1)^{3/2} + c}{\sqrt{1}}$

$\therefore c = 3$

m/c

hence, $y = \frac{\frac{1}{3}(x^2+1)^{\frac{3}{2}} + 3}{\sqrt{x^2+1}}$

9.

9. [Maximum mark: 6]

Consider the expansion of $(1+x)^n$ in ascending powers of x, where $n \geq 3$.

The coefficients of the second, third and fourth terms of the expansion are consecutive terms of an arithmetic sequence.

(a) Show that $n^3 - 9n^2 + 14n = 0$. [4]

(b) Hence find the value of n. [2]

a) $(1+x)^n = 1 + {}^nC_1 x + {}^nC_2 x^2 + {}^nC_3 x^3 + {}^nC_4 x^4 + \dots$

$\therefore u_1 \; \cancel{u_2} = {}^nC_1 x = \frac{n!}{1!(n-1)!} x$

$u_2 \; \cancel{u_3} = {}^nC_2 x^2 = \frac{n!}{2!(n-2)!} x^2$

$u_3 \; \cancel{u_4} = {}^nC_3 x = \frac{n!}{3!(n-3)!} x^3$

$\therefore d = \frac{n!}{2(n-2)!} x^2 - \frac{n!}{(n-1)!} x$

$= \frac{n!}{2(n-2)!} x^2 - \frac{n!}{(n-1)(n-2)!} x$

$= \frac{n!}{(n-2)!} x \left(\frac{x}{2} - \frac{1}{n-1} \right)$

$= \frac{n!}{(n-2)!} x \left(\frac{x(n-1) - 2}{2n-2} \right)$

$= \frac{n!}{(n-2)!} x \left(\frac{xn - x - 2}{2n-2} \right)$

b) not attempted.

Note: (b) can be done easily even if you can't do (a)

→ updated attempt in answer booklet 14/9

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 12]

Points A(0 , 0 , 10) , B(0 , 10 , 0) , C(10 , 0 , 0) , V(p , p , p) form the vertices of a tetrahedron.

(a) Show that $\overrightarrow{\mathrm{AB}} \times \overrightarrow{\mathrm{AV}} = -10\begin{pmatrix} 10-2p \\ p \\ p \end{pmatrix}$ and find a similar expression for $\overrightarrow{\mathrm{AC}} \times \overrightarrow{\mathrm{AV}}$. [3]

(b) Hence, show that, if the angle between the faces ABV and ACV is θ, then $\cos\theta = \frac{p(3p-20)}{6p^2-40p+100}$. [5]

(c) Consider the case where the faces ABV and ACV are perpendicular. Find the two possible coordinates of V. [3]

(d) Comment on the positions of V in relation to the plane ABC. [1]

$2(k+1)-3$
$= 2k+2-3$
$= 2k-1$

$k+1-2 = k-1$

$(2k-3)! = (2k-3)(2k-4)(2k-5)!$

$(2k-1)(2k-2)(2k-3)!$

11. [Maximum mark: 20]

Let $f(x) = \sqrt{1+x}$ for $x > -1$.

(a) Show that $f''(x) = -\frac{1}{4\sqrt{(1+x)^3}}$. [3]

(b) Use mathematical induction to prove that

$$f^{(n)}(x) = \left(-\frac{1}{4}\right)^{n-1}\frac{(2n-3)!}{(n-2)!}(1+x)^{\frac{1}{2}-n} \text{ for } n \in \mathbb{Z},\ n \geq 2.$$ [9]

(c) Let $g(x) = \mathrm{e}^{mx}$, $m \in \mathbb{Q}$.

Consider the function h defined by $h(x) = f(x) \times g(x)$ for $x > -1$.

It is given that the x^2 term in the Maclaurin series for $h(x)$ has a coefficient of $\frac{7}{4}$.

Find the possible values of m. [8]

$(10-2p)^2 = 100-40p$

Do **not** write solutions on this page.

12. [Maximum mark: 18]

(a) Express $-3+\sqrt{3}\mathrm{i}$ in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r>0$ and $-\pi<\theta\le\pi$. [5]

(b) Let the roots of the equation $z^3=-3+\sqrt{3}\mathrm{i}$ be u, v and w.

Find u, v and w expressing your answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$,

where $r>0$ and $-\pi<\theta\le\pi$. [5]

(c) On an Argand diagram, u, v and w are represented by the points U, V and W respectively.

Find the area of triangle UVW. [4]

(d) By considering the sum of the roots u, v and w, show that

$\cos\frac{5\pi}{18}+\cos\frac{7\pi}{18}+\cos\frac{17\pi}{18}=0.$ [4]

$(\cancel{12})^{1/3}$ $\quad 12^{1/6} =$

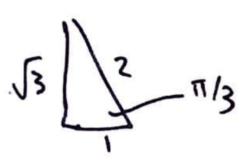
$12^{\frac{1}{6}} \times 12^{\frac{1}{6}}$

$= 12^{1/3}$

End of paper 1

$\left((12)^{1/2}\right)^{1/3}$

$= \sqrt[6]{12}$



$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$= \pi/6$

$= 5\pi/6$

$\frac{5\pi}{6} + 2k\pi$

$= \frac{5\pi + 12k\pi}{6}$

$= \frac{5\pi + 12k\pi}{18}$

$2^6 = 64$

64×12: 128, 640, 768

$k=0: \frac{5\pi}{18}$ $\quad k=1: \frac{5\pi+12\pi}{18} = \frac{17\pi}{18}$

$k=-1: \frac{5\pi-12\pi}{18} = -\frac{7\pi}{18}$

IB practice exams 2022

ANSWER BOOKLET 1

4 PAGES 22 T R L P 1 M A H L

ST ANDREW'S CATHEDRAL SCHOOL

FOUNDED 1885

Candidate session number

003376-0041

Candidate Name

JAMES SULLIVAN

At the start of each answer to a question, write the question number in the box using your normal handwriting

Example 27 | 2 | 7 |

Example 3 | | 3 |

10

a) $A(0,0,10)$ $B(0,10,0)$ $C(10,0,0)$

$V(p,p,p)$

$\therefore \overrightarrow{AB} \times \overrightarrow{AV} = \begin{pmatrix} 0 \\ 10 \\ -10 \end{pmatrix} \times \begin{pmatrix} p-10 \\ p \\ p \end{pmatrix}$

$= \begin{pmatrix} 10p-(-10p) \\ p-10-0 \end{pmatrix}$

$\overrightarrow{AB} = \begin{pmatrix} 0-0 \\ 10-0 \\ 0-10 \end{pmatrix} = \begin{pmatrix} 0 \\ 10 \\ -10 \end{pmatrix}$

$\overrightarrow{AV} = \begin{pmatrix} p-0 \\ p-0 \\ p-10 \end{pmatrix}$

$\therefore \overrightarrow{AB} \times \overrightarrow{AV} = \begin{pmatrix} 0 \\ 0 \\ -10 \end{pmatrix} \times \begin{pmatrix} p \\ p \\ p-10 \end{pmatrix}$

$= \begin{pmatrix} 0+10p \\ -10p-0 \\ 0 \end{pmatrix}$

started on following page.

heart mind life

10

a) $A = \begin{pmatrix} 0 \\ 0 \\ 10 \end{pmatrix}$ $B = \begin{pmatrix} 0 \\ 10 \\ 0 \end{pmatrix}$

$\therefore \overrightarrow{AB} = B - A$

$= \begin{pmatrix} 0 \\ 10 \\ -10 \end{pmatrix}$

$A = \begin{pmatrix} 0 \\ 0 \\ 10 \end{pmatrix}$ $V = \begin{pmatrix} p \\ p \\ p \end{pmatrix}$

$\therefore \overrightarrow{AV} = V - A$

$= \begin{pmatrix} p \\ p \\ p-10 \end{pmatrix}$

$\therefore \overrightarrow{AB} \times \overrightarrow{AV} = \begin{pmatrix} 0 \\ 10 \\ -10 \end{pmatrix} \times \begin{pmatrix} p \\ p \\ p-10 \end{pmatrix}$

2

$= \begin{pmatrix} 10(p-10) - (-10p) \\ -10p - 0 \\ 0 - 10p \end{pmatrix}$

$= \begin{pmatrix} 10p - 100 + 10p \\ -10p \\ -10p \end{pmatrix}$

$\therefore \overrightarrow{AB} \times \overrightarrow{AV} = -10 \begin{pmatrix} 10 - 2p \\ p \\ p \end{pmatrix}$

$$\vec{AC} = C - A$$

$$= \begin{pmatrix} 10-0 \\ 0 \\ 0-10 \end{pmatrix}$$

$$\therefore \quad AC \times AV = \begin{pmatrix} 10 \\ 0 \\ -10 \end{pmatrix} \times \begin{pmatrix} p \\ p \\ p-10 \end{pmatrix}$$

$$= \begin{pmatrix} 0(p-10) - (-10p) \\ -10p - 10(p-10) \\ 10p - 0 \end{pmatrix}$$

$$= \begin{pmatrix} 10p \\ -10p + 100 - 10p \\ 10p \end{pmatrix}$$

$$= \begin{pmatrix} 10p \\ 100 - 20p \\ 10p \end{pmatrix}$$

$$\therefore \vec{AC} \times \vec{AV} = 10 \begin{pmatrix} p \\ 10 - 2p \\ p \end{pmatrix}$$

b) $AC \times AU = 10\begin{pmatrix} p \\ 10-2p \\ p \end{pmatrix} = v$

$AB \times AU = -10\begin{pmatrix} 10-2p \\ p \\ p \end{pmatrix} = w$

$$\therefore \cos\theta = \frac{|v \cdot w|}{|v||w|}$$

$$= \frac{|p(10-2p) + (10-2p)p + p^2|}{\sqrt{p^2+(10-2p)^2+p^2}\sqrt{(10-2p)^2+p^2+p^2}}$$

$$= \frac{|10p - 2p^2 + 10p - 2p^2 + p^2|}{2p^2 + (10-2p)^2}$$

$$= \frac{|-3p^2 + 20p|}{2p^2 + 100 - 40p + 4p^2}$$

$$= \frac{|-p(3p-20)|}{6p^2 - 40p + 100}$$

$$= \frac{p(3p-20)}{6p^2-40p+100}$$

5

c) – always write "next page".

heart
mind
life

IB practice exams 2022

ANSWER BOOKLET 2

4 PAGES

ST ANDREW'S CATHEDRAL SCHOOL

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Candidate session number

0 0 3 3 7 6 - 0 0 4 1

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At the start of each answer to a question, write the question number in the box using your normal handwriting

Example 27 [2][7] Example 3 [ ][3]

c) if ABU and ACU are perpendicular,
then $\theta = 90$, $\cos\theta = 0$

$\therefore p(3p-20) = 0$

$\therefore p = 0$ OR $3p-20=0$

$\therefore p = 20/3$

$\therefore V$ is $(0,0,0)$

OR

V is $\left(\frac{20}{3}, \frac{20}{3}, \frac{20}{3}\right)$

3

d) not completed

11

Should be 11?

heart mind life

11

a) $f(x) = (1+x)^{1/2}$

$\therefore f'(x) = \frac{1}{2}(1+x)^{-1/2}$

$\therefore f''(x) = (\frac{1}{2})(-\frac{1}{2})(1+x)^{-3/2}$ 3

$= -\frac{1}{4\sqrt{(1+x)^3}}$

b) $f^{(n)}(x) = \left(-\frac{1}{4}\right)^{n-1} \frac{(2n-3)!}{(n-2)!}(1+x)^{\frac{1}{2}-n}$

Step 1: prove for $n = 2$:

$LHS = f''(x) = -\frac{1}{4\sqrt{(1+x)^3}}$

$RHS = \left(-\frac{1}{4}\right)\frac{(4-3)!}{(2-2)!}(1+x)^{\frac{1}{2}-2}$

$= \left(-\frac{1}{4}\right)\left(\frac{1}{1}\right)(1+x)^{-3/2}$

$= -\frac{1}{4\sqrt{(1+x)^3}} = LHS$

$\therefore$ true for $n = 2$

Step 2: assume true for $n = k$

$f^{(k)}(x) = \left(-\frac{1}{4}\right)^{k-1} \frac{(2k-3)!}{(k-2)!}(1+x)^{\frac{1}{2}-k}$

Step 3: prove true for $n = k+1$ when $n = k$ is assumed to be true.

$f^{(k+1)}(x) = \frac{d}{dx} f^{(k)}(x)$

$= \frac{d}{dx}\left[\left(-\frac{1}{4}\right)^{k-1} \frac{(2k-3)!}{(k-2)!}(1+x)^{\frac{1}{2}-k}\right]$

$= \left(\frac{1}{2}-k\right)\left(-\frac{1}{4}\right)^{k-1} \frac{(2k-3)!}{(k-2)!}(1+x)^{\frac{1}{2}-k-1}$

$$= \left(\frac{1-2k}{2}\right)\left(-\frac{1}{4}\right)^{k}\left(-\frac{4}{1}\right)\frac{(2k-3)!}{(k-2)!}(1+x)^{\frac{1}{2}-(k+1)}$$

$$= \left(\frac{(2k-1)}{2}\right)\left(-\frac{4}{1}\right)^{k}\frac{(2k-2)(2k-3)!}{(2k-2)(k-2)!}(1+x)^{\frac{1}{2}-(k+1)}$$

$$= \left(\frac{1}{2}\right)\left(-\frac{1}{4}\right)^{k}\frac{(2k-1)!}{(2k-2)(k-2)!}(1+x)^{\frac{1}{2}-(k+1)}$$

$$= \left(\frac{1}{2}\right)\left(-\frac{1}{4}\right)^{k}\frac{(2k-1)!(k-1)}{(2k-2)(k-1)!}(1+x)^{\frac{1}{2}-(k+1)}$$

$$= \left(\frac{1}{2}\right)\left(-\frac{1}{4}\right)^{k}\frac{(2k-1)!}{2(k-1)!}(1+x)^{\frac{1}{2}-(k+1)}$$

$$= \left(\frac{1}{4}\right)\left(-\frac{1}{4}\right)^{k}\frac{(2k-1)!}{2(k-1)!}(1+x)^{\frac{1}{2}-(k+1)}$$

$$= \left(\frac{1-2k}{2}\right)\left(-\frac{1}{4}\right)^{k}\left(-\frac{4}{1}\right)\frac{(2k-3)!}{(k-2)!}(1+x)^{\frac{1}{2}-(k+1)}$$

$$= (+2)\left(-\frac{1}{4}\right)^{k}\frac{(2k-1)(2k-2)(2k-3)!}{(2k-2)(k-2)!}(1+x)^{\frac{1}{2}-(k+1)}$$

$$= (2)\left(-\frac{1}{4}\right)^{k}\frac{(2k-1)!(k-1)}{2(k-1)(k-1)!}(1+x)^{\frac{1}{2}-(k+1)}$$

$$= \left(-\frac{1}{4}\right)^{k}\frac{(2k-1)!}{(k-1)!}(1+x)^{\frac{1}{2}-(k+1)}$$

$$= \left(-\frac{1}{4}\right)^{(k+1)-1}\frac{(2(k+1)-3)!}{((k+1)-2)!}(1+x)^{\frac{1}{2}-(k+1)}$$

$\therefore$ true for $n=k+1$ whenever $n=k$ is true

Step 4: as it is true for $n=2$, and is true for $n=k+1$ whenever $n=k$ is also true, then true for all $n \geq 2$, $n \in \mathbb{Z}$ by mathematical induction.

9.

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c

$g(x) = e^{mx}$

$h(x) = f(x) \times g(x) \quad , x > -1$

finding the maclaurin series :

$h'(x) = f(x)g'(x) + f'(x)g(x)$ product rule

$h''(x) = f(x)g''(x) + f'(x)g'(x) + f'(x)g'(x) + f''(x)g(x)$

$f(x) = (1+x)^{-1} \rightarrow f(0) = 1$

$\cancel{f''(x)}\ f'(x) = \frac{1}{2\sqrt{1+x}} \rightarrow f'(0) = 1/2$

$f''(x) = -\frac{1}{4\sqrt{(1+x)^3}} \rightarrow f''(0) = -1/4$

$g(x) = e^{mx} \rightarrow g(0) = 1$

$g'(x) = me^{mx} \rightarrow g'(0) = m$

$g''(x) = m^2 e^{mx} \rightarrow g''(0) = m^2$

$\therefore h(0) = 1.1$

$= 1$

$\therefore h'(0) = f(0)g'(0) + f'(0)g(0)$

$= 1.m + \frac{1}{2}$

$= m + 1/2$

$h''(0) = f(0)g''(0) + 2f'(0)g'(0) + f''(0)g(0)$

$= 1.m^2 + 2(\frac{1}{2}m) + \cancel{f''(0)g(0)}(-\frac{1}{4})$

$= m^2 + m - 1/4$

$\therefore$ maclaurin $= h(0) + xh'(0) + \frac{x^2}{2}h''(0)$

$= 1 + (m+\frac{1}{2})x + \frac{m^2+m-1/4}{2}x^2$

~~equate~~ $\therefore$ coefficients : $\frac{m^2+m-1/4}{2} = \frac{7}{4}$

$\therefore 2m^2 + 2m - 1/2 = 7$

$\therefore 4m^2 + 4m - 14 = 0$

continued on following booklet 3

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ANSWER BOOKLET (3)

4 PAGES

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Candidate session number

0	0	3	3	7	6	-	0	0	4	1

Candidate Name: JAMES SULLIVAN

At the start of each answer to a question, write the question number in the box using your normal handwriting

Example 27 [2][7]   Example 3 [ ][3]

$\therefore \quad 4m^2 + 4m - 15 = 0$

$\therefore \quad 4m^2 + 10m - 6m - 15 = 0$

$\therefore \quad 2m(2m-3) + 5(2m-3) = 0$

$\therefore \quad (2m+5)(2m-3) = 0$

$\therefore$

$2m+5=0 \qquad\qquad 2m-3=0$

$\therefore \quad \underline{\underline{m = -5/2}} \qquad \therefore \quad \underline{\underline{m = 3/2}}$

✓ 8

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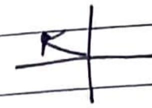
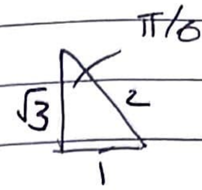
12

a) $-3+\sqrt{3}\,i$



$$r = \sqrt{(-3)^2+(\sqrt{3})^2}$$
$$= \sqrt{9+3}$$
$$= \sqrt{12}$$
$$= 2\sqrt{3}$$



$$\arg = \tan^{-1}\left(\frac{\sqrt{3}}{\cancel{-}3}\right)$$
$$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$
$$= \pi/6 \quad \{\text{acute}\}$$
$$= \pi - \pi/6$$
$$= 5\pi/6 \quad \{Q2\}$$

5

$$\therefore\ -3+\sqrt{3}\,i = 2\sqrt{3}\ \text{cis}(5\pi/6)$$
$$= 2\sqrt{3}\,e^{\frac{5\pi}{6}i}$$

b) $z^3 = 2\sqrt{3}\ \text{cis}\left(\frac{5\pi}{6}+2k\pi\right)$, $k\in\mathbb{Z}$

$$\therefore z = (2\sqrt{3})^{1/3}\ \text{cis}\left(\frac{5\pi}{18}+\frac{2k\pi}{3}\right)$$
$$= \left(2^{1/3}\right)\left(3^{1/6}\right)\ \text{cis}\left(\frac{5\pi+12k\pi}{18}\right)$$

$k=-1$: $= \sqrt[6]{12}\ \text{cis}\left(\frac{5\pi-12\pi}{18}\right) = \sqrt[6]{12}\left(-\frac{7\pi}{18}\right)$

$k=0$: $= \sqrt[6]{12}\ \text{cis}\left(\frac{5\pi}{18}\right)$

$k=1$: $= \sqrt[6]{12}\ \text{cis}\left(\frac{5\pi+12\pi}{18}\right) = \sqrt[6]{12}\left(\frac{17\pi}{18}\right)$

continued on following page

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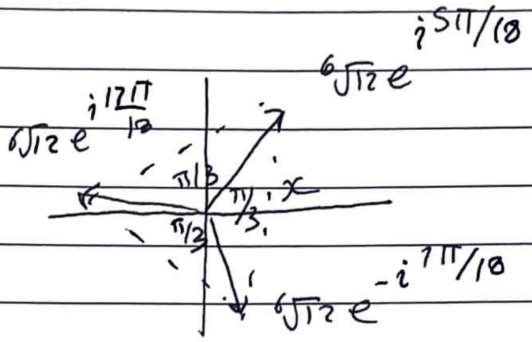
The 3 roots of z are:

$u = \sqrt[6]{12}\, e^{-i7\pi/18}$

$v = \sqrt[6]{12}\, e^{i5\pi/18}$ 5.

$w = \sqrt[6]{12}\, e^{i17\pi/18}$

c)



$$x^2 = \sqrt[6]{12} + \sqrt[6]{12} - 2(\sqrt[6]{12})(\sqrt[6]{12})\cos(\pi/3)$$

$$\therefore x^2 = 12^{1/3} - 2(12^{1/12})$$

$$= \sqrt[6]{12} + \sqrt[6]{12} - 2 \times 12^{1/3} \times 1/2$$

$$= 2 \times \sqrt[6]{12} - \sqrt[3]{12}$$

$$\therefore x = \sqrt{2 \times \sqrt[6]{12} - \sqrt[3]{12}}$$

$$\therefore A = \frac{1}{2} x^2$$

$$A = 3 \times \text{Area of minor triangle}$$

$$= 3 \times \tfrac{1}{2}(|z|)(|z|)\sin(\pi/3)$$

$$= \tfrac{3}{2}(\sqrt[6]{12})(\sqrt[6]{12}) \times \sqrt{3}/2$$

$$= \frac{3\sqrt{3}}{4}(\sqrt[3]{12}) \text{ units}^2$$

4

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d) $u + v + w = \sqrt[6]{12}\left(e^{-i7\pi/18} + e^{i5\pi/18} + e^{i17\pi/18}\right)$

$= \sqrt[6]{12}\left(\cos\frac{7\pi}{18} - i\sin\frac{7\pi}{18} + \cos\frac{5\pi}{18} + i\sin\frac{5\pi}{18} + \cos\frac{17\pi}{18} + i\sin\frac{17\pi}{18}\right)$

$\therefore \operatorname{Re}(u+v+w) = 0$

$\therefore \cos\frac{7\pi}{18} + \cos\frac{5\pi}{18} + \cos\frac{17\pi}{18} = 0$ ✓4.

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IB practice exams 2022

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4 PAGES

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003376-0041

Candidate Name

JAMES SULLIVAN

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Example 27 | 2 | 7 | Example 3 | | 3 |

2

$x^2 - 5x < -6$

$\therefore x^2 - 5x + 6 < 0$

$\therefore (x-3)(x-2) < 0$

$\therefore x < 3, \quad x < 2$

hence, ~~#~~ $|x^2 - 5x| < 6$ for:

$-4 < x < 2$ and $3 < x < 9$

3

heart mind life

3

$$1 + \log_6 x = \log_6 (15y - 25)$$

$$\therefore \quad \log_6 x = \log_6 (15y - 25) - 1 \quad \ldots (1)$$

$$\log_2 6x = 1 + \log_2 y$$

$$\therefore \quad \frac{\log_6 6x}{\log_6 2} = 1 + \log_2 y$$

$\therefore$

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5

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Candidate name: / Nom du candidat: / Nombre del alumno: JAMIE SULLIVAN

At the start of each answer to a question, write the question number in the box using your normal hand writing / Avant de répondre à une question, inscrivez son numéro à la main dans la case appropriée / Al comienzo de cada respuesta, escriba a mano el número de pregunta en la casilla.

Example Ejemplo 27 | 2 7 | Example Ejemplo 3 | 3 |

9

(a) $(1+x)^n = 1 + {}^nC_1 x + {}^nC_2 x^2 + {}^nC_3 x^3 + {}^nC_4 x^4 + \dots$

$\to u_1 = {}^nC_1 = \frac{n!}{(n-1)!}$

$\to u_2 = {}^nC_2 = \frac{n!}{2(n-2)!}$

$\to u_3 = {}^nC_3 = \frac{n!}{3!(n-3)!}$

$\to d = \frac{n!}{2(n-2)!} - \frac{n!}{(n-1)!}$

~~6~~

$\therefore \left(\frac{n!}{6(n-3)!} - \frac{n!}{2(n-2)!}\right) = \left(\frac{n!}{2(n-2)!} - \frac{n!}{(n-1)!}\right)$

$\therefore \frac{n(n-1)(n-2)}{6} - \frac{n(n-1)}{2} = \frac{n(n-1)}{2} - \frac{n}{1}$

$\therefore n(n-1)(n-2) - 3n(n-1) = 3n(n-1) - 6n$

$\therefore n(n^2-3n+2) - 3n^2 + 3n = 3n^2 - 3n - 6n$

$\therefore n^3 - 3n^2 + 2n - 3n^2 + 3n = 3n^2 - 3n - 6n$

$\therefore n^3 - 9n^2 + 14n = 0$

(b) $n(n^2 - 9n + 14) = 0$ ~~(div by n)~~ → ~~$n \neq 0$~~

$\therefore n(n-7)(n-2) = 0$

$\therefore n = 0$

or $n = 7$

or $n = 2$

$\therefore n = 7$, $n \geq 3$

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