

4 PAGES / PÁGINAS

Candidate session number: / Numéro de session du candidat: / Número de convocatoria del alumno:

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Candidate name: / Nom du candidat: / Nombre del alumno:

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At the start of each answer to a question, write the question number in the box using your normal hand writing / Avant de répondre à une question, inscrivez son numéro à la main dans la case appropriée / Al comienzo de cada respuesta, escriba a mano el número de pregunta en la casilla.

Example
Ejemplo

27

2	7
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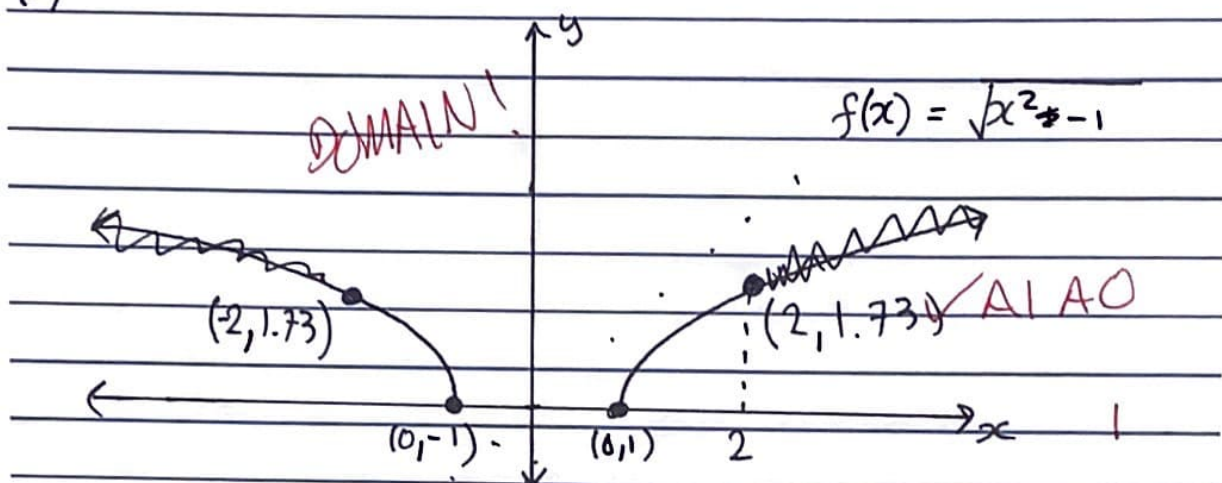
Example
Ejemplo

3

	3
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1	0
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(a)



(b)(i) $x = \sqrt{y^2 - 1}$ ✓

$\therefore y^2 - 1 = x^2$

$\therefore y^2 = x^2 + 1$ ✓

$\therefore y = \sqrt{x^2 + 1}$

$\therefore f^{-1}(x) = \sqrt{x^2 + 1}$ ✓

~~108~~ 3

(ii) Domain: $\{x \mid x \geq 1\}$ ✓

Range: $\{y \mid y \leq -1, y \geq 1\}$

Domain: $\{x \mid 0 \leq x \leq 1.73\}$ ✓

Range: $\{y \mid -2 \leq y \leq -1, 1 \leq y \leq 2\}$ ✓

(c)(i)

$$V = \pi \int_0^h (y^2 + 1) dy$$

$$= \pi \left[\frac{1}{3} y^3 + y \right]_0^h$$

$$= \pi \left(\frac{1}{3} h^3 + h - 0 \right)$$

$$= \pi \left(\frac{1}{3} h^3 + h \right)$$

(ii)

$$V = \pi \left(\frac{1}{3} h^3 + h \right)$$

$$\therefore \frac{dV}{dh} = \pi h^2 + \pi$$

$$V_{\text{MAX}} = \pi \lim_{h \rightarrow \infty} \left(\frac{1}{3} h^3 + h \right)$$

$$= \pi \lim_{h \rightarrow \infty} \left(\frac{h^3 + 3h}{3} \right)$$

$$= \pi \lim_{h \rightarrow \infty} \left(\frac{1 + 3/h^2}{3/h^3} \right)$$

$$\therefore V_{\text{MAX}} = \infty$$

V_{MAX} occurs at $h = 1.73$

$$\therefore V_{\text{MAX}} = \pi \left(\frac{1}{3} (1.73)^3 + 1.73 \right)$$

$$= 10.857$$

$$\therefore V_{\text{MAX}} \approx 10.9 \text{ m}^3$$

$$(d) \quad \frac{dV}{dt} = 0.4$$

$$\text{at } t=0, V=0$$

$$\text{If } \frac{dV}{dt} = 0.4, \text{ then}$$

$$V = 0.4t$$

$$\therefore 0.4t = 10.857$$

$$\therefore t = 27.1425 \text{ s}$$

$$\therefore t = 27.1 \text{ s}$$

$$(e) \quad 0.4t = \frac{10.857}{2}$$

$$\therefore t = 13.57125$$

$$V = \pi \left(\frac{1}{3} h^3 + h \right)$$

$$\therefore \frac{dV}{dt} = \pi \left(\frac{1}{3} h^3 + h \right) \frac{dh}{dt} + \pi \frac{dh}{dt}$$

$$= \frac{dh}{dt} \left(\pi h^2 + \pi \right)$$

$$\therefore 0.4 = \frac{dh}{dt} (\pi h^2 + \pi)$$

$$\text{When } V = 5.4285, \quad 5.4285 = \pi \left(\frac{1}{3} h^3 + h \right)$$

$$\therefore h = 1.18011 \text{ m} \quad \{\text{Solve}\}$$

$$= 0.4 = \frac{dh}{dt} (\pi (1.18011)^2 + \pi)$$

$$\therefore \frac{dh}{dt} = 0.0532 \text{ ms}^{-1}$$