Markscheme

ID: 3012

Mathematics: analysis and approaches

Higher level

1. (a)

(i) θs A1

(ii) $\dfrac{\theta s^2}{2}$ A1

(b) $2\pi r$ A1

(c) We have

$$2\pi r = \theta s \quad \text{M1}$$

So

$$\theta = \frac{2\pi r}{s} \quad \text{A1}$$

The curved surface area is therefore

$$\frac{2\pi r}{s} \cdot \frac{s^2}{2} = \pi r s \quad \text{A1}$$

(d)

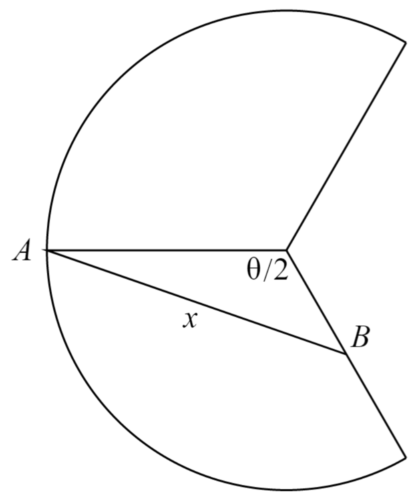
(i) $2\pi \times 1 = 2\pi$ A1

(ii) $\sqrt{1^2 + 1^2} = \sqrt{2}$ M1A1

(e) We have

$$\theta = \frac{2\pi}{2\pi\sqrt{2}} \times 2\pi = \pi\sqrt{2} \quad \text{M1A1}$$

(f) We need to calculate the length of x in the diagram below



Use the cosine rule M1

$$x^2 = \left(\sqrt{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 - 2 \cdot \sqrt{2} \cdot \frac{\sqrt{2}}{2} \cdot \cos\left(\frac{\pi\sqrt{2}}{2}\right) \quad \text{A1}$$

So

$$x = 1.93 \quad \text{A1}$$

(g) We need to find the closest point on line AB to the center of the circle sector. R1

The area of the triangle is

$$\frac{\sqrt{2}/2 \times \sqrt{2} \times \sin\left(\pi\sqrt{2}/2\right)}{2} = 0.3978466$$ A1

If the closest distance to the centre is d we have

$$\frac{1.93d}{2} = 0.3978466$$ M1

Giving

$$d = 0.412276$$ A1

The distance walked is therefore

$$\sqrt{\left(\sqrt{2}\right)^2 - 0.412276^2} = 1.35\,\text{km}$$ A1

The height above sea level is

$$\frac{h}{\sqrt{2} - 0.412276} = \frac{1}{\sqrt{2}}$$ M1

So the height is 0.708 km. A1

2. (a)

(i) Let b represent the base. Using similarity we have

$$\frac{b}{a} = \frac{a}{c}$$ M1

So

$$b = \frac{a^2}{c}$$ A1

(ii) Let h represent the height. Using similarity we have

$$\frac{h}{a} = \frac{b}{c}$$ M1

So

$$h = \frac{ab}{c}$$ A1

(b)

(i) We have

$$\frac{h_n}{b_n} = \frac{b}{a}$$ M1

So

$$h_n = \frac{b}{a} b_n$$ A1

(ii) We have

$$\frac{b_{n+1}}{h_n} = \frac{a}{c}$$ M1

So

$$b_{n+1} = \frac{a}{c} h_n$$ A1

(iii) We have

$$\frac{h_{n+1}}{h_n} = \frac{b}{c}$$ M1

So

$$h_{n+1} = \frac{b}{c} h_n$$ A1

Using the result from (b) part (i) this gives

$$h_{n+1} = \frac{b^2}{ac} b_n$$ A1

(c) For $n = 1$ the area of T_1 is equal to

$$\frac{a^2}{c} \times \frac{ab}{c} = \frac{a^3 b}{c^2}$$ M1A1

So it is true for $n = 1$. A1

Assume it is true for $n = k$. So the area of T_k is equal to

$$\frac{a^3 b^{2k-1}}{2c^{2k}}$$ A1

Let b_n and h_n represent the base and length of T_n.

The area of T_{k+1} is

$$\frac{b_{k+1} h_{k+1}}{2} = \frac{1}{2} \times \frac{a}{c} h_k \times \frac{b^2}{ac} b_k = \frac{b^2}{c^2} \times \frac{1}{2} b_k h_k$$ M1A1

Using our inductive hypothesis this is equal to

$$\frac{b^2}{c^2} \times \frac{a^3 b^{2k-1}}{2c^{2k}} = \frac{a^3 b^{2(k+1)-1}}{2c^{2(k+1)}}$$ M1A1

So it is true for $n = k + 1$. A1

By the principle of mathematical induction it must be true for all positive integers n. R1

(d) The area of all of the triangles is equal to

$$\sum_{i=1}^{\infty} \frac{a^3 b^{2i-1}}{2c^{2i}} \qquad \text{A1}$$

This is an infinite geometric series with first term $\frac{a^3 b}{2c^2}$ and common ratio $\frac{b^2}{c^2}$. R1

Its value is therefore

$$\frac{\frac{a^3 b}{2c^2}}{1 - \frac{b^2}{c^2}} \qquad \text{M1}$$

This simplifies to

$$\frac{a^3 b}{2(c^2 - b^2)} \qquad \text{A1}$$

The area of the large triangle is also equal to $\frac{ab}{2}$. A1

So we have

$$\frac{a^3 b}{2(c^2 - b^2)} = \frac{ab}{2} \qquad \text{M1}$$

This simplifies to

$$\frac{a^2}{c^2 - b^2} = 1 \qquad \text{A1}$$

This rearranges to

$$a^2 + b^2 = c^2 \qquad \text{A1}$$