Mathematics: analysis and approaches

Higher level

Additional Practice

Counting Principles (Calculator)

ID: 4005

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[140 marks]**.

Document name: Taskmaker-ID

Scanned document name: Taskmaker-ID-MAHL-DD.MM

1. [Maximum points: 4]

Six identical mathematics books and three identical science books are placed randomly on a shelf. Find in how many ways this can be done if no science books are placed next to each other.

2. [Maximum points: 5]

Four different math books and six different science books are placed randomly on a shelf. Find in how many ways can this be done so that no two math books are next to each other.

3. [Maximum points: 5]

Two friends are waiting in line for a bus. There are six more people in front of them. The bus arrives and the first six people each sit in a random seat. There are four sets of two seats altogether. Determine the probability that the two friends are able to sit together in the same set of seats.

4. [Maximum points: 5]

Five grade 10 students and six grade 11 students stand randomly in a line. Determine the probability that no students of the same grade are standing together.

5. [Maximum points: 5]

Six different mathematics books and three different science books are placed randomly on a shelf. Determine the probability that all three science books are together.

6. [Maximum points: 6]

Four mathematics books, three science books, two English books and a dictionary are all placed on a shelf. Determine in how many ways this can be done so that all books of the same type are together.

7. [Maximum points: 6]

The prime factorisation of 20 is $2 \times 2 \times 5$. The prime factorisation of 34 is 2×17.

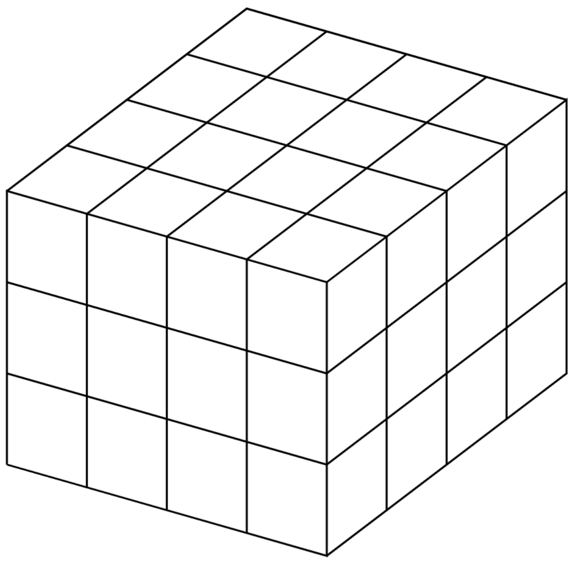
(a) Determine the prime factorisation of [3]

(i) 14

(ii) 18

(iii) 30

A large rectangular prism consists of many smaller identical cubes. For example the rectangular prism below consists of 48 cubes.



(b) Determine how many different rectangular prisms can be constructed which consist of 18 cubes. You may assume a $2 \times 3 \times 3$ prism is identical to, for example, a $3 \times 2 \times 2$ prism etc. [3]

8. [Maximum points: 6]

Five Mathematics books and three English books are places randomly on a shelf. Determine the probability that no English books are placed next to each other.

9. [Maximum points: 6]

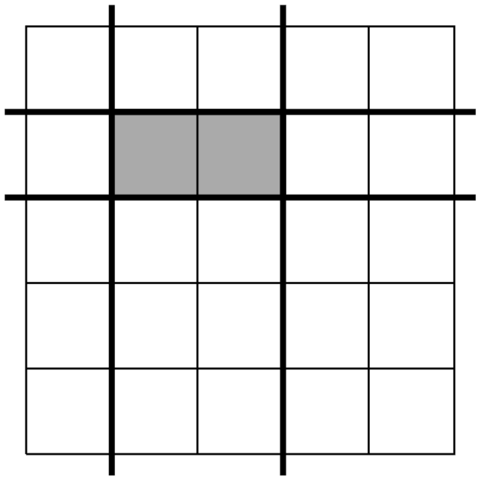
Two friends and seven other people each sit at a desk. The desks are arranged in three rows of three. Find the total number of ways in which this can happen if

(a) they each sit on the same row [4]

(b) they each sit on a different row [2]

10. [Maximum points: 6]

A 5×5 grid of squares contains many rectangles (a square is also a rectangle). A rectangle is formed by the area completely bound by two vertical lines and two horizontal lines. This is shown in the diagram below.



(a) Find the total number of rectangles in the 5×5 grid. [3]

(b) Find the total number of rectangles in an $n \times n$ grid in terms of n. Factorise your answer. [3]

11. [Maximum points: 6]

Four children stand randomly in a line. Determine the probability that

(a) they are stood in order of height (either ascending or descending) [4]

(b) they are not stood in order of height [2]

12. [Maximum points: 6]

Five grade 11 students and three grade 12 students are standing randomly in a line. Find the probability that all members of the same grade are standing together.

13. [Maximum points: 6]

Five mathematics books and three science books are placed randomly on a shelf. Calculate the probability that all books of the same subject are together.

14. [Maximum points: 7]

A teacher creates a mathematics test by selecting 10 questions from 5 calculus questions and 8 trigonometry questions. Find the number of different tests that can be created if the test contains at least 3 calculus questions.

15. [Maximum points: 10]

A bicycle lock is secured using a three digit code. Each digit is randomly assigned using integers from 0 to 9 with repetition allowed.

(a) When interpretted as a single number, for example 3,6,1 is interpretted as 361 and 0,4,3 is interpretted as 43, find the probability that the code is [3]

(i) less than 200

(ii) a **positive** multiple of 5

The owner of the bicycle forgets the code and therefore tries random codes in the hope of finding the correct one. The owner is forgetful and does not remember any previous codes which don't work.

(b) Find the probability the correct code is found in a maximum of 50 guesses. [3]

Suppose now the code is randomly assigned with the repetition of digits not allowed. For example 2,3,2 and 0,0,5 are not allowed.

(c) Find the total number of codes that are a multiple of 5. [4]

16. [Maximum points: 16]

Four regular six-sided dice are rolled. Let X represent the total number of distinct values shown. For example, if the dice show 1,3, 2 and 1 then $X = 3$. If the dice show 3, 5, 3 and 3 then $X = 2$.

(a) Write down all of the outcomes for which $X = 1$. [2]

(b) Hence show that $P(X = 1) = \frac{1}{216}$. [2]

(c) To five significant figures calculate the probability of the dice showing [4]

(i) exactly three 4s

(ii) two 3s and two 6s

(d) Hence show that $P(X = 2) = 0.162$ to three significant figures. [4]

(e) Find [4]

(i) $P(X = 4)$

(ii) $P(X = 3)$

17. [Maximum points: 17]

Let $f(x) = \dfrac{(x-a)(x-b)}{x-c}$ where $a, b, c \in \mathbb{Z}$ and $a < b$.

Three integers are randomly chosen from 1 to 10 with no repetition. One of these integers is randomly assigned to the value of c. The smallest of the remining two integers is assigned to the value of a and the remaining integer is assigned to the value of b.

(a) Determine how many different functions can be formed. [3]

Consider the graph of $y = f(x)$.

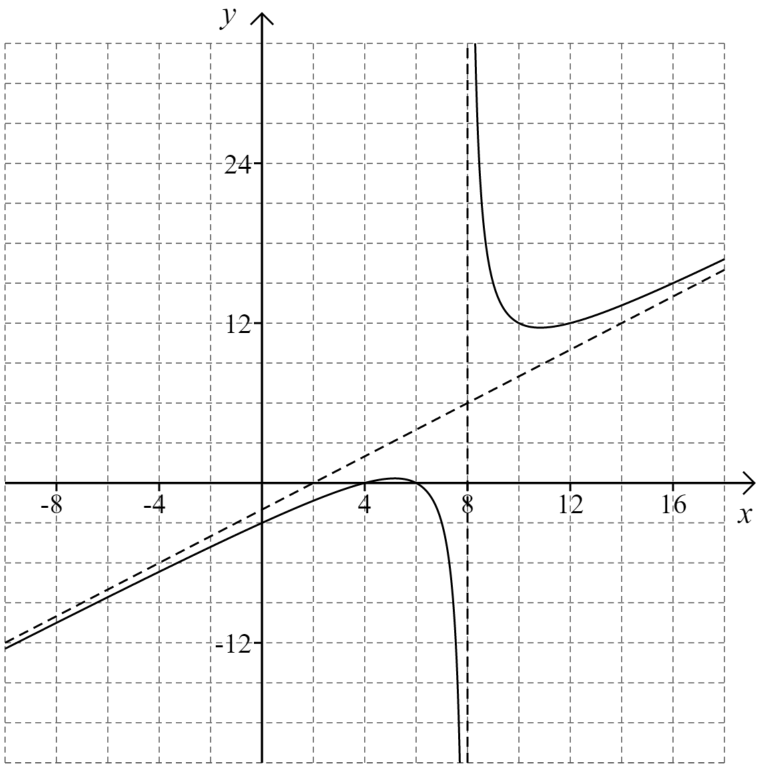
(b) Find the equation of [4]

(i) the vertical asymptote

(ii) the linear oblique asymptote

(c) Write down the coordinates of the axes intercepts. [2]

The diagram below shows that graph of $y = f(x)$ when $a = 4$, $b = 6$ and $c = 8$.



(d) Sketch the graph of $y = f(x)$ when [4]

(i) $a = 1$, $b = 4$ and $c = 2$

(ii) $a = 2$, $b = 5$ and $c = 1$

(e) Find any restrictions on the values of a, b and c if the graph of $y = f(x)$ has no turning points. [1]

(f) Hence find the probability the graph of $y = f(x)$ has no turning points. [3]

18. [Maximum points: 18]

Let $f(x) = \dfrac{x-a}{(x-b)(x-c)}$ where $a, b, c \in \mathbb{Z}$ and $b < c$.

Three integers are randomly chosen from 1 to 10 with no repetition. One of these integers is randomly assigned to the value of a. The smallest of the remaining two integers is assigned to the value of b and the remaining integer is assigned to the value of c.

(a) Find how many different functions can be formed. [3]

Consider the graph of $y = f(x)$.

(b) Find the equations of [4]

(i) the vertical asymptotes

(ii) the horizontal asymptote

(c) Write down the cordinates of the x-intercept. [1]

Consider the equation $f(x) = k$ where $k \in \mathbb{R}$.

(d) Explain why the equation can have a maximum of two solutions. [1]

(e) If the range of $f(x)$ is $\mathbb{R}$ explain, with the aid of graphs of example functions, why we must have $b < a < c$. [4]

(f) Hence find the probability that the range of $f(x)$ is $\mathbb{R}$. [5]