ANSWER BOOKLET ①
LIVRET DE RÉPONSES
CUADERNILLO DE RESPUESTAS

Q1: $\frac{29}{30}$ Q2: $\frac{20}{24}$ $\frac{49}{54} = 91\%$

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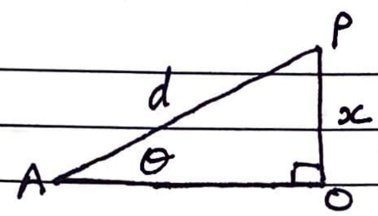
1 2 3 4 5 6 7 8 9 10

1 (a)



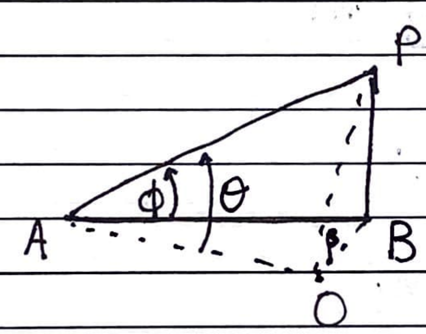
②

$\sin\theta = x/d$

$\therefore \quad x = d\sin\theta$

$\therefore |OP| = d\sin\theta$

(b)



$\sin\theta = \frac{|OP|}{|AP|}$ (1)

$\sin\phi = \frac{|BP|}{|AP|}$ (2)

$\sin\beta = \frac{|BP|}{|OP|}$ (3)

$\therefore \quad |OP| = \overset{(1)}{|AP|\sin\theta} = \overset{(3)}{\frac{|BP|}{\sin\beta}}$ ③

$\therefore \quad \sin\beta\sin\theta = \frac{|BP|}{|AP|}$

$\therefore \quad \sin\beta\sin\theta = \sin\phi$ using (2)

$\therefore \quad \sin\phi = \sin\theta\sin\beta$

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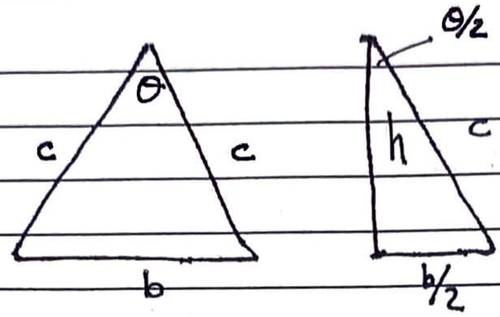
(c)



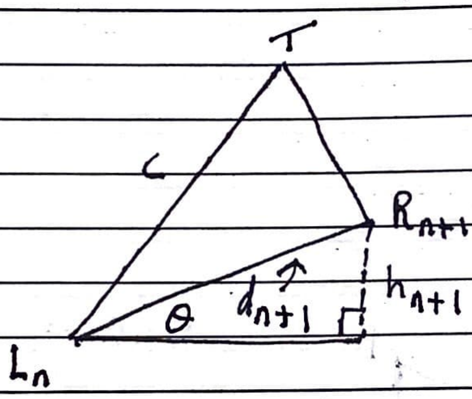
(2)

$\therefore \quad c^2 = \left(\frac{b}{2}\right)^2 + h^2$

$\therefore \quad h^2 = c^2 - b^2/4$

$\therefore \quad h = \sqrt{c^2 - \frac{b^2}{4}}$

(d)



$\therefore \quad \cancel{\sin\theta = \frac{h}{d_n}}$

$\therefore \quad \cancel{d_n = h/\sin\theta}$

$\therefore \quad \sin\theta = \frac{h_{n+1}}{d_{n+1}} = \frac{h_n}{d_n}$

(2)

$\therefore \quad d_n = h_n/\sin\theta$

$\therefore$

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(e)

$$\sum_{n=1}^{\infty} d_n = d_1 + d_2 + d_3 + \cdots + d_\infty$$

$$= \frac{h_1 + h_2 + h_3 + \cdots + h_\infty}{\sin\theta}$$ {similar}

$$= \frac{\sqrt{c^2 - b^2/4}}{\sin\theta}$$ ✓

[as $h_1 + h_2 + h_3 + \cdots$ = total h of the triangle

$$= \frac{\sqrt{4c^2 - b^2}}{2\sin\theta}$$ ✓

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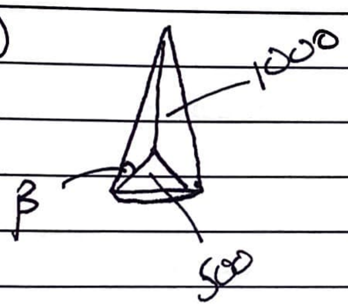
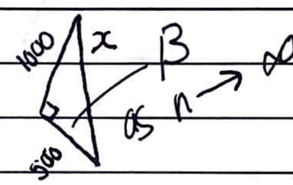
(f) cone ✓ 1

(g)





$\therefore$ ~~sin~~ $x = \sqrt{500^2 + 1000^2}$

$= \sqrt{5^2 + 10^2}$

$= \sqrt{125}$ ✓

$\lim_{n\to\infty}$ $\therefore$ ~~tan~~ $\sin\beta = \frac{10}{\sqrt{125}}$ ✓

$= \frac{10}{5\sqrt{5}}$

$= 2/\sqrt{5}$ ✓

$\therefore \lim_{n\to\infty} \beta = \arcsin(2 \cdot 5^{-1/2})$

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(h) $\lim_{n\to\infty} |L_0R_0| = 0$ ✓ 1

(i) $\lim_{n\to\infty} \sum_{n=1}^{\infty} d_n = \lim_{n\to\infty}\left(\frac{\sqrt{4c^2-b^2}}{2\sin\theta}\right)$

$b = \lim_{n\to\infty} |L_0R_0| = 0$

$c = |TL_0| = |TR_0|$

$= \lim_{n\to\infty}\left(\frac{\sqrt{5000000}}{2\sin\theta}\right)$

$= \sqrt{500^2+1000^2}$

$= \sqrt{1250000}$

$= \frac{500\sqrt{5}}{\sin\theta}$ 1

(j) speed $= \frac{\text{distance}}{\text{time}}$

$\therefore t = d/s$

$= \frac{\frac{\sqrt{5000000}}{2\sin\theta}}{\frac{1}{2}\cos\phi}$

$= \frac{\sqrt{5000000}}{2\sin\theta} \times \frac{2}{\cos\phi}$

$= \frac{\sqrt{5000000}}{\sin\theta\cos\phi}$

$= \frac{\sqrt{5000000}}{\frac{2\sin\phi\cos\phi}{\sqrt{5}}}$

$\Rightarrow \sin\phi = \sin\theta\sin\beta$

$\therefore \sin\theta = \sin\phi / 2.5^{-1/2}$

$= \frac{\sin\phi\sqrt{5}}{2}$

$\therefore \sin\phi = 2\sin\theta/\sqrt{5}$

$\therefore \cos\phi = \frac{2\cos\theta}{\sqrt{5}}$

$= \frac{\frac{\sqrt{5000000}}{2\sin\theta}}{0.5\cos\phi}$ ✓

$= \frac{\sqrt{5000000}}{\sin\theta\cos\phi}$ ✓

$= \frac{\sqrt{5000000}}{\frac{\sqrt{5}}{2}\sin\phi\cos\phi}$

$= \frac{2000}{\sin\phi\cos\phi}$ ✓

$= \frac{4000}{\sin 2\phi}$ ✓

$\sin\phi = \sin\theta\sin\beta$

$\therefore \sin\phi = \sin\theta \times 2/\sqrt{5}$

$\therefore \sin\theta = \frac{\sqrt{5}}{2}\sin\phi$

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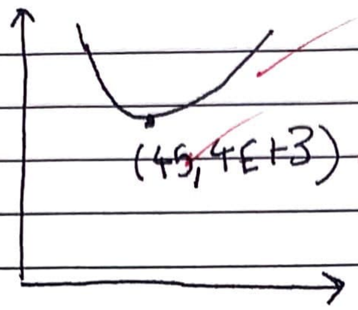
(K) G.D.C



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$\therefore$ at $\phi = 45°$
$t = 4000$ s

(l) The path converges on a single dot, which would be impractical in the real world.

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2

(a) $\cancel{\frac{d}{dx}(0.9^x) =}$ (1) → not really proving

$d/dx\,(a^x) = a^x(\ln a)$ $\quad 0.9^x = e^{x\ln 0.9}$

$\therefore \frac{d}{dx}(e^{x\ln 0.9})$

$\therefore d/dx\,(0.9^x) = 0.9^x(\ln 0.9)$ $\quad = \ln 0.9\, e^{x\ln 0.9}$

$= \ln 0.9 \times 0.9^x$

(b) $\int 0.9^x \sin x\, dx$ $\quad$ let $u = 0.9^x$ $\quad du = 0.9^x(\ln 0.9)$

$= -0.9^x\cos x - \int -0.9^x(\ln 0.9)\cos x\,dx$ $\quad$ let $dv = \sin x$ $\quad v = -\cos x$

$= -0.9^x\cos x + (\ln 0.9)\int 0.9^x \cos x\,dx$ $\quad$ let $u = 0.9^x$ $\quad du = 0.9^x(\ln 0.9)$

$= -0.9^x\cos x + (\ln 0.9)\left[0.9^x\sin x - \int 0.9^x(\ln 0.9)\sin x\,dx\right]$ $\quad$ let $dv = \cos x$ $\quad v = \sin x$

$= -0.9^x\cos x + 0.9^x(\ln 0.9)\sin x - (\ln 0.9)^2\int 0.9^x \sin x\,dx$

$\therefore \int 0.9^x \sin x\,dx + (\ln 0.9)^2\int 0.9^x\sin x\,dx = 0.9^x(\ln 0.9\sin x - \cos x)$

$\therefore (1 + (\ln 0.9)^2)\int 0.9^x \sin x\,dx = 0.9^x(\ln 0.9\sin x - \cos x)$

$\therefore \int 0.9^x \sin x\,dx = \dfrac{0.9^x(\ln 0.9\sin x - \cos x)}{1 + (\ln 0.9)^2} + C$

Note: using the graph above, the constant C can be regarded as 0 (8)

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(c)(i) $\int_{n\pi}^{(n+1)\pi} |0.9^x \sin x| dx = \left| \int_{n\pi}^{(n+1)\pi} (0.9^x \sin x) dx \right|$

$= \left| \left[\frac{0.9^x(\ln 0.9 \sin x - \cos x)}{1+(\ln 0.9)^2} \right]_{n\pi}^{(n+1)\pi} \right|$

$= \left| \frac{0.9^{(n+1)\pi}(\ln 0.9 \sin[(n+1)\pi] - \cos[(n+1)\pi])}{1+(\ln 0.9)^2} - \frac{0.9^{n\pi}(\ln 0.9 \sin(n\pi) - \cos(n\pi))}{1+(\ln 0.9)^2} \right|$

$= \left| \frac{0.9^{(n+1)\pi}(0-(-1)^{n+1}) - 0.9^{n\pi}(0-(-1)^n)}{1+(\ln 0.9)^2} \right|$

$= \left| \frac{-0.9^{\pi} \cdot 0.9^{n\pi} \cdot (-1)^{n+1} + 0.9^{n\pi} \cdot (-1)^n}{1+(\ln 0.9)^2} \right|$

$= \left| \frac{0.9^{\pi} \cdot 0.9^{n\pi} \cdot (-1)^{n+1} + 0.9^{n\pi} \cdot (-1)^n}{1+(\ln 0.9)^2} \right|$

$= \left| \frac{-0.9^{n\pi}(-1)^n(0.9^{\pi}+1)}{1+(\ln 0.9)^2} \right|$ correct

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(ii) $\int_{(n+1)\pi}^{(n+2)\pi} |0.9 \sin x| dx = \left| \int_{(n+1)\pi}^{(n+2)\pi} (0.9 \sin x) dx \right|$

$= \left| \left[\frac{0.9^{n\pi+2\pi}(\ln 0.9 \sin((n+2)\pi) - \cos((n+2)\pi)) - 0.9^{n\pi+\pi}(\ln 0.9 \sin((n+1)\pi) - \cos((n+1)\pi))}{1+(\ln 0.9)^2} \right] \right|$

$= \left| \frac{0.9^{2\pi} \cdot 0.9^{n\pi}(0-(-1)^{n+2}) - 0.9^{\pi} \cdot 0.9^{n\pi}(0-(-1)^{n+1})}{1+(\ln 0.9)^2} \right|$

~~$= -0.9^{\pi}\, 0.9^{n\pi}$~~

$= \left| \frac{-0.9^{2\pi}\, 0.9^{n\pi}(-1)^{n+2} + 0.9^{\pi}\, 0.9^{n\pi}(-1)^{n+1}}{1+(\ln 0.9)^2} \right|$

$= \left| \frac{0.9^{2\pi}\, 0.9^{n\pi}(-1)^{n+1} + 0.9^{\pi}\, 0.9^{n\pi}(-1)^{n+1}}{1+(\ln 0.9)^2} \right|$

$= \left| \frac{0.9^{\pi}\, 0.9^{n\pi}(-1)^{n+1}(0.9^{\pi}+1)}{1+(\ln 0.9)^2} \right|$

$= \left| \frac{0.9^{\pi}\, 0.9^{n\pi}(-1)^{n}(0.9^{\pi}+1)}{1+(\ln 0.9)^2} \right|$ correct

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(d) $u_1 = \left| \dfrac{0.9^{\pi}(-1)(0.9^{\pi}+1)}{1+(\ln 0.9)^2} \right|$

$u_2 = \dfrac{0.9^{2\pi}(0.9^{\pi}+1)}{1+(\ln 0.9)^2}$

$\therefore \ r = 0.9^{\pi}$ ← show in more detail. ②

$\therefore$ forms a G.P

(e) $\lim_{n \to \infty} \int_0^{n\pi} |0.9 \sin x| \, dx = \dfrac{u_1}{1-r}$

$= \dfrac{\dfrac{0.9^{\pi}(0.9^{\pi}+1)}{1+(\ln 0.9)^2}}{1-0.9^{\pi}}$ → u_1 has $n=0$

$= \dfrac{0.9^{\pi}(0.9^{\pi}+1)}{1+(\ln 0.9)^2} \times \dfrac{1}{(1-0.9^{\pi})}$ ECF

$= \dfrac{0.9^{\pi}(0.9^{\pi}+1)}{(1-0.9^{\pi})(1+(\ln 0.9)^2)}$

②

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