

Mathematics: analysis and approaches
Higher level
Paper 2 Practice Set B (Hodder)

Candidate session number

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2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in an answer booklet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A copy of the mathematics: analysis and approaches formula book is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

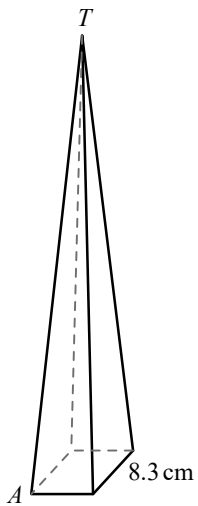
1 [Maximum mark: 5]

In an arithmetic sequence, the fifth term is 7 and the tenth term is 81. Find the sum of the first 20 terms.

[illegible]

2

A flag pole has the shape of a square-based pyramid shown in the diagram. The side length of the base is 8.3 cm. The edge AT makes an angle of 89.8° with the base.



Find the height of the flag pole. Give your answer in centimetres, in standard form, correct to two significant figures.

[illegible]

3 [Maximum mark: 5]

In this question, an *outlier* is defined as a piece of data which is more than two standard deviations above or below the mean.

The heights of eight children, in centimetres, are:

122	124	127	131	134	134	136	147
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Determine whether any of the heights are outliers.

[illegible]

4 [Maximum mark: 5]

Prove that

$$\frac{\sin\left(x+\frac{\pi}{3}\right)-\sin\left(x-\frac{\pi}{3}\right)}{\cos\left(x+\frac{\pi}{3}\right)-\cos\left(x-\frac{\pi}{3}\right)} \equiv -\cot x$$

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

5 [Maximum mark: 6]

Find

$$\int \frac{2}{x(x-1)} \, dx$$

Write your answer in the form $\ln(f(x)) + c$.

[illegible]

6 [Maximum mark: 6]

The normal to the graph of $y = 4 \sin^{-1}\left(\frac{x}{2}\right)$ at the point where $x = 1.5$ intersects the x -axis at the point A and the y -axis at the point B . Find the area of triangle AOB , where O is the origin.

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

7 [Maximum mark: 7]

Find the coordinates of the points on the curve

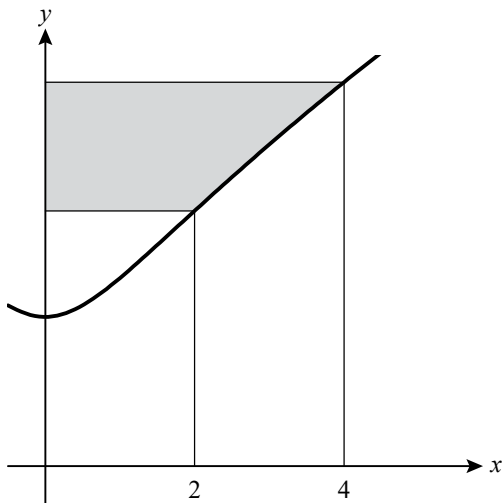
$$3x^2 + 2xy - y^2 + 24 = 0$$

where the tangent is parallel to the x -axis.

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

8

The curve in the diagram has equation $y = \sqrt[3]{x^2 + 1}$. The shaded region is bounded by the curve, the y -axis and two horizontal lines.



- a** Find the area of the shaded region. [4]
- b** Find the volume generated when the shaded region is rotated 2π radians about the y -axis. [3]

[illegible]

9 [Maximum mark: 5]

One of the roots of the equation $x^3 - 7x^2 + bx + c = 0$ is $2 - i$. Find the value of c .

[illegible]

10 [Maximum mark: 5]

The constant term in the binomial expansion of $\left(x^2 + \frac{1}{x}\right)^n$ is 495. Find the value of n .

[illegible]

Section B

Answer **all** questions in an answer booklet. Please start each question on a new page.

11 [Maximum mark: 18]

Battery life of a certain brand of smartphone can be modelled by a normal distribution with mean μ hours and standard deviation σ hours. It is known that 5% of the batteries last less than 24 hours, while 20% last more than 72 hours.

- a i Show that $\mu + 0.8416\sigma = 72$ and find another similar equation connecting μ and σ . [7]
- ii Show that approximately 65.7% of the batteries last longer than 48 hours. [7]
- b Find the interquartile range of battery life, giving your answer to the nearest hour. [2]
- c Find the probability that, out of 20 randomly selected batteries, at least 10 last more than 48 hours. [3]
- d Given that a battery has lasted for 48 hours, what is the probability that it will last for another 24 hours? [2]
- e A customer buys a new smartphone and tests the battery. If the battery lasts less than 24 hours they return the phone with probability 0.9. If it lasts between 24 and 72 hours they return the phone with probability 0.2. Otherwise they do not return the phone.

Given that a customer keeps the phone, what is the probability that its battery lasts more than 72 hours? [4]

12 [Maximum mark: 18]

Two of the sides of a triangle have length x cm and $2x$ cm, and the angle between them is θ° . The perimeter of the triangle is 10 cm.

- a In the case $x = 2$, find the area of the triangle. [4]
- b Explain why x must be less than $\frac{10}{3}$. [2]
- c i Show that $\cos \theta = \frac{15x - x^2 - 25}{x^2}$. [7]
- ii Sketch the graph of $y = \frac{15x - x^2 - 25}{x^2}$ for $x > 0$. [7]
- iii Hence find the range of possible values of x . [7]
- d Find the value of x for which the triangle has the largest possible area, and state the value of that area. [5]

13 [Maximum mark: 19]

Consider the differential equation $\frac{dy}{dx} = \frac{y}{x+y}$.

- a Find and simplify an expression for $\frac{d^2y}{dx^2}$ in terms of x and y . [7]
- b Show that the substitution $y = xv$ transforms this equation into $x \frac{dv}{dx} = f(v)$, where $f(v)$ is a function to be found. [4]
- c Hence find the particular solution of the equation $\frac{dy}{dx} = \frac{y}{x+y}$ for which $y = 1$ when $x = 1$. Give your answer in the form $x = g(y)$. [8]