Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 2

Tuesday 14 November 2017 (morning)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.

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13 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

Boxes of mixed fruit are on sale at a local supermarket.
Box A contains 2 bananas, 3 kiwifruit and 4 melons, and costs $\$6.58$.
Box B contains 5 bananas, 2 kiwifruit and 8 melons and costs $\$12.32$.
Box C contains 5 bananas and 4 kiwifruit and costs $\$3.00$.

Find the cost of each type of fruit.

2. [Maximum mark: 6]

Events A and B are such that $\mathrm{P}(A \cup B) = 0.95$, $\mathrm{P}(A \cap B) = 0.6$ and $\mathrm{P}(A \mid B) = 0.75$.

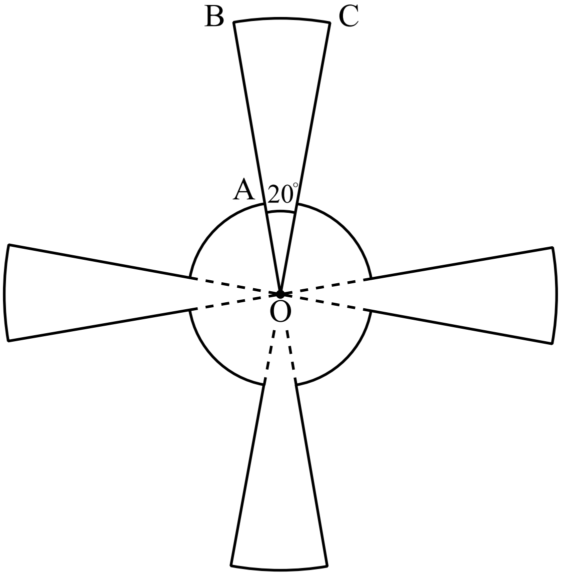
(a) Find $\mathrm{P}(B)$. [2]

(b) Find $\mathrm{P}(A)$. [2]

(c) Hence show that events A' and B are independent. [2]

3. [Maximum mark: 4]

This diagram shows a metallic pendant made out of four equal sectors of a larger circle of radius $\mathrm{OB} = 9\,\mathrm{cm}$ and four equal sectors of a smaller circle of radius $\mathrm{OA} = 3\,\mathrm{cm}$.
The angle $\mathrm{BOC} = 20^\circ$.



Find the area of the pendant.

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4. [Maximum mark: 6]

It is given that one in five cups of coffee contain more than $120\,\text{mg}$ of caffeine.
It is also known that three in five cups contain more than $110\,\text{mg}$ of caffeine.

Assume that the caffeine content of coffee is modelled by a normal distribution.
Find the mean and standard deviation of the caffeine content of coffee.

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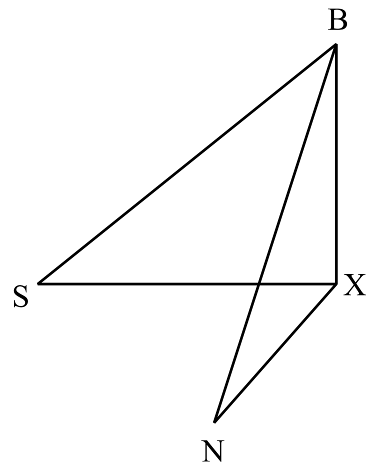
5. [Maximum mark: 6]

Barry is at the top of a cliff, standing $80\,\text{m}$ above sea level, and observes two yachts in the sea.
"*Seaview*" (S) is at an angle of depression of 25°.
"*Nauti Buoy*" (N) is at an angle of depression of 35°.
The following three dimensional diagram shows Barry and the two yachts at S and N.
X lies at the foot of the cliff and angle $\text{SXN} = 70^\circ$.



Find, to 3 significant figures, the distance between the two yachts.

6. [Maximum mark: 6]

The number of bananas that Lucca eats during any particular day follows a Poisson distribution with mean 0.2.

(a) Find the probability that Lucca eats at least one banana in a particular day. [2]

(b) Find the expected number of weeks in the year in which Lucca eats no bananas. [4]

7. [Maximum mark: 5]

In the quadratic equation $7x^2 - 8x + p = 0$, $(p \in \mathbb{Q})$, one root is three times the other root. Find the value of p.

8. [Maximum mark: 7]

By using the substitution $x^2 = 2\sec\theta$, show that $\int \frac{\mathrm{d}x}{x\sqrt{x^4 - 4}} = \frac{1}{4}\arccos\left(\frac{2}{x^2}\right) + c$.

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9. [Maximum mark: 6]

Twelve students are to take an exam in advanced combinatorics.
The exam room is set out in three rows of four desks, with the invigilator at the front of the room, as shown in the following diagram.

INVIGILATOR

Desk 1	Desk 2	Desk 3	Desk 4
Desk 5	Desk 6	Desk 7	Desk 8
Desk 9	Desk 10	Desk 11	Desk 12

(a) Find the number of ways the twelve students may be arranged in the exam hall. [1]

Two of the students, Helen and Nicky, are suspected of cheating in a previous exam.

(b) Find the number of ways the students may be arranged if Helen and Nicky must sit so that one is directly behind the other (with no desk in between). For example Desk 5 and Desk 9. [2]

(c) Find the number of ways the students may be arranged if Helen and Nicky must not sit next to each other in the same row. [3]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 17]

Consider the function $f(x) = \frac{\sqrt{x}}{\sin x}, 0 < x < \pi$.

(a) (i) Show that the x-coordinate of the minimum point on the curve $y = f(x)$ satisfies the equation $\tan x = 2x$.

(ii) Determine the values of x for which $f(x)$ is a decreasing function. [7]

(b) Sketch the graph of $y = f(x)$ showing clearly the minimum point and any asymptotic behaviour. [3]

(c) Find the coordinates of the point on the graph of f where the normal to the graph is parallel to the line $y = -x$. [4]

Consider the region bounded by the curve $y = f(x)$, the x-axis and the lines $x = \frac{\pi}{6}, x = \frac{\pi}{3}$.

(d) This region is now rotated through 2π radians about the x-axis. Find the volume of revolution. [3]

Do **not** write solutions on this page.

11. [Maximum mark: 18]

Consider the function $f(x) = 2\sin^2 x + 7\sin 2x + \tan x - 9$, $0 \le x < \frac{\pi}{2}$.

(a) (i) Determine an expression for $f'(x)$ in terms of x.

(ii) Sketch a graph of $y = f'(x)$ for $0 \le x < \frac{\pi}{2}$.

(iii) Find the x-coordinate(s) of the point(s) of inflexion of the graph of $y = f(x)$, labelling these clearly on the graph of $y = f'(x)$. [8]

(b) Let $u = \tan x$.

(i) Express $\sin x$ in terms of u.

(ii) Express $\sin 2x$ in terms of u.

(iii) Hence show that $f(x) = 0$ can be expressed as $u^3 - 7u^2 + 15u - 9 = 0$. [7]

(c) Solve the equation $f(x) = 0$, giving your answers in the form $\arctan k$ where $k \in \mathbb{Z}$. [3]

Do **not** write solutions on this page.

12. [Maximum mark: 15]

Phil takes out a bank loan of $\$150\,000$ to buy a house, at an annual interest rate of 3.5%. The interest is calculated at the end of each year and added to the amount outstanding.

(a) Find the amount Phil would owe the bank after 20 years. Give your answer to the nearest dollar. [3]

To pay off the loan, Phil makes annual deposits of $\$P$ at the end of every year in a savings account, paying an annual interest rate of 2%. He makes his first deposit at the end of the first year after taking out the loan.

(b) Show that the total value of Phil's savings after 20 years is $\frac{\left(1.02^{20}-1\right)P}{\left(1.02-1\right)}$. [3]

(c) Given that Phil's aim is to own the house after 20 years, find the value for P to the nearest dollar. [3]

David visits a different bank and makes a single deposit of $\$Q$, the annual interest rate being 2.8%.

(d) (i) David wishes to withdraw $\$5000$ at the end of each year for a period of n years. Show that an expression for the minimum value of Q is
$\frac{5000}{1.028}+\frac{5000}{1.028^2}+\ldots+\frac{5000}{1.028^n}$.

(ii) Hence or otherwise, find the minimum value of Q that would permit David to withdraw annual amounts of $\$5000$ indefinitely. Give your answer to the nearest dollar. [6]

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