Diploma Programme
Programme du diplôme
Programa del Diploma

# Mathematics
# Higher level
# Paper 2

Wednesday 4 November 2020 (morning)

2 hours

Candidate session number

| 20 | N | TZ | 0 | P | 2 | M | A | H | L |
|---|---|---|---|---|---|---|---|---|---|

## Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.

$\frac{80}{96} = 83\%$ 29/9/22

13 pages

8820–7202

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

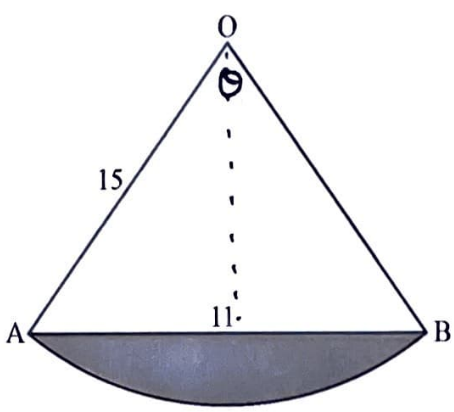
## Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

**1.** [Maximum mark: 5]

The following diagram shows a sector OAB of radius 15 cm. The length of [AB] is 11 cm.

**diagram not to scale**




Find the area of the shaded region.

$\cancel{A = \frac{1}{2}r^2 \theta}$

$\sin \theta/2 = \frac{11/2}{15}$

$\therefore \theta/2 = \arcsin(11/30)$

$\therefore \theta = 2\arcsin(11/30)$

Hence, $A = \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin(\theta)$

$= \frac{1}{2}(15^2)(2\arcsin\frac{11}{30}) - \frac{1}{2}(15^2)(\sin(2\arcsin\frac{11}{30}))$

$= 7.71624 \text{ cm}^2$

$\approx 7.72 \text{ cm}^2$

5

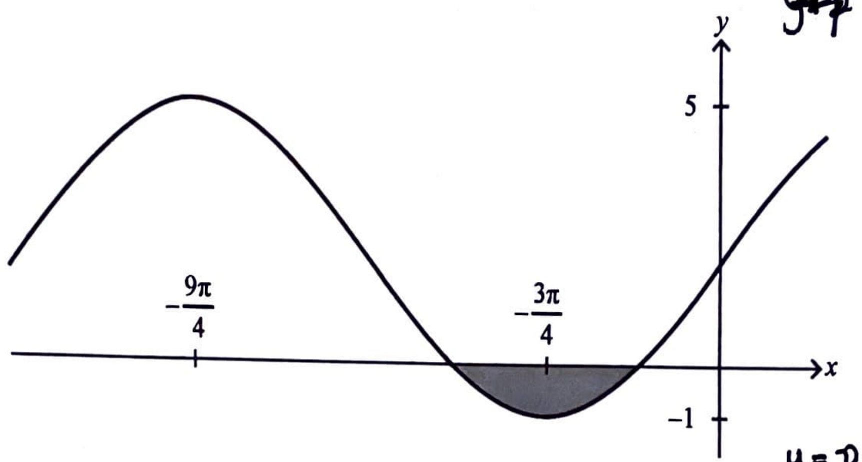
**2.** ~~[Maximum mark: 4]~~

Jenna is a keen book reader. The number of books she reads during one week can be modelled by a Poisson distribution with mean $2.6$.

Determine the expected number of weeks in one year, of $52$ weeks, during which Jenna reads at least four books.

**3.** [Maximum mark: 8]

The following diagram shows part of the graph of $y = p + q\sin(rx)$. The graph has a local maximum point at $\left(-\frac{9\pi}{4}, 5\right)$ and a local minimum point at $\left(-\frac{3\pi}{4}, -1\right)$.




$y = p + q\sin rx$

(a) Determine the values of $p$, $q$ and $r$. [4]

(b) Hence find the area of the shaded region. [4]

(a) $T = 2(-3\pi/4 - -9\pi/4)$
$= 2(6\pi/4)$
$= 6\pi/2$
$= 3\pi \rightarrow \boxed{r = 2\pi/3\pi = 2/3}$

$\frac{\text{Max} - \text{Min}}{2} = q$

$\therefore q = \frac{5 - -1}{2}$

$\therefore \boxed{q = 3}$

4

$\frac{\text{Max} + \text{Min}}{2} = p$

$\therefore \boxed{p = 2}$

(b) Zeros from GDC: $x = -1.09$, $x = -3.62$

4

$\therefore A = \int_{-3.62}^{-1.09} \left(2 + 3\sin \tfrac{2}{3}x\right) dx = -1.66177$

$\approx +1.66$ units$^2$ 8

**4.** [Maximum mark: 6]

Find the term independent of $x$ in the expansion of $\frac{1}{x^3}\left(\frac{1}{3x^2}-\frac{x}{2}\right)^9$.

$$\left(\frac{1}{3x^2}-\frac{x}{2}\right)^9 \rightarrow T_{r+1} = \binom{9}{r}\left(\frac{1}{3x^2}\right)^{9-r}\left(-\frac{x}{2}\right)^r$$

$$= \binom{9}{r}\left(-\frac{1}{2^r}\right)\left(3x^{-18+2r+r}\right)$$

$\therefore$ $x^3$ term occurs at $-18+2r=3$

$\therefore$ $3r=21$

$\therefore r=7$

Hence, $T_8 = \binom{9}{7}\left(\frac{1}{3x^2}\right)^2\left(-\frac{x}{2}\right)^7$

$$= -\frac{1}{32} \times \frac{1}{x^4} \times x^7$$

$$= -\frac{1}{32} \times x^3$$

Hence, the term independent in expansion

$$= \frac{1}{x^3}\left(-\frac{1}{32}x^3\right)$$

$$= -\frac{1}{32}$$

6

**5.** [Maximum mark: 7]

A survey of British holidaymakers found that $15\%$ of those surveyed took a holiday in the Lake District in 2019.

(a) A random sample of 16 British holidaymakers was taken. The number of people in the sample who took a holiday in the Lake District in 2019 can be modelled by a binomial distribution.

(i) State two assumptions made in order for this model to be valid.

(ii) Find the probability that at least three people from the sample took a holiday in the Lake District in 2019. [4]

(b) From a random sample of $n$ holidaymakers, the probability that at least one of them took a holiday in the Lake District in 2019 is greater than $0.999$.

Determine the least possible value of $n$. [3]

(a)(i) · The sample is large enough to negate sample size error

· The sample is diverse enough to ~~the~~ represent the entire population

0

(ii) $X \sim B(16, 0.15)$

$\therefore P(X \geq 3) = \text{binomCdf}(16, 0.15, 3, 16)$

$= 0.438621$

$\approx \cancel{0.437}\ 0.439$

2

(b) $\cancel{0.999 = 0.15^n + \binom{n}{1} 0.15^{n-1} 0.85 + \dots}$

$Y \sim B(N, 0.15)$

$0.999 = P(Y \geq 1)$

$= 1 - P(Y = 0)$

$\therefore 0.999 = 1 - 0.85^n$

$\therefore 0.85^n = 0.001$

$\therefore n = 42.5043$

$\therefore n = 43$

2

**6.** [Maximum mark: 7]

Use mathematical induction to prove that $\frac{d^n}{dx^n}\left(xe^{px}\right) = p^{n-1}(px+n)e^{px}$ for $n \in \mathbb{Z}^+$, $p \in \mathbb{Q}$.

1) Prove $n=1$:

$LHS = \frac{d}{dx}(xe^{px})$

$= xe^{px} \cdot p + e^{px}$

$= (px+1)e^{px}$

$RHS = p^0(px+1)e^{px}$

$= LHS$

$\therefore$ True for $n=1$

2) Inductive hypothesis: {assume}

$\frac{d^k}{dx^k}(xe^{px}) = p^{k-1}(px+k)e^{px}$

3) Prove for $n=k+1$:

$\frac{d^{k+1}}{dx^{k+1}}(xe^{px}) = p^k(px+k+1)e^{px}$

$LHS = \frac{d}{dx}\left(\frac{d^k}{dx^k}(xe^{px})\right)$

$= \frac{d}{dx}\left(p^{k-1}(px+k)e^{px}\right)$ [IH]

$= p^{k-1} \frac{d}{dx}\left(pxe^{px} + ke^{px}\right)$

$= p^{k-1}\left(p(pxe^{px} + pe^{px}) + pke^{px}\right)$

$= p^{k-1}\left(p^2xe^{px} + pe^{px} + pke^{px}\right)$

$= p^k\left(pxe^{px} + e^{px} + ke^{px}\right)$

$= p^k(px+k+1)e^{px}$

$= RHS$

4) $\therefore$ true for ~~and~~ $n=1$ and true for $n=k+1$ whenever $n=k$ is assumed to be true. Therefore true for all $n \in \mathbb{Z}^+$ by mathematical induction.

7

**7.** [Maximum mark: 7]

At a gathering of 12 teachers, seven are male and five are female. A group of five of these teachers go out for a meal together. Determine the possible number of groups in each of the following situations:

(a) There are more males than females in the group. [4]

(b) Two of the teachers, Gary and Gerwyn, refuse to go out for a meal together. [3]

(a) 5M OR 4M1F OR 3M2F

$= {}^7C_5 + {}^7C_4 \times {}^5C_1 + {}^7C_3 \times {}^5C_2$

$= 546$

4

(b) $\underbrace{\bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet}$

CHOOSE FROM 11

$\therefore x = \cancel{12}\ {}^{11}C_5$

$= 462$

$\binom{10}{5} + \binom{10}{4} + \binom{10}{4}$

$= 674$

4

**8.** [Maximum mark: 6]

A small bead is free to move along a smooth wire in the shape of the curve $y = \dfrac{10}{3-2e^{-0.5x}}$ $(x \geq 0)$.

(a) Find an expression for $\dfrac{dy}{dx}$. [3]

At the point on the curve where $x = 4$, it is given that $\dfrac{dy}{dt} = -0.1\,\text{m s}^{-1}$.

(b) Find the value of $\dfrac{dx}{dt}$ at this exact same instant. [3]

(a) $y = \dfrac{10}{3-2e^{-0.5x}}$ $x \geq 0$

$= 10(3-2e^{-0.5x})^{-1}$

$\therefore \dfrac{dy}{dx} = 10(-1)(3-2e^{-0.5x})^{-2}\left((-2)(-0.5)e^{-0.5x}\right)$

$= \dfrac{-10}{(3-2e^{-0.5x})^2} \times \dfrac{1}{e^{0.5x}}$

$= \dfrac{-10}{(3-2e^{-0.5x})^2 e^{0.5x}}$ 3

(b) $\cancel{\dfrac{dy}{dx} = \dfrac{dy}{dt} \times \dfrac{dt}{dx}}$ $\dfrac{dx}{dt} = \dfrac{dx}{dy} \cdot \dfrac{dy}{dt}$

$= -0.1 \times \dfrac{(3-2e^{-2})^2 e^2}{-10}$

$= 0.5504$

$\approx 0.550$

3

Alternative: $\dfrac{dy}{dt} = \dfrac{dy}{dx} \times \dfrac{dx}{dt}$

6

Do **not** write solutions on this page.

## Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

**9.** [Maximum mark: 14]

The weights, in grams, of individual packets of coffee can be modelled by a normal distribution, with mean $102\text{ g}$ and standard deviation $8\text{ g}$.

(a) Find the probability that a randomly selected packet has a weight less than $100\text{ g}$. [2]

(b) The probability that a randomly selected packet has a weight greater than $w$ grams is $0.444$. Find the value of $w$. [2]

(c) A packet is randomly selected. Given that the packet has a weight greater than $105\text{ g}$, find the probability that it has a weight greater than $110\text{ g}$. [3]

(d) From a random sample of $500$ packets, determine the number of packets that would be expected to have a weight lying within $1.5$ standard deviations of the mean. [3]

(e) Packets are delivered to supermarkets in batches of $80$. Determine the probability that at least $20$ packets from a randomly selected batch have a weight less than $95\text{ g}$. [4]

Do **not** write solutions on this page.

**10.** [Maximum mark: 16]

The plane $\Pi_1$ has equation $3x - y + z = -13$ and the line $L$ has vector equation

$$\boldsymbol{r} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ -1 \\ 4 \end{pmatrix}, \ \lambda \in \mathbb{R}.$$

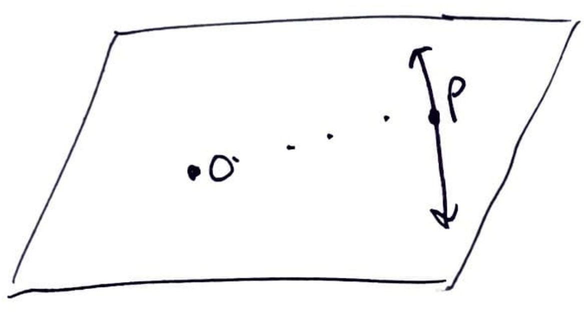
(a) Given that $L$ meets $\Pi_1$ at the point P, find the coordinates of P. [4]

(b) Find the shortest distance from the point $\mathrm{O}(0, 0, 0)$ to $\Pi_1$. [4]

The plane $\Pi_2$ contains the point O and the line $L$.

(c) Find the equation of $\Pi_2$, giving your answer in the form $\boldsymbol{r}.\boldsymbol{n} = d$. [3]

(d) Determine the acute angle between $\Pi_1$ and $\Pi_2$. [5]



Do **not** write solutions on this page.

**11.** [Maximum mark: 20]

A particle $P$ moves in a straight line such that after time $t$ seconds, its velocity, $v$ in $\mathrm{m\,s^{-1}}$, is given by $v = \mathrm{e}^{-3t} \sin 6t$, where $0 < t < \frac{\pi}{2}$.

(a) Find the times when $P$ comes to instantaneous rest. [2]

At time $t$, $P$ has displacement $s(t)$; at time $t = 0$, $s(0) = 0$.

(b) Find an expression for $s$ in terms of $t$. [7]

(c) Find the maximum displacement of $P$, in metres, from its initial position. [2]

(d) Find the total distance travelled by $P$ in the first $1.5$ seconds of its motion. [2]

At successive times when the acceleration of $P$ is $0\,\mathrm{m\,s^{-2}}$, the velocities of $P$ form a geometric sequence. The acceleration of $P$ is zero at times $t_1$, $t_2$, $t_3$ where $t_1 < t_2 < t_3$ and the respective velocities are $v_1$, $v_2$, $v_3$.

(e) (i) Show that, at these times, $\tan 6t = 2$.

(ii) Hence show that $\frac{v_2}{v_1} = \frac{v_3}{v_2} = -\mathrm{e}^{-\frac{\pi}{2}}$. [7]

ANSWER BOOKLET
LIVRET DE RÉPONSES
CUADERNILLO DE RESPUESTAS

1

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

4 PAGES / PÁGINAS

Candidate session number: / Numéro de session du candidat: / Número de convocatoria del alumno: 20 N TZ 0 P 2 - M A H L

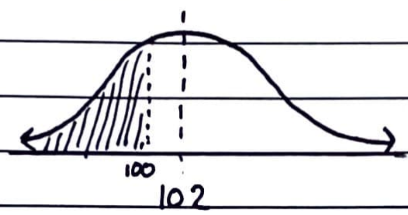
Candidate name: / Nom du candidat: / Nombre del alumno:

Please write question numbers in the following format: / Veuillez numéroter les questions en utilisant la présentation suivante: / Sírvase escribir los números de las preguntas en el siguiente formato:

1 2 3 4 5 6 7 8 9 10

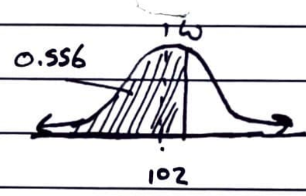
9

(a) $X \sim N(102, 8^2)$




$\therefore P(X \leq 100) = \text{normcdf}(-9E999, 100, 102, 8)$
$= 0.401294$
$\approx 0.401$ ✓✓ 2

(b) $P(X \geqslant w) = 0.444$
$\therefore P(X \leq w) = 0.556$ ✓
$\therefore w = \text{invNorm}(0.556, 102, 8)$ 2
$= 103.127$ ✓
$\approx 103 \text{ g}$




~~(c) P(W > 105) =~~

(c) $P(W > 105) = \cancel{0.35383}$ normcdf(105, 9E999, 102, 8)

$= 0.35383$

$P(W > 110) =$ normcdf(110, 9E999, 102, 8)

$= 0.158655$

$\therefore$ ~~P(W > 110~~ $P(W > 110 \mid W > 105) = \dfrac{P(W > 110)}{P(W > 105)}$

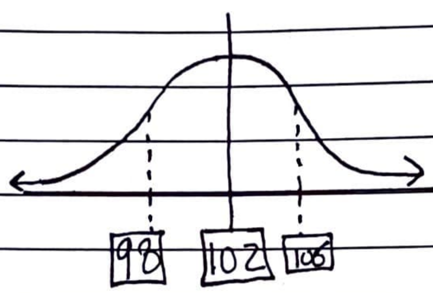
$= \dfrac{0.158655}{0.35383}$

$= 0.44839$

3

$\approx 0.448$

(d)




1.5, not 0.5!

$\therefore$ P(success) = binomcdf(98, 106, 102, 8)

$= 0.382925$

~~Y ~ B(500, 0.382925)~~

ECF

$\therefore E(X) = 0.382925 \times 500$

$= 191.463$

$\approx 191$ packets

~~4~~ 2

(e) $\cancel{P(X<20)} =$

$P(X<95) = \text{binomCdf}(-9E999, 95, 102, 8)$
$= 0.190787$

$\therefore Y \sim B(80, 0.190787)$

$\therefore P(Y \geq 20) = \text{binomCdf}(80, 0.190787, 20, 80)$

$= 0.115986$
$\approx 0.116$

4

13

10

(a) $\Pi_1 : 3x - y + z = 13$ (1)

$r = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ -1 \\ 4 \end{pmatrix}$ $\begin{cases} x = 1 - 3\lambda \\ y = 2 - \lambda \\ z = -2 + 4\lambda \end{cases}$ (2)

Substituting (2) into (1)

$\therefore 3(1-3\lambda) - (2-\lambda) + (-2+4\lambda) = -13$

$\therefore 3 - 9\lambda - 2 + \lambda - 2 + 4\lambda = -13$

$\therefore -4\lambda = -13 + 4 - 3$

$\therefore -4\lambda = \cancel{14} - 12$

$\therefore \cancel{\lambda = -14/4}$

$\cancel{= -7/2}$

$\therefore \lambda = 3$

$\cancel{\therefore P : x = 1 - 3(-7/2)}$

$\cancel{= 25/4 \quad 23/2}$

$\cancel{y = 2 - (-7/2)}$

$\cancel{= 11/2}$

$\cancel{z = -2 + 4(-7/2)}$

$\cancel{= -16}$

$\cancel{\therefore P : \left(\frac{23}{2}, \frac{11}{2}, -16\right)}$

$\therefore P : x = 1 - 3(3)$

$= 1 - 9$

$= -8$

$y = 2 - 3$

$= -1$

$z = -2 + 12$

$= 10$

$\therefore P(-8, -1, 10)$

4

**ANSWER BOOKLET**
**LIVRET DE RÉPONSES**
**CUADERNILLO DE RESPUESTAS** 2

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

**4 PAGES / PÁGINAS**

Candidate session number: / Numéro de session du candidat: / Número de convocatoria del alumno: 20 N TZ 0 P 2 - M A H L

Candidate name: / Nom du candidat: / Nombre del alumno:

Please write question numbers in the following format: / Veuillez numéroter les questions en utilisant la présentation suivante: / Sírvase escribir los números de las preguntas en el siguiente formato:

1 2 3 4 5 6 7 8 9 10

10

(b) $\vec{n} = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$

$\therefore r_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$ must intersect $\Pi_1$

$x = 3\mu$; $y = -\mu$; $z = \mu$

$\therefore 3(3\mu) - (-\mu) + \mu = -13$

$\therefore 9\mu + \mu + \mu = -13$

$\therefore \mu = -13/11$

$\therefore r_2 = -\frac{13}{11}\begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$

$\therefore |r_2| = \left|\begin{pmatrix} -39/11 \\ 13/11 \\ -13/11 \end{pmatrix}\right|$

$= \sqrt{\left(\frac{39}{11}\right)^2 + \left(\frac{13}{11}\right)^2 + \left(\frac{13}{11}\right)^2}$

$= 3.91965$

$\approx 3.92$ units

4

(c) ~~$Q(0,0,0)$~~ ~~$P\left(\frac{23}{2}, \frac{11}{2}, -16\right)$~~

~~$\vec{b} = \begin{pmatrix} -3 \\ -1 \\ 4 \end{pmatrix}$~~ ~~$\vec{OP} = (23/2, 11/2, -16)$~~

$O(0,0,0)$ $P(-8,-1,10)$ ×

$\therefore \vec{b} = \begin{pmatrix} -3 \\ -1 \\ 4 \end{pmatrix}$ $\therefore \vec{OP} = \begin{pmatrix} -8 \\ -1 \\ 0 \end{pmatrix}$ ECF.

Hence, $\vec{n} = \begin{pmatrix} -3 \\ -1 \\ 4 \end{pmatrix} \times \begin{pmatrix} -8 \\ -1 \\ 10 \end{pmatrix}$

$= \begin{pmatrix} (-1)(10) - (-1)(4) \\ (-8)(4) - (-3)(10) \\ (-3)(-1) - (-8)(-1) \end{pmatrix}$

$= \begin{pmatrix} 4 \\ -32 \\ -5 \end{pmatrix}$ $\begin{pmatrix} -6 \\ -2 \\ -5 \end{pmatrix}$ 1

$\therefore 4x - 32y - 5z = d$ ×

As $O(0,0,0)$ lies on the plane, $d = 0$

$\therefore 4x - 32y - 5z = 0$

(d) $\theta = \arccos\left(\frac{|n_1 \cdot n_2|}{|n_1||n_2|}\right)$ M1 (A1)(A1)

$= \arccos\left(\frac{3 \cdot 4 + -1 \cdot -32 + 1 \cdot -5}{\sqrt{9+1+1} \times \sqrt{16+1024+25}}\right)$

$= 1.20^c$ 2

$= 68.9^\circ$

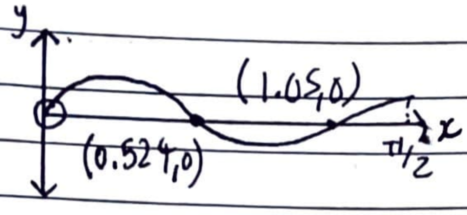
12

$v = e^{-3t} \sin 6t \qquad 0 < t < \pi/2$

(a) $e^{-3t} \sin 6t = 0$

$\therefore \quad t = 0.524, \quad 1.05 \ (s)$

[G.D.C]




2

(b) $s(t) = \int e^{-3t} \sin 6t \, dt$

$\therefore \int e^{-3t} \sin 6t \, dt = -\frac{1}{6} e^{-3t} \cos 6t - \int -\frac{1}{6} \cdot -3e^{-3t} \cos 6t \, dt$

$$\boxed{\begin{array}{ll} u = e^{-3t} & du = -3e^{-3t} \\ dv = \sin 6t & v = -\frac{1}{6}\cos 6t \end{array}}$$

$\therefore \int e^{-3t} \sin 6t \, dt = -\frac{1}{6} e^{-3t} \cos 6t - \frac{1}{2} \int e^{-3t} \cos 6t \, dt$

$$\boxed{\begin{array}{ll} u = e^{-3t} & du = -3e^{-3t} \\ dv = \cos 3t & v = \frac{1}{6}\sin 6t \end{array}}$$

$\therefore \int e^{-3t} \sin 6t \, dt = -\frac{1}{6} e^{-3t} \cos 6t - \frac{1}{2}\left[\frac{1}{6} e^{-3t} \sin 6t - \int \frac{1}{6} \sin 6t \cdot -3e^{-3t} \, dt\right]$

$\therefore \int e^{-3t} \sin 6t \, dt = -\frac{1}{6} e^{-3t} \cos 6t - \frac{1}{12} e^{-3t} \sin 6t - \frac{3}{12} \int e^{-3t} \sin 6t \, dt$

$\therefore \frac{5}{4} \int e^{-3t} \sin 6t \, dt = -\frac{1}{12}\left(2e^{-3t} \cos 6t + e^{-3t} \sin 6t\right) + c$

$\therefore \int e^{-3t} \sin 6t \, dt = -\frac{4}{60}\left(2e^{-3t} \cos 6t + e^{-3t} \sin 6t\right) + c$

$\therefore \int e^{-3t} \sin 6t \, dt = -\frac{1}{15}\left(2e^{-3t} \cos 6t + e^{-3t} \sin 6t\right) + c$

as $s(0) = 0$,

$$0 = -\frac{1}{15}\left(2e^{0}\cos 0 + e^{0}\sin 0\right) + c$$
$$= -\frac{1}{15}(2) + c$$
$$\therefore\ c = 2/15$$

7

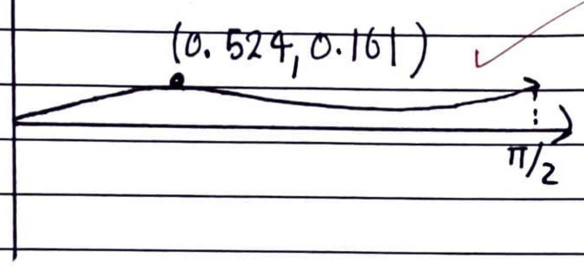
$\therefore$ ~~$s(t) = -\frac{1}{15}(2e^{x}$~~

$$\therefore\ s(t) = -\frac{1}{15}\left(2e^{-3t}\cos 6t + e^{-3t}\sin 6t\right) + \frac{2}{15}$$

(c) ~~Maximum displacement occurs when $v(t) = 0$~~

~~$\therefore\ t = 0.524,\ 1.05\ (s)$ {part (a)}~~

G.D.C $\quad f_3(x) = \frac{-1}{15}\cdot\left(2\cdot e^{-3x}\cdot\cos(6\cdot x) + e^{-3\cdot x}\cdot\sin(6\cdot x)\right) + \frac{2}{15}$




2

$\therefore$ Max displacement at $t = 0.524$

is $S(0.524) = 0.161$ m

ANSWER BOOKLET
LIVRET DE RÉPONSES
CUADERNILLO DE RESPUESTAS

3

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

4 PAGES / PÁGINAS

Candidate session number: / Numéro de session du candidat: / Número de convocatoria del alumno:

| 20 | N | TZ | 0 | P | 2 | - | M | A | H | L |
|---|---|---|---|---|---|---|---|---|---|---|

Candidate name: / Nom du candidat: / Nombre del alumno:

Please write question numbers in the following format: / Veuillez numéroter les questions en utilisant la présentation suivante: / Sírvase escribir los números de las preguntas en el siguiente formato:

1 2 3 4 5 6 7 8 9 10

(d)

$$\text{Distance} = \int_0^{1.5} \left| e^{-3\cdot x} \cdot \sin(6\cdot x) \right| dx$$

$$= 0.201336 \text{ m}$$

$$\approx 0.201 \text{ m}$$

2

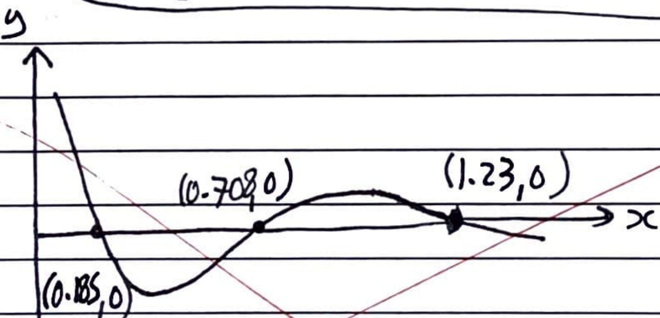
(e) (i) $v = e^{-3t} \sin 6t$

$$\therefore a = \sin 6t\,(-3e^{-3t}) + e^{-3t}\,(6 \cancel{\sin}\cos 6t)$$

$$\cancel{= -3e^{-3t}}$$

$$= -3e^{-3t}\sin 6t + 6e^{-3t}\cancel{\sin}\cos 6t$$

~~[GDC]:~~




~~$\therefore t_1 = 0.185$ s~~

~~$t_2 = 0.708$ s~~

~~$t_3 = 1.23$ s~~

following page

$3e^{-3t}\sin 6t = 6e^{-3t}\cos 6t$

$\therefore \sin 6t = 2\cos 6t$

$\therefore \dfrac{\sin 6t}{\cos 6t} = 2$

$\therefore \tan 6t = 2$

2

(ii) ~~$6t = \arctan(2)$~~

~~$\therefore t = \frac{1}{6}\arctan(2)$~~

~~$\therefore \dfrac{v_2}{v_1} = \dfrac{e^{-3(\frac{1}{6}\arctan(2))}}{\sin(\arctan(2))}$~~

$\therefore \tan\left(6t + k\pi\right) = 2$

$\therefore 6t + k\pi = \arctan(2)$

$\therefore t = \dfrac{\arctan(2) - k\pi}{6}, k = 0, -1, -2$

$= \dfrac{\arctan(2) + k\pi}{6}, k = 0, 1, 2.$

$\therefore v_1 =$

2

17

$$\cancel{\frac{dy}{dt} = -0.1}$$

$$\therefore \cancel{y = -0.1t + C}$$

Q11 (e) (ii) $\frac{v_2}{v_1} = \frac{v_3}{v_2} = -e^{-\pi/2}$

$$\tan 6t = 2$$

$$\therefore \tan(6t + k\pi) = 2$$

$$\therefore 6t + k\pi = \arctan(2)$$

$$\therefore 6t = \arctan(2) + k\pi, \quad k = 0, 1, 2.$$

$$\therefore t_1 = \frac{1}{6}\arctan(2)$$

$$t_2 = \frac{1}{6}(\arctan(2) + \pi)$$

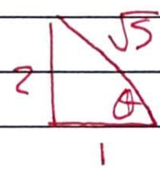
$$t_3 = \frac{1}{6}(\arctan(2) + 2\pi)$$

$$v_1(t_1) = e^{-3(\frac{1}{6}\arctan(2))} \sin(\arctan(2))$$

$$= e^{-\frac{1}{2}\arctan 2} \sin(\arctan(2))$$

$$v_2(t_2) = e^{-3(\frac{1}{6}\arctan 2 + \pi/6)} \sin(\arctan(2) + \pi/6)$$

$$\therefore \frac{v_2}{v_1} = \frac{e^{-\frac{1}{2}\arctan 2 - \pi/2} \sin(\arctan 2 + \pi)}{e^{-\frac{1}{2}\arctan 2} \sin(\arctan 2)}$$




$$= \frac{e^{-\pi/2} \cancel{\sin}(2/\sqrt{5})}{-\sin(2/\sqrt{5})}$$

$$= -e^{-\pi/2}$$

Note: same derivation for $v_3/v_2$