Practice Set C: Paper 1 Mark scheme

SECTION A

1 **a** Attempt to find x-coordinate of turning point:

$\frac{dy}{dx} = 0 : 4x + 10 = 0$ M1

$x = -\frac{5}{2}$

So required domain: $x \leqslant -\frac{5}{2}$ A1

b $y = 2\left[\left(x + \frac{5}{2}\right)^2 - \frac{25}{4}\right] + 7$ (M1)

$= 2\left(x + \frac{5}{2}\right)^2 - \frac{11}{2}$ A1

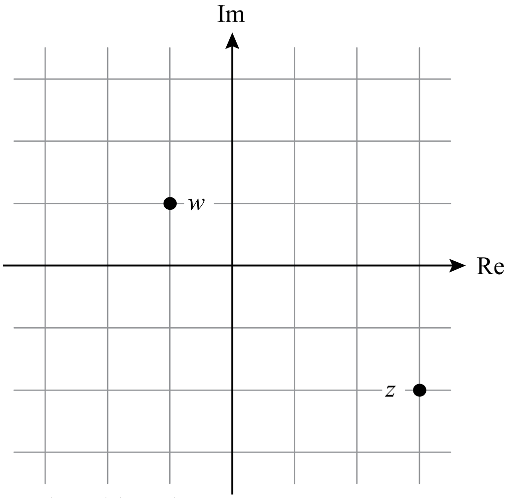
Since $x \leqslant 1$, $f^{-1}(x) = \frac{-5 - \sqrt{2x+11}}{2}$ M1

Domain of $f^{-1} : x \geqslant -\frac{11}{2}$ A1

[6 marks]

2 **a** z correct A1

w correct A1



b $\frac{(-1+i)(3+2i)}{9+4}$ M1

$= -\frac{5}{13} + \frac{1}{13}i$ A1

c Compare real and imaginary parts: M1

$3p - q = 6, -2p + q = 0$

$p = 6, q = 12$ A1

[6 marks]

3 Find the intersection points:

$2x + 1 = x - 3$ OR $2x + 1 = -x + 3$

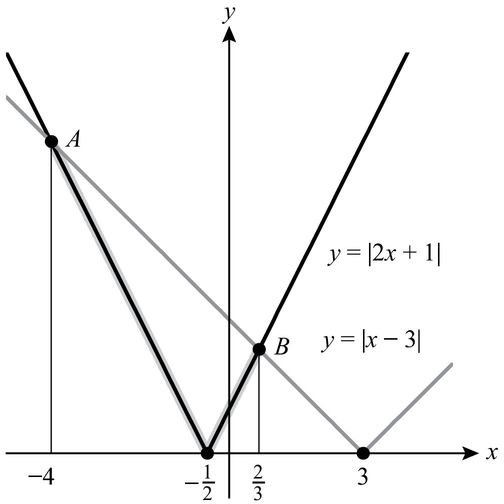
OR

square to get $4x^2 + 4x + 1 = x^2 - 6x + 9$ M1

$x = -4$ A1

$x = \frac{2}{3}$ A1

Graph sketch (or consider signs of factors) M1



$-4 < x < \frac{2}{3}$ A1

[5 marks]

4 To be strictly increasing for all x, f must have no stationary points M1
$f'(x) = 3x^2 + 2kx + k$ A1
$3x^2 + 2kx + k = 0$ has no solutions when $(2k)^2 - 4 \times 3k < 0$ M1
$k(k - 3) < 0$ A1
$0 < k < 3$ A1

[5 marks]

5 Attempt to use partial fractions

$$\frac{3x-16}{(3x-2)(x+4)} = \frac{A}{3x-2} + \frac{B}{x+4}$$

$3x - 16 = A(x + 4) + B(3x - 2)$ M1
$x = -4$: $-28 = B(-14)$
$B = 2$ A1
$x = \frac{2}{3}: -14 = A\left(\frac{14}{3}\right)$
$A = -3$ A1
$\int_1^6 \frac{2}{x+4} - \frac{3}{3x-2}\,dx = \left[2\ln|x+4| - \ln|3x-2|\right]_1^6$ A1ft
Substitute in limits
$= 2\ln 10 - \ln 16 - 2\ln 5 + \ln 1$ M1
$= \ln\frac{1}{4}$ A1

[6 marks]

6 **a** Use $\sin x \approx x$ M1

$\frac{1}{10}\sin 3x \approx \frac{3}{10}x$ A1

b $\frac{3}{10}x \approx x^2$ M1

$x = 0$ A1
$x \approx 0.3$ A1

[5 marks]

7 Use $\dfrac{u_1}{(1-r)} = 5$ M1

Use $u_1 + u_1 r = 3$ M1

Express u_1 from both equations and equate:

$5(1-r) = \dfrac{3}{1+r}$ M1

$1 - r^2 = \dfrac{3}{5}$ A1

$r = \sqrt{\dfrac{2}{5}}$ A1

[5 marks]

8 EITHER

$\log_4(3-2x) = \dfrac{\log_{16}(3-2x)}{\log_{16}4} = \dfrac{\log_{16}(3-2x)}{\frac{1}{2}}$ M1A1

$2\log_{16}(3-2x) = \log_{16}(6x^2 - 5x + 12)$

$\log_{16}(3-2x)^2 = \log_{16}(6x^2 - 5x + 12)$ A1

OR

$\log_{16}(6x^2 - 5x + 12) = \dfrac{\log_4(6x^2 - 5x + 12)}{\log_4 16} = \dfrac{\log_4(6x^2 - 5x + 12)}{2}$ M1A1

$2\log_4(3-2x) = \log_4(6x^2 - 5x + 12)$

$\log_4(3-2x)^2 = \log_4(6x^2 - 5x + 12)$ A1

$(3-2x)^2 = 6x^2 - 5x + 12$ M1

$2x^2 + 7x + 3 = 0$ A1

$(2x+1)(x+3) = 0$

$x = -\dfrac{1}{2}, -3$ A1

Checks their solutions in equation:

$x = -\dfrac{1}{2}$: $3 - 2x = 4 > 0$ and $6x^2 - 5x + 12 = 16 > 0$

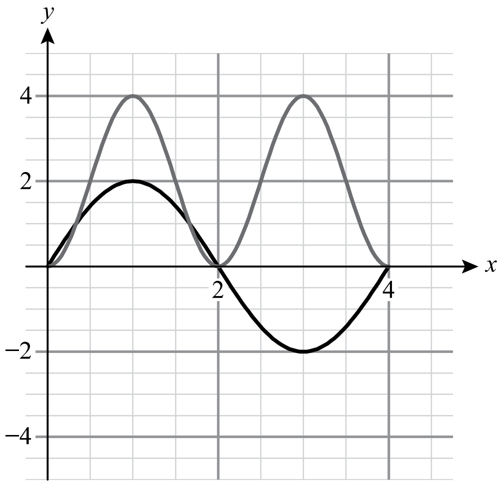
$x = -3$: $3 - 2x = 9 > 0$ and $6x^2 - 5x + 12 = 81 > 0$

So solutions are $x = -\dfrac{1}{2}, -3$

Note: Award A1 if conclusion consistent with working A1

[7 marks]

9 **a**



y in the range 0 to 4 A1

Intersections at $y = 0$ A1

Intersections at $y = 1$ A1

b Domain: $1 \leqslant x \leqslant 5$ A1

Range: $-4 \leqslant g(x) \leqslant 4$ A1

[5 marks]

10 a $\sin y = x$ (M1)

$\cos\left(\frac{\pi}{2} - y\right) = x$ (M1)

$\arccos x = \frac{\pi}{2} - y$ A1

b $\arcsin x + \arccos x = y + \frac{\pi}{2} - y$ M1

So $\arcsin x + \arccos x \equiv \frac{\pi}{2}$ A1

[5 marks]

SECTION B

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11 a i Find $\overrightarrow{AB} = \begin{pmatrix} 1 \\ 5 \\ -4 \end{pmatrix}$ A1

$\mathbf{r} = \begin{pmatrix} 1 \\ -4 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 5 \\ -4 \end{pmatrix}$

OR

$\mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 5 \\ -4 \end{pmatrix}$ A1A1

ii $\mathbf{r} = \begin{pmatrix} -1 \\ 3 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 5 \\ -4 \end{pmatrix}$ A1A1ft

[5 marks]

b i $AB = \sqrt{1^2 + 5^2 + (-4)^2}$ M1

$= \sqrt{42}$ A1

ii $\mathbf{c} = \mathbf{d} \pm 2\overrightarrow{AB}$ (M1)

So the coordinates of C are (1, 13, –5) A1

OR (–3, –7, 11) A1

iii Consider $\overrightarrow{AC_1} \cdot \overrightarrow{AC_2}$ M1

$= 0 - 51 - 64$ $[= -115]$ A1

< 0 so obtuse A1

[8 marks]

c i Use $\overrightarrow{AD} = \begin{pmatrix} -2 \\ 7 \\ 0 \end{pmatrix}$

$\overrightarrow{AB} \times \overrightarrow{AD} = \begin{pmatrix} 1 \\ 5 \\ -4 \end{pmatrix} \times \begin{pmatrix} -2 \\ 7 \\ 0 \end{pmatrix}$ M1

$= \begin{pmatrix} 28 \\ 8 \\ 17 \end{pmatrix}$ A1

ii Scalar product of $\begin{pmatrix} 28 \\ 8 \\ 17 \end{pmatrix}$ with **a**, **b** or **d** attempted $A(1, -4, 3)$ M1

$28x + 8y + 17z$ M1

$= 47$ A1

[5 marks]

Total [18 marks]

12 a $\cos(2\theta + \theta) = \cos(2\theta)\cos\theta - \sin(2\theta)\sin\theta$ M1

$= (2\cos^2\theta - 1)\cos\theta - 2\sin^2\theta\cos\theta$ A1

$= 2\cos^3\theta - \cos\theta - 2(1 - \cos^2\theta)\cos\theta$ (M1)

$= 4\cos^3\theta - 3\cos\theta$ A1

[4 marks]

b i $8\cos^3\theta - 6\cos\theta + 1 = 0$ M1

$2(4\cos^3\theta - 3\cos\theta) = -1$ (M1)

$\cos 3\theta = -\dfrac{1}{2}$ A1

ii $3\theta = \dfrac{2\pi}{3},$ A1

$\dfrac{4\pi}{3}, \dfrac{8\pi}{3}$ A1

$\theta = \dfrac{2\pi}{9}, \dfrac{4\pi}{9}, \dfrac{8\pi}{9}$ A1

Hence $x = \cos\left(\dfrac{2\pi}{9}\right), \cos\left(\dfrac{4\pi}{9}\right), \cos\left(\dfrac{8\pi}{9}\right)$ A1

[7 marks]

c Product of the roots of the cubic equation is $-\dfrac{1}{8}$ M1

$\cos\left(\dfrac{2\pi}{9}\right)\cos\left(\dfrac{4\pi}{9}\right)\cos\left(\dfrac{8\pi}{9}\right) = -\dfrac{1}{8}$ M1

$8\cos\left(\dfrac{2\pi}{9}\right)\cos\left(\dfrac{4\pi}{9}\right) = -\dfrac{1}{\cos\left(\dfrac{8\pi}{9}\right)}$

$= -\sec\left(\dfrac{8\pi}{9}\right)$ A1(AG)

[3 marks]

d State 0 A1

It is the sum of the roots of the equation, the coefficient of x^2 is 0 A1

[2 marks]

Total [16 marks]

13 a $f(-x) = \dfrac{-x}{1+(-x)^2}$ M1

$= -\dfrac{x}{1+x^2}$

$= -f(x)$ A1

So f is an odd function A1

[3 marks]

b $\displaystyle\int_0^{\sqrt{3}} \frac{kx}{1+x^2}\,dx = 1$ M1

$\left[\dfrac{k}{2}\ln(1+x^2)\right]_0^{\sqrt{3}} = 1$ A1

$\dfrac{k}{2}\ln(4) = 1$ A1

$k\ln 4^{\frac{1}{2}} = 1$ M1

$k = \dfrac{1}{\ln 2}$ AG

[4 marks]

c $\frac{1}{\ln 2}\int_0^m \frac{x}{1+x^2}\,dx = \frac{1}{2}$ (M1)

$\frac{1}{\ln 2}\frac{1}{2}\ln(1+m^2) = \frac{1}{2}$ A1

$\ln(1+m^2) = \ln 2$ A1

$1 + m^2 = 2$

$m = 1$ A1

[4 marks]

d $g'(x) = \frac{1}{\ln 2}\left(\frac{1(1+x^2) - x(2x)}{(1+x^2)}\right) = 0$ M1A1

$1 - x^2 = 0$

$x = 1$ A1

$g(0) = 0$ and $g(1) = \frac{1}{2\ln 2} > 0$ so $x = 1$ is local maximum (or alternative justification) M1

So $x = 1$ is the mode A1

[5 marks]

e $E(X) = \frac{1}{\ln 2}\int_0^{\sqrt{3}} \frac{x^2}{1+x^2}\,dx$ (M1)

$\frac{x^2}{1+x^2} = \frac{1+x^2-1}{1+x^2} = 1 - \frac{1}{1+x^2}$ M1

$E(X) = \frac{1}{\ln 2}\left[x - \arctan x\right]_0^{\sqrt{3}}$ A1

$= \frac{1}{\ln 2}\left(\sqrt{3} - \arctan\sqrt{3}\right)$ (M1)

$= \frac{1}{\ln 2}\left(\sqrt{3} - \frac{\pi}{3}\right)$ A1

[5 marks]

Total [21 marks]