Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 2

Thursday 3 May 2018 (morning)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.

2218 – 7204

12 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 7]

The 3rd term of an arithmetic sequence is 1407 and the 10th term is 1183.

(a) Find the first term and the common difference of the sequence. [4]

(b) Calculate the number of positive terms in the sequence. [3]

2. [Maximum mark: 6]

The equation $x^2 - 5x - 7 = 0$ has roots α and β. The equation $x^2 + px + q = 0$ has roots $\alpha + 1$ and $\beta + 1$. Find the value of p and the value of q.

3. [Maximum mark: 5]

Let $f(x) = \tan(x + \pi)\cos\left(x - \frac{\pi}{2}\right)$ where $0 < x < \frac{\pi}{2}$.

Express $f(x)$ in terms of $\sin x$ and $\cos x$.

4. [Maximum mark: 5]

The age, L, in years, of a wolf can be modelled by the normal distribution $L \sim \mathrm{N}(8\,,\,5)$.

(a) Find the probability that a wolf selected at random is at least 5 years old. [2]

Eight wolves are independently selected at random and their ages recorded.

(b) Find the probability that more than six of these wolves are at least 5 years old. [3]

5. [Maximum mark: 7]

(a) Given that $2x^3 - 3x + 1$ can be expressed in the form $Ax(x^2 + 1) + Bx + C$, find the values of the constants A, B and C. [2]

(b) Hence find $\int \frac{2x^3 - 3x + 1}{x^2 + 1} \mathrm{d}x$. [5]

6. [Maximum mark: 5]

The mean number of squirrels in a certain area is known to be 3.2 squirrels per hectare of woodland. Within this area, there is a 56 hectare woodland nature reserve. It is known that there are currently at least 168 squirrels in this reserve.

Assuming the population of squirrels follow a Poisson distribution, calculate the probability that there are more than 190 squirrels in the reserve.

7. [Maximum mark: 8]

It is known that the number of fish in a given lake will decrease by 7% each year unless some new fish are added. At the end of each year, 250 new fish are added to the lake. At the start of 2018, there are 2500 fish in the lake.

(a) Show that there will be approximately 2645 fish in the lake at the start of 2020. [3]

(b) Find the approximate number of fish in the lake at the start of 2042. [5]

8. [Maximum mark: 7]

Each of the 25 students in a class are asked how many pets they own. Two students own three pets and no students own more than three pets. The mean and standard deviation of the number of pets owned by students in the class are $\frac{18}{25}$ and $\frac{24}{25}$ respectively.

Find the number of students in the class who do not own a pet.

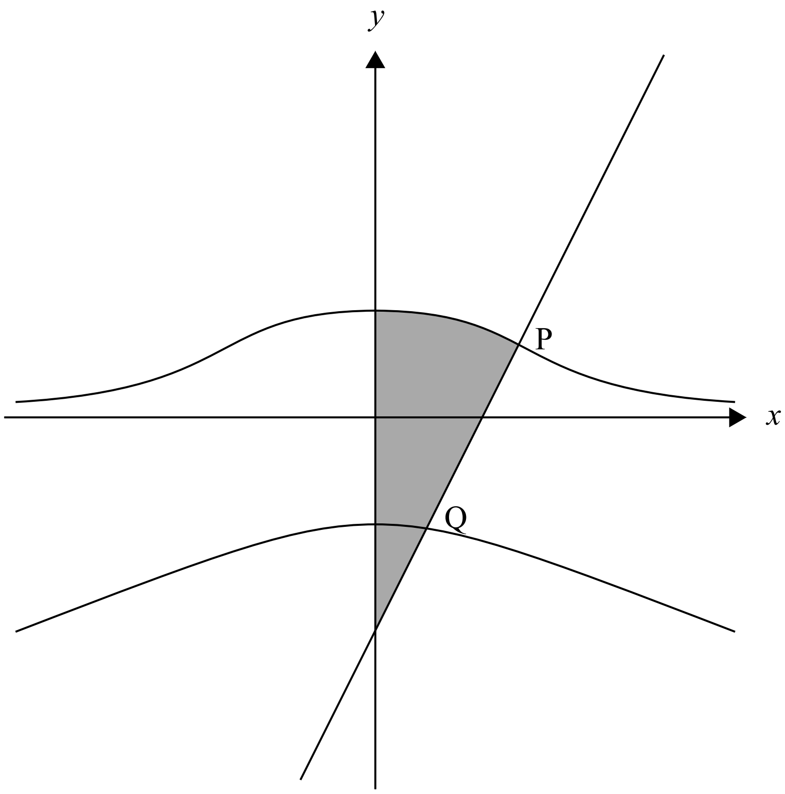
Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 22]

The following graph shows the two parts of the curve defined by the equation $x^2y = 5 - y^4$, and the normal to the curve at the point $\mathrm{P}(2, 1)$.



(a) Show that there are exactly two points on the curve where the gradient is zero. [7]

(b) Find the equation of the normal to the curve at the point P. [5]

(c) The normal at P cuts the curve again at the point Q. Find the x-coordinate of Q. [3]

(d) The shaded region is rotated by 2π about the y-axis. Find the volume of the solid formed. [7]

Do **not** write solutions on this page.

10. [Maximum mark: 13]

The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} 3ax & , \quad 0 \le x < 0.5 \\ a(2-x) & , \quad 0.5 \le x < 2 \\ 0 & , \quad \text{otherwise} \end{cases}$$

(a) Show that $a = \frac{2}{3}$. [3]

(b) Find $\mathrm{P}(X < 1)$. [3]

(c) Given that $\mathrm{P}(s < X < 0.8) = 2 \times \mathrm{P}(2s < X < 0.8)$, and that $0.25 < s < 0.4$, find the value of s. [7]

Do **not** write solutions on this page.

11. [Maximum mark: 15]

Two submarines A and B have their routes planned so that their positions at time t hours, $0 \le t < 20$, would be defined by the position vectors $\boldsymbol{r}_A = \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} + t\begin{pmatrix} -1 \\ 1 \\ -0.15 \end{pmatrix}$ and $\boldsymbol{r}_B = \begin{pmatrix} 0 \\ 3.2 \\ -2 \end{pmatrix} + t\begin{pmatrix} -0.5 \\ 1.2 \\ 0.1 \end{pmatrix}$ relative to a fixed point on the surface of the ocean (all lengths are in kilometres).

(a) Show that the two submarines would collide at a point P and write down the coordinates of P. [4]

To avoid the collision submarine B adjusts its velocity so that its position vector is now given by

$$\boldsymbol{r}_B = \begin{pmatrix} 0 \\ 3.2 \\ -2 \end{pmatrix} + t\begin{pmatrix} -0.45 \\ 1.08 \\ 0.09 \end{pmatrix}.$$

(b) (i) Show that submarine B travels in the same direction as originally planned.

(ii) Find the value of t when submarine B passes through P. [3]

(c) (i) Find an expression for the distance between the two submarines in terms of t.

(ii) Find the value of t when the two submarines are closest together.

(iii) Find the distance between the two submarines at this time. [8]