ANSWER BOOKLET
LIVRET DE RÉPONSES
CUADERNILLO DE RESPUESTAS

1

Attempt 1: $\frac{38}{50} = 76\%$.

International Baccalaureate® Baccalauréat International Bachillerato Internacional

4 PAGES / PÁGINAS

Candidate session number: / Numéro de session du candidat: / Número de convocatoria del alumno: P30004 - MA HL

Candidate name: / Nom du candidat: / Nombre del alumno:

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Example Ejemplo 27 [27]   Example Ejemplo 3 [3]

1

(a) $\frac{1}{x^2-1} = \frac{1}{(x+1)(x-1)}$

$= \frac{A}{x+1} + \frac{B}{x-1}$

$\therefore \quad 1 = A(x-1) + B(x+1)$

$\hookrightarrow \quad 1 = -2A$

$\therefore A = -1/2$

$\hookrightarrow \quad 1 = 2B$

$\therefore B = 1/2$

4

$\therefore \frac{1}{x^2-1} = -\frac{1}{2x+2} + \frac{1}{2x-2}$

(b) $= \frac{(\sqrt{1+x^2}+x)}{(\sqrt{1+x^2}-x)} \times \frac{(\sqrt{1+x^2}+x)}{(\sqrt{1+x^2}+x)}$

$= \frac{(1+x^2) + 2x\sqrt{1+x^2} + x^2}{(1+x^2) - x^2}$

2

$= 2x^2 + 2x\sqrt{1+x^2} + 1$

(c) $\cancel{\int \frac{x^2}{(x^2-1)^2} dx} =$

$\cancel{u = x \quad du = 1}$
$\cancel{dv = \frac{x}{(x^2-1)^2} \quad v =}$
$\cancel{= \frac{x}{}}$

let $u = x \rightarrow du = dx$
let $dv = \frac{x}{(x^2-1)^2} \rightarrow v = \int \frac{x}{(x^2-1)^2} dx$

$= \frac{1}{2} \int \frac{1}{s^2} ds$

$= \frac{1}{2} \int s^{-2} ds$

$= \frac{1}{2}(-1) s^{-1}$

$= -\frac{1}{2s}$

$= -\frac{1}{2(x^2-1)}$

$\therefore \int \frac{x^2}{(x^2-1)^2} dx = -\frac{x}{2x^2-2} - \int -\frac{1}{2x^2-2} dx$

$= -\frac{x}{2x^2-2} + \frac{1}{2} \int \frac{1}{x^2-1} dx$

$= -\frac{x}{2(x^2-1)} + \frac{1}{2} \int \left(\frac{1}{2x-2} - \frac{1}{2x+2} \right) dx$

$= -\frac{x}{2(x^2-1)} + \frac{1}{4} \int \left(\frac{1}{x-1} - \frac{1}{x+1} \right) dx$

$= -\frac{x}{2(x^2-1)} + \frac{1}{4} \left(\ln|x-1| - \ln|x+1| \right)$

$= -\frac{x}{2(x^2-1)} + \frac{1}{4} \ln\left| \frac{x-1}{x+1} \right| + C$

5

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(d)

$$t^2 = \frac{1+x^2}{x^2}$$

$$\therefore x^2t^2 = 1 + x^2$$

$$\therefore x^2t^2 - x^2 = 1$$

$$\therefore x^2(t^2-1) = 1$$

$$\therefore x^2 = \frac{1}{t^2-1}$$

2

(e)

$$\int \sqrt{1+x^2}\,dx = \int \frac{x\sqrt{1+x^2}}{x}\,dx$$

$$\cancel{t = \sqrt{\frac{1+x^2}{x^2}}} \quad \therefore \frac{dt}{dx} =$$

$$x^2 = \frac{1}{t^2-1}$$

$$= (t^2-1)^{-1}$$

$$\therefore dx/dt = -(t^2-1)^{-2}(2t)$$

$$= -2t(t^2-1)^{-2}$$

$$\therefore dx = \frac{-2t}{(t^2-1)^2}\,dt$$

$$\therefore \int\sqrt{1+x^2}\,dx = \int\left(\sqrt{1+\frac{1}{t^2-1}} \times \frac{-2t}{(t^2-1)^2}\right)dt$$

$$= -2\int\left(\sqrt{\frac{t^2-1+1}{t^2-1}} \times \frac{t}{(t^2-1)^2}\right)dt$$

$$= -2\int \frac{t^2}{(t^2-1)^{1/2}(t^2-1)^2}\,dt$$

$$= -2\int \frac{t^2}{(t^2-1)^{5/2}}\,dt$$

(e)

$$x^2 = \frac{1}{t^2-1}$$

$$\therefore\ x = \left(\sqrt{t^2-1}\right)^{-1}$$

$$\therefore\ x = (t^2-1)^{-1/2}$$

$$\therefore\ dx/dt = -\tfrac{1}{2}(t^2-1)^{-3/2}\ (2t)$$

$$\therefore\ dx = -t(t^2-1)^{-3/2}\ dt$$

$$\therefore\ \int (1+x^2)^{\frac{1}{2}} dx = \int \left(1+\frac{1}{t^2-1}\right)^{1/2}\left(-t\left((t^2-1)\right)^{3/2}\right) dt$$

$$= -\int\left(\sqrt{\frac{t^2-1+1}{(t^2-1)}} \times \frac{t}{(t^2-1)^{3/2}}\right) dt$$

$$= -\int \frac{t^2}{(t^2-1)^{1/2}(t^2-1)^{3/2}} dt$$

$$= -\int \frac{t^2}{(t^2-1)^2} dt$$

$$= -\left(-\frac{t}{2(t^2-1)} + \frac{1}{4}\ln\left|\frac{x-1}{x+1}\right|\right) + D$$

$$= \frac{t}{2(t^2-1)} - \frac{1}{4}\ln\left|\frac{x-1}{x+1}\right| + D$$

$$= \frac{t}{2(t^2-1)} + \frac{1}{4}\ln\left|\frac{x+1}{x-1}\right| + D$$

7

(f) $t = \sqrt{\frac{1+x^2}{x^2}} = \frac{\sqrt{1+x^2}}{x}$

$$\therefore \int\sqrt{1+x^2}\,dx = \frac{\sqrt{1+x^2}/x}{2\left(\frac{1+x^2}{x^2}-1\right)} + \frac{1}{4}\ln\left|\frac{\sqrt{1+x^2}/x+1}{\sqrt{1+x^2}/x-1}\right| + E$$

$$= \frac{x\sqrt{1+x^2}}{2(1+x^2-x^2)} + \frac{1}{4}\ln\left|\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}-x}\right| + E$$

$$= \frac{x\sqrt{1+x^2}}{\cancel{2x^2}+2} + \frac{1}{4}\ln\left|\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}-x}\right|$$

1

use part (b) to simplify

(g) $y = \dfrac{e^{x} - e^{-x}}{2}$

$\Rightarrow$ for the inverse: $x = \dfrac{e^{y} - e^{-y}}{2}$ 1

$\therefore e^{y} - e^{-y} = 2x$

$\therefore e^{2y} - 1 - 2xe^{y} = 0$

$\therefore e^{y} = \dfrac{2x \pm \sqrt{4x^2+4}}{2}$

$= x \pm \sqrt{x^2+1}$

$\therefore y = \ln(x + \sqrt{x^2+1})$

(h) $\cosh^2 x - \sinh^2 x = \dfrac{(e^{x}+e^{-x})^2}{4} - \dfrac{(e^{x}-e^{-x})^2}{4}$

$= \dfrac{\cancel{e^{2x}} + 2e^{x}e^{-x} + \cancel{e^{-2x}} - (\cancel{e^{2x}} - 2e^{x}e^{-x} + \cancel{e^{-2x}})}{4}$

$= \dfrac{4}{4}$

$= 1$ 2

(i) $f(x) = \sinh x$ $g(x) = \cosh x$

(i) $f'(x) = \dfrac{d}{dx}\left(\dfrac{e^{x} - e^{-x}}{2}\right)$

$= \dfrac{1}{2}(e^{x} + e^{-x})$

$= \dfrac{e^{x} + e^{-x}}{2}$

$= g(x)$

(ii) $g'(x) = \frac{d}{dx}\left(\frac{e^{x}+e^{-x}}{2}\right)$

$= \frac{1}{2}\left(e^{x}-e^{-x}\right)$

$= \frac{e^{x}-e^{-x}}{2}$

$= f(x)$

(iii) LHS $= \cosh 2x$

$= \frac{e^{2x}+e^{-2x}}{2}$

RHS $= 2\cosh^2 x - 1$

$= 2\left(\frac{(e^{x}+e^{-x})^2}{2^2}\right) - 1$

$= 2\left(\frac{e^{2x}+2e^{x}e^{-x}+e^{-2x}}{4}\right) - 1$

$= \frac{e^{2x}+e^{-2x}+2}{2} - 1$

$= \frac{e^{2x}+e^{-2x}+2-2}{2}$

$= \frac{e^{2x}+e^{-2x}}{2}$

$=$ LHS

6

(j) $\sinh 2x = \dfrac{e^{2x} - e^{-2x}}{2}$

$= \dfrac{(e^{x} + e^{-x})(e^{x} - e^{-x})}{2}$

$= e^{x} + e^{-x} \times e^{x} - e^{-x} \times \dfrac{1}{2}$

$= \dfrac{e^{x} + e^{-x}}{2} \times \dfrac{e^{x} - e^{-x}}{2} \times 2$

$= \cosh x \ \sinh x \times 2$

$= 2 \sinh x \cosh x$

2

(k) $\sinh(2\sinh^{-1}(x)) = \sinh\left(2\ln(x + \sqrt{1+x^2})\right)$ 0

$\sinh(2\sinh^{-1}x) = 2\sinh(\sinh^{-1}x)\cosh(\sinh^{-1}x)$
$= 2x(\sqrt{1+\sinh^2 x})$
$= 2x\sqrt{1+x^2}$

$= \dfrac{e^{2\ln(x+\sqrt{1+x^2})} - e^{-2\ln(x+\sqrt{1+x^2})}}{2}$

$= \dfrac{(x+\sqrt{1+x^2})^2 - (x+\sqrt{1+x^2})^{-2}}{2}$

$= \dfrac{x^2 + 2x\sqrt{1+x^2} + (1+x^2) - \dfrac{1}{x^2 + 2x\sqrt{1+x^2} + (1+x^2)}}{2}$

$= \dfrac{2x^2 + 2x\sqrt{1+x^2} + 1 - \dfrac{1}{2x^2 + 2x\sqrt{1+x^2} + 1}}{2}$

$= \dfrac{(2x^2 + 2x\sqrt{1+x^2} + 1)^2 - 1}{2}$

$= \left(4x^4 + 4x^3\sqrt{1+x^2} + 2x^2 + 4x^3\sqrt{1+x^2} + 4x^2(1+x^2) + 2x\sqrt{1+x^2} + 2x^2 + 2x\sqrt{1+x^2} + 1\right)/2$

$= \dfrac{4x^4 + 8x^3\sqrt{1+x^2} + 4x^2 + 4x\sqrt{1+x^2} + 1}{2}$

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Example Ejemplo 27 | 27

Example Ejemplo 3 | 3

1

~~(l)~~ $\cancel{\int \sqrt{1+x^2}\,dx = \int \sqrt{1+\sinh^2 u}\cosh u\,du}$

$\cancel{= \frac{1}{2}\int \sqrt{S}\,dS}$ $\cancel{S = 1+\sinh^2 u}$, $ds = 2\sinh u\cosh u$

let $x = \sinh u$

$\therefore dx = \cosh u\,du$ $\cancel{= \frac{1}{2}\int S^{1/2}\,dS}$

if $x = \sinh u$
$dx = \cosh u\,du$

$\sin^2 x + \cos^2 x = 1$
$\therefore \sin^2 x = 1 - \cos^2 x$

$\therefore \int \sqrt{1+x^2}\,dx = \int \sqrt{1+\sinh^2 u}\cosh u\,du$

$= \frac{1}{2}\int 2\cosh u\sqrt{1+\sinh^2 u}\,du$

$\cancel{= \frac{1}{2}\int 2\cosh u\sqrt{1+\cosh^2 u}\,du}$

$\cancel{= \frac{1}{2}\int 2\cosh u\sqrt{\cosh^2 u}\,du}$

$= \int \cosh u \times \cosh u\,du$

$= \int \cosh^2 u\,du$

$= \frac{1}{2}\int (\cosh^2 u + 1 + 1)\,du$

$= \frac{1}{2}\int (\cosh 2u + 1)\,du$

$= \frac{1}{2}(\sinh 2u \times 2 + u)$

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$= \sinh 2u + u$ ECF

$\cancel{}$

$$\left[\begin{array}{l} x = \sinh u \\ \therefore u = \sinh^{-1} x \end{array}\right]$$

$= \sinh(2\sinh^{-1}x) + \sinh^{-1}x$ ✓

$= 2x\sqrt{1+x^2} + \sinh^{-1}x$ ✓

$= 2x\sqrt{1+x^2} + \ln(x+\sqrt{1+x^2}) + C$ ✗

6