

Mathematics: analysis and approaches
Higher level
Paper 1 Practice Set A (Hodder)

Candidate session number

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2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in an answer booklet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A copy of the mathematics: analysis and approaches formula book is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1 [*Maximum mark: 5*]

On his way to school, Suresh stops for coffee with probability 0.8. If he stops for coffee, the probability that he is late for school is 0.4; otherwise, the probability that he is late is 0.1. Given that on a particular day Suresh is late for school, what is the probability that he did not stop for coffee?

[illegible]

2 [Maximum mark: 7]

Use the substitution $u = x - 3$ to find the exact value of $\int_3^7 5x\sqrt{x-3} \, dx$.

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

3 [Maximum mark: 6]

z is the complex number which satisfies the equation $3z - 4z^* = 18 + 21i$. Find $\left|\frac{z}{3}\right|$.

[illegible]

4 [Maximum mark: 6]

a Show that $(2x + 1)$ is a factor of $f(x) = 2x^3 - 13x^2 + 17x + 12$.

[2]

b Solve the inequality $2x^3 - 13x^2 + 17x + 12 > 0$.

[4]

This image shows a full page of white paper with ten horizontal rows of small black dots, used as guides for handwriting practice. The dots are evenly spaced and extend across the width of the page.

5 [Maximum mark: 6]

Given the functions

$$f(x) = \frac{2-x}{x+3} \ (x \neq -3) \text{ and } g(x) = \frac{2}{x-1} \ (x \neq 1)$$

find $(f \circ g)^{-1}$ in the form $\frac{ax+b}{cx+d}$.

[illegible]

6 [Maximum mark: 6]

Find the possible values of x such that $45e^x$, $7e^{2x}$ and e^{3x} are consecutive terms of an arithmetic sequence.

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

7 [Maximum mark: 6]

Use L'Hôpital's rule to find

$$\lim_{x \rightarrow \pi} \frac{x \sin x}{\ln \left(\frac{x}{\pi} \right)}.$$

[illegible]

9 [Maximum mark: 6]

a Prove that $\log_2 5$ is an irrational number.

[4]

b Aron says that $\log_2 n$ is an irrational number for every integer $n \geq 10$. Give a counterexample to disprove this statement.

[2]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and extend across the entire width of the page. There are no margins, text, or other markings present.

Section B

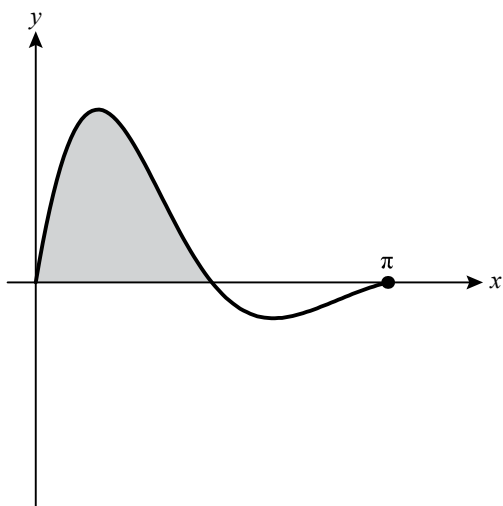
Answer **all** questions in an answer booklet. Please start each question on a new page.

10 [Maximum mark: 20]

- a** Sketch the graph of $y = x^2 + 3x - 10$, showing clearly the axes intercepts and the coordinates of the vertex. [4]
- b i** Show that the line $y = 2x - 20$ does not intersect the graph of $y = x^2 + 3x - 10$. [7]
- ii** Find the set of values of k for which the line $y = 2x - k$ intersects the graph of $y = x^2 + 3x - 10$ at two distinct points. [7]
- c** Describe fully a sequence of transformations which transforms the graph of $y = x^2 + 3x - 10$ to the graph of $y = \left(2x + \frac{3}{2}\right)^2 + 2$. [4]
- d** Sketch the following graphs, indicating clearly all axes intercepts, asymptotes and turning points:
- i** $y = |x^2 + 3x - 10|$
- ii** $y = \frac{1}{x^2 + 3x - 10}$ [5]

11 [Maximum mark: 16]

The graph of $y = e^{-x} \sin 2x$ for $0 \leq x \leq \pi$ is shown below.



The graph has a maximum point at P, a minimum point at Q and points of inflection at R and S.

- a** Show that the x -coordinates of point P and point Q satisfy $\tan 2x = 2$. [4]
- b** Show that the x -coordinates of points R and S satisfy $\tan 2x = -\frac{4}{3}$. [4]
- c** Show that the area of the shaded region enclosed below the curve and above the x -axis is given by $a + be^c$, where a , b and c are constants to be found. [8]

12 [Maximum mark: 19]

- a** State and prove de Moivre's theorem. [5]
- b** Use de Moivre's theorem to prove that $\cos 5\theta = 16 \cos^5 \theta - 20 \cos^3 \theta + 5 \cos \theta$. [4]
- c** Solve the equation $\cos 5\theta = 0$ for $0 \leq \theta \leq \pi$. [2]
- d** By considering the equation $16c^5 - 20c^3 + 5c = 0$, where $c = \cos \theta$, find the exact value of $\cos\left(\frac{\pi}{10}\right)$. [6]
- Justify your choice. [6]
- e** Find the exact value of $\cos\left(\frac{\pi}{10}\right) \cos\left(\frac{7\pi}{10}\right)$. [2]