Practice Set B: Paper 2 Mark scheme

SECTION A

1 $a + 4d = 7,\ a + 9d = 81$	M1A1	
Solving:		
$a = –52.2,\ d = 14.8$	A1	
$S_{20} = \frac{20}{2}(–104.4 + 19 \times 14.8)$	(M1)	
$= 1768$	A1	
		[5 marks]
2 Find the diagonal of the square base: $\sqrt{8.3^2 + 8.3^2}$	M1	
Height $= \frac{\sqrt{8.3^2 + 8.3^2}}{2} \tan(89.8°)$	M1	
$= 1681$	A1	
$= 1.7 \times 10^3$ cm	A1	
		[4 marks]
3 mean = 131.9, SD = 7.41	A1	
Boundaries for outliers: mean ± SD	(M1)	
= 117.1, 146.7	A1A1ft	
147 is an outlier	A1	
		[5 marks]
4 At least one correct use of compound angle formula	M1	
Correct values of $\sin\left(\frac{\pi}{3}\right)$ and $\cos\left(\frac{\pi}{3}\right)$ used	A1	
LHS $\equiv \dfrac{\left(\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x\right) - \left(\frac{1}{2}\sin x - \frac{\sqrt{3}}{2}\cos x\right)}{\left(\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x\right) - \left(\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x\right)}$	A1	
$\equiv \dfrac{\sqrt{3}\cos x}{-\sqrt{3}\sin x}$	A1	
$\equiv -\cot x$	A1(AG)	
		[5 marks]
5 $\frac{2}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1}$	M1	
$2 = A(x - 1) + Bx$	M1	
Using $x = 0$: $A = –2$		
Using $x = 1$: $B = 2$ (both correct)	A1	
$\int -\frac{2}{x} + \frac{2}{x-1}\,dx = –2 \ln x + 2 \ln(x – 1) + c$	M1A1	
$= \ln\left(\frac{x-1}{x}\right)^2 + c$	A1	
		[6 marks]
6 gradient = 3.024	(M1)	
normal gradient $= -\frac{1}{\text{their gradient}}$ [–0.3307]	(M1)	
y-coordinate = 3.392	A1	
Equation of normal: $y – 3.392 = – 0.3307(x – 1.5)$	A1	
A: $y = 0$, B: $x = 0$ [$x_A = 11.76,\ y_B = 3.888$]	(M1)	
Area = 22.9	A1	
		[6 marks]
7 Differentiate implicitly: at least one term containing y correct	M1	
$6x + 2xy' + 2y – 2yy' = 0$	A1	
$y' = 0 \Rightarrow y = –3x$	M1	
Substitutes their expression for x or y back into curve:		
$3x^2 + 2x(–3x) – (–3x)^2 + 24 = 0$	M1A1	
$12x^2 = 24 \Rightarrow x = \pm\sqrt{2}$	A1	
$(\sqrt{2}, -3\sqrt{2}),\ (-\sqrt{2}, 3\sqrt{2})$	A1	
		[7 marks]

8	**a**	Limits $\sqrt[3]{5}, \sqrt[3]{17}$ (seen in either part)	A1
		$x = \sqrt{y^3 - 1}$	A1
		$\int \sqrt{y^3 - 1}\, dy$	M1
		$= 2.57$	A1
	b	Using x^2	M1
		$\int \pi (y^3 - 1)\, dy$	M1
		$= 24.9$	A1
			[7 marks]
9		Another root is $2 + i$	A1
		Consider sum of roots:	
		$(2 + i) + (2 - i) + x_3 = 7$ (allow -7)	M1
		$x_3 = 3$	A1
		Product of roots: $3(2 + i)(2 - i)$	M1
		$c = -15$	A1
			[5 marks]
10		The rth term is	
		$nC_r x^{2r} \left(\frac{1}{x}\right)^{n-r}$	(M1)
		For constant term: $2r - (n - r) = 0$	(M1)
		$n = 3r$	A1
		So need $(3r)C_r = 495$	(M1)
		Using GDC: $r = 4$ so $n = 12$	A1
			[5 marks]

SECTION B

11	**a** **i**	$\dfrac{72 - \mu}{\sigma} = 0.8416$	M1
		$72 - \mu = 0.8416\sigma$	A1
		$\mu + 0.8416\sigma = 72$	(AG)
		$\dfrac{24 - \mu}{\sigma} = \dots$	M1
		$\dots -1.645$	A1
		$\mu - 1.645\sigma = 24$	A1
	ii	(From GDC) $\mu = 55.8$, $\sigma = 19.3$	A1
		$P(>48) = 0.657$	A1
			[7 marks]
	b	Use inverse normal with $p = 0.25$ or $p = 0.75$	
		($Q_1 = 42.8$ or $Q_3 = 68.8$)	M1
		IQR = 26 (hours)	A1
			[2 marks]
	c	Use $B(20, 0.656)$	(M1)
		$1 - P(\leqslant 9)$	(M1)
		$= 0.953$	A1
			[3 marks]
	d	$\dfrac{P(>72)}{P(>48)}$	(M1)
		$= 0.305$	A1
			[2 marks]
	e	P(keep phone) = $1 - (0.05 \times 0.9 + 0.75 \times 0.2)$	(M1M1)
		$\dfrac{0.2}{\text{P(keep phone)}}$	M1
		$= 0.248$	A1
			[4 marks]
			Total [18 marks]
12	**a**	$\cos\theta = \dfrac{2^2 + 4^2 - 4^2}{2 \times 2 \times 4}$ [= 0.25]	(M1)
		$\sin\theta = \sqrt{\dfrac{15}{16}}$ [= 0.968]	M1
		Area $= \dfrac{1}{2}(2 \times 4) \times$ their $\sin\theta$	M1
		$= 3.87$ [cm^2]	A1
			[4 marks]

b The third side is $10 - 3x$ … M1

… which must be positive. A1

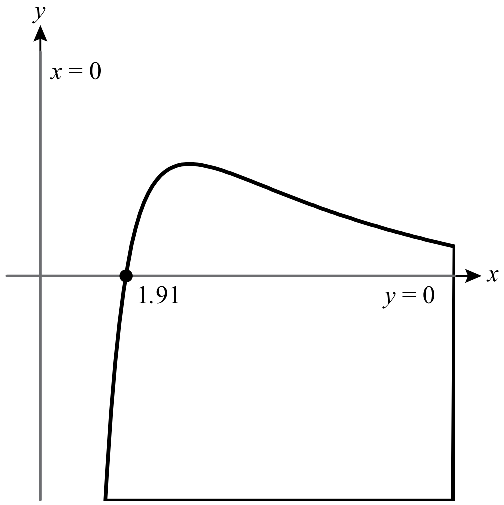
[2 marks]

c **i** $(10 - 3x)^2 = x^2 + (2x)^2 - 2x(2x)\cos\theta$ M1

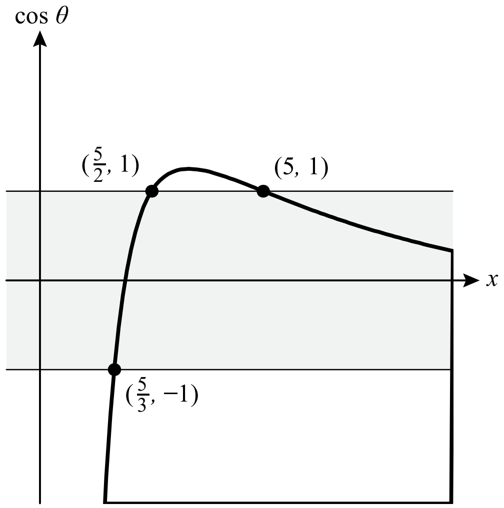
$$\cos\theta = \frac{60x - 4x^2 - 100}{4x^2}$$

$$= \frac{15x - x^2 - 15}{x^2}$$ A1(AG)

ii A2



iii Need $-1 < \cos\theta < 1$ (allow $\leqslant$ here) M1



Intersections at $x = \frac{5}{3}, \frac{5}{2}, 5$ A1

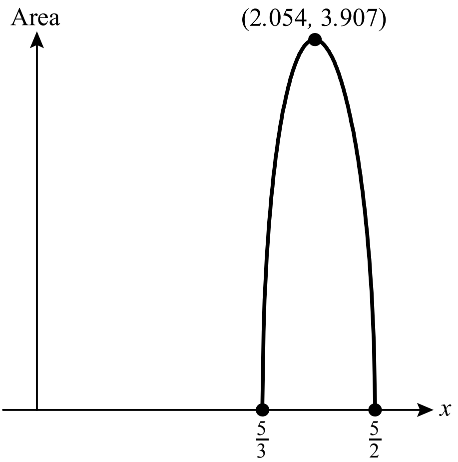
So $\frac{5}{3} < x < \frac{5}{2}$ A1

[7 marks]

d State or use $\sin\theta = \sqrt{1 - \cos^2\theta}$ M1

State or use Area $= \frac{1}{2}x(2x)\sin\theta$ M1

Sketch area as a function of x: M1



Max area for $x = 2.05$ A1

Max area = 3.91 [cm^2] A1

[5 marks]

Total [18 marks]

13 a Use quotient rule M1

Use implicit differentiation M1

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{\frac{\mathrm{d}y}{\mathrm{d}x}(x+y) - y\left(1+\frac{\mathrm{d}y}{\mathrm{d}x}\right)}{(x+y)^2}$$ A1

$$= \frac{x\frac{\mathrm{d}y}{\mathrm{d}x} - y}{(x+y)^2}$$ A1

Substitute $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y}{x+y}$:

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{\frac{xy}{x+y} - y}{(x+y)^2}$$ M1

$$= \frac{xy - y(x+y)}{(x+y)^3}$$ M1

$$-\frac{y^2}{(x+y)^3}$$ A1

[7 marks]

b $\frac{\mathrm{d}y}{\mathrm{d}x} = v + x\frac{\mathrm{d}v}{\mathrm{d}x}$ M1

$v + x\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{xv}{x+xv}$ M1

$x\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{v}{1+v} - v$ M1

$= -\frac{v^2}{1+v}$ A1

[4 marks]

c Separate variables: $\frac{1+v}{v^2}\frac{\mathrm{d}v}{\mathrm{d}x} = -\frac{1}{x}$ or equivalent M1

$\int \frac{1+v}{v^2}\,\mathrm{d}v = \int -\frac{1}{x}\,\mathrm{d}x$ M1

$-\frac{1}{v} + \ln v = -\ln x + c$ A1

Using $x = 1, y = 1, v = 1$: $-1 + 0 = 0 + c$ M1

$c = -1$ A1

$-\frac{1}{v} + \ln(xv) = -1$ (M1)

$\frac{x}{y} = \ln y + 1$ (M1)

$x = y(\ln y + 1)$ A1

[8 marks]

Total [19 marks]