Mathematics: analysis and approaches
Higher level
Paper 1 Practice Set A (Hodder)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in an answer booklet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A copy of the mathematics: analysis and approaches formula book is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

$\frac{96}{110} = 87.3\%$

19/10/22

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1 *[Maximum mark: 5]*

On his way to school, Suresh stops for coffee with probability 0.8. If he stops for coffee, the probability that he is late for school is 0.4; otherwise, the probability that he is late is 0.1. Given that on a particular day Suresh is late for school, what is the probability that he did not stop for coffee?

$P(C' \mid L) = \dfrac{P(C' \cap L)}{P(L)}$

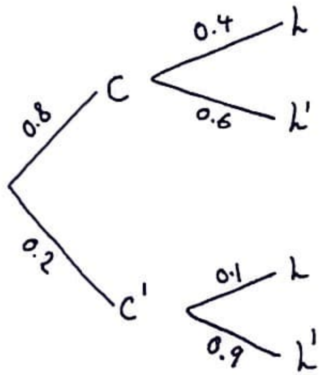
$= \dfrac{(0.2)(0.1)}{(0.8)(0.4) + (0.2)(0.1)}$ ✓M1 ✓M1

$= \dfrac{2\times10^{-2}}{32\times10^{-2} + 2\times10^{-2}}$ ✓A1

$= \dfrac{2}{32+2}$

$= 2/34$

$= 1/17$ ✓A1



✓M1

5

2 *[Maximum mark: 7]*

Use the substitution $u = x - 3$ to find the exact value of $\int_3^7 5x\sqrt{x-3}\,dx$.

$u = x - 3 \rightarrow x = u + 3$

$\therefore du = dx$ ✓M1

Limits: $u = 7 - 3 = 4$ ✓M1

$u = 3 - 3 = 0$

$\therefore \int_3^7 5x\sqrt{x-3}\,dx = \int_0^4 5(u+3)\sqrt{u}\,du$ ✓A1

$= \int_0^4 (5u\sqrt{u} + 15\sqrt{u})\,du$ ✓M1

$= \int_0^4 5u^{3/2}\,du + \int_0^4 15u^{1/2}$

$= 5(\tfrac{2}{5})[u^{5/2}]_0^4 + 15(\tfrac{2}{3})[u^{3/2}]_0^4$ ✓A1

$= 2(4^2\sqrt{4}) + 10(4\sqrt{4})$ ✓M1

$= 2(32) + 80$

$= 144$ ✓A1

7

3 *[Maximum mark: 6]*

z is the complex number which satisfies the equation $3z - 4z^* = 18 + 21i$. Find $\left|\frac{z}{3}\right|$.

let $z = a + bi$

$\therefore z^* = a - bi$ ✓M1

$\therefore 3z - 4z^* = 3a + 3bi - 4a + 4bi$

$= \cancel{-a + 7bi}$

$= -a + 7bi = 18 + 21i$ ✓A1

$\therefore a = -18$ and $7b = 21 \rightarrow b = 3$ ✓M1

$\therefore \left|\frac{z}{3}\right| = \left|\frac{-18+3i}{3}\right|$ ✓A1 ✓M1

$= |-6 + i|$

$= \sqrt{36+1}$

$= \sqrt{37}$ ✓A1

6

4 *[Maximum mark: 6]*

a Show that $(2x + 1)$ is a factor of $f(x) = 2x^3 - 13x^2 + 17x + 12$. [2]

b Solve the inequality $2x^3 - 13x^2 + 17x + 12 > 0$. [4]

(a) If $2x+1$ is a factor, then:

$f(-\frac{1}{2}) = 0$ ✓M1

$\therefore 2(-\frac{1}{8}) - 13(\frac{1}{4}) + 17(-\frac{1}{2}) + 12 = 0$

$-\frac{1}{4} - \frac{13}{4} - \frac{17}{2} + 12 = 0$

$\therefore -1 - 13 - 2(17) + 4(12) = 0$

$\therefore -14 - 34 + 48 = 0$

$\therefore f(-\frac{1}{2}) = -48 + 48$

$= 0$ ✓A1

2

(b) $(2x+1)(x^2+bx+c) = 2x^3 + 2bx^2 + 2cx + x^2 + bx + c$

$= 2x^3 + (2b+1)x^2 + (2c+b)x + c$

Equating coefficients:

$\therefore 2b+1 = -13$

$\therefore 2b = -14$

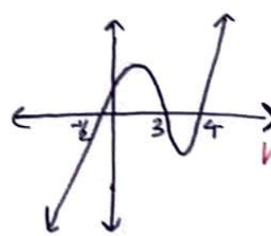
$\therefore b = -7$

$c = 12$

$\therefore$ other intercepts are $(x^2 - 7x + 12) = 0$

$\therefore (x-4)(x-3) = 0$ ✓M1

$\therefore x = 4, 3$



✓M1

✓M1 ✓A1

$\therefore f(x) > 0$ for $-\frac{1}{2} < x < 3$ AND $x > 4$

4

6

5 *[Maximum mark: 6]*

Given the functions

$f(x) = \frac{2-x}{x+3}$ $(x \neq -3)$ and $g(x) = \frac{2}{x-1}$ $(x \neq 1)$

find $(f \circ g)^{-1}$ in the form $\frac{ax+b}{cx+d}$.

$f \circ g = f(g(x))$

$= f(2/x-1)$

$= \frac{2 - 2/x-1}{2/x-1 + 3}$ ✓M1

$= \frac{2x-2-2}{2+3x-3}$ ✓M1

$= \frac{2x-4}{3x-1}$ ✓A1

$(f \circ g)^{-1}$: $x = \frac{2y-4}{3y-1}$ ✓M1

$\therefore 3yx - x = 2y - 4$

$\therefore 3yx - 2y = x - 4$

$\therefore y(3x-2) = x - 4$ ✓M1

$\therefore (f \circ g)^{-1} = \frac{x-4}{3x-2}$ ✓A1

$\Rightarrow a = 1$

$b = -4$

$c = 3$

$d = -2$

6

6 *[Maximum mark: 6]*

Find the possible values of x such that $45e^x$, $7e^{2x}$ and e^{3x} are consecutive terms of an arithmetic sequence.

AP: $d = 7e^{2x} - 45e^x = e^{3x} - 7e^{2x}$ ✓M1

$\therefore -e^{3x} + 14e^{2x} - 45e^x = 0$ ✓A1

$\therefore -e^{2x} + 14e^x - 45 = 0$ $\{e^x \neq 0, x \in \mathbb{R}\}$ ✓R1

let $u = e^x \rightarrow u^2 - 14u + 45 = 0$

$\therefore u^2 - 9u - 5u + 45 = 0$

$\therefore u(u-9) - 5(u-9) = 0$

$\therefore (u-5)(u-9) = 0$ ✓✓M1 A1

$\therefore u = 5, 9$

$\therefore e^x = 5$, $e^x = 9$

$\therefore x = \ln 5$, $x = \ln 9$ ✓A1

6

7 *[Maximum mark: 6]*

Use L'Hôpital's rule to find

$\lim_{x\to\pi} \frac{x \sin x}{\ln\left(\frac{x}{\pi}\right)}$.

$$\lim_{x\to\pi} \frac{x\sin x}{\ln(x/\pi)} = \frac{\pi \sin \pi}{\ln(1)}$$ ✓M1

$= 0/0$ = indeterminate.

$$= \lim_{x\to\pi} \left(\frac{x\cos x + \sin x}{(\pi/x)(1/\pi)} \right)$$ ✓M1 ✓A1 {L'Hs}

$$= \lim_{x\to\pi} \left(\frac{x\cos x + \sin x}{1/x} \right)$$ ✓A1

$$= \lim_{x\to\pi} \left(x^2\cos x + x\sin x \right)$$ ✓M1

$= \pi^2 \cos \pi + \pi \sin \pi$ ✓A1

$= -\pi^2$

(6)

8 *[Maximum mark: 7]*

Sketch the graph of

$$y = \frac{2x^2 + 5x - 12}{x + 3}$$

State the coordinates of all axis intercepts and the equations of all asymptotes.

vert asymptote: $x = -3$

oblique asymptote:

$$\begin{array}{r|l} & 2x - 1 \\ x+3 & 2x^2 + 5x - 12 \\ & (2x^2 + 6x) - \\ & -x - 12 \\ & (-x - 3) - \\ & -9 \end{array}$$ ✓M1 ✓A1

horiz asymptote: $y = \lim_{x\to\infty} \left(\frac{2x^2+5x-12}{x+3} \right)$

$= \lim_{x\to\infty} \left(\frac{4x+5}{3} \right)$

=

$\therefore$ $2x-1$ is the oblique asymptote.

x-int: $2x^2 + 5x - 12 = 0$

$\therefore 2x^2 + 8x - 3x - 12 = 0$

$\therefore 2x(x+4) - 3(x+4) = 0$

$\therefore (2x-3)(x+4) = 0$

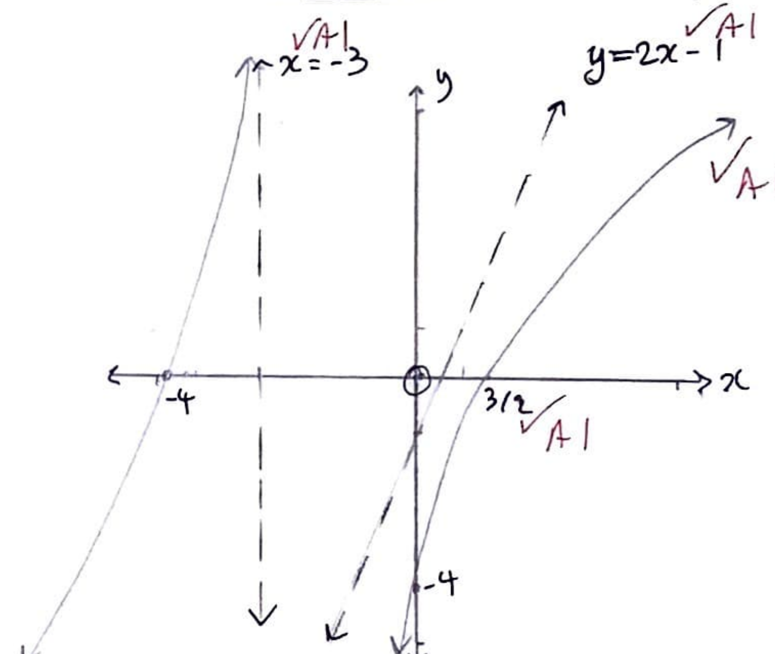
$\therefore$ $x = 3/2, -4$ ✓M1

y-int: $y = -12/3$

$\therefore y = -4$

~~$y = 2x - 1 - \frac{9}{2x^2+5x-12}$~~

$\Rightarrow$ as $x \to \pm\infty$,



(7)

9 *[Maximum mark: 6]*

a Prove that $\log_2 5$ is an irrational number. *[4]*

b Aron says that $\log_2 n$ is an irrational number for every integer $n \geqslant 10$. Give a counterexample to disprove this statement. *[2]*

(a) assume $\log_2 5 = a/b$ ✓M1 $a, b \in \mathbb{Z}$ is true

$\therefore 2^{a/b} = 5$

$\therefore 2 = \cancel{}\ \sqrt[b]{5^a}$

~~which is a contradiction as no root of 5^a gives 2~~ 2

$\therefore 2^b = 5^a$ ✓M1

$\therefore b = \log_2 5^a = a \log_2 5$ X

=> which is a contradiction ~~as~~ $\log_2 5$ is irrational.

=> even LHS & odd RHS.

(b) let $n = 16 = 2^4$ ✓M1

$\therefore \log_2 2^4 = 4$ which is a disproof by counterexample as 4 is not an irrational number. A1 2

4

Do **not** write solutions on this page

Section B

Answer **all** questions in an answer booklet. Please start each question on a new page.

10 *[Maximum mark: 20]*

a Sketch the graph of $y = x^2 + 3x - 10$, showing clearly the axes intercepts and the coordinates of the vertex. *[4]*

b i Show that the line $y = 2x - 20$ does not intersect the graph of $y = x^2 + 3x - 10$.

ii Find the set of values of k for which the line $y = 2x - k$ intersects the graph of $y = x^2 + 3x - 10$ at two distinct points. *[7]*

c Describe fully a sequence of transformations which transforms the graph of $y = x^2 + 3x - 10$ to the graph of $y = \left(2x + \frac{3}{2}\right)^2 + 2$. *[4]*

d Sketch the following graphs, indicating clearly all axes intercepts, asymptotes and turning points:

i $y = |x^2 + 3x - 10|$

ii $y = \dfrac{1}{x^2 + 3x - 10}$ *[5]*

$(-\frac{3}{2})^2 + 3(-\frac{3}{2}) - 10$

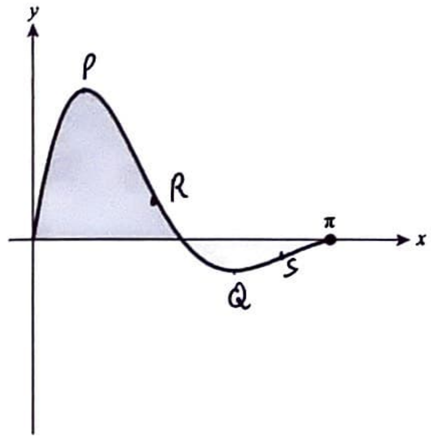
$= \frac{9}{4} - \frac{9}{2} - 10$

$= \frac{9 - 18}{4} - 10$

$= -\frac{9}{4} - 10$

11 *[Maximum mark: 16]*

The graph of $y = e^x \sin 2x$ for $0 \leqslant x \leqslant \pi$ is shown below.



The graph has a maximum point at P, a minimum point at Q and points of inflection at R and S.

a Show that the x-coordinates of point P and point Q satisfy $\tan 2x = 2$. *[4]*

b Show that the x-coordinates of points R and S satisfy $\tan 2x = -\frac{4}{3}$. *[4]*

c Show that the area of the shaded region enclosed below the curve and above the x-axis is given by $a + be^c$, where a, b and c are constants to be found. *[8]*

12 *[Maximum mark: 19]*

a State and prove de Moivre's theorem. *[5]*

b Use de Moivre's theorem to prove that $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$. *[4]*

c Solve the equation $\cos 5\theta = 0$ for $0 \leqslant \theta \leqslant \pi$. *[2]*

d By considering the equation $16c^5 - 20c^3 + 5c = 0$, where $c = \cos\theta$, find the exact value of $\cos\left(\frac{\pi}{10}\right)$. Justify your choice. *[6]*

e Find the exact value of $\cos\left(\frac{\pi}{10}\right)\cos\left(\frac{7\pi}{10}\right)$. *[2]*

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At the start of each answer to a question, write the question number in the box using your normal hand writing / Avant de répondre à une question, inscrivez son numéro à la main dans la case appropriée / Al comienzo de cada respuesta, escriba a mano el número de pregunta en la casilla.

Example / Ejemplo 27 [2 7]   Example / Ejemplo 3 [3]

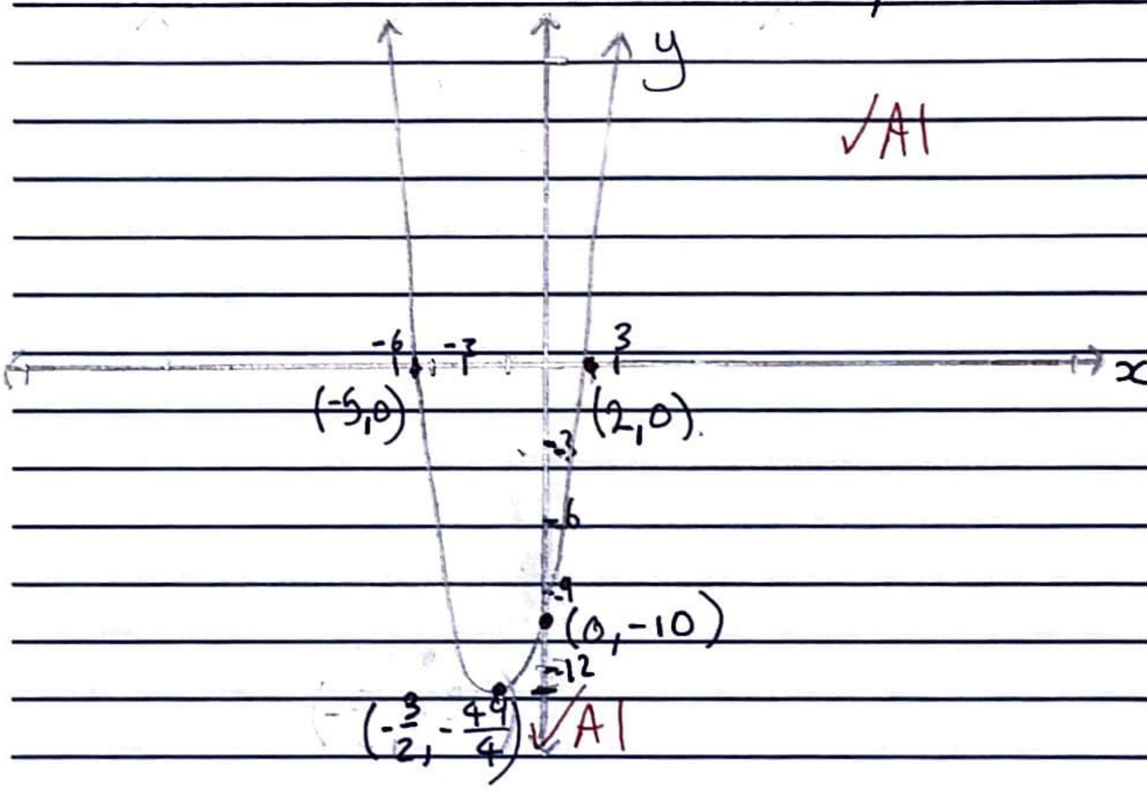
10

(a) $y = x^2 + 3x - 10$

y-int @ $y = -10$

x-int @ $x^2 + 3x - 10 = 0$

$\therefore \quad x = -5, \quad x = +2$ ✓M1



✓A1

✓A1

AOS $= -b/2a = -\cancel{3}3/2$

$= \cancel{} -1\frac{1}{2}$ ✓M1 4

@ $x = -3/2$, $y = (9/4) - 3(3/2) - 10$

$= 9/4 - 18/4 - 40/4$

$= -49/4 = -12\frac{1}{4}$

(b)(i)

$$2x-20 = x^2+3x-10$$

$$\therefore \quad 0 = x^2+x+10$$

$$\therefore x = \frac{-1 \pm \sqrt{1-4(10)}}{2} \qquad \checkmark M1$$

$$= -\frac{1}{2} \pm \frac{\sqrt{39}\,i}{2}$$

∴ as the roots are imaginary, $y=2x-20$ does not intersect $y=x^2+3x-10$ ✓M1 ✓A1

=> just use discriminant

3

(ii) $y = \cancel{2x}\,2x - k = x^2+3x-10$

$$\therefore \quad 0 = x^2+x-10+k$$

$$\therefore 0 = x^2+x+(k-10) \qquad \checkmark M1A1$$

$$\therefore \Delta = 1-4(k-10) \qquad \checkmark M1 \qquad 3$$

$$= 1-4k+40$$

$$= 11-4k \qquad \times A0$$

Two distinct points when $11-4k>0$

$$\therefore 4k<11$$

$$\therefore k<11/4$$

(c) $y = x^2+3x-10$

$$= x^2+3x+(3/2)^2-10-(3/2)^2$$

$$= x^2+3x+(3/2)^2-10-9/4$$

$$= x^2+3x+(3/2)^2-49/4$$

$$= (x+3/2)^2-49/4$$

(c)

$$\therefore y = (x+3/2)^2-49/4 \qquad \checkmark M1$$

↓ horizontal stretch, 1/2

$$y = (2(x+3/2))^2-49/4$$

$$= (2x+3/2)^2-49/4$$

↓ translation through $\begin{pmatrix}3/2\\57/4\end{pmatrix}$ A1

$$y = (2x+3-3/2)^2-49/4+(49/4+2)$$

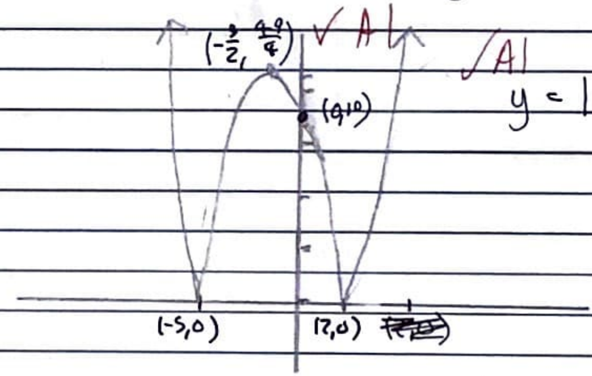
$$= (2x+3/2)^2-49/4+(57/4) \qquad A0A0$$

$$\therefore y = (2x+3/2)^2+2$$

2

(d)(i) $y = |x^2+3x-10| \rightarrow$ max @ $(-\frac{3}{2}, \frac{49}{4})$

y-int @ $(0,10)$



✓A1 ✓A1

$y = |f(x)|$

2

(d)(ii)

$y = \frac{1}{x^2+3x-10}$

$\Rightarrow$ vert asymptotes @ $x = 2, -5$

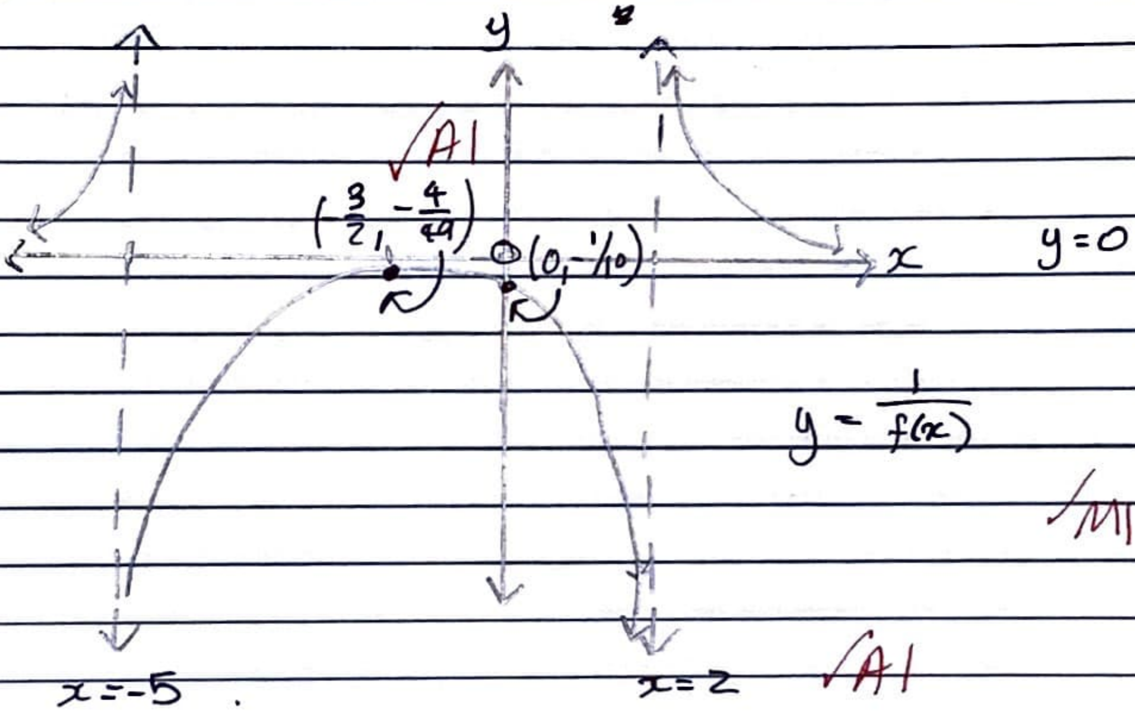
$\Rightarrow$ y-int @ $y = -1/10$

$\Rightarrow$ horiz asymptote @ $y = 0$ $\therefore$ no x-int.

$\Rightarrow$ maximum @ $x = -3/2$

$\therefore y = -1/49/4$

$= -4/49$



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Example Ejemplo 27 | 2 | 7 | Example Ejemplo 3 | | 3 |

11

(a) $y = e^{-x} \sin 2x \qquad 0 \le x \le \pi$

$\therefore dy/dx = e^{-x} \cos 2x (2) + (-1) e^{-x} \sin 2x$ ✓M1

$= 2e^{-x} \cos 2x - e^{-x} \sin 2x$

$= e^{-x}(2\cos 2x - \sin 2x)$ ✓A1

$\therefore e^{-x}(2\cos 2x - \sin 2x) = 0$ ✓M1 {turning point}

$\therefore 2\cos 2x - \sin 2x = 0 \qquad \{e^{-x} \neq 0\}$

$\therefore \dfrac{\sin 2x}{\cos 2x} = 2$ ✓A1

$\therefore \tan 2x = 2$

4

(b) $d^2y/dx^2 = e^{-x}(2(-\sin 2x)(2) - 2\cos 2x) + (2\cos 2x - \sin 2x)(-1)e^{-x}$ ✓M1

$= e^{-x}(-4\sin 2x - 2\cos 2x - 2\cos 2x + \sin 2x)$

$= e^{-x}(-3\sin 2x - 4\cos 2x)$ ✓A1

$\therefore d^2y/dx^2 = e^{-x}(-3\sin 2x - 4\cos 2x) = 0$ ✓M1 {inflexion}

$\therefore -3\sin 2x - 4\cos 2x = 0 \qquad \{e^{-x} \neq 0\}$

$\therefore 3\sin 2x = -4\cos 2x$ ✓A1

$\therefore \tan 2x = -4/3$ ✓

4

04AX01

(c) x-intercepts occur @ $e^{-x}\sin 2x = 0$

$\therefore e^{-x} = 0 \quad \{x \neq 0, \pi/2, \pi\}$ ✓

$\therefore x$ DNE

$\sin 2x = 0 \quad \{e^{-x} \neq 0\}$

$\therefore 2x = 0, \pi, 2\pi, 3\pi, 4\pi$

$\therefore x = 0, \pi/2, \pi, 3\pi/2, 2\pi, \ldots$

$\therefore x = 0, \pi/2, \pi, \quad \{0 \le x \le \pi\}$

$$A = \int_0^{\pi/2} e^{-x}\sin 2x\,dx = -\left[e^{-x}\sin 2x\right]_0^{\pi/2} - \int -e^{-x}\cdot 2\cos 2x\,dx \quad ✓M1$$

$\left[\begin{array}{l} \text{let } u = \sin 2x;\ du = 2\cos 2x \\ \text{let } dv = e^{-x};\ v = -e^{-x} \end{array}\right]$

$$= -\left(e^{-\pi/2}\sin\pi - 0\right) + 2\int_0^{\pi/2} e^{-x}\cos 2x\,dx$$

$$= 0 + 2\int_0^{\pi/2} e^{-x}\cos 2x\,dx$$

$\left[\begin{array}{ll} u = \cos 2x & du = -2\sin 2x \\ dv = e^{-x} & v = -e^{-x} \end{array}\right]$

$$= 2\left(\left[-e^{-x}\cos 2x\right]_0^{\pi/2} - \int_0^{\pi/2} -e^{-x}(-2\sin 2x)\,dx\right) \quad ✓A1$$

$$= 2\left(\left(+e^{-\pi/2} - -e^{0}\right) - 2\int_0^{\pi/2} e^{-x}\sin 2x\,dx\right)$$

$$= 2\left(+e^{-\pi/2} + 1 - 2\int_0^{\pi/2} e^{-x}\sin 2x\,dx\right)$$

$$\therefore \int_0^{\pi/2} e^{-x}\sin 2x\,dx = +2e^{-\pi/2} + 2 - 4\int_0^{\pi/2} e^{-x}\sin 2x\,dx$$

✓M1

$$\therefore 5\int_0^{\pi/2} e^{-x}\sin 2x\,dx = 2 + 2e^{-\pi/2}$$

$$\therefore \int_0^{\pi/2} e^{-x}\sin 2x\,dx = \frac{2 + 2e^{-\pi/2}}{5}$$

$$= \frac{2}{5} - \frac{2}{5}e^{-\pi/2}$$

$\therefore a = 2/5$ ✓A1 $\quad b = -2/5$ ✗A0 $\quad c = -\pi/2$ ✓A1

6

14

12

(a) $[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$

$\rightarrow$ for $n \in \mathbb{Z}^+$ A0

Prove for $n=1$: LHS $= r(\cos\theta + i\sin n\theta)$

RHS $= r(\cos\theta + i\sin n\theta)$

$=$ LHS ✓A1

$\therefore$ true for $n=1$

~~Prove for +~~

Step 2: Assume true for $n=k$:

$[r(\cos\theta + i\sin\theta)]^k = r^k(\cos k\theta + i\sin k\theta)$ ✓M1

Step 3: Consider ~~AB~~ $n=k+1$:

$[r(\cos\theta + i\sin\theta)]^{k+1} = r^{k+1}(\cos(k+1)\theta + i\sin(k+1)\theta)$

$\Rightarrow$ LHS $= [r(\cos\theta + i\sin\theta)]^k\,[r(\cos\theta + i\sin\theta)]$

$= r^k(\cos k\theta + i\sin k\theta)(r(\cos\theta + i\sin\theta))$ {by assumption}

$= r^{k+1}(\cos\theta\cos k\theta + i\cos\theta\sin k\theta + i\sin\theta\cos k\theta + i^2\sin\theta\sin k\theta)$

~~$= r^{k+1}(\cos\theta(\cos k\theta + i\sin k\theta) + i\sin\theta(\cos k\theta + i\sin k\theta))$~~

~~$= r^{k+1}(\cos\theta + i\sin\theta)(\cos k\theta + i\sin k\theta)$~~

$= r^{k+1}(\cos\theta\cos k\theta - \sin\theta\sin k\theta + i(\cos\theta\sin k\theta + \sin\theta\cos k\theta))$

$= r^{k+1}(\cos(k+1)\theta + i(\sin(k+1)\theta))$

$= r^{k+1}(\cos(k+1)\theta + i\sin(k+1)\theta)$ ✓A1

$=$ RHS.

$\therefore$ true for $n=k+1$

4

Step 4: as true for $n=1$ and true for $n=k+1$ whenever $n=k$ is assumed to be true, then true for all $n \in \mathbb{Z}^+$ by mathematical induction. ✓R1

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Example Ejemplo 27 | 2 | 7 |

Example Ejemplo 3 | | 3 |

| 1 | 2 |

(b) $\cos 5\theta = \cos(2\theta + 3\theta)$

$= \cos 2\theta \cos 3\theta - \sin 2\theta \sin 3\theta$

$= \cos 2\theta \cos(\theta + 2\theta) - \sin 2\theta \sin(\theta + 2\theta)$

$= \cos 2\theta(\cos\theta\cos 2\theta - \sin\theta\sin 2\theta) - \sin 2\theta(\sin\theta\cos 2\theta + \cos\theta\sin 2\theta)$

$= \cos\theta(\cos 2\theta)^2 - \sin\theta\sin 2\theta\cos 2\theta - \sin\theta\sin 2\theta\cos 2\theta - \cos\theta(\sin 2\theta)^2$

$= \cos\theta(\cos 2\theta)^2 - 2\sin\theta\sin 2\theta\cos 2\theta - \cos\theta(\sin 2\theta)^2$

$= \cos\theta(1 - \sin^2 2\theta) - 2\sin\theta\sin 2\theta\cos 2\theta - \cos\theta(1 - \cos^2 2\theta)$

$= \cos\theta - \cos\theta\sin^2 2\theta - 2\sin\theta\sin 2\theta\cos 2\theta - \cos\theta + \cos\theta\cos^2 2\theta$

$= \cos\theta\cos^2 2\theta - \cos\theta\sin^2 2\theta - 2\sin\theta\sin 2\theta\cos 2\theta$

$= \cos(\cos^2 2\theta - \sin^2 2\theta) - \sin\theta(2\sin 2\theta\cos 2\theta)$

$= \cos(\cos 4\theta) - \sin\theta(\sin 4\theta)$

$= \cos(\cos 4\theta) - \sin(\sin 4\theta)$

(b) let $z = \cos 5\theta + i\sin 5\theta$

$= (\cos\theta + i\sin\theta)^5$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

let $a = \cos\theta$ and $b = \sin\theta$

~~$\therefore (a+ib)^5 = a^5 + 4a^4(ib) + 6a^3(ib)^2 + 4a^2(ib)^3 +$~~

√A1

$\therefore (a+ib)^5 = a^5 + 5a^4(ib) + 10a^3(ib)^2 + 10a^2(ib)^3 + 5a(ib)^4 + (ib)^5$

$= a^5 - 10a^3b^2 + 5ab^4 \quad + i(5a^4b - 10a^2b^3 + b^5)$

$\therefore Re(z) = \cos 5\theta = a^5 - 10a^3b^2 + 5ab^4$ √M1

$= \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$

$= \cos^5\theta - 10\cos^3\theta(1-\cos^2\theta) + 5\cos\theta(\sin^2\theta)^2$ √M1

$= \cos^5\theta - 10\cos^3\theta + 10\cos^5\theta + 5\cos\theta(1-\cos^2\theta)^2$

$= 11\cos^5\theta - 10\cos^3\theta + 5\cos\theta(1 - 2\cos^2\theta + \cos^4\theta)$

$= 11\cos^5\theta - 10\cos^3\theta + 5\cos\theta - 10\cos^3\theta + 5\cos^5\theta$

$\therefore \cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$ √A1

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~~(c) $\cos 5\theta = 0 \rightarrow 5\theta = \pi/2 \quad \therefore \theta = \pi/10$~~

~~$\therefore 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta = 0$~~

~~$\therefore 16\cos^4\theta - 20\cos^2\theta + 5 = 0$~~

~~$\therefore \cos^4\theta = \dfrac{20 \pm \sqrt{400 - 20(16)}}{32}$~~

~~$= \dfrac{20 \pm \sqrt{80}}{32}$~~

~~$= \dfrac{20}{32} \pm \dfrac{2\sqrt{10}}{32}$~~

~~$= \dfrac{5}{8} \pm \dfrac{\sqrt{10}}{16}$~~ ✗

$\therefore \cos 5\theta = 0$

$\therefore 5\theta = \pi/2, 3\pi/2, 5\pi/2, \ldots \{0 \le \theta \le \pi\}$

$\therefore \theta = \pi/10, 3\pi/10, 5\pi/10, 7\pi/10, 9\pi/10$

$= \pi/10, 3\pi/10, \pi/2, 7\pi/10, 9\pi/10$ √M1

2 √A1

(d) $16c^5 - 20c^3 + 5c = 0$

$\therefore 16c^4 - 20c^2 + 5 = 0$ √A1

$\therefore 16c^4 - 20c^2 + 10^2 + 5 - 10^2 = 0$

$\therefore 16c^4 - 20c^2 + 10^2 = 100 - 5$

$\therefore (4c^2 \mp 10)^2 = 95$

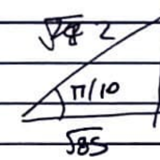
$\therefore 4c^2 \mp 10 = \pm\sqrt{95}$

$\therefore 4c^2 = 85, \quad -105$

$\therefore 4c^2 = \sqrt{95} + 10 =$

$\therefore c = \pm\sqrt{85/4}$

$\therefore c = \sqrt{85}/2 \qquad \{\cos \pi/10 > 0\}$ √B1



2

(e) $\cos(7\pi/10) = \cos(\pi - 3\pi/10)$ ✗ 0

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(d) $16c^5 - 20c^3 + 5c = 0$

$\therefore 16c^4 - 20c^2 + 5 = 0$

$\therefore c^2 = \dfrac{20 \pm \sqrt{400 - 4(16)(5)}}{32}$

$= \dfrac{20 \pm \sqrt{80}}{32}$

$= \dfrac{10}{16} \pm \dfrac{4\sqrt{5}}{32}$

$= \dfrac{5 \pm \sqrt{5}}{8}$

$\therefore c = \pm\sqrt{\dfrac{5 \pm \sqrt{5}}{8}} = \cos\left(\dfrac{\pi}{10}\right)$

$\therefore \cos\left(\dfrac{\pi}{10}\right) = +\sqrt{\dfrac{5+\sqrt{5}}{8}}$

(e) $\cos\left(\dfrac{7\pi}{10}\right) = -\sqrt{\dfrac{5-\sqrt{5}}{8}}$

$\therefore \left(\sqrt{\dfrac{5+\sqrt{5}}{8}}\right)\left(-\sqrt{\dfrac{5-\sqrt{5}}{8}}\right)$

$= -\sqrt{\dfrac{25-5}{64}}$

$= -\dfrac{\sqrt{20}}{8}$

$= -\sqrt{5}/4$