Mathematics: analysis and approaches

Higher level

Paper 3

ID: 3012

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[50 marks]**.

$\frac{35}{50} = 70\%$

ANSWER BOOKLET ①
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Example / Ejemplo 27 | 27

Example / Ejemplo 3 | 3

1

~~(a)(i)~~ $\cancel{l = s(2\pi - \theta)s}$

$\cancel{= 2\pi s - \theta s}$

~~(ii)~~

(a)(i) $l = s\theta$ ✓ A1 $\{\theta$ in radians$\}$

(ii) $A = \frac{1}{2}s^2\theta$ ✓ A1 $\{\theta$ in radians$\}$

2

(b) $c = 2\pi r$ ✓ A1 1

~~(c)~~

(c) $\cancel{c = l}$

$\cancel{\therefore 2\pi r = s\theta}$

$\cancel{\therefore s = 2\pi r/\theta}$

$\cancel{\therefore A = \frac{1}{2}(4\pi^2 r^2)(\theta)/\theta^2}$

$\cancel{= 2\pi^2 r^2.}$

By formula: ✗

$A = \pi r l$

$\therefore A = \pi r s$

M0A0A1

1

04AX01

(d)(i) $c = 2\pi r$

$= 2\pi (1)$ ✓

$\therefore c = 2\pi$ km ✓ A1

$\therefore c = 6.28$ km

(ii) $s = \sqrt{r^2 + h^2}$

$= \sqrt{1+1}$ ✓ M1

$\therefore s = \sqrt{2}$ km ✓ A1 3

~~$= 1.41$ km~~

$\therefore s = 1.41$ km

(e) $l = s\theta$, where l = circumference

$\therefore 2\pi = \sqrt{2}\,\theta$ ✓ M1

$\therefore \theta = 2\pi/\sqrt{2}$

$\therefore \theta = \pi\sqrt{2}^c$ ✓ A1 2

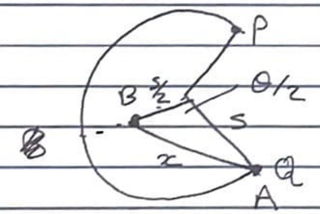
$\therefore \theta = 4.44^c$

(f) half way - up $= \frac{1}{2}\sqrt{2}$

$= \sqrt{2}/2$

$= 1/\sqrt{2}$ km

M1



$\therefore x^2 = \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\sqrt{2}\right)^2 - 2\left(\frac{1}{\sqrt{2}}\right)\left(\sqrt{2}\right)\cos\left(\frac{\pi\sqrt{2}}{2}\right)$

$= \frac{1}{2} + 2 - \cos\left(\frac{\pi}{\sqrt{2}}\right)$

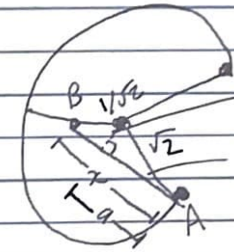
$= 5/2 - \cos(\pi/\sqrt{2})$

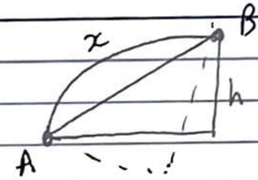
$\therefore x = \sqrt{5/2 - \cos(\pi/\sqrt{2})}$

$= 1.7623$ km ✗

1

(g)

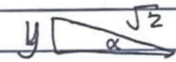




R1

a occurs at ⊥ section as indicated

M1 M1



$\frac{\sin\alpha}{\sqrt{2}/2} = \frac{\sin(\theta/2)}{1.7623}$

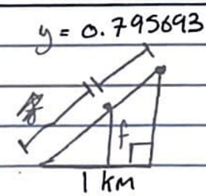
$\therefore \sin\alpha = \frac{\sqrt{2}}{2}\sin\left(\pi/\sqrt{2}\right)$

$\therefore \alpha = 0.597576^c$

ECF

$\therefore \sin(0.597576) = y/\sqrt{2}$

$\therefore y = 0.795693$

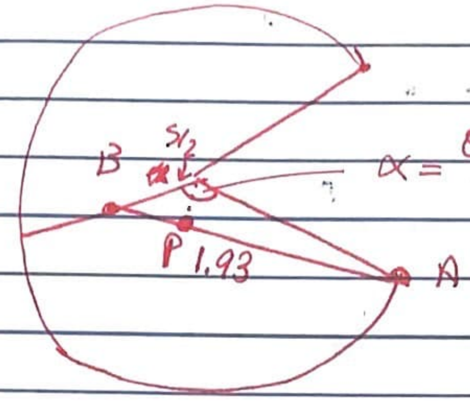


f = height above sea

3

13

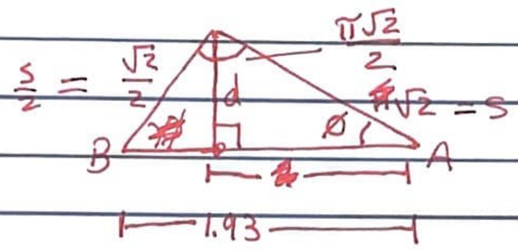
(g) → Model solution



$\alpha = \theta/2 = \frac{\pi\sqrt{2}}{2}$

$s/2 = \sqrt{2}/2$

Point P is closest to the peak / centre of the cone.

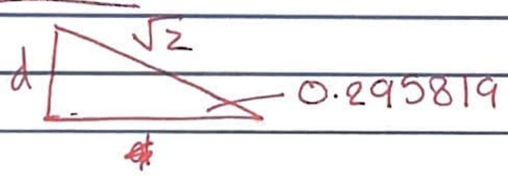


$\frac{\sin Ø}{\sqrt{2}/2} = \frac{\sin\alpha}{1.93}$

$\therefore \sin Ø = \frac{\sqrt{2}}{2} \times \frac{\sin(\pi\sqrt{2}/2)}{1.93}$

$\therefore Ø = 0.295819^c$

Distance



$\therefore \sin Ø = d/\sqrt{2}$

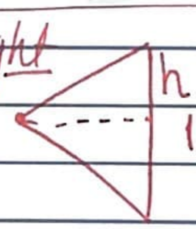
$\therefore d = \sqrt{2}\sin Ø$

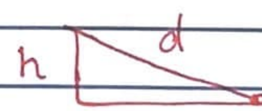
$\therefore \boxed{d = 0.412276}$

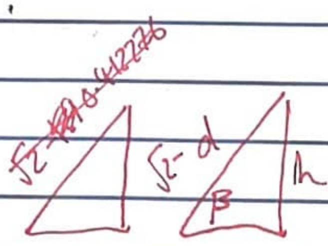
$\therefore$ distance walked $= \sqrt{2 - d^2}$

$= 1.35$ km

height







$\beta = \pi/4 \rightarrow h = \sin\left(\frac{\pi}{4}\right)(\sqrt{2} - d)$

$\therefore h = 0.708$ km

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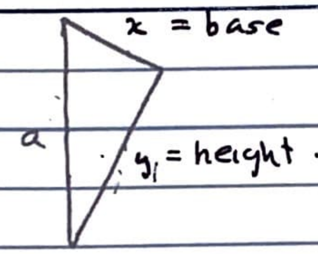
Example / Ejemplo 27 | 2 7 | Example / Ejemplo 3 | 3 |

| 2 | (a)(i) Similar triangles:



$\therefore \; x/y = a/b \quad \ldots (1)$ M1

$y/a = b/c$

$\therefore \; y = ab/c \quad \ldots (2)$

$(1) \to (2): \quad x = \frac{a}{b}\left(\frac{ab}{c}\right)$

$= \frac{a^2}{c}$ A1 $= \cancel{\text{base}}$

(ii) $\cancel{x=} \; y = \frac{bx}{a}$ M1 $\ldots \cancel{(2)} \; (3)$

$x/a = a/c$

$\therefore \; x = a^2/c \quad \ldots (4)$

$(3) \to (4): \quad y = \frac{ba^2}{ca}$

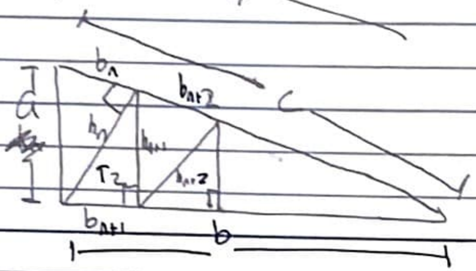
$= ab/c$ A1 $=$ height

4

04AX01

(b)(i) ~~let $h_1 = ab/c$~~

~~let $b_1 = a^2b/c$~~



∴ ~~b_n~~

for T_n, $\frac{h_n}{b_n} = \frac{b}{a}$ ✓M1 {similar △s}

∴ $h_n = \frac{b}{a} b_n$ ✓A1

(ii) for b_{n+1} on the diagram:

$\frac{b_{n+1}}{h_n} = \frac{a}{c}$ ✓M1 {similar △s}

∴ $b_{n+1} = \frac{a}{c} h_n$ ✓A1

(iii) for h_{n+1} on the triangle:

$\frac{h_{n+1}}{h_n} = \frac{b}{c}$ ✓M1 7

∴ $h_{n+1} = \frac{b}{c} h_n$ ✓A1 $= \frac{b^2}{ac} b_n$ ✓A1 {from part (b)(i)}

(c) $T_n = \frac{a^3 b^{2n-1}}{2c^{2n}}$

Step 1: prove for $n=1$:

$T_1 = \frac{a^3 b^{2-1}}{2c^2}$

$T_1 = \frac{a^3 b}{2c^2}$ ✓M1A1

Using side lengths from part (a):

$A_1 = T_1 = \frac{1}{2} xy$

$= \frac{1}{2}\left(\frac{a^2}{c}\right)\left(\frac{ab}{c}\right)$

$= \frac{a^3 b}{2c^2}$

∴ true for $n=1$ ✓A1

Step 2: assume true for $n=k$:

$T_k = \frac{a^3 b^{2k-1}}{2c^{2k}}$. ✓A1

Step 3: consider $n=k+1$:

$T_{k+1} = \frac{a^3 b^{2(k+1)-1}}{2c^{2(k+1)}}$ ✓

$= \frac{a^3 b^{2k+1}}{2c^{2k+2}}$

$= \frac{a^3 b^{2k-1+2}}{2c^{2k+2}}$

$= \frac{a^3 b^{2k-1}}{2c^{2k}}\left(\frac{b^2}{c^2}\right)$ ✓

$= T_k \times b^2/c^2$

M1A1

~~from part (b)~~

~~$T_{k+1} = \frac{1}{2} b_{n+1} h_{n+1}$~~

~~$= \frac{1}{2}\left(\frac{a}{c} h_n\right)\left(\frac{b^2}{ac} b_n\right)$~~

~~$= \frac{ab^2}{2ac^2} h_n b_n$~~

from part (b) :

$$T_{k+1} = \tfrac{1}{2}(b_{n+1})(h_{n+1})$$ ✓ M1

$$= \frac{1}{2}\left(\frac{a}{c}h_n\right)\left(\frac{b^2}{ac}b_n\right)$$

$$= \left(\frac{1}{2}h_n b_n\right)\left(\frac{a\,b^2}{ac^2}\right)$$

$$= T_n\left(b^2/c^2\right)$$ ✓ A1

$\therefore$ True for $n = k+1$ ✓ A1

Step 4: as true for $n=1$ and true for $n=k+1$ whenever $n=k$ is true, true for all $n \in \mathbb{Z}^+$ by mathematical induction. ✓ R1

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(d) $$\frac{a^3 b^{2n-1}}{\cancel{2}c^{2n}} = \frac{1}{\cancel{2}}a\,b$$ ✓ M1

1

$$\therefore \frac{a^3 b^{2n-1}}{c^{2n}} = ab$$

$$\therefore \frac{a^2 b^{2n-2}}{c^{2n}} = 1$$

$$\therefore a^2 b^{2n-2} = c^{2n}$$

$$\therefore a^2 b^{2n} = b^2 c^{2n}$$

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2

$$\frac{1}{2}ab = \lim_{n\to\infty} \frac{a^3 b^{2n-1}}{2c^{2n}}$$

$$= \lim_{n\to\infty} \frac{a^3 b^{2n}}{2b\,c^{2n}}$$

$$= \lim_{n\to\infty} \left(\frac{b^{2n}}{c^{2n}}\right) \times \frac{a^3}{2b}$$

$$= \cancel{\lim_{n\to\infty} \left(b^{2n} \ln b \; d/dn\right.}$$

$$= \lim_{n\to\infty} \left(\frac{b^n b^n}{c^n c^n}\right) \times \frac{a^3}{2b}$$

$$\frac{1}{2}ab = \sum_{i=1}^{\infty} \frac{a^3 b^{2n-1}}{2c^{2n}}$$

$$\therefore \frac{ab}{a^3} = \frac{1}{b}\sum_{i=1}^{\infty} \frac{b^{2n}}{c^{2n}} \quad = \quad u_1 = \frac{b^{2n}}{c^{2n}} \quad r = \frac{b^2}{c^2}$$

$$\therefore \frac{b^2}{a^2} = \frac{b^2}{c^2} + \frac{b^4}{c^4} + \frac{b^6}{c^6} + \dots = \quad \therefore S_\infty = \frac{b^2/c^2}{-b^2/c^2 + 1}$$

$$= \frac{b^2}{-b^2 + c^2}$$

$$\therefore \frac{b^2}{a^2} = \frac{b^2}{b^2 - c^2}$$

$$\therefore a^2 = -b^2 + c^2$$

$$\therefore a^2 + b^2 = c^2$$