Mathematics: analysis and approaches

Higher level

Paper 3

ID: 3012

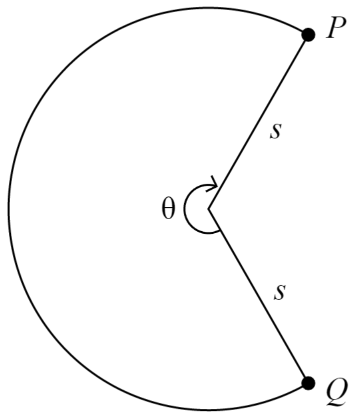
Instructions to candidates

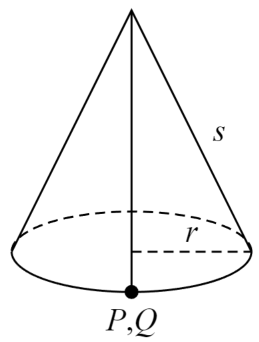
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[50 marks]**.

1. [Maximum points: 21]

In this problem you will investigate area and distance on the curved surfaces of cones.

The diagram on the left shows a circle sector of radius s and angle θ. Point P is pulled towards point Q to create a cone with a base of radius r. This is shown in the diagram on the right.





(a) In terms of s and θ use the diagram on the left to write down an expression for [2]

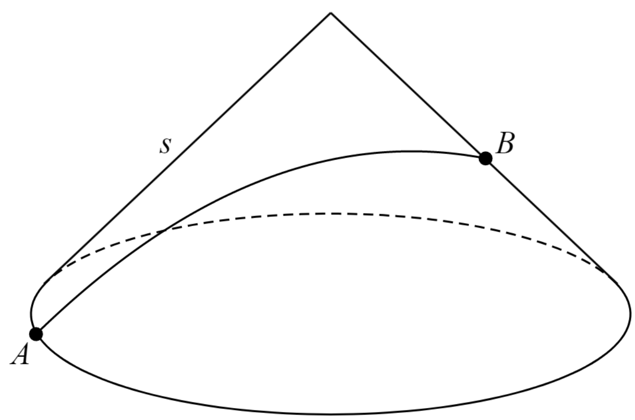
(i) the arc length from P to Q

(ii) the area of the sector

(b) Write down an expression for the circumference of the base of the cone in terms of r. [1]

(c) Hence show that the area of the curved surface of the cone is equal to πrs. [3]

A mountain is in the shape of a cone with a base of radius 1 km and a height of 1 km. A hiker walks from point A (at sea level) to point B which is on the opposite side of the mountain and half way up. This is shown in the diagram below.



(d) Find the exact value of [3]

(i) the circumference of the base

(ii) the slope length s

(e) Hence find the value θ in the circle sector which produces this cone. [2]

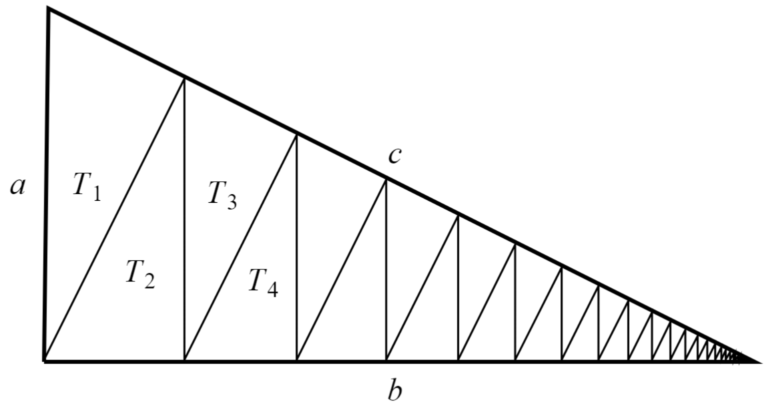
(f) Find the shortest distance the hiker walks from point A to points B. [3]

(g) As the hiker walks the distance from part (f) find the maximum height of the hiker above sea level and the distance walked to this point. [7]

2. [Maximum points: 29]

In this problem you will prove the Pythagorean theorem by dividing a triangle into an infinite number of smaller triangles.

A right-angled triangle with sides of length a, b and c is divided into infinitely many similar right-angled triangles. Starting from the left the triangles are labelled T_1, T_2, T_3 etc. The first four triangles have been labelled in the diagram below.



Let the base of a triangle be defined as the length of its shortest side, and the height be defined as the length of the side which is perpendicular to the base.

(a) Determine an expression in terms of a, b and/or c for [4]

(i) the base of T_1

(ii) the height of T_1

(b) Let b_n represent the base of T_n and h_n represent its height. Show that [7]

(i) $h_n = \frac{b}{a} b_n$

(ii) $b_{n+1} = \frac{a}{c} h_n$

(iii) $h_{n+1} = \frac{b^2}{ac} b_n$

(c) Prove by induction that the area of triangle T_n is equal to $\frac{a^3 b^{2n-1}}{2c^{2n}}$. [10]

(d) By considering the sum of the areas of all of the triangles prove the Pythagorean theorem $a^2 + b^2 = c^2$. [8]