ST ANDREW'S CATHEDRAL SCHOOL
FOUNDED 1885

Diploma Programme

Candidate session number

Mathematics

Higher level

Paper 1

Trial Examination 2020

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 9]

Given the function $f(x) = \ln x - \ln(1-x)$,

(a) Find:

(i) the domain

(ii) the range

(iii) the inverse function $f^{-1}(x)$ [5]

(b) Sketch $y = f(x)$ and $y = f^{-1}(x)$, labelling any intercepts and asymptotes. [4]

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2. [Maximum mark: 6]

(a) Prove the identity

$$\frac{\cos\theta}{1+\sin\theta}+\frac{1+\sin\theta}{\cos\theta}=2\sec\theta.$$ [2]

(b) Solve the equation $\sec^2 x+2\tan x=0, \quad -\pi\le x\le\pi$. [4]

3. [Maximum mark: 7]

(a) Write the first three derivatives of $f(x) = x^2 e^x$. [3]

(b) Use mathematical induction to prove that

$$f^{(n)}(x) = e^x[x^2 + 2nx + n(n-1)]$$

where $n \in \mathbb{Z}^+$ and $f^{(n)}(x)$ represents the n^{th} derivative. [4]

4. [Maximum mark: 8]

(a) Factorise $2x^2 - 3x - 5$. [2]

(b) Hence, or otherwise, find the coefficient of x^{23} in the expansion of $(2x^2 - 3x - 5)^{12}$, writing your answer in the form $k \times 2^m$ where $k, m \in \mathbb{Z}$. [6]

5. [Maximum mark: 6]

(a) Find $\int x^2 \sin x \, dx$. [4]

(b) Evaluate $\int_{-1}^{1} x^2 \sin x \, dx$. [2]

6. [Maximum mark: 9]

Let the probability that it rains on any one day be p and the weather on any day is independent of the weather on any other day.

(a) Using $p = 0.5$, find the probability that during a period of one week:

(i) it will rain on at least five ;

(ii) it will rain on the last day;

(iii) raining and non-raining days will alternate. [5]

(b) Find p, if during a full week period, it is equally likely that there will be five raining days as there will be six raining days. [4]

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7. [Maximum mark: 5]

Use Mathematical Induction to prove $1 + \text{cis}\ \theta + \text{cis}\ 2\theta + \ldots + \text{cis}\ n\theta = \frac{1-\text{cis}(n+1)\theta}{1-\text{cis}\theta}$.

Do **NOT** write solutions on this page.

Section B

Answer **all** the questions on the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 12]

A triangle has sides of length $(x+1), (2x+1)$ and $(2x+3)$ cm.

(a) Show that $x > 1$. [3]

(b) Find the value of x for which that triangle is right-angled. [3]

(c) (i) Find, in terms of x, the cosine of the largest angle;
(ii) hence find the value of x for which one angle of the triangle is 120^0. [3]

(d) Find the value of x for which one angle of the triangle is 60^0. [3]

9. [Maximum mark: 12]

(a) Find the x-coordinates of the two stationary points on the curve $y = x^3 - 3x^2 - 2x - 6$. [3]

(b) Show that the x-coordinate of the point of inflexion is the mean of the x-coordinates of the two stationary points. [2]

(c) Write the equation of the tangent to the curve $y = x^3 - 3x^2 - 2x - 6$ at the point where it crosses the y-axis. [2]

(d) Evaluate the area enclosed by the tangent from (c) and the cubic curve. [5]

Do **NOT** write solutions on this page.

10. [Maximum mark: 26]

(a) (i) Show that the line l given by: [3]

$$\frac{x-1}{2} = y+2 = \frac{z+1}{3}$$

and the line m, defined by the parametric equations:

$x = 3\lambda + 2,\ y = -2\lambda + 2,\ z = \lambda + 4$ intersect.

(ii) Hence, find the coordinates of their point of intersection C. [3]

(b) Find the equation of the plane Π containing the lines l and m. [6]

(c) Determine the normal vector of the plane Π which has unit length. [4]

(d) Line k is perpendicular to the plane Π and passes through the point C. Find the coordinates of points P_1 and P_2 which lie on the line k and at a distance of 5 units from C. [5]

(e) Determine the equation of the plane which is parallel to the plane Π and passes through P_1. [5]