Mathematics: analysis and approaches
Higher level
Paper 2 Practice Set C (Hodder)

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in an answer booklet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A copy of the mathematics: analysis and approaches formula book is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1 *[Maximum mark: 5]*

Sacha is investigating the relationship between time spent doing homework and time spent on social media. In her year at school, at the time of the survey, 60% of students are aged 17 and the rest are aged 18. Sacha wants to represent both age groups fairly, so she takes a random sample of six 17-year-olds and four 18-year-olds.

a State the name of this sampling technique. *[1]*

The results are shown in the table, showing the number of hours per day spent on each activity.

Student	1	2	3	4	5	6	7	8	9	10
Time spent on social media (*x*)	1.7	3.5	2.6	1.7	2.1	3.2	3.8	2.5	3.1	3.6
Time spent on homework (*y*)	4.2	2.1	3.2	3.5	4.2	2.5	0.6	2.5	2.7	1.5

Sacha finds that there is a strong negative correlation between the two variables, and decides to use a linear regression line to model the relationship between them.

b Find the equation of the regression line in the form $y = ax + b$. *[2]*

c Interpret, in the context of this question, the meaning of the coefficients a and b. *[2]*

2 *[Maximum mark: 5]*
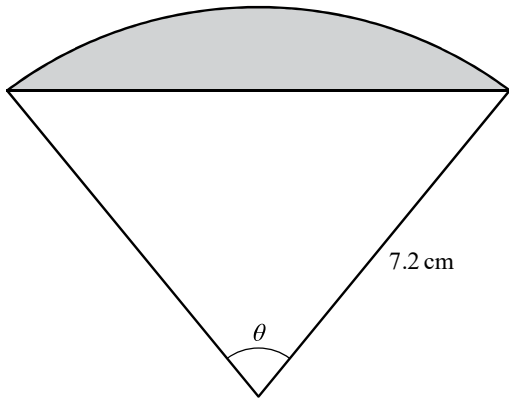
The diagram shows a sector of a circle with radius 7.2 cm. The angle at the centre is θ radians.

7.2 cm

θ

The area of the shaded region is 9.7 cm². Find the value of θ.

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

3 *[Maximum mark: 8]*

The discrete random variable X has the following probability distribution:

x	1	2	4	7
$P(X = x)$	k	$2k$	$3k$	$4k$

a Find the value of k.   *[2]*

b Find the variance of X.   *[4]*

c Find the variance of $20 - 5X$.   *[2]*

....

....

....

....

....

....

....

....

....

....

....

....

....

4 *[Maximum mark: 5]*

Prove the identity

$$\frac{\sec\theta\ \sin\theta}{\tan\theta + \cot\theta} \equiv \sin^2\theta.$$

5 *[Maximum mark: 6]*

Ilya decorates small cakes in his bakery. The time it takes him to decorate x cakes is modelled by $T(x) = 0.003x^3 + 10x + 200$ minutes.

a If Ilya wants to work at most 12 hours in a day, what is the largest number of cakes he can decorate? *[2]*

b Ilya can choose to decorate between 20 and 40 cakes per day, inclusive. Find the minimum and maximum time he can take per cake on average. *[4]*

6 *[Maximum mark: 6]*

Ibrahim has 20 different books on his bookshelf. He wants to select six books to take on holiday.

a How many different selections can he make? *[2]*

Out of the 20 books, 12 are fiction and 8 non-fiction. Ibrahim wants to take at least three fiction and at least two non-fiction books.

b How many different selections of six books can he make? *[4]*

7 *[Maximum mark: 7]*

Use the substitution $u = e^x$ to find

$$\int \frac{e^x}{e^{2x} + e^x - 2} \, dx.$$

Give your answer in the form $\ln(f(x))$.

8 *[Maximum mark: 7]*

Prove by contradiction that there exists no function
f(x) = $2x^3 + bx^2 + cx + 3$, where $b, c \in \mathbb{Z}$
such that $(2x + 3)$ is a factor of f(x), and when f(x) is divided by $x - 2$ the remainder is 5.

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

9 *[Maximum mark: 6]*

The total surface area of a cone with radius r and height h is given by $S = \pi r^2 + \pi r \sqrt{r^2 + h^2}$. A cone has height 5 cm and the radius increases at the rate of 2 cm per second. Find the rate at which the surface area is increasing when the radius equals 10 cm.

Section B

Answer **all** questions in an answer booklet. Please start each question on a new page.

10 *[Maximum mark: 20]*

  a Stella is planning to start a small business selling cosmetics gift boxes. She plans to start by selling 30 boxes in the first month. In each subsequent month she plans to sell 10 more boxes than in the previous month.

    i According to Stella's plan, how many boxes will she sell in the 12th month?

    ii How many boxes will she sell in the first year?

    iii In which month will she sell her 2000th box? *[8]*

  b Giulio also sells cosmetics gift boxes. He also sells 30 boxes in the first month, but expects to increase his sales by 10% each month.

    i How many boxes will Giulio sell in the first year?

    ii In which month will Giulio first sell more than 100 boxes? *[6]*

  c Stella makes a profit of £2.20 per box and Guilio makes a profit of £3.10 per box.

    i Find the profit each person makes in the first year.

    ii In which month will Giulio's **total** profit first overtake Stella's? *[6]*

11 *[Maximum mark: 17]*

The velocity (in $\mathrm{m\,s^{-1}}$) of an object at *t* seconds is given by

$$v(t) = \frac{8 - 3t}{t^2 - 6t + 10}, \, 0 \leq t \leq 10.$$

Find

  a the initial speed *[1]*

  b the maximum speed *[2]*

  c the length of time for which the speed is greater than $1\,\mathrm{m\,s^{-1}}$ *[3]*

  d the time at which the object changes direction *[2]*

  e the length of time for which the object is decelerating *[2]*

  f the acceleration after 5 seconds *[2]*

  g the distance travelled after 10 seconds *[2]*

  h the time when the object returns to its starting position. *[3]*

12 *[Maximum mark: 18]*

  a Show that

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln|\sec x + \tan x|\right) = \sec x.$$ *[3]*

  b Find the general solution to the differential equation

$$\cos x \frac{\mathrm{d}y}{\mathrm{d}x} + y = 1, \, -\frac{\pi}{2} < x < \frac{\pi}{2}.$$ *[7]*

Consider now the differential equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \cos x \frac{\mathrm{d}y}{\mathrm{d}x} + y = 1, \, -\frac{\pi}{2} < x < \frac{\pi}{2}.$$

  c **i** Show that

$$\frac{\mathrm{d}^3 y}{\mathrm{d}x^3} = (\sin x - 1)\frac{\mathrm{d}y}{\mathrm{d}x} - \cos x \frac{\mathrm{d}^2 y}{\mathrm{d}x^2}.$$

    ii Given that $y = 2$ and $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 1$ when $x = 0$, find the Maclaurin series solution up to and including the term in x^3. *[8]*