Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 2

Tuesday 14 May 2019 (morning)

2 hours

Candidate session number

1	9	M	T	Z	2	P	2	M	A	H	L

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.

$\frac{59}{100} = 59\%$

1:36:00

11 pages

2219–7206

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

In triangle ABC, $AB = 5$, $BC = 14$ and $AC = 11$.
Find all the interior angles of the triangle. Give your answers in degrees to one decimal place.

let $a = 5$ $b = 14$ $c = 11$ ~~$A\hat{B}C$ $A\hat{C}B = C$~~

$A\hat{B}C = C$

$c^2 = a^2 + b^2 - 2ab\cos C$

$\therefore \cos C = (a^2+b^2-c^2)/2ab$

$= (5^2+14^2-11^2)/2(5)(14)$

$= 5/7$

$\therefore C = \cos^{-1}(5/7)$

$= 44.4153°$

$\therefore C \approx 44.4°$

~~$\frac{\sin 44.4}{11} = \frac{\sin A}{14}$~~

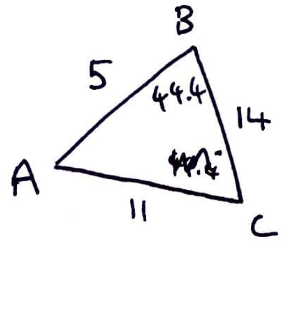
~~$\therefore \sin A = \frac{14\times\sin 44.4}{11}$~~

$\frac{\sin 44.4}{11} = \frac{\sin A}{14}$

$\therefore A = \sin^{-1}\left(\frac{14\sin 44.4}{11}\right.$

$= 62.9337°$ ✗

$c = 72.6663°$ ✓ ECF



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2. [Maximum mark: 5]

Timmy owns a shop. His daily income from selling his goods can be modelled as a normal distribution, with a mean daily income of \$820, and a standard deviation of \$230. To make a profit, Timmy's daily income needs to be greater than \$1000.

(a) Calculate the probability that, on a randomly selected day, Timmy makes a profit. [2]

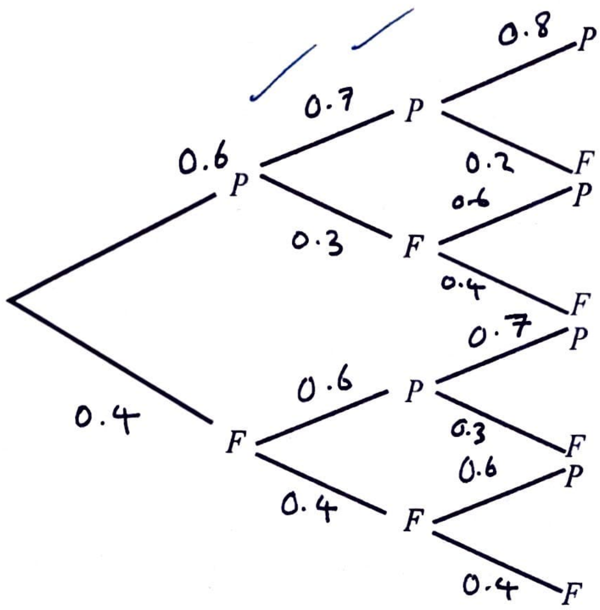
The shop is open for 24 days every month.

(b) Calculate the probability that, in a randomly selected month, Timmy makes a profit on between 5 and 10 days (inclusive). [3]

3. [Maximum mark: 8]

Iqbal attempts three practice papers in mathematics. The probability that he passes the first paper is 0.6. Whenever he gains a pass in a paper, his confidence increases so that the probability of him passing the next paper increases by 0.1. Whenever he fails a paper the probability of him passing the next paper is 0.6.

(a) Complete the given probability tree diagram for Iqbal's three attempts, labelling each branch with the correct probability. [3]



(b) Calculate the probability that Iqbal passes at least two of the papers he attempts. [2]

(c) Find the probability that Iqbal passes his third paper, given that he passed only one previous paper. [3]

b) $P = P(PPP) + P(PFP) + P(FPP)$

$= 0.6 \times 0.7 \times 0.8 + 0.6 \times 0.3 \times 0.6 + 0.4 \times 0.6 \times 0.7$

$= 0.612$

c) $P = \cancel{P(PFP) + P(FFP)}$ $P(PF) = 0.6 \times 0.3 = 0.18$

$\therefore P(P|PF) = 0.6/0.18$

$= 10/3$

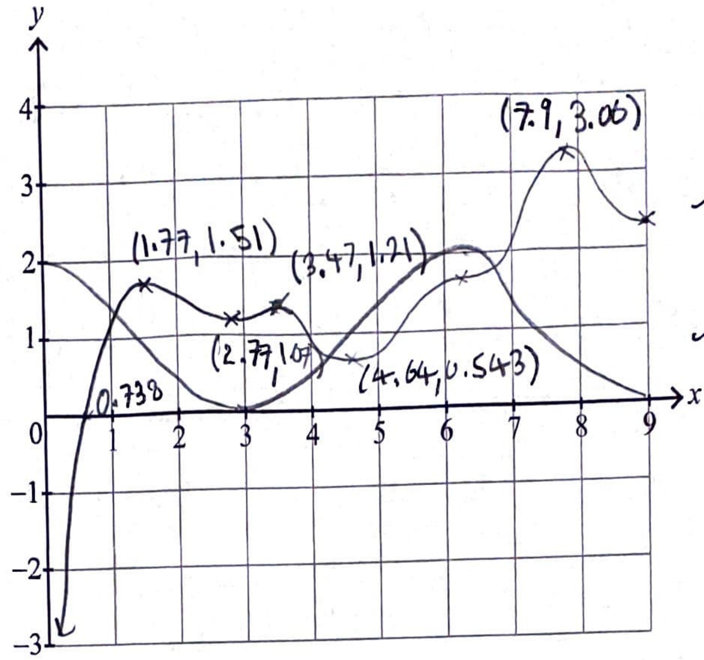
$P(FP) = 0.4 \times 0.6 = 0.24$

$P(P|FP) = \cancel{0.7/0.24}$

3/

4. [Maximum mark: 6]

(a) Sketch the graphs of $y = \sin^3 x + \ln x$ and $y = 1 + \cos x$ on the following axes for $0 < x \leq 9$. [2]



(b) Hence solve $\sin^3 x + \ln x - \cos x - 1 < 0$ in the range $0 < x \leq 9$. [4]

~~$0 < x < 6.86$~~ {By G.D.C}

point of intersection = $x = 1.35, 4.35, 6.64$

$\therefore x < 1.35$

$4.35 < x < 6.64$

6

5. [Maximum mark: 6]

(a) Prove the identity $\dfrac{1+\sin 2x}{\cos 2x} \equiv \dfrac{1+\tan x}{1-\tan x}$. [4]

(b) Solve the equation $\dfrac{1+\sin 2x}{\cos 2x} = \sqrt{3}$ for $0 \le x < 2\pi$. [2]

a) $\cancel{\dfrac{1+\sin 2x}{\cos 2x} = \dfrac{1+2\sin x\cos x}{2\sin^2 x\ \cos^2 x - \sin^2 x}}$

$=$

$$\frac{1+\tan x}{1-\tan x} = \frac{1}{1-\frac{\sin x}{\cos x}} + \frac{\frac{\sin x}{\cos x}}{1-\frac{\sin x}{\cos x}}$$

$$= \frac{\cos x}{\cos x - \sin x} + \frac{\sin x}{\cos x - \sin x}$$

$$= \frac{\cos x + \sin x}{\cos x - \sin x} \times \frac{\frac{1}{\cos x}}{\frac{1}{\cos x}}$$

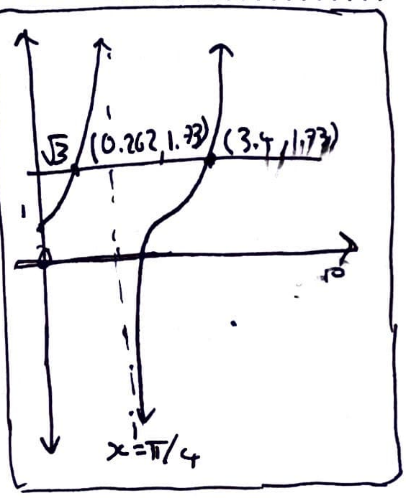
$$= \frac{1+\tan x}{1-\tan x}$$

b) $\cancel{\dfrac{1+2\sin 2x}{\cos 2x} = \sqrt{3}}$

$\cancel{\therefore \dfrac{1+\tan x}{1-\tan x} = \sqrt{3}}$

$\cancel{\therefore 1+\tan x = \sqrt{3} - \sqrt{3}\tan x}$

$\cancel{\therefore \tan x(1+\sqrt{3})}$



GDC.

$\cancel{\therefore \left\{x \,\middle|\, \begin{array}{l} 0 < x < 0.262 \\ \frac{\pi}{4} < x < 3.4 \end{array}\right\}}$

$x = 0.262$

$x = 3.40$

6

6. [Maximum mark: 6]

A particle moves along a horizontal line such that at time t seconds, $t \geq 0$, its acceleration a is given by $a = 2t - 1$. When $t = 6$, its displacement s from a fixed origin O is 18.25 m. When $t = 15$, its displacement from O is 922.75 m. Find an expression for s in terms of t.

$a = 2t - 1$ $\qquad s = 18.25$ @ $t = 6$

$v = \int (2t-1)\,dt$ $\qquad s = 922.75$ @ $t = 15$

$= \frac{2}{3}t^2 - t + c$

$\therefore s = \int (\frac{2}{3}t^2 - t + c)\,dt$

$= \frac{2}{12}t^4 - \frac{t^2}{2} + ct + d$

$= \frac{1}{6}t^4 - \frac{1}{2}t^2 + ct + d$

sub $s = \cancel{922.75}$ 18.25, $t = 6$:

~~Asolve (18.25 = $\frac{1}{6}(6^4)$ –~~

$18.25 = \frac{1}{6}(6^4) - \frac{1}{2}(6^2) + 6c + d$

$\therefore 18.25 = 6^3 - 18 + 6c + d$ .. (1)

Sub $s = 922.75$ @ $t = 15$:

$922.75 = \frac{1}{6}(15^4) - \frac{1}{2}(15^2$

$a = 2t - 1$

$\therefore v = \int (2t-1)\,dt$

$= t^2 - t + c$

$s = \int (t^2 - t + c)\,dt$

$= \frac{1}{3}t^3 - \frac{1}{2}t^2 + ct + d$

sub $s = 18.25$, $t = 6$:

$18.25 = \frac{1}{3}(6^3) - \frac{1}{2}(6^2) + 6c + d$

sub $s = 922.75$, $t = 15$

$922.75 = \frac{1}{3}(15^3) - \frac{1}{2}(15^2) + 15c + d$

G.D.C : linSolve$\left(\begin{cases} 18.25 = \frac{1}{3}\cdot 6^3 - \frac{1}{2}\cdot 6^2 + 6\cdot c + d \\ 922.75 = \frac{1}{3}\cdot 15^3 - \frac{1}{2}\cdot 15^2 + 15\cdot c + d \end{cases}, \{c, d\}\right)$

$\{-6, 0.25\}$

$\therefore s = \frac{1}{3}t^3 - \frac{1}{2}t^2 - 6t + \frac{1}{4}$

6

Turn over

7. [Maximum mark: 7]

Suppose that u_1 is the first term of a geometric series with common ratio r. Prove, by mathematical induction, that the sum of the first n terms, S_n is given by

$S_n = \frac{u_1(1-r^n)}{1-r}$, where $n \in \mathbb{Z}^+$.

u_1 $\cancel{t}$ $\quad u_2 = u_1 r \quad u_3 = u_1 r^2 \quad u_4 = u_1 r^3 \quad u_n = u_1 r^{n-1}$

$\therefore u_1 + u_1 r + u_1 r^2 + \cdots + u_1 r^{n-1} = \frac{u_1(1-r^n)}{1-r}$

=> Step 1: Prove for $n=1$:

$LHS = u_1$

$RHS = \frac{u_1(1-r)}{1-r}$

$= u_1$

$= LHS$ $\quad\therefore$ true for P_1

=> Step 2: Assume true for $n=k$:

$u_1 + u_1 r + u_1 r^2 + \cdots + u_1 r^{k-1} = \frac{u_1(1-r^k)}{1-r}$

=> Step 3: Prove true for $n=k+1$.

$u_1 + u_1 r + u_1 r^2 + \cdots + u_1 r^{k-1} + u_1 r^k = \frac{u_1(1-r^{k+1})}{1-r}$

$LHS = \frac{u_1(1-r^k)}{1-r} + u_1 r^k$ {by assumption}

$= \frac{u_1(1-r^k) + (1-r)u_1 r^k}{1-r}$

$= \frac{u_1(1-r^k + (1-r)r^k)}{1-r}$

$= \frac{u_1(1-r^k + r^k - r^{k+1})}{1-r}$

$= \frac{u_1(1-r^{k+1})}{1-r}$

$= RHS$

=> Step 4: as ~~P~~ LHS = RHS is true for $n=1$, and true for $n=k+1$ whenever $n=k$ is true, then LHS = RHS is true for all $n \in \mathbb{Z}^+$ by mathematical induction

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8. [Maximum mark: 7]

(a) Find the roots of the equation $w^3 = 8\mathrm{i}$, $w \in \mathbb{C}$. Give your answers in Cartesian form. [4]

One of the roots w_1 satisfies the condition $\mathrm{Re}(w_1) = 0$.

(b) Given that $w_1 = \dfrac{z}{z-\mathrm{i}}$, express z in the form $a + b\mathrm{i}$ where $a, b \in \mathbb{Q}$. [3]

a) $w^3 = 8i$

$= 8\,\mathrm{cis}\left(\frac{\pi}{2} + 2k\pi\right)$

$= 8\,\mathrm{cis}\left(\frac{\pi + 4k\pi}{2}\right)$

$\therefore w = 2\,\mathrm{cis}\left(\frac{\pi + 4k\pi}{6}\right)$

$\hookrightarrow w_1 = 2\,\mathrm{cis}\left(\frac{\pi - 4\pi}{6}\right)$ $\{k=-1\}$ $\rightarrow = 2\left(\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right)$

$= 2\,\mathrm{cis}(-\pi/2)$ $= 2(0 - i)$

$w_1 = -2i$

$\rightarrow w_2 = 2\,\mathrm{cis}(\pi/6)$ $\{k=0\}$ $\rightarrow = 2\cos(\pi/6) + 2i\sin(\pi/6)$

$= 2(\sqrt{3}/2) + i \cdot 2(1/2)$

$w_2 = \sqrt{3} + i$

$\rightarrow w_3 = 2\,\mathrm{cis}\left(\frac{\pi + 4\pi}{6}\right)$ $\{k=1\}$

$= 2\,\mathrm{cis}(5\pi/6) \longrightarrow = -2\cos(\pi/6) + i2\sin(\pi/6)$

$= -2(\sqrt{3}/2) + i \cdot 2(1/2)$

$w_3 = -\sqrt{3} + i$

b) if $\mathrm{Re}(w_1) = 0$, then $w_1 = -2i$

$\therefore -2i = \frac{z}{z - i}$

$= \frac{a + bi}{a + bi - i}$

$= \frac{a + bi}{a + i(b-1)}$

$= \frac{(a+bi)(a - i(b-1))}{a^2 - i^2(b-1)^2}$

$= \frac{a^2 - ia(b-1) + abi + b(b-1)}{a^2 + (b-1)^2}$

$\therefore -2i = \mathrm{Im}$

$\therefore -2 = \frac{ab - a(b-1)}{a^2 + (b-1)^2}$

$\therefore -2(a^2 + b^2 - 2b + 1) = ab - ab - a$

$\therefore -2a^2 - 2b^2 + 4b - 2 = -a$

$\therefore$

$-2i = \frac{z}{z+i}$

$\therefore (z+i)(-2i) = z$

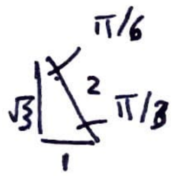
$\therefore -2zi + 2i^2 = z$

$\therefore -2zi + -2 = z$

$\therefore -2 = z(1 + 2i)$

$\therefore z = \frac{-2}{1+2i}$

$= -\frac{2}{5} + \frac{4}{5}i$



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Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 15]

Consider the polynomial $P(z) \equiv z^4 - 6z^3 - 2z^2 + 58z - 51$, $z \in \mathbb{C}$.

(a) Express $P(z)$ in the form $(z^2 + az + b)(z^2 + cz + d)$ where $a, b, c, d \in \mathbb{R}$. [7]

(b) Sketch the graph of $y = x^4 - 6x^3 - 2x^2 + 58x - 51$, stating clearly the coordinates of any maximum and minimum points and intersections with axes. [6]

(c) Hence, or otherwise, state the condition on $k \in \mathbb{R}$ such that all roots of the equation $P(z) = k$ are real. [2]

10. [Maximum mark: 16]

Steffi the stray cat often visits Will's house in search of food. Let X be the discrete random variable "the number of times per day that Steffi visits Will's house". The random variable X can be modelled by a Poisson distribution with mean 2.1.

(a) Find the probability that on a randomly selected day, Steffi does not visit Will's house. [2]

Let Y be the discrete random variable "the number of times per day that Steffi is fed at Will's house". Steffi is only fed on the first four occasions that she visits each day.

(b) Copy and complete the probability distribution table for Y. [4]

y	0	1	2	3	4
$P(Y=y)$					

(c) Hence find the expected number of times per day that Steffi is fed at Will's house. [3]

(d) In any given year of 365 days, the probability that Steffi does not visit Will for at most n days in total is 0.5 (to one decimal place). Find the value of n. [3]

(e) Show that the expected number of occasions per year on which Steffi visits Will's house and is not fed is at least 30. [4]

Do **not** write solutions on this page.

11. [Maximum mark: 19]

The plane Π_1 contains the points $\mathrm{P}(1, 6, -7)$, $\mathrm{Q}(0, 1, 1)$ and $\mathrm{R}(2, 0, -4)$.

(a) Find the Cartesian equation of the plane containing P, Q and R. [6]

The Cartesian equation of the plane Π_2 is given by $x - 3y - z = 3$.

(b) Given that Π_1 and Π_2 meet in a line L, verify that the vector equation of L can be given by $\boldsymbol{r} = \begin{pmatrix} \frac{5}{4} \\ 0 \\ -\frac{7}{4} \end{pmatrix} + \lambda \begin{pmatrix} \frac{1}{2} \\ 1 \\ -\frac{5}{2} \end{pmatrix}$. [3]

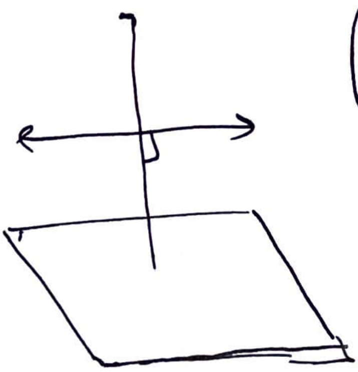
The Cartesian equation of the plane Π_3 is given by $ax + by + cz = 1$.

(c) Given that Π_3 is parallel to the line L, show that $a + 2b - 5c = 0$. [1]

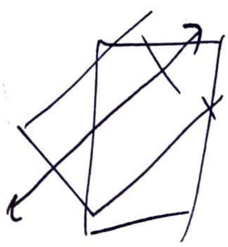
Consider the case that Π_3 contains L.

(d) (i) Show that $5a - 7c = 4$.

(ii) Given that Π_3 is equally inclined to both Π_1 and Π_2, determine two distinct possible Cartesian equations for Π_3. [9]



$\begin{pmatrix} \frac{5}{4} \\ 0 \\ -\frac{7}{4} \end{pmatrix} + \lambda \begin{pmatrix} \frac{1}{2} \\ 1 \\ -5/2 \end{pmatrix}$



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4 PAGES / PÁGINAS (1)

Candidate session number: / Numéro de session du candidat: / Número de convocatoria del alumno: 19 M TZ 2 P 2 - M A H L

Candidate name: / Nom du candidat: / Nombre del alumno:

Please write question numbers in the following format: / Veuillez numéroter les questions en utilisant la présentation suivante: / Sírvase escribir los números de las preguntas en el siguiente formato:

1 2 3 4 5 6 7 8 9 10

9

a) $P(z) = z^4 - 6z^3 - 2z^2 + 58z - 51, \quad z \in \mathbb{C}$

$$(z^2 + az + b)(z^2 + cz + d) = z^4 + cz^3 + dz^2 + az^3 + acz^2 + adz + bz^2 + bcz + bd$$
$$= z^4 + (c+a)z^3 + (ac+d+b)z^2 + (ad+bc)z + db$$

$\therefore$ $c + a = -6$
$db = -51$
$ac + d + b = -2$
$ad + bc = 58$

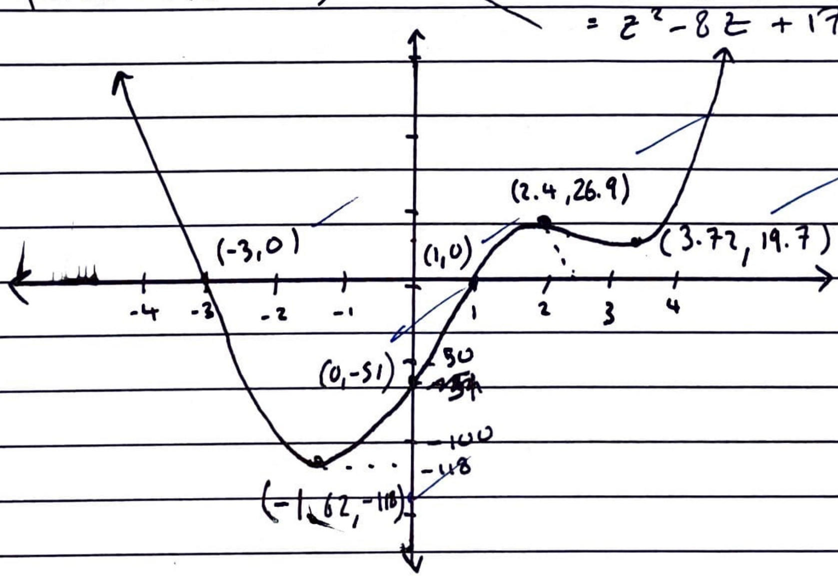
Roots: $-3, 1, 4-i, 4+i$ (6)

$\therefore (z+3)(z-1) = z^2 + 2z - 3$

$(z-(4-i))(z-(4+i))$
$= z^2 - z(4+i) - z(4-i) + (4^2 - i^2)$
$= z^2 - 4z - 4z + zi - zi + 17$
$= z^2 - 8z + 17$

$P(z) = (z^2 + 2z - 3)(z^2 - 8z + 17)$

b)



(6)

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c) $-118 < k < 19.7$

$19.7 \le k \le 26.9$

{ this is where k will intersect the polynomial four times }

Π_1 : $P(1,6,-7)$ $Q(0,1,1)$ $R(2,0,-4)$

a) $\vec{PQ} = \begin{pmatrix} 0-1 \\ 1-6 \\ 1-(-7) \end{pmatrix} = \begin{pmatrix} -1 \\ -5 \\ 8 \end{pmatrix}$

$\vec{PR} = \begin{pmatrix} 2-1 \\ 0-6 \\ -4-(-7) \end{pmatrix} = \begin{pmatrix} 1 \\ -6 \\ 3 \end{pmatrix}$

$\therefore \vec{n} = \begin{pmatrix} -1 \\ -5 \\ 8 \end{pmatrix} \times \begin{pmatrix} 1 \\ -6 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} (-5)(3)-(-6)(8) \\ (1)(8)-(-1)(3) \\ (-1)(-6)-(1)(-5) \end{pmatrix}$

$= \begin{pmatrix} -15+48 \\ 8+3 \\ 6+5 \end{pmatrix}$

$= \begin{pmatrix} 33 \\ 11 \\ 11 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$

$\therefore \Pi_1 : 3x+y+z = d$

sub $a(0,1,1)$ into $3x+y+z=d$.

$\therefore \quad d=2$

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hence, $3x+y+z=2 \quad \ldots (1)$

b)

$$L: \cancel{r} = \begin{cases} x = 5/4 + \frac{1}{2}\lambda \\ y = 0 + \lambda \\ z = -\frac{7}{4} - \frac{5}{2}\lambda \end{cases} \quad \ldots (2)$$

sub $(2) \rightarrow (1)$

$$3\left(5/4 + \tfrac{1}{2}\lambda\right) + \lambda - \tfrac{7}{4} - \tfrac{5}{2}\lambda = 2$$

$\therefore LHS = \cancel{3} \; 15/4 + 3/2\,\lambda + \lambda - 7/4 - 5/2\,\lambda$
$= 2 - 0\lambda$
$= 2$
$= RHS$

$\therefore L$ fits on π_1

sub $(2) \rightarrow x-3y-z=3$:

$\therefore \quad \frac{5}{4} + \frac{1}{2}\lambda - 3(\lambda) - \left(-\frac{7}{4} - \frac{5}{2}\lambda\right) = 3$

$\therefore LHS = \left(\frac{5}{4}+\frac{7}{4}\right) + \left(\frac{1}{2} - 3 + \frac{5}{2}\right)\lambda$
$= 3$
$= RHS$

$\therefore L$ fits on Π_1 and Π_2

3

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c) $\Pi_3 : ax + by + cz = 1$

$\therefore n_3 = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$

If $\Pi_3 \parallel$ to L, then $\vec{n} \cdot \vec{b} = 0$ {b = direction vector of L}

$\therefore \begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} 1/2 \\ 1 \\ -5/2 \end{pmatrix} = 0$ ✓

1

$\therefore \frac{1}{2}a + b - \frac{5}{2}c = 0$

$\therefore a + 2b - 5c = 0$ ✓

d) i) ~~eqs~~ Parametric eqs of L: $x = \frac{5}{4} + \frac{1}{2}\lambda$
$y = \lambda$
$z = -\frac{7}{4} - \frac{5}{2}\lambda$

$\therefore a\left(\frac{5}{4} + \frac{1}{2}\lambda\right) + b\lambda + c\left(-\frac{7}{4} - \frac{5}{2}\lambda\right) = 0$

$\therefore \frac{5}{4}a - \frac{7}{4}c + \left(\frac{1}{2}a + b - \frac{5}{2}c\right)\lambda = 0$ ✗

hence, for infinite solutions of λ:

$5/4\, a - 7/4\, c = 1$

$\therefore 5a - 7c = 4$

ii) $\vec{n}_3 \perp \vec{n}_1 \rightarrow \vec{n}_3 \cdot \vec{n}_1 = 0$

$\therefore \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0$ ✗

$\therefore 3a + b + c = 0$

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1 2 3 4 5 6 7 8 9 10

$\vec{n}_3 \perp \vec{n}_2 \rightarrow \vec{n}_3 \cdot \vec{n}_2 = 0$

$\therefore \begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -3 \\ -1 \end{pmatrix} = 0$

$\therefore a - 3b - c = 0$

$5a - 7c = 4$

$\therefore$ ~~$a + 2b - 5c = 0$~~ $\{given\}$

$a - 3b - c = 0 \quad \{n_3 \cdot n_2 = 0\}$

$3a + b + c = 0 \quad \{n_3 \cdot n_1 = 0\}$

$\therefore a, b, c = \frac{1}{10}, \frac{1}{5}, -\frac{1}{2} \; \{G.D.C\}$.

hence, $\frac{1}{10}x + \frac{1}{5}y - \frac{1}{2}z = 1$

OR

$-\frac{1}{10}x - \frac{1}{5}y + \frac{1}{2}z = 1$

dii) θ between Π_3 and Π_1

$= \theta$ between Π_3 and Π_2

$\therefore \quad n_3 = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$

$n_2 = \begin{pmatrix} 1 \\ -3 \\ -1 \end{pmatrix}$

$n_1 = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$

$$\therefore \frac{n_1 \cdot n_3}{|n_1||n_3|} = \frac{n_2 \cdot n_3}{|n_2||n_3|}$$

$$\therefore \frac{3a+b+c}{\sqrt{11}\sqrt{a^2+b^2+c^2}} = \frac{a-3b-c}{\sqrt{11}\sqrt{a^2+b^2+c^2}}$$

$\therefore \; 3a+b+c = a-3b-c$

$\therefore \; 2a+4b+2c = 0 \quad \ldots (1)$

$5a-7c = 4 \quad \ldots (2)$

$a+2b-5c = 0$

$\therefore \; a = 4/5 \qquad b = -2/5 \qquad c = 0$

$\therefore \; \frac{4}{5}x - \frac{2}{5}y = 1$