

Mathematics: analysis and approaches
Higher level
Paper 2 Practice Set B (Hodder)

Candidate session number

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2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in an answer booklet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A copy of the mathematics: analysis and approaches formula book is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

$$A: \frac{50}{55} - B: \frac{45}{55}$$

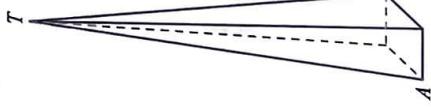
$$\Rightarrow \frac{95}{110} = 0.864 \\ = 86.4\%$$

16/10/22

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

2 [Maximum mark: 4]

A flag pole has the shape of a square-based pyramid shown in the diagram. The side length of the base is 8.3 cm. The edge AT makes an angle of 89.8° with the base.



Section A

Answer all questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1 [Maximum mark: 5]

In an arithmetic sequence, the fifth term is 7 and the tenth term is 81. Find the sum of the first 20 terms.

$$U_5 = 7 \quad U_1 + 4d \quad M1 \quad U_{10} = 81 \quad U_1 + 9d \quad A1$$

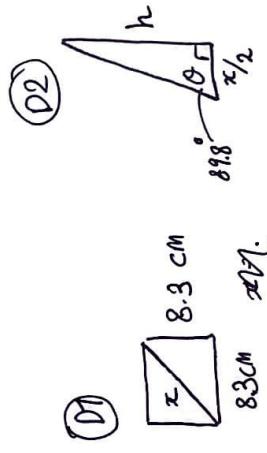
$$\therefore U_1 = -\frac{261}{5} \quad \checkmark A1 \quad d = \frac{74}{5} \quad [GDC \quad InSolve]$$

$$\begin{aligned} S_{20} &= \frac{10}{2} (2U_1 + 9d) \quad MPAO \\ &\equiv 5 \left(2\left(-\frac{261}{5}\right) + 9\left(\frac{74}{5}\right)\right) \\ &= 144 \end{aligned}$$

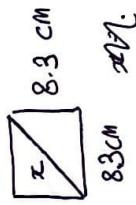
Find the height of the flag pole. Give your answer in centimetres, in standard form, correct to two significant figures.

$$\text{Base diagonal} = x = \sqrt{8.3^2 + 8.3^2} = 11.738 \text{ cm} \quad (D1)$$

$$\begin{aligned} \tan \theta &= \frac{h}{11.738/2} \quad M1 \\ \therefore h &= 5.869 \tan(89.8) \\ &= 1681.34 \text{ cm} \quad A1 \\ &\approx 1700 \text{ cm} \quad A1 \end{aligned}$$



(3)



(4)

3 [Maximum mark: 5]

In this question, an *outlier* is defined as a piece of data which is more than two standard deviations above or below the mean.

The heights of eight children, in centimetres, are:

122	124	127	131	134	134	136	147
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Determine whether any of the heights are outliers.

[OneVar stats on GDC]: $\bar{x} = 131.875$ ✓ A1
 $\sigma x = 7.406711$

$$\therefore \text{Lower fence} = \bar{x} - \sigma x \approx 117.061578 \quad \checkmark \text{M1}$$

$$\therefore \text{Upper fence} = \bar{x} + \sigma x \quad \checkmark \text{A1}$$

$$= 146.688422$$

Hence, 147 cm is an outlier (upper) ✓ A1

4 [Maximum mark: 5]

Prove that

$$\frac{\sin(x + \frac{\pi}{3}) - \sin(x - \frac{\pi}{3})}{\cos(x + \frac{\pi}{3}) - \cos(x - \frac{\pi}{3})} \equiv -\cot x$$

$$\sin(x + \frac{\pi}{3}) - \sin(x - \frac{\pi}{3}) = \sin x \cos \frac{\pi}{3} + \sin \frac{\pi}{3} \cos x - (\sin x \cos \frac{\pi}{3} - \sin \frac{\pi}{3} \cos x) \quad \checkmark \text{M1}$$

$$\cos(x + \frac{\pi}{3}) - \cos(x - \frac{\pi}{3}) = \cos x \cos \frac{\pi}{3} - \cos x \sin \frac{\pi}{3} - (\cos x \cos \frac{\pi}{3} + \cos x \sin \frac{\pi}{3})$$

$$= \frac{2 \sin \frac{\pi}{3} \cos x}{-2 \sin x \sin \frac{\pi}{3}} \quad \checkmark \text{A1}$$

$$= -\frac{\cos x}{\sin x} \quad \checkmark \text{A1 A1}$$

$$= -\cot x \quad \checkmark \text{A1}$$

(5)

(5)

5 [Maximum mark: 6]

Find

$$\int \frac{2}{x(x-1)} dx$$

Write your answer in the form $\ln(f(x)) + c$.

Partial fractions:

$$\frac{2}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1} \quad \checkmark M1$$

$$\Rightarrow 2 = A(x-1) + Bx \quad \checkmark M1$$

$$\Rightarrow x=0; 2=-A$$

$$\therefore A = -2$$

$$\Rightarrow x=1; 2=B$$

$$\therefore B=2$$

$$\left. \begin{aligned} &= -\frac{2}{x} + \frac{2}{x-1} \\ &= \frac{2}{x(x-1)} \end{aligned} \right\} \checkmark A1$$

$$\text{Hence, } \int \frac{2}{x(x-1)} dx = -\int \frac{2}{x} dx + \int \frac{2}{x-1} dx$$

$$= -2\ln|x| + 2\ln|x-1| + C \quad \checkmark MIA1$$

$$= 2(\ln|x-1| - \ln|x|)$$

$$\equiv 2 \left(\ln \left| \frac{x-1}{x} \right| \right) + C$$

$$= \ln \left(\frac{x^2-2x+1}{x^2} \right) + C \quad \checkmark A1$$

$$\left\{ \frac{x-1}{x} \right\}^2 = \left(\frac{x-1}{x} \right)^2$$

$$\text{Hence, } f(x) = \frac{x^2-2x+1}{x^2}$$

(6)

6 [Maximum mark: 6]

The normal to the graph of $y = 4 \sin^{-1}\left(\frac{x}{2}\right)$ at the point where $x = 1.5$ intersects the x -axis at the point A and the y -axis at the point B . Find the area of triangle AOB , where O is the origin.

$$\Rightarrow y = 4 \sin^{-1}\left(\frac{x}{2}\right) \rightarrow \frac{dy}{dx} = 4 \left(\frac{1}{\sqrt{1-\frac{x^2}{4}}} \right) \left(\frac{1}{2} \right) \Rightarrow \text{make life easier}$$

$$= \frac{2}{\sqrt{1-x^2/4}} \Rightarrow \text{using chain rule}$$

$$= \frac{2}{\sqrt{4-x^2}} \Rightarrow \text{more or less 2111}$$

$$\Rightarrow \text{when } x=1.5, \frac{dy}{dx} = \frac{4}{\sqrt{4-1.5^2}} = \frac{4}{\sqrt{1.75}} \quad \checkmark M1$$

$$\Rightarrow \text{normal, } n = -\frac{\sqrt{1.75}}{4} \quad \checkmark M1$$

$$\Rightarrow \text{at } x=1.5, y = 4 \sin^{-1}(1.5/2)$$

$$= 3.392248 + 4 \arcsin(3/4) \quad \checkmark A1$$

$$\Rightarrow y - 4 \arcsin(3/4) = -\frac{\sqrt{1.75}}{4} (x - 1.5)$$

$$\therefore y = -\frac{\sqrt{1.75}x}{4} + \frac{3\sqrt{1.75}}{8} + 4 \arcsin(3/4) \quad \checkmark A1$$

$$\Rightarrow \text{Intercepts, } x = \frac{-3\sqrt{1.75}/8 - 4 \arcsin(3/4)}{-2\sqrt{1.75}}$$

$$A(0, 3.88833) \quad \checkmark M1$$

$$(11.75719, 0) \quad \checkmark M1$$

$$\therefore \text{Area} = \frac{1}{2} (3.88833)(11.75719) \quad \checkmark A1$$

$$\approx 22.857917 \quad \checkmark A1$$

$$\approx 22.9 \text{ units}^2 \quad \checkmark A1$$

(6)

7 [Maximum mark: 7]

Find the coordinates of the points on the curve

$$3x^2 + 2xy - y^2 + 24 = 0$$

where the tangent is parallel to the x-axis.

$$\Rightarrow 3x^2 + 2xy - y^2 + 24 = 0$$

$$\therefore 6x + 2y(x \frac{dy}{dx} + y) - 2y \frac{dy}{dx} = 0$$

$$\therefore 6x + 2x \frac{dy}{dx} + 2y - 2y \frac{dy}{dx} = 0$$

$$\therefore \frac{dy}{dx}(2x - 2y) = -6x - 2y$$

$$\frac{dy}{dx} = \frac{-6x - 2y}{2x - 2y}$$

$$= \frac{6x + 2y}{2y - 2x}$$

$$\checkmark A1$$

\Rightarrow When $\frac{dy}{dx}$ is \parallel to x-axis, $\frac{dy}{dx}$ is ~~not~~ 0

$$6x + 2y = 0$$

$$\therefore 3x = -y$$

$$\therefore y = -3x \checkmark M1$$

\Rightarrow Substitute into original equation:

$$3x^2 + 2x(-3x) - (-3x)^2 + 24 = 0 \checkmark M1$$

$$\therefore 3x^2 - 6x^2 - 9x^2 + 24 = 0$$

$$-12x^2 + 24 = 0$$

$$\therefore -x^2 = -2$$

$$\therefore x = \pm\sqrt{2} \checkmark A1$$

\Rightarrow When $x = \sqrt{2}$,

$$3(2) + 2\sqrt{2}y - y^2 + 24 = 0$$

$$\therefore y^2 - 2\sqrt{2}y - 30 = 0$$

$$\therefore y = -4.2426, 7.071$$

\Rightarrow When $x = -\sqrt{2}$, $3(2) - 2\sqrt{2}y - y^2 + 24 = 0$

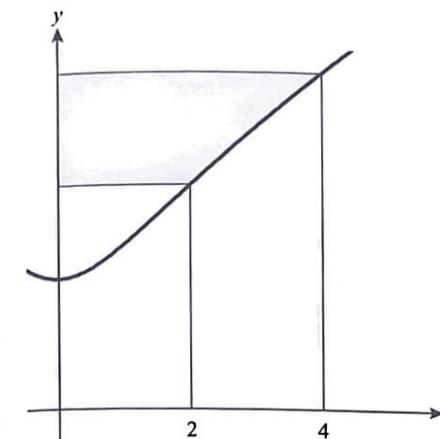
$$\therefore y^2 + 2\sqrt{2}y - 30 = 0$$

$$\therefore y = -7.071, 4.2426$$

\Rightarrow coordinates: $(\sqrt{2}, -4.24), (\sqrt{2}, 7.07), (-\sqrt{2}, 4.24), (-\sqrt{2}, -7.07)$

8 [Maximum mark: 7]

The curve in the diagram has equation $y = \sqrt{x^2 + 1}$. The shaded region is bounded by the curve, the y-axis and two horizontal lines.



a Find the area of the shaded region.

b Find the volume generated when the shaded region is rotated 2π radians about the y-axis.

[4]

[3]

$$(a) y = \sqrt[3]{x^2 + 1} \rightarrow x^2 + 1 = y^3 \quad | \text{y-axis bounds}$$

$$\therefore x = \pm \sqrt[3]{y^3 - 1} \quad | \text{M1}$$

$$y = \sqrt[3]{2^2 + 1} = \sqrt[3]{5} \quad | \text{A1}$$

$$y = \sqrt[3]{4^2 + 1} = \sqrt[3]{17} \quad | \text{A1}$$

$$\therefore \text{Hence, } A = \int_{\sqrt[3]{5}}^{\sqrt[3]{17}} (y^3 - 1) dy \quad | \text{M1}$$

$$\approx 168.518477 \text{ units}^2$$

$$\approx 169 \text{ units}^2 \quad \times \text{Calculation error} \quad | \text{AO}$$

$$(b) V = \pi \int_{\sqrt[3]{5}}^{\sqrt[3]{17}} (y^3 - 1) dy \quad | \text{M1}$$

$$= 16777.169415 \text{ units}^3$$

$$\approx 16800 \text{ units}^3 \quad \times \text{Calculation error} \quad | \text{AO}$$

2

5

6

9 [Maximum mark: 5]

One of the roots of the equation $x^3 - 7x^2 + bx + c = 0$ is $2 - i$. Find the value of c .

let the roots be α, β and γ
If $\alpha = 2 - i$, then $\beta = 2 + i$ ✓ A1

$$\Rightarrow \text{sum} = 2 - i + 2 + i + \gamma = -\frac{7}{1} \quad \text{M1}$$

$$\therefore 7 = 4 + \gamma$$

$$\therefore \gamma = 3 \quad \text{✓ A1} \quad \{ \text{third root} \}$$

$$\Rightarrow \text{product} = (2-i)(2+i)(3) = -c/1$$

$$\therefore -c = (4 - i^2)(3)$$

$$= (4+1)(3) \quad \text{M1}$$

$$= 15$$

$$\therefore c = -15 \quad \text{✓ A1}$$

5

5

10 [Maximum mark: 5]

The constant term in the binomial expansion of $(x^2 + \frac{1}{x})^n$ is 495. Find the value of n .

$$\Rightarrow T_{r+1} = \binom{n}{r} (x^2)^{n-r} \left(\frac{1}{x}\right)^r$$

$$= \binom{n}{r} (x^{2n-2r}) (x^{-r})$$

$$= \binom{n}{r} x^{2n-3r} \quad \text{✓ M1}$$

$$\therefore r = \frac{2}{3}n \quad \text{A1}$$

\Rightarrow constant term occurs when $2n-3r = 0$... (1)
 $\therefore n = \frac{3r}{2}$

$$\Rightarrow 495 = \binom{n}{r} \rightarrow \frac{n!}{r!(n-r)!} = 495 \quad \text{✓ M1} \quad \text{(2)}$$

$\Rightarrow (1) \rightarrow (2)$:

$$\frac{\cancel{(3r)!}}{r!} \cdot \frac{n!}{(\frac{2}{3}n)!(n-\frac{2}{3}n)!} = 495$$

$$\therefore \frac{n!}{(\frac{2}{3}n)!(\frac{1}{3}n)!} = 495$$

$\therefore n = 12$ ✓ A1 { trial & error
with the
knowledge
that $\frac{2}{3}n$
must be
divisible
by 3 }

5

Do not write solutions on this page

Section B

Answer all questions in an answer booklet. Please start each question on a new page.

11 [Maximum mark: 18]

Battery life of a certain brand of smartphone can be modelled by a normal distribution with mean μ hours and standard deviation σ hours. It is known that 5% of the batteries last less than 24 hours, while 20% last more than 72 hours.

- a i Show that $\mu + 0.8416\sigma = 72$ and find another similar equation connecting μ and σ . [7]
- ii Show that approximately 65.7% of the batteries last longer than 48 hours. [7]
- b Find the interquartile range of battery life, giving your answer to the nearest hour. [2]
- c Find the probability that, out of 20 randomly selected batteries, at least 10 last more than 48 hours. [3]
- d Given that a battery has lasted for 48 hours, what is the probability that it will last for another 24 hours? [2]
- e A customer buys a new smartphone and tests the battery. If the battery lasts less than 24 hours they return the phone with probability 0.9. If it lasts between 24 and 72 hours they return the phone with probability 0.2. Otherwise they do not return the phone.

Given that a customer keeps the phone, what is the probability that its battery lasts more than 72 hours? [4]

12 [Maximum mark: 18]

Two of the sides of a triangle have length x cm and $2x$ cm, and the angle between them is θ° .

The perimeter of the triangle is 10 cm.

- a In the case $x = 2$, find the area of the triangle. [4]
- b Explain why x must be less than $\frac{10}{3}$. [2]
- c i Show that $\cos \theta = \frac{15x - x^2 - 25}{x^2}$.
ii Sketch the graph of $y = \frac{15x - x^2 - 25}{x^2}$ for $x > 0$.
iii Hence find the range of possible values of x . [7]
- d Find the value of x for which the triangle has the largest possible area, and state the value of that area. [5]

13 [Maximum mark: 19]

Consider the differential equation $\frac{dy}{dx} = \frac{y}{x+y}$.

- a Find and simplify an expression for $\frac{d^2y}{dx^2}$ in terms of x and y . [7]
- b Show that the substitution $y = xv$ transforms this equation into $x \frac{dv}{dx} = f(v)$, where $f(v)$ is a function to be found. [4]
- c Hence find the particular solution of the equation $\frac{dy}{dx} = \frac{y}{x+y}$ for which $y = 1$ when $x = 1$. Give your answer in the form $x = g(y)$. [8]

4 PAGES / PÁGINAS

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Hodder MA P2 Set B

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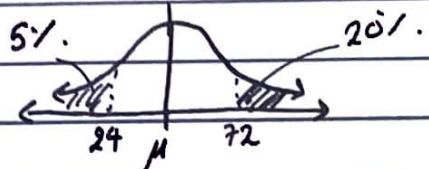
Example
Ejemplo

27

Example
Ejemplo

3

(a) (i)



$$B \sim N(\mu, \sigma^2)$$

$$\therefore P(B \leq 24) = 0.05$$

$$\therefore \frac{24 - \mu}{\sigma} = \text{invNorm}(0.05)$$

$$= -1.644854$$

$$\therefore 24 - \mu = -1.6449 \sigma$$

$$\therefore \mu - 1.6449 \sigma = 24 \quad \dots \text{eq.(1)}$$

$$P(B > 72) = 1 - P(B \leq 72) = 0.20$$

$$\therefore P(B \leq 72) = 0.80$$

$$\therefore \frac{72 - \mu}{\sigma} = \text{invNorm}(0.80)$$

$$\therefore 72 - \mu = 0.8416 \sigma$$

$$\therefore \mu + 0.8416 \sigma = 72 \quad \text{eq.(2)}$$

5

$$(ii) P(X > 48) = \text{normcdf} \quad \text{InSolve: } \mu = 55.7535$$

$$\sigma = 19.3042$$

$$\therefore P(X > 48) = \text{bnormcdf}(48, 99999, 55.8, 19.3) \quad \text{A1}$$

$$= 0.656947$$

$$\approx 0.657$$

$$\approx 65.7\% \quad \text{A1}$$

2



$$(b) IQR = \text{invNorm}\left(\frac{3}{4}, 55.75, 19.30\right) - \text{invNorm}\left(\frac{1}{4}, 55.75, 19.30\right)$$

$$= 26.035304 \quad \checkmark M1$$

≈ 26.0

$\therefore IQR \approx 26 \text{ hours} \quad \checkmark A1 \quad 2$

$$(c) X \sim B(20, 0.65)$$

$$\therefore P(X \geq 10) = \text{binomcdf}(20, 0.65, 10, 20)$$

$$= 0.999849 \quad \checkmark M1$$

≈ 0.994

$$\therefore P(X \geq 10) \approx 99.4\% \quad \checkmark A1$$

3

$$(d) P(B \geq 72) = 0.20$$

$$P(B \geq 48) = 0.657$$

$$P(B \geq 72 \cap B \geq 48) = 0.20$$

$$\therefore P(B \geq 72 \mid B \geq 48) = \frac{P(B \geq 72 \cap B \geq 48)}{P(B \geq 48)}$$

$$\Rightarrow \frac{0.20}{0.657} \quad \checkmark M1$$

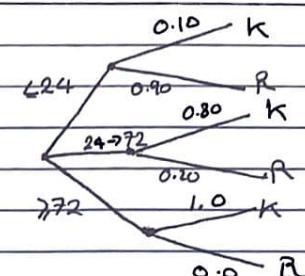
$$= 0.304414$$

≈ 0.304

$$\therefore P(B \geq 72 \mid B \geq 48) \approx 30.4\% \quad \checkmark A1$$

2

(e)



$$P(X \leq 24) = 0.05$$

$$P(24 \leq X \leq 72) = 1 - 0.05 - 0.20$$

$$= 0.75$$

$$P(X \geq 72) = \underline{0.20}$$

$\checkmark M1$

$$\begin{aligned} P(\text{keep} \mid \text{keep}) &= \frac{0.2 \times 1.0}{0.2 \times 1.0 + 0.75 \times 0.8} \quad (0.2)(1) \\ &= 0.248447 \\ &\approx 0.248 \end{aligned}$$

$\checkmark M1$

$$\therefore P(X \geq 72 \mid \text{keep}) \approx 24.8\% \quad \checkmark A1$$

4

18



04AX02

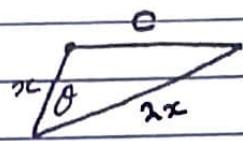


04AX03

31 2

(a)

$$3x + c = 10 \quad (\text{cm}) \dots \text{eq.(1)}$$



$$c^2 = x^2 + 4x^2 - 2x(2x) \cos \theta$$

$$\therefore (10-3x)^2 = 5x^2 - 4x^2 \cos \theta$$

$$\therefore 4x^2 \cos \theta = 5x^2 - (10-3x)^2$$

$$\therefore \cos \theta = \frac{5x^2 - (10-3x)^2}{4x^2} \dots \text{eq.(2)}$$

Using eq (2) when $x = 2$,

$$\theta = \arccos \left(\frac{5(4) + 3(2) - 10}{4(4)} \right)$$

$$\theta = \arccos \left(\frac{5(4) - (4)^2}{4(4)} \right)$$

$$\therefore \theta = \arccos \left(\frac{1}{4} \right) \checkmark M1$$

$$\begin{array}{c} 4 \\ \backslash \\ 1 \end{array} \sqrt{15} \rightarrow \sin \theta = \frac{\sqrt{15}}{4} \checkmark M1$$

$$\begin{aligned} \text{Hence, Area} &= \frac{1}{2}(x)(2x) \sin \theta \\ &= \frac{1}{2}(2)(2^2) \cancel{\sin \frac{\sqrt{15}}{4}} \times \text{MOAO} \\ &= 3.295571 \\ &\approx 3.30 \text{ cm}^2 \end{aligned}$$

2



04AX04

4 PAGES / PÁGINAS

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Hodder MA P2 Set B

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Example Ejemplo 27

2	7
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Example Ejemplo 3

	3
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1 2

$$(b) \text{ As } \text{perimeter} = x + 2x + c = 10, \\ 3x + c = 10$$

~~c > 0~~ C70
M1

$\Rightarrow x$ will be greatest when $c=0$:

$$3x + 0 = 10$$

$$\therefore x = 10/3 \quad \cancel{\text{is the}}$$

$$\therefore x \leq 10/3$$

\Rightarrow however, as $c > 0$, $x < 10/3$ ✓ A1

\Rightarrow Hence, x must be less than $10/3$

\Rightarrow a simple $10 - 3x \neq 0$
could suffice

(c)(i) from part (a), eq(1): $c = 10 - 3x$

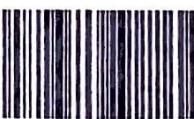
$$\text{eq (2): } \cos \theta = \frac{5x^2 - (10 - 3x)^2}{4x^2}$$

$$= \frac{5x^2 - (100 - 60x + 9x^2)}{4x^2}$$

$$= \frac{-4x^2 + 60x - 100}{4x^2}$$

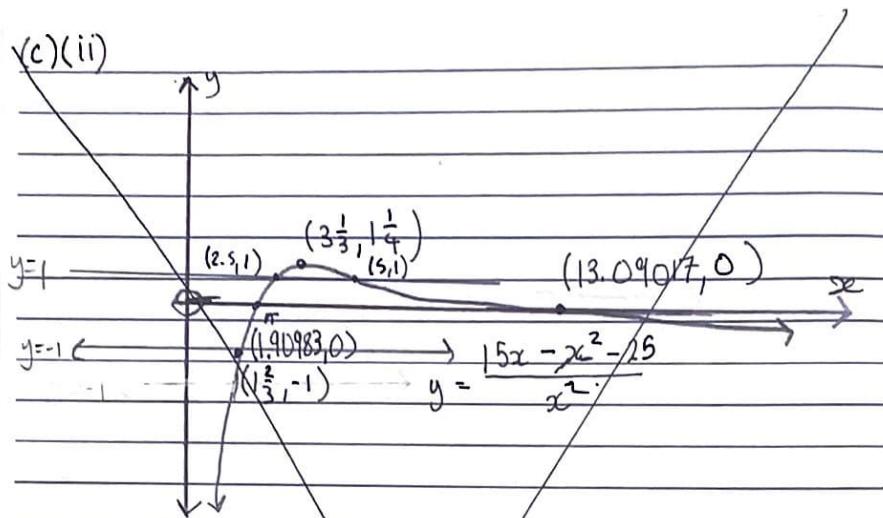
$$\therefore \cos \theta = \frac{16x - x^2 - 25}{x^2} \quad \checkmark \text{A1}$$

2



04AX01

(c)(ii)



(iii) As $|\cos\theta| \leq 1$, $-1 \leq y \leq 1$

$$\text{Hence } -1 \leq \frac{15x - x^2 - 25}{x^2} \leq 1$$

∴ from graph and intersections in part (c)(ii),

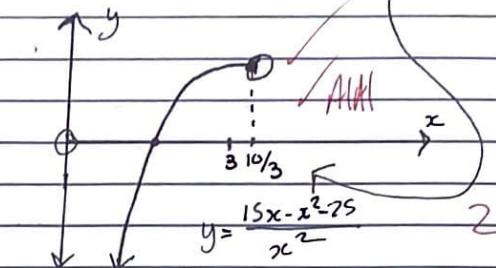
$$\begin{cases} \frac{5}{3} \leq x \leq 2\frac{1}{7} \text{ and } x \geq 1\frac{1}{7} & \{\text{asymptotic}\} \\ \frac{5}{3} \leq x \leq 5/2 \text{ and } x \geq 5/4 \end{cases}$$

$$\lim_{x \rightarrow \infty} \left(\frac{15x - x^2 - 25}{x^2} \right) = -1$$

Hence, as $y \rightarrow x \rightarrow \infty$,
 $y \rightarrow -1$, but never
crosses. Hence ~~less~~
 y will be valid as $x \rightarrow \infty$

(f) (c)(ii)

$$x < 10/3.$$



$$(c)(iii) \text{ GAC: } \begin{array}{c} \text{A} \\ \text{M} \\ \text{B} \\ \text{C} \end{array} \quad \begin{array}{c} y=1 \\ y=-1 \end{array} \quad \begin{array}{c} \text{M} \\ \text{A} \\ \text{B} \\ \text{C} \end{array}$$

Mence, as $|\cos\theta| \leq 1$,
Using the graph,
 $|y| \leq 1$
∴ therefore, $\frac{5}{3} \leq x \leq \frac{5}{2}$
 $\checkmark \times \text{AO}$

(d) on following bracket.

θ cannot be 90°
in a triangle.

1 3

$$(a) \frac{dy}{dx} = \frac{y}{x+y}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{(x+y)\frac{dy}{dx} - y(1 + \frac{dy}{dx})}{(x+y)^2}$$

$$= \frac{\frac{dy}{dx}x + \frac{dy}{dx}y - y - y\frac{dy}{dx}}{x^2 + 2xy + y^2}$$

$$= \frac{\frac{dy}{dx}x - y}{x^2 + 2xy + y^2}$$

$$= \frac{\frac{y}{x+y}x - y}{x^2 + 2xy + y^2}$$

$$= \frac{xy - y(x+y)}{(x+y)^3}$$

$$= \frac{xy - xy - y^2}{(x+y)^3}$$

$$= -\frac{y^2}{(x+y)^3}$$

(b)

$$\frac{dy}{dx} = \frac{y}{x+y}$$

[let $y = ux$ ~~then~~ $\therefore \frac{dy}{dx} = x\frac{du}{dx} + u$] $\rightarrow x\frac{du}{dx} - u = \frac{ux}{x+ux} \checkmark M$
~~x~~ $\therefore x\frac{du}{dx} = \frac{u}{1+u} + u \checkmark M$

$$\therefore x\frac{du}{dx} = 2u + 1 \quad \text{XAO}$$

Hence, $x\frac{du}{dx} = f(u)$, where $f(u) = 2u + 1$

2

(c) $\frac{dx}{dx} = \frac{1}{2u+1} \frac{du}{dx} = \frac{1}{2u+1}$ $\checkmark M$

$$\therefore \int \frac{1}{2u+1} du = \int \frac{1}{x} dx$$

$$\therefore \ln|2u+1| = \ln|x| + \ln(e^c) \ln(\pm e^c)$$

$$\therefore \ln|2u+1| = \ln|x \pm e^c| \checkmark M$$

$$= \ln|Ax| \quad \{ A = \pm e^c \}$$

$$\therefore 2u+1 = Ax$$

$$\therefore \frac{y}{x} + 1 = Ax$$

$$\therefore y = Ax^2 - x$$

3

when $y=1, x=1 \checkmark M$ $1 = A - 1$
 $\therefore A = 2$

Hence, $y = 2x^2 - x$

12





4 PAGES / PÁGINAS

Candidate session number: / Numéro de session du candidat: / Número de convocatoria del alumno:

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Candidate name: / Nom du candidat: / Nombre del alumno:

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At the start of each answer to a question, write the question number in the box using your normal hand writing / Avant de répondre à une question, inscrivez son numéro à la main dans la case appropriée / Al comienzo de cada respuesta, escriba a mano el número de pregunta en la casilla.

Example
Ejemplo

27

2	7
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Example
Ejemplo

3

	3
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1 2

(d)

$$\text{Area} = \frac{1}{2} \pi(2x) \sin \theta$$

$$= x^2 \sin \theta$$

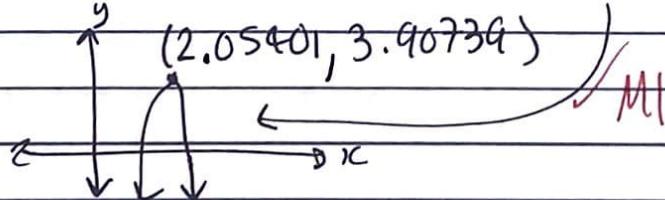
$$\sin \theta = \cos(90 - \theta)$$

$$\cos \theta = \frac{15x - x^2 - 2s}{x^2}$$

$$\therefore \theta = \arccos \left(\frac{15x - x^2 - 2s}{x^2} \right)$$

$$\therefore \sin \theta = \sin \left(\arccos \left(\frac{15x - x^2 - 2s}{x^2} \right) \right) \checkmark \text{M1}$$

$$\therefore \text{Area} = x^2 \sin \left(\arccos \left(\frac{15x - x^2 - 2s}{x^2} \right) \right) \checkmark \text{M1}$$



$$\therefore x = 2.05401 \text{ cm} \quad \text{when } A = 3.90739 \text{ cm}^2 \checkmark \text{A1}$$

(15)

5

