# Mathematics: analysis and approaches
# Higher level
# Paper 1 Practice Set C (Hodder)

Candidate session number

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2 hours

**Instructions to candidates**

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in an answer booklet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A copy of the mathematics: analysis and approaches formula book is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

## Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

**1** *[Maximum mark: 6]*

Let $f(x) = 2x^2 + 10x + 7$, $x \in \mathbb{R}$.

**a** Find the largest possible domain of the form $x \leqslant k$ for which the inverse function, $f^{-1}$, exists. *[2]*

**b** For the value of $k$ from part **a**, find the inverse function $f^{-1}(x)$, stating its domain. *[4]*

**2** *[Maximum mark: 6]*

Let $z = 3 - 2\mathrm{i}$ and $w = -1 + \mathrm{i}$.

**a** Represent $z$ and $w$ on an Argand diagram. *[2]*

**b** Find $\dfrac{w}{z}$ in the form $a + b\mathrm{i}$. *[2]*

**c** Find the real numbers $p$ and $q$ such that $pz + qw = 6$. *[2]*

**3** *[Maximum mark: 5]*

Solve the inequality $|2x+1| < |x-3|$.

**4** *[Maximum mark: 5]*

Find the set of values of $k$ for which the function $f(x) = x^3 + kx^2 + kx - 2$ is strictly increasing for all $x \in \mathbb{R}$.

**5** *[Maximum mark: 6]*
Evaluate

$$\int_1^6 \frac{3x - 16}{3x^2 + 10x - 8}\,\mathrm{d}x$$

Give your answer in the form $\ln k$.

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**6** *[Maximum mark: 5]*

**a** Use Maclaurin series to find constant $a$ such that $\frac{1}{10}\sin 3x \approx ax$ when $x \approx 0$. *[2]*

**b** Hence find the approximate solutions of the equation $\frac{1}{10}\sin 3x = x^2$. *[3]*

**7** *[Maximum mark: 5]*

The sum of the first two terms of a geometric series is 3 and its sum to infinity is 5.

Given that all terms of the series are positive, find the common ratio of the series.

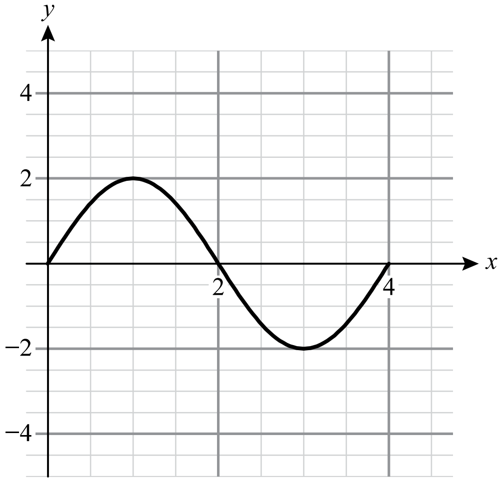
**8** *[Maximum mark: 7]*

Solve the equation

$$\log_4(3-2x)=\log_{16}(6x^2-5x+12).$$

**9** *[Maximum mark: 5]*

The graph of $y = \mathrm{f}(x)$ is shown in the diagram. The domain of f is $0 \leqslant x \leqslant 4$.




**a** On the same grid, sketch the graph of $y = [\mathrm{f}(x)]^2$. *[3]*

**b** Find the domain and range of the function $\mathrm{g}(x) = 2\mathrm{f}(x - 1)$. *[2]*

**10** *[Maximum mark: 5]*

Let $y = \arcsin x$.

**a** Express $\arccos x$ in terms of $y$. *[3]*

**b** Hence show that $\arcsin x + \arccos x \equiv k$, where $k$ is a constant to be found. *[2]*

## Section B

Answer **all** questions in an answer booklet. Please start each question on a new page.

**11** *[Maximum mark: 18]*

**a** Points $A$, $B$ and $D$ have coordinates $A(1, -4, 3)$, $B(2, 1, -1)$ and $D(-1, 3, 3)$.

**i** Find the equation of the line $l_1$ through $A$ and $B$.

**ii** Write down the equation of the line $l_2$ which passes through $D$ and is parallel to $AB$. *[5]*

**b** **i** Find the exact distance $AB$.

**ii** Find the coordinates of two possible points $C$ on the line $l_2$ such that $CD = 2AB$.

**iii** Denote the two possible points $C$ by $C_1$ and $C_2$. Determine whether angle $C_1AC_2$ is acute, right or obtuse. *[8]*

**c** **i** Find $\overrightarrow{AB} \times \overrightarrow{AD}$.

**ii** Hence find the equation of the plane containing the points $A$, $B$ and $D$. *[5]*

**12** *[Maximum mark: 16]*

**a** Use compound angle identities to express $\cos 3\theta$ in terms of $\cos \theta$. *[4]*

**b** Consider the equation $8x^3 - 6x + 1 = 0$.

**i** Given that $x = \cos \theta$, for $0 \leqslant \theta \leqslant \pi$, find the value of $\cos 3\theta$.

**ii** Hence find the possible values of $x$ and show that they are all distinct. *[7]*

**c** Show that $8 \cos\left(\frac{2\pi}{9}\right) \cos\left(\frac{4\pi}{9}\right) = -\sec\left(\frac{8\pi}{9}\right)$. *[3]*

**d** State, with a reason, the value of $\cos\left(\frac{2\pi}{9}\right) + \cos\left(\frac{4\pi}{9}\right) + \cos\left(\frac{8\pi}{9}\right)$. *[2]*

**13** *[Maximum mark: 21]*

Let $f(x) = \dfrac{x}{1 + x^2}$ for $x \in \mathbb{R}$.

**a** Determine algebraically whether f is an even function, an odd function or neither. *[3]*

The continuous random variable $X$ has probability density function given by

$$g(x) = \begin{cases} \dfrac{kx}{1+x^2} & \text{for } 0 \leqslant x \leqslant \sqrt{3} \\ 0 & \text{otherwise} \end{cases}.$$

**b** Show that $k = \dfrac{1}{\ln 2}$. *[4]*

**c** Find the median of $X$. *[4]*

**d** Find the mode of $X$. *[5]*

**e** Find the mean of $X$. *[5]*