



Candidate session number

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**Mathematics  
Higher Level  
Paper 2**

August 2020

2 hours

~~1:40:00~~ 1:32:00

**Instructions to candidates**

- Write your session number in the boxes above
- Do not open this examination paper until instructed to do so.
- A graphics display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklets provided. Start each question in a new booklet
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correctly to three significant figures.
- The maximum mark for this examination paper is **[100 marks]**.

83

87 73

$$\frac{47}{73} = 64\%$$

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

### Section A

Answer all questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

Two dice are rolled four times and the sums of the scores were: 2, 5, 6, 9. The dice are then rolled twice more. If the new mean score sum was 7 and the standard deviation of the score sum was 3. Find the values of the last two score sums.

$$\frac{2+5+6+9+x+y}{6} = 7 \rightarrow x+y = 20 \quad (1)$$

~~$$\sqrt{\frac{2+5+6+9+x+y}{6}} = 7 \rightarrow 3$$~~

$$3^2 = \frac{(7-2)^2 + (7-5)^2 + (7-6)^2 + (7-9)^2 + (7-x)^2 + (7-y)^2}{6}$$

$$= \frac{34 + (7-x)^2 + (7-y)^2}{6} \quad (2)$$

(1)  $\rightarrow$  (2) :

$$9 = \frac{(34 + (7-x)^2 + (7-(20-x))^2)}{6}$$

$$= \frac{(34 + 49 - 14x + x^2 + (x-13)^2)}{6}$$

$$\therefore 56 = 34 + 49 + 169 + 2x^2 - 40x$$

$$\therefore 0 = 2x^2 - 40x + 198$$

$$\begin{aligned} \therefore x &= 9, & x &= 11 \\ y &= 11 & \text{by } y = 9 \end{aligned} \quad \left. \begin{array}{l} \text{two numbers are 9 and 11} \end{array} \right\}$$

[Maximum mark: 4]

Find the values of  $m$  such that this system of equations has no unique solution

$$mx + 2y = 1$$

$$4x + (m+2)y = 4$$

$$mx + 2y = 1 \rightarrow y = \frac{1-mx}{2}$$

Attempt 1:

$$\therefore 4x + (m+2)\left(\frac{1-mx}{2}\right) = 4$$

$$\therefore 4x + \frac{1}{2}(m+2)(1-mx) = 4$$

$$\therefore 4x + \frac{1}{2}(m - m^2x + 2 - 2mx) = 4$$

$$\therefore 4x + \frac{1}{2}m - \frac{1}{2}m^2x + 1 - mx = 4$$

$$\therefore 8x + m - m^2x + 2 - 2mx = 8$$

$$\therefore -m^2x - 2mx + m + 8x - 8 = 0$$

$$\therefore m^2x + 2mx - m - 8x + 6 = 0$$

$$\begin{array}{l} mx + 2y = 1 \\ (4x + (m+2)y = 4) + \end{array}$$

$$(M+4)x + (2+m+2)y = 5$$

$$\therefore (M+4)x + (4+m)y = 5$$

Attempt 2:

$$mx + 2y = 1$$

$$\cancel{4x + (m+2)y = 4} -$$

$$(m-4)x + (m)y = -3$$

$$\therefore m=4, M=0$$

NO solutions when parallel.

$$mx + 2y = 1$$

$$\therefore y = -\frac{1}{2}mx + \frac{1}{2}$$

$$4x + (m+2)y = 4$$

$$\therefore y = \frac{-1}{m+2}4x + \frac{4}{m+2}$$

$$\therefore -\frac{1}{2}m = -\frac{4}{m+2}$$

$$\therefore -m = -\frac{8}{m+2}$$

$$\therefore -m(m+2) = -8$$

$$\therefore m^2 + 2m = 8$$

$$\therefore m^2 + 2m - 8 = 0$$

$$\therefore m = -4, m = 2$$

O

## 3. [Maximum mark: 6]

Let  $p(z) = z^4 + az^3 + bz^2 + cz + d$  be a polynomial with real coefficients. Given that  $z = 3+i$  and  $z = \sqrt{2} \operatorname{cis} \left(-\frac{\pi}{4}\right)$  are two complex zeros of  $p(z)$ , find the values of the real coefficients.

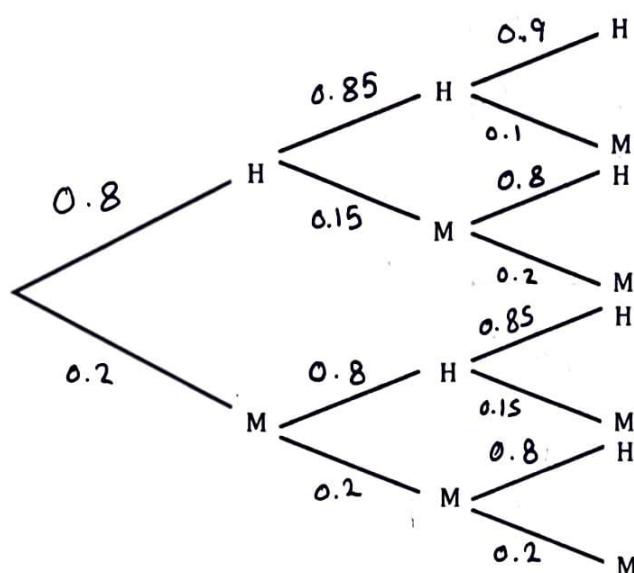
$$\begin{aligned}
 & (\cancel{z - (3+i)}) \quad \sqrt{2} \operatorname{cis} \left(-\frac{\pi}{4}\right) = \sqrt{2} \left(\frac{1}{\sqrt{2}} - i \frac{1}{\sqrt{2}}\right) = 1 - i \\
 & \therefore (3+i)(1-i) \cancel{(\cancel{z - (3+i)})} \cancel{(\cancel{z - (1-i)})} \cancel{(\cancel{z - (3-i)})} \cancel{(\cancel{z - (1+i)})} \\
 & = (\cancel{z^2 - 2(3+i)}) \\
 & = ((z-3)-i)((z-3)+i)((z-(1)+i)((z-(1)-i)) \\
 & = [(z-3)^2 - i^2] [ (z-1)^2 - i^2 ] \\
 & = [z^2 - 6z + 9 + 1] [ z^2 - 2z + 1 + 1 ] \\
 & = (z^2 - 6z + 10)(z^2 - 2z + 2) \\
 & = z^4 - 2z^3 + 2z^2 \\
 & \quad - 6z^3 + 12z^2 - 12z \\
 & \quad + 10z^2 - 20z + 20
 \end{aligned}$$

$$= z^4 - 8z^3 + 24z^2 - 32z + 20$$

$$\therefore a = -8; \quad b = 24; \quad c = -32; \quad d = 20$$

Klay attempts three free throws in a basketball game. The probability that he hits (H) his first free throw is 0.8. Whenever he hits a free throw, his confidence increases so that the probability of him making the next free throw increases by 0.05. Whenever he misses (M) a free throw the probability of him making the next one is 0.8.

- a) Complete the given probability tree diagram for Klay's three attempts, labelling each branch with the correct probability.



- b) Calculate the probability that he hits at least two free throws.  
 c) Calculate the probability that he hits his last (third) free throw, given that he hit exactly one previous free throw.

b)  $P(\text{at least 2 hits}) = P(HHM) + P(HMN) + P(MHH) + P(HMM)$

$$= (0.8 \times 0.85 \times 0.1) + (0.8 \times 0.15 \times 0.8) + (0.2 \times 0.8 \times 0.85) + (0.8 \times 0.85 \times 0.9)$$

$$= 0.912$$

c)  $P(\text{third } | \text{ exactly 1 previous}) = P(MMH | HM) + P(MMH | MM)$

$$= \frac{0.2 \times 0.8 \times 0.85}{0.8 \times 0.15} + \frac{0.8 \times 0.15 \times 0.8}{0.8 \times 0.85}$$

## 5. [Maximum mark: 5]

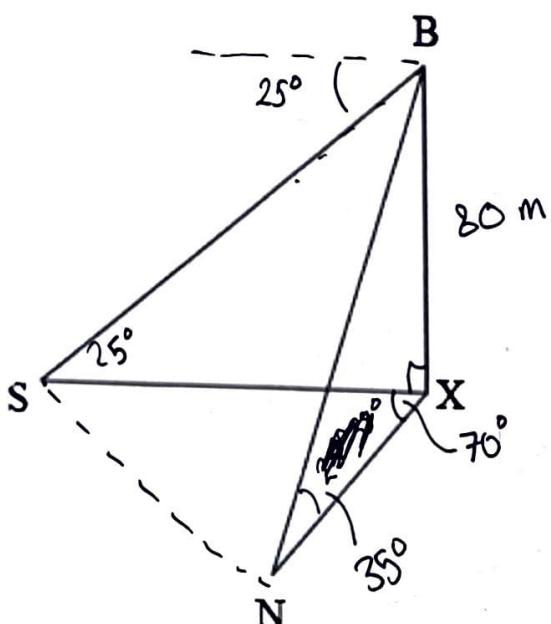
Brett is at the top of a cliff, standing 80 m above sea level, and observes two yachts in the sea.

"Seaview" (S) is at an angle of depression of  $25^\circ$ .

"Nauti Buoy" (N) is at an angle of depression of  $35^\circ$ .

The following three dimensional diagram shows Brett and the two yachts at S and N.

X lies at the foot of the cliff and angle SXN =  $70^\circ$ .



Find, to 3 significant figures, the distance between the two yachts.

$$(1) \tan 25 = \frac{80}{SX} \rightarrow \cancel{\text{X}}$$

$$\therefore SX \approx 171.561 \text{ m}$$

$$(2) \tan 35 = \frac{80}{NX} \checkmark$$

$$\therefore NX \approx 144.252 \text{ m}$$

$$(3) \text{ By } [SN]^2 = [SX]^2 + [NX]^2 - 2[SX][NX] \cos 70 \checkmark$$

$$\therefore [SN] = \sqrt{171.561^2 + 144.252^2 - 2(171.561)(144.252) \cos 70}$$

$$\approx 182.519 \text{ m}$$

$$\approx 183 \text{ m } \times$$

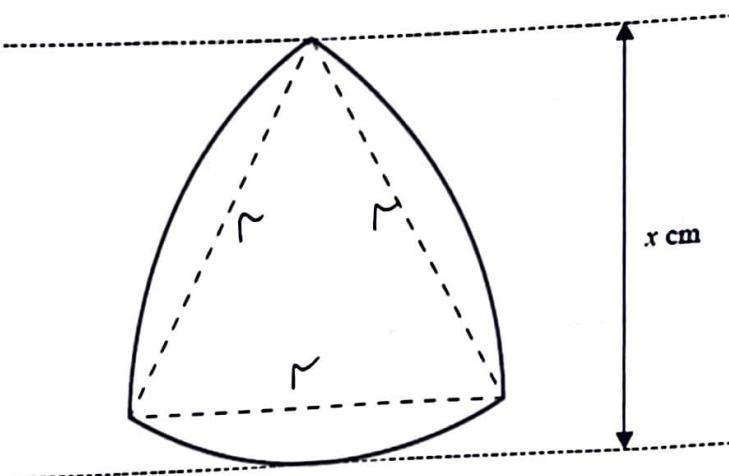
CAREFUL WITH  
ROUNDING  
→ KEEP IN FRACTION  
FORM

(4)

(Maximum mark: 7)

The following shape consists of three arcs of a circle, each with centre at the opposite vertex of an equilateral triangle as shown in the diagram.

diagram not to scale



Given that the perimeter and area have the same value (in cm and  $\text{cm}^2$  respectively), find  $x$  to 2 decimal places. [7]

$$\begin{aligned}\text{Perimeter} &= 3r\theta \\ &= 3r(\pi/3) \\ &= \pi r\end{aligned}$$

$$\begin{aligned}\text{Area} &= 3 \times \text{area of sector} - 2 \times \text{area of } \triangle \\ &= 3 \times \frac{1}{2}r^2\theta - 2 \times \frac{1}{2}r^2 \sin\theta \\ &= \frac{3}{2}r^2\theta - r^2 \sin\theta \\ &= r^2 \left[ \frac{3}{2}\theta - \sin\theta \right] \\ &= r^2 \left[ \frac{3}{2} \times \frac{\pi}{3} - \sin \frac{\pi}{3} \right] \\ &= r^2 \left[ \frac{\pi}{2} - \frac{\sqrt{3}}{2} \right]\end{aligned}$$

$$\text{Perimeter} = \text{Area}$$

$$\pi r = r^2 \left[ \frac{\pi - \sqrt{3}}{2} \right]$$

$$\pi = \frac{1}{2}r(\pi - \sqrt{3})$$

$$r = \frac{(2\pi)}{(\pi - \sqrt{3})}$$

$$r \approx 4.46$$

$$x \approx 4.46 \text{ cm}$$

$$\begin{array}{l} \sqrt{3} \\ \diagdown \\ 1 \\ \diagup \end{array} \quad \pi/3$$

7

## 7. [Maximum mark: 7]

Consider the geometric sequence  $\ln b, \ln \sqrt{b}, \ln \sqrt[4]{b}, \ln \sqrt[8]{b}, \dots$  where  $b > 1$ .

a) Show that the series  $\ln b + \ln \sqrt{b} + \ln \sqrt[4]{b} + \ln \sqrt[8]{b} + \dots$  converges and find its sum. [3]

b) A data set consists of the first  $n$  terms of the sequence. Find an expression for the mean of the data set in terms of  $n$  and  $\ln b$ . [2]

c) Hence find the least value of  $n$  for which the mean is less than 5% of the first term of the sequence. [2]

$$a) r = \frac{\ln \sqrt{b}}{\ln b}$$

$$= \frac{1}{2} \times \frac{\ln b}{\ln b}$$

$$= \frac{1}{2}$$

$\therefore |r| < 1$  as  $\frac{1}{2} < 1$   $\therefore$  converges

$$\therefore S_{\infty} = \frac{u_1}{1-r}$$

$$= \frac{\ln b}{1 - \frac{1}{2}}$$

$$= \frac{\ln b}{\frac{1}{2}}$$

$$= 2 \ln b$$

b)

3/

- a) Given that the coefficients of  $x^{r-1}, x^r, x^{r+1}$  in the expansion of  $(1+x)^n$  are in an arithmetic sequence, show that,  $n^2 + 4r^2 - 2 - n(4r+1) = 0$ . [6]
- b) Hence find three consecutive coefficients of the expansion of  $(1+x)^{14}$  which form an arithmetic sequence. [2]

$$\text{a). } (1+x)^n = 1 + {}^n C_1 x + \cdots + {}^n C_{r-1} x^{r-1} + {}^n C_r x^r + {}^n C_{r+1} x^{r+1} + \cdots + {}^n C_n x^n$$

$$\therefore \text{the coefficients are: } x^{r-1} : {}^n C_{r-1} = \frac{n!}{(r-1)!(n-(r-1))!} \\ = \frac{n!}{(r-1)!(n-r)!}$$

$$1 + n x + \cancel{x^2} + \cdots + \cancel{\frac{n(n-1)}{(r-1)!(n-r)!}}$$

$$x^r : {}^n C_r = \frac{n!}{r!(n-r)!}$$

$$x^{r+1} : {}^n C_{r+1} = \frac{n!}{(r+1)!(n-r-1)!}$$

Completed in answer booklet.

Do not write solutions on the following pages.

## Section B

Answer all questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 17]

A survey is conducted in a large office building. It is found that 28% of the office workers weigh less than 70 kg and that 15% of the office workers weigh more than 92.5 kg.

The weights of the office workers may be modelled by a normal distribution with mean  $\mu$  and standard deviation  $\sigma$ .

- a) (i) Determine two simultaneous linear equations satisfied by  $\mu$  and  $\sigma$ . [3]
- (ii) Find the values of  $\mu$  and  $\sigma$ . [3]

- b) Find the probability that an office worker weighs more than 100 kg. [1]

There are elevators in the office building that take the office workers to their offices.

- c) Given that there are 10 workers in a particular elevator, find the probability that at least four of the workers weigh more than 100 kg. [2]
- d) Given that there are 10 workers in an elevator and at least one weighs more than 100 kg, find the probability that there are fewer than four workers exceeding 100 kg. [3]

The arrival of the elevators at the ground floor between 08:00 and 09:00 can be modelled by a Poisson distribution. Elevators arrive on average every 36 seconds.

- e) Find the probability that in any half hour period between 08:00 and 09:00 more than 60 elevators arrive at the ground floor. [3]

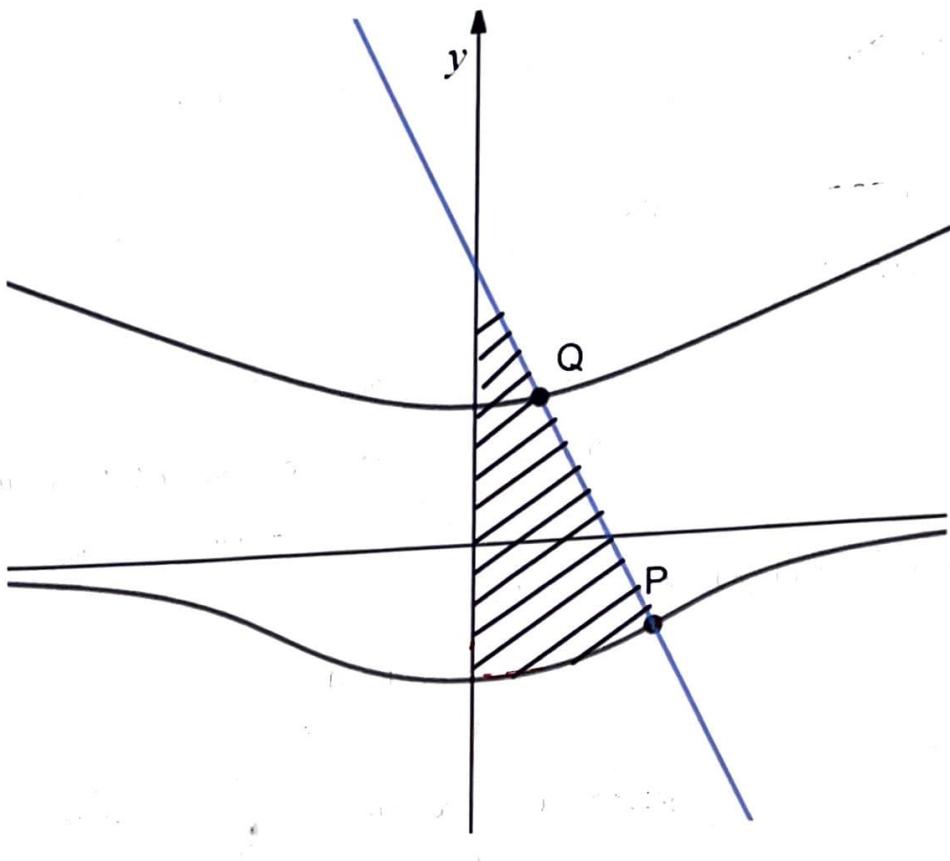
An elevator can take a maximum of 10 workers. Given that 400 workers arrive in a half hour period independently of each other,

- f) find the probability that there are sufficient elevators to take them to their offices. [2]

not write solutions on the following pages.

1. [Maximum mark: 21]

The following graph shows the two parts of the curve defined by the equation  $x^2y = y^4 - 5$ , and the normal to the curve at the point P(2, -1).



- (a) Show that there are exactly two points on the curve where the gradient is zero. [7]

- (b) Find the equation of the normal to the curve at the point P. [5]

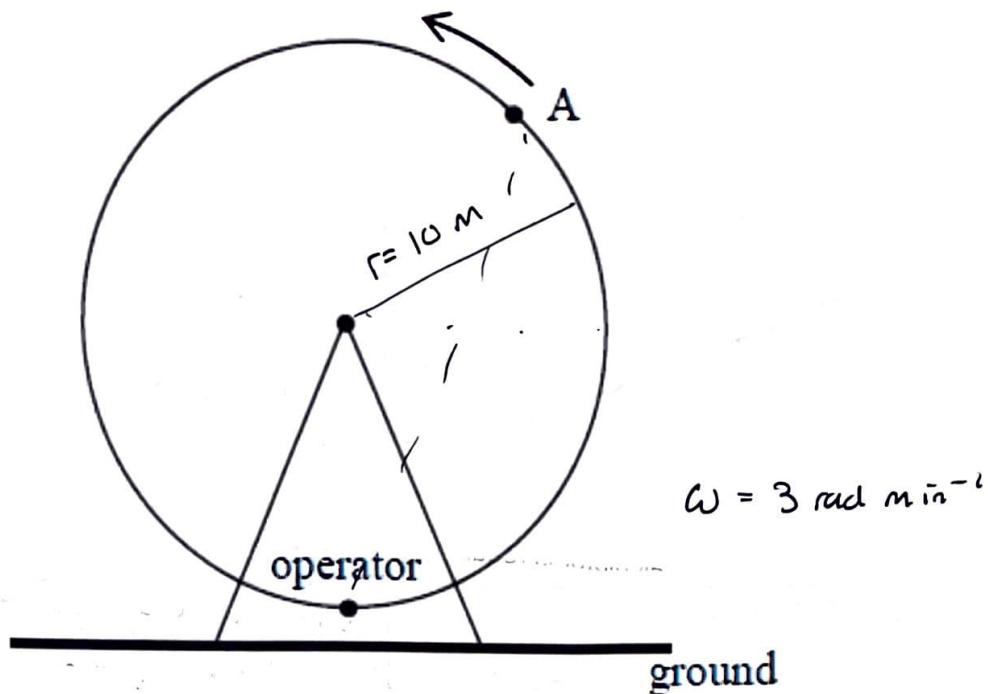
- (c) The normal at P cuts the curve again at the point Q. Find the x-coordinate of Q. [3]

- (d) The shaded region is rotated by about the y-axis. Find the volume of the solid formed. [6]

Do not write solutions on the following pages.

**12. [Maximum mark: 12]**

Below is a sketch of a Ferris wheel, an amusement park ride carrying passengers around the rim of the wheel.



- a) The circular Ferris wheel has a radius of 10 metres and is revolving at a rate of 3 radians per minute. Determine how fast a passenger (in metres per minute) on the wheel is going vertically upwards when the passenger is at point A, 6 metres higher than the centre of the wheel, and is rising. [6]
- b) The operator of the Ferris wheel stands directly below the centre such that the bottom of the Ferris wheel is level with his eyeline. As he watches the passenger his line of sight makes an angle  $\alpha$  with the horizontal. Find the rate of change of  $\alpha$  at point A (in radians per minute). [3]

**End of Paper Two**



$$\frac{47}{73} = 64\%$$

Candidate session number: / Numéro de session du candidat: / Número de convocatoria del alumno:

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Candidate name: / Nom du candidat: / Nombre del alumno:

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Please write question numbers in the following format: / Veuillez numérotter les questions en utilisant la présentation suivante: / Sírvase escribir los números de las preguntas en el siguiente formato:

1      2      3      4      5      6      7      8      9      10

a)  $x^2y = y^4 - 5$

$$\therefore 2xy + x^2 \frac{dy}{dx} = 4y^3 \frac{dy}{dx}$$

$$\therefore x^2 \frac{dy}{dx} - 4y^3 \frac{dy}{dx} = -2xy$$

$$\therefore \frac{dy}{dx} (x^2 - 4y^3) = -2xy$$

$$\therefore \frac{dy}{dx} = -2xy / (x^2 - 4y^3)$$

If  $\frac{dy}{dx} = 0$ , then  $-2xy = 0$

$$\therefore x = 0 \quad \{ \text{as } y \neq 0 \}$$

When  $x=0$ ,  $0^2y = y^4 - 5$

$$\therefore y^4 = 5$$

$$\therefore y = \pm \sqrt[4]{5}$$

(7)

∴ two points: ~~(0, 4)~~  $(0, 4\sqrt{5})$  and  $(0, -4\sqrt{5})$

b) at  $P(2, -1)$ ,  $\frac{dy}{dx} = \frac{-2xy}{x^2 - 4y^3}$

$$= \frac{-2(2)(-1)}{2^2 - 4(-1)^3}$$

$$= 1/2$$

$$\therefore m_{PA} = -1/m = -2$$

continued on following page

7/

∴ If  $m_L = -2$  and  $P(2, -1)$

$$\begin{aligned}y - (-1) &= -2(x - 2) \quad \checkmark \\∴ y &= -2x + 4 + 1 \\∴ y &= -2x + 3 \quad \checkmark\end{aligned}$$

(5)

c)  $-2x + 3 = y$

$$\begin{aligned}\therefore 2x &= 3 - y \\∴ x &= \frac{3-y}{2} \dots (1)\end{aligned}$$

Sub (1) into the equation:

$$\left(\frac{3-y}{2}\right)y = y^4 - 5 \quad \checkmark$$

$$\therefore \frac{1}{4}(9-6y+y^2)y = y^4 - 5$$

$$\therefore (9-6y+y^2)y = 4y^4 - 20$$

$$\therefore 9y - 6y^2 + y^3 = 4y^4 - 20$$

$$\therefore 0 = 4y^4 - y^3 + 6y^2 - 9y - 20$$

$$\therefore \cancel{y = 1.55275}$$

$$\therefore \text{at } y = 1.55275, x = \frac{3 - 1.55275}{2}$$

$$\begin{aligned}&= 0.723625 \\&\{x\text{-coord of } P\}\end{aligned}$$

8/

$$d) V = \pi \int_a^b x^2 dy$$

$$y = -2x + 3 \rightarrow$$

$\downarrow$   
y-int at  $y=3 \dots (1)$

inverse:

$$x = -2y + 3$$

$$\therefore 2y = 3 - x$$

$$\therefore y = (3 - x)/2$$

$$\therefore f^{-1}(x) = (3 - x)/2 \dots (2)$$

$$y \cancel{x} = -2x + 3$$

$$\therefore 2x = 3 - y$$

$$\therefore x = (3 - y)/2 \dots (2)$$

$$x^2 y = y^4 - 5$$

$$\therefore x^2 = y^3 - 5y^{-1}$$

$$\therefore x = \sqrt{y^3 - 5y^{-1}} \dots (3)$$

(1)  $\rightarrow$  (2)  $\rightarrow$  (3)

$$\therefore V = \pi \int_{-1}^3 \left( \frac{3-y}{2} \right)^2 dy + \pi \int_{-4\sqrt{5}}^{-1} \left( \sqrt{y^3 - 5y^{-1}} \right)^2 dy$$

$$= 19.9338 \quad \{\text{G.D.C}\} \quad \text{part (a)}$$

$$\approx 19.9 \text{ units}^3$$

~~3~~

⑥

6

a) let the ~~origin~~ origin be  $x=0$ .

$$f(x) = a \sin(b(x-c)) + d$$

$$\rightarrow a = \text{amplitude} = 10 \text{ m}$$

$$\frac{2\pi}{T} \leftarrow$$

$\rightarrow$  In a minute,  $\frac{3}{2}\pi$  revolutions

$$\therefore T = \frac{2\pi}{\frac{3}{2}\pi}$$

$$\therefore T = \frac{4}{3}$$

$$\therefore f = \frac{2\pi}{\frac{4}{3}}$$

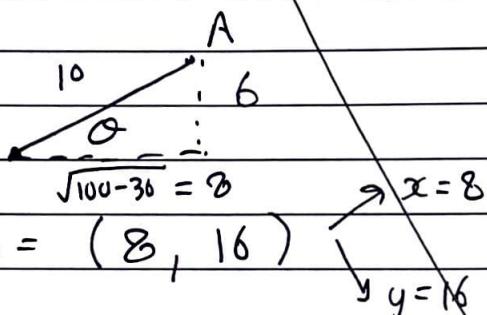
$$= 3$$

$$= b$$

$\rightarrow d, c = 0$ :

$$\therefore f(x) = 10 \sin(3x) - 10 \cos(3x)$$

at A:



$$\therefore A = (8, 6)$$

4 PAGES / PÁGINAS

Candidate session number: / Numéro de session du candidat: / Número de convocatoria del alumno:

20 T R L P Z - M A H L

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1 2 3 4 5 6 7 8 9 10

a) Attempting to model ~~to~~ vertical height using a cos wave:

$$f(x) = a \cos(bx) \quad \{c, d = 0\}$$

$$\rightarrow a = -10 \quad \{\text{begin below origin}\}$$

$$\rightarrow b = 2\pi/3$$

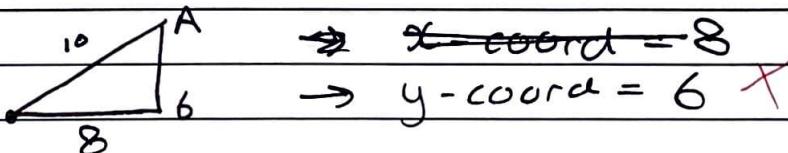
$$\therefore f(x) = -10 \cos\left(\frac{2\pi}{3}x\right) \quad \times$$

$$\therefore f'(x) = 10 \sin\left(\frac{2\pi}{3}x\right) \cdot \frac{2\pi}{3} \quad \checkmark$$

$$\therefore f'(x) = -10 \sin\left(\frac{2\pi}{3}x\right) \cdot \frac{2\pi}{3}$$

$$= -\frac{20\pi}{3} \sin\left(\frac{2\pi}{3}x\right) \quad \{\text{speed}\}$$

finding the ~~xy~~ coordinate of point A:



$$\rightarrow x\text{-coord} = 8$$

$$\rightarrow y\text{-coord} = 6 \quad \times$$

at  $y = 6$ ,  $x = 1.94 \quad \{\text{time}\} \quad \times$

GDC:

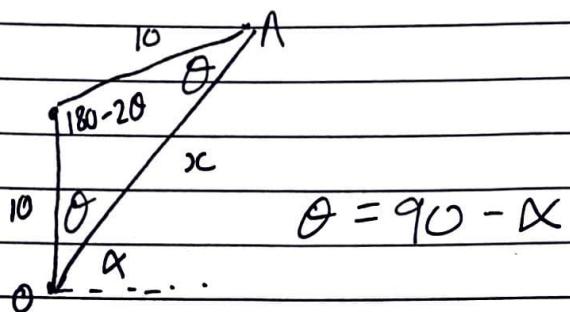
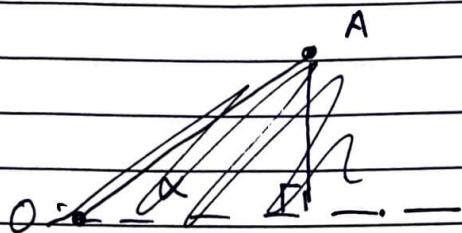


21

at  $t = 1.94$  at  $x = 1.94$ ,

$$\begin{aligned}f'(1.94) &= -\frac{20\pi}{3} \sin\left(\frac{2\pi}{3} \times 1.94\right) \\&= 16.6825 \text{ m min}^{-1} \\&\approx 16.7 \text{ m min}^{-1}\end{aligned}$$

b)



$$\begin{aligned}180 - 2\theta &= 180 - 2(90 - \alpha) \\&= 180 - 180 + 2\alpha \\&= 2\alpha\end{aligned}$$

$\therefore x$

$$\frac{d\theta}{dt} = 3 \text{ rad min}^{-1}$$

4 PAGES / PÁGINAS

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a)  $(1+x)^n = 1 + {}^nC_1 x + \dots + {}^nC_{r-1} x^{r-1} + {}^nC_r x^r + {}^nC_{r+1} x^{r+1} + \dots + x^n$

$\therefore \text{coefficients: } u_1 = {}^nC_{r-1}$

$u_2 = {}^nC_r$

$u_3 = {}^nC_{r+1}$

$\therefore d = {}^nC_r - {}^nC_{r-1} = {}^nC_{r+1} - {}^nC_r$

$$\therefore \frac{n!}{r!(n-r)!} - \frac{n!}{(r-1)!(n-r+1)!} = \frac{n!}{(r+1)!(n-r-1)!} - \frac{n!}{r!(n-r)!}$$

$$\therefore \frac{2n!}{r!(n-r)!} - \frac{n!}{(r-1)!(n-r+1)!} - \frac{n!}{(r+1)!(n-r-1)!} = 0$$

$$\therefore \frac{2}{r(r-1)!(n-r+1)!} - \frac{1}{(r-1)!(n-r+1)!} - \frac{1}{(r+1)(r)(r-1)!(n-r-1)!} = 0$$

$$\therefore \frac{2}{r(n-r)(n-r-1)!} - \frac{1}{(n-r+1)(n-r)(n-r-1)!} - \frac{1}{(r+1)(r)(n-r-1)!} = 0$$

$$\therefore 2(n-r+1)(r+1) - r(\cancel{n-r})(\cancel{r+1}) - (n-r)(n-r+1) = 0$$

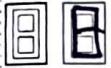
$$\therefore 2(nr+n-r^2-r+r+1) - r^2\cancel{+r} - (n^2\cancel{+r}-nr+n-nr+r^2-r) = 0$$

$$\cancel{+2nr+2n} \cancel{+r^2+2-r^2+r} \cancel{-n^2+2nr} - \cancel{n-r^2+r} = 0$$

$$\therefore 2nr+2n-2r^2+2-r^2-n^2+2nr-\cancel{n-r^2+r} = 0$$

$$\therefore 4nr+n-4r^2-2-n=0$$

$$\therefore n^2+4r^2-2-n(4r+1)=0$$



b) When  $n = 14$ ,

$$14^2 + 4r^2 - 2 - 14(4r+1) = 0$$
$$\therefore 4r^2 - 14 \cdot 4r + 14^2 - 2 - 14 = 0$$

$$\therefore r = 5, 9 \quad \{G.D.C \text{ poly Roots}\}$$

$\Rightarrow$  for  $r=5$ , the coefficients are:

$${}^{14}C_4 = 1001$$

$${}^{14}C_5 = 2002$$

$${}^{14}C_6 = 3003$$

(note,  $r=9$  will be the same  
due to symmetry in  ${}^nC_r$ )