Practice Set C: Paper 2 Mark scheme

SECTION A

1	**a**	Stratified sampling	A1
	b	Correct regression line attempted	M1
		$y = -1.33x + 6.39$	A1
	c	For every extra hour spent on social media, 1.33 hours less spent on homework.	A1
		No social media gives around 6.39 hours for homework.	A1
			[5 marks]
2		Shaded area $\frac{1}{2}(7.2)^2\,\theta\ (= 25.92\,\theta)$	M1
		Triangle area $\frac{1}{2}(7.2)^2 \sin\theta\ (= 25.92 \sin\theta)$	M1
		$\frac{1}{2}(7.2)^2\,\theta - \frac{1}{2}(7.2)^2 \sin\theta = 9.7$ or equivalent (e.g. $\theta - \sin\theta = 0.3742$)	A1
		Solve their equation using GDC	M1
		$\theta = 1.35$	A1
			[5 marks]
3	**a**	$k + 2k + 3k + 4k = 1$	(M1)
		$k = 0.1$	A1
	b	$\mathrm{E}(X) = k + 4k + 12k + 28k$	(M1)
		$\mathrm{E}(X^2) = k + 8k + 48k + 196k\ (= 25.3)$	M1
		$\mathrm{Var}(X) = 25.3 - [4.5]^2$	(M1)
		$= 5.05$	A1
	c	$25 \times \mathrm{Var}(X)$	(M1)
		$= 126(.25)$	A1
			[8 marks]

4 METHOD 1

Use of $\cot\theta = \dfrac{1}{\tan\theta}$

$\text{LHS} \equiv \dfrac{\sec\theta\sin\theta}{\tan\theta + \frac{1}{\tan\theta}}$ M1

$\equiv \dfrac{\sec\theta\sin\theta\tan\theta}{\tan^2\theta + 1}$ A1

Use of $\sec^2\theta \equiv \tan^2\theta + 1$

$\equiv \dfrac{\sec\theta\sin\theta\tan\theta}{\sec^2\theta}$ M1

$\equiv \dfrac{\sin\theta\tan\theta}{\sec\theta}$ A1

Express in terms of $\sin\theta$ and $\cos\theta$

$\equiv \sin\theta\dfrac{\sin\theta}{\cos\theta} \times \cos\theta$ M1

$\equiv \sin^2\theta$ AG

[5 marks]

METHOD 2

Use of $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$\text{LHS} \equiv \dfrac{\sec\theta\sin\theta}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}}$ M1

Add fractions in denominator (or multiply through by $\sin\theta\cos\theta$)

$\equiv \dfrac{\sec\theta\sin\theta}{\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}}$ M1

$\equiv \dfrac{\sin^2\theta\sec\theta\cos\theta}{\sin^2\theta + \cos^2\theta}$ A1

$\equiv \dfrac{\sin^2\theta}{\sin^2\theta + \cos^2\theta}$ A1

Use of $\sin^2\theta + \cos^2\theta \equiv 1$

$$\equiv \frac{\sin^2\theta}{1} \qquad \text{M1}$$

$$\equiv \sin^2\theta \qquad \text{AG}$$

[5 marks]

5 **a** Solve $0.003x^3 + 10x + 200 = 720$ using GDC — M1
36 cakes — A1

b Sketch graph of $y = \frac{T(x)}{x}$ — M1
Minimum point marked at $x = 32.2$ — M1
Minimum = 19.3 minutes — A1
Maximum = 21.2 minutes — A1

[6 marks]

6 **a** $20\,C\,6$ — (M1)
$= 38\,760$ — A1

b Consider two cases: (3 F and 3 NF) or (4 F and 2 NF) — M1
$12C3 \times 8C3$ (= 12 320) or $12C4 \times 8C2$ (= 13 860) — M1
Both of the above terms seen (not necessarily added for this mark) — A1
26 180 selections — A1

[6 marks]

7 $\frac{du}{dx} = e^x \Rightarrow dx = \frac{1}{e^x}\,du$ — M1

$\int \frac{u}{u^2+u-2}\frac{1}{e^x}\,du = \int \frac{1}{u^2+u-2}\,du$ — A1

Attempt to use partial fractions

$\frac{1}{u^2+u-2} = \frac{A}{u-1} + \frac{B}{u+2}$ — M1

$1 = A(u+2) + B(u-1)$

$A = \frac{1}{3}, B = -\frac{1}{3}$ — A1

$\int \frac{\frac{1}{3}}{u-1} - \frac{\frac{1}{3}}{u+2}\,du = \frac{1}{3}(\ln|u-1| - \ln|u+2|)$ — M1

$\ln\left|\frac{u-1}{u+2}\right|^{\frac{1}{3}}$ — A1

$\int \frac{e^x}{e^{2x}+e^x-2}\,dx = \ln\left|\frac{e^x-1}{e^x+2}\right|^{\frac{1}{3}}(+c)$ — A1

[7 marks]

8 Assume there does exist such a function — M1

By factor theorem $f\left(-\frac{3}{2}\right) = 0$:

$2\left(-\frac{3}{2}\right)^3 + b\left(-\frac{3}{2}\right)^2 + c\left(-\frac{3}{2}\right) + 3 = 0$

Note: award M1 for $f\left(\pm\frac{3}{k}\right) = 0$ where $k = 1$ or 2. — M1

$3b - 2c - 5 = 0$ — A1

By remainder theorem $f(2) = 5$

$2(2)^3 + b(2)^2 + c(2) + 3 = 5$

Note: award M1 for $f(\pm 2) = 5$ — M1

$2b + c + 7 = 0$ — A1

Solving (1) and (2) simultaneously:

$b = -\frac{9}{7}, c = -\frac{31}{7}$ — A1

This is a contradiction as b, c were assumed to be integers.
So, there exists no such function. — A1

[7 marks]

9 $\frac{dS}{dt} = 2\pi r\frac{dr}{dt}\ldots$ — M1

$\ldots + \pi\frac{dr}{dt}\sqrt{r^2+25}$ — A1

$\ldots + \pi r\frac{2r\frac{dr}{dt}}{2\sqrt{r^2+25}}$ — M1A1

Substitute $r = 10, \frac{dr}{dt} = 2$ into their expression — M1

$\frac{dS}{dt} = 252\,\text{cm}^2\,\text{sec}^{-1}$ — A1

[6 marks]

SECTION B

10 a i Arithmetic sequence, $u_1 = 30$, $d = 10$ (M1)

$u_{12} = 30 + 11 \times 10$ M1

$= 140$ A1

ii $S_{12} = 6(60 + 11 \times 10)$ or $\frac{12(30 + 140)}{2}$ M1

$= 1020$ A1

iii $\frac{N}{2}\left(60 + 10(N - 1)\right) = 2000$

OR Create table of values M1

$N = 17.7$

OR $S_{17} = 1870$, $S_{18} = 2070$ A1

In the 18th month A1

[8 marks]

b i Geometric sequence, $u_1 = 30$, $r = 1.1$ (M1)

$S_{12} = \frac{30(1.1^{12} - 1)}{1.1 - 1}$ M1

$= 642$ A1

ii $30 \times 1.1^{N-1} > 100$ M1

$N = 13.6$ (M1)

In the 14th month A1

[6 marks]

c i Multiply answer to **a(ii)** or **b(i)** by the profit at least once M1

Stella: $1020 \times 2.20 = £2244$ A1

Giulio: $642 \times 3.10 = £1990$ A1

ii $\frac{30(1.1^N - 1)}{0.1} \times 3.10 > \frac{N}{2}\left(60 + 10(N - 1)\right) \times 2.20$ M1

$N = 22.9$ (M1)

In the 22nd month A1

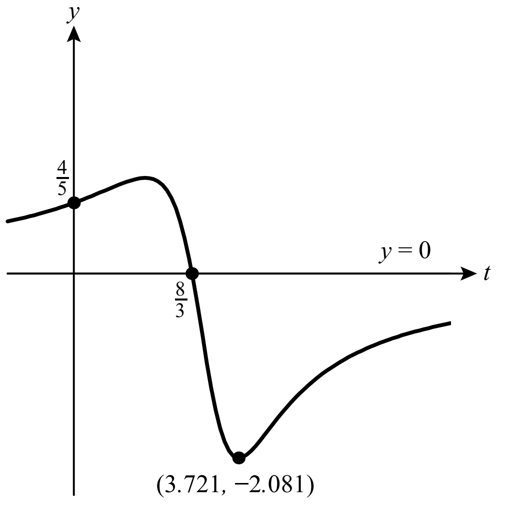
[6 marks]

Total [20 marks]

11 a $v(0) = \frac{8}{10} = 0.8\ \text{m s}^{-1}$ A1

[1 mark]

b Sketch graph $y = v(t)$ and identify minimum point. (M1)



Maximum speed $= |-2.08| = 2.08\ \text{m s}^{-1}$

Note: Award M1A0 for $-2.08\ \text{m s}^{-1}$ A1

[2 marks]

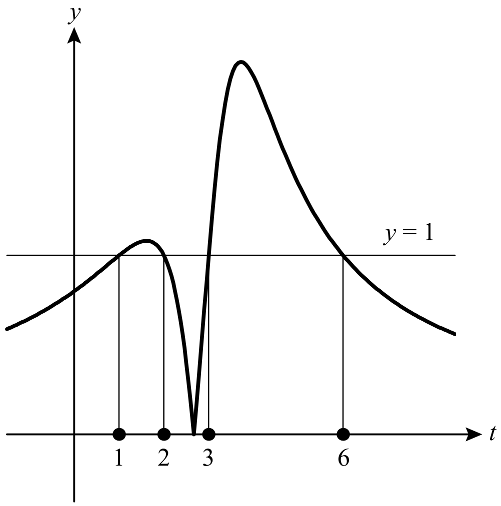
c EITHER

$v > 1$ for $1 < t < 2$ M1

$v < 1$ for $3 < t < 6$ M1

OR

Graph $y = |v(t)|$ M1



$|v| > 1$ for $1 < t < 2$ or $3 < t < 6$ M1

So speed > 1 for 4 seconds A1

[3 marks]

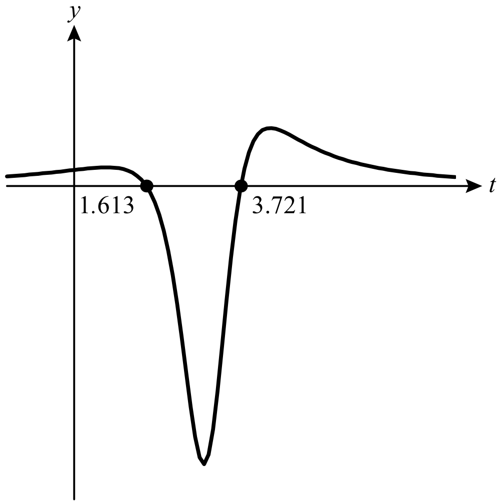
d Object changes direction when $v = 0$ (M1)

$t = \frac{8}{3} = 2.67\,\text{s}$ A1

[2 marks]

e EITHER

Sketch graph of $y = \frac{dv}{dt}$: $y < 0$ for $1.61 < t < 3.72$ (M1)



OR

Use graph of $y = v(t)$: gradient negative for $1.61 < t < 3.72$ (between turning points) (M1)

So $a < 0$ for 2.11 seconds A1

[2 marks]

f From GDC, $\frac{dv}{dt}$ at $t = 5$ … (M1)

… gives $a = 0.52\,\text{m}\,\text{s}^{-2}$ A1

[2 marks]

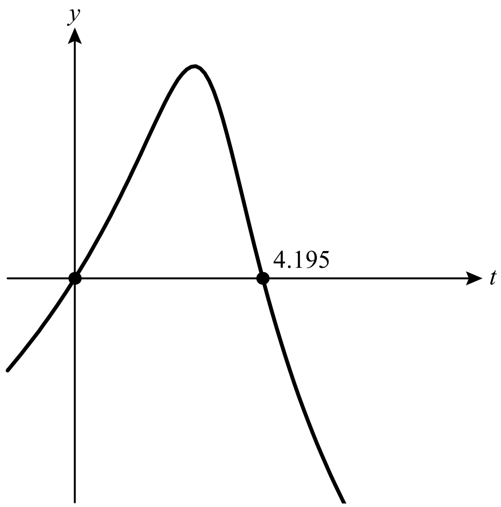
g From GDC:

distance $= \int_0^{10} \left| \frac{8 - 3t}{t^2 - 6t + 10} \right| dt$ M1

$= 9.83\,\text{m}$ A1

[2 marks]

h Sketch graph of $y = \int_0^x v\,dt$ (M1)



Identify x-intercept as being point at which object back at start (M1)

$t = 4.20$ seconds A1

[3 marks]

Total [17 marks]

12 a $\frac{d}{dx}(\ln|\sec x + \tan x|) = \frac{1}{\sec x + \tan x}(\sec x \tan x + \sec^2 x)$ M1A1

$= \frac{\sec x(\sec x + \tan x)}{\sec x + \tan x}$ A1

$= \sec x$ AG

[3 marks]

b $\frac{dy}{dx} + \sec x y = \sec x$ M1

Integrating factor:

$e^{\int \sec x\,dx} = e^{\ln|\sec x + \tan x|}$ M1

$= \sec x + \tan x$ A1

$\frac{d}{dx}\left(y(\sec x + \tan x)\right) = \sec^2 x + \sec x \tan x$ M1A1

$y(\sec x + \tan x) = \int \sec^2 x + \sec x \tan x\,dx$

$y(\sec x + \tan x) = \tan x + \sec x + c$ A1

$y = 1 + \frac{c}{\sec x + \tan x}$ A1

[7 marks]

c i $\frac{d^3y}{dx^3} - \sin x\frac{dy}{dx} + \cos x\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$ M1A1

$\frac{d^3y}{dx^3} = (\sin x - 1)\frac{dy}{dx} - \cos x\frac{d^2y}{dx^2}$ AG

ii Substitute given values into differential equation:

When $x = 0$

$\frac{d^2y}{dx^2} + \cos 0\,(1) + 2 = 1$ M1

$\frac{d^2y}{dx^2} = -2$ A1

Substitute their value into expression for $\frac{d^3y}{dx^3}$:

When $x = 0$

$\frac{d^3y}{dx^3} = (\sin 0 - 1)(1) - \cos 0\,(-2)$ M1

$= 1$ A1

Substitute their values into Maclaurin series

$y = 2 + x - \frac{2}{2!}x^2 + \frac{1}{3!}x^3 + \cdots$ M1

$2 + x - x^2 + \frac{1}{6}x^3 + \cdots$ A1

[8 marks]

Total [18 marks]