Practice Set B: Paper 1 Mark scheme

SECTION A

1	$k\ln(x^2+3)$	M1
	$2\ln(x^2+3)$	A1
	Limits: $2\ln(a^2+3) - 2\ln 3$	M1
	$2\ln\left(\frac{a^2+3}{3}\right) = \ln 16$ or $2\ln(a^2+3) = \ln(16\times 9)$	A1
	$\left(\frac{a^2+3}{3}\right)^2 = 16$ or $(a^2+3)^2 = 16\times 9$	M1
	$\frac{a^2+3}{3} = 4$ or $a^2+3 = 12$ only	A1
	$a = 3$	A1
		[7 marks]
2	**a** $\frac{1}{4}$ or 10 seen	A1
	$\frac{10}{40}\times\frac{9}{39}$	(M1)
	$=\frac{3}{52}$	A1
	b $\frac{10}{40}\times\frac{20}{39}$	(M1)
	$\times 2$	(M1)
	$=\frac{10}{39}$	A1
		[6 marks]
3	Attempt quotient rule:	
	$\frac{dy}{dx} = \frac{x\cos x - \sin x}{x^2}$	M1A1
	$= \frac{-\pi - 0}{\pi^2} = -\frac{1}{\pi}$	A1
	$y = 0$	A1
	$y = \pi(x-\pi)$	A1
		[5 marks]
4	Sketch both graphs or consider four cases	M1
	Attempts to find intersection points:	M1
	$x - 3 = 2x + 1 \Rightarrow x = -4$	A1
	$-x + 3 = 2x + 1 \Rightarrow x = \frac{2}{3}$	A1
	$x \leqslant -4$ or $x \geqslant \frac{2}{3}$	A1
		[5 marks]
5	$P(B\|A) = \frac{P(A\cap B)}{P(A)}$: $0.6 = \frac{P(A\cap B)}{0.3}$	(M1)
	$P(A\cap B) = 0.18$	A1
	$P(A\cup B) = P(A) + P(B) - P(A\cap B)$: $0.8 = 0.3 + P(B) - 0.18$	M1
	$P(B) = 0.68$	A1
	$P(B\|A) = \frac{P(A\cap B)}{P(B)}\left[=\frac{0.18}{0.68}\right]$	M1ft
	$=\frac{9}{34}$	A1
		[6 marks]

6 $A = -1$ A1

$x = 0$: $A + B = 8$ M1

$\Rightarrow B = 9$ A1

$-1 + 9e^{-2k} = 0 \Rightarrow e^{-2k} = \frac{1}{9}$ M1

Attempt taking logarithm of both sides, e.g. $2k = -\ln\left(\frac{1}{9}\right)$ M1

$k = \ln 3$ A1

[6 marks]

7 $7^1 + 3^0 = 8$, so true for $n = 1$ A1

Assume that, for some k, $7^k + 3^{k-1} = 4A$ M1

Then

$7^{k+1} + 3^k = 7 \times 7^k + 3 \times (4A - 7^k)$ M1

$= 4 \times 7^k + 12A$ A1

So $7^{k+1} + 3^k$ is divisible by 4 A1

The statement is true for $n = 1$, and if it is true for some $n = k$ then it is also true for $n = k + 1$. Therefore it is true for all integers $n \geqslant 1$ [by the principle of mathematical induction]. A1

[6 marks]

8 $\left|4 - 4\sqrt{3}\,i\right| = \sqrt{16+48} = 8$ M1

$\Rightarrow |z| = 2$ A1

$\arg(4 - 4\sqrt{3}\,i) = \arctan\left(-\sqrt{3}\right)$ M1

$= -\frac{\pi}{3}\left[\text{or } \frac{5\pi}{3}\right]$ A1

$\Rightarrow \arg z = -\frac{\pi}{9}, \ldots$ M1

$\ldots \frac{5\pi}{9}$ or $\frac{11\pi}{9}$ A1

$\therefore z = 2e^{-\frac{\pi}{9}i}, 2e^{\frac{5\pi}{9}i}, 2e^{\frac{11\pi}{9}i}, \left[\text{or } 2e^{\frac{5\pi}{9}i}, 2e^{\frac{11\pi}{9}i}, 2e^{\frac{17\pi}{9}i}\right]$ A1

[7 marks]

9 $\cos x \approx 1 - \frac{x^2}{2} + \frac{x^4}{24}$ A1

$\left(1 - x^2\right)^{-\frac{1}{2}}$ M1

$\approx 1 + \frac{x^2}{2}$ A1

$+\frac{3x^4}{8}$ A1

Multiply the two expansions, using at least two terms in each one M1

Attempt to simplify, obtain at least $1 + 0x^2$ A1

$1 + \frac{x^4}{6}$ A1

[7 marks]

SECTION B

10 a Use product rule

$f'(x) = e^{-kx} + x(-k)e^{-kx}$ M1

$= (1 - kx)e^{-kx}$ A1AG

Use product rule again, $u' = -k$, $v' = -ke^{-kx}$ M1

$f''(x) = (-k)e^{-kx} + (1 - kx)(-k)e^{-kx}$ A1

$= (k^2x - 2k)e^{-kx}$ A1

[5 marks]

b $f'(x) = 0$: $(1 - kx)e^{-kx} = 0$ M1

$e^{-kx} \neq 0$ A1

$x = \dfrac{1}{k}$ A1

$f''\left(\dfrac{1}{k}\right) = \left(\dfrac{k^2}{k} - 2k\right)e^{-\frac{k}{k}}$ M1

$= -ke^{-1} < 0 \therefore$ local maximum A1

[5 marks]

c $f''(x) = 0$: $k^2x - 2k = 0$ M1

$x = \dfrac{2}{k}$ A1

The coordinates are

$\left(\dfrac{2}{k}, \dfrac{2}{k}e^{-2}\right)$ A1

[3 marks]

Integration by parts:

$$\int_{\frac{1}{k}}^{\frac{2}{k}} x\,e^{-kx}\,dx = \left[-\frac{x}{k}e^{-kx}\right]_{\frac{1}{k}}^{\frac{2}{k}} + \int_{\frac{1}{k}}^{\frac{2}{k}} \frac{1}{k}e^{-kx}\,dx$$ M1A1

$$= \left[-\frac{2}{k^2}e^{-2} + \frac{1}{k^2}e^{-1}\right] - \left[\frac{1}{k^2}e^{-kx}\right]_{\frac{1}{k}}^{\frac{2}{k}}$$ A1

$$= \frac{2}{k^2}e^{-1} - \frac{3}{k^2}e^{-2}$$ A1

$$= \frac{2}{k^2 e} - \frac{3}{k^2 e^2}$$ A1

$$= \frac{2e - 3}{k^2 e^2}$$ AG

[5 marks]

Total [18 marks]

11 a Eliminate a variable between two equations, e.g. x between equations (2) and (3):

$$\begin{cases} 6x + ky + 2z = a \\ 6x - y - z = 7 \\ 2y - z = 1 \end{cases}$$ M1

Eliminate the same variable between another pair of equations, e.g. x between (1) and (2):

$$\begin{cases} 6x + ky + 2z = a \\ (k+1)y + 3z = a - 7 \\ 2y - z = 1 \end{cases}$$ M1

Eliminate a variable between the pair of equations in two variables, e.g. z between (2) and (3):

$$\begin{cases} 6x + ky + 2z = a \\ (k+1)y + 3z = a - 7 \\ (k+7)y = a - 4 \end{cases}$$ M1

Leading to:

$(2k + 14)x = a + 10 + 2k$

OR

$(k + 7)y = a - 4$

OR

$(k + 7)z = 2a - 15 - k$ A1

Their coefficient of $x/y/z = 0$ (M1)

$k = -7$ A1

[6 marks]

b **i** Their RHS = 0 (with their value of k) (M1)

$a = 4$ A1

ii Let $z = \lambda$ M1

$2y - z = 1$ (M1)

$6x - y - z = 7$ (M1)

At least one of

$y = \dfrac{1+\lambda}{2}$, $x = \dfrac{5+\lambda}{4}$ A1ft

$$\mathbf{r} = \begin{pmatrix} \frac{5}{4} \\ \frac{1}{2} \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$$ A1

[7 marks]

c Normal vectors to each plane are

$$\begin{pmatrix} 6 \\ -7 \\ 2 \end{pmatrix}, \begin{pmatrix} 6 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$$

Since none of these are multiples of each other, no two planes are parallel M1

So the planes form a triangular prism A1

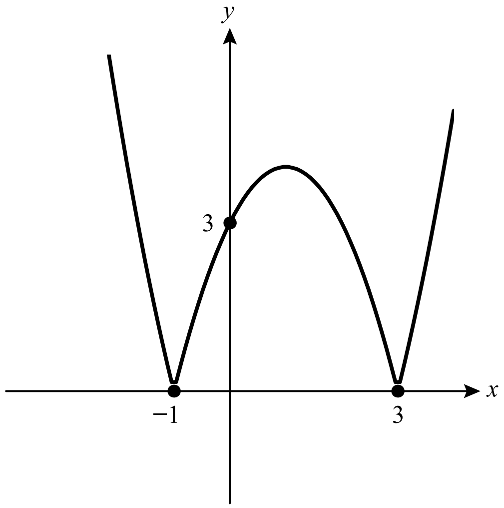
[2 marks]

Total [15 marks]

12 **a** Factorize to find x-intercepts: $(x - 3)(x + 1)$ M1

$(-1, 0)$ and $(3, 0)$ A1

Correct shape – reflected above x-axis A1



[3 marks]

b Solve $f(x) = -\frac{1}{2}x + 4$

$x^2 - 2x - 3 = -\frac{1}{2}x + 4$ (M1)

$2x^2 - 3x - 14 = 0$

$(2x - 7)(x + 2) = 0$

$x = \frac{7}{2}, -2$ A1

Solve $-f(x) = -\frac{1}{2}x + 4$

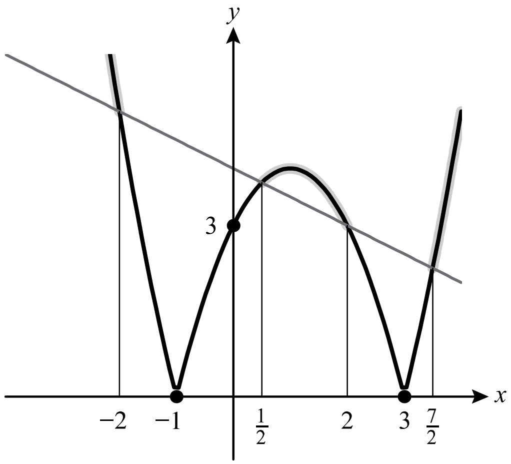
$-(x^2 - 2x - 3) = -\frac{1}{2}x + 4$ (M1)

$2x^2 - 5x + 2 = 0$

$(2x - 1)(x - 2) = 0$

$x = \frac{1}{2}, 2$ A1

Sketch of $y = |f(x)|$ and $y = -\frac{1}{2}x + 4$



$x < -2$ or $\frac{1}{2} < x < 2$ or $x > \frac{7}{2}$

Note: Award A1 for two correct regions A1A1

[6 marks]

c $x \in \mathbb{R}, x \neq -1, x \neq 3$ A1

[1 mark]

d $g'(x) = \dfrac{2(x^2 - 2x - 3) - (2x - 7)(2x - 2)}{(x^2 - 2x - 3)^2}$

Note: Award M1 for attempt at quotient rule M1A1

For turning points, $g'(x) = 0$:

$2(x^2 - 2x - 3) - (2x - 7)(2x - 2) = 0$ M1

$x^2 - 7x + 10 = 0$

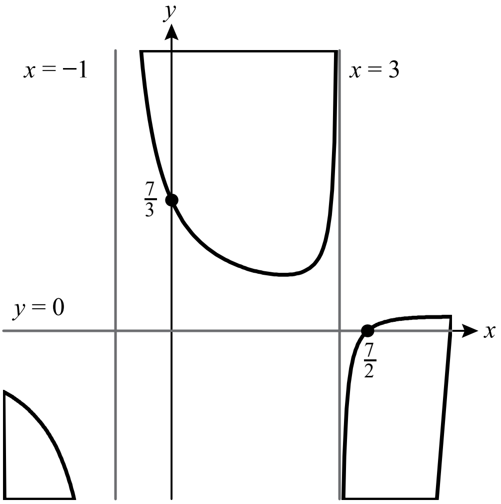
$(x - 2)(x - 5) = 0$

$x = 2, 5$ A1

So, coordinates $(2, 1)$ and $\left(5, \frac{1}{4}\right)$ A1

[5 marks]

e



Correct shape between vertical asymptotes A1

Correct shape outside vertical asymptotes A1

Vertical asymptotes: $x = -1$, $x = 3$ A1

Horizontal asymptote: $y = 0$ A1

Axis intercepts at $\left(\frac{7}{2}, 0\right)$ and $\left(0, \frac{7}{3}\right)$ A1

[5 marks]

f $g(x) \in \left(-\infty, \frac{1}{4}\right] \cup [1, \infty)$ A2

[2 marks]

Total [22 marks]