ANSWER BOOKLET
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$Q_1: \frac{22}{25}$

$Q2: \frac{25}{30}$

$\frac{47}{55} = 85\%$

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1 2 3 4 5 6 7 8 9 10

(a) $T(0,0,h)$ ✓ 1

(b) $T = \begin{pmatrix} 0 \\ 0 \\ h \end{pmatrix}$ $P_1 = \begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix}$ $P_2 = \begin{pmatrix} r\cos\theta \\ r\sin\theta \\ 0 \end{pmatrix}$

(i) $\overrightarrow{TP_1} = \begin{pmatrix} r-0 \\ 0-0 \\ 0-h \end{pmatrix} = \begin{pmatrix} r \\ 0 \\ -h \end{pmatrix}$ ✓✓ 2

(ii) $\overrightarrow{TP_2} = \begin{pmatrix} r\cos\theta - 0 \\ r\sin\theta - 0 \\ 0 - h \end{pmatrix} = \begin{pmatrix} r\cos\theta \\ r\sin\theta \\ -h \end{pmatrix}$ ✓✓ 2

(c) $\overrightarrow{TP_1} \times \overrightarrow{TP_2} = \begin{pmatrix} r \\ 0 \\ -h \end{pmatrix} \times \begin{pmatrix} r\cos\theta \\ r\sin\theta \\ -h \end{pmatrix}$

$= \begin{pmatrix} 0(-h) - (r\sin\theta)(-h) \\ (r\cos\theta)(-h) - r(-h) \\ r(r\sin\theta) - 0(r\cos\theta) \end{pmatrix}$

$= \begin{pmatrix} hr\sin\theta \\ -hr\cos\theta + hr \\ r^2\sin\theta \end{pmatrix}$ ✓✓

As area of triangle $= \frac{1}{2}\left|\overrightarrow{TP_1} \times \overrightarrow{TP_2}\right|$,

~~$A = \frac{1}{2}\left(\sqrt{h^2r^2\sin^2\theta + h^2r^2\cos^2}\right.$~~

$$A = \frac{1}{2}\left(\sqrt{(hr\sin\theta)^2 + (hr - hr\cos\theta)^2 + (r^2\sin\theta)^2}\right)$$

$$= \frac{1}{2}\left(r\sqrt{(h^2\sin^2\theta) + (h - h\cos\theta)^2 + r^2\sin^2\theta}\right)$$

$$= \frac{r}{2}\sqrt{h^2(1-\cos^2\theta) + h^2(1-\cos\theta)^2 + r^2\sin^2\theta}$$

$$= \frac{r}{2}\sqrt{h^2 - h^2\cos^2\theta + h^2 - 2h^2\cos\theta + h^2\cos^2\theta + r^2\sin^2\theta}$$

$$= \frac{r}{2}\sqrt{2h^2 - 2h^2\cos\theta + r^2\sin^2\theta}$$

$$\therefore A = \frac{r\sqrt{2h^2(1-\cos\theta) + r^2\sin^2\theta}}{2}$$

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(d) $\theta = 360/n$ OR $2\pi/n$

1

(e) There will be $n \times A$ for the SA of all triangles.

$\therefore \quad S = nA$

However, $n = 2\pi/\theta$ (part (d))

$$\therefore \quad S = \frac{\pi r\sqrt{2h^2(1-\cos\theta)+r^2\sin^2\theta}}{\theta}$$

2

(f) $\theta = 2\pi/n$

$$\therefore \lim_{n\to\infty} \theta = \lim_{n\to\infty}\left(2\pi/n\right)$$

$$\cancel{= \lim_{n\to\infty}\left(\frac{2\pi/n}{n}\right)}$$

$$= 0$$

1

(g) As $\theta \to 0$, $n \to \infty$

$\sqrt{2h^2(1-\cos\theta)+r^2\sin^2\theta}$

will continue creating $0/0$

(h) $$\lim_{\theta \to 0} \frac{2h^2 - 2h^2\cos\theta + r^2\sin^2\theta}{\theta^2}$$

$$= \lim_{\theta \to 0}\left(\frac{+2h^2\sin\theta + 2r^2\sin\theta\cos\theta}{2\theta}\right) \quad \{LH's\}$$

$$= \lim_{\theta \to 0}\left(\frac{2h^2\cos\theta + 2r^2(\sin\theta \cdot -\sin\theta + \cos\theta\cos\theta)}{2}\right) \quad \{LH's\}$$

$$= \lim_{\theta \to 0}\left(\frac{2h^2\cos\theta - 2r^2\sin^2\theta + 2r^2\cos^2\theta}{2}\right)$$

$$= \frac{2h^2\cos 0 - 2r^2\sin^2 0 + 2r^2\cos^2 0}{2}$$

$$= \frac{2h^2 + 2r^2}{2}$$

$$= h^2 + r^2$$

$$\rightarrow \lim_{\theta \to 0} S = \pi r \cdot \sqrt{\lim_{\theta \to 0}\left(\frac{2h^2(1-\cos\theta) + r^2\sin^2\theta}{\theta^2}\right)}$$

$$= \pi r\sqrt{h^2 + r^2}$$

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(i) $\sqrt{h^2+r^2}$ is the length of a diagonal

1

$\Rightarrow$ SA of ~~a~~ the curved surface of a cone.

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2

$$B(m,n) = \int_0^1 x^m (1-x)^n \, dx \qquad m,n \in \mathbb{N}$$

(a) (i) $B(0,0) = \int_0^1 x^0 (1-x)^0 \, dx$

$= \int_0^1 dx$

$= x\Big|_0^1$

$= 1$

2

(ii) $B(1,0) = \int_0^1 x(1-x)^0 \, dx$

$= \int_0^1 x \, dx$

$= \frac{1}{2}x^2\Big|_0^1$

$= \frac{1}{2}(1^2 - 0^2)$

$= 1/2$

2

(iii) $B(0,1) = \int_0^1 x^0 (1-x) \, dx$

$= \int_0^1 (1-x) \, dx$

$= \left[x - \frac{1}{2}x^2\right]_0^1$

$= (1 - 1/2) - 0$

$= 1/2$

2

(b) $B(m,0) = \int_0^1 x^m (1-x)^0 \, dx$

$= \cancel{\left[\frac{1}{m}x^{m+1}\right.}$

$= \left[\frac{1}{m+1} x^{m+1}\right]_0^1$

$= \frac{1}{m+1}$

2

(c) $B(m,n) = \int_0^1 x^m (1-x)^n \, dx$

$$\left[\begin{array}{l} \text{let } u = (1-x)^n \rightarrow du = n(1-x)^{n-1}(-1) \\ \text{let } dv = x^m \rightarrow v = \frac{1}{m+1} x^{m+1} \end{array}\right]$$

$$\therefore B(m,n) = \left[\left(\frac{1}{m+1}\right)(1-x)^n\left(x^{m+1}\right)\right]_0^1 - \int_0^1 \frac{n}{m+1} x^{m+1} (1-x)^{n-1} \, dx$$

$$= \left(\frac{1}{m+1}\right)(0)^n\left(1^{m+1}\right) - \left(\frac{1}{m+1}\right)\left(1^n\right)\left(0^{m+1}\right) - \frac{n}{m+1} \times B(m+1, n-1)$$

$$= - \frac{n\,B(m+1, n-1)}{m+1}$$

from above.

$$= \frac{n\,B(m+1, n-1)}{m+1}$$

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$$B(4,0) = \frac{0\,B(5,-1)}{5}$$

AND

$$B(4,1) = \frac{B(5,0)}{4}$$

$$B(4,2) = \frac{B(5,1)}{2}$$

$$B(4,3) = \frac{3B(5,2)}{4}$$

$$B(4,4) = B(5,3)$$

(d) Using G.D.C

$$f(n,m) := \int_0^1 \left(x^m \cdot (1-x)^n\right) dx$$ Done

$\Rightarrow f(4,0) = 0.2 = 1/5$

$\Rightarrow f(4,1) = 0.0\dot{3} = 1/30$

$\Rightarrow f(4,2) = 0.009524 \approx 1/105$

$\Rightarrow f(4,3) = 0.003571 \approx 1/280$

$\Rightarrow f(4,4) = 0.001587 \approx 1/630$

MS unclear 2B0D

n	0	1	2	3	4
$B(4,n)$	0.2	0.0333	0.00952	0.00357	0.00159

3

(e) $B(m,0) = \frac{1}{m+1}$ (part (b))

$$RHS = \frac{1}{(m+0+1) \cdot {}^{m}C_{0}}$$

$$= \frac{1}{m+1}$$

$= LHS$

2

$\therefore$ true for $n=0$

(f) ~~$B(m,k) = \int_0^1 x^m (1-x)^k dx$~~

~~$\therefore LHS = \frac{k B(m+1, k-1)}{m+1}$~~

~~$RHS = 1$~~

ASSUME : $B(m,k) = \frac{1}{(m+k+1) \cdot {}^{m+k}C_k}$ [IH]

PROVE : $B(m,k+1) = \frac{1}{(m+k+2) \cdot {}^{m+k+1}C_{k+1}}$

$$\therefore LHS = \frac{(k+1)\, B(m+1, k)}{m+1}$$

$$= \frac{(k+1)}{(m+1)} \times \frac{1}{(m+1+k+1) \cdot {}^{m+k+1}C_k}$$ (by [IH])

$$= \frac{(k+1)}{(m+k+2) \cdot \boxed{\frac{(m+k+1)!(m+1)}{k!(m+k+1-k)!}}}$$

unclear ×

$$= \frac{\cancel{(k+1)}}{(m+k+2) \cdot \frac{(m+k+1)!(m+1)}{(k+1)!\,(m+1)!}}$$

$$= \frac{1}{(m+k+2) \cdot \frac{(m+k+1)!}{(k+1)!\,(m)!}}$$

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$$= \frac{1}{(m+k+2)\cdot\frac{(m+k+1)!}{(k+1)(m+k+1-(k+1))!}}$$

$$= \frac{1}{(m+k+2)\cdot {}^{m+k+1}C_{k+1}}$$

5

$$= RHS$$

$\therefore$ true for $n=k+1$ whenever $n=k$ is assumed true.

(g) As true for $n=0$, and true for $n=k+1$, whenever $n=k$ is assumed to be true, it holds true for all $n \geq 0$, $n \in \mathbb{Z}$ by mathematical induction.

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