Practice Set A: Paper 1 Mark scheme

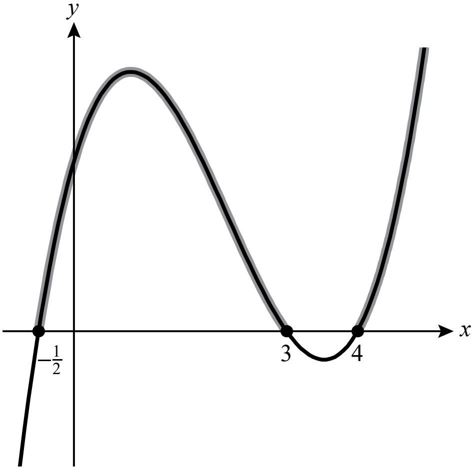
SECTION A

1	P(late) $= 0.8 \times 0.4 + 0.2 \times 0.1$ (= 0.34)	(M1)
	P(late and not coffee) $= 0.2 \times 0.1$ (= 0.02)	(M1)
	P(not coffee|late)	M1
	$= \dfrac{0.02}{0.34}$	A1
	$= \dfrac{1}{17}$	A1
		[5 marks]
2	Substitute $\mathrm{d}x = \mathrm{d}u$, $5x = 5(u+3)$	M1
	Change limits	M1
	Obtain $\int_0^4 5(u+3)\sqrt{u}\,\mathrm{d}u$	A1
	Expand the brackets before integrating: $\int_0^4 5u^{\frac{3}{2}} + 15u^{\frac{1}{2}}\,\mathrm{d}u$	M1
	$= \left[2u^{\frac{5}{2}} + 10u^{\frac{3}{2}}\right]_0^4$	A1
	$= 2 \times 2^5 + 10 \times 2^3$	(M1)
	$= 144$	A1
		[7 marks]
3	Write $z = x + \mathrm{i}y$	(M1)
	Then $3x + 3\mathrm{i}y - 4x + 4\mathrm{i}y = 18 + 21\mathrm{i}$	A1
	Compare real and imaginary parts	M1
	$z = -18 + 3\mathrm{i}$	A1
	$\left\|\dfrac{z}{3}\right\| = \sqrt{6^2 + 1^2}$	M1
	$= \sqrt{37}$	A1
		[6 marks]
4 a	EITHER	
	Use factor theorem:	
	$\mathrm{f}\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right)^3 - 13\left(-\frac{1}{2}\right)^2 + 17\left(-\frac{1}{2}\right) + 12$	M1
	$= -\frac{1}{4} - \frac{13}{4} - \frac{34}{4} + \frac{48}{4}$	
	$= 0$	
	So $(2x+1)$ is a factor	A1
	OR	
	Compare coefficients or long division:	
	$2x^3 - 13x^2 + 17x + 12 = (2x+1)(x^2 - 7x + 12)$	M1A1

b $(2x + 1)(x - 3)(x - 4) = 0$

$x = -\frac{1}{2}, 3, 4$ (M1)

Sketch graph or consider sign of factors (M1)



$-\frac{1}{2} < x < 3$ or $x > 4$

Note: Award M1A0 for correct region from their roots M1A1

[6 marks]

5 $$f \circ g(x) = \frac{2 - \frac{2}{x-1}}{\frac{2}{x-1} + 3}$$ M1

$$= \frac{2(x-1) - 2}{2 + 3(x-1)}$$ (M1)

$$= \frac{2x - 4}{3x - 1}$$ A1

$$x = \frac{2y - 4}{3y - 1}$$

$3xy - x = 2y - 4$ (M1)

$3xy - 2y = x - 4$ M1

$$y = \frac{x - 4}{3x - 2}$$ A1

[6 marks]

6 $7e^{2x} - 45e^{x} = e^{3x} - 7e^{2x}$ M1

$e^{3x} - 14e^{2x} + 45e^{3x} = 0$ A1

$e^{x}(e^{x} - 9)(e^{x} - 5) = 0$ M1A1

Reject $e^{x} = 0$ R1

$x = \ln 5$ or $\ln 9$ A1

[6 marks]

7 Attempt to differentiate both top and bottom. M1

Top: $\sin x + x \cos x$ M1A1

Bottom: $\frac{1}{x}$ A1

$\lim_{x \to \pi}(x \sin x + x^2 \cos x)$ M1

$= -\pi^2$ A1

[6 marks]

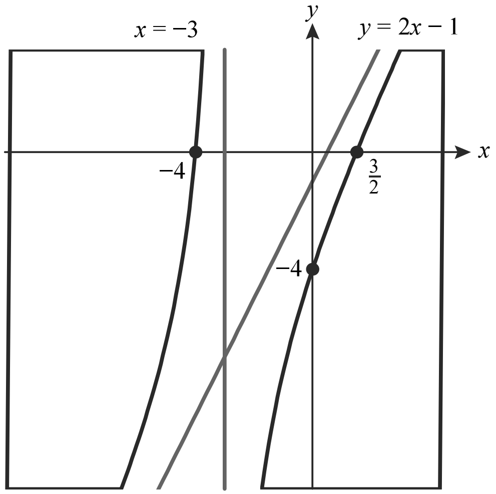
8 Factorize denominator to find x-intercepts: $(2x-3)(x+4)$ (M1)

Long division or compare coefficients:

$\dfrac{2x^2+5x-12}{x+3} = \dfrac{(x+3)(2x-1)-9}{x+3}$ M1

$= 2x - 1 - \dfrac{9}{x+3}$ A1

Correct shape A1



Axis intercepts: $\left(\frac{3}{2}, 0\right)$, $(-4, 0)$, $(0, -4)$ A1

Vertical asymptote: $x = -3$ A1

Oblique asymptote: $y = 2x - 1$ A1

[7 marks]

9 **a** Suppose that $\log_2 5$ is rational, and write $\log_2 5 = \dfrac{p}{q}$. M1

Then $2^{\frac{p}{q}} = 5$, so $2^p = 5^q$. M1

e.g. LHS is even and RHS is odd. A1

This is a contradiction, so $\log_2 5$ is irrational A1

b Any suitable example, e.g. $n = 16$ M1

Complete argument, e.g. $\log_2 16 = 4$, which is rational A1

[6 marks]

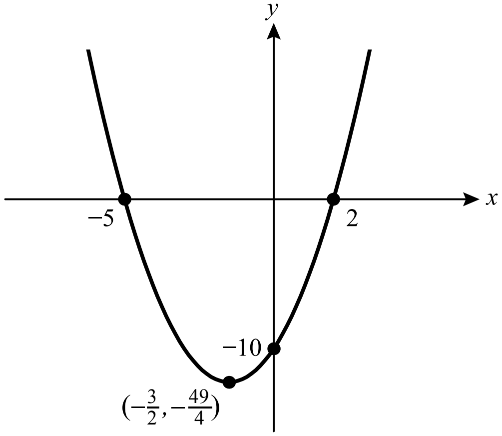
SECTION B

10 **a** Factorize to find x-intercepts: $(x+5)(x-2)$ (M1)

Complete the square for vertex (or half-ways between intercepts):

$\left(x+\frac{3}{2}\right)^2-\frac{49}{4}$ (M1)

Correct shape and all intercepts A1



Correct vertex $\left(-\frac{3}{2}, -\frac{49}{4}\right)$ A1

[4 marks]

b **i** $x^2+3x-10=2x-20$

$\Leftrightarrow x^2+x+10=0$ M1

discriminant $= 1-40$ $(=-39)$ M1

< 0 so no intersections A1

ii $x^2+x+(k-10)=0$ M1A1

$1-4(k-10)>0$ M1

$k<\frac{41}{4}$ A1

[7 marks]

c Compare to $\left(x+\frac{3}{2}\right)^2-\frac{49}{4}$ (M1)

Vertical translation $\frac{57}{4}$ units up A1

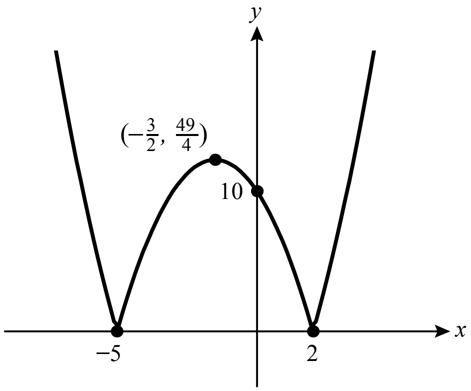
Horizontal stretch A1

Scale factor $\frac{1}{2}$ A1

[4 marks]

d **i** Correct shape A1

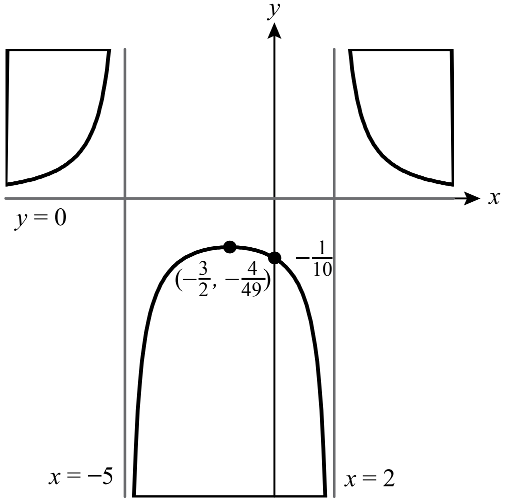
Correct intercepts and turning point labelled A1



ii Vertical asymptotes at x = −5, 2, y-int −0.1 A1

Parts of curve in correct quadrants M1

Turning point $\left(-\frac{3}{2}, -\frac{4}{49}\right)$ A1



[5 marks]
Total [20 marks]

11 a $\frac{dy}{dx} = -e^{-x}\sin 2x + 2e^{-x}\cos 2x$ M1A1

M1 for attempt at product rule or quotient rule

$-e^{-x}\sin 2x + 2e^{-x}\cos 2x = 0$

$-\sin 2x + 2\cos 2x = 0$ M1

$\frac{\sin 2x}{\cos 2x} = 2$ A1

$\tan 2x = 2$ AG

[4 marks]

b $\frac{d^2y}{dx^2} = e^{-x}\sin 2x - 2e^{-x}\cos 2x - 2e^{-x}\cos 2x - 4e^{-x}\sin 2x$ (M1)

M1 for attempt at product rule or quotient rule on their $\frac{dy}{dx}$

$= -3e^{-x}\sin 2x - 4e^{-x}\cos 2x$ A1

$-3e^{-x}\sin 2x - 4e^{-x}\cos 2x = 0$ M1

$\frac{\sin 2x}{\cos 2x} = -\frac{4}{3}$ A1

$\tan 2x = -\frac{4}{3}$ AG

[4 marks]

c x-intercept $= \frac{\pi}{2}$ A1

$\int e^{-x}\sin 2x\,dx = -e^{-x}\sin 2x - \int -2e^{-x}\cos 2x\,dx$ (M1)

$= -e^{-x}\sin 2x + 2\int e^{-x}\cos 2x\,dx$ A1

$= -e^{-x}\sin 2x + 2(-e^{-x}\cos 2x - \int 2e^{-x}\sin 2x\,dx)$

$= -e^{-x}\sin 2x - 2e^{-x}\cos 2x - 4\int e^{-x}\sin 2x\,dx$ A1

$5\int e^{-x}\sin 2x\,dx = -e^{-x}\sin 2x - 2e^{-x}\cos 2x$ (M1)

$\int_0^{\frac{\pi}{2}} e^{-x}\sin 2x\,dx = \frac{1}{5}[-e^{-x}\sin 2x - 2e^{-x}\cos 2x]_0^{\frac{\pi}{2}}$

$= \frac{1}{5}\left(2e^{-\frac{\pi}{2}} + 2\right)$

$= \frac{2}{5} + \frac{2}{5}e^{-\frac{\pi}{2}}$ A1A1A1

[8 marks]
Total [16 marks]

12 a For any positive integer n, $(\cos\theta + \mathrm{i}\sin\theta)^n = \cos n\theta + \mathrm{i}\sin n\theta$ A1

True when $n = 1$: $(\cos\theta + \mathrm{i}\sin\theta)^1 = \cos\theta + \mathrm{i}\sin\theta$ A1

Assume it is true for $n = k$:

$(\cos\theta + \mathrm{i}\sin\theta)^k = \cos k\theta + \mathrm{i}\sin k\theta$ M1

Then

$(\cos\theta + \mathrm{i}\sin\theta)^{k+1} = (\cos k\theta + \mathrm{i}\sin k\theta)(\cos\theta + \mathrm{i}\sin\theta)$

$= \cos(k+1)\theta + \mathrm{i}\sin(k+1)\theta$ A1

The statement is true for $n = 1$ and if it is true for some $n = k$ then it is also true for $n = k + 1$; it is therefore true for all integers $n > 1$ [by the principle of mathematical induction]. R1

[5 marks]

b [Writing $c = \cos\theta$, $s = \sin\theta$:]

$(\cos\theta + \mathrm{i}\sin\theta)^5 = c^5 + 5\mathrm{i}c^4s - 10c^3s^2 - 10\mathrm{i}c^2s^3 + 5cs^4 + \mathrm{i}s^5$ A1

Equating real parts of $\cos 5\theta + \mathrm{i}\sin 5\theta$ and $(\cos\theta + \mathrm{i}\sin\theta)^5$:

$\cos 5\theta = c^5 - 10c^3s^2 + 5cs^4$ M1

Using $s^2 = 1 - c^2$

$\cos 5\theta = c^5 - 10c^3(1 - c^2) + 5c(1 - c^2)^2$ (M1)

$= c^5 - 10c^3 + 10c^5 + 5c(1 - 2c^2 + c^4)$ A1

$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$ AG

[4 marks]

c $5\theta = \frac{\pi}{10}, \frac{3\pi}{10}, \frac{\pi}{2}, \frac{7\pi}{10}, \frac{9\pi}{10}$ M1

Obtain at least $\theta = \frac{\pi}{10}$ A1

[2 marks]

d The roots of the equation are cos(values above) (M1)

Either $c = 0$, in which case $\theta = \frac{\pi}{2}\ldots$ A1

… or $16c^4 - 20c^2 + 5 = 0$ A1

$c^2 = \frac{5 \pm \sqrt{5}}{8}$ A1

$\cos\left(\frac{\pi}{10}\right)$ is positive and the largest of the roots R1

So $\cos\left(\frac{\pi}{10}\right) = \sqrt{\frac{5+\sqrt{5}}{8}}$ A1

[6 marks]

e [$\cos\left(\frac{7\pi}{10}\right)$ is negative and not equal to $-\cos\left(\frac{\pi}{10}\right)$]

$\cos\left(\frac{\pi}{10}\right)\cos\left(\frac{7\pi}{10}\right) = \left(\sqrt{\frac{5+\sqrt{5}}{8}}\right)\left(-\sqrt{\frac{5-\sqrt{5}}{8}}\right)$ M1

$\left[= -\sqrt{\frac{25-5}{64}}\right] = -\frac{\sqrt{5}}{4}$ A1

[2 marks]

Total [19 marks]