Mathematics: analysis and approaches

Higher level

Paper 3

ID: 3010

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[55 marks]**.

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1. [Maximum points: 27]

Mike, Bill and Paul play a game where they each take turns throwing a ball at each other. Each player gets one throw per turn and if a person is hit by the ball they are eliminated from the game.

Mike hits his target 50% of the time, Bill hits his target 70% of the time, and Paul always hits his target.

Mike throws first, then Bill, then Paul. They repeat this until one player is the winner.

(a) If each player always aims at the most skillful player still in the game determine the probability of [15]

(i) Mike winning

(ii) Paul winning

(iii) Bill winning

(b) If each player always aims at the most skillful player still in the game, but Mike makes an intentional miss on his first throw, determine the probability of [12]

(i) Mike winning

(ii) Paul winning

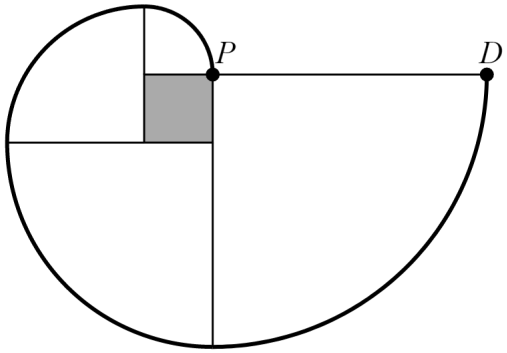
(iii) Bill winning

2. [Maximum points: 28]

A dog is attached to the top-right corner of a square building of perimeter 8 m with a piece of rope also of length 8 m. The initial position of the dog D is such that it is collinear with the top two corners of the building.

The dog walks clockwise around the building, keeping the rope tight at all times, until he can do so no more.

This is shown in the diagram below.



(a) The dog walks through four different arcs. Determine [6]

(i) the angle subtended by each arc

(ii) the radius of each arc in descending order

(iii) the total distance the dog walks.

The building is now in the shape of a regular octagon with perimeter 8 m. The dog walks clockwise around the building, keeping the rope tight at all times, until he can do so no more.

This is shown in the diagram below.

(b) The dog walks through eight different arcs. Determine [6]

(i) the angle subtended by each arc

(ii) the radius of each arc in descending order

(iii) the total distance the dog walks

The dog is now tied to a tree with a trunk of circumference 8 m. The initial position of the dog is such that the rope is tangential to the top of the trunk. The dog walks clockwise around the tree, keeping the rope tight at all times, until he can do so no more.

This is shown in the diagram below.

(c) By considering a regular n-gon as $n \to \infty$, show that the distance the dog walks around the tree is equal to 8π metres. [11]

(d) After reaching point P on the tree the dog decides to walk anti-clockwise, keeping the rope tight at all times, until he can do so no more. Determine the distance that he walks, excluding the initial distance calculated in part (c). [5]

1. (a)

(i) If Mike wins then he either eliminates Paul then Bill and Mike aim at each other until Mike wins, or he misses Paul then Bill eliminates Paul then Bill and Mike aim at each other until Mike wins, or Mike and Bill both miss then Paul eliminates Bill then Mike eliminates Paul. R1

The probability of the first case is

$$0.5 \times \left(0.3 \times 0.5 + (0.3 \times 0.5)^2 + (0.3 \times 0.5)^3 + \ldots\right)$$ A1

Use the infinite geometric series formula to calculate the value. M1

$$0.5 \times \frac{0.3 \times 0.5}{1 - 0.3 \times 0.5} = 0.088235$$ A1

The probability of the second case is

$$0.5 \times 0.7 \times \left(0.5 + 0.5^2 \times 0.3 + 0.5^3 \times 0.3^2 + \ldots\right)$$ A1

Use the infinite geometric series formula to calculate the value. M1

$$0.5 \times 0.7 \times \frac{0.5}{1 - 0.3 \times 0.5} = 0.205882$$ A1

The probability of the third case is

$$0.5 \times 0.3 \times 1 \times 0.5 = 0.075$$ A1

Add all of these probabilities together to determine the probability of Mike winning. M1

$$0.088235 + 0.205882 + 0.075 = 0.369$$ A1

(ii) If Paul wins then Mike and Bill both miss Paul then Paul eliminates Bill then Mike misses Paul then Paul eliminates Mike. R1

The probability of this happening is

$$0.5 \times 0.3 \times 1 \times 0.5 \times 1 = 0.075$$ M1A1

(iii) Bill winning is complementary to either Mike or Paul winning, so subtract the answers to parts (i) and (ii) from 1. M1

$$1 - (0.369 + 0.075) = 0.556.$$ A1

(b)

(i) If Mike makes an intentional miss then either Bill needs to eliminate Paul and then Mike and Bill aim at each other until Mike wins, or Bill misses Paul then Paul eliminates Bill then Mike eliminates Paul. R1

The probability of the first case is

$$0.7 \times \left(0.5 + 0.5^2 \times 0.3 + 0.5^3 \times 0.3^2 + \ldots\right)$$ A1

Use the infinite geometric series formula to calculate the value. M1

$$0.7 \times \frac{0.5}{1 - 0.3 \times 0.5} = 0.411765$$ A1

The probability of the second case is $0.3 \times 1 \times 0.5 = 0.15$ A1

Add these probabilities together. M1

$$0.411765 + 0.15 = 0.562$$ A1

(ii) If Paul wins then Bill misses him then Paul eliminates Bill then Mike misses Paul then Paul eliminates Mike. R1

The probability of this happening is

$$0.3 \times 1 \times 0.5 \times 1 = 0.15$$ M1A1

(iii) Bill winning is complementary to either Mike or Paul winning, so subtract the answers to part (i) and (ii) from 1. M1

$$1 - (0.562 + 0.15) = 0.288$$ A1

2. (a)

(i) $\frac{360}{4} = 90°$ M1A1

(ii) The square has sides of length 2 m. R1

So the radii are 8 m, 6 m, 4 m and 2 m. A1

(iii) Use the arc length formula to determine the total distance the dog walks. M1

$\frac{90}{360} \times 2\pi \times 8 + \frac{90}{360} \times 2\pi \times 6 + \frac{90}{360} \times 2\pi \times 4 + \frac{90}{360} \times 2\pi \times 2 = 10\pi$ A1

(b)

(i) $\frac{360}{8} = 45°$ M1A1

(ii) The octagon has sides of length 1 m. R1

So the radii are 8 m, 7 m, 6 m, 5 m, 4 m, 3 m, 2 m and 1 m. A1

(iii) Use the arc length formula to determine the total distance the dog walks. M1

$$\frac{45}{360} \times 2\pi \times \sum_{k=1}^{8} k = 9\pi$$ A1

(c) If we consider a regular n-gon then as $n \to \infty$ the shape gets closer to being a circle. R1

The angle subtended by each arc is equal to $\frac{360}{n}$. A1

The regular n-gon has sides of length $\frac{8}{n}$. A1

Use the arc length formula to determine the total distance the dog walks. M1

$$\frac{360}{n} \div 360 \times 2\pi \times \sum_{k=1}^{n} 8 - (k-1) \times \frac{8}{n}$$ A1

The sigma notation is an arithmetic series with $t_1 = 8$ and $d = -\frac{8}{n}$. R1

Use the arithmetic series formula to evaluate the sigma notation. M1

$$\frac{2\pi}{n} \times \frac{n}{2} \times \left[2 \times 8 + (n-1) \times \left[-\frac{8}{n}\right]\right]$$

This simplifies to

$$\pi\left[16 - \frac{8(n-1)}{n}\right]$$ A1

Consider the limit as $n \to \infty$. M1

$$\lim_{n \to \infty} \pi\left[16 - \frac{8(n-1)}{n}\right] = \lim_{n \to \infty} \pi\left[16 - \frac{8\left[1 - \frac{1}{n}\right]}{1}\right]$$ A1

This is equal to 8π. A1

(d) The distance back to point D is 8π. A1

The dog then walks through a semi-circle of radius 8 metres. This distance is equal to

$$\frac{1}{2} \times 2\pi \times 8 = 8\pi$$ M1A1

The dog can then walk another 8π metres around the tree until he arrives at point P again. A1

So altogether he walks 24π metres. He is a good boy. A1