ST ANDREW'S CATHEDRAL SCHOOL
FOUNDED 1885

Diploma Programme

Candidate session number

~~0~~	~~0~~	~~3~~	~~3~~	~~7~~	~~6~~	~~0~~	~~0~~	4	1

20	T	R	L	P1	1	M	A	H	L

Mathematics

Higher level

Paper 1

$\frac{78}{100} = 78\%$.

Trial Examination 2020

2:00:00

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

[Maximum

(a)

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 9]

Given the function $f(x) = \ln x - \ln(1 - x)$,

(a) Find:

(i) the domain

(ii) the range

(iii)the inverse function $f^{-1}(x)$ [5]

(b) Sketch $y = f(x)$ and $y = f^{-1}(x)$, labelling any intercepts and asymptotes. [4]

ai) $x > 0$ $1 - x > 0$

$\therefore$ ~~$x > -1$~~ $1 > x$

→ ~~domain : $\{x \mid x \geq -1\}$~~ $\therefore x < 1$

aii) range: $\{y \mid y \in \mathbb{R}\}$ $\therefore \{x \mid 0 < x < 1\}$

aiii) ~~$x = \ln y - \ln(1-y)$~~

~~$\therefore e^x = e^{\ln y - \ln(1-y)}$~~

~~$= \frac{y}{1-y}$~~

~~$\therefore x =$~~

$f^{-1}(x)$: $x = \ln y - \ln(1-y)$

$\therefore e^x = e^{\ln y - \ln(1-y)}$

$\therefore e^x = y/1-y$

~~$\therefore \frac{1-y}{y} = e^{-x}$~~

~~$\therefore \frac{1}{y} - 1 = e^{-x}$~~

~~$\therefore \frac{1}{y} = \frac{1}{e^x} + 1$~~

~~$\therefore y = e^x + 1$~~

$e^x(1-y) = y$

$\therefore e^x y + e^x = y$

$\therefore y(e^x + 1) = e^x$

$\therefore y = e^x / e^x + 1$

$\therefore f^{-1}(x) = \frac{e^x}{e^x + 1}$

7 8 /

2. [Maximum mark: 6]

(a) Prove the identity

$$\frac{\cos\theta}{1+\sin\theta}+\frac{1+\sin\theta}{\cos\theta}=2\sec\theta.$$ [2]

(b) Solve the equation $\sec^2 x+2\tan x=0,\quad -\pi\le x\le\pi$. [4]

~~a)~~ $\cancel{\dfrac{\cos\theta\cos\theta+(1+\sin\theta)^2}{\cos\theta(1+\sin\theta)}=\dfrac{\cos^2\theta+1+2\sin\theta+\sin^2\theta}{\cos\theta+\cos\theta\sin\theta}}$

$\cancel{=\dfrac{2+2\sin\theta}{\cos\theta+\cos\theta\sin\theta}}$

~~b)~~ $\cancel{\dfrac{1}{\cos^2\theta}+2\dfrac{\sin\theta}{\cos\theta}=0}$

$\cancel{\therefore\ \dfrac{1+2\sin\theta\cos\theta}{\cos^2\theta}=0}$

a) $\dfrac{\cos^2\theta+(1+\sin\theta)^2}{\cos\theta(1+\sin\theta)}=\dfrac{1+1+2\sin\theta}{\cos\theta+\cos\theta\sin\theta}$

$=\dfrac{2}{\cos\theta}$

$=2\sec\theta=\text{RHS}.$

b) $\cancel{\sec^2 x+2\tan x=0}$

$\cancel{\therefore\ \dfrac{1}{\cos^2 x}+\dfrac{2\sin x}{\cos x}=0}$

$\cancel{\therefore\ \dfrac{1+2\sin x\cos x}{\cos^2 x}=0}$

$\cancel{\therefore\ \dfrac{1+\sin 2x}{\cos^2 x}=0}$

$\sec^2 x+2\tan x=0$

$\therefore\ 1+\tan^2 x+2\tan x=0$

$\therefore\ \tan^2 x+\tan x+\tan x+1=0$

$\therefore\ \tan x(\tan x+1)+(\tan x+1)=0$

$\tan x=\pm 1$

$\therefore$

$\therefore\ x=\frac{\pi}{4}$ $\cancel{\frac{\pi}{4}}$

$\therefore\ x=-\frac{\pi}{4},\ \frac{3\pi}{4}$ $\{Q4, 2\}$

6/

3. [Maximum mark: 7]

(a) Write the first three derivatives of $f(x) = x^2e^x$. [3]

(b) Use mathematical induction to prove that

$$f^{(n)}(x) = e^x[x^2 + 2nx + n(n-1)]$$

where $n \in \mathbb{Z}^+$ and $f^{(n)}(x)$ represents the n^{th} derivative. [4]

a) $f(x) = x^2e^x$

$\therefore f'(x) = x^2e^x + e^x(2x)$

$= (x^2+2x)e^x$

$\therefore f''(x) = e^x(2x+2) + (x^2+2x)e^x$

$= (x^2+4x+2)e^x$

$\therefore f'''(x) = e^x(2x+4) + (x^2+4x+2)e^x$

$= (x^2+6x+6)e^x$

b) Prove $n=1$: LHS $= f'(x)$ $= (x^2+2x)e^x$

RHS $= e^x(x^2+2x+1(0))$ $= (x^2+2x)e^x$ $=$ LHS

$\therefore$ true for $n=1$

~~Prov~~ assume $n=k$: $f^{(k)}(x) = e^x(x^2+2kx+k(k-1))$

prove $n=k+1$: $f^{(k+1)}(x) = \frac{d}{dx}(f^{(k)})$

$= \frac{d}{dx}[(x^2+2kx+k(k-1))e^x]$

$= e^x(2x+2k) + (x^2+2kx+k(k-1))e^x$

$= (x^2+2k+2kx+2x+k(k-1))e^x$

$= (x^2+2(k+1)x+k^2-k+2k)e^x$

$= (x^2+2(k+1)x+(k+1)k)e^x$

$\therefore$ true for $n=k+1$ when $n=k$

$\therefore$ since true for $n=1$, and true for $n=k+1$ when $n=k$ is true, then true for all $n \in \mathbb{Z}^+$, by mathematical induction

7/

4. [Maximum mark: 8]

(a) Factorise $2x^2 - 3x - 5$. [2]

(b) Hence, or otherwise, find the coefficient of x^{23} in the expansion of $(2x^2 - 3x - 5)^{12}$, writing your answer in the form $k \times 2^m$ where $k, m \in \mathbb{Z}$. [6]

a) $= 2x^2 - 5x + 2x - 5$

$= 2x(x+1) - 5(x+1)$

$= (2x-5)(x+1)$

$2^6 = 64$

6

$64 \times 64 = 3816$

$2 \overline{)3816}$ 1908

$2 \overline{)3816}$

b) in $(2x-5)^{12} = (2x)^{12} + {}^{12}C_1 (2x)^{11}(-5) + \cdots + (-5)^{12}$

in $(x+1)^{12} = x^{12} + {}^{12}C_1 (x^{11})(1) + \cdots + 1^{12}$

$\therefore$ x^{23} will occur with: $(2x)^{12}({}^{12}C_1 x^{11}) + x^{12}({}^{12}C_1 (2x)^{11})$

$= 3816 . 12 x^{23} + 1908 x^{23}$

$3816 \times 12 = 9932$

$= 48000$

$= 24 \times 2^{23}$

3816×12: 7932, 38160, 46092, + 1908 = 48000

$T_{n+1} = {}^nC_r (2x)^{n-r}(-5)^{r} \times {}^nC_k (x)^{n-k}(1)^k$

$x^{23} = (2x)^{n-r}(x)^{n-k}$

$\therefore 23 = 12 - r - k$

$\therefore r = 1$

3/

5. [Maximum mark: 6]

(a) Find $\int x^2 \sin x \, dx$. [4]

(b) Evaluate $\int_{-1}^{1} x^2 \sin x \, dx$. [2]

a) $\int x^2 \sin x$

$u = x^2$ $\quad \frac{du}{} = 2x$

$dv = \sin x$ $\quad v = -\cos x$

$= -x^2 \cos x - \int -2x \cos x \, dx$

$= -x^2 \cos x + 2 \int x \cos x \, dx$

$u = x$ $\quad du = 1$

$= -x^2 \cos x + 2\left[x \sin x - \int \sin x \, dx\right]$

$dv = \cos x$ $\quad v = \sin x$

$= -x^2 \cos x + 2x \sin x - 2\int \sin x \, dx$

$= -x^2 \cos x + 2x \sin x + 2\cos x + c$

$= 2\cos x + 2x \sin x - x^2 \cos x + c$

b) $\int_{-1}^{1} x^2 \sin x = \left[2\cos x + 2x \sin x - x^2 \cos x\right]_{-1}^{1}$

$= 2\cos(1) + 2\sin(1) - \cos(1) - \left[2\cos(-1) + 2(-1)\sin(-1) - \cos(-1)\right]$

$= 2\cos(1) + 2\sin(1) - \cos(1) - 2\cos(1) \mp 2\sin(-1) + \cos(1)$

$= 0$

6

Q6

6. [Maximum mark: 9]

Let the probability that it rains on any one day be p and the weather on any day is independent of the weather on any other day.

(a) Using $p = 0.5$, find the probability that during a period of one week:

(i) it will rain on at least five ;

(ii) it will rain on the last day;

(iii) raining and non-raining days will alternate. [5]

(b) Find p, if during a full week period, it is equally likely that there will be five raining days as there will be six raining days. [4]

~~ai) $^7C_5 \times \frac{1}{2^7}$~~

~~$= \frac{7!}{5!\,2!} \times \frac{1}{128}$~~

~~$= \frac{7\times 6}{2} \times \frac{1}{128}$~~

~~$= 21/128$~~

~~$2^5 = 32$~~

~~$2^7 = 128$~~

~~aii) p(last) = 1/2~~

~~aiii) p(alternating) $= 2/128$~~

~~$= 1/64$~~

~~b)~~

COMPLETED IN ANSWER BOOKLET

7. [Maximum mark: 5]

Use Mathematical Induction to prove $1+\text{cis}\,\theta+\text{cis}\,2\theta+\ldots+\text{cis}\,n\theta=\dfrac{1-\text{cis}(n+1)\theta}{1-\text{cis}\theta}$.

prove for $n=0$: $LHS = 1$, ~~$RHS = \frac{1-\text{cis}(1)\theta}{1-\text{cis}\theta}$~~ $\cancel{\frac{1-(\cos\theta + i\sin\theta)\theta}{1-(\cos\theta - i\sin\theta)}}$

prove for $n=1$: $LHS = 1+\text{cis}\,\theta$

$RHS = \frac{1-\text{cis}\,2\theta}{1-\text{cis}\,\theta}$

$= \frac{1-\text{cis}^2\theta}{1-\text{cis}\,\theta}$

$= \frac{(1+\text{cis}\,\theta)(1-\text{cis}\,\theta)}{1-\text{cis}\,\theta}$

$= 1+\text{cis}\,\theta$ $\therefore$ true for $n=1$

assume $n=k$:

$1+\text{cis}\,\theta+\cdots+\text{cis}\,k\theta = \frac{1-\text{cis}[(k+1)\theta]}{1-\text{cis}\,\theta}$

prove for $n=k+1$:

$1+\text{cis}\,\theta+\cdots+\text{cis}\,k\theta + \text{cis}((k+1)\theta) = \frac{1-\text{cis}(k+2)\theta}{1-\text{cis}\,\theta}$

$= \frac{1-\text{cis}[(k+1)\theta]}{1-\text{cis}\,\theta} + \text{cis}((k+1)\theta)$ From assumption

$= \frac{1-\text{cis}[(k+1)\theta]}{1-\text{cis}\,\theta} + \cancel{\text{cis}\,k\theta\ \text{cis}\,\theta}$ {De MS}

$= \frac{1-\text{cis}[(k+1)\theta]+(1-\text{cis}\,\theta)(\text{cis}((k+1)\theta))}{1-\text{cis}\,\theta}$

$= \frac{1-\text{cis}[(k+1)\theta]+\text{cis}[(k+1)\theta] - \text{cis}\,\theta\,\text{cis}[(k+1)\theta]}{1-\text{cis}\,\theta}$

$= \frac{1-\text{cis}[(k+2)\theta]}{1-\text{cis}\,\theta} = RHS$

$\therefore$ true for $n=1$, $n=k+1$ when $n=k$ is true. Therefore true by mathematical induction

5/

Do NOT write solutions on this page.

Section B

Answer **all** the questions on the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 12]

A triangle has sides of length $(x+1)$, $(2x+1)$ and $(2x+3)$ cm.

(a) Show that $x > 1$. [3]

(b) Find the value of x for which that triangle is right-angled. [3]

(c) (i) Find, in terms of x, the cosine of the largest angle;
(ii) hence find the value of x for which one angle of the triangle is 120^0. [3]

(d) Find the value of x for which one angle of the triangle is 60^0. [3]

9. [Maximum mark: 12]

(a) Find the x-coordinates of the two stationary points on the curve $y = x^3 - 3x^2 - 2x - 6$. [3]

(b) Show that the x-coordinate of the point of inflexion is the mean of the x-coordinates of the two stationary points. [2]

(c) Write the equation of the tangent to the curve $y = x^3 - 3x^2 - 2x - 6$ at the point where it crosses the y-axis. [2]

(d) Evaluate the area enclosed by the tangent from (c) and the cubic curve. [5]

Do **NOT** write solutions on this page.

10. [Maximum mark: 26]

(a) (i) Show that the line l given by: [3]

$$\frac{x-1}{2} = y+2 = \frac{z+1}{3}$$

and the line m, defined by the parametric equations:

$x = 3\lambda + 2,\ y = -2\lambda + 2,\ z = \lambda + 4$ intersect.

(ii) Hence, find the coordinates of their point of intersection C. [3]

(b) Find the equation of the plane Π containing the lines l and m. [6]

(c) Determine the normal vector of the plane Π which has unit length. [4]

(d) Line k is perpendicular to the plane Π and passes through the point C. Find the coordinates of points P_1 and P_2 which lie on the line k and at a distance of 5 units from C. [5]

(e) Determine the equation of the plane which is parallel to the plane Π and passes through P_1. [5]

ANSWER BOOKLET
LIVRET DE RÉPONSES
CUADERNILLO DE RESPUESTAS

1

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

4 PAGES / PÁGINAS

Candidate session number: / Numéro de session du candidat: / Número de convocatoria del alumno:

20	T	R	L	P	1	-	M	A	H	L

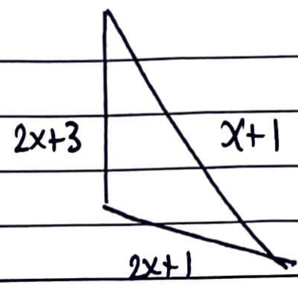
Candidate name: / Nom du candidat: / Nombre del alumno:

Please write question numbers in the following format: / Veuillez numéroter les questions en utilisant la présentation suivante: / Sírvase escribir los números de las preguntas en el siguiente formato:

1 2 3 4 5 6 7 8 9 10

8

a)



$c^2 = a^2 + b^2 - 2ab\cos C$

$\therefore (2x+1)^2 = (x+1)^2 + (2x+3)^2 - 2(x+1)(2x+3)\cos C$

$\therefore 4x^2+4x+1 = x^2+2x+1+4x^2+12x+9-2(2x^2+3x+2x+3)\cos C$

$\therefore \cancel{3x^2+1=2x+}$

$\therefore 4x+1 = x^2+2x+1+12x+9-2(2x^2+5x+3)\cos C$

$\therefore -10x = x^2+9-(4x^2+10x+6)\cos C$

$$\therefore \cos C = -\frac{-x^2-10x-9}{4x^2+10x+6}$$

$$= \frac{x^2+10x+9}{4x^2+10x+6} < 1$$

Simpler way:
triangle inequality:
side a + side b > side c

$\therefore x^2+10x+9 < 4x^2+10x+6$

$\therefore 0 < 3x^2-3$

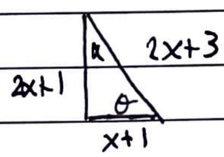
3

$\therefore x^2 > 1$

$\therefore \cancel{-1<x}$ $\cancel{x<1}$ $x>1, x<-1$

$\therefore x > 1$ $\{x>0\}$

b)



$\therefore \sin\alpha = \frac{x+1}{2x+3}$

$\cos\theta = x+1/2x+3$

$(x+1)^2+(2x+1)^2 = (2x+3)^2$

$\cancel{x+1} \; x^2+2x+1+4x^2+4x+1 = 4x^2+12x+9$

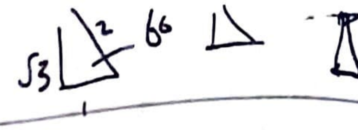
2

$\cancel{5x^2}$ $\therefore 5x^2+6x+2 = 4x^2+12x+9$

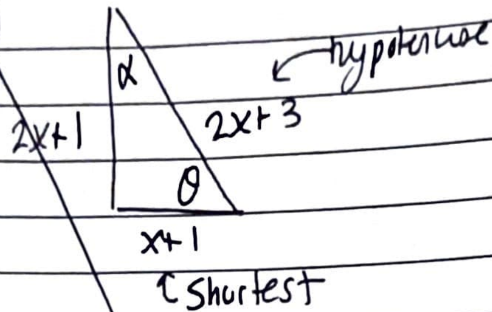
$\therefore x^2-6x-7=0$

$\therefore (x-7)(x+1)=0 \rightarrow x=7, \cancel{x=-1}$

6/



c(i)



$\theta > \alpha$ for all x

$\therefore \cos\theta = \cancel{\frac{2x+1}{x}} \; \frac{x+1}{2x+3}$

c(ii) $\frac{x+1}{2x+3} = \cos(120)$

$= -\cos(60)$

$= -1/2$

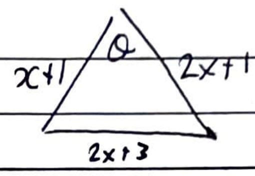
$\therefore 2x+2 = -2x-3$

$\therefore 4x = -5$

$\therefore x = -5/4$

d) cos

ci) $\cos\theta = \dfrac{(x+1)^2 + (2x+1)^2 - (2x+3)^2}{2(x+1)(2x+1)}$



$$= \frac{x^2+2x+1+4x^2+4x+1-4x^2-12x-9}{2(2x^2+x+2x+1)}$$

$$= \frac{x^2-6x-7}{4x^2+6x+2}$$

$\therefore\ 2(x^2-6x-7) = -(4x^2+6x+2)$

$\therefore\ 2x^2-12x-14 = -4x^2-6x-2$

$\therefore\ 6x^2-6x-12 = 0$

$\therefore\ x^2-x-2 = 0$

$\therefore\ (x-2)(x+1) = 0$

$\therefore\ x = -1,\ x = 2$

$\therefore\ x = 2 \quad \{x > 1\}$

a) $y = x^3 - 3x^2 - 2x - 6$

$\therefore dy/dx = 3x^2 - 6x - 2$

stationary : $3x^2 - 6x - 2 = 0$

$\therefore$ ~~3x~~ $x = \frac{6 \pm \sqrt{36 - 4(3)(-2)}}{6}$

$= \frac{6 \pm \sqrt{36 + 24}}{6}$

$= \frac{6 \pm \sqrt{60}}{6}$

$\therefore$ x-coordinates are $\frac{6 + \sqrt{60}}{6}$ and $\frac{6 - \sqrt{60}}{6}$

3

b) $d^2y/dx^2 = 6x - 6$

$\therefore 6x - 6 = 0$

$\therefore x = 1$

mean of stationary points $= \frac{\frac{6 + \sqrt{60} + 6 - \sqrt{60}}{6}}{2}$

$= \frac{12/6}{2}$

$= 2/2$

$= 1$

2

5/8

ANSWER BOOKLET
LIVRET DE RÉPONSES
CUADERNILLO DE RESPUESTAS

2

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

4 PAGES / PÁGINAS

Candidate session number: / Numéro de session du candidat: / Número de convocatoria del alumno: 20TRLP1-MAHL

Candidate name: / Nom du candidat: / Nombre del alumno:

Please write question numbers in the following format: / Veuillez numéroter les questions en utilisant la présentation suivante: / Sírvase escribir los números de las preguntas en el siguiente formato:

1 2 3 4 5 6 7 8 9 10

9

c) ~~$y = x^3 - 3x^2 - 2x - 6 = 0$~~

~~$\therefore x(x-3x) - 2(x+3) = 0$~~

~~$\therefore (x-2)$~~

$\therefore$ ~~$x($~~

$y = x^3 - 3x^2 - 2x - 6$ @ $x = 0$:

$\therefore y = -6$

when $y = -6$, $x = 0$,

therefore $m = dy/dx = 3(0)^2 - 6(0) - 2$
$= -2$

$\therefore y - y_1 = m(x - x_1)$

$\therefore y - (-6) = -2(x)$

$\therefore y = -2x - 6$ ✓ 2

2/

d) ~~tangent-intercept~~ : $\cancel{-2x-6=0}$

$\cancel{\therefore\ x = 3}$

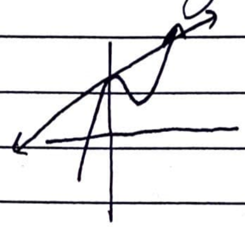
tangent-intercept w/ y :

$$-2x-6 = x^3-3x^2-2x-6$$

$$\therefore\ x^3-3x^2 = 0$$

$$\therefore\ x^2(x-3) = 0$$

$$\therefore\ \underbrace{x = 0, \quad x = 3}_{\text{bounds of the integral}}$$



$$\therefore A = \int_0^3 \left[(-2x-6)-(x^3-3x^2-2x-6)\right] dx$$

$$= \int_0^3 \left[3x^2-x^3\right] dx$$

$$= \left[x^3-\tfrac{1}{4}x^4\right]_0^3 \quad (3) \qquad 2$$

$$= 3^3-\tfrac{1}{4}(3^4)$$

$$= 3^3 - 81/4$$

81-12

$$= \cancel{12-81} \quad \frac{12-81}{4} \qquad \text{ECF}$$

$$= \cancel{69/4}\ -69/4$$

$\therefore A = 69/4$ units^2

3/

ANSWER BOOKLET
LIVRET DE RÉPONSES
CUADERNILLO DE RESPUESTAS

3

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

4 PAGES / PÁGINAS

Candidate session number: / Numéro de session du candidat: / Número de convocatoria del alumno:

20	T	R	L	P	1	-	M	A	H	L

Candidate name: / Nom du candidat: / Nombre del alumno:

Please write question numbers in the following format: / Veuillez numéroter les questions en utilisant la présentation suivante: / Sírvase escribir los números de las preguntas en el siguiente formato:

1 2 3 4 5 6 7 8 9 10

10

a i) $\frac{x-1}{2} = y+2 = \frac{z+1}{3}$ 3

$\therefore \quad x = 1 + 2\mu \quad .. (1)$

$y = -2 + \mu \quad .. (2)$

$z = -1 + 3\mu \quad .. (3)$

$x = 2 + 3\lambda \quad ... (4)$

$y = 2 - 2\lambda \quad ... (5)$

$z = 4 + \lambda \quad ... (6)$

$(1) \to (4): \quad 1 + 2\mu = 2 + 3\lambda$

$\therefore \quad 2\mu = 1 + 3\lambda \quad ... (7)$

$(2) \to (5): \quad -2 + \mu = 2 - 2\lambda$

$\therefore \quad \mu = 4 - 2\lambda \quad ... (8)$

$(7) \to (8): \quad 2(4 - 2\lambda) = 1 + 3\lambda$

$\therefore \quad 8 - 4\lambda = 1 + 3\lambda$

$\therefore \quad 7 \cancel{4\lambda} = 7\lambda$

$\therefore \quad \lambda = 1$ ✓ $... (9)$

~~(4) → (6)~~: $\therefore \quad \mu = 4 - 2 = 2$ ✓

test : (3) and (6).

$z = -1 + 3(2) = 5, \quad z = 4 + 1 = 5$ ✓

3

aii) at $\lambda = 1$, $x = 5$

$y = 0$

$z = 5$

3

$\therefore$ C: $(5, 0, 5)$

b) on following page

c) $b_1 = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$

$b_2 = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$

$\therefore \vec{n} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} -6-1 \\ 2-9 \\ 3+4 \end{pmatrix}$

$= \begin{pmatrix} -7 \\ -7 \\ 7 \end{pmatrix} \rightarrow = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$

$\therefore -7x - 7y + 7z = d$

5

at point C: $-7(5) - 7(0) + 7(5) = d$

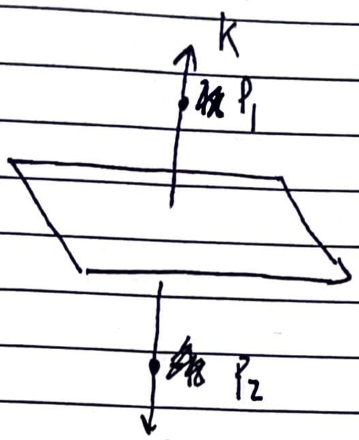
$\therefore d = 0$

hence, $7x - 7y + 7z = 0$

8/

b) $r = \begin{pmatrix}5\\0\\5\end{pmatrix} + \begin{pmatrix}3\\-2\\1\end{pmatrix}\lambda + \begin{pmatrix}2\\1\\3\end{pmatrix}\mu$ 1

d)



line $k = \begin{pmatrix}5\\0\\5\end{pmatrix} + m\begin{pmatrix}-7\\-7\\7\end{pmatrix}$

$\left|\begin{pmatrix}-7\\-7\\7\end{pmatrix}\right| = \sqrt{49+49+49}$ 5

$= 7\sqrt{3}$

$\therefore 7m\sqrt{3} = 5$

$\therefore m = 5/7\sqrt{3}$

$\therefore P_1 : \begin{pmatrix}5\\0\\5\end{pmatrix} + \frac{5}{7\sqrt{3}}\begin{pmatrix}-7\\-7\\7\end{pmatrix}$

$= \begin{pmatrix}5 - 5/\sqrt{3}\\0 - 5/\sqrt{3}\\5 + 5/\sqrt{3}\end{pmatrix}$

$= \left(5 - \frac{5}{\sqrt{3}}, -\frac{5}{\sqrt{3}}, 5 + \frac{5}{\sqrt{3}}\right)$

$\therefore P_2 : \left(5 + \frac{5}{\sqrt{3}}, \frac{5}{\sqrt{3}}, 5 - \frac{5}{\sqrt{3}}\right)$

6

e) $$r = \begin{pmatrix} 5-\frac{5}{\sqrt{3}} \\ -5/\sqrt{3} \\ 5+5/\sqrt{3} \end{pmatrix} + \lambda\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} + \mu\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$$

5

ANSWER BOOKLET
LIVRET DE RÉPONSES
CUADERNILLO DE RESPUESTAS

4

4 PAGES / PÁGINAS

Candidate session number: / Numéro de session du candidat: / Número de convocatoria del alumno:

20 T R L P 1 - M A H L

Candidate name: / Nom du candidat: / Nombre del alumno:

Please write question numbers in the following format: / Veuillez numéroter les questions en utilisant la présentation suivante: / Sírvase escribir los números de las preguntas en el siguiente formato:

1 2 3 4 5 6 7 8 9 10

6

ai.) $\cancel{\frac{1}{2^5} + \frac{1}{2^6} + \frac{1}{2^7}}$ $P(x \geq 5) = P(x=5) + P(x=6) + P(x=7)$

$= {}^7C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^2 + {}^7C_6 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right) + {}^7C_7 \left(\frac{1}{2}\right)^7$

aii). $1/2$ $= \left(\frac{1}{2}\right)^7 \left({}^7C_5 + {}^7C_6 + {}^7C_7\right)$

$= \frac{1}{2^7} \times 29$

aiii) $2/2^7 = 1/64$ $= 29/128$

$\frac{7!}{5!\,2!} = \frac{7 \times 6}{2} = \frac{42}{2}$

$= 21$

b.) $\cancel{P(5) = P(6)}$

$\therefore {}^7C_5\, p^5 = {}^7C_6\, p^6$

$\therefore \cancel{21p\; 7p}$

$\therefore p = 21/7$

$= 3$

$P(x=5) = P(x=6)$

$\therefore {}^7C_5\, p^5(1-p)^2 = {}^7C_6\, p^6(1-p)$

$\therefore 21(1-p) = 7p$

$\therefore 3 - 3p = p$

$\therefore 4p = 3$

$\therefore p = 3/4$

3/