Markscheme

Additional Practice

Counting Principles (Non-Calculator)

ID: 4003

Mathematics: analysis and approaches

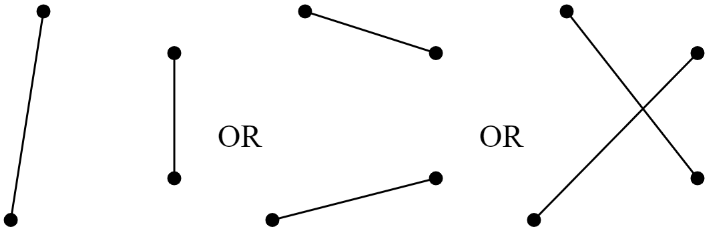
Higher level

1. The total number of ways of doing this is ${}^5C_3 = 10$. A1

To make a triangle we need to choose $\{2,3,4\}$, $\{2,4,5\}$ or $\{3,4,5\}$. A1A1

So the probability is $\frac{3}{10}$. A1

2. From the four points that are chosen there are three ways in which the two lines can be drawn. These are shown below. M1 A1A1



The probability is therefore $\frac{1}{3}$. A1

3. (a) $\frac{1}{5}$ A1A1

(b) $\frac{2}{5!}=\frac{1}{60}$ M1A1

4. The total number of ways of choosing 3 digits from 10 is ${}^{10}C_3 = \dfrac{10!}{3!7!} = 120$. M1A1

There is only one way to arrange these in increasing order. R1

The total number of ways of choosing 3 digits is $10^3 = 1000$. A1

The probability is therefore $\dfrac{120}{1000} = 0.12$. A1

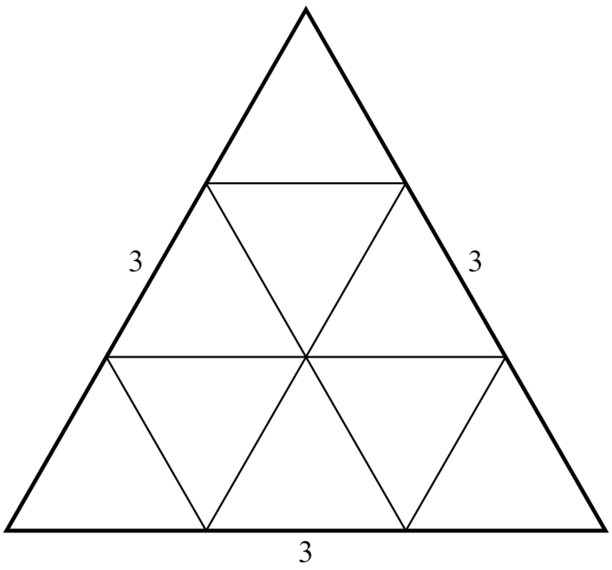
5. The number of combinations of horror and comedy is $5 \times 6 = 30$. A1

The number of combinations of comedy and drama is $6 \times 3 = 18$. A1

The number of combinations of horror and drama is $5 \times 3 = 15$. A1

The total number of combinations is therefore $30 + 18 + 15 = 63$. M1A1

6. (a) A1A1



(b) If we add 10 points then at least two of them must be inside the same smaller triangle. R1

Since each smaller triangle has sides of length 1 then at least two points must be a maximum of 1 apart. R1 A1

7. (a) 1, 2, 3 and 6 A1

(b) ${}^4P_2 = 12$ M1A1

(c) Members of set B that have rational roots. A1

(d) $x^2 + 3x + 2 = 0$ A1

$x^2 + 2x + 1 = 0$ A1

8. (a) We have

$$\frac{N!}{r!(N-r)!} \cdot \frac{(N-r)!}{(n-r)!(N-n)!} = \frac{N!}{r!(n-r)!(N-n)!} \qquad \text{M1A1}$$

(b) Use binomial expansion

$$(1+1)^n = \sum_{r=0}^{n} {}^{n}\mathrm{C}_r = 2^n \qquad \text{M1A1}$$

(c) We have

$$\sum_{r=0}^{N} \frac{N!}{r!(n-r)!(N-n)!} = \frac{N!}{(N-n)!n!} \sum_{r=0}^{n} \frac{n!}{r!(n-r)!} = \frac{2^n \cdot {}^{N}\mathrm{P}_n}{n!} \qquad \text{M1A1}$$

9. (a) $6! = 720$ M1A1

(b) $\dfrac{2}{3!} = \dfrac{1}{3}$ M1A1

(c) $\dfrac{1}{3} \times 720 = 240.$ M1A1

10. (a) We have

$$(1+1)^n = \sum_{k=0}^{n} {}^nC_k = 2^n \qquad \text{M1A1}$$

(b) One of the factors is 1. A1

All other factors are formed by the product of k prime factors. R1

The total number of factors is therefore

$$1 + \sum_{k=1}^{n} {}^nC_k = 1 + 2^n - 1 = 2^n \qquad \text{M1A1}$$

11. The number of ways of placing the mathematics books is $5!$ A1

This creates six spaces where we can place a single science book. R1

The number of ways of placing the three science books in these six spaces is

$$6 \cdot 5 \cdot 4 = 120$$ M1A1

The number of ways of placing all the books in any order is $8!$ A1

The overall probability is therefore

$$\frac{5! \times 120}{8!} = \frac{5}{14} \approx 0.357$$ M1A1

12. (a) The final digit must be a 0 or a 5. R1

So the total number of ways is $2 \times 9 \times 8 = 144$. M1A1

(b) The final two digits must be a multiple of 4. R1

The possibilities are 04, 08, 12, 16, 20, 24, 28, 32, 36, 40, 48, 52, 56, 60, 64, 68, 72, 80, 76, 84, 92, 96. A1

So the total number of ways is $8 \times 22 = 176$. M1A1

13. If they all finish at different times then this can happen in $4!$ ways. A1

Two of the runners could finish at the same time. There are ${}_4C_2$ ways of choosing these two runners. There are $3!$ ways in which this group of two, and the other two runners could finish. A1 A1

Two of the runners could finish at the same time, and the other two could also finish at the same time. There are ${}_4C_2$ ways of choosing the two runners who finish first (the other two runners automatically finish next). A1

Three of the runners could finish at the same time. There are ${}_4C_3$ ways of choosing these three runners. There are 2 ways in which this group of three and the other runner could finish. A1 A1

All four runners could finish at the same time. There is 1 way in which this can happen. A1

Combining all of these values together gives M1

$$4! + {}_4C_2 \times 3! + {}_4C_2 + {}_4C_3 \times 2 + 1 = 75$$ A1

14. (a) $>$ A1

(b) 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 A1

(c) ${}^{10}C_3 = 120$ M1A1

(d) If a triangle has a side of length 1 then we cannot choose two other lengths so that the inequality in part (a) is satisfied because the minimum difference in the lengths of each side is 1. R1 R1

(e)

(i) If the shortest length is 2 then the difference between the other two sides must be 1. R1

So we have $\{2,3,4\}$, $\{2,4,5\}$, $\{2,5,6\}$, $\{2,6,7\}$, $\{2,7,8\}$, $\{2,8,9\}$ and $\{2,9,10\}$ (or $10 - 2 - 1 = 7$ sets). A1

(ii) If the shortest length is 3 then the difference between the other two sides must by 1 or 2. R1

So we have $(10 - 3 - 1) + (10 - 4 - 1) = 6 + 5 = 11$ sets. A1

(f) If the shortest side is 4 then we have $5 + 4 + 3 = 12$ triangles. A1

If the shortest side is 5 then we have $4 + 3 + 2 + 1 = 10$ triangles. A1

If the shortest side is 6 then we have $3 + 2 + 1 = 6$ triangles. A1

If the shortest side is 7 then we have $2 + 1 = 3$ triangles. A1

If the shortest side is 8 then we have 1 triangle. A1

The total is therefore $7 + 11 + 12 + 10 + 6 + 3 + 1 = 50$ triangles. A1

(g) $\frac{50}{120} = \frac{5}{12}$ A1

(h) The only right angled triangles are the ones with sides of lengths $\{3,4,5\}$ and $\{6,8,10\}$. M1

So the probability is $\frac{2}{50} = \frac{1}{25}$. A1