Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 2

Friday 11 November 2016 (morning)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[120 marks]**.

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

A random variable X has a probability distribution given in the following table.

x	0.5	1.5	2.5	3.5	4.5	5.5
$\mathrm{P}(X=x)$	0.12	0.18	0.20	0.28	0.14	0.08

(a) Determine the value of $\mathrm{E}(X^2)$. [2]

(b) Find the value of $\mathrm{Var}(X)$. [3]

2. [Maximum mark: 5]

Find the acute angle between the planes with equations $x + y + z = 3$ and $2x - z = 2$.

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3. [Maximum mark: 6]

A discrete random variable X follows a Poisson distribution $\mathrm{Po}(\mu)$.

(a) Show that $\mathrm{P}(X = x + 1) = \dfrac{\mu}{x + 1} \times \mathrm{P}(X = x), x \in \mathbb{N}$. [3]

(b) Given that $\mathrm{P}(X = 2) = 0.241667$ and $\mathrm{P}(X = 3) = 0.112777$, use part (a) to find the value of μ. [3]

4. [Maximum mark: 5]

Find the constant term in the expansion of $\left(4x^2 - \dfrac{3}{2x}\right)^{12}$.

5. [Maximum mark: 9]

Consider the function f defined by $f(x) = 3x \arccos(x)$ where $-1 \le x \le 1$.

(a) Sketch the graph of f indicating clearly any intercepts with the axes and the coordinates of any local maximum or minimum points. [3]

(b) State the range of f. [2]

(c) Solve the inequality $|3x \arccos(x)| > 1$. [4]

6. [Maximum mark: 6]

An earth satellite moves in a path that can be described by the curve $72.5x^2 + 71.5y^2 = 1$ where $x = x(t)$ and $y = y(t)$ are in thousands of kilometres and t is time in seconds.

Given that $\frac{dx}{dt} = 7.75 \times 10^{-5}$ when $x = 3.2 \times 10^{-3}$, find the possible values of $\frac{dy}{dt}$.
Give your answers in standard form.

7. [Maximum mark: 8]

In a triangle ABC, $\mathrm{AB} = 4\,\mathrm{cm}$, $\mathrm{BC} = 3\,\mathrm{cm}$ and $\mathrm{B\hat{A}C} = \frac{\pi}{9}$.

(a) Use the cosine rule to find the two possible values for AC. [5]

(b) Find the difference between the areas of the two possible triangles ABC. [3]

8. [Maximum mark: 8]

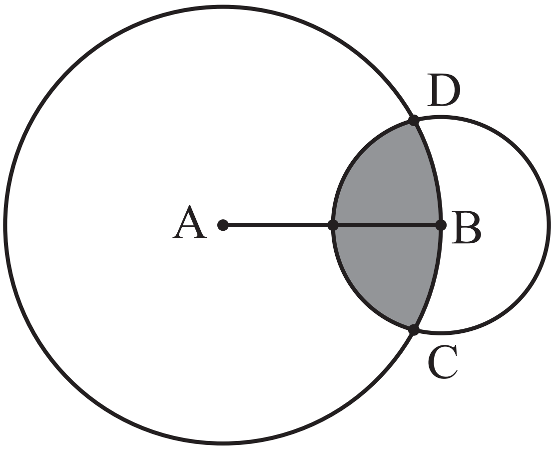
A random variable X is normally distributed with mean μ and standard deviation σ, such that $\mathrm{P}(X < 30.31) = 0.1180$ and $\mathrm{P}(X > 42.52) = 0.3060$.

(a) Find μ and σ. [6]

(b) Find $\mathrm{P}(|X - \mu| < 1.2\sigma)$. [2]

9. [Maximum mark: 8]

The diagram shows two circles with centres at the points A and B and radii $2r$ and r, respectively. The point B lies on the circle with centre A. The circles intersect at the points C and D.



Let α be the measure of the angle CAD and θ be the measure of the angle CBD in radians.

(a) Find an expression for the shaded area in terms of α, θ and r. [3]

(b) Show that $\alpha = 4\arcsin\frac{1}{4}$. [2]

(c) Hence find the value of r given that the shaded area is equal to 4. [3]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 22]

Let the function f be defined by $f(x) = \dfrac{2 - e^x}{2e^x - 1}$, $x \in D$.

(a) Determine D, the largest possible domain of f. [2]

(b) Show that the graph of f has three asymptotes and state their equations. [5]

(c) Show that $f'(x) = -\dfrac{3e^x}{(2e^x - 1)^2}$. [3]

(d) Use your answers from parts (b) and (c) to justify that f has an inverse and state its domain. [4]

(e) Find an expression for $f^{-1}(x)$. [4]

(f) Consider the region R enclosed by the graph of $y = f(x)$ and the axes.
Find the volume of the solid obtained when R is rotated through 2π about the y-axis. [4]

Do **not** write solutions on this page.

11. [Maximum mark: 20]

A Chocolate Shop advertises free gifts to customers that collect three vouchers. The vouchers are placed at random into 10% of all chocolate bars sold at this shop. Kati buys some of these bars and she opens them one at a time to see if they contain a voucher. Let $\mathrm{P}(X = n)$ be the probability that Kati obtains her third voucher on the nth bar opened.

(It is assumed that the probability that a chocolate bar contains a voucher stays at 10% throughout the question.)

(a) Show that $\mathrm{P}(X = 3) = 0.001$ and $\mathrm{P}(X = 4) = 0.0027$. [3]

It is given that $\mathrm{P}(X = n) = \dfrac{n^2 + an + b}{2000} \times 0.9^{n-3}$ for $n \geq 3$, $n \in \mathbb{N}$.

(b) Find the values of the constants a and b. [5]

(c) Deduce that $\dfrac{\mathrm{P}(X = n)}{\mathrm{P}(X = n-1)} = \dfrac{0.9(n-1)}{n-3}$ for $n > 3$. [4]

(d) (i) Hence show that X has two modes m_1 and m_2.

(ii) State the values of m_1 and m_2. [5]

Kati's mother goes to the shop and buys x chocolate bars. She takes the bars home for Kati to open.

(e) Determine the minimum value of x such that the probability Kati receives at least one free gift is greater than 0.5. [3]

Do **not** write solutions on this page.

12. [Maximum mark: 18]

On the day of her birth, 1st January 1998, Mary's grandparents invested $\$x$ in a savings account. They continued to deposit $\$x$ on the first day of each month thereafter.
The account paid a fixed rate of 0.4% interest per month. The interest was calculated on the last day of each month and added to the account.
Let $\$A_n$ be the amount in Mary's account on the last day of the nth month, immediately after the interest had been added.

(a) Find an expression for A_1 and show that $A_2 = 1.004^2x + 1.004x$. [2]

(b) (i) Write down a similar expression for A_3 and A_4.

(ii) Hence show that the amount in Mary's account the day before she turned 10 years old is given by $251(1.004^{120} - 1)x$. [6]

(c) Write down an expression for A_n in terms of x on the day before Mary turned 18 years old showing clearly the value of n. [1]

(d) Mary's grandparents wished for the amount in her account to be at least $\$20\,000$ the day before she was 18. Determine the minimum value of the monthly deposit $\$x$ required to achieve this. Give your answer correct to the nearest dollar. [4]

(e) As soon as Mary was 18 she decided to invest $\$15\,000$ of this money in an account of the same type earning 0.4% interest per month. She withdraws $\$1000$ every year on her birthday to buy herself a present. Determine how long it will take until there is no money in the account. [5]

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