Mathematics: analysis and approaches

Higher level

Paper 3

ID: 3013

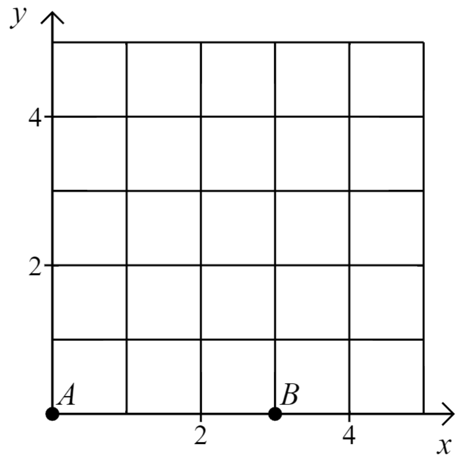
Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[55 marks]**.

1. [Maximum points: 28]

In this problem you will investigate the probability of two people walking past each other when walking from one corner of a grid to the other.

The diagram below represents a map of Alice's and Bob's neighbourhood. (A)lice is located at point (0,0) and (B)ob is located at point (3,0).



Alice wishes to travel to point (3,3) randomly travelling east and north at each intersection (if there is a choice). For example if she walks north, then east, then north, then north, she will have no choice but to go east and east to reach her destination.

Bob wishes to travel to point (0,3) randomly travelling west and north at each intersection (if there is a choice). Alice and Bob leave at the same time and walk at the same speed.

(a) **Copy the diagram** above and label the only four road segments where it is possible for Alice and Bob to pass each other. [2]

(b) Calculate the number of ways Alice can reach point [7]

(i) (1,0)

(ii) (1,1)

(iii) (1,2)

(iv) (1,3)

(c) List all of the ways Alice can reach point (1,3). [2]

(d) Explain why the probabilities of Alice travelling each of the routes in part (c) are not all equal. [3]

(e) Show that the probability that Alice and Bob pass each other is equal to 0.258 to 3 significant figures. [5]

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The start and finish points are now changed according to the table below.

	Alice	Bob
Start	(0,0)	(5,0)
Finish	(5,5)	(0,5)

They continue to travel randomly in the same directions as described previously. They leave at the same time and walk at the same speed.

(f) Calculate the probability that Alice and Bob pass each other. [9]

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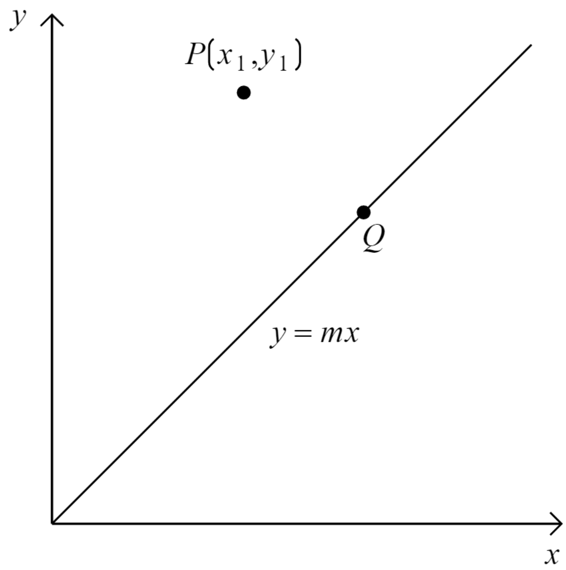
2. [Maximum points: 27]

The following graph shows the point P with coordinates (x_1, y_1) and the line $y = mx$. The point Q represents the closest point to P which lies on the line $y = mx$.



(a) Write down the gradient of line PQ. Give a reason for your answer. [2]

(b) In terms of x_1, y_1 and m determine [7]

(i) the x-coordinate of point Q

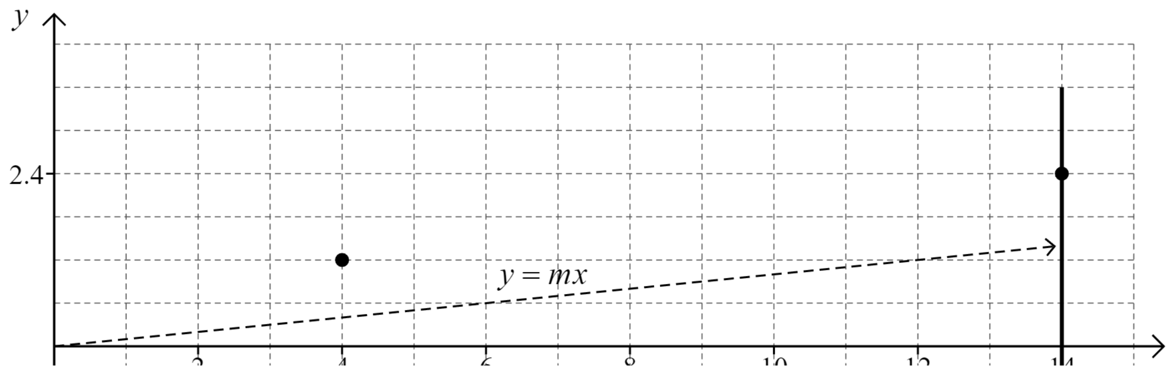
(ii) the y-coordinate of point Q

(c) Let the length of PQ be equal to d. Show that [6]

$$d^2 = \frac{(y_1 - x_1 m)^2}{m^2 + 1}$$

In a football match a player, located at point (0,0), is in control of the ball. The goal stretches from point (14,3.6) to (14,-3.6). Between the player and the goal there are four players from the opposing team. The player in control of the ball shoots at the goal and the ball follows the line $y = mx$.

This is shown in the diagram below. The four points represent the four players of the opposing team.



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(d) Determine an expression for the sum of the square of the distances from each player to the path of the ball in terms of *m*. [4]

(e) Determine the value of *m* that ensures the ball meets the goal, but also maximises the value of the expression found in part (d). [4]

(f) Comment on whether you think the value found in part (e) represents the path of a ball that has the best chance of scoring a goal. Explain why you think this model produces such a result. [4]

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