QUANTITATIVE EQUITY STRATEGY BASED ON BOOK-TO-MARKET RATIO

ACFI 234 - Theory of Finance II

Date: 15/05/2025 Word Count: 3000

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Executive Summary

Methodology

Fama-Macbeth two-step regression

- Run monthly cross-sectional regressions (OLS), regressing excess stock returns on the set of predictors.
- Transformed skewed variables by using natural logarithms.
- Take time-series average of monthly regression coefficients- compute t-stats and p-values

$$R_{it} - R_{ft} = \alpha_i + \beta_{i1}\mathrm{F}_{1t} + \beta_{i2}\mathrm{F}_{2t}$$

Equally weighted quintile portfolios

- Monthly, and based on their B/M ratios we sort all stocks into five quintiles (Q1-Q5).
- Compute each quintile, equally weighted average return for the next month.
- Construct zero-cost long-short portfolio by going long Q5 and short Q1. Calculate the return of this portfolio.

Factor Attribution Analysis

- Regress monthly returns of the long-short portfolio on the Fama-French factor returns.

$$\alpha_p + \beta_{MKT} \cdot \mathrm{MKT}_t + \beta_{SMB} \cdot \mathrm{SMB}_t + \beta_{HML} \cdot \mathrm{HML}_t + \mathrm{RMW}_t \cdot \beta_{RMW} + \mathrm{CMA}_t \cdot \beta_{CMA}$$

Value Weighted Portfolio Analysis and Factor Attribution

- Same steps as equally weighted quintile, instead of weighting equally, each stock was weighted by its lagged market capitalisation from the previous month.
- Again, a long-short strategy was created, and we ran time-series regressions using the Fama-French 3-factor and 5-factor models.

Dependent Double Sorting Analysis

- All stocks sorted into five quintiles based on lagged investment levels at the end of each month. Then further sorted into five quintiles based on their B/M ratio. Creating 25 portfolios representing different combinations of investment and B/M, all equally weighted and rebalanced monthly.
- Each portfolio, I computed average excess returns, average B/M and average investment levels.
- Created a long-short strategy of long the average of the top five portfolios with the highest B/M and short the 5 lowest B/M

Key Findings

Fama-Macbeth regression

The results show us that the Book-to-Market ratio is statistically significant when predicting the future returns, (t-stat = 3.78, p-value=0.002). A positive coefficient indicates value stocks tend to earn higher excess returns than growth stocks.

Additionally, the size predictor shows negative relationship with expected returns, profitability predictor is positively related and have a significant relationship and investment shows a negative significant relationship. These results are all consistent with Fama-French findings.

Equally weighted quintile portfolios

The results of the portfolio analysis are consistent with the findings from Fama-Macbeth regression. Average returns increase as we go from Q1 (low B/M) to Q5 (High B/M). The long-short portfolio generates a positive average return with a statistically significant t-stat.

Factor Attribution Analysis

The Fama-French 3 factor regression shows that the long-short strategy produces strong and statistically significantly excess returns, with an alpha (abnormal returns) of 0.73% and a t-stat of 7.25, which indicates returns are not explained by market, size or value factors. The 3-factor model explain 67.7% of the variation in the long-short returns.

The Fama-French 5 factor model regression is still statistically significant with an alpha of 0.53%, but smaller than the 3-factor model, suggesting that adding profitability and investment factors reduce abnormal returns. This model explains 72.5% of the variation of the returns.

Value Weighted Portfolio Analysis

The value weighted returns were lower than the equally weighted portfolio, especially in Q1 and Q5. The long-short strategy stayed statistically significant but excess returns decreased slightly. Indicating a weaker performance when portfolio weights account for firm size. When observing the factor model regressions, they showed similar patterns, as the alpha remained positive and significant, and the investment and profitability factors explained some excess returns. Overall, it suggests small cap stocks play a crucial role in the profitability of the B/M strategy.

Dependent Double Sorting Analysis

The value effect was persistent across all investment quintiles, suggesting it is a robust anomaly. However, the magnitude of the value effects varied across investment quintiles as higher investment stocks showed lower returns across all B/M portfolios. The enhanced strategy underperformed when comparing to the original strategy as it generated a lower average return and t-statistic, suggesting the conditioning on investment reduced the profitability of the B/M strategy. overall, the analysis highlighted the importance of considering multiple factors.

Introduction

This report will dive into the efficient market hypothesis (EMH), which is a cornerstone of financial theory, which states that asset prices are fully reflective of all available information, which makes attaining consistent abnormal returns impossible. However, evidence has suggested against this as there have been anomalies that propose the idea that some factors can have some predictive power in future stock returns.

One factor that this report will look at closely is Book-to-market ratio (B/M). The evidence suggests value stocks (high B/M) outperforms growth stocks (low B/M). this was initially recognised by Fama and French (1992) and has stayed true through many market cycles justifying its validity. This anomaly disregards the EMH and leads to the potential for active investment strategies, which is what this research is aimed at.

The investigation into the B/M ratio's predictive power will address certain questions; does the B/M ratio reliably predict stock returns after controlling for other established factors? Is the B/M premium strong across equally and value weighted portfolios? Can standard factor models, such as the Fama-French 3-factor and 5-factor models explain returns associated with the B/M predictor?

Theoretical Background

In financial models, there are observations that firm characteristics appear to the predict future stock returns, this challenges the traditional financial theory of market efficiency, which suggests all information available is priced into the stock already, and abnormal returns are impossible to maintain. This section tries to explain reason for this predictability, does it stem from market inefficiency or does it reflect underlying risk factors.

Market inefficiencies arise due to behavioural biases as investors aren't always rational. Overconfidence and herding are common mistakes that can lead to mispricing of stocks. For example, a good earnings release may lead to an overreaction and driving making the stocks overpriced, and vice versa. Even if some investors are completely rational, they may face financial constraints that prevent them from benefit from the arbitrage situation, such as transaction costs and short-selling restrictions. This results in the mispricing of assets continuing and allowing predictable patterns to arise.

On the other hand, are risk-related explanations, some characteristics may be proxies for underlying risk factors that aren't captures in models. Which explains why some characteristics predict higher returns. Additionally, risk may vary over time, for example small cap stocks usually see higher risk in times of economic downturn.

Empirical Results

Fama-Macbeth Regression

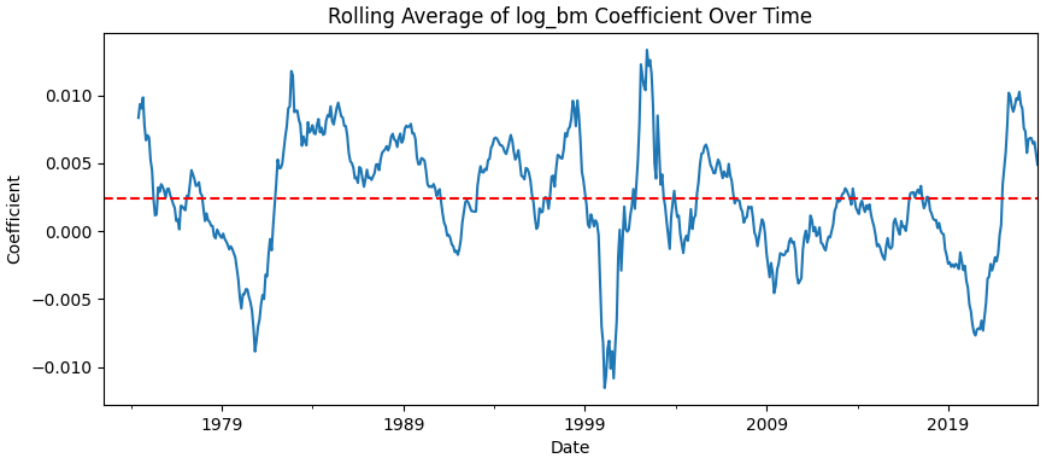
A Fama-Macbeth two-step methodology that was employed to understand the predictive power of the Book-to-Market (B/M) ratio.

Table 1	*Coefficient*	*Std. error*	*t-stat*	*p-value*
Intercept	0.014	0.0038	3.8044	0.0002
Size	-0.0013	0.0004	-2.9429	0.0034
Profitability	0.0045	0.0011	4.2918	<0.0001
Investment	-0.0013	0.0003	-4.2862	<0.0001
B/M ratio	**0.002**	**0.006**	**3.7794**	**0.002**

The positive and statistically significant coefficient on B/M ratio leads to the conclusion that firms with higher book-to-market ratios have higher expected returns which aligns with the Fama-French literature. This effect remains statistically significant after controlling for size, profitability and investment.

Additionally, the negative and statistically significant coefficient on size also aligns with the 'size effect' where smaller firms outperform larger ones. Profitability has a positive impact on future excess returns, whilst investment shows a negative relationship with returns.

Overall, the regression output supports the claim that B/M ratio has predictive power over future excess returns. The following figure shows the rolling average of the regression coefficient for the book-to-market ratio, demonstrating constant predictive power.

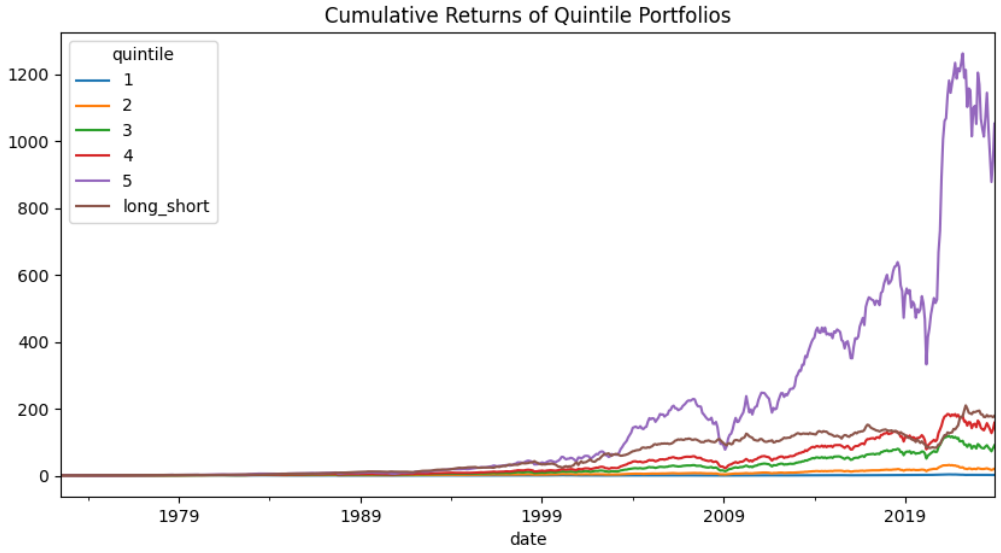


Equally weighted quintile portfolios

Firms were sorted into quintiles based on the natural logarithms of their book-to-market ratio each month. Portfolios were equally weighted to avoid large-cap dominance. Table 2 represents the average excess returns of each portfolio. Short-long portfolio is created to exploit mispricing's and hedges market risk.

Table 2	*Q1(low b/m)*	*Q2*	*Q3*	*Q4*	*Q5(High b/m)*	*Long Short*
Avg Monthly Returns (%)	0.42	0.59	0.78	1.01	1.23	0.81

The output shows a clear pattern, average returns rise with increasing B/M. Stocks with greater B/M outperform low B/M stocks by 0.81% on average shown through the long-short ratio (9.7% annually) . The t-statistic (5.83 and p-value <0.0001) shows statistical significance meaning it is robust over time. The long-short portfolio exploits the positive relationship between B/M and returns. The graph below plots the cumulative returns of the quintile portfolios and the long-short strategy, highlighting how the higher B/M stocks consistently outperform the lower.



Factor Attribution Analysis

To determine whether the returns generated from the long-short portfolio are compensation of taking on well-known risks, we run time-series regressions of the long-short portfolios returns on the Fama-French factors. There are two models, a 3-factor model which is made up of; market risk that represents the excess return of the overall market over the risk-free rate, size risk that captures the historical tendency of small-cap stocks outperforming large-cap stocks, and value risk which captures the historical tendency of value stocks to outperform growth stocks. The 5-factor model has an additional two factors, profitability risk that represents how profitable companies tend to outperform less profitable companies, and investment risk that captures how firms who tend to invest les outperform those who do.

Table 3 ***Fama-French 3-Factor Model***

	Coefficient	Std. Error	t-stat	p- value
Alpha	0.0073	0.001	7.254	0.000
Market	-0.0018	0.000	-6.204	0.000
Size	0.0003	0.000	0.706	0.480
Value	0.0095	0.001	18.833	0.000
R-squared	**0.677**			

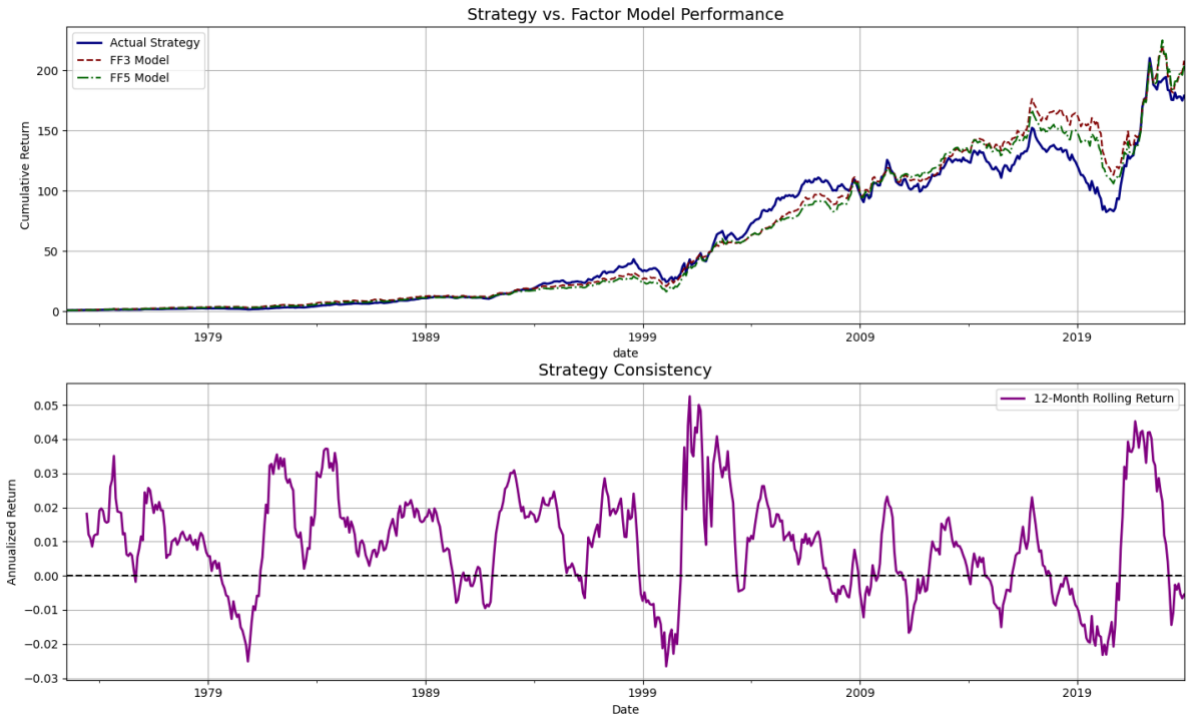
The 3-factor model regression revealed a highly statistically significant alpha (abnormal returns) of 0.73% per month, indicating how the book-to-market based long-short portfolio generated returns what is explained by the market, size and value factors. The market beta (-0.0018 and significant) suggests that the strategy tends to perform well when the overall market declines, indicating defensive characteristic. A significant positive value beta is consistent with theory of value stocks outperforming growth stocks. However, small and insignificant size beta indicates returns aren't related to the size factor. The 3-factor model can explain 67.7% of the variance in the strategy's returns, revealed by the R-squared value.

Table 4 ***Fama-French 5-Factor Model***

	Coefficient	Std. Error	t-stat	p- value
Alpha	0.0053	0.001	5.364	0.000
Market	-0.0013	0.000	-5.427	0.000
Size	0.0013	0.000	2.641	0.008
Value	0.0077	0.000	15.698	0.000
Profitability	0.0036	0.001	6.429	0.000
Investment	0.0033	0.001	3.645	0.000
R-squared	**0.725**			

Extending the analysis to Fama-French 5-factor model allows for more detailed and potentially more accurate results. We saw a reduction of the monthly alpha to 0.53%, although it kept statistically significant, this suggests the addition of profitability and investment factors accounted for some of the abnormal returns observed in the 3-factor model. The positive market beta and negative value beta continued, however at a reduced magnitude. The model showed positive coefficients for profitability and investment, which indicates that our strategy performs when profitable and conservative firms perform better. The 5-factor model explains 72.5% of the variations, which is an improvement on the 3-factor model, suggesting profitability and investment factors are relevant when explaining the strategies returns.

These findings indicate that there is still Prescence of abnormal returns which aren't covered by the F-F 3 and F-F 5 models, suggesting for potential market inefficiencies or unmodeled risks. The figure below illustrates the cumulative performance of the actual long-short strategy vs the Fama-French 3 and 5 factor models. The lower plot shows the 12-month rolling returns, highlighting the strategies varying consistency and ability to deliver excess returns.



Value Weighted Portfolio Analysis

To understand how weighting affects the strategy's results, we must redo the previous steps in this report, create value-weighted quintiles, create a long-short strategy, calculate performance statistics and then run Fama-French 3-Factor and 5-Factor regressions on the value-weighted long-short portfolio. Value-weighting reflects market reality as higher market cap stocks often dominate portfolios; value-weighting reduces the bias towards small stocks. Comparing the results of equally and value weighted portfolios serves as a robustness checker, if the B/M anomaly is robust then it should appear across different weighting schemes.

Table 5 *Equally Weighted vs Value-Weighted Quintiles*

Avg Monthly Returns (%)

Portfolio	Q1(low b/m)	Q2	Q3	Q4	Q5(High b/m)	Long Short
Equally Weighted	0.41	0.69	0.90	0.98	1.33	0.92
Value Weighted	0.53	0.64	0.66	0.70	0.83	0.30

The value-weighted long-short portfolio sees lower average monthly returns (0.30%) than the equally weighted returns (0.92%), and its statistical significance was marginal (t-stat = 1.8033, p= 0.0718). This difference suggests that the profitability of the B/M strategy is influenced heavily by small-cap stocks, as the value weighted strategy puts less power on small-cap stocks.

Table 6 *Fama-French **3-Factor** Model*

Value Weighted

	Coefficient	Std. Error	t-stat	p- value
Alpha	-0.0011	0.001	-1.517	0.129
Market	0.0001	0.000	0.266	0.790
Size	0.0035	0.000	8.585	0.000
Value	0.0117	0.000	29.821	0.000
R-squared	**0.797**			

For the value-weighted strategy, following the 3-factor regression, there was a negative, statistically insignificant alpha of -0.11% per month, suggesting no excess returns after accounting for the market, size and value factors. Which contrasts to the equally weighted strategy which showed a positive and statistically significant alpha (0.73%). Additionally, the market beta was found insignificant when testing on the value weighted strategy which differed from the equally weighted strategy. The size factor, which was insignificant and small for the equally weighted strategy, was highly significant for the value weighted strategy. the value factor was positive and significant in both strategies, confirming the value effect. The R-squared was greater in the value-weighted, indicating a better fit.

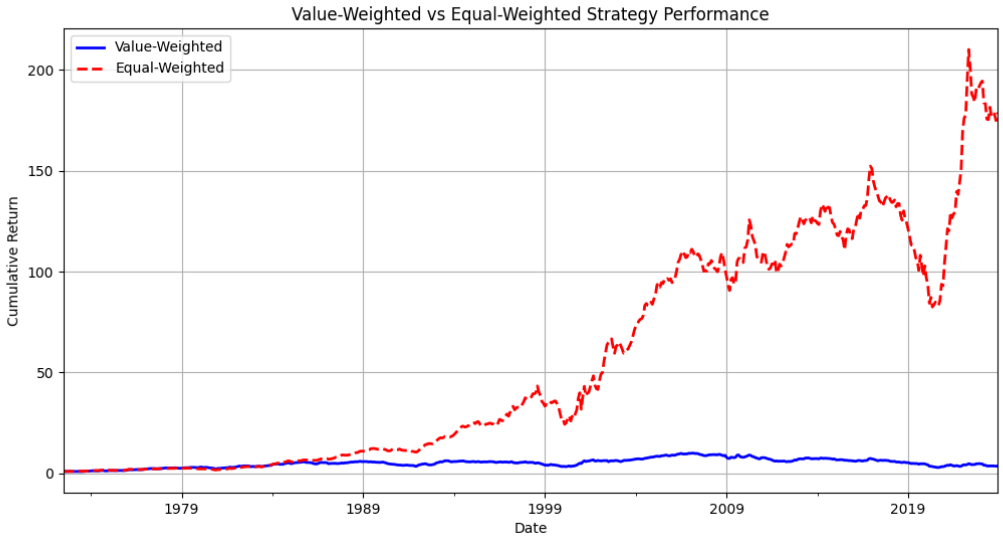
Table 7 *Fama-French **5-factor** Model*

Value Weighted

	Coefficient	Std. Error	t-stat	p- value
Alpha	-0.0006	0.001	-1.517	0.129
Market	0.0000	0.000	0.266	0.790
Size	0.0029	0.000	8.585	0.000
Value	0.0108	0.000	29.821	0.000
Profitability	-0.0022	0.000	-4.619	0.000
Investment	0.0012	0.001	1.678	0.093
R-squared	**0.815**			

When looking at the 5-factor model results, we saw that the differences observed in the 3-factor model persisted. The value weighted strategy continued to show a negative and insignificant alpha, indicating there are no excess returns after accounting for all the 5 factors, which contrasts from the equally weighted strategy. However, the magnitude did decrease for both. The market beta remained negative and significant for equally weighted and insignificant for the value weighted. The size beta was positive and significant in both strategies but slightly lower in the equally weighted. Positive and significant coefficients on profitability and investment factors were seen on the equally weighted, whereas they were significant and negative and insignificant negative for the value weighted. The R-squared value was higher again for the value weighted.

The considerable difference in returns between the two strategies underlines the role of small-cap stocks in the B/M anomaly and highlights the sensitivity of the B/M strategy to portfolio construction. The following figure illustrates the cumulative returns of the long-short strategy using value weighted vs equal-weighted portfolios. Emphasising the performance of the equally weighted portfolio over the value weighted portfolio.



Dependent double sorting Analysis

The double sorting analysis aims to assess robustness of the B/M anomaly by looking at the conditional relationship between B/M ratio and stock returns, when controlling for the influence of investment. We create an enhanced long-short portfolio to potentially improve the performance on the original long-short portfolio.

Table 8	*Book-to-Market Ratio Quintile*				
Investment Quintile	**Quintiles**	**1 (Low B/M)**	**2**	**3**	**4**	**5 (High B/M)**
	1 (Low Inv)	0.0095	0.0119	0.137	0.0142	0.0193
	2	0.0087	0.0091	0.0095	0.0116	0.0145
	3	0.0071	0.0091	0.0093	0.0100	0.0107
	4	0.0054	0.0071	0.0082	0.0086	0.0095
	5 (High Inv)	-0.0025	0.0001	0.0014	0.0042	0.0053

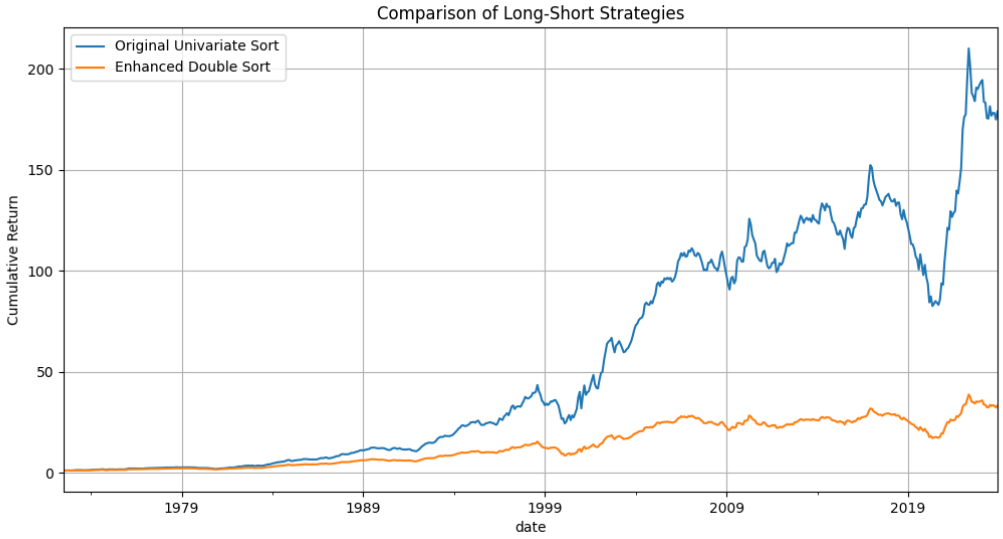
Table 8 shows the average monthly returns for the 25 portfolios. A constant pattern forms where returns decrease as investment increases throughout the all the B/M portfolios which aligns with the established financial theory, which states that increased levels of investments lead to lower future returns. Additionally returns tend to increase as B/M increases, supporting the value effect across various investment levels reinforcing its robustness. However, it should be said that the magnitude of the value effect differs across investment quintiles, as in the high investment quintiles lower returns are seen across all B/M quintiles.

Table 9	*Original (B/M only)*	*Enhanced (B/M & Investment)*
Average Return (%)	0.92	0.62
t-statistic	5.8277	4.7061
p-value	0.0000	0.0000

To further explore the relationship between the two characteristics, an enhanced long-short strategy was created, that buys the average of the highest B/M quintile and sells the lowest B/M quintile across all investment portfolios. The performance of this strategy is compared to the original strategy which was only based on B/M ratio (Table 9).

The enhanced strategy returned 0.62% monthly and shows statistical significance. However, this performance is worse than the original strategy, which returned 0.92% a month with stronger statistical significance. This indicates that conditioning for investment reduces the profitability of the B/M strategy.

These findings lead us to the conclusion that firstly, there is a persistent value effect across different investment levels which strengthens the significance in future asset pricing. Additionally, its highlights the importance of considering multiple factors when creating investment strategies. The analysis demonstrates the conditional nature of the B/M anomaly, whilst the value effect Is prominent, the size effect is determined by other factors, in this case investment levels. And the figure below presents the comparison shown in Table 9, in the form of cumulative returns of each strategy. it highlights that while both yield returns, the original strategy is superior over the sample period, suggesting investment as a sorting category reduces the effectiveness of the original strategy.



Conclusion

This report pulled apart the Book-to-Market ratio anomaly, and we conclude that it has consistent predictive power, and the strategies created were profitable. While the Fama-Macbeth regressions and equally weighted portfolios were significant, the value-weighted portfolio highlighted the ability of small-cap stocks to drive returns, it also signified the importance of weighting the portfolios. Factor model regressions showed the Prescence of unexplained abnormal returns, suggesting market inefficiencies or unmodeled risks.

Furthermore, dependent double sorting showed the conditional nature of B/M anomaly, as investment levels had a significant impact on returns of the strategy. Ultimately this report highlights the importance of multiple factors being considered, and weighting schemes to predict returns.