BANGABANDHU SHEIKH MUJIBUR RAHMAN SCIENCE AND TECHNOLOGY UNIVERSITY

GOPALGANJ-8100

Assignment on

VECTOR

Course Title: Vector, Matrix & Fourier Analysis

Course Code: MAT205

Submitted by	Submitted to
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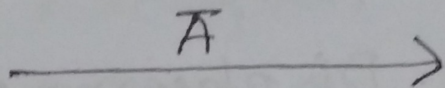
Vector

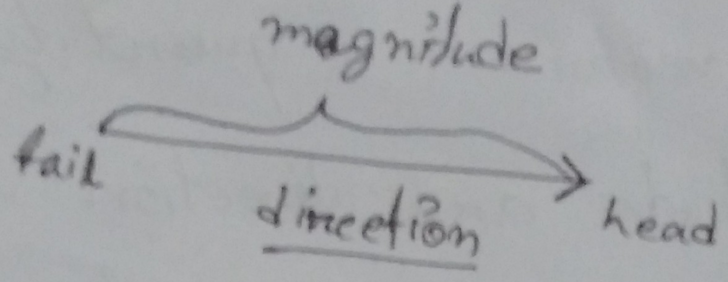
A vector is a quantity having both magnitude and direction such as displacement, velocity, force, and acceleration.



(Fig. 1)

- Graphically, a vector is represented by an arrow defining the direction. The magnitude of the vector being indicated by the length of the arrow.
- Analytically, a vector is represented by a letter with an arrow over it. And its magnitude is denoted

by $|\bar{A}|$ or A.

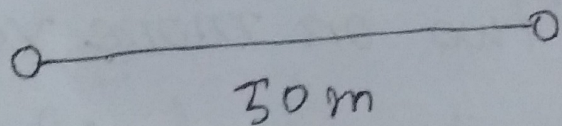


Some example of Vector.

1. Force
2. Displacement
3. Velocity.
4. Accelaration.
5. Momentum etc.

Scalar

A scalar is a quantity having magnitude but no direction. This are indicated by letters in ordinary type as in elementary algebra.



Distance between two ~~bodi~~ ball showing sealar representation.

Scalars Example:

(1) Mass.
(2) Speed
(3) Distance
(4) Time
(5) Area
(6) Volume.
(7) Temperature.

Equal vectors:

Two or more vectors are said to be equal if they have the same length or magnitude, and they point in the same direction. Any two or more vectors will be equal if they are collinear, codirected and have the same magnitude.

Mathematically, we can say that two vectors say A and B are equal if they satisfy the following conditions:

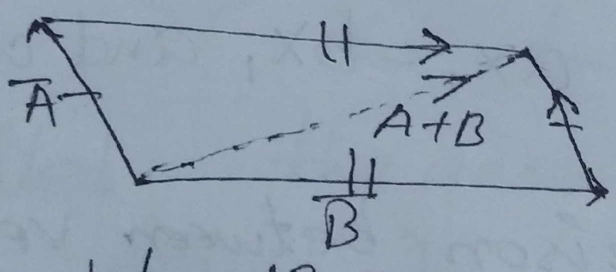
$\vec{A} = \vec{B}$ (Vector $\bar{A}$ and $\bar{B}$ are equal.)

If and only if.

$|\underline{A}| = |\underline{B}|$ (Equal magnitude)

Addition and Subtraction of Vector:

Vector addition: It is the operation of adding two or more vectors together into a vector sum. The so-called parallelogram law gives the rule for vector addition of two or more vectors.



Vector subtraction:

Vector subtraction is the process of taking a vector difference. Inverse operation to vector addition.

and $\vec{A} \uparrow\uparrow \vec{B}$ (same direction)

Two or more vectors are equal if their co-ordinates are equal.

For example, consider the vectors

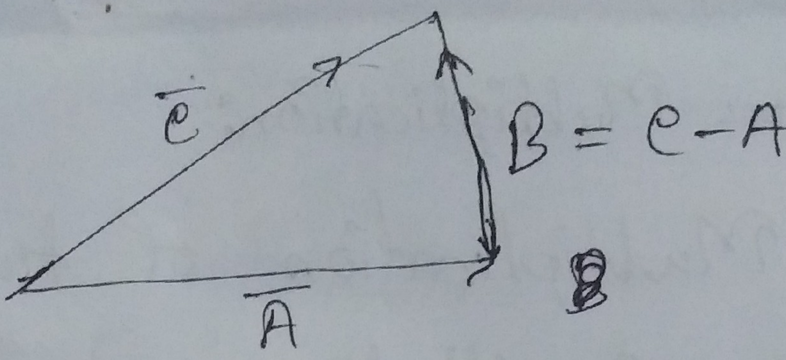
$\overline{A} = (ax_1, ay_1)$ and

$\overline{B} = (bx_1, by_1)$

If these two vectors are equal

then: $ax_1 = bx_1$ and $ay_1 = by_1$

Comparison between vectors is essentially a comparison of the vectors magnitudes and directions.



Add the corresponding components

let $\underline{u} = \langle u1, u2 \rangle$ and $\underline{v} = \langle v1, v2 \rangle$ be two vectors,

then, the sum of $\underline{u}$ and $\underline{v}$ is the vector.

$$\underline{u} + \underline{v} = \langle u1 + v1 , u2 + v2 \rangle$$

The sum of the two vectors is called the resultant.

We can found it using either the parallelogram method or the triangle method.

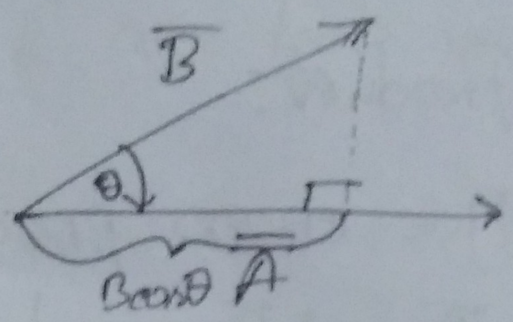
Vector Multiplication:

Multiplication of two (or more) vector ~~s~~ with themselves is called vector multiplication. The product of the projection of the first vector on to the second vector. and the magnitude of the second vector.

There are two kinds of multiplication for vector.

- Dot product/Scalar product
- Cross Product/ Vector product

Dot: The dot product of two vector $\vec{A}$ and $\vec{B}$ is defined as the scalar value $AB\cos\theta$



Where θ is the angle between them such that $0 \leq \theta \leq \pi$.

It is denoted by A.B by placing a dot sign between the vectors.

So we have the equation,

$$\underline{A}.\underline{B} = AB\cos\theta$$

Another name of dot product is Scalar product.

If $\overline{A}$ and $\overline{B}$ are two vectors of form,

$$\overline{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$$

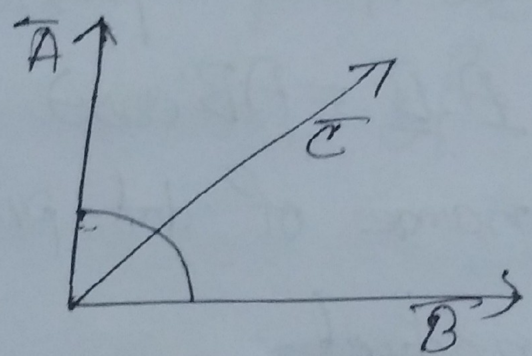
$$\overline{B} = B_1\hat{i} + B_2\hat{j} + B_3\hat{k}$$

then the dot product of $\overline{A}$ and $\overline{B}$ is,

$$\overline{A}.\overline{B} = A_1B_1 + A_2B_2 + A_3B_3$$

Cross Product:

The cross product of two vector $\overline{A}$ and $\overline{B}$ is defined as $AB\sin\theta$ with a direction prependicular to $\overline{A}$ and $\overline{B}$ in right hand system. where θ is the angle betwen them such that $0 \le \theta \le \pi$



$$\overline{C} = \overline{A} \times \overline{B}$$

$$|C| = |A||B| \sin\theta$$

Its denoted by $\overline{A} \times \overline{B}$ by placing a cross sign between the vectors so we have the equation,

$$\overline{A} \times \overline{B} = \eta AB \sin\theta = \overline{C}$$

If. $\bar{A}$ and $\bar{B}$ are two vectors of form.

$$\bar{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$$

$$\bar{B} = B_1\hat{i} + B_2\hat{j} + B_3\hat{k}$$

then the cross product of $\bar{A}$ and $\bar{B}$ is

$$A \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$$

Differentiation of a vector:

A vector function of u is called differentiable of order n, if its nth derivative exists. A function which is differentiable is necessarily continuous but the converse is not true.

Example-

If $A = 5t^2\hat{i} + t\hat{j} - t^3\hat{k}$ and $B = \sin t\hat{i} - \cos t\hat{j}$ find

(a) $\frac{d}{dt}(A.B)$

(b) $\frac{d}{dt}(A \times B)$

Answer:

a: $A.B = (5t^2\hat{i} + t j - t^3 k).(\sin t\, i - \cos t\, j)$

$= 5t^2 \sin t - t\cos t$

$\frac{d}{dt}(A.B) = \frac{d}{dt}(5t^2 \sin t - t\cos t)$

$= 5t^2\cos t + 10t\sin t + t\sin t - \cos t$

$= (5t^2 - 1)\cos t + 11t\sin t$

(Ans)

6. $A \times B = (5t^2\hat{i} + t\hat{j} - t^2\hat{k}) \times (\sin t\,\hat{i} - \cos t\,\hat{j})$

$$= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5t^2 & t & -t^2 \\ \sin t & -\cos t & 0 \end{bmatrix}$$

$$= -t^2\cos t\,\hat{i} - t^2\sin t\,\hat{j} + (-5t^2\cos t - t\sin t)\hat{k}$$

$$\frac{d}{dt}(A \times B) = \frac{d}{dt}\{-t^2\cos t\,\hat{i} - t^2\sin t\,\hat{j} + (-5t^2\cos t - t\sin t)k$$

$$= (t^2\sin t - 3t^2\cos t)\hat{i} - (t^3\cos t + 3t^2\sin t)\hat{j}$$

$$+ (5t^2\sin t - 11t\cos t - \sin t)k$$

(Ans)

Gradient:

Let $\phi(x,y,z)$ be defined and differentiable at each point (x,y,z) in a certain region of space. Then the gradient of ϕ, written $\nabla\phi$ or grad ϕ, is defined by

$$\nabla\phi = \left(\frac{\delta}{\delta x}\hat{i} + \frac{\delta}{\delta y}\hat{j} + \frac{\delta}{\delta z}\hat{k}\right)\phi$$

$$= \frac{\delta\phi}{\delta x}\hat{i} + \frac{\delta\phi}{\delta y}\hat{j} + \frac{\delta\phi}{\delta z}\hat{k}$$

Divergence

Let $V(x,y,z) = V_1\hat{i} + V_2\hat{j} + V_3\hat{k}$ be defined and differentiable at each point (x,y,z) in a certain region of space. Then the divergence of $\vec{v}$, written $\nabla.\vec{V}$ or div $\vec{v}$, is defined by

$$\nabla.\vec{V} = \left(\frac{\delta}{\delta x}\hat{i} + \frac{\delta}{\delta y}\hat{j} + \frac{\delta}{\delta z}\hat{k}\right)\left(V_1\hat{i} + V_2\hat{j} + V_3\hat{k}\right)$$

$$= \left(\frac{\delta}{\delta x}V_1 + \frac{\delta}{\delta y}V_2 + \frac{\delta}{\delta z}V_3\right)$$

Curl.

If $V(x,y,z)$ is a differentiable vector field then the curl or rotation of $\vec{v}$, written $\nabla \times \vec{V}$, curl $\vec{V}$ or rot $\vec{V}$ is defined by.

$$\nabla \times \vec{V} = \left(\frac{\delta}{\delta x}\hat{i} + \frac{\delta}{\delta y}\hat{j} + \frac{\delta}{\delta z}\hat{k} \right) \times \left(v_1 \hat{i} + v_2 \hat{j} + v_3 \hat{k} \right)$$

$$= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\delta}{\delta x} & \frac{\delta}{\delta y} & \frac{\delta}{\delta z} \\ v_1 & v_2 & v_3 \end{bmatrix}$$

$$= \left(\frac{\delta v_3}{\delta y} - \frac{\delta v_2}{\delta z} \right)\hat{i} + \left(\frac{\delta v_1}{\delta z} - \frac{\delta v_3}{\delta x} \right)\hat{j}$$

$$+ \left(\frac{\delta v_2}{\delta x} - \frac{\delta v_2}{\delta y} \right)\hat{k}$$