

* Central tendency:

Measures of central tendency are numerical indices that attempts to answer the question: What is the typical value of the observations in the distribution?

* popular measures of central Tendency

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- ① Mean ————— Arithmetic mean
- ② Median (अधिकांश) Geometric u
- ③ Mode Harmonic u

Arithmetic mean:- Let us consider n observations x_1, x_2, \dots, x_n

$$\begin{aligned} \text{Arithmetic mean, } \bar{X} &= \frac{x_1 + x_2 + \dots + x_n}{n} \\ &= \frac{\sum_{i=1}^n x_i}{n} \end{aligned}$$

$$* \bar{X} = \frac{16 + 18 + 19 + 20 + 22}{5} = \frac{95}{5} = 19$$

n n class mark numerical of statistics (BBS)

Age at first marriage x_i	Number of women f_i	$f_i x_i$
11	17	187
12	28	336
13	37	481
14	52	728
15	70	1050
16	48	768
17	36	612
18	23	414
19	11	209
20	8	160
	$\sum f_i = n = 330$	$\sum f_i x_i = 4945$

$$\begin{aligned}
 \text{Arithmetic mean} &= \frac{\sum f_i x_i}{\sum f_i} \\
 &= \frac{4945}{330} \\
 &= 14.98
 \end{aligned}$$

Let us have a grouped data as follows :-

Class limits	Frequency f_i	x_i	$f_i x_i$
14.5 - 19.5	1	17	17
19.5 - 24.5	6	22	132
24.5 - 29.5	9	27	243
29.5 - 34.5	2	32	64
34.5 - 39.5	2	37	74

→ $\frac{\text{Sum of data value } x_i}{\text{upper limit} + \text{lower limit}}$
 $270 \quad \frac{\quad}{2}$

class average (mean) $x = \frac{14.5 + 39.5}{2} = 27$

So, Arithmetic mean $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$
 $= \frac{530}{20}$
 $= 26.5$

~~H.W. for data of frequency of workers~~
~~2000~~

Some properties of Arithmetic mean:-

- (1) Arithmetic mean is dependent on both origin and scale of measurement.

Proof:- Let x be a quantitative variable taking on values x_1, x_2, \dots, x_n . Let d be a new variable taking on values d_1, d_2, \dots, d_n such that

$$d_i = \frac{x_i - a}{h} \quad \text{where } a \text{ and } h \text{ are the change of origin and scale respectively.}$$

$$\Rightarrow x_i = a + h d_i$$

$$\Rightarrow \sum x_i = \sum a + h \sum d_i$$

$$\Rightarrow \frac{\sum x_i}{n} = \frac{\sum a}{n} + h \frac{\sum d_i}{n}$$

$$\Rightarrow \bar{x} = \frac{na}{n} + h \cdot \bar{d}$$

$$\Rightarrow \bar{x} = a + h \bar{d}$$

So, arithmetic mean is dependent on both origin and scale.

$\sum \rightarrow$ Summation. $\sum_{i=1}^n$ means sum of n terms, $\sum_{i=1}^n x_i$ means sum of n terms, x_i means x_1, x_2, \dots, x_n and i is the index.

(2) The algebraic sum of the deviations of the values x_1, x_2, \dots, x_n from their arithmetic mean is zero. that is $\sum_{i=1}^n (x_i - \bar{x}) = 0$

Proof:- L.H.S $\sum_{i=1}^n (x_i - \bar{x})$

$$= (x_1 - \bar{x}) + (x_2 - \bar{x}) + \dots + (x_n - \bar{x})$$

$$= (x_1 + x_2 + \dots + x_n) - (\bar{x} + \bar{x} + \dots + \bar{x})$$

$$= \sum_{i=1}^n x_i - n\bar{x}$$

$$= \sum_{i=1}^n x_i - n \cdot \frac{\sum_{i=1}^n x_i}{n}$$

$$= \sum_{i=1}^n x_i - \sum_{i=1}^n x_i$$

$$= 0$$

$$= R.H.S$$

(proved)

Median

Median is that value in a data set which divides the set into two equal parts where the data set is arranged in ascending or descending order.

Formula of median:-

When n is not divisible by 2

$$\text{Median} = \left(\frac{n+1}{2}\right)^{\text{th}} \text{ value}$$

Example:-

Let us have the values

13, 17, 12, 16, 19

At first we have to arrange the values either in ascending order or descending order.

12, 13, 16, 17, 19

Here, $n = 5$ which is not divisible by 2

$$\text{So, median} = \left(\frac{n+1}{2}\right)^{\text{th}} \text{ value} = \left(\frac{5+1}{2}\right)^{\text{th}} \text{ value}$$

$$= \left(\frac{6}{2}\right)\text{th value}$$

$$= 3\text{rd value}$$

our 3rd value is 16.

$$\text{So Median} = 16.$$

Let us add another value with the data set which is 10. So the new data set is

10, 12, 13, 16, 17, 19

Here, $n = 6$ which is divisible by 2.

$$\text{So median} = \frac{\left(\frac{n}{2}\right)\text{th value} + \left(\frac{n}{2} + 1\right)\text{th value}}{2}$$

$$= \frac{\left(\frac{6}{2}\right)\text{th value} + \left(\frac{6}{2} + 1\right)\text{th value}}{2}$$

$$= \frac{3\text{rd value} + 4\text{th value}}{2}$$

$$= \frac{13 + 16}{2}$$

$$= \frac{29}{2} = 14.5 \text{ is the}$$

median.