Bangabandhu Sheikh Mujibur Rahman Science and Technology University

Gopalganj-8100

ASSIGNMENT ON

Home Work Solving

Course Title :Applied Statistics and Queuing Theory

Course Code:STAT205

Submitted by	Submitted to
Student ID:18CSE058 Year:2nd Semester:1st Session: 2018-19 Department of Computer Science of Engineering BSMRSTU,Gopalganj-8100.	Kakoli Khatun, Lecturer, Department of Statistics BSMRSTU,Gopalganj-8100

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①

The weight of 11 mothers in kg are recorded as follows 47, 44, 42, 41, 58, 52, 55, 39, 40, 43, 61

Find median.

Add another value 60 with the above data set, then find median again.

Solution:

At first we have to arrange the values either in ascending or descending order.

39, 40, 41, 42, 43, 44, 47, 52, 55, 58, 61

Here $n = 11$ which is not divisible by 2

So, median $= \left(\frac{n+1}{2}\right)$ th value

$= \left(\frac{11+1}{2}\right)$ th value

$= 6$ th value

Our 6th value is 44

Hence median $= 44$

Ans.

(2)

Let us add another value with the data set which is 10. So, the new data set is,

39, 40, 41, 42, 43, 44, 47, 52, 55, 58, 60, 61

Here, $n=12$ which is divisible by 2

So, median $= \dfrac{\left(\frac{n}{2}\right) \text{th value} + \left(\frac{n}{2}+1\right) \text{th value}}{2}$

$= \dfrac{\left(\frac{12}{2}\right) \text{th value} + \left(\frac{12}{2}+1\right) \text{th value}}{2}$

$= \dfrac{6 \text{th value} + 7 \text{th value}}{2}$

$= \dfrac{44+47}{2}$

$= 45.5$

Hence, median is 45.5.

Ans.

③

x	5	10	15	20	25
Y	2	6	10	15	20

Find r and Interprete your result.

x_i	y_i	$x_i y_i$	x_i^2	y_i^2
5	2	10	25	4
10	6	60	100	36
15	10	150	225	100
20	15	300	400	225
25	20	500	625	400
$\Sigma x_i = 75$	$\Sigma y_i = 53$	$\Sigma x_i y_i = 1020$	$\Sigma x_i^2 = 1375$	$\Sigma y_i^2 = 765$

we know that, $r = \dfrac{\Sigma x_i y_i - n\bar{x}\bar{y}}{\sqrt{\left(\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}\right)\left(\Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}\right)}}$

Here, $n = 5$

$\bar{x} = 15$

$\bar{y} = 10.6$

$$\therefore r = \frac{1020 - (5 \times 15 \times 10.6)}{\sqrt{\left(1375 - \frac{(75)^2}{5}\right)\left(765 - \frac{(53)^2}{5}\right)}}$$

$$= 0.07$$

Thus $r = 0.07$

Hence, There is weak positive correlation between X and Y.

Q. Is father's height dependent on son's height or son's height dependent on father's height?

Answer: Son's height dependent on father's height.

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Show that regression coefficient is independent of ~~both~~ origin ~~and~~ but dependent on the scale of measurement.

Proof: For n pairs of values $(x_1, y_1), (x_2, y_2), \ldots (x_n, y_n)$ of the variables x and y, the regression coefficient of y on x, b as defined earlier is

$$b_{y|x} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2} \quad \text{------} (1)$$

Now let us change the variable x to u and y to v where

$$u = \frac{x-a}{h} \text{ and } v = \frac{y-b}{k} \quad [h>0, k>0]$$

So that for the ith pair of the variables

$$x_i = a + hu_i \text{ and } y_i = b + kv_i$$

The corresponding mean values are

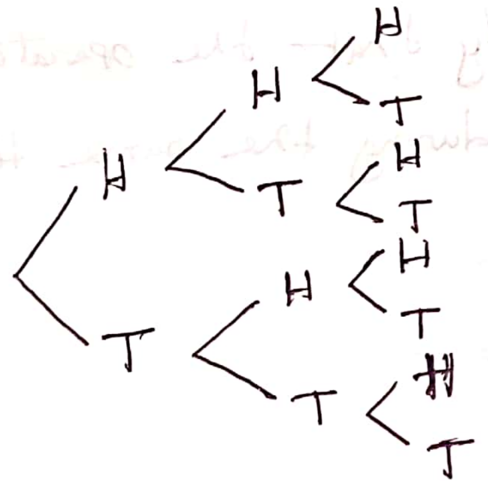
$$\bar{x} = a + h\bar{u} \text{ and } \bar{y} = b + k\bar{v}$$

Setting these values in 1

$$b_{y|x} = \frac{hk \sum (u_i-\bar{u})(v_i-\bar{v})}{h^2 \sum (u_i-\bar{u})^2} = \left(\frac{k}{h}\right) b_{v|u}$$

The absence of the factors ~~a~~ and b Proves that regression coefficient independent of origin, while the Presence of the factors h and k confirm that regression coefficient is dependent on the scale of measurement.

Let us consider that a coin is tossed three times, obtain the Sample space. Find the probability of exactly two heads will be occured.



A sample space for this experiment is

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

The event of exactly two heads (2H) is

$$2H = \{HHT, HTH, THH\}$$

Hence, the probability of exactly two heads

$$P(2H) = \frac{n(2H)}{n(S)} = \frac{3}{8}$$

Ans.

Example 7.24: A newly married couple plans to have two children, and suppose that each child is equally likely to be a boy or a girl. In order to find a sample space for this experiment, let B denote that a child is a boy and G denote that a child is a girl. Then one possible sample space that can be formed is

$$S = \{BB, BG, GB, GG\}$$

The double BG, for instance represents the outcome 'the older child is a boy' while 'the younger one is a girl'.

(a) What is the probability that the couple will have two boys?

(b) what is the probability that the couple will have one boy and one girl ?

Sol^n:

(a) Given that,

$$S = \{BB, BG, GB, GG\}$$

Let, the event of two boy is $2B = \{BB\}$

$\therefore$ The probability of two boy is.

$$P(2B) = \frac{1}{4} \quad \text{Ans.}$$

(b) Let, the event of one boy and one Girl is $1B1G = \{BG, GB\}$

$\therefore$ The probability of one boy and one Girl is $P(1B\,1G) = \frac{2}{4}$

$$= \frac{1}{2} \quad \text{Ans.}$$

Example 7.27: A businessman has a stock of 8400 baby wears imported from 5 different countries. The distribution of the wears was as follows:

Country	Number of wears
USA	1500
India	1200
China	2700
Korea	1000
Thailand	2000
Total	8400

A piece of baby were was selected at random. what is the probability that it was imported from (i) USA, from (ii) china, and (iii) either from India or from Thailand?

Sol^n: Using classical definition of probability we find that

$$P(USA) = \frac{1500}{8400} = 0.18$$

$$P(China) = \frac{2700}{8400} = 0.32$$

$$P(\text{India or Thailand}) = \frac{1200}{8400} + \frac{2000}{8400} = 0.13 + 0.24 = 0.37$$

Ans.

Example 7.28: A leap year consists of 366 days with 29 days in february. If a leap year is selected at random, what is the probability that the selected leap year will consist of 53 Saturdays ?

Sol^n:

The probability of 53 Saturdays in a leap year is, $P(\text{Saturday}) = \frac{2}{7}$

Ans.

Example 7.36: In an office 100 employees 75 read English, 50 read Bangla dailies and 40 read both. An employee is selected at random. what is the probability that the selected employee

(a) Reads English newspaper?

(b) Reads at least one of the papers?

(c) Reads none?

(d) Reads English but not Bangla?

Solⁿ: Let us define the above events:

E = Reads English

B = Reads Bangla

$\bar{E} \cap \bar{B}$ = Reads none

$B \cap \bar{E}$ = Reads Bangla but not English

The number of cases favoriable to the above events can be placed in a tabular from as follows:

	E	$\bar{E}$	Total
B	$n(B\cap E)=40$	$n(B\cap\bar{E})=?$	$n(B)=50$
$\bar{B}$	$n(\bar{B}\cap E)=?$	$n(\bar{B}\cap\bar{E})=?$	$n(\bar{B})=50$
Total	$n(E)=75$	$n(\bar{E})=25$	$n(S)=100$

(a) The probability that the selected employee read English is

$$P(E)=\frac{n(E)}{n(S)}=\frac{75}{100}=0.75$$

(b) The probability that the selected employee reads at least one (either E or B or Both) is

$$P(E\cup B)=P(E)+P(B)-P(E\cap B)$$

$$=\frac{75}{100}+\frac{50}{100}-\frac{40}{100}$$

$$=0.85$$ Ans.

(c) The probability that the selected employee reads none (neither E nor B) is $P(\bar{E} \cap \bar{B}) = P(\overline{E \cup B})$

$= 1 - P(E \cup B)$

$= 1 - 0.85$

$= 0.15$ Ans.

(d) The probability that the selected employee reads Bangla but not English is $P(B \cap \bar{E}) = \dfrac{n(B) - n(B \cap E)}{n(S)}$

$= \dfrac{50 - 40}{100}$

$= 0.10$

Ans.

Example-7.38: The probability that a married man watches a certain TV show is 0.4 that his wife watches the show is 0.5. The probability that a man watches the show, given that his wife does is 0.7 find -

(a) The probability that a married couple watches the show.

(b) The probability that a wife watches the show given that her husband does.

(c) The probability that at least one of the partners will watch the show.

Sol^n:→ Let us define two events H and w a follow:

H : Husband watches the show

w : wife watches the show

Given that,

$$P(H) = 0.4, P(w) = 0.5, P(H/w) = 0.7$$

(a) The probability that the couple watches the show is,

$$P(W \cap H) = P(W).\ P(H/W) = 0.5 \times 0.7 = 0.35$$

(b) The conditional probability that a wife watches the show given that her husband also watches

$$P(W/H) = \frac{P(W \cap H)}{P(H)} = \frac{0.35}{0.4} = 0.875$$

(c) The probability that at least one (either H or W or both) watches,

$$P(W \cup H) = P(W) + P(H) - P(W \cap H)$$

$$= 0.40 + 0.50 - 0.35$$

$$= 0.55.$$

Ans.

Example 7.44: Two ideal coins are tossed. Let A be the event 'head on the first coin' and B the event that 'head on the second coin. A sample space for this experiment is.

$$S = \{HH, HT, TH, TT\}$$

$\therefore A = \{HH, HT\}$ and $B = \{HH, TH\}$

$\therefore A \cap B = \{HH\}$

Now, $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{2}$

And, $P(A \cap B) = \frac{1}{4}$

$\therefore P(A \cap B) = P(A) \times P(B)$

Hence, we say that the events A and B are independent.

~~Example~~

Example 7.45: Three coins are tossed. Show that the events "heads on the first coin" and the event tails on the last two" are independent.

Sol^n: we construct a Sample space S for the above experiment.

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

Let A denote the event "head on the first coin" and B denote the event "tails on the last two coin"

Then, $A = \{HHH, HHT, HTH, HTT\}$

$B = \{HTT, TTT\}$

$A \cap B = \{HTT\}$

Hence, $P(A) = \frac{4}{8} = \frac{1}{2}$

$P(B) = \frac{2}{8} = \frac{1}{4}$

$P(A \cap B) = \frac{1}{8}$

Since, $P(A) \times P(B) = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8} = P(A \cap B)$

Hence, The events "heads on the first coin" and "tails on the last two coin" are independent.

(showed).

Example 7.46: A fire brigade has two fire engines operating independently. The probability that a specific fire engine is available when needed is 0.99.

(a) What is the probability that an engine is available when needed?

(b) What is the probability that neither is available when needed?

Sol^n: Let A be the event that the first engine is available when needed and B be the event that the second engine is available when needed. Then $P(A) = P(B) = 0.99$.

Given this, the probability that both of them will be available when needed is

$P(A \cap B) = 0.99 \times 0.99 = 0.9801$

Since they operate independently.

(a) Here the event of interest is $A \cup B$.

So that, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.99 + 0.99 - 0.9801$

$= 0.9999$

(b) In set notation, this event is $\bar{A} \cap \bar{B}$, which equals $\overline{A \cup B}$. So that the required probability is $P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - 0.9999$

$= 0.0001$

Ans.

A department store has 6 television sets of which 2 are defective. A purchaser makes a random choice of three sets. If x is the number of defective sets purchased, find the probability distribution of x.

Solⁿ: If x denote the number of defective set choice, Then clearly x can assume values ~~0, 1, 2~~ 0, 1 & 2. To obtain the probabilities a function can be written in the from

$$f(x) = P(X=x) = \frac{{}^{2}C_{x}\ {}^{4}C_{3-x}}{{}^{6}C_{3}}\ ;\ x = 0,1,2$$

Now, we need to compute probabilities associated with 0, 1, 2

$$f(0) = P(X=0) = \frac{{}^{2}C_{0}\ {}^{4}C_{3}}{{}^{6}C_{3}} = 1/5$$

$$f(1) = P(x=1) = \frac{{}^2C_1 \, {}^4C_2}{{}^6C_3} = 3/5$$

$$f(2) = P(x=2) = \frac{{}^2C_2 \, {}^4C_1}{{}^6C_3} = 1/5$$

So, in tabular from

x	0	1	2
$f(x)$	1/5	3/5	1/5

Ans.

4. A continuous random variable x has the following density function:

$$f(x) = \frac{2}{27}(1+x) \; ; \; 2 < x < 5$$
$$= 0 \text{, elsewhere}$$

(a) Verify that it satisfies the condition $\int_{-\infty}^{\infty} f(x)\,dx = 1$

(b) find $P(x<4)$ and

(c) find $P(3<x<4)$

Soln:

(a) Integration between 2 and 5

$$\int_2^5 f(x)\,dx = \frac{2}{27}\int_2^5 (1+x)\,dx = \frac{2}{27}\left[\left(x+\frac{x^2}{2}\right)\right]_2^5$$

$$= \frac{2}{27}\left[\left(5+\frac{25}{2}-2-\frac{4}{2}\right)\right]$$

$$= \frac{2}{27}\left(\frac{10+25-4-4}{2}\right)$$

$$= \frac{2}{27} \times \frac{27}{2}$$

$$= 1$$

$\therefore$ That the given function is a density function.

(b) Since the lower limit is 2, we integrate between 2 and 4, to evaluate $P(x<4)$

$$P(x<4) = \frac{2}{27}\int_2^4 (1+x)\,dx = \frac{2}{27}\left[x+\frac{x^2}{2}\right]_2^4$$

$$= \frac{2}{27}\left(4+\frac{16}{2}-2-\frac{4}{2}\right)$$

$$= \frac{2}{27}\left(\frac{8+16-4-4}{2}\right)$$

$$= \frac{16}{27}$$

(c) Evaluating the integral between 3 and 4, we obtain $P(3<x<4)$

$$P(3<x<4) = \frac{2}{27}\int_3^4 (1+x)\,dx$$

$$= \frac{2}{27}\left[x+\frac{x^2}{2}\right]_3^4$$

$$= \frac{2}{27}\left(4+\frac{16}{2}-3-\frac{9}{2}\right)$$

$$= \frac{2}{27}\left(\frac{8+16-6-9}{2}\right)$$

$$= \frac{9}{27} = \frac{1}{3} \text{ Ans.}$$

A traffic control officer reports that 75% of the tracks passing through a check post are from within Dhaka city. What is the probability that at least three of the next five tracks are from out of the Dhaka city?

Sol^n: Let x be the number of tracks that pass through are from out of Dhaka city. The probability of such an event is then

$P = 1 - 0.75 = 1/4$. Hence,

$$P(x \geq 3) = \sum_{x=3}^{5} b(x, 5, 1/4)$$

$$= b(3, 5, 1/4) + b(4, 5, 1/4) + b(5, 5, 1/4)$$

$$= {}^5C_3 (1/4)^3 (3/4)^2 + {}^5C_4 (1/4)^4 (3/4)^1 + {}^5C_5 (1/4)^5 (3/4)^0$$

$$= \frac{90}{1024} + \frac{15}{1024} + \frac{1}{1024}$$

$$= \frac{106}{1024} = 0.1035$$

Ans.

[1] The average number of calls recived by a telephone operator during a time interval of 10 minutes during 5 P.m to 5.10 P.m daily is 3. what is the probability that the operator will recieve tomorrow during the same time interval.

i) No call ?

ii) Exactly one call ?

iii) At least two calls ?

(i) Here, the average number of recieved calls, $\mu = 3$

If x is the random variable, we want to compute $f(x, \mu) = f(0, 3)$

$$= \frac{e^{-3} \cdot 3^0}{0!}$$

$$= 0.05$$

Hence, the probability of no call 0.05.

(ii) if x is the random variable, we want to compute, $f(x, \mu) = f(1, 3)$

$$= \frac{e^{-3} \cdot 3^1}{1!}$$

$$= 0.15$$

Hence, the probability of one call 0.15

(iii) (i) and (ii) we get the probability of one call $P(x=1) = 0.15$ and probability of no call & $P(x=0) = 0.05$.

$\therefore$ The probability of at least two call is $P(x \geq 2) = 1 - P(x<2)$

$$= 1 - [P(x=0) + P(x=1)]$$

$$= 1 - (0.05 + 0.15)$$

$$= 0.8$$

Ans.

Suppose that there are, on the average four vehicle accidents per day on the Asian highway running from Dhaka to Manikganj. what is the probability that on a given day.

i) There ~~are~~ is no vehicle accident?

ii) There are three or fewer accidents?

iii) There are three or more accidents?

i) Here the average number of accident, $\mu = 4$

If x is the random variable, we want to compute $f(x, \mu) = f(0, 4)$

$$= \frac{e^{-4} \cdot 4^0}{0!}$$

$$= 0.018$$

Hence, The probability of no accident 0.018 - Ans.

(ii) if x is the random variable, we want to compute, $f(x, \mu) = f(x \leq 3, 4)$

$$= f(0,4) + f(1,4) + f(3,4) + f(2,4)$$

$$= \frac{e^{-4} \cdot 4^0}{0!} + \frac{e^{-4} \cdot 4^1}{1!} + \frac{e^{-4} \cdot 4^2}{2!} + \frac{e^{-4} \cdot 4^3}{3!}$$

$$= 0.433$$

Hence, the probability of three or fewer accidents 0.43. Ans.

(iii) if x is the random variable, we want to compute, $f(x, \mu) = f(x \geq 3, \mu)$

$$= 1 - f(x < 3, \mu)$$

$$= 1 - [f(0,4) + f(1,4) + f(2,4)]$$

$$= 1 - \left[\frac{e^{-4} \cdot 4^0}{0!} + \frac{e^{-4} \cdot 4^1}{1!} + \frac{e^{-4} \cdot 4^2}{2!}\right]$$

$$= 0.762$$

Hence, the probability of three or more accident 0.76 Ans.

11 The average number of emergency patients in a given day in a private clinic is 10. Find the probability that in a specified day, the clinic will receive

(i) 5 emergency patients

(ii) at least 3 emergency patients

(iii) between 5 and 10 emergecy patients, if the number of patients is assumed to follow the poisson distribution ?

Sol^n: Let x denote the number of patients. Thus x is a poisson variable with mean $\mu = 10$, we now use Appendix V to compute the required probabilities.

(i) Hence, $x = 5$. So that,

$$P(x=5) = f(5,10) = \frac{e^{-10} \cdot 10^5}{5!}$$

$$= \sum_{x=0}^{5} f(x,10) - \sum_{x=0}^{4} f(x,10)$$

$$= 0.067 - 0.029 = 0.038$$ Ans.

(ii) For at least three cases,

$$P(x \geq 3) = 1 - P(x \leq 2)$$

$$= 1 - \sum_{x=0}^{2} f(x,10)$$

$$= 1 - 0.003 = 0.0997$$ Ans.

(iii) Here x lies between 5 and 10 inclusive,

So that, $$P(5 \leq x \leq 10) = \sum_{x=0}^{10} f(x,10) - \sum_{x=0}^{4} f(x,10)$$

$$= 0.583 - 0.029$$

$$= 0.554$$

Ans.