~~Joint probability :-~~

Axioms of Probability :-

Suppose S is a sample space associated with an experiment. To every event A in S, let $P(A)$ is the probability of A. Then the following axioms hold :

(I) $P(A) \geq 0$

(II) $P(S) = 1$.

(III) If $A_1, A_2, A_3, \ldots A_n$ form a sequence of mutually exclusive events in S, then

$$P(A_1 \cup A_2 \cup \ldots \cup A_n) = \sum_{i=1}^{n} P(A_i)$$

$$= P(A_1) + P(A_2) + \cdots + P(A_n).$$

Joint Probability :- Two or more events form a joint event if all of them occur simultaneously and probability of these events are called the joint probability.

(2)

Thus all the events of the form $A \cap B$, $A \cap B \cap C$, $A \cap B \cap C \cap D$ or $A_1 \cap A_2 \cap \cdots \cap A_n$ are joint events.

Example :- If A is the event of "smoker" and B is the event of "heart disease patient" then $A \cap B$ is the joint event describing that a randomly chosen person is a smoker who suffers from heart disease.

Problem :- suppose a sample space consists of 500 persons and are distributed according to their gender and employment status as shown in the table

	Employment Status		
Gender	Employed (E)	Unemployed U	Total
Male (M)	255	20	275
Female (F)	80	145	225
Total	335	165	500

(i) What is the probability that a randomly chosen person will be a male?

(ii) " " " " " employed?

(iii) " " " male and at the same time unemployed?

Solution:- (i) Total number of male $n(M) = 275$

$n(S) = 500$

$$P(M) = \frac{275}{500}.$$

(ii) $$P(E) = \frac{n(E)}{n(S)} = \frac{255}{500}.$$

(iii) $$P(M \cap U) = \frac{n(M \cap U)}{n(S)} = \frac{20}{500}$$

H.W

Example 7.36

<u>Conditional Probability</u> :- The probability of an event A when it is known that some other event B has been occured is called a conditional probability and is denoted by $P(A/B)$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B/A) = \frac{P(A \cap B)}{P(A}$$

$$P(A \cap B) = P(A)\ P(B/A)$$

$$P(A \cap B) = P(B)\ P(A/B)$$

<u>Example</u>:- A pair of dice is thrown. Find the probability that sum of the points on the two dice is 10 or greater if a five appears on the first die.

Solⁿ:- let A be the event that sum of the points on the two dice is 10 or greater and B be the event that a 5 appears on the first toss. Symbolically we want to evaluate $P(A|B)$.

$A = \{(4,6), (5,5), (5,6), (6,4), (6,5), (6,6)\}$

$B = \{(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)\}$

$A \cap B = \{(5,5), (5,6)\}$

$$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{2/36}{6/36} = \frac{1}{3}.$$

Note:- why the denominate of each probability is 36? Answer: If we throw two dice there are 36 outcomes

$(1,1), (1,2), (1,3)\ (1,4)\ (1,5)\ (1,6)$

$(2,1)$. - - - —

⋮

$(6,1)$ - - — — — - $(6,6)$

H.W Example 7.38

Independence of two events:-

Suppose two events A and B occur in a manner that occurance or non occurance of either of them has no relation and no influence on the occurance and non-occurance of the other. Under this condition, we say that events A and B occur independently of one another. Given this situation, the probability that both A and B will occur is equal to the product of their individual probabilities.

$$\cancel{P(A \cap B) = P(A) \times P(B)}.$$

$$P(A \cap B) = P(A) \times P(B).$$

Problem:- Two ideal coins are tossed. Let A denote the event 'head on the first coin' and B the event 'head on the second coin'. then S

$$S = \{HH, HT, TH, TT\}$$

$$A = \{HH, HT\} \quad B = \{HH, TH\}$$

$$A \cap B = \{HH\}$$

$$P(A \cap B) = \frac{1}{4}$$

$$P(A) = \frac{1}{2}, \quad P(B) = \frac{1}{2}.$$

$$P(A \cap B) = P(A) \times P(B)$$

So, A & B are independent.

H.W Example 7.45, 7.46