1 Using Java:

Develop a function poly(double x, int n, double[] a) to evaluate the n-degree polynomial of x (array a stores the coefficients of polynomial).

$$poly(x, n, a) = a[0] + a[1] * x + a[2] * x^2 + ... + a[n] * x^n$$

...then discuss the time and storage complexity of your algorithm.

1.1 Code:

```java
// Poly.java

public void poly(double x, int n, double[] a) {
   double runningTotal = 0.0;
   for (int i = 0; i < n; i++) {
      runningTotal = runningTotal + ( a[i] * Math.pow(x,i) );
   }
   System.println(runningTotal);
}

// end of code
```

1.2 Time and Storage Complexity:

In the above example, there is one static operation outside of the loop (the instantiation of var runningTotal), which is constant, therefore 1. Inside the loop lay one primitive operation (the accumulation of values within var runningTotal) which occurs n times for the life of the program. Additionally, in this example, the Storage Complexity follows the Time Complexity due to the simplicity of the algorithm.

THEREFORE, the Analytic Time Complexity and Storage Complexity of the program are both:

$$n + 1$$

2 Inductively, prove:

$$1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$$

FUNDAMENTALLY, when $n = 1$:

$$1^2 = \frac{1(1+1)((2*1)+1)}{6}$$
$$\therefore 1 = \frac{1(2)(3)}{6}$$
$$\therefore 1 = \frac{6}{6}$$
$$\therefore 1 = 1$$

...which is true.

THEREFORE, by induction, we can show, when $n = k$:

$$1^2 + 2^2 + 3^2 + ... + k^2 + (k+1)^2 = \frac{k+1(k+1+1)(2(k+1)+1)}{6}$$
$$\therefore \frac{k(k+1)(2k+1)}{6} + (k+1)^2 = \frac{k+1(k+2)(2k+3)}{6}$$
$$\therefore \frac{k(2k^2+3k+1)}{6} + \frac{6(k+1)^2}{6} = \frac{k+1(2k^2+7k+6)}{6}$$
$$\therefore \frac{2k^3+3k^2+k+6k^2+12k+6}{6} = \frac{2k^3+9k^2+13k+6}{6}$$
$$\therefore \frac{2k^3+9k^2+13k+6}{6} = \frac{2k^3+9k^2+13k+6}{6}$$

...QED.

3 Inductively, prove:

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + ... - \frac{(-1)^n}{n} > 0$$

FUNDAMENTALLY, when $n = 1$:

$$-\frac{(-1)^1}{1} > 0$$
$$\therefore -\frac{-1}{1} > 0$$
$$\therefore 1 > 0$$

...which is true.

THEREFORE, by induction, we can show, when $n = k$:

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + ... - \frac{(-1)^k}{k} - \frac{(-1)^k + 1}{k+1} > 0$$

$$\therefore \mathbb{Z}_{>0} - \frac{-1}{k+1} > 0$$

$$\therefore \mathbb{Z}_{>0} + \frac{1}{k+1} > 0$$

...QED.

4 Inductively, prove:

$$1 + 3 + 5 + ... + (2n - 1) = n^2$$

FUNDAMENTALLY, when $n = 1$:

$$(2(1) - 1) = 1^2$$

$$\therefore 2 - 1 = 1$$

$$\therefore 1 = 1$$

...which is true.

THEREFORE, by induction, we can show, when $n = k$:

$$1 + 3 + 5 + ... + (2k - 1) + (2(k + 1) - 1) = (k + 1)^2$$

$$\therefore k^2 + (2k + 2 - 1) = k^2 + 2k + 1$$

$$\therefore k^2 + 2k + 1 = k^2 + 2k + 1$$

...QED.

5 Inductively, prove:

$$n^2 + n + 1 = 2l + 1; n \in \mathbb{Z}_{>0}, l \in \mathbb{Z}_{>0}$$

...WHEREAS $2l + 1$ represents any odd number, or:

$$2l + 1 = \mathbb{N}_{odd}$$

...WHEREAS $\mathbb{N}_{odd} + 1$ represents any even number, or:

$$\mathbb{N}_{odd} + 1 = \mathbb{N}_{even}$$

FUNDAMENTALLY, when $n = 1$:

$$1^2 + 1 + 1 = \mathbb{N}_{odd}$$
$$\therefore 3 = \mathbb{N}_{odd}$$
$$\therefore 3 = \mathbb{N}_{odd}$$

...which is true.

THEREFORE, by induction, we can show, when $n = k$:

$$(k+1)^2 + (k+1) + 1 = \mathbb{N}_{odd}$$
$$\therefore k^2 + 2k + 1 + k + 1 + 1 = \mathbb{N}_{odd}$$
$$\therefore (k^2 + k + 1) + 2k + 2 = \mathbb{N}_{odd}$$
$$\therefore \mathbb{N}_{odd} + 2k + 2 = \mathbb{N}_{odd}$$
$$\therefore \mathbb{N}_{odd} + (\mathbb{N}_{odd} + 1) = \mathbb{N}_{odd}$$
$$\therefore \mathbb{N}_{odd} + \mathbb{N}_{even} = \mathbb{N}_{odd}$$

THEREFORE, because an even number plus $\mathbb{N}_{odd}$ is odd,

...QED.