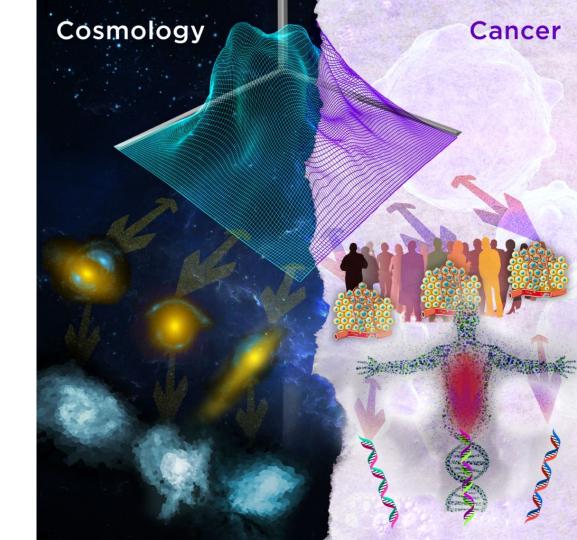
PyAutoFit: Classy
Probabilistic
Programming for
Data Science

James Nightingale

Richard Hayes, Matthew Griffiths

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Overview

PyAutoFit & Probabilistic Programming:

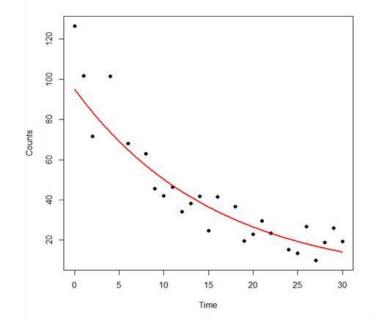
- What is probabilistic programming?
- What is PyAutoFit?
- Why PyAutoFit?

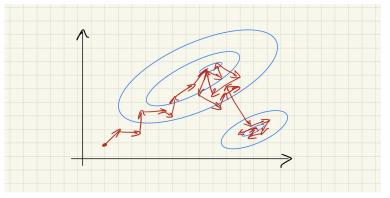
Cosmology:

- Description of example use-case strong gravitational lensing.
- Application to Astronomy data.
- Building multi-level models via Python classes.

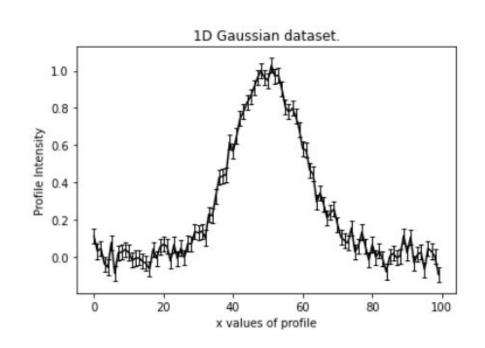
Given some data and a model, finding the set of model parameters that provide the best fit to the data.

$$P(A|B) = \frac{P(B|A) P(A)}{P(B)}$$

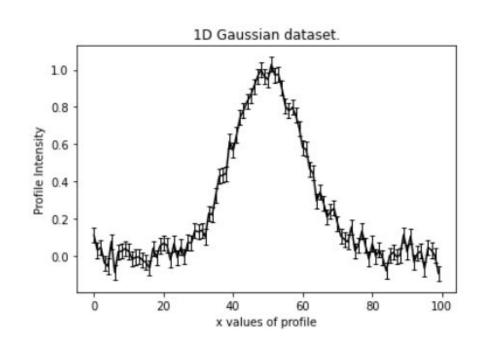




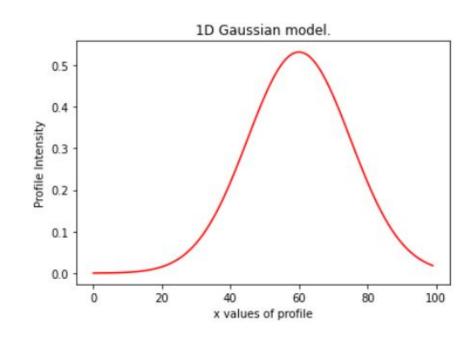
- Centre
- Normalization
- Sigma
- 1) Draw a set of parameters.
- 2) Create Model Gaussian.
- 3) Fit to Dataset.
- 4) Compute Likelihood.
- 5) Repeat using non-linear search.



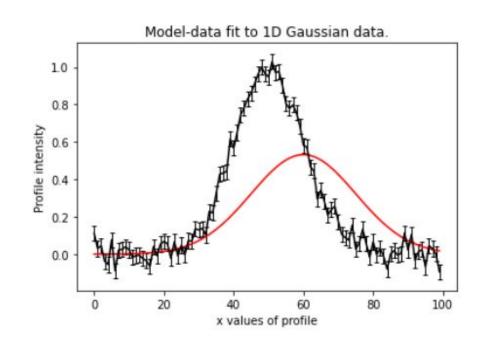
- Centre = 60.0
- Intensity = 20.0
- Sigma = 15.0
- 1) Draw a set of parameters.
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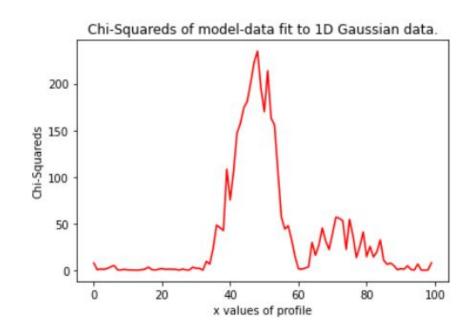
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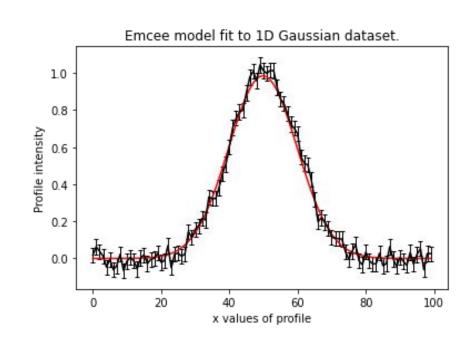
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- Centre = **50.0**
- Intensity = 10.0
- Sigma = **5.0**
- 1) Draw a set of parameters.
- 2) Create Model Gaussian.
- 3) Fit to Dataset.
- 4) Compute Likelihood.
- 5) Repeat using non-linear search.



Probabilistic Programming

What is Probabilistic Programming?

Probabilistic programming languages (PPL) provide a framework that allows users to easily specify a probabilistic model and perform inference automatically.

- There are a plethora of PPL's available (e.g. **PyMC3**, **STAN**, **Pyro**).
- All are suited to different problems, have different core features, etc.

They are some of the Github mega projects, so why on Earth are we developing our own PPL?

Existing PPL's not suited to the model fitting challenges we faced when in Astronomy, for example:



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 Fitting large and homogenous datasets with an identical model fitting procedure, with tools for processing the large libraries of results output.



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- Problems where likelihood evaluations are expensive
 (e.g. run times of days per model-fit), necessitating
 highly customizable model-fitting pipelines with support
 for massively parallel computing.



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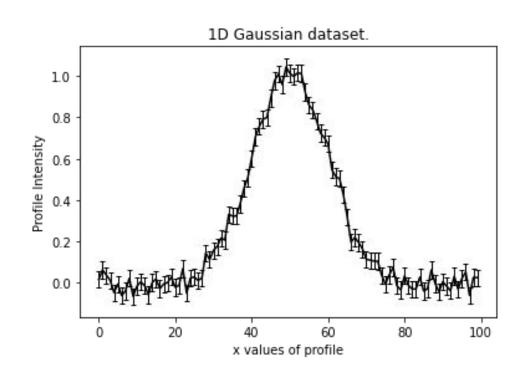
PyAutoFit: highly customizable model-fitting software, for big data challenges in the many model regime.



PyAutoFit: Classy Interface

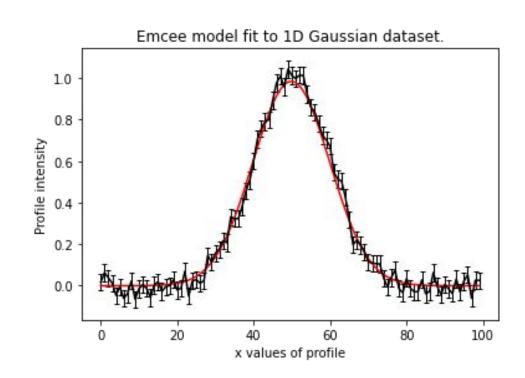
Illustrative example, fitting noisy 1D data of a Gaussian.

Aim: use **PyAutoFit** to fit a Gaussian to the dataset via a non-linear search.



Illustrative example, fitting noisy 1D data of a Gaussian.

Aim: use **PyAutoFit** to fit a Gaussian to the dataset via a non-linear search.



Illustrative example, fitting noisy 1D data of a Gaussian.

 Write a Python class to define the model component.

```
class Gaussian:
   def init (
       self,
       centre=0.0,
                      # <- PvAutoFit recognises these
       intensity=0.1, # <- constructor arguments are
       sigma=0.01, # <- the Gaussian's parameters.
   ):
       self.centre = centre
       self.intensity = intensity
       self.sigma = sigma
   An instance of the Gaussian class will be available during model fitting.
   This method will be used to fit the model to ``data`` and compute a likelihood.
   def profile from xvalues(self, xvalues):
       transformed xvalues = xvalues - self.centre
       return (self.intensity / (self.sigma * (2.0 * np.pi) ** 0.5)) * \
               np.exp(-0.5 * transformed xvalues / self.sigma)
```

Illustrative example, fitting noisy 1D data of a Gaussian.

- Write a Python class to define the model component.
- Write an Analysis class with the data and likelihood function.

```
class Analysis(af.Analysis):
    def init (self, data, noise map):
        self.data = data
        self.noise map = noise map
    def log likelihood function(self, instance):
        The 'instance' that comes into this method is an instance of the Gaussian class
        above, with the parameters set to values chosen by the non-linear search.
        print("Gaussian Instance:")
       print("Centre = ", instance.centre)
       print("Intensity = ", instance.intensity)
       print("Sigma = ", instance.sigma)
       We fit the ``data`` with the Gaussian instance, using its
        "profile from xvalues" function to create the model data.
       xvalues = np.arange(self.data.shape[0])
       model_data = instance.profile_from_xvalues(xvalues=xvalues)
       residual map = self.data - model data
       chi squared map = (residual map / self.noise map) ** 2.0
       log_likelihood = -0.5 * sum(chi_squared_map)
        return log likelihood
```

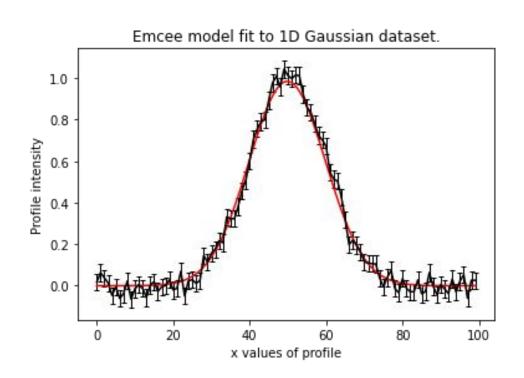
Illustrative example, fitting noisy 1D data of a Gaussian.

- Write a Python class to define the model component.
- Write an Analysis class with the data and likelihood function.
- Combine with your favourite non-linear search to fit the model to the data.

```
model = af.Model(Gaussian)
analysis = Analysis(data=data, noise_map=noise_map)
emcee = af.Emcee(nwalkers=50, nsteps=2000)
result = emcee.fit(model=model, analysis=analysis)
```

Illustrative example, fitting noisy 1D data of a Gaussian.

Aim: use **PyAutoFit** to fit a Gaussian to the dataset via a non-linear search.



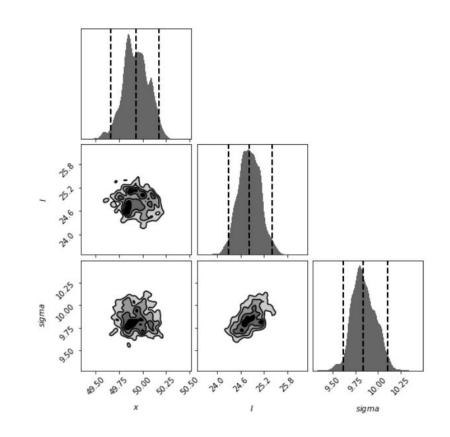
Illustrative example, fitting noisy 1D data of a Gaussian.

- Write a Python class to define the model component.
- Write an Analysis class with the data and likelihood function.
- Combine with your favourite non-linear search to fit the model to the data.
- Result object contains all the information you need on your model-fit.

```
samples = result.samples
print("Final 10 Parameters:")
print(samples.parameter lists[-10:])
print("Sample 10's third parameter value (Gaussian -> sigma)")
print(samples.parameter lists[9][2], "\n")
median pdf vector = samples.median pdf vector
vector at upper sigma = samples.vector at upper sigma(sigma=3.0)
vector at lower sigma = samples.vector at lower sigma(sigma=3.0)
print("Upper Parameter values w/ error (at 3.0 sigma confidence):")
print(vector at upper sigma)
print("lower Parameter values w/ errors (at 3.0 sigma confidence):")
print(vector at lower sigma, "\n")
```

Illustrative example, fitting noisy 1D data of a Gaussian.

- Write a Python class to define the **model component**.
- Write an Analysis class with the data and likelihood function.
- Combine with your favourite non-linear search to fit the model to the data.
- Result object contains all the information you need on your model-fit.



PyAutoFit: Links / Overview

PyAutoFit

GitHub: https://github.com/rhayes777/PyAutoFit

Readthedocs: https://pyautofit.readthedocs.io/en/latest/

JOSS Paper: https://joss.theoj.org/papers/10.21105/joss.02550

Binder: https://mybinder.org/v2/gh/Jammy2211/autofit_workspace/HEAD

HowToFit

Teach **anyone** how to compose and fit a probabilistic model with **PyAutoFit**.

We can also use it to get a model instance of the median_pdf model, which is the model where each parameter is the value estimated from the probability distribution of parameter space.

Median PDF Model:

Centre = 49.92285569756167

Intensity = 24.974961843717058

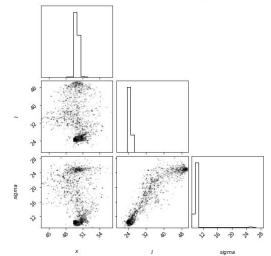
Sigma = 9.969794911012947

The Probability Density Functions (PDF's) of the results can be plotted using the Emcee's visualization tool corner.py, which is wrapped via the EmceePlotter object.

The PDF shows the 1D and 2D probabilities estimated for every parameter after the model-fit. The two dimensional figures can show the degeneracies between different parameters, for example how increasing σ and decreasing the intensity I can lead to similar likelihoods and probabilities.

```
In [15]: M emcee_plotter = aplt.EmceePlotter(samples=result.samples)
emcee_plotter.corner()

2021-07-26 16:42:47,675 - root - WARNING - Too few points to create valid contours
2021-07-26 16:42:47,712 - root - WARNING - Too few points to create valid contours
2021-07-26 16:42:47,737 - root - WARNING - Too few points to create valid contours
```



Cosmology: Strong Gravitational Lensing

Strong Gravitational Lensing

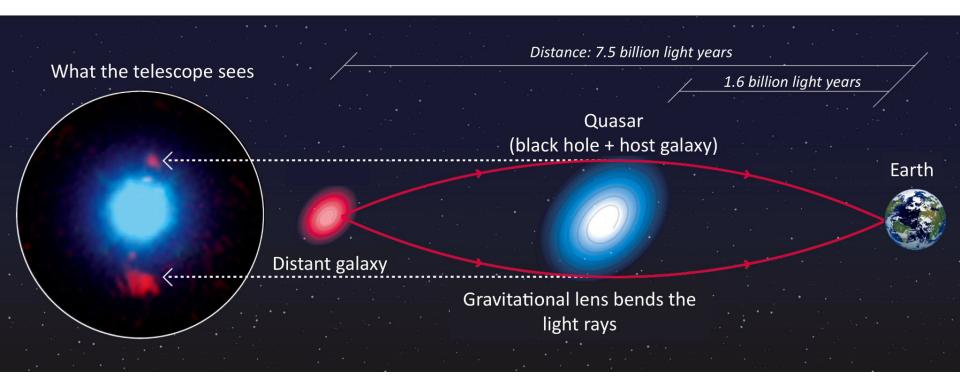
"Normal" Galaxy:



Strong Gravitational Lens:



Strong Gravitational Lensing



PyAutoFit: Model Composition

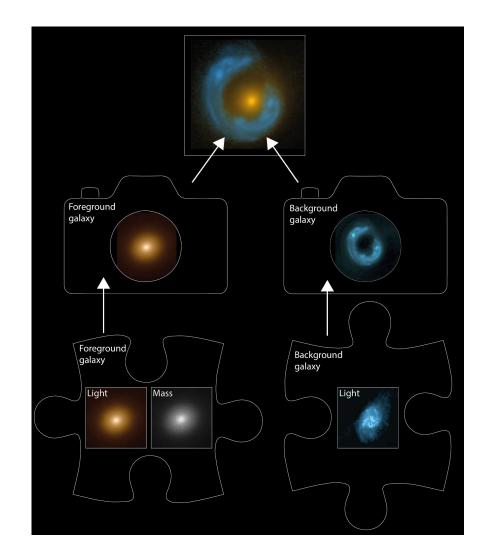
Model Composition

Break strong lens system into different model components:

Lens Galaxy: Light +

Mass

Source Galaxy: Light



Light and Mass Profile classes

Write the **model components** of the problem as **Python classes** using the same API shown previously.

Note how the **model specific** calculations of this problem are functions of the classes.

Light Profile:

```
class LightDeVaucouleurs:
    def __init__(
        self.
        centre: typing.Tuple[float, float] = (0.0, 0.0),
        axis_ratio : float = 1.0.
        angle : float = 0.0,
       intensity: float = 0.1,
        effective_radius: float = 0.6,
        """The De Vaucouleurs light profile representing the bulge of galaxies....""
        self.centre = centre
        self.axis ratio = axis ratio
        self.angle = angle
       self.intensity = intensity
       self.effective_radius = effective_radius
    def transform_grid_to_reference_frame(self, grid : np.ndarray):...
    def grid_to_elliptical_radii(self, grid : np.ndarray) -> np.ndarray:...
    def image_from_grid(self, grid : np.ndarray) -> np.ndarray:...
```

Light and Mass Profile classes

Mass Profile:

class MassIsothermal: def __init__(self. centre: typing.Tuple[float, float] = (0.0, 0.0), axis_ratio : float = 1.0, angle : float = 0.0, mass: float = 1.0,): """Represents an elliptical isothermal mass distribution....""" self.centre = centre self.axis_ratio = axis_ratio self.angle = angle self.mass = mass def transform_grid_to_reference_frame(self, grid : np.ndarray):... def rotate_grid_from_reference_frame(self, grid : np.ndarray) -> np.ndarray:... def psi_from(self, grid : np.ndarray) -> np.ndarray:... def deflections_from_grid(self, grid : np.ndarray) -> np.ndarray:...

Light Profile:

```
class LightExponential:
   def init (
        self.
       centre: typing.Tuple[float, float] = (0.0, 0.0),
        axis ratio : float = 1.0.
       angle : float = 0.0.
       intensity: float = 0.1,
        effective radius: float = 0.6.
       """The Exponential light profile representing the disk of galaxies...."""
        self.centre = centre
        self.axis ratio = axis ratio
        self.angle = angle
        self.intensity = intensity
        self.effective radius = effective radius
   def transform_grid_to_reference_frame(self, grid : np.ndarray) -> np.ndarray:...
   def grid_to_elliptical_radii(self, grid : np.ndarray) -> np.ndarray:...
   def image_from_grid(self, grid : np.ndarray) -> np.ndarray:...
```

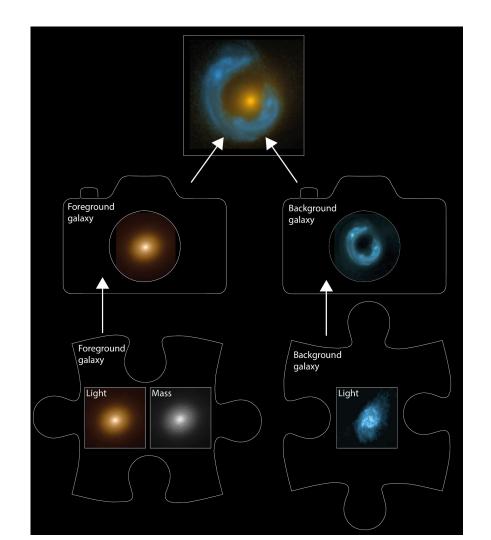
Model Composition

Break strong lens system into different model components:

Lens Galaxy: Light +

Mass

Source Galaxy: Light



Python Classes

The use of Python Classes to define the has a crucial additional benefit.

It allows for multi-level model composition.

Core for PyAutoFit's graphical modeling and hierarchical modeling functionality.

Galaxy Class

Combine the mass and light profiles at a specific redshift to make the lens galaxy and source galaxy.

Note how the **image_from_grid** and **deflections_from_grid** methods are included, which use the methods of the individual light and mass profiles.

Redshift = Distance from us in the Universe.

```
class Galaxv:
   def __init__(
           self.
           redshift: float,
           light_profiles: Optional[List] = None,
           mass_profiles: Optional[List] = None,
       """A qalaxy, which contains light and mass profiles at a specified redshift...
        self.redshift = redshift
       self.light_profiles = light_profiles
       self.mass_profiles = mass_profiles
   def image_from_grid(self, grid : np.ndarray) -> np.ndarray:
        if len(self.light_profiles) > 0:
            return sum(
                map(lambda p: p.image_from_grid(grid=grid), self.light_profiles)
       return np.zeros((grid.shape[0],))
   def deflections_from_grid(self, grid : np.ndarray) -> np.ndarray:
       if len(self.mass_profiles) > 0:
            return sum(
               map(lambda p: p.deflections from grid(grid=grid), self.mass profiles)
       return np.zeros((grid.shape[0], 2))
```

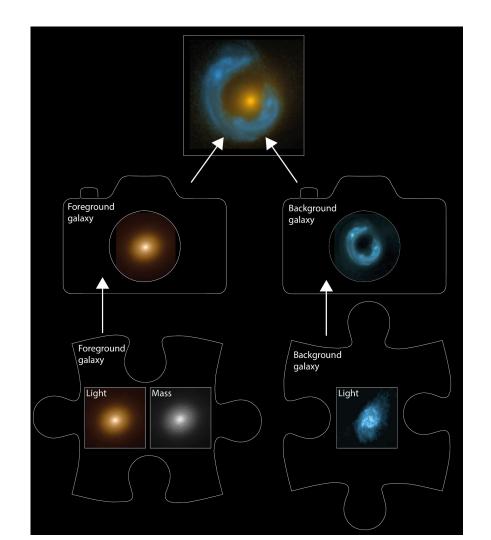
Model Composition

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Lens Galaxy: Light +

Mass

Source Galaxy: Light



Composing the Model

Scans every light and mass profile to determine this model has 16 free parameters that the non-linear search fits.

- A user can easily extend the model with more light profiles, mass profiles, etc.

This is the API a user of your model-fitting software is greeted with!

```
import autofit as af
lens galaxy model = af.Model(
   Galaxy,
   redshift=0.5,
    bulge=LightDeVaucouleurs,
   mass=MassIsothermal
source galaxy model = af.Model(
   Galaxy,
   redshift=1.0.
   disk=LightExponential
model = af.Collection(
   lens=lens galaxy model,
    source=source galaxy model
```

Writing the Analysis

By using Python classes as the model components, this means we can write a concise likelihood function.

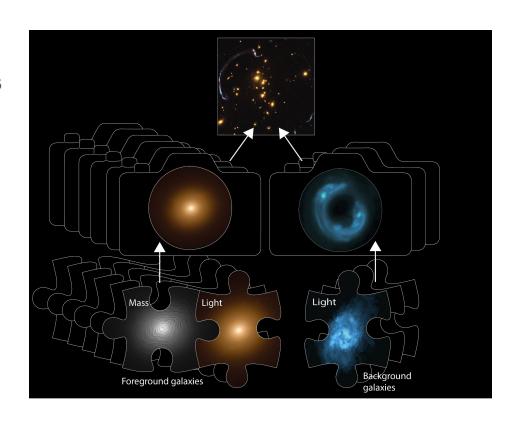
- Cleanly separate the model-specific code (e.g. light profiles, mass profiles, lensing) from the model-fitting code.
- Easy to extend and customize the Analysis class for bespoke model-fitting.

```
class Analysis(af.Analysis):
   def init (self, image, noise map, psf, grid):
        self.image = image
       self.noise map = noise map
       self.psf = psf
       self.grid = grid
   def log likelihood function(self, instance):
       The 'instance' that comes into this method contains the `Galaxy`'s
        we setup in the model.
       print("Lens Model Instance:")
       print("Lens Galaxy = ", instance.lens)
       print("Lens Galaxy Bulge = ", instance.lens.bulge)
       print("Lens Galaxy Bulge Centre = ", instance.lens.bulge.centre)
       print("Lens Galaxy Mass Centre = ", instance.lens.mass.centre)
       print("Source Galaxy = ", instance.sources)
       The methods of the 'Galaxy' class are available, making it easy to fit
        the lens model.
       lens image = instance.lens.image from grid(grid=self.grid)
       deflections = instance.lens.deflections from grid(grid=self.grid)
        source grid = self.grid - deflections
        source image = instance.source.image from grid(grid=source grid)
       model image = lens image + source image
       model image = self.psf.convolve(model image)
       residual map = self.image - model image
       chi squared map = (residual map / self.noise map) ** 2.0
       log likelihood = -0.5 * sum(chi squared map)
       return log likelihood
```

Model Composition

Straightforward for complex models to be composed and fitted in a scalable and streamlined way:

- Easy to extend model galaxies with many light and mass profiles.
- Or extend the model with many more galaxies.



PyAutoLens: Open Source Strong Gravitational Lensing

All code publically available (pip / conda), object oriented design, extensive documentation.

GitHub: https://github.com/Jammy2211/PyAutoLens

Readthedocs: https://pyautolens.readthedocs.io/en/latest/

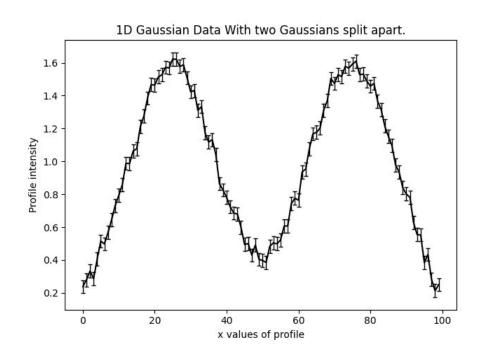
JOSS paper: https://joss.theoj.org/papers/10.21105/joss.02825

The HowToLens Jupyter notebook lectures teach strong lens modeling to beginners (pitched at undergrads and above)!

Features

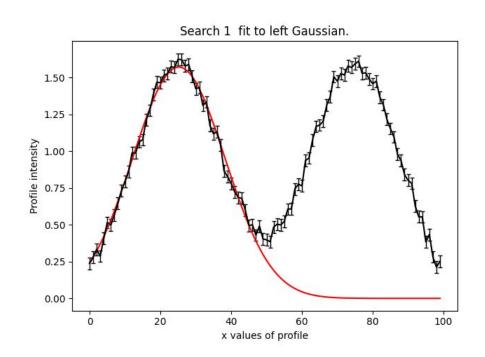
PyAutoFit: Advanced

Break a model-fit into a chained sequence of searches:



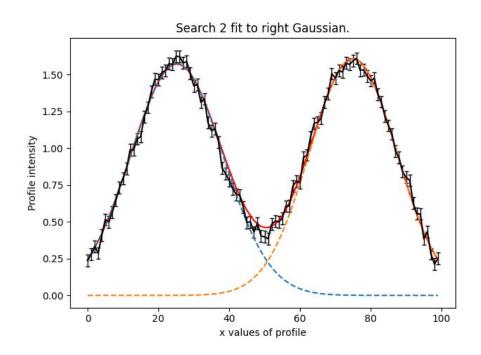
Break a model-fit into a chained sequence of searches:

Search 1: fit model to left Gaussian with fast non-linear search.



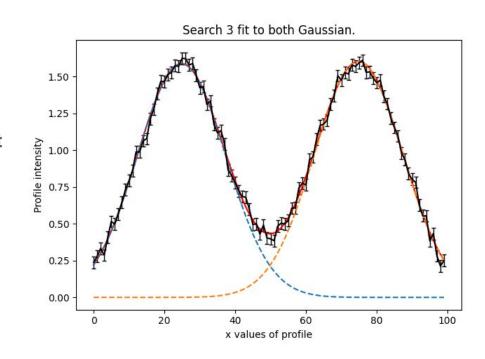
Break a model-fit into a chained sequence of searches:

- Search 1: fit model to left Gaussian with fast non-linear search.
- Search 2: fit model to right Gaussian with fast non-linear search and result of search 1.



Break a model-fit into a chained sequence of searches:

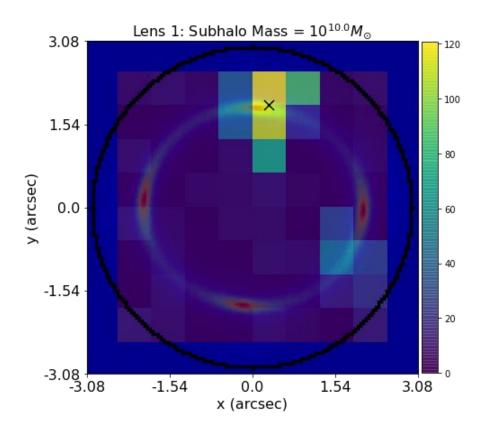
- Search 1: fit model to left Gaussian with fast non-linear search.
- Search 2: fit model to right Gaussian with fast
 non-linear search and result of search 1.
- Search 3: Fit both Gaussians simultaneously with thorough non-linear search and a parameter space starting point inferred from first two searches.



Grid Search of Non-linear Searches

Break a model-fit into a grid search of searches:

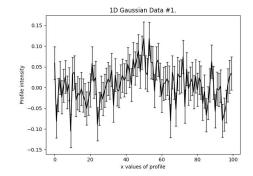
- Support for massively parallel fits.
- Database provides tools for analysing results efficiently.

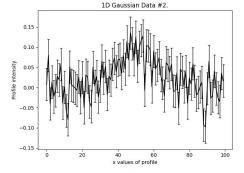


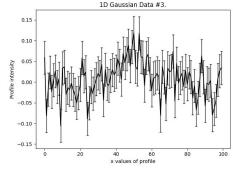
Graphical and Hierarchical Models

Compose multi-level models for fitting many datasets:

- Simple example of fitting three low signal to noise Gaussians simultaneously.
- Can assume all three Gaussians have same centre, but different intensity / sigma.







Graphical and Hierarchical Models

Can set up unique **Analysis** class, which will pair every dataset with components of the multi-level model:

```
analysis_0 = a.Analysis(data=data_0, noise_map=noise_map_0)
analysis_1 = a.Analysis(data=data_1, noise_map=noise_map_1)
analysis_2 = a.Analysis(data=data_2, noise_map=noise_map_2)
```

Model is built out of individual model components like before.

```
centre_shared_prior = af.GaussianPrior(mean=50.0, sigma=30.0)
gaussian_0 = af.Model(m.Gaussian)
gaussian_0.centre = centre_shared_prior
gaussian_0.intensity = af.GaussianPrior(mean=10.0, sigma=10.0)
gaussian_0.sigma = af.GaussianPrior(mean=10.0, sigma=10.0) # This prior is used by all 3 Gaussians!
prior_model_0 = af.Collection(gaussian=gaussian_0)
gaussian_1 = af.Model(m.Gaussian)
gaussian_1.centre = centre_shared_prior
gaussian_1.intensity = af.GaussianPrior(mean=10.0, sigma=10.0)
gaussian_1.sigma = af.GaussianPrior(mean=10.0, sigma=10.0) # This prior is used by all 3 Gaussians!
prior_model_1 = af.Collection(gaussian=gaussian_1)
```

PyAutoFit: Customization

Python Classes

The use of Python Classes to define the model, analysis and non-linear searches has downsides relative to other PPLs:

- It is a less concise interface.
- It requires a basic understanding of Python classes and object oriented programming (albeit good documentation can alleviate this).

The benefit is it provides a far more customizable model-fitting experience.

Customizing the Model

Full customization of the model parameterization, priors and valid regions of parameter space.

 Default priors can be specified in easy to set up configuration files, so a new user does not need to 'think' about them.

```
Compose model with multiple-components.
gaussian 0 = af.Model(Gaussian)
gaussian 1 = af.Model(Gaussian)
Manually set prior on each parameter.
gaussian 0.centre = af.UniformPrior(lower limit=0.0, upper limit=100.0)
gaussian 0.intensity = af.LogUniformPrior(lower limit=0.0, upper limit=1e2)
gaussian 0.sigma = af.GaussianPrior(mean=10.0, sigma=5.0, lower limit=0.0, upper limit=np.inf)
Fix a parameter to a value (reducing dimensionality of parameter space by 1).
gaussian 0.sigma = 0.5
Link two parameters in a model (reducing dimensionality of parameter space by 1).
gaussian 0.centre = gaussian 1.centre
Make assertions removing regions of parameter space.
gaussian 1.add assertion(gaussian 1.sigma > 5.0)
To make a model with multiple components we use a `Collection` object.
model = af.Collection(gaussian 0=gaussian 0, gaussian 1=gaussian 1)
```

Customizing the Model

Full customization of the model parameterization, priors and valid regions of parameter space.

- Straightforward to add many different model-components via inheritance.
- Composition makes this concise and scalable.

```
class Gaussian:
    def __init__(
            self.
            centre=0.0.
           intensity=0.1,
            sigma=0.01,
   ):
        self.centre = centre
        self.intensity = intensity
        self.sigma = sigma
class GaussianKurtosis(Gaussian):
   def init (
            self.
            centre=0.0,
           intensity=0.1,
           sigma=0.01.
            kurtosis=0.1.
   ):
        super(). init (
            centre=centre,
           intensity=intensity,
            sigma=sigma
        self.kurtosis = kurtosis
class Exponential:
   def init (
        self.
        centre=0.0.
       intensity=0.1,
        rate=0.01,
   ):
        self.centre = centre
        self.intensity = intensity
        self.rate = rate
```

Customizing the Analysis

The Analysis class can be extended or provide model-specific on-the-fly visualization of the model-fit so far.

- Uses the maximum likelihood model of the search so far.
- For long model-fits can inform you if the fitting has gone wrong early.

```
class Analysis(af.Analysis):
   def init (self, data, noise map):
        self.data = data
        self.noise map = noise map
    def log likelihood function(self, instance):
        . . .
    def visualize(self, paths, instance):
        During a model-fit, the 'visualize' method is called throughout the
        non-linear search. The `instance` is maximum log likelihood solution
        obtained so far and is used to output on-the-fly images.
        xvalues = np.arange(self.data.shape[0])
        model data = instance.profile from xvalues(xvalues=xvalues)
        residual map = self.data - model data
        plt.errorbar(
            x=xvalues, v=residual map, color="k", ecolor="k",
        plt.title("1D Residual Map")
        plt.xlabel("x value of profile")
        plt.ylabel("Residual")
        plt.savefig(path.join(paths.image path, "residual map.png"))
        plt.clf()
```

Customizing the Search

PyAutoFit supports many non-linear searches (MCMC, nested sampling, optimizers, etc.).

- Full customization of their settings.
- Defaults to configuration file values if not specified.

```
emcee = af.Emcee(
   name="example_mcmc",
   nwalkers=50,
   nsteps=2000,
   initializer=af.InitializerBall(lower_limit=0.49, upper_limit=0.51),
   auto_correlations_settings=af.AutoCorrelationsSettings(
        check_for_convergence=True,
        check_size=100,
        required_length=50,
        change_threshold=0.01,
   ),
}
```

PyAutoFit: Features

Database

Results of many model fits are output in an sqlite relational database:

- Allocated a unique identifier based on the model-fit, such that you can trivially fit many models.
- Database supports advanced queries (e.g. find all results, where this parameter is in this range).
- Results use memory-light Python generators.

You can therefore fit (very) large datasets on a HPC and access the results efficiently via a Jupyter notebook.

```
agg = af.Aggregator.from_database("database.sqlite")
bulge = agg.lens.bulge
agg_query = agg.query(bulge == LightDeVaucouleurs)

for samples in agg_query.values("samples"):
    print("Maximum Log Likelihood Instance:")
    print(samples.max_log_likelihood_instance)
```

Advanced Modeling Tools

Search Grid Search: Massively parallel grid searches of non-linear searches.

Search Chaining: Write highly customizable model-fitting pipelines that chain together multiple non-linear searches.

Sensitivity Mapping: Simulate and fit many datasets to determine when a more complex model would be accepted via model comparison.

Graphical / Hierarchical Models: Fit for global trends in large datasets by composing and fitting graphical models.