1.25. $\int\left(\sqrt[5]{x}-\frac{4}{x^5}+2\right)dx=\int x^{\frac{1}{5}}dx-4\int x^{-5}dx+\int 2\,dx=$

$=\frac{x^{\frac{1}{5}+1}}{\frac{1}{5}+\frac{5}{5}}+4\cdot\frac{x^{-4}}{+4}+2x+C=\frac{5}{6}x^{\frac{6}{5}}+x^{-4}+2x+C$

2.25. $\int\sqrt[4]{2-5x}\,dx=\left|\begin{matrix}2-5x=t & dx=\frac{-dt}{5}\\ -5dx=dt & \end{matrix}\right|=\int\sqrt[4]{t}\cdot\left(\frac{-dt}{5}\right)=$

$=-\frac{1}{5}\int t^{\frac{1}{4}}dt=-\frac{1}{5}\cdot\frac{4t^{\frac{5}{4}}}{5}=-\frac{4}{25}\cdot\sqrt[4]{t^5}+C=$

$=-\frac{4}{25}\cdot\sqrt[4]{(2-5x)^5}+C$

3.25. $\int\frac{dx}{7-3x}=\left|\begin{matrix}7-3x=t & dx=-\frac{dt}{3}\\ -3dx=dt & \end{matrix}\right|=\int-\frac{1}{3t}dt=-\frac{1}{3}\ln|t|+C$

$=-\frac{1}{3}\ln|7-3x|+C$

4.25 $\int\cos(3x-7)\,dx=\left|\begin{matrix}3x-7=t & dx=\frac{dt}{3}\\ 3dx=dt & \end{matrix}\right|=\frac{1}{3}\int\cos t\,dt=$

$=\frac{1}{3}\sin t+C=\frac{1}{3}\cdot\sin(3x-7)+C$

5.25. $\int\frac{3x\,dx}{9x^2+2}=\left|\begin{matrix}9x^2+2=t & dx=\frac{dt}{18x}\\ 18x\,dx=dt & dx=\frac{dt}{3x\cdot 6}\end{matrix}\right|=\int\frac{1}{6t}dt=\frac{1}{6}\ln|t|+C=$

$=\frac{1}{6}\ln|9x^2+2|+C$

6.25. $\int\frac{dx}{3x^2+4}=\frac{1}{3}\int\frac{dx}{x^2+\frac{4}{3}}=\frac{1}{3}\int\frac{dx}{(x)^2+\left(\frac{2}{\sqrt{3}}\right)^2}=\frac{1}{3}\cdot\frac{1}{\frac{2}{\sqrt{3}}}\cdot\operatorname{arctg}\frac{x}{\frac{2}{\sqrt{3}}}+C=$

$=\frac{\sqrt{3}}{6}\cdot\operatorname{arctg}\frac{x\sqrt{3}}{2}+C$

7.25. $\int e^{4-5x}dx=\left|\begin{matrix}4-5x=t & dx=\frac{-dt}{5}\\ -5dx=dt & \end{matrix}\right|=-\frac{1}{5}\int e^t dt=$

$-\frac{1}{5}e^t+C=-\frac{1}{5}e^{4-5x}+C$

8.25 $\int\frac{dx}{(x+3)\ln^4(x+3)}=\int\frac{d(\ln(x+3))}{\ln^4(x+3)}=\int\frac{dt}{t^4}=-\frac{1}{3t^3}+C=$

$=-\frac{1}{3\ln^3(x+3)}+C$

9.25 $\int \frac{dx}{\sin^2 x \cdot \sqrt[5]{ctg^4 x}} = -\int \frac{d(ctg x)}{ctg^{\frac{4}{5}} x} = -\int \frac{dt}{t^{\frac{4}{5}}} = -\int t^{-\frac{4}{5}} dt =$

$= -\frac{t^{\frac{1}{5}}}{\frac{1}{5}} = -5 \cdot \sqrt[5]{t} = -5 \cdot \sqrt[5]{ctg x} + C$

10.25 $\int \frac{\arcsin^2 5x}{\sqrt{1-25x^2}} dx = \left| \begin{array}{l} \arcsin 5x = t \\ \frac{5}{\sqrt{1-25x^2}} dx = dt \end{array} \right| = \int \frac{t^2}{5} dt = \frac{t^3}{15} + C =$

$= \frac{1}{15} \cdot \arcsin^3 5x + C$

11.25 a) $\int_0^{\sqrt{6}} \sqrt{6-x^2}\, dx = \left(\frac{x}{2} \sqrt{6-x^2} + \frac{6}{2} \arcsin \frac{x}{\sqrt{6}} + C \right) \Big|_0^{\sqrt{6}} =$

$= \frac{\sqrt{6}}{2} \cdot 0 + \frac{6}{2} \cdot \arcsin \frac{\sqrt{6}}{\sqrt{6}} - 0 - \arcsin 0 = \frac{6\pi}{4}$

12.25. $r = 5(1+\cos\varphi)$

$\int_0^{10} (5 + 5\cos\varphi) d\varphi = (5\varphi + 5\sin\varphi) \Big|_0^{10} =$

$\max(\cos\varphi) = 1 \quad r_1 = 10$

$= 50 + 5\sin 10$ kv. birlik.

$\min(\cos\varphi) = -1 \quad r_2 = 0$

11.25 b) $\int_1^3 \ln(2x+3) dx = \left| \begin{array}{ll} \ln(2x+3) = u & dx = dv \\ \frac{2}{2x+3} dx = du & v = x \end{array} \right| \Rightarrow$

$\int u\, dv = uv - \int v\, du$ ga asosan

$\Rightarrow x \ln(2x+3) \Big|_1^3 - \int_1^3 \frac{2x}{2x+3} dx = \left(x \ln(2x+3) \right) \Big|_1^3 - \left| \begin{array}{l} 2x+3 = t \\ 2dx = dt \\ dx = \frac{dt}{2} \end{array} \right| =$

$= (x \ln(2x+3)) \Big|_1^3 - \int_1^3 \frac{dt}{t} = (x \ln(2x+3) - \ln|2x+3|) \Big|_1^3 = 4 \ln 3$

13.25 $\begin{cases} x = 5\sin^3 t \\ y = 5\cos^3 t \end{cases}, 0 \le t \le \pi, \quad l = ?$

$l = \int_{t_1}^{t_2} \sqrt{(x'(t))^2 + (y'(t))^2} dt \qquad \begin{array}{l} x'(t) = 15\sin^2 t \cdot \cos t \\ y'(t) = -15\cos^2 t \cdot \sin t \end{array}$

$l = \int_0^{\pi} \sqrt{15^2 \sin^4 t \cdot \cos^2 t + 15^2 \cdot \cos^4 t \cdot \sin^2 t}\, dt = \int_0^{\pi} 15 \sqrt{(\cos^2 t)^2 + (\sin^2 t)^2} =$

$= \int_0^{\pi} 15 \sqrt{\sin^2 t \cos^2 t \cdot \sin^2 t + \cos^2 t \cdot \cos^2 t \cdot \sin^2 t}\, dt = 15 \int_0^{\pi} \sin t \cos t\, dt = \frac{15}{2} \int_0^{\pi} \sin 2t\, dt$

$= -\frac{15}{2 \cdot 2} \cos 2t \Big|_0^{\pi} = -\frac{15}{2 \cdot 2} - \frac{15}{2 \cdot 2} = -\frac{15}{2}$

uzunlik manfiy emas

$l = 7{,}5$

Yo'nalishli fetkari

14.25. $y=(x-1)^2,\ x=3,\ y=0$

Oy: $\Rightarrow$ $x=1$ da $y=0$

$$V=\pi\int_{t_1}^{t_2} xy\,dt=\pi\int_1^3 3(x-1)^2dx=\pi\cdot\frac{(x-1)^3}{3}\Big|_1^3=\frac{\pi\cdot 2^3}{3}-0=$$

$$=\frac{8\pi}{3}$$ kub. birlik

15.25. a) $$\int_0^\infty x^3e^{-x^2}dx=\int_0^\infty\left|\begin{matrix}x^3=u & e^{-x^2}dx=d\vartheta\\ 3x^2dx=du & \vartheta=-\frac{1}{2x}\cdot e^{-x^2}\end{matrix}\right|=$$

$$=\int\frac{te^{-t}}{2}dt=\frac{1}{2}\left(t\cdot(-e^{-t})+\int e^{-t}dt\right)=\frac{1}{2}\left(x^2\cdot(-e^{-x^2})-e^{-x^2}\right)=$$

$$=-\frac{x^2+1}{2e^{x^2}}\Big|_0^\infty=-\frac{1+\frac{1}{x^2}}{\frac{2}{x^2}\cdot e^{x^2}}\Big|_0^\infty=\frac{1}{2}$$

b) $$\int_0^{1/5}\frac{3x\,dx}{\sqrt[5]{1-25x^2}}=\left|\begin{matrix}1-25x^2=t & 3x\,dx=\frac{-3}{50}dt\\ -\frac{50}{3}\cdot 3x\,dx=dt & \end{matrix}\right|=\int_0^{1/5}\frac{-3\,dt}{50t^{\frac{1}{5}}}=$$

$$-\frac{3}{\cancel{50}_{10}}\cdot\frac{t^{\frac{4}{5}}}{4}\cdot\cancel{5}\Big|_0^{1/5}=-\frac{3}{40}\cdot\sqrt[5]{(1-25x^2)^4}\Big|_0^{1/5}=-\frac{3}{40}\cdot\sqrt[5]{\left(1-25\cdot\frac{1}{25}\right)^4}+$$

$$+\frac{3}{40}\cdot\sqrt[5]{1}=\frac{3}{40}$$