1-shaxsiy topshiriq.

1.25) $Z = -4 + 4i\sqrt{3}$ $\quad n = 6,\ k = 4.\quad z^n, \sqrt[k]{z} = ?$

$r = \sqrt{x^2 + y^2} = \sqrt{16 + 48} = 8$

$x = r\cos\varphi$ $\qquad z = 8\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

$\cos\varphi = \frac{x}{r}$

$\qquad z^6 = 8^6\left(\cos\frac{16\pi}{3} + i\sin\frac{16\pi}{3}\right)$

$\cos\varphi = -\frac{1}{2}$

$\varphi = \frac{2\pi}{3}$ $\qquad \sqrt[4]{z} = \sqrt[4]{8}\cdot\left(\cos\frac{\frac{2\pi}{3} + 2\pi k}{4} + i\sin\frac{\frac{2\pi}{3} + 2\pi k}{4}\right)$

2.25)
$$\lim_{x\to\infty}\frac{2x^2 + 8x - 11}{-x^2 + 3x + 4} = \lim_{x\to\infty}\frac{2 + \frac{8}{x} - \frac{11}{x^2}}{-1 + \frac{3}{x} + \frac{4}{x^2}} = \boxed{-2}$$

3.25)
$$\lim_{x\to 2}\frac{(\sqrt{3x-2} - x)(\sqrt{3x-2} + x)}{(3x^2 - x - 10)(\sqrt{3x-2} + x)} = \lim_{x\to 2}\frac{3x - 2 - x^2}{(3x^2 - x - 10)(\sqrt{3x-2} + x)} =$$

$$= \lim_{x\to 2}\frac{\cancel{(x-2)}(1-x)}{(3x+5)\cancel{(x-2)}(\sqrt{3x-2} + x)} = \lim_{x\to 2}\frac{1-x}{(3x+5)(\sqrt{3x-2} + x)} = \boxed{-\frac{1}{44}}$$

4.25)
$$\lim_{x\to 0}\frac{\operatorname{tg}x - \sin x}{x(1 - \cos 2x)} = \lim_{x\to 0}\frac{\frac{\sin x(1 - \cos x)}{\cos x}}{2x\sin^2 x} =$$

$$= \lim_{x\to 0}\frac{1 - \cos x}{x\cdot\sin 2x} = \lim_{x\to 0}\frac{\sin x}{\sin 2x + 2x\cdot\cos 2x} =$$

$$= \lim_{x\to 0}\frac{\cos x}{4\cos 2x - 4x\sin 2x} = \boxed{\frac{1}{4}}$$

5.25) $\lim\limits_{x\to 3}\left(\frac{x-2}{2x-5}\right)^{\frac{2}{3-x}} = \lim\limits_{x\to 3}\left(1+\frac{1}{\frac{2x-5}{3-x}}\right)^{\frac{2}{3-x}\cdot\frac{2x-5}{3-x}\cdot\frac{3-x}{2x-5}} =$

$= e^{\lim\limits_{x\to 3}\frac{2}{x-3}\cdot\frac{x-3}{2x-5}} = e^{\lim\limits_{x\to 3}\frac{2}{2x-5}} = e^2$

6.25) a) $f(x) = \begin{cases} x-5, & x<-2 \\ -2x^2+1, & -2\le x\le 2 \\ 2x+3, & x>2 \end{cases}$

$\lim\limits_{x\to -2-0} x-5 = -7$ $\qquad$ $\lim\limits_{x\to 2-0} -2x^2+1 = -7$

$\lim\limits_{x\to -2+0} -2x^2+1 = -7$ $\qquad$ $\lim\limits_{x\to 2+0} 2x+3 = 7$

$f(-2) = -7$ $\qquad$ $f(2) = 7$

$x=-2$ da uzluksiz $\qquad$ 1-tur sakrash

b) $f(x) = \frac{1}{3^{\frac{1}{2-x}}} + 1$ $\qquad$ $\lim\limits_{x\to 2+0} \frac{1}{3^{\frac{1}{2-x}}}+1 = \infty$

$\lim\limits_{x\to 2-0} \frac{1}{3^{\frac{1}{2-x}}}+1 = 0$ $\qquad$ 2-tur uzulish

7.25) $(y)' = \left(\sin(tg^2 3x)\cdot \ln\sqrt{1+2x^2}\right)' =$

$= \cos(tg^2 3x)\cdot 2tg3x\cdot\frac{3}{\cos^2 3x}\cdot\ln\sqrt{1+2x^2} + \sin(tg^2 3x)\cdot\frac{4x}{2\sqrt{1+2x^2}} =$

$= \frac{6\cdot\cos(tg^2 3x)\cdot tg\,3x\cdot\ln\sqrt{1+2x^2}}{\cos^2 3x} + \frac{2x\cdot\sin(tg^2 3x)}{1+2x^2}$

8.25) $x^4 + y^4 = x^2 \cdot y^2$

$4x^3 + 4y^3 = 2xy^2 + 2yx^2$

$12x^2 + 12y^2 = 2y^2 + 4xy + 4xy + 2x^2y'$

$$y' = \frac{12x^2 + 12y^2 - 2y^2 - 8xy}{2x^2}$$

$$y' = \frac{6x^2 + 5y^2 - 4xy}{x^2}$$

9.25) $y = \frac{1+x}{\sqrt{x}}$

$y_0 = x^{-\frac{1}{2}}$ $\qquad$ $y_1 = x^{\frac{1}{2}}$

$y_0' = -\frac{1}{2}x^{-\frac{3}{2}}$ $\qquad$ $y_1' = \frac{1}{2}x^{-\frac{1}{2}}$

$y_0'' = \left(-\frac{1}{2}\right)\cdot\left(-\frac{3}{2}\right)\cdot x^{-\frac{5}{2}}$ $\qquad$ $y_1'' = \frac{1}{2}\left(-\frac{1}{2}\right)\cdot x^{-\frac{3}{2}}$

$y''' = \left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\cdot\left(-\frac{5}{2}\right)\cdot x^{-\frac{7}{2}}$ $\qquad$ $y_1''' = \frac{1}{2}\cdot\left(-\frac{1}{2}\right)\cdot\left(-\frac{3}{2}\right)\cdot x^{-\frac{5}{2}}$

$$y^{(n)}(x) = \frac{(-1)^n}{2^n}(2n-1)\cdot x^{-\frac{1}{2}-n} + \frac{(-1)^{n+1}}{2^n}(2n-1)\cdot x^{\frac{1}{2}-n}$$

10.25) $\begin{cases} x = \operatorname{arctg} t \\ y = t^2/2 \end{cases}$

$$y_{xx}'' = \frac{y_t''}{x_t'} = \frac{\left(\left(\frac{t^2}{2}\right)'\right)'}{(\operatorname{arctg} t)'} =$$

$$= \frac{t'}{\frac{1}{1+t^2}} = \frac{1}{\frac{1}{1+t^2}} = t^2 + 1$$

$$y_x' = \frac{\frac{t^2}{2}}{\operatorname{arctg} t} = t(1+t^2)$$

$$y_{xx}'' = 1 + t^2$$

1,25) a) $\sqrt{1{,}37^2+5}$ $\sqrt{x+h} \approx \sqrt{x} + \frac{1}{2\sqrt{x}} \cdot h$

$f(x_0+h) \approx f(x_0) + f'(x_0) \cdot h$

$\sqrt{1{,}37^2+5} = \sqrt{2{,}87^2 + 0{,}644} \approx 2{,}87 + \frac{1}{2 \cdot 2{,}87} \cdot 0{,}644 \approx$

$\approx 2{,}98008$

0,01

$= 2{,}98$

b) $\cos 61° = \cos(60°+1°) \approx \cos 60° + (-\sin 60°) \cdot 1°$

$= \frac{1}{2} - \frac{\sqrt{3}}{2} \cdot \frac{3{,}14}{180} = 0{,}484893 = 0{,}48$

0,01

2,25) a) $\lim\limits_{x\to 0} \frac{e^x - e^{-x} - 2x}{x - \sin x} = \lim\limits_{x\to 0} \frac{\frac{e^{2x}+1-2e^x}{e^x}}{1-\cos x} = \lim\limits_{x\to 0} \frac{\frac{e^{2x}-1}{e^x}}{\sin x} =$

$= \lim\limits_{x\to 0} \frac{e^{2x}+1}{e^x \cos x} = 2$

b) $\lim\limits_{x\to 1} (\operatorname{tg} \pi x)^{2\operatorname{arctg} x - 1} = \lim\limits_{x\to 1} (\operatorname{tg} \pi \cdot 1)^{2\operatorname{arctg} 1 - 1} =$

$= \lim\limits_{x\to 1} (\operatorname{tg} \pi)^{\frac{\pi}{2}-1} = 0$

3(2s) $y=(x-1)\,e^{-x}$ $[0;3]$

$f'(x)=2\cdot e^{-x}-x\cdot e^{-x}$ $e^{-x}(2-x)=0$

$f'(x)=e^{-x}(2-x)$ $x=2$.

$f(0)=-1$ eng kichik

$f(3)=\frac{2}{e^3}=\frac{2}{e}\cdot\frac{1}{e^2}$

$f(2)=\frac{1}{e^2}$ eng katta

4(2s) $y=\frac{x^4}{x^3-1}$

1) $D(y)=(-\infty;1)\cup(1;\infty)$

2) $E(y)=R$

3). Na juft, na toq.

4) $x=1$ da uzilishga ega

$\lim\limits_{x\to 1+0}\frac{x^4}{x^3+1}=0$

$\lim\limits_{x\to 1-0}\frac{x^4}{x^3-1}=0$

5) $x=0$ da $y=0$

$y=0$ da $x=0$ $(0;0)$

6) Asimptotalari: $x=1\to$ vertikal.

$y=\frac{x^4}{x^3-1}$

$k=\lim\limits_{x\to\infty}\left(\frac{f(x)}{x}\right)=\lim\limits_{x\to 1}\frac{x^3}{x^3-1}=\lim\limits_{x\to\infty}\frac{x^3-1}{x^3}=\left(1-\frac{1}{x^3}\right)=1$.

$b=\lim\limits_{x\to\infty}\left(\frac{x^4}{x^3-1}-x\right)=\lim\limits_{x\to 1}\left(\frac{x^4-x^4+x}{x^3-1}\right)=\lim\limits_{x\to 1}\frac{x}{x^3-1}=\left(\frac{x^3-1}{x}\right)^{-1}=\frac{1}{\infty}$

$y=x$ gorizontal

5.25

$$L_3(x) = \frac{0{,}167\cdot\left(x-\frac{\pi}{12}\right)\left(x-\frac{\pi}{9}\right)\left(x-\frac{5\pi}{36}\right)}{\left(\frac{\pi}{18}-\frac{\pi}{12}\right)\left(\frac{\pi}{18}-\frac{\pi}{9}\right)\left(\frac{\pi}{18}-\frac{5\pi}{36}\right)} +$$

$$+\frac{0{,}268\cdot\left(x-\frac{\pi}{18}\right)\left(x-\frac{\pi}{9}\right)\left(x-\frac{5\pi}{36}\right)}{\left(\frac{\pi}{12}-\frac{\pi}{9}\right)\left(\frac{\pi}{12}-\frac{\pi}{18}\right)\left(\frac{\pi}{12}-\frac{5\pi}{36}\right)} + \frac{0{,}364\left(x-\frac{\pi}{18}\right)\left(x-\frac{\pi}{12}\right)\left(x-\frac{5\pi}{36}\right)}{\left(\frac{\pi}{9}-\frac{\pi}{18}\right)\left(\frac{\pi}{9}-\frac{\pi}{12}\right)\left(\frac{\pi}{9}-\frac{5\pi}{36}\right)} +$$

$$+\frac{0{,}466\left(x-\frac{\pi}{18}\right)\left(x-\frac{\pi}{12}\right)\left(x-\frac{\pi}{9}\right)}{\left(\frac{5\pi}{36}-\frac{\pi}{18}\right)\left(\frac{5\pi}{36}-\frac{\pi}{12}\right)\left(\frac{5\pi}{36}-\frac{\pi}{9}\right)}.$$

$$L_3(0{,}3) = \operatorname{arctg} 0{,}3 = 16^\circ\,.\,11'$$

1.25. $\int\left(\sqrt[5]{x}-\frac{4}{x^5}+2\right)dx=\int x^{\frac{1}{5}}dx-4\int x^{-5}dx+\int 2\,dx=$

$=\frac{x^{\frac{1}{5}+1}}{\frac{1}{5}+\frac{5}{5}}+4\cdot\frac{x^{-4}}{-4}+2x+C=\frac{5}{6}x^{\frac{6}{5}}+x^{-4}+2x+C$

2.25. $\int\sqrt[4]{2-5x}\,dx=\left|\begin{matrix}2-5x=t & dx=\frac{-dt}{5}\\ -5dx=dt & \end{matrix}\right|=\int\sqrt[4]{t}\cdot\left(\frac{-dt}{5}\right)=$

$=-\frac{1}{5}\int t^{\frac{1}{4}}dt=-\frac{1}{5}\cdot\frac{4t^{\frac{5}{4}}}{5}=-\frac{4}{25}\cdot\sqrt[4]{t^5}+C=$

$=-\frac{4}{25}\cdot\sqrt[4]{(2-5x)^5}+C$

3.25. $\int\frac{dx}{7-3x}=\left|\begin{matrix}7-3x=t & dx=-\frac{dt}{3}\\ -3dx=dt & \end{matrix}\right|=\int-\frac{1}{3t}dt=-\frac{1}{3}\ln|t|+C$

$=-\frac{1}{3}\ln|7-3x|+C$

4.25 $\int\cos(3x-7)\,dx=\left|\begin{matrix}3x-7=t & dx=\frac{dt}{3}\\ 3dx=dt & \end{matrix}\right|=\frac{1}{3}\int\cos t\,dt=$

$=\frac{1}{3}\sin t+C=\frac{1}{3}\cdot\sin(3x-7)+C$

5.25. $\int\frac{3x\,dx}{9x^2+2}=\left|\begin{matrix}9x^2+2=t & dx=\frac{dt}{18x}\\ 18x\,dx=dt & dx=\frac{dt}{3x\cdot 6}\end{matrix}\right|=\int\frac{1}{6t}dt=\frac{1}{6}\ln|t|+C=$

$=\frac{1}{6}\ln|9x^2+2|+C$

6.25. $\int\frac{dx}{3x^2+4}=\frac{1}{3}\int\frac{dx}{x^2+\frac{4}{3}}=\frac{1}{3}\int\frac{dx}{(x)^2+\left(\frac{2}{\sqrt{3}}\right)^2}=\frac{1}{3}\cdot\frac{1}{\frac{2}{\sqrt{3}}}\cdot\operatorname{arctg}\frac{x}{\frac{2}{\sqrt{3}}}+C=$

$=\frac{\sqrt{3}}{6}\cdot\operatorname{arctg}\frac{x\sqrt{3}}{2}+C$

7.25. $\int e^{4-5x}dx=\left|\begin{matrix}4-5x=t & dx=\frac{-dt}{5}\\ -5dx=dt & \end{matrix}\right|=-\frac{1}{5}\int e^{t}dt=$

$-\frac{1}{5}e^{t}+C=-\frac{1}{5}e^{4-5x}+C$

8.25 $\int\frac{dx}{(x+3)\ln^4(x+3)}=\int\frac{d(\ln(x+3))}{\ln^4(x+3)}=\int\frac{dt}{t^4}=-\frac{1}{3t^3}+C=$

$=-\frac{1}{3\ln^3(x+3)}+C$

9.25 $\int \frac{dx}{\sin^2 x \cdot \sqrt[5]{ctg^4 x}} = -\int \frac{d(ctg\,x)}{ctg^{\frac{4}{5}} x} = -\int \frac{dt}{t^{\frac{4}{5}}} = -\int t^{-\frac{4}{5}} dt =$

$= -\frac{t^{\frac{1}{5}}}{\frac{1}{5}} = -5 \cdot \sqrt[5]{t} = -5 \cdot \sqrt[5]{ctg\,x} + C$

10.25 $\int \frac{\arcsin^2 5x}{\sqrt{1-25x^2}} dx = \left| \begin{matrix} \arcsin 5x = t \\ \frac{5}{\sqrt{1-25x^2}} dx = dt \end{matrix} \right| = \int \frac{t^2}{5} dt = \frac{t^3}{15} + C =$

$= \frac{1}{15} \cdot \arcsin^3 5x + C$

11.25 a) $\int_0^{\sqrt{6}} \sqrt{6-x^2}\, dx = \left(\frac{x}{2}\sqrt{6-x^2} + \frac{6}{2} \arcsin \frac{x}{\sqrt{6}} + C \right) \Big|_0^{\sqrt{6}} =$

$= \frac{\sqrt{6}}{2} \cdot 0 + \frac{6}{2} \cdot \arcsin \frac{\sqrt{6}}{\sqrt{6}} - 0 \cdot \arcsin 0 = \frac{6\pi}{4}$

12.25 $r = 5(1+\cos\varphi)$

$\max(\cos\varphi) = 1 \quad r_1 = 10$

$\min(\cos\varphi) = -1 \quad r_2 = 0$

$\int_0^{10} (5 + 5\cos\varphi)\, d\varphi = (5\varphi + 5\sin\varphi) \Big|_0^{10} =$

$= 50 + 5\sin 10$ kv. birlik.

11.25 b) $\int_1^3 \ln(2x+3)\, dx = \left| \begin{matrix} \ln(2x+3) = u & dx = dv \\ \frac{2}{2x+3} dx = du & v = x \end{matrix} \right| \Rightarrow$

$\int u\, dv = uv - \int v\, du$ ga asosan

$\Rightarrow x \ln(2x+3) \Big|_1^3 - \int_1^3 \frac{2x}{2x+3} dx = \left(x \ln(2x+3) \right)\Big|_1^3 - \left| \begin{matrix} 2x+3 = t \\ 2dx = dt \\ dx = \frac{dt}{2} \end{matrix} \right| =$

$= \left(x \ln(2x+3) \right)\Big|_1^3 - \int_1^3 \frac{dt}{t} = \left(x \ln(2x+3) - \ln|2x+3| \right)\Big|_1^3 = 4\ln 3$

13.25 $\begin{cases} x = 5\sin^3 t \\ y = 5\cos^3 t \end{cases}, \quad 0 \le t \le \pi, \quad l = ?$

$$l = \int_{t_1}^{t_2} \sqrt{(x'(t))^2 + (y'(t))^2}\,dt \qquad \begin{aligned} x'(t) &= 15\sin^2 t \cdot \cos t \\ y'(t) &= -15\cos^2 t \cdot \sin t \end{aligned}$$

$$l = \int_0^{\frac{\pi}{4}} \sqrt{15^2 \sin^4 t \cdot \cos^2 t + 15^2 \cdot \cos^4 t \cdot \sin^2 t}\,dt = \int_0^{\frac{\pi}{4}} 15 \sqrt{(\cos^2 \cdot t)^2 + (\sin^2 t)^2} =$$

$$= \int_0^{\frac{\pi}{4}} 15 \sqrt{\sin^2 t \cos^2 t \cdot \sin^2 t + \cos^2 t \cdot \cos^2 t \cdot \sin^2 t}\,dt = 15\int_0^{\frac{\pi}{4}} \sin t \cos t\,dt = \frac{15}{2}\int_0^{\frac{\pi}{4}} \sin 2t\,dt$$

$$= -\frac{15}{2 \cdot 2} \cdot \cos 2t \Big|_0^{\frac{\pi}{4}} = -\frac{15}{2\cdot 2} - \frac{15}{2 \cdot 2} = -\frac{15}{2}$$

uzunlik manfiy

$l = 7{,}5$ emas [illegible] [illegible]

15.25. a) $\int_0^\infty x^3 e^{-x^2} dx = \int_0^\infty \left/ \begin{matrix} x^3 = u & e^{-x^2} dx = d\vartheta \\ 3x^2 dx = du & \vartheta = -\frac{1}{2x} \cdot e^{-x^2} \end{matrix} \right/ =$

$= \int \frac{te^{-t}}{2} dt = \frac{1}{2}\left(t \cdot (-e^{-t}) + \int e^{-t} dt\right) = \frac{1}{2}\left(x^2 \cdot (-e^{-x^2}) - e^{-x^2}\right) =$

$= -\frac{x^2+1}{2e^{x^2}}\Big|_0^\infty = -\frac{1+\frac{1}{x^2}}{\frac{2}{x^2} \cdot e^{x^2}}\Big|_0^\infty = \frac{1}{2}$

в) $\int_0^{1/5} \frac{3x\,dx}{\sqrt[5]{1-25x^2}} = \left| \begin{matrix} 1-25x^2 = t & 3x\,dx = \frac{-3}{50} dt \\ -\frac{50}{3} \cdot 3x\,dx = dt & \end{matrix} \right| = \int_0^{1/5} \frac{-3\,dt}{50 t^{\frac{1}{5}}} =$

$-\frac{3}{50} \cdot \frac{t^{\frac{4}{5}}}{4} \cdot 5 \Big|_0^{1/5} = -\frac{3}{40} \cdot \sqrt[5]{(1-25x^2)^4}\Big|_0^{1/5} = -\frac{3}{40} \cdot \sqrt[5]{\left(1-25 \cdot \frac{1}{25}\right)^4} +$

$+ \frac{3}{40} \cdot \sqrt[5]{1} = \frac{3}{40}$

1.25 $z = \operatorname{arctg}(x^2+y^2)$

$\frac{\partial z}{\partial x} = ?$ $\frac{\partial z}{\partial y} = ?$ $dz = ?$

$$\frac{\partial z}{\partial x} = \frac{1}{1+(x^2+y^2)^2} \cdot 2x$$

$$\frac{\partial z}{\partial y} = \frac{1}{1+(x^2+y^2)^2} \cdot 2y$$

$$dz = z_x' dx + z_y' dy = \frac{2x}{1+(x^2+y^2)^2} dx + \frac{2y}{1+(x^2+y^2)^2} dy$$

2.25 $z = \sqrt{u+v+3}$, $u = \ln x$, $v = x^2$, $x_0 = 1$

$z_x'(1) = ?$ $z = \sqrt{\ln x + x^2 + 3}$

$$z_x' = \frac{1}{2\sqrt{\ln x + x^2 + 3}} \cdot \frac{1}{x} \cdot (1 + 2x^2)$$

$$z_x'(1) = \frac{1}{2\sqrt{0+1+3}} \cdot \frac{1}{1} \cdot (1 + 1 \cdot 1^2) =$$

$$= \frac{1}{2 \cdot 2} \cdot 1 \cdot 2 = \frac{1}{2}$$

$\boxed{3{,}25}$ $x^2+2y^2+3z^2=59$

$$F(x,y,z)=x^2+2y^2+3z^2-59$$

$\frac{\partial z}{\partial x}=?$ $\frac{\partial z}{\partial y}=?$

$F_x'=2x$, $F_y'=4y$, $F_z'=6z$

$$\frac{\partial z}{\partial x}=\frac{-F_x'}{F_z'}=\frac{-2x}{6z}=-\frac{x}{3z}$$

$$\frac{\partial z}{\partial y}=\frac{-F_y'}{F_z'}=\frac{-4y}{6z}=\frac{-2y}{3z}$$

4.25 $z = 5x^2 + y^2 - 3xy + 4$

$\begin{cases} z_x' = 0 \\ z_y' = 0 \end{cases}$ $\quad z_x' = 10x - 3y = 0 \quad\quad 2y = 3x$

$\quad\quad\quad\quad z_y' = 2y - 3x = 0 \quad\quad y = 1{,}5x$

$10x - 4{,}5x = 0 \quad x = 0 \quad y = 0$

$P_0(0;0)$

$z''_{xx}(0;0) = 10 = A$

$z''_{xy}(0;0) = -3 = B$

$z''_{yy}(0;0) = 2 = C$

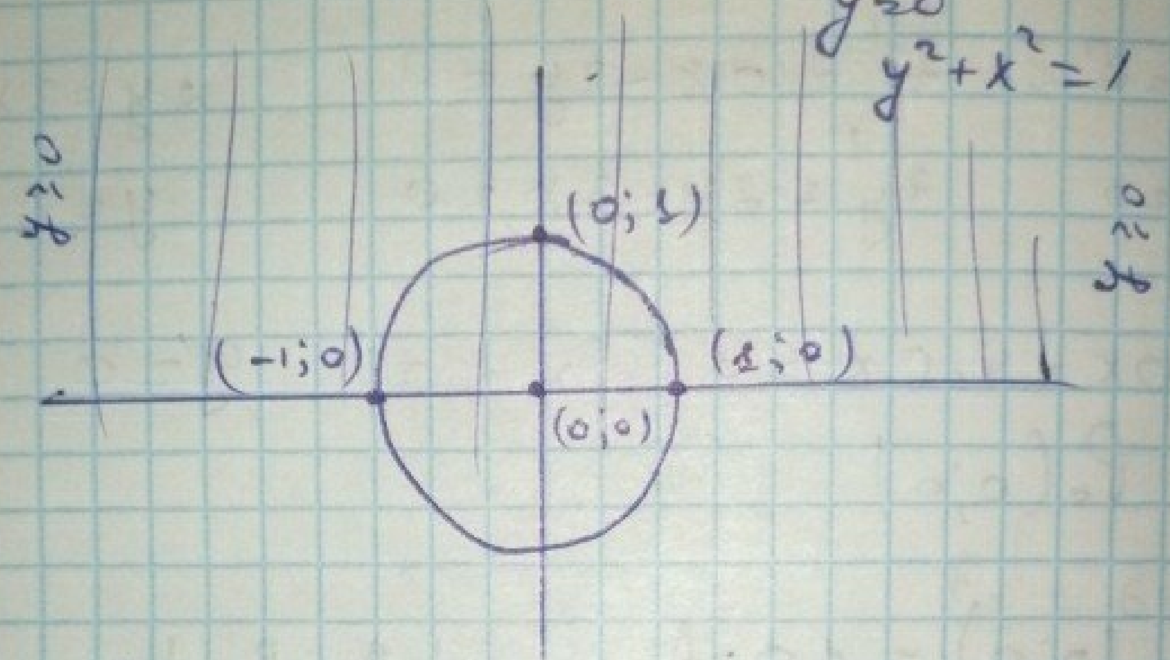
$\Delta = \begin{vmatrix} A & B \\ B & C \end{vmatrix} = AC - B^2 = \begin{vmatrix} 10 & -3 \\ -3 & 2 \end{vmatrix} = 20 - 9 = 11 > 0$

$\Delta > 0 \quad P \to \min$

$z_{min}(0;0) = 4$

5,25

$z = 4 - 2x^2 - y^2$, $\overline{D}$: $0 \le y \le \sqrt{1 - x^2}$



$z(0;0) = 4$

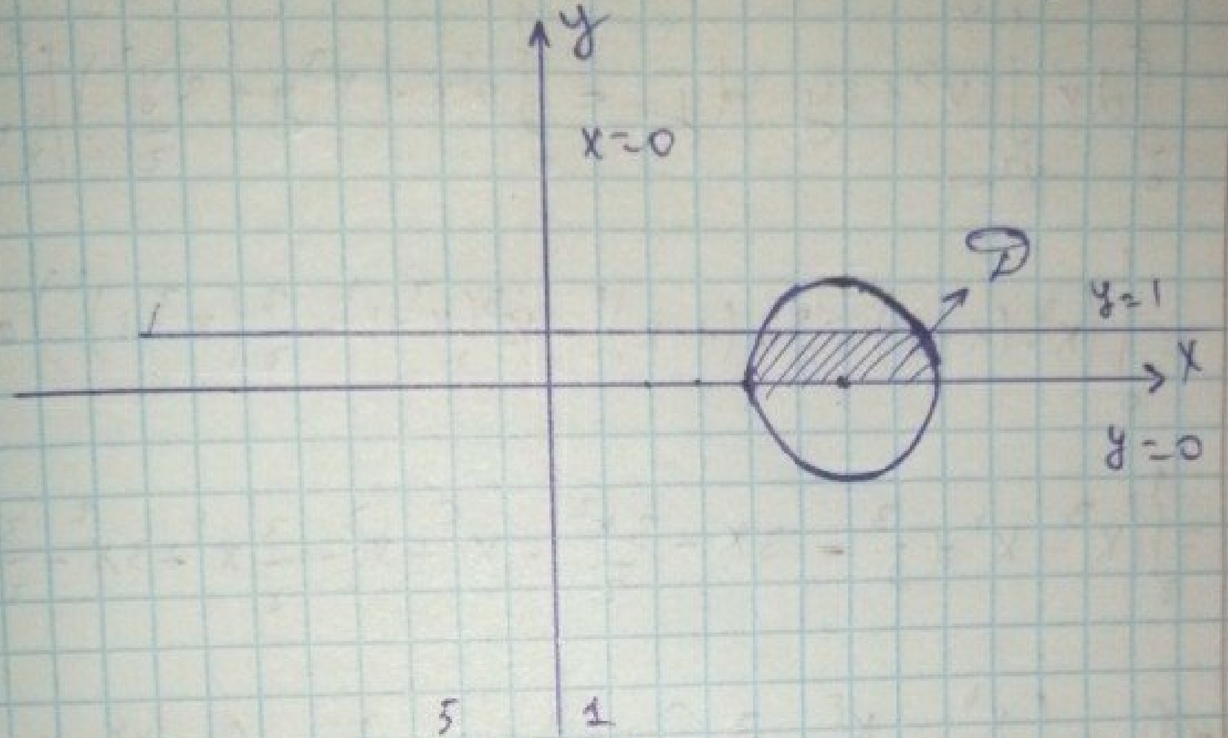
$z(1;0) = 4 - 2 - 0 = 2$

$z(-1;0) = 4 - 2 - 0 = 2$

$z(0;1) = 4 - 0 - 1 = 3$

$z_{max}(0;0) = 4$, $z_{min}(-1;0) = z(1;0) = 2$

6,25 $\overline{D}$: $x=0$, $y=0$, $y=1$ $(x-3)^2+y^2=1$



$$\iint_D f(x,y)\,dx\,dy = \int_3^5 dx \int_0^1 f(x,y)\,dy$$

7.25 $\iint_D (x^3+3y)\,dx\,dy$, $\overline{D}$: $x+y=1$

$y=x^2-1$, $x \geq 0$

$y=1-x$ $y=x^2-1$

x	1	0	2	-1	-2
y	0	-1	3	0	3

$$\int dx \int f(x,y)\, dy \Rightarrow$$

$$\int_0^1 dx \int_{x^2-1}^{1-x} (x^3+3y)\,dy = \int_0^1 dx \cdot \left(x^3 y + 3\frac{y^2}{2}\right)\Big|_{x^2-1}^{1-x} =$$

$$= \int_0^1 \left(x^3\cdot(1-x) + \frac{3}{2}\cdot(1-x)^2 - x^3(x^2-1) - \frac{3}{2}(x^2-1)^2\right) dx =$$

$$= \int_0^1 \left(x^3 - x^4 + \frac{3}{2} - 3x - \frac{3x^2}{2} - x^5 + x^3 - \frac{3}{2}x^4 + 3x^2 - \frac{3}{2}\right) dx$$

$$= \left(\frac{x^4}{4} - \frac{x^5}{5} + \cancel{\frac{3}{2}x} - \frac{3x^2}{2} - \frac{3}{2}\frac{x^3}{3} - \frac{x^6}{6} + \frac{x^4}{4} - \frac{3}{2}\frac{x^5}{5} +\right.$$

$$\left. + \frac{3x^3}{3} - \cancel{\frac{3}{2}x}\right)\Big|_0^1 = \frac{1}{4} - \frac{1}{5} + 1 - \frac{3}{2} - \frac{1}{2} - \frac{1}{6} + \frac{1}{4} -$$

$$- \frac{3}{10} + 1 = -\frac{1}{6}$$

8.25 $\int_0^3 dx \int_0^{\sqrt{9-x^2}} \ln(1+x^2+y^2)\, dy =$

$x = R\cos\varphi$
$y = R\sin\varphi$
$dx = -R\sin\varphi\, d\varphi$
$dy = R\cos\varphi\, d\varphi$

$$= \int_0^3 -R\sin\varphi\, d\varphi \int_0^{\sqrt{9-9\cos^2\varphi}} \ln(1+R^2)\, R\cos\varphi\, d\varphi =$$

$$= \int_0^3 -R\sin\varphi\, d\varphi \cdot \int_0^{3\sin\varphi} \ln 10 \cdot 3\cos\varphi\, d\varphi =$$

$$-3\int_0^3 \sin\varphi\, d\varphi \cdot \int_0^{3\sin\varphi} 3\ln 10 \cos\varphi\, d\varphi =$$

$$= 3\cos\varphi \Big|_0^3 \cdot 3\ln 10 \cdot \sin\varphi \Big|_0^{3\sin\varphi} =$$

$$= 3(\cos 3 - 1) \cdot 3\ln 10 \left(\sin 3\sin\varphi\right)$$