SLP - Notes

KT Prajwal Prathiksh

Department of Aerospace,
Indian Institute of Technology-
-Bombay

1) Fluid

2) Solid

3) Body Surface

4) Dynamics of rigid body (Center of gravity)

RK-4 Integration (?)

Boundary condition (?)
$(\rho, \vec{u}, P)$

Forces Evaluation (?)

$$\text{div}(u_i) = \sum_j (u_j - u_i) \cdot \nabla W_{ji} \frac{m_j}{\rho_j}$$

δ^+- SPH : (Non- linear water ....) + (Delta SPH model for...) + (The δ-plus -...)
+ (Free-surface flows ....)

$$\frac{D\rho_i}{Dt} = -\rho_i \sum_j (\vec{u}_j - \vec{u}_i) \cdot \nabla_i W_{ij} V_j + \delta h c_0 \sum_j \psi_{ij} \cdot \frac{(\vec{r}_j - \vec{r}_i) \cdot \nabla_i W_{ij}}{\|\vec{r}_j - \vec{r}_i\|^2} V_j$$

$\delta = 0.1$
$c_0 = 150$

→ Not there in (Free-surface flows....)

$$\psi_{ij} = 2(\rho_j - \rho_i) - \left\{ \left[(\nabla \rho)^L_i + (\nabla \rho)^L_j \right] \cdot (\vec{r}_j - \vec{r}_i) \right\}$$

↳ Renormalized density gradients

$$(\nabla \rho)^L_a = \sum_b (\rho_b - \rho_a) \underline{\underline{L_a}} \nabla_a W_{ab} \, dV_b$$

$$\underline{\underline{L_a}} = \left[\sum_b (r_b - r_a) \otimes \nabla_a W_{ab} \, dV_b \right]^{-1}$$

$$\frac{D\vec{u}_i}{Dt} = -\frac{1}{\rho_i} \sum_j F_{ji} \nabla_i W_{ij} V_j + \vec{f}_i + K \frac{\mu}{\rho_i} \sum_j \pi_{ij} \nabla_i W_{ij} V_j$$

↳ Body force (= 0)

$$F_{ji} = \begin{cases} P_j + P_i , & P_i \geq 0 \\ P_j - P_i , & P_i < 0 \end{cases}$$

$K = 2(\text{dim} + 2)$

μ = Dynamic viscosity $(= \rho_0 \nu)$; ν = kinematic viscosity

$$\pi_{ij} = \frac{(\vec{u}_j - \vec{u}_i) \cdot (\vec{r}_j - \vec{r}_i)}{\|\vec{r}_i . \vec{r}_j\|^2}$$

$$\frac{D\vec{r}_i}{Dt} = \vec{u}_i$$

$$P_i = c_0^2 (\rho_i - \rho_0)$$

ρ_0 = Ref density when $(P = 0)$

- δ^+-SPH → Reformulation of PST for weakly compressible framework, combined with δ-SPH & APR for modelling:
  - → confined,
  - → free-surface flows

- Validated for flow past immersed bodies
- Needs testing for free surface intersecting the body wall

Main features:

① Includes diffusive term in continuity equation

→ stabilizes density & pressure fields
→ avoids spurious high-freq. noise
} Typical in WCSPH

② PST - correction of particle positions

→ maintains uniform particle distribution
→ allows higher numerical accuracy

Implementation Features

m_i → remains constant throughout the simulation

Wendland - C2 Kernel (Properties - ?)

→ $h_i = 2\Delta x_i$ (2D) ; Δx_i = initial particle spacing

Viscosity is negligible ∴ Artificial viscosity is used (Helps numerical stability)

CFL = 1.0 [Allows for complex free surface evolution]

PST:

(Prevents tensile, pairing instability) → [Incompressible smoothed particle hydro....]

Artificial pressure → $f_{ij} = R\left(\frac{W_{ij}}{W(dx)}\right)^n$

★ $\frac{D\vec{r}_i}{Dt} = \vec{u}_i$ $\quad$ $\vec{r}_i \rightarrow (\vec{r}_i + \delta\vec{r}_i)$

$$\delta\vec{r}_i := -CFL.Ma.(2h_i)^2 \sum_j \left[1 + 0.2\left(\frac{W_{ij}}{W(\Delta x_i)}\right)^4\right]\nabla_i W_{ij}\,\frac{m_j}{\rho_i+\rho_j}$$

Δx_i → Avg. particle spacing in the neighbourhood of (i)
↳ [SPH without a Tensile....]

Calculation? ← $CFL.Ma = \frac{\Delta t\, U_{max}}{h_i}$

Observed: $\frac{|\delta r_i|}{\Delta x_i} < 0.05$, → 0 as resolution increases

Derivation:

$J = -D'\nabla C$

$\delta r_i = -D'\nabla C$ $\quad$ $\nabla C_i \sim \sum_j V_j(1+f_{ij})\nabla W_{ij}$

$\Delta t \leq 0.5\frac{h^2}{D'}$ ⇝ Von Neumann Stability analysis

$\Delta t = \frac{c\Delta x}{u}$

$\therefore D' \sim \frac{h^2}{2} \times \frac{1}{\Delta t}$

Particles close to free surface (or solid walls) → generate incorrect (δr_i)

Corrections: [The δ-plus - SPH model: Simple ....]

1) Accurate detection of free surface particles

2) Evaluate the normal vector to the free surface (or solid boundary)

$$L(\vec{r}_i) := \left[\sum_j (\vec{r}_j - \vec{r}_i) \otimes \nabla_i W_{ij} V_j\right]^{-1}$$

Renormalization tensor

λ (eigenvalue) of L^{-1} → 0 for free surface $(\lambda_i < 0.2)$

→ 1 for inside particles $(\lambda_i > 0.75)$

→ Second step if $\lambda_i \in [0.25, 0.75]$

$$\vec{n}(\vec{r}_i) = \frac{\langle \nabla \lambda_i \rangle}{|\langle \nabla \lambda_i \rangle|},$$

$$\langle \nabla \lambda_i \rangle := \sum_j (\lambda_j - \lambda_i) \otimes L_i \nabla_i W_{ij} V_j$$

$$\therefore \delta \hat{r}_i = \begin{cases} 0 & \lambda_i < 0.4 \\ (\mathbb{I} - n_i \otimes n_i)\delta r_i & \lambda_i > 0.4 \\ \delta r_i & \end{cases}$$

} $i \in$ Free surface

} $i \notin$ Free surface

↳ How to allow only tangential shifting for free-surface particles?

Boundary Conditions: [An accurate and efficient way...]

Solid wall boundaries → Fixed Ghost Particle

Pressure of ghost particle: (Shepard Kernel)

$$P_j = \frac{\sum_i \left(P_i + \rho_i (\vec{a}_j - \vec{f}) \cdot (r_i - r_j)\right) W_{ij}}{\sum_i W_{ij}}, \quad j \in \text{ghost}, \; i \in \text{fluid}$$

$\vec{a}_j$ = accl. of ghost

$\vec{f}$ = body-force

"Free-slip" : Viscous-force b/w fluid & ghost $= 0$

Interpolation of $\vec{u}$ of ghost not reqd. (use $\vec{u}_j$)

Avoids evaluation of normals of solid surfaces

Evaluation of Forces & Torques using ghost-fluid technique

[Nonlinear water wave....]

→ Net global force exerted by fluids on solids:

$$\vec{F}_{f\text{-}s} = \sum_i \left[\sum_j \underbrace{\left[-(p_j + p_i)\right.}_{\text{Pressure component}} + \underbrace{\left.\mu \Pi_{ij} \right]}_{\text{Viscous component of the stress tensor}} \nabla_i W_{ij} \, V_i V_j \right]$$

$i \in$ fluid
$j \in$ solid

- Does not require interpolation on body nodes

→ Net global torque:

$$\vec{T}_{f\text{-}s} = \sum_i \left[\sum_j \left\{ (\vec{r}_j - \vec{r}_c) \times \left[\left(-p_i + \frac{\mu \Pi_{ij}}{2} \right) \nabla_i W_{ij} \right] + (\vec{r}_i - \vec{r}_c) \times \left[\left(-p_j + \frac{\mu \Pi_{ij}}{2} \right) \nabla_i W_{ij} \right] \right\} V_i V_j \right]$$

→ Fluid-Body Coupling:

$$M \frac{d \vec{V}_c}{dt} = M \vec{f} + \vec{F}_{f\text{-}s}$$

$\vec{V}_c$ = Velocity of the CoM
M = Mass of the body
$\vec{f}$ = Body-force

$$\frac{d}{dt} \left(I_c(t) \, \omega_c(t) \right) = \vec{T}_{f\text{-}s} \cdot \hat{k}$$

$I_c(t)$ = Moment of Inertia wrt. CoM
$\omega_c(t)$ = Angular velocity wrt CoM

Fish - Body

NACA0012 Aerofoil

$$y = \pm 5 T_k c \left[k_1 \left(\frac{x}{c}\right)^{1/2} + k_2 \left(\frac{x}{c}\right) + k_3 \left(\frac{x}{c}\right)^2 + k_4 \left(\frac{x}{c}\right)^3 + k_5 \left(\frac{x}{c}\right)^4 \right]$$

$T_k = 0.12$
$c = 0.37$

Undulating Equation

$$h = h_{max} \, a_1 a_2 \sin \left[2\pi \left(\frac{x}{K} - \frac{t}{T} \right) \right]$$

$$a_1 = \begin{cases} \left(\dfrac{x/L - 0.2}{1 - 0.2} \right)^2 & s \in (0.2L, L] \\ 0 & s \in [0, 0.2L] \end{cases}$$

$$a_2 = \begin{cases} t/t_0 - \left(1/2\pi\right) \sin\left(2\pi t/t_0\right) & t \in [0, t_0] \\ 1 & t \in (t_0, \infty) \end{cases}$$