Supervised Learning Project - 2020

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Abstract

This project is aimed primarily at replicating the research of Sun, Peng-Nan, et al. on the self-propulsive fish-like swimming hydrodynamics [1]

The work done by Sun is based on $\delta^{+}-$SPH scheme, which involves the $\delta-$SPH framework along with a particle shifting technique (PST), and adaptive particle refinement (APR) which is a numerical technique adopted to refine the particle resolution in the local region and de-refine particles outside that region. These modifications impart the newly developed $\delta^{+}-$ SPH scheme with higher numerical accuracy and efficiency. The fish is modelled using a NACA0012 aerofoil that performs periodic oscillations, similar to that of a fish in water.

Contents

1 Concerns

1. The integrator mentioned in all of the resources is RK4 (fourth-order) - Can this be solved by a $1^{st}, 2^{nd}$ order integrator with small Δt?

2. Incomplete clarity - boundary conditions (Fixed Ghost Particle); Is there any better resource that has explained this?

3. Evaluation of renormalization tensor as well it's eigenvalues at every Δt instant for all particles - Understood the theory, but having issues implementing it in PySPH

4. Adaptive Particle Refinement - This part can be included at the end, since it is meant to make the scheme more efficient with lower resolution **(Can be forgone in the initial model, by compensating with higher resolution)**

2 Scope for original work

Once the preliminary model is complete, the following domains can be worked on immediately without significant modifications to the scheme to answer additional unanswered questions:

1. Include Torque effects, as well as the transversal and rotational motions of the foil which are ignored [1]

2. The transverse and angular velocities due to the tail flapping of the foil which were left for the future studies can be studied

3. Extend $\delta^{+}-$SPH scheme to fishlike swimming problems in 3D

4. Complex interactions among more than two fishes in different positions can be investigated, in order to further explain the mechanism of fish schooling in nature

5. The study of the interaction between fish and free-surface can be carried out

6. Compare the results using $\delta-$SPH and $\delta^{+}-$SPH scheme - verify the superiority of the accuracy and stability wrt to $\delta-$SPH, since introduction of PST causes the loss of exact momenta and energy conservation; check is this drawback has negligible effects on the result [2]

7. Water entry of spheres or cylinders at higher entry speeds can be conducted. For these cases the air phase has to be considered and a multi-phase SPH model capable of considering the real sound speed of the air and the water will be needed.
   If the water entry speed is large enough, then cavitation can be generated. This can be solved with a cavitation model [3]

8. Analyze the impact of viscosity and vorticity generation [4]

9. Optimize this SPH scheme [5]

3 Governing Equations

3.1 Equation of State

$$p_i = c_o^2(\rho_i - \rho_o) \tag{1}$$

where,

1. ρ_o - The reference density when pressure is zero initially

2. c_o - The artificial speed that is based on the weakly-compressible hypothesis. Here, c_o is a constant in the whole simulation and it is determined as: $15U$, where U is the reference velocity

Refer - [3], [1]

3.2 Renormalization Tensor

$$\mathbb{L}_i = \left[\sum_j \mathbf{r_{ji}} \otimes \nabla_i W_{ij} V_j\right]^{-1} \tag{2}$$

$$\lambda_i = \min\big(\text{eigenvalue}(\mathbb{L}_i^{-1})\big) \tag{3}$$

Refer - [2], [6]

3.3 Continuity Equation

$$\frac{D\rho_i}{Dt} = \sum_j \rho_i \mathbf{u_{ij}}.\nabla_i W_{ij} V_j + \delta h c_o \Psi_{ij} \frac{\mathbf{r_{ji}}.\nabla_i W_{ij}}{|\mathbf{r_{ji}}|^{\mathbf{2}}} V_j \tag{4}$$

where,

$$\Psi_{ij} = 2(\rho_j - \rho_i) \tag{5}$$

1. δ - The density diffusion parameter ($= 0.1$)

Refer - [7], [1]

3.4 Continuity Equation - RDGC

$$\frac{D\rho_i}{Dt} = \sum_j \rho_i \mathbf{u_{ij}}.\nabla_i W_{ij} V_j + \delta h c_o \Psi_{ij} \frac{\mathbf{r_{ji}}.\nabla_i W_{ij}}{|\mathbf{r_{ji}}|^{\mathbf{2}}} V_j \tag{6}$$

where,

$$\Psi_{ij} = 2(\rho_j - \rho_i) - (\langle \nabla \rho \rangle_i^L + \langle \nabla \rho \rangle_j^L).\mathbf{r_{ji}} \tag{7}$$

$$\langle \nabla \rho \rangle_i^L = \sum_j (\rho_j - \rho_i) \mathbb{L}_i.\nabla_i W_{ij} V_j \tag{8}$$

This equation includes the renormalized density gradient correction (RDGC) term as well.

Refer - [4], [5]

3.5 Momentum Equation

$$\frac{D\mathbf{u_i}}{Dt} = \frac{1}{\rho_i} \sum_j \Big(F_{ij} \nabla_i W_{ij} V_j + K \mu \pi_{ij} \nabla_i W_{ij} V_j \Big) + \mathbf{f_i} \tag{9}$$

where,

$$F_{ij} = \begin{cases} -(p_j + p_i), & p_i \geq 0 \\ -(p_j - p_i), & p_i < 0 \end{cases} \tag{10}$$

$$K = 2(\text{dim} + 2) \tag{11}$$

$$\pi_{ij} = \frac{\mathbf{u_{ji}}.\mathbf{r_{ji}}}{|\mathbf{r_{ji}}|^2} \tag{12}$$

1. $\mathbf{f_i}$ - Body-forces

2. μ - The dynamic viscosity ($\mu = \rho_o \nu$), where ν is the kinematic viscosity

Refer - [5], [1]

3.6 Displacement Equation

$$\frac{D\mathbf{r_i}}{Dt} = \mathbf{u_i} \tag{13}$$

4 Particle Shifting Technique

Once the particle positions are advected through the time, a repositioning is performed as follows:

$$\mathbf{r_i}^* = \mathbf{r_i} + \delta\hat{\mathbf{r}}_\mathbf{i} \tag{14}$$

$$\delta\hat{\mathbf{r}}_\mathbf{i} = \begin{cases} 0 & , \lambda_i \in [0, 0.4) \\ (\mathbb{I} - \mathbf{n_i} \otimes \mathbf{n_i})\delta\mathbf{r_i} & , \lambda_i \in [0.4, 0.75] \\ \delta\mathbf{r_i} & , \lambda_i \in (0.75, 1] \end{cases} \tag{15}$$

$$\delta\mathbf{r_i} = \frac{-\Delta t c_o (2h)^2}{h_i} . \sum_j \left[1 + R\left(\frac{W_{ij}}{W(\Delta p)}\right)^n \right] \nabla_i W_{ij} \left(\frac{m_j}{\rho_i + \rho_j}\right) \tag{16}$$

$$\mathbf{n_i} = \frac{\langle \nabla \lambda_i \rangle}{|\langle \nabla \lambda_i \rangle|} \tag{17}$$

$$\langle \nabla \lambda_i \rangle = -\sum_j (\lambda_j - \lambda_i) \mathbb{L}_i \nabla_i W_{ij} V_j \tag{18}$$

1. Δp = Average particle spacing in the neighbourhood of i. Refer - [8]

Refer - [2], [9]

5 Boundary Conditions

Solid wall boundaries are modelled using the **'Fixed Ghost Particle Technique'** [3]. The pressure of the ghost particle is interpolated from the fluid using the Shepard kernel as follows:

$$p_j = \frac{\sum_i (p_i + \rho_i(\mathbf{a_j} - \mathbf{f}).\mathbf{r_{ij}}) W_{ij}}{\sum_i W_{ij}} \tag{19}$$

where,

- $j \in$ ghost
- $i \in$ fluid
- $\mathbf{a_j}$ = acceleration of ghost particle
- $\mathbf{f}$ = body-force

In a *"free-slip"* boundary condition the viscous force between the fluid particle and the ghost particle gets forced to zero. The interpolation of the velocity to the ghost particles is not needed since the own velocity u_j of the ghost particle is used. With this solid wall boundary treatment, the evaluation of the normals on the solid surface can be avoided and therefore it is more convenient to model 3D boundaries with complex configurations.

6 Forces & Torques

6.1 Force

The net global force exerted by fluids on solids can be calculated as follows [5]:

$$\mathbf{F}_{f-s} = \sum_i \Big[\sum_j [-(p_i + p_j) + \mu\pi_{ij}] \nabla_i W_{ij} V_i V_j \Big] \tag{20}$$

where,

- $i \in$ fluid
- $j \in$ solid
- $(p_i + p_j) \rightarrow$ Pressure component
- $\mu\pi_{ij} \rightarrow$ Viscous component of the stress tensor

This implementation does not require interpolation on body nodes.

6.2 Torque

The net global torque exerted by fluids on solids can be calculated as follows [5]:

$$\mathbf{T}_{f-s} = \sum_i \left[\sum_j \left((\mathbf{r_j} - \mathbf{r_c}) \times \left[\left(-p_i + \frac{\mu\pi_{ij}}{2} \right) \nabla_i W_{ij} \right] + (\mathbf{r_i} - \mathbf{r_c}) \times \left[\left(-p_j + \frac{\mu\pi_{ij}}{2} \right) \nabla_i W_{ij} \right] \right) V_i V_j \right] \tag{21}$$

where,

- $\mathbf{r_c} \rightarrow$ A fixed point *(eg: center of mass)*

6.3 Fluid-Body Coupling

$$M \frac{d\vec{V_c}}{dt} = M\mathbf{f} + \mathbf{F}_{f-s} \tag{22}$$

where,

1. $V_c \rightarrow$ Velocity of the center of mass
2. $M \rightarrow$ Mass of the body
3. $f \rightarrow$ Body-force

$$\frac{d}{dt}\Big(I_c(t)\omega_c(t)\Big) = \mathbf{T}_{f-s}.\hat{\mathbf{k}} \tag{23}$$

where,

1. $I_c(t) \rightarrow$ Moment of inertia wrt. center of Mass
2. $\omega_c(t) \rightarrow$ Angular velocity wrt. center of Mass

   Refer - [5]

7 Fish

7.1 Foil Model

The body of the two-dimensional fish (Saithe) is modelled after a NACA0012 foil. The foil is defined as [10], [11]:

x' = x$_{\bar{c}}$

where,

- $T_k = 0.12 \rightarrow$ The maximum thickness as a fraction of the chord
- $c = 0.37m \rightarrow$ Chord length
- $x \rightarrow$ Position along the chord
- $y_t \rightarrow$ Half thickness at a given value of x (centerline to surface)

7.2 Undulation Model

The deformation of the fish is described as the undulation of the centerline, as approximately fitted as follows [12]:

$$h(x,t) = h_{max}.a_1(x).a_2(t).\sin\left[2\pi\Big(\frac{x}{\lambda} - \frac{t}{T}\Big)\right] \tag{24}$$

where,

$$a_1 = \begin{cases} 0 & , x \in [0, 0.2L] \\ \left(\frac{x/L-0.2}{1-0.2}\right)^2 & , x \in (0.2L, L] \end{cases} \tag{25}$$

$$a_2(t) = \begin{cases} \frac{t}{t_o} - \frac{1}{2\pi}\sin\left(\frac{2\pi t}{t_o}\right) & , t \in [0, t_o] \\ 1 & , t \in (t_o, \infty) \end{cases} \tag{26}$$

- $L = 0.37m \rightarrow$ Body length (same as the chord length)
- $T = 0.278s \rightarrow$ Cycle of undulation
- $\lambda = 1.04L \rightarrow$ Wavelength of undulation
- $h_{max} = 0.083L \rightarrow$ Maximum half-amplitude of the deflection at tail tip

This model causes the posterior part of the fish to undulate from rest $(t = 0)$ to constant periodic undulation $(t > t_o = 1.0T)$

8 Taylor-Green Vortex

The 2-dimensional decaying vortex is defined in a square domain of [13]:

$$x, y \in [0, \pi]$$

The governing equations are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{27}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{28}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{29}$$

The Taylor-Green Vortex solution is given by:

$$u = \sin(x)\cos(y).F(t)$$
$$v = -\cos(x)\sin(y).F(t)$$
$$F(t) = e^{-2\nu t}$$
$$p = \frac{\rho}{4}\big(\cos(2x) + \sin(2y)\big).F^2(t)$$

A periodic boundary condition is used:

$$x = \begin{cases} x, & x \in [0, \pi] \\ \pi - x, & x > \pi \end{cases}$$
$$y = \begin{cases} y, & y \in [0, \pi] \\ \pi - y, & y > \pi \end{cases}$$

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