1) a) $\pi_n(\underbrace{T}_{T=S^1\times S^1}) = \pi_n(S^1)\times\pi_n(S^1) = \begin{cases}\mathbb{Z}\times\mathbb{Z}, & n=1\\ 0, & n\neq 1\end{cases} \rightsquigarrow (S^1)^{\times i}$ is $K(\mathbb{Z}^i, 1)$

b) $\pi_1(D^1, S^0, 1\} = \left\{\begin{matrix}\text{const}_1\\ \text{Weg von } 1 \text{ nach } -1\end{matrix}\right\}$ zwei Abbildungen (nicht $\pi_1(\underbrace{D^1/S^0}_{=S^1}) \cong \mathbb{Z}$)

2) a) z.z. $\pi_n(X\vee Y, (x_0,y_0)) \cong \pi_n(X,x_0)\oplus\pi_n(Y,y_0)\oplus\pi_{n+1}(X\times Y, X\wedge Y, (x_0,y_0))$, $n\geq 2$

$$\pi_{n+1}(X\vee Y,(x_0,y_0)) \underset{\varphi}{\overset{i_*}{\rightleftarrows}} \pi_{n+1}(X\times Y,(x_0,y_0)) \xrightarrow{0} \pi_{n+1}(X\times Y, X\wedge Y,(x_0,y_0)) \longrightarrow \pi_n(X\vee Y,(x_0,y_0)) \underset{\varphi}{\overset{i_*}{\rightleftarrows}} \pi_n(X\times Y,(x_0,y_0))$$

$$\varphi:\ \pi_n(X\times Y,(x_0,y_0)) \cong \pi_n(X,x_0)\times\pi_n(Y,y_0) \longrightarrow \pi_n(X\vee Y,(x_0,y_0))$$
$$([f],[g]) \longmapsto [f]+[g]$$

Homomorphismus: $([f],[g]) + ([f'],[g']) \longmapsto [f]+[g]+[f']+[g']$

$\qquad\qquad\qquad\quad \| \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\ =$ weil $n\geq 2$, d.h. die Gruppen sind abelsch!

$\qquad\qquad\quad ([f]+[f'], [g]+[g']) \longmapsto [f]+[f']+[g]+[g']$

$$(i_*\circ\varphi)([f],[g]) = i_*([f]+[g]) = ([f],0)+(0,[g]) = ([f],[g])$$

$\Rightarrow \mathrm{id}_{\pi_n(X\times Y,(x_0,y_0))} = i_*\circ\varphi \quad \Rightarrow i_*$ surj

$\rightsquigarrow$ erhalte kurze exakte Sequenzen

$$0\to\pi_{n+1}(X\times Y, X\vee Y,(x_0,y_0))\to\pi_n(X\vee Y,(x_0,y_0)) \underset{\varphi}{\rightleftarrows} \pi_n(X\times Y,(x_0,y_0))\to 0$$

$$\Rightarrow \pi_n(X\vee Y,(x_0,y_0)) \cong \pi_n(X\times Y,(x_0,y_0))\oplus\pi_{n+1}(X\times Y, X\vee Y,(x_0,y_0))$$

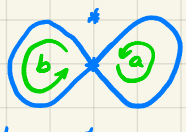
b) Die Aussage ist falsch für $n=1$

$$\pi_1(S^1\vee S^1)\cong\mathbb{Z}*\mathbb{Z} \not\cong \mathbb{Z}^3 \cong \pi_1(S^1)\oplus\pi_1(S^1)\oplus\pi_2(\underbrace{S^2}_{\cong S^1\wedge S^1 = (S^1\times S^1)/(S^1\vee S^1)})$$

3) z.z. es gibt keine Gruppenstruktur auf $\pi_1(S^1\vee S^1, S^1, *)$ ...

$$\underbrace{\pi_1(S^1\vee S^1, *)}_{\cong\mathbb{Z}*\mathbb{Z}} \xrightarrow{\varphi} \underbrace{\pi_1(S^1\vee S^1, S^1, *)}_{=\{a^{r_1}b^{s_1}\ldots a^{r_n}\mid r_i,s_i\in\mathbb{Z},\ r_n\neq 0 \text{ oder } a^{r_1}, r_1\in\mathbb{Z}\}}$$

$=\{a^{r_1}b^{s_1}\ldots a^{r_n}b^{s_n}\mid r_i,s_i\in\mathbb{Z}\}$



$S^1\subseteq S^1\vee S^1$

$$0=\varphi(0)=\text{const}_*=\varphi(b)$$

$\varphi(aba) = \varphi(a)\cdot\varphi(ba)$

$\quad \|$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad \lightning \qquad \varphi(ba)\neq\varphi(a)$

$\varphi(ab)\cdot\varphi(a) = \varphi(a)\cdot\varphi(a)$

4) a) $\pi_n(S^1)\to\underbrace{\pi_n(D^2)}_{=0}\to\pi_n(D^2,S^1)\to\underbrace{\pi_{n-1}(S^1)}_{=0,\ n\geq 3}\to\underbrace{\pi_{n-1}(D^2)}_{=0}$

$$\Rightarrow \pi_n(D^2,S^1) = \begin{cases} * , & n=1 \quad \text{direkt}\\ \mathbb{Z}, & n=2\\ 0, & n\geq 3\end{cases} \Big\} \text{mit der Sequenz}$$

b) z.z. $\pi_n(D^2,S^1)\not\cong\pi_n(\underbrace{D^2/S^1}_{\cong S^2})$, $\pi_3(D^2,S^1)=0\not\cong\mathbb{Z}\cong\pi_3(S^3)\cong\pi_3(S^2)$

c) $\pi_n(\mathbb{RP}^{n-1}) \to \underbrace{\pi_n(\mathbb{RP}^n)}_{=\pi_n(S^n)\cong\mathbb{Z}} \to \pi_n(\mathbb{RP}^n, \mathbb{RP}^{n-1}) \to \underbrace{\pi_{n-1}(\mathbb{RP}^{n-1})}_{=\pi_{n-1}(S^{n-1})\cong\mathbb{Z}} \to \underbrace{\pi_{n-1}(\mathbb{RP}^n)}_{=\pi_{n-1}(S^n)=0}$

$$\begin{tikzcd}
\overbrace{\pi_n(S^0)}^{=0} \arrow[r] & \overbrace{\pi_n(S^n)}^{=\mathbb{Z}} \arrow[r, "\cong"] & \overbrace{\pi_n(\mathbb{RP}^n)}^{=\mathbb{Z}} \arrow[r] & \overbrace{\pi_{n-1}(S^0)}^{=0} \\
\underbrace{\pi_n(S^0)}_{=0} \arrow[u] \arrow[r] & \pi_n(S^{n-1}) \arrow[u, "0"] \arrow[r, "\cong"'] \arrow[r, phantom, shift left=6, "\Rightarrow"] & \pi_n(\mathbb{RP}^{n-1}) \arrow[u, "0"] \arrow[r] & \underbrace{\pi_{n-1}(S^0)}_{=0} \arrow[u]
\end{tikzcd}$$

$\rightsquigarrow \quad 0 \to \underbrace{\pi_n(\mathbb{RP}^n)}_{=\mathbb{Z}} \to \pi_n(\mathbb{RP}^n, \mathbb{RP}^{n-1}) \to \underbrace{\pi_{n-1}(\mathbb{RP}^{n-1})}_{=\mathbb{Z}} \to 0$

$\Rightarrow \pi_n(\mathbb{RP}^n, \mathbb{RP}^{n-1}) \cong \mathbb{Z}\oplus\mathbb{Z}$