Modeling Non-Linear Psychological Processes: Reviewing and Evaluating (Non-) parametric Approaches and Their Applicability to Intensive Longitudinal Data

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Abstract

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Modeling Non-Linear Psychological Processes: Reviewing and Evaluating (Non-) parametric Approaches and Their Applicability to Intensive Longitudinal Data

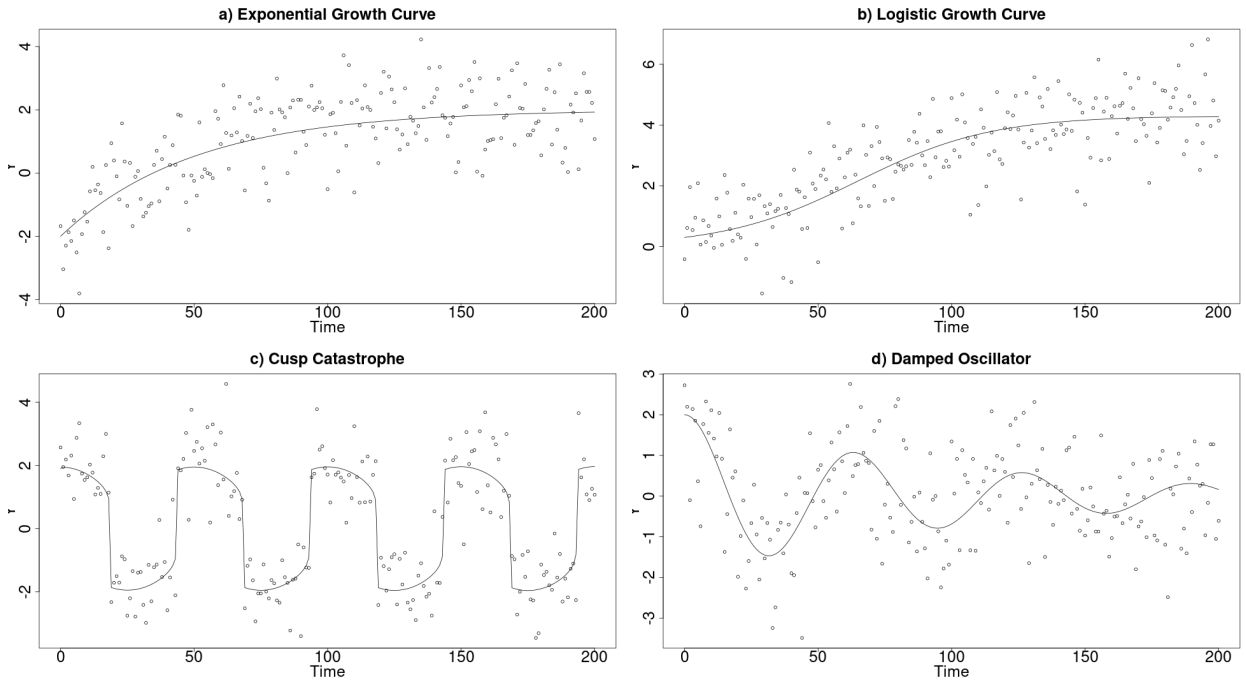
Psychological constructs are increasingly recognized as components of complex dynamic systems (Nesselroade & Ram, 2004; Wang et al., 2012). This perspective emphasizes how these constructs fluctuate over time and within individuals. To study these variations and the underlying processes, researchers are increasingly collecting intensive longitudinal data (ILD) using ecological momentary assessment (EMA), experience sampling, or similar methods (Fritz et al., 2023). These studies assess several individuals at high frequencies (up to multiple times per day) using brief questionnaires or passive measurement devices. This rich data allows researchers to examine variations in latent psychological variables within an ecologically valid context and to explain them through (between-person differences) in within-person processes (citation).

Many psychological phenomena and processes have been shown to be non-linear, leading many researchers to believe that the underlying systems likely exhibit similar complexity. Clear examples of this are the learning and growth curves observed in intellectual and cognitive development (Kunnen, 2012; McArdle et al., 2002). In these cases, an individual's latent ability increases over time from a (person specific) starting point toward an (person specific) asymptote, which reflects their maximum ability. Additionally, examples of asymptotic growth over the shorter time spans that are typically studied with ILD include motor skill development (Newell et al., 2001) and second language acquisition (De Bot et al., 2007). Figure 1 illustrates general growth curves through an exponential growth function (a) and a logistic growth function (b), both of which are common model choices for these processes.

Another commonly observed non-linear phenomenon involves the construct under study switching between multiple distinct states, which often reflect different means. This occurs, for example, during the sudden perception of cognitive flow, where individuals abruptly switch from a 'normal' state to a flow state and back (Ceja & Navarro, 2012). Another example is alcohol use relapse, where patients suddenly switch from an abstinent state to a relapsed state (Witkiewitz & Marlatt, 2007). This sudden switching behavior is exemplified in Figure 1 (c) through a cusp

Figure 1

Examples of non-linear processes demonstrated to occur in psychological time series.



Note. This figure shows four demonstrated psychological non-linear processes. Panels (a) and (b) show exponential and logistic growth curves, respectively, which have been shown to describe intellectual and cognitive development (Kunnen, 2012; McArdle et al., 2002), motor skill learning (Newell et al., 2001), and second language acquisition (De Bot et al., 2007). Panel (c) shows a cusp model from catastrophe theory that exhibits apparent jumps between two stable states. As such, it has been used to describe the perception of mental flow (Ceja & Navarro, 2012) or alcohol use relapse (Witkiewitz & Marlatt, 2007). Lastly, panel (d) shows a damped oscillator, which describes the return of a perturbed system to baseline, which has been proposed as a model for affect regulation (Chow et al., 2005).

catastrophe model. This model, drawn from catastrophe theory, naturally leads to mean level switches when varying one of its parameters (Chow et al., 2015; van der Maas et al., 2003).

As a final example, one may consider (self-) regulatory systems, which maintain a desired state by counteracting perturbations. These systems regulate adaptively, meaning that the regulation strength depends on the distance between the current and desired states. The common autoregressive model describes such a system with a linear relationship between this distance and

regulation strength. However, this relationship may also be non-linear, such that the regulatory force increases disproportionately with larger mismatches. Such models have been proposed to describe for example emotion regulation (Chow et al., 2005). *An example involving depression could be included here, but a suitable reference has not yet been found.* Figure 1 (d) illustrates a (self-) regulatory system exemplified as a damped oscillator.

Although initial evidence for non-linearity in psychological research exists, theories about the nature and form of non-linear psychological processes remain scarce (Tan et al., 2011). This gap is largely due to the lack of advanced statistical methods that are flexible enough to study the complex behavior of these processes adequately. As a result, researchers are often ill-equipped to infer the functional characteristics of non-linear processes from ILD, which hinders the development and evaluation of guiding theories for subsequent studies. Due to this lack of adequate available statistical methods, non-linear trends are most often addressed in psychology through polynomial regression or regression splines.

Polynomial regression (Jebb et al., 2015) uses higher-order terms (e.g., squared or cubed time) as predictors in a standard multiple linear regression model. While effective for relatively simple non-linear relationships, particularly those that can be represented as polynomials, this method has significant limitations and likely leads to invalid results when applied to more complex latent processes, such as mean switching or (self-) regulatory systems (e.g., Figure 1 c & d). In these cases, polynomial approximations require many higher-order terms to capture the process's high variability, which raises the problem of over- or underfitting the data, causes model instability, and leads to nonsensical inferences (e.g., interpolating scores outside the scale range; Boyd and Xu (2009) and Harrell (2001)).

An alternative approach is spline regression, which constructs a complex non-linear trend by joining multiple simple piecewise functions at specified points, known as knots (e.g., combining multiple cubic functions into a growth curve with plateaus; Tsay and Chen (2019)). However, spline regression requires careful selection of the optimal piecewise functions and knot locations. This can be problematic in practice because, as mentioned, precise guiding theories

about the functional form of most psychological processes are lacking (Tan et al., 2011). This absence of clear guidance can easily lead to misspecified models and invalid results.

These limitations in current practices underscore the need for alternative statistical methods to study non-linear processes. Various such advanced statistical methods, such as kernel regression, Gaussian processes, smoothing splines, and latent change score models, are available outside of psychology. However, these methods have rarely been applied in psychology because they have not been reviewed for an applied audience, nor have their assumptions and inference possibilities been evaluated in the context of ILD. As a result, psychological researchers struggle to select the most suitable method for a specific context. This challenge is further complicated by the fact that the ideal statistical method may depend on the characteristics of the underlying non-linear process, which are generally unknown. Additionally, the smooth processes for which many of these methods were originally developed are unlikely to occur in psychological research.

To address this important gap, this article reviews four advanced non-linear analysis methods and evaluates their applicability to typical ILD. Specifically, we compare how well each method can recover different latent processes under typical ILD conditions through a simulation study. We also demonstrate the conclusions that can be drawn from each method by applying them to an existing dataset. The methods reviewed in this article range from data-driven non-parametric techniques to a flexible parametric modeling framework. These approaches were selected to accommodate varying degrees of prior knowledge, as precise theories about the nature of non-linear psychological processes are scarce (Tan et al., 2011). Further, to introduce these methods accessibly and apply them under conditions where software implementations are available, this article focuses on the univariate single-subject design.

Method

Data structure

Generally, any psychological construct under study follows a (possibly non-linear) function over time, as represented by the lines in Figure 1. However, since these psychological

processes are typically unobservable or latent, they are measured through observable indicators, such as questionnaire items or passive measurements. The observations of these indicators (Figure 1, dots) differ from the true values of the latent process due to measurement error, which may result from an imperfect measurement instrument. For this introduction, we assume that all time-point-specific measurement errors are independent and normally distributed. The model for the observations of a single indicator can then be written as:

$$Y_t = f(t) + \varepsilon_t; \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \tag{1}$$

where $f(t)$ represents the potentially non-linear latent process, and ε_t represents the time-point-specific measurement error.

In this context, researchers typically aim to infer the underlying process and draw conclusions about its functional form. The following sections will introduce four methods to achieve these goals, two non-parametric techniques, one semi-parametric approach, and one parametric modeling framework, using the (self-) regulatory process depicted in Figure 1 (d) as a running example.

Local polynomial regression

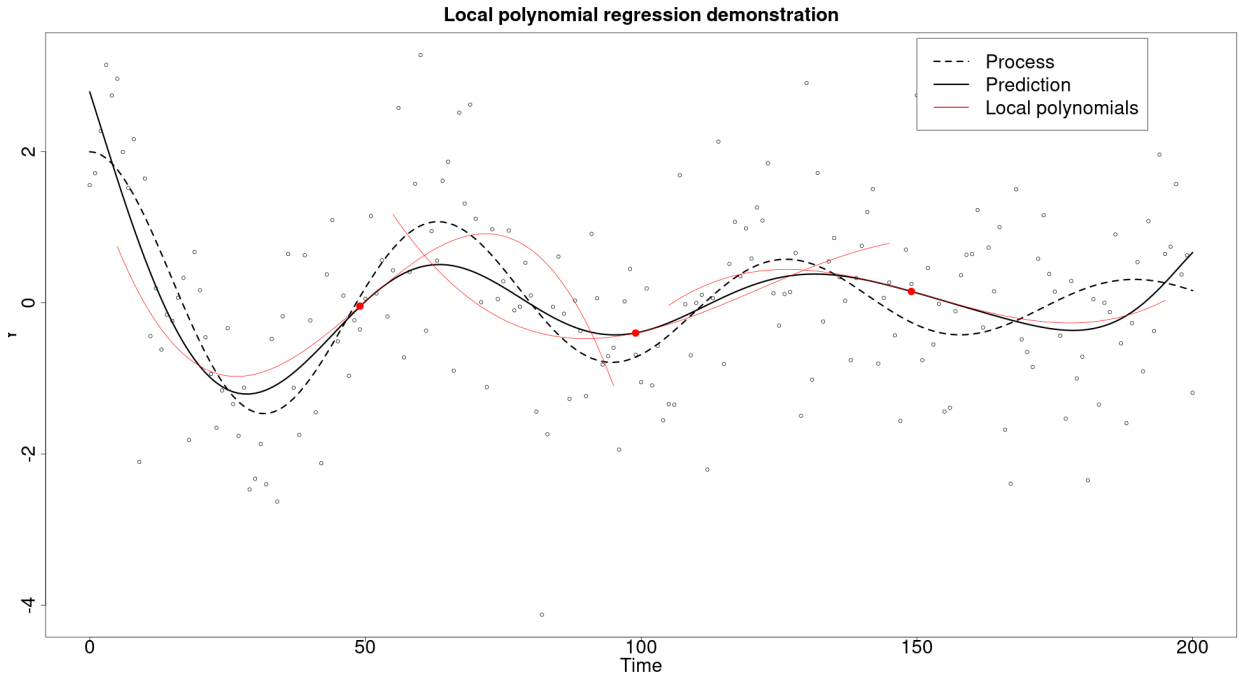
The first technique is called local polynomial regression (LPR). Similarly to regular polynomial regression, LPR approximates the process using polynomial basis functions (e.g., squared or cubed time). However, instead of using one large polynomial to approximate the entire process, LPR estimates smaller, local polynomials at each point in time. These local polynomials are then combined into a single non-linear function over the entire set of observations (Fan & Gijbels, 2018; Fan & Gijbels, 1995; Ruppert & Wand, 1994).

To determine the value of the LPR at a specific time point, the data is centered around that point (by shifting the data along the time axis so that the chosen time point is at zero), and a low-order polynomial is fitted around it. Additionally, to account for the idea that data points closer in time are more related, a weighting function is applied during the polynomial estimation.

This function assigns weights to each data point based on its distance from the point of interest. The intercept of this locally weighted polynomial provides the value of the LPR for the chosen timepoint. To find the LPR value at a different time point, the same procedure is repeated, centering the data around the new point of interest. The theoretical ability to repeat this process at infinitely many time points makes LPR a non-parametric technique. Figure 2 shows the estimated LPR for the example process depicted in Figure 1 (d). In this figure, three truncated examples of local cubic regressions that contribute to the LPR for this process are shown in red.

Figure 2

Demonstration of how local polynomial regression (solid black) estimates the underlying process (dotted black). Here, three local cubic functions (red) are shown as examples at the time points 50, 100, and 150, which provide the values of the local polynomial regression at these time points.



When fitting an LPR, several decisions must be made regarding the degree of the local polynomials and the optimal weighting of the data. Typically, the degree of these local polynomials is kept low and odd. This choice reflects a bias-variance tradeoff, where higher-order

polynomials reduce bias but increase variance, when transitioning from an odd to an even power (citation). Data weighting in LPR is achieved through a kernel function, which is usually centered and symmetric, assigning weights based on the distance from the origin. Common kernel choices include the Gaussian and symmetric Beta distributions. Most commonly used kernel functions yield similar inferences and a specific kernel can be selected to minimize a chosen criterion function. The kernel is further defined by a bandwidth parameter, which determines its width and effectively controls the influence of more distant data points. The bandwidth parameter, in practice, represents the wiggliness of the estimated process. Several methods are available to find the optimal bandwidth by optimizing a data-dependent criterion function, such as cross-validation or the mean integrated squared error (Debruyne et al., 2008; Köhler et al., 2014).

Due to its non-parametric nature, LPR makes minimal assumptions about the data. However, it does require that the underlying process is smooth or differentiable with regards to the covariate, which is a necessary condition for polynomial approximation. Another key assumption is that the process has constant wiggliness, represented by a single bandwidth parameter. However, this assumption can be relaxed by using a time-varying bandwidth.

Gaussian process regression

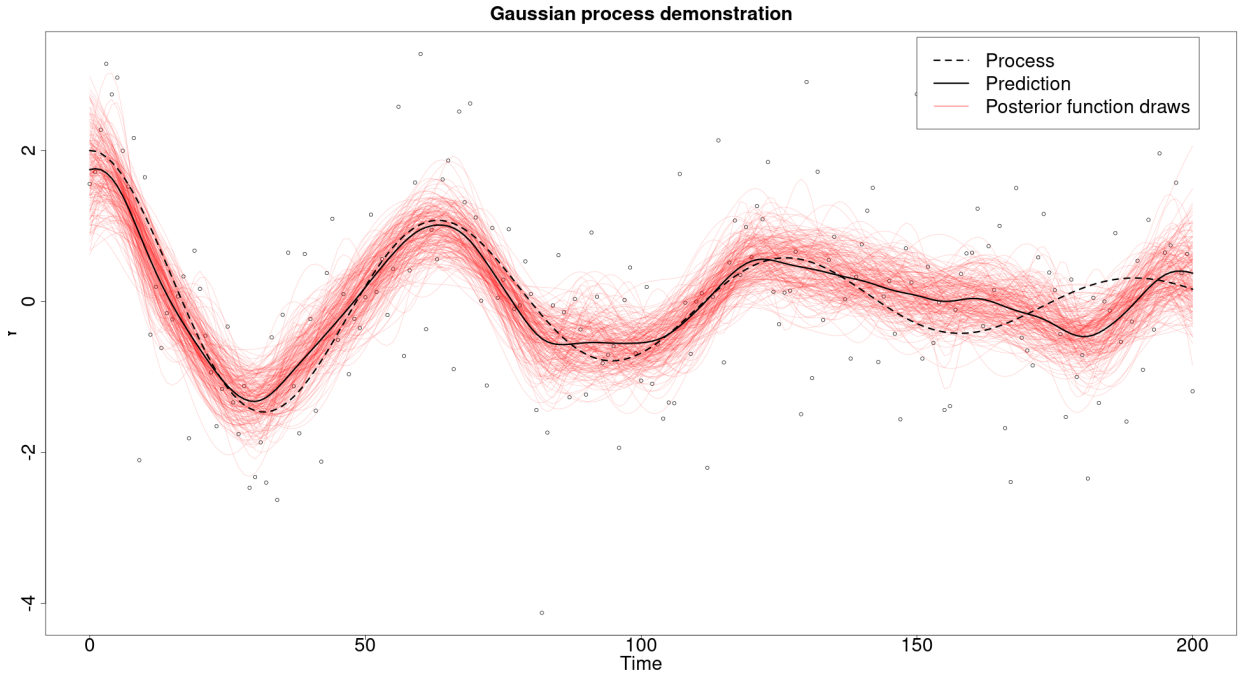
The second non-parametric technique is Gaussian process (GP) regression, a Bayesian approach that directly imposes a probability distribution over an entire family of non-linear functions flexible enough to capture many complex processes effectively (Betancourt, 2020; Rasmussen & Williams, 2006; Roberts et al., 2013). Unlike regular probability distributions (e.g., normal distributions) that specify the plausibility of single values, Gaussian processes determine the plausibility of entire (non-linear) functions. In a Bayesian framework, one can use a GP to define a prior distribution for the latent process. This prior is then combined with an appropriate likelihood for the observed data to obtain a posterior distribution for the latent process given the observed data.

The posterior distribution represents an updated belief about which functions describe the

latent process well, allowing one to draw inferences about the process. Figure 3 illustrates such a posterior distribution for the runnign example process. The red lines represent a sample of non-linear functions drawn from the posterior distribution, with the pointwise average of these functions providing a mean estimate for the underlying process.

Figure 3

Demonstration of how Gaussian process regression estimates the underlying process (dotted black). Here, a sample of functions drawn from the posterior Gaussian process probability distribution is shown (red). The predicted value for the underlying process is then obtained by averaging the drawn functions.



The GP prior is parameterized by a mean function and a covariance function, which are continuous extensions of the mean vector and covariance matrix of a multivariate normal distribution. These functions can be selected based on domain knowledge or through data-driven model selection (Abdessalem et al., 2017; Richardson et al., 2017). In practice, the mean function is often set to zero when no specific prior knowledge is available, which does not constrain the

posterior mean to zero but instead indicates a lack of prior information about its deviations from zero. The covariance function is typically based on a kernel function, which assigns covariances between time points only depending on their distance (e.g., quadratic exponential, Matern class). Finally, appropriate hyperpriors for the parameters of the mean and covariance functions are used to generate the corresponding posterior distribution. These distributions reflect prior beliefs about the hyperparameters and can be used to constrain them to sensible values.

The functional behavior of a GP is entirely determined by the posterior mean and covariance function. To accurately capture a process, it is crucial that the functional familiy generated by a chosen mean and covariance function is similar to the actual process. Common choices for the covariance function result in smooth and covariance-stationary GPs with constant wiggliness. Unlike LPR, GP regression provides more interpretable information about the process through the posterior distributions of hyperparameters. One typical hyperparameter, the characteristic length scale of the covariance function, is analogous to the bandwidth parameter in LPR, as it also describes the wiggliness of the estimated process. However, GP regression can include additional hyperparameters, allowing for more specific theories to be tested through model comparison.

Generalized additive models

Generalized additive models (GAMs) are a semi-parametric modeling framework that utilizes smooth terms, which are non-linear functions that are inferred from the data through smoothing splines (Hastie & Tibshirani, 1999; Wood, 2006, 2020). Smoothing splines are an extension of regular spline regression, which circumvent the knot placement issue by placing a knot at each observation (Tsay & Chen, 2019). This approach is equivalent to a basis function regression with as many basis functions as there are observations. However, using this many knots would lead to overfitting. To prevent this, smoothing splines use a penalty term, similar to the penalty used in a lasso or ridge regression, which controls the smooth term's wigglyness (Gu, 2013; Wahba, 1980). This penalty makes it possible to balance the flexibility and overfitting of

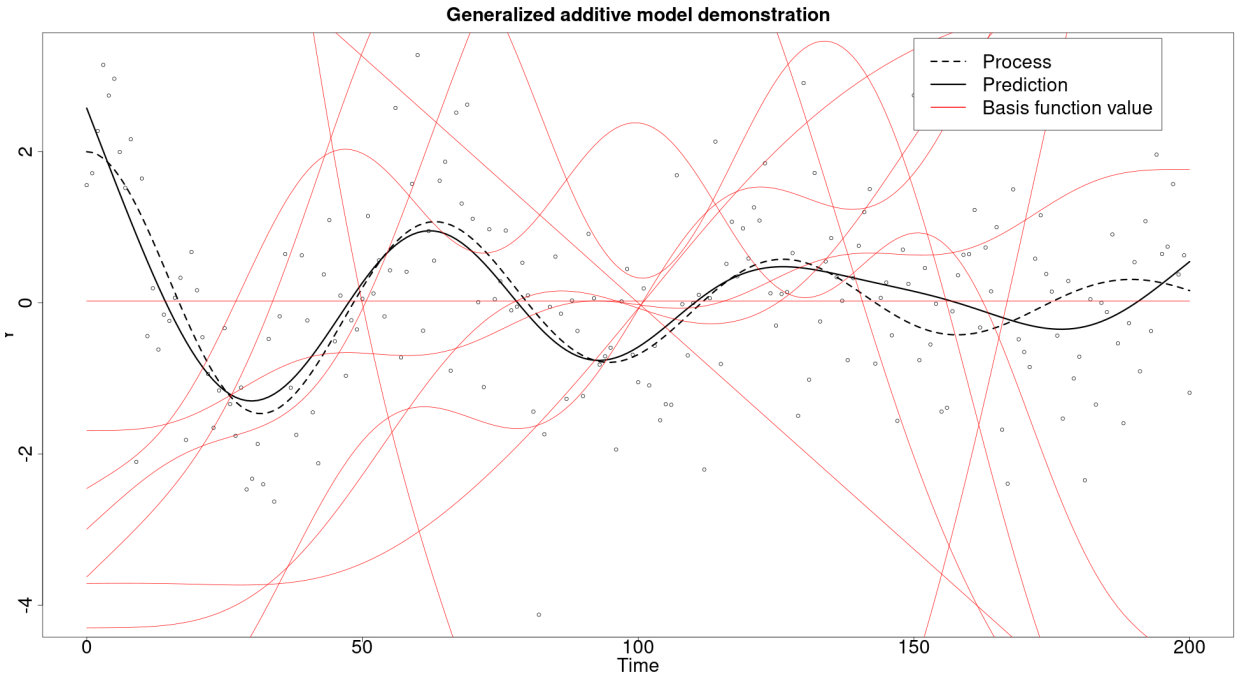
the smooth term, ensuring that the model captures the process accurately without excessive complexity. The optimal weight of the penalty is determined by minimizing a criterion function, such as the generalized cross-validation criterion (Golub & von Matt, 1997; Wood, 2006). While researchers must still choose a set of basis functions, cubic (Tsay & Chen, 2019) and thin plate spline bases (Wood, 2003) are optimal for many applications.

These smooth terms can then be combined in an additive regression model, where each smooth term essentially functions as a predictor within a regular regression analysis. In this model, smooth terms can be multiplied by covariates and summed into a single overall non-linear function to explain the data. This approach enables the formulation of models such as a time-varying autoregressive model, where the intercept and autoregressive parameters are smooth functions that vary over time (Bringmann et al., 2015; Bringmann et al., 2017). By integrating non-parametric smooth terms into a broader parametric model, GAMs become semi-parametric models that are well-suited for testing specific hypotheses while maintaining the flexibility needed to accurately capture the underlying process. Figure 4 illustrates a simple GAM with a single smooth term for time, fitted to the example process. Some of the scaled thin-plate smoothing spline basis functions that constitute the smooth term are shown in red.

Compared to previous methods, GAMs offer a more accessible modeling framework, enabling the specific modeling and testing of partial theories. For instance, a GAM can model a linear trend while adding a smooth term around it to capture non-linear deviations. This makes it possible gain insight into both the linear trend and the necessity of the smooth term, which can be examined through model comparison. Additionally, GAMs also provide an estimate of the wigglyness of the process through the penalty weight term. In contrast, to LPR and GP this penalty weight does not assume constant wigglyness. Lastly, the basis function coefficients within each smooth term can be interpreted. However, the specific interpretations depend on the spline basis used.

Figure 4

Demonstration of how generalized additive models (solid black) estimate the underlying process (dotted black). Here, the predicted values for the process at any point in time correspond to the weighted average of the basis functions (red).



Parametric models

Finally, several methods to parametrically model non-linear processes will be disucussed. The most direct approach for this involves defining a non-linear regression model for the process itself. As with all parametric models, this approach requires a thorough prior theory about the functional form of the process. However, such theories are often difficult or even impossible to find in practice due to the assumed complexity of the underlying dynamic systems.

However, in the case of the running example, such a formulation exists. Considering the oscillatory nature of this (self-) regulatory process, a cosine function is a natural candidate to model it. Additionally, since the oscillations diminish over time, the model should account for the decreasing amplitude. An example of such a model is:

$$
\begin{aligned}
f(t) &= Ae^{-ct}\cos(\omega t - \delta) \\
\omega &= \sqrt{k - \frac{c^2}{4}}
\end{aligned}
\tag{2}
$$

where the parameters control the initial amplitude of the wave function (A), the frequency (ω), the rate at which the amplitude decreases (c), and the phase (δ). Even for this relatively simple system, this parametric form is already quite complex. For more intricate processes, finding an appropriate model of this form can be nearly impossible.

Instead it is often more practical to define a model for how a process changes over time using a differential equation model (Boker, 2012). These models describe the relationship between the current value of the process and its instantaneous rate of change. By combining a model of the process's change with information about its initial state, it is possible to infer the entire trajectory of the process. Differential equations form a general class of models capable of representing various dynamic processes. For the running example, we use a damped oscillator model. This is a classic differential equation model originally designed to describe the behavior of an oscillating spring with resistance. This model can be expressed as:

$$
\begin{aligned}
\frac{\partial f}{\partial t} &= v \\
\frac{\partial v}{\partial t} &= -cv + -kf
\end{aligned}
\tag{3}
$$

This model relates the rate of change of the position of the spring f to its velocity v. In turn, the rate of change of the velocity is determined by the velocity itself through the damping coefficient c and by the position of the spring through the spring constant k. Here, the spring constant is related to the frequency of the oscillations since a stiffer spring oscillates faster and the damping coefficient is related to how quickly the spring returns to its basis position after being perturbed. Chow et al. (2005) show how this model can be applied in a psychological context to model emotional self-regulation. Such that a perturbed emotional state is regulated back to the desired baseline faster the larger the perturbation is.

In this model, f epresents the position of the spring, and v denotes its velocity, which is

the rate of change of the position. The rate of change of the velocity is influenced by both the velocity itself, through the damping coefficient c and the position of the spring, through the spring constant k. The spring constant k influences the frequency of oscillations, since a stiffer spring oscillates faster, while the damping coefficient c determines how quickly the spring returns to its equilibrium position after being disturbed. Chow et al. (2005) demonstrate how this model can be applied to psychological contexts, such as emotional self-regulation. In their application, a perturbed emotional state is regulated back to the desired baseline faster the larger the perturbation is.

Alternatively, when working with equally spaced time points, one can model dynamic local changes in discrete time using difference equations (Durbin & Koopman, 2012). A common example of this approach is the classic AR(1) model. These equations express the current value of the latent construct as a function of its previous value. While the smooth nature of the running example process cannot be perfectly captured by difference equations, it can be approximated by these equations:

$$\begin{aligned} f_t &= f_{t-1} + v_{t-1} \\ v_t &= (1-c)v_{t-1} + -kf_{t-1} \end{aligned} \tag{4}$$

Lastly, when modeling a process through local dynamics, two types of errors should be taken into account. The first is dynamic error, which comprises external perturbations to the latent construct value that are carried forward over time. For instance, if a participant experiences an unusually pleasant conversation that elevates their true positive affect, this change represents an error effect if such external influences are not accounted for by the model. However, since the true positive affect level has increased, this will influence future measurements due to emotional inertia. Dynamic errors can be incorporated into the model in different ways, with the most common approach being the addition of a normally distributed error term to the deterministic dynamic change models described above.

The second type of error that should be considered is measurement error, which represents

the difference between observed measurements and the latent construct values, which may be introduced by an imperfect measurement instrument. These errors are typically modeled using a factor or item response model that links the observations to the latent construct. Combining a dynamic model with a dynamic error component and a factor model yields a state space model (discrete time; Durbin & Koopman, 2012) or a stochastic differential equation model (continuous time). These models can then be used to infer the process and estimate the parameters of the dynamic equations using the Kalman filter and its extensions (Chow et al., 2007).

Simulation

Problem

To assess the effectiveness of the introduced parametric and non-parametric statistical methods in recovering different non-linear processes commonly encountered in experience sampling research (Figure 1) a simulation study was conducted. Further, to apply these models under the conditions described in the introduction, and within the constraints of available software implementations, we focused on a univariate single-subject design. Thus, the simulated data represented repeated measurements of a single variable for one individual.

Design and Hypotheses

For the non-parametric analysis methods (i.e., LPR, GP), we expect that each method in its default configuration will most accurately infer processes that are (a) continuous (i.e., without sudden jumps), (b) have constant wigglyness (i.e., constant second derivative), and (c) are smooth (i.e., differentiable). This expectation arises because both methods by default produce continuous, smooth estimates with a single constant bandwidth or lengthscale. For the GAMs, we expect that only criteria (a) and (c) will influence inference, as GAMs do not assume constant wigglyness. The parametric modeling approach is expected to provide the most accurate inferences, serving as a benchmark for comparison with the other methods. We also expect that the performance of parametric models will decrease as the model complexity increases (e.g., with jumps or reduced

smoothness), though likely less so than with non-parametric and semi-parametric models. Additionally, we expect that larger sample sizes will lead to more accurate inferences, with both (d) the overall length of the sampling period and (e) the sampling frequency being varied.

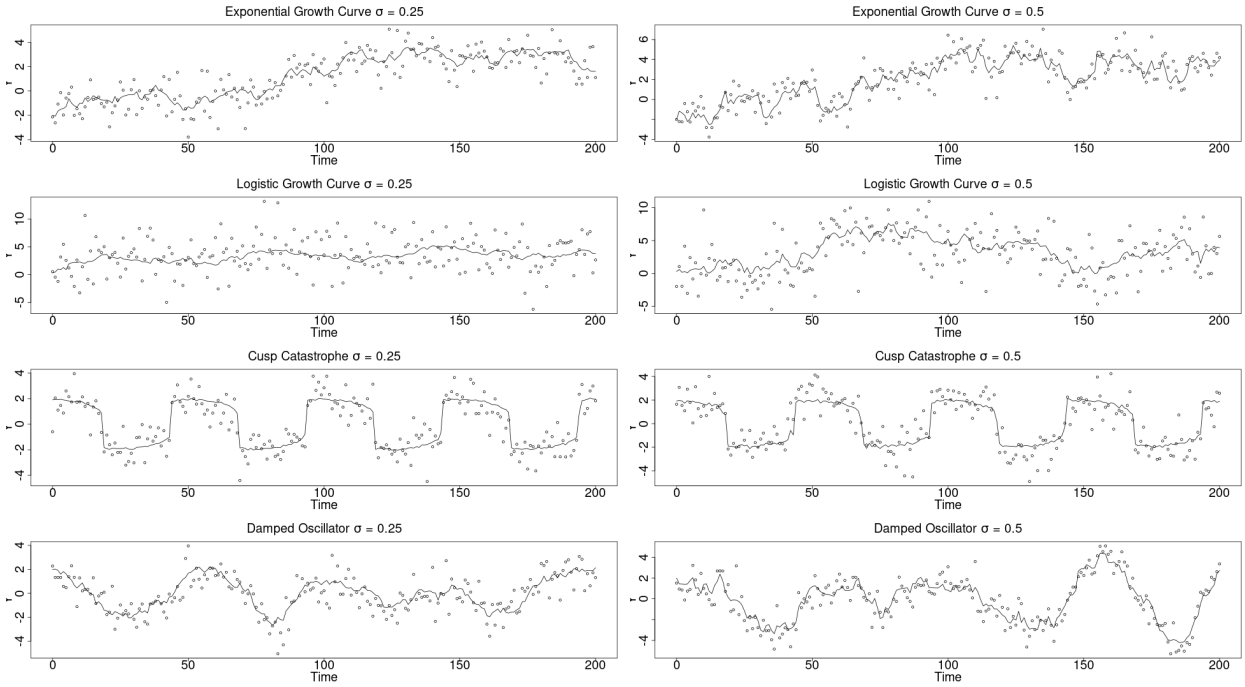
To conduct the simulation with processes that might be encountered in real EMA studies, we selected the exemplar processes illustrated in Figure 1 as a basis. These include two growth curves, modeled as an exponential and a logistic growth function, a mean-level switching process, modeled as a cusp catastrophe, and a self-regulatory process, represented by a damped oscillator. These processes allow us to test the impact of (a) sudden jumps and (b) changing wigglyness on the four methods. First, we hypothesize that the cusp catastrophe model, which is the only process featuring jumps, will be least accurately inferred by all methods. Second, all four processes exhibit changes in wigglyness (i.e., changes in the second derivative) over time. However, while the wiggliness of the exponential and logistic growth functions and the damped oscillator decreases monotonically, the cusp catastrophe's wigglyness changes cyclically. Therefore, we hypothesize that longer sampling periods for the exponential and logistic growth curves and the damped oscillator will reduce the inference accuracy of the LPR and the GP, as these rely on a single bandwidth or lengthscale parameter, which may become increasingly inadequate in capturing the changing wigglyness over time. We do not expect this effect to occur for the cusp catastrophe process, or when using GAMs or parametric models.

To manipulate the (c) smoothness of the processes, a dynamic noise component was added to the data-generating models. This perturbed the processes at each point in time by a normally distributed error, resulting in non-smooth (i.e., non-differentiable or rough) trajectories. The degree of roughness was controlled by the variance of these dynamic errors and we considered variances of 0.5, 1, and 2 reasonable relative to the process range. Figure 5 illustrates one possible realization of the exemplar processes with dynamic noise. Importantly, we intentionally omitted a condition without dynamic noise from this simulation, as dynamic noise is reasonably expected to be present in all psychological intensive longitudinal data (ILD).

Additionally, the sample size was varied during the simulation by manipulating both (d)

Figure 5

Non-linear exemplar processes



the sampling period and (e) the sampling frequency, as these methodological choices are expected to impact the performance of the analysis methods differently. Specifically, for the LPR and the GP, which rely only on data in local neighborhoods, we expected that extending the sampling period beyond this neighborhood will not increase the inference accuracy. In fact, if the process exhibits changing wigglyness over the extended period, as previously discussed, increasing the sampling period might even negatively affect the inference accuracy. In contrast, we expeced GAMs and parametric models, which incorporate the entire dataset in their estimations, to perform better with a longer sampling period. Since there is no inherent scaling to the time axis in this simulation, we chose to simulate the first half of each process in one condition and the full process in another, referred to as sampling periods of one or two weeks. This scaling is arbitrary and could be changed to any time frame. Lastly, we expected that increasing the sampling frequency will generally improve the inference accuracy across all methods, as it provides more

information about the latent processes. We tested sampling frequencies of three, six, and nine measurements per day, to align these choices with typical experience sampling study designs (Wrzus & Neubauer, 2023).

Procedure

To simulate the data, each exemplar process was represented as a generative stochastic differential equation. This means that, at each time point, the rate of change of the process was determined by a nonlinear function of its current value together with an additive dynamic error component modeled by a Wiener process. Data from these generative models were then simulated using the Euler-Maruyama method, ensuring accurate dynamic error variance, sampling frequency, and sampling period for each condition. Measurement errors were added to the latent process data at each time point using a standard normal distribution, generating the final sets of observations. For a detailed technical explanation of the data generation process, see Appendix A.

To determine the required number of data sets per condition, a power simulation was conducted based on an initial pilot sample of 30 generated data sets per condition. Based on these data sets, the outcome measures (e.g., MSE, GCV, and confidence interval coverage scores) and their corresponding standard deviations were calculated by condition. These standard deviations were then used to predict the Monte Carlo standard errors of the means of each outcome measure across increasing sample sizes. Monte Carlo standard errors reflect the expected variation in an outcome statistic due to random processes within the simulation. We selected the number of data sets per condition for the full simulation, so that the maximum expected Monte Carlo error across all outcome measures and conditions was 0.05. This criterion was met with **N** data sets per condition.

Model estimation

After simulating the data, all introduced methods were applied to each data set using the statistical software R (R Core Team, 2024). First, the LPR was estimated using the nprobust package (Calonico et al., 2019), which allows to correction for the bias inherent in LPRs. Second,

GPs were estimated in STAN (Gabry et al., 2024) with a zero mean and a squared exponential covariance function, following common practice. Third, GAMs with a single smooth term for time were fitted using the MGCV package (Wood, 2011). Finally, parametric differential equation models corresponding to the true data-generating models were created and estimated using the Dynr package (Ou et al., 2019). While the same non-parametric models were used across all conditions, the parametric models were tailored to each specific data-generating process. A detailed description of each model fitting procedure is provided in Appendix B.

To ensure reliable model fit and reasonable inferences, the fitting procedures for each method were validated on pilot samples within each condition. After this an initial run of the simulation was performed, which revealed that the GAMs over- or linearly underfit some data sets and that the parametric models overfit some data sets. To prevent this, the fitting procedure of both methods was adjusted and the simulation rerun. Further, if any models failed to converge during the simulation, the corresponding outcome measures were excluded from the following analyses. After fitting each model to the data, they were used to obtain point and interval estimates (i.e., 95% confidence and credible intervals) for the latent process at each time point.

Outcome measures

To evaluate and compare the performance of the different analysis methods, we focused on three outcome measures. The first two assessed each method's accuracy in predicting the process values at or between the observed time points. These predictive accuracy measures indicate how well each method captures the underlying non-linear process. The third outcome measure evaluated the accuracy of the uncertainty estimates provided by each method. Specifically, whether the confidence or credible intervals produced by each method correctly included the true state value the expected proportion of times.

Capturing the non-linear process

To assess how effectively each method captured the non-linear process at the observed time points, we calculated the mean squared error (MSE) between the estimated and generated

process values. Additionally, to evaluate how well each method interpolated the process between the observed time points, we computed the generalized cross-validation ((GCV; Golub et al., 1979)) criterion for each method and data set. The GCV is a more computationally efficient and rotation-invariant version of the ordinary leave-one-out cross-validation criterion, with a similar interpretation. The latter is calculated by removing one data point, refitting the model while keeping certain parameters fixed, predicting the left-out observation, and calculating the squared prediction error. By repeating this procedure for each data point and averaging the squared errors, leave-one-out cross-validation provides an estimate of how accurately the model predicts unobserved values within the design range.

Subsequently, two ANOVAs were conducted to analyze differences in MSE and GCV values across simulation conditions and analysis methods. To simultaneously identify both factors that likely do and do not affect MSE and GCV values, an exhaustive model search was performed. In this search, AIC and BIC model weights were used to determine the model, which provides the optimal balance between data fit and model complexity. Notably, AIC weights can be interpreted as conditional model probabilities when the true model is among the tested models (Wagenmakers & Farrell, 2004). The specific hypotheses stated earlier were tested using post hoc tests and marginal comparisons with appropriate corrections for multiple comparisons. Model assumptions were assessed on the final selected model. Given the large sample sizes in this simulation, the ANOVAs were robust to moderate violations of normality, and any potential violations of homoscedasticity were addressed using heteroscedasticity-consistent standard errors. However, if both assumptions were severely violated, results were instead presented descriptively, using means and standard errors for the respective conditions.

Uncertainty quantification

To evaluate the uncertainty estimates provided by each method, we recorded whether the true generated process was located within the confidence or credible intervals at each time point. Subsequently, the average confidence interval coverage proportion for each method and data set

was obtained, by averaging over all time points. Given that all confidence or credible intervals were set at a 95% confidence level, the expected coverage proportion should ideally be close to 95%. Due to Monte Carlo error in the simulation, average coverage proportions between 93% and 97% were also deemed acceptable. Average coverage proportions above 97% suggested overestimated standard errors, while those below 93% indicated either a poor approximation of the underlying process or an underestimation of the standard errors.

Results (Template)

Capturing the non-linear process

For both outcome measures both the AIC and BIC model weights indicated the most complex model including all interactions as the best fitting model with a model weight of 1. While the residuals of both models showed considerable deviations from normality for both outcome measures by being platykurtic, the residual distributions were unimodel and approximately symmetric. Thus, we concluded that due to the large sample sizes in this simulation, the ANOVAs will be robust against these deviations. However, a Breush-Pagan test revealed that for both outcome measures the resiudals were heteroscedastic, which were corrected for in the following. Using these corrections and Bonferroni corrections for fitting two seperate ANOVAs the type-III ANOVA for the MSE showed significant main and interaction effects everywhere except for the five way interaction and one four way interaction. The type-III ANOVA fo the GCV values indicated that all effects were significant. However, most higher order interactions had partial-η^2 values very close to zero. Table 1 shows the partial-η^2 values of all effects that indicated at least a small effect size. These effect sizes indicate that the largest effects are associated with the main effects, and interactions with the analysis method and the latent process. Therefore, the following section focuses on analysing these effects. For a full overview of all the effects in the model, see Appendix C.

Main and second-order interaction effects. Figure 6 illustrates the mean effect of the different analysis methods for each process by averaging over the sampling period, frequency, and

Table 1

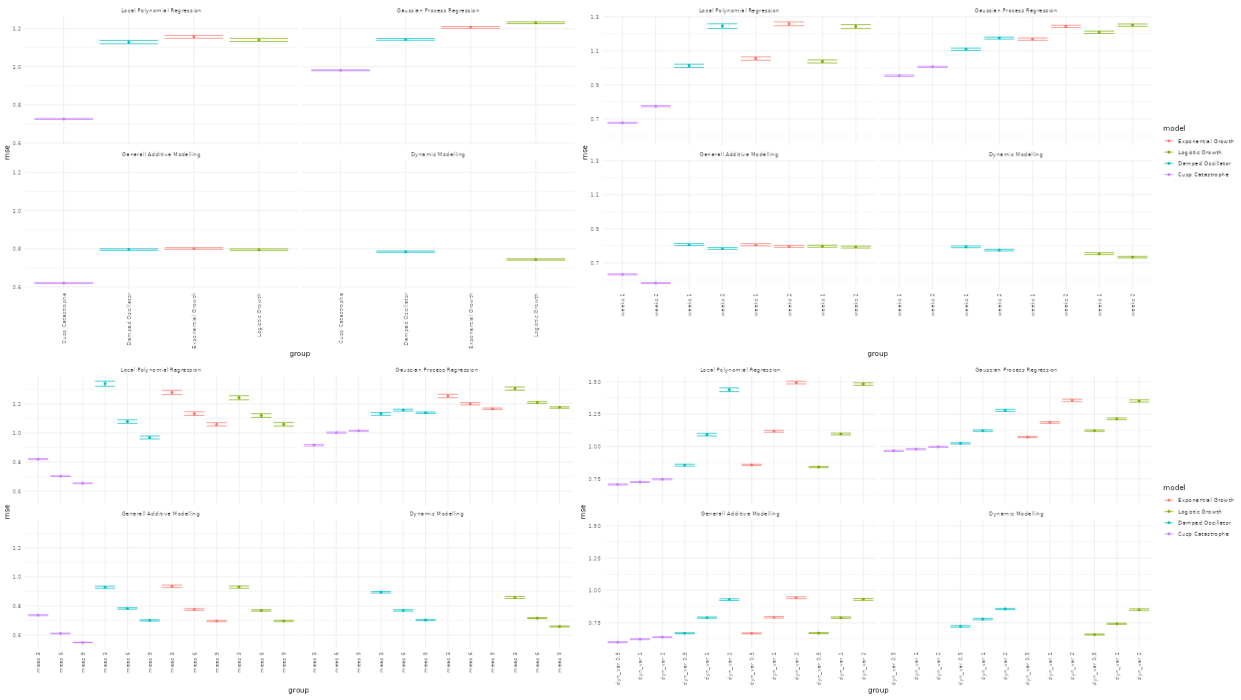
All effect sizes that are least samll (0.01)

Factor	MSE partial-η^2
Method	0.65
Process	0.43
Sampling period (SP)	0.07
Sampling frequency (SF)	0.23
Dynamic error variance (DE)	0.47
Method:Process	0.11
Method:SP	0.10
Method:SF	0.08
Method:DE	0.18
Process:SF	0.02
Process:DE	0.18
SF:DE	0.02
Method:Process:SF	0.03
Method:Process:DE	0.03

the dynamic error variances. This shows that as expected on average the parametric models have the lowest MSE and GCV across all processes. Further, the GAMs appear to perform best among the semi- and non-parametric techniques, in term of the MSE and GCV, while the LPRs and GPs infer the latent process less accurately. Further, while there are only small differencess in how accurately both growth curves and the damped oscillator were inferred, the cusp-catastrophe process has been inferred with a lower MSE and GCV ny all methods, contradicting our expectation. For sampling period Figure 6 shows that sampling over the entire process

Figure 6

Non-linear exemplar processes



An Empirical Example

In the following, the four analysis methods previously introduced were applied to depression data from the Leuven clinical study. This study employed experience sampling measures to study the dynamics of anhedonia in individuals with major depressive disorder (Heininga et al., 2019). This study was selected for its heterogeneous sample, which includes participants with major depressive disorder, borderline personality disorder, and healthy controls. This diversity increases the likelihood of the data exhibiting a range of non-linear dynamics and processes. Specifically, Houben et al. (2015) found in their meta-analysis that individuals with lower psychological well-being tend to experience greater emotional variability, less emotional stability, and higher emotional inertia. However, this finding did not replicate in an analysis of positive affect within the Leuven clinical study (Heininga et al., 2019). Additionally, emotional inertia, the extent to which an emotional state carries over across observations, has been shown to

vary within individuals over time, which makes it likely that the processes underlying this data are non-stationary. Combined with the previously outlined hypothesis that emotion regulation may function as a self-regulating system, this data is optimal for demonstrating the introduced methods..

To maintain consistency with how the methods were introduced and to avoid using measurement models with multiple indicators, we analyzed momentary depression, which was measured using a single item. This item was chosen over affect measures because it displays sufficient variability, has a relatively low proportion of participants with strong floor or ceiling effects, and is measured on a broad response scale (0 to 100), making it ideal for illustrating the introduced methods.

Sample and data description

The participants in the clinical sample of the Leuven clincial study were screened by clinicians during the intake in three Belgian psychiatric wards (Heininga et al., 2019). Patients who met the DSM criteria for mood disorders or borderline personality disorder during the intake were eligible for enrollment, while those presenting with acute psychosis, mania, addiction, or (neuro-)cognitive symptoms were excluded. Following the screening, 90 patients enrolled in the study. Additionally, 44 control participants were matched to the clinical sample by gender and age, resulting in a total sample size of 134. Within the clinical sample, three patients withdrew during the baseline assessment, two were excluded due to faulty devices, and seven were removed for responding to less than half of the EMA measures. In the control sample, one participant was excluded for responding to less than half of the EMA measures, and three were removed for meeting the criteria for a current psychiatric disorder. Consequently, the final published data set included 78 participants in the clinical sample and 40 participants in the control sample.

During the study, all participants completed a baseline assessment, followed by seven days of semi-random EMA assessments, with 10 equidistant assessments per day. However, the starting date of the EMA measures varied between people. During each assessment, participants

responded to 27 questions covering emotions, social expectancies, emotion regulation, context, and psychiatric symptoms. This analysis focused on the item assessing momentary depressive mood (i.e., "How depressed do you feel at the moment?") rated on a scale from 0 to 100. One additional participant was removed from the analysis for consistently reporting a score of zero across all assessments, resulting in a final sample of 117 participants. For a more thorough sample and data description, see Heininga et al. (2019).

The published data set was obtained from the EMOTE database. The initial study procedure was approved by the KU Leuven Social and Societal Ethics Committee and the KU Leuven Medical Ethics Committee. This secondary data analysis was approved by the Ethics Review Board of the Tilburg School of Social and Behavioral Sciences.

Analysis Plan

Exploratorty idiographic analysis

First, the LPR, GP, and GAM methods were applied to explore the idiographic latent processes underlying the data. Each method was applied separately to the time-series of each participant, using the same specifications as in the simulation study (Appendix B). However, for the LPR, only local cubic polynomials were considered to keep the interpretation of the bandwidth consistent across participants. Since all participants were assessed over seven days, but not during the same period, the time-series for each participant was centered so that the first measurement time point served as the zero point. The LPR bandwidth, GP lengthscale, and GAM smoothing parameter were then analyzed to assess the wigglyness of the idiographic processes. Additionally, the GCV values produced by each method were evaluated to determine which method provided the most accurate interpolations. Lastly, the mean squared error was calculated for each method and data set to estimate the expected measurement error.

Multilevel analysis

After the idiographic analysis, mixed-effects GAMs were fitted to the data to assess the extent to which common and idiographic smooth are necessacary for accurately modeling the data. The baseline model was a random intercept model, where each participant's data was modelled by a flat line. The second model fitted a random smooth, in which each participant's data was modelled by an individual smooth function with a common smoothing parameter through factor-smooth interactions. To determine if there was evidence of a common non-linear trend, this model was compared to an extended model, which combined fixed non-linear smooth with individual random non-linear deviations. These deviations were again modelled using factor-smooth interactions with a common smothing parameter. Finally, to test potential heterogeneity in the smoothing parameters across participants, a fourth model was fitted, which allowed for individual smooth terms with varying smoothing parameters.

Parametric analysis

Lastly, three parametric differential equation models were applied to explore the data. The first model is a random-walk model, where the rate of change in the latent depression state does not depend on its current value, such that future values are only influenced by random perturbations in form of the dynamic errors. The second model is a continuous-time autoregressive model, where the depression score reverts to an individual's mean at a rate linearly dependent on the distance from this mean. Both the individual mean and autoregressive effect were treated as random effects by incorporating them as time-invariant state variables, similar to how random effects are included in dynamic structural equation models. The third model is the damped oscillator model discussed earlier. To adapt this model to the data structure, a random mean was added to account for between-person differences in the baseline level around which the depression score oscillates. Random effects were also included for the spring constant and damping coefficient to accommodate individual differences in oscillation patterns. All three models included components for measurement and dynamic error variance. Lastly, the model fit

was compared between the models using the AIC and BIC.

Results

Exploratorty idiographic analysis

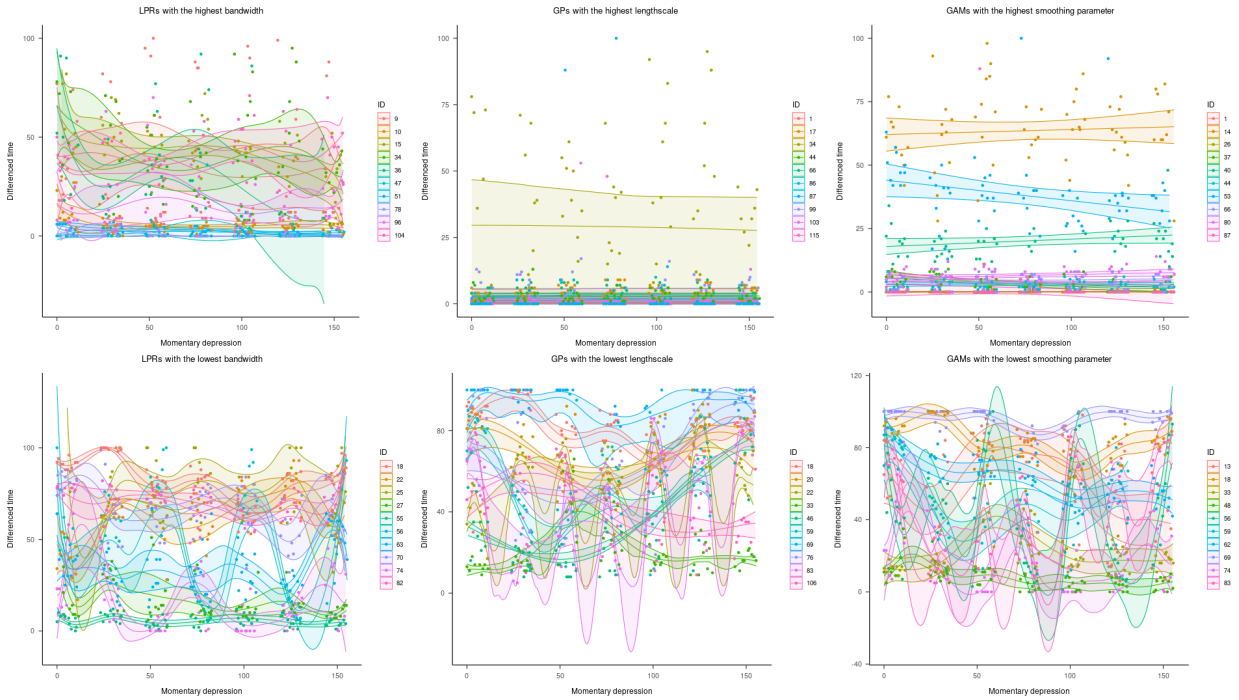
First, the three non-parametric and semi-parametric methods were used to estimate the individual latent depression processes. For the local cubic regression, the median optimal bandwidth was 21.28 hours (*IQR*: 5.52). For the GP, the median optimal length scale was 22.57 standard deviations (*IQR*: 15.09). Lastly, for the GAMs, the median optimal smoothing parameter was $8.18 * 10^9$ (*IQR*: $1.76 * 10^{10}$). Unfortunately, these three measures of wiggliness are not only on different scales, but there is also only a moderate correlation between the bandwidth and lengthscale parameters (r = 0.33). The smoothing parameter of the GAMs shows little to no correlation with the other two measures (bandwidth: r = -0.03; length scale: r = -0.08). This discrepancy arises because, while all three parameters reflect the wiggliness of the estimate, they capture different aspects of it. For example, in data with a linear trend, the bandwidth of the local cubic regression and the smoothing parameter of the GAMs would theoretically be infinite, while the length scale parameter of the GP would have a finite value. Additionally, the interpretation of each wiggliness parameter depends in the model configurations chosen and would change for different configurations of these methods.

Because of this, there is not much value in interpreting the absolute values of these parameters. Instead, we explored the range of functional behaviours inferred by the most extreme values of each parameters. Figuer 7 shows the ten least and most wiggly processes inferred by each method. This figure reveals considerably heterogeneity in the functional behaviour inferred by each method. Most interestingly, the least wiggly processes inferred by both the GPs and the GAMs are linear trends, indicating the absence of any dynamic errors for these individuals. In contrast to this, the processes with the highest inferred wigglyness, display either large dynamic errors around their respective person means or in addition to this a different non-linear dynamic.

Lastly, a cross-validation was conducted using the generalized cross-validation criterion to

Figure 7

The ten least and most wiggly processes as inferred by each method



investigate which method predicted the latent processes most accurately. The median GCV for the GAMs was 125.29 (*IQR*: 201.04), the Go it was 248.27 (*IQR*: 578.00), and for the LPR it was 131.57 (*IQR*: 213.73). In addition to this, the mean squared error was calculated between the predictions of each method and the data. Here the median MSE of the GAMs was 114.04 (*IQR*: 171.59), for the GP it was 197.54 (*IQR*: 388.40), and for the LPR it was 101.74 (*IQR*: 153.77). Together with the GCV this indicates that the GAMs inferred the latent processes most accurately, whereas the LPR slightly overfit the data and the GPs tended to underfit the data.

Multilevel analysis (Numbers here are still wrong)

First, the baseline model with only the random intercepts had an adjusted R^2 of 0.75 (*GCV* = 231). Extending the model to include random smooths for each individual with a common smoothing penalty increased the adjusted R^2 to 0.81 (*GCV* = 194.89). However, adding a common smooth to this model reduced adjusted R^2 to 0.80 (*GCV* = 197.44). Lasltly, using

person-specific smooth terms for the random smooth increased the adjusted R^2 further to 0.81 (GCV = 185.29). Table 2 compares the four models using the AIC and BIC. The AIC indicates the most complex model, providing evidence for the necessacity of individual smooth parameters. However, the BIC, which given the sample size uses a heavier penalty, indicates the random intercpet model as the best fitting model, highlighting that the majority of the variance in the data is due to mean level differences. Overall, there is conflicting evidence regarding to which model fits the data best.

Table 2

Model information criteria for the four mixed effects GAMs

Model	Degrees of freedom	AIC	BIC
Random intercept	117.3599	58997.31	59803.73
Random smooth w. common penalty	708.5276	57712.79	62581.35
Common smooth w. deviations	714.1653	57804.00	62711.29
Random smooth w. individual penalties	565.1936	57381.04	61264.69

Parametric analysis

Not working yet

Discussion

Conclusion

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