

**Modeling Non-Linear Psychological Processes: Reviewing and Evaluating Non-parametric
Approaches and Their Applicability to Intensive Longitudinal Data**

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Abstract

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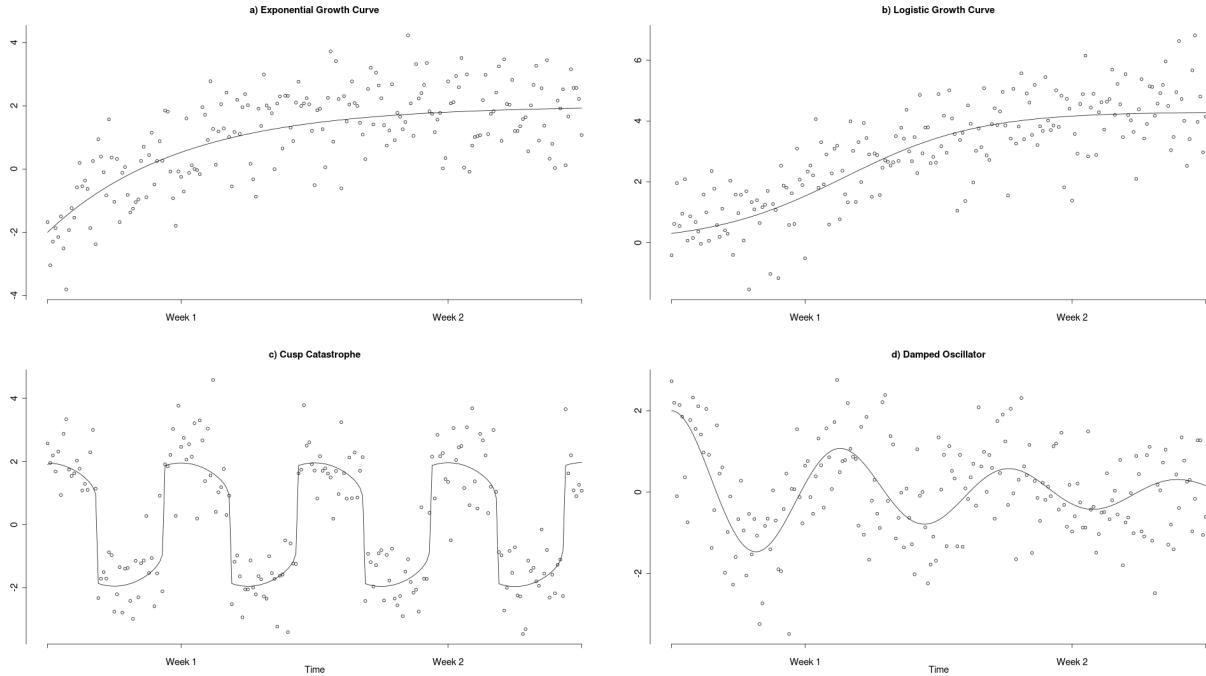
Psychological constructs are increasingly understood as components of complex dynamic systems (Nesselroade & Ram, 2004; Wang et al., 2012). This perspective emphasizes that these constructs fluctuate over time and within individuals. To study these variations and the underlying processes, researchers are increasingly collecting intensive longitudinal data (ILD) using ecological momentary assessment (EMA), experience sampling, or similar methods (Fritz et al., 2023). In these studies, one or more individuals are assessed at a high frequency (multiple times per day) using brief questionnaires or passive measurement devices. These rich data allow researchers to examine complex temporal variations in the underlying psychological variables within an ecologically valid context and to explain them through (between-person differences) in within-person processes.

Due to these ILD studies, many non-linear psychological phenomena and processes have been discovered in recent years. Clear examples of this are the learning and growth curves observed in intellectual and cognitive development (Kunnen, 2012; McArdle et al., 2002). In these cases, an individual's latent ability increases over time, following an intricate non-linear trajectory from a (person-specific) starting point towards a (person-specific) asymptote, which reflects the individual's maximum ability. Additional examples of asymptotic growth over shorter time spans that are typically studied with ILD include motor skill development (Newell et al., 2001) and second language acquisition (De Bot et al., 2007). Figure 1 shows common model choices for these kinds of processes in the form of an exponential growth function (a) and a logistic growth function (b).

Another common non-linear phenomenon is switching between seemingly distinct states that differ, for instance, in their means. This occurs, for example, during the sudden perception of cognitive flow, where individuals abruptly switch from a "normal" state to a flow state and back (Ceja & Navarro, 2012). Another example is alcohol use relapse, where patients suddenly switch from an abstinent state to a relapsed state (Witkiewitz & Marlatt, 2007). This sudden switching

Figure 1

Examples of non-linear processes demonstrated to occur in psychological time series



Note. This figure shows four demonstrated psychological non-linear processes. Panels (a) and (b) show exponential and logistic growth curves, respectively. Panel (c) shows a cusp catastrophe model. Lastly, panel (d) shows a damped oscillator.

behavior has been modelled using a cusp catastrophe model. This dynamic model, drawn from catastrophe theory, naturally leads to mean level switches when varying one of its parameters (Chow et al., 2015; van der Maas et al., 2003) and has been exemplified in Figure 1 (c).

As a final example, one may consider (self-) regulatory systems, which maintain a desired state by counteracting external perturbations. In these systems, the degree of regulation often depends on the distance between the current and the desired states. The common autoregressive model describes such a system in which the regulation strength depends linearly on this distance. However, this relationship may also be non-linear, such that the degree of regulation changes disproportionately with larger distances. Such a (self-) regulatory model has been used to model, for example, emotion regulation (Chow et al., 2005) using a damped oscillator model. This model is exemplified in Figure 1 (d).

Although initial evidence for non-linearity in psychological research exists, theories about the nature and form of non-linear psychological processes remain scarce (Tan et al., 2011). Frequently, psychological theories are too general to result in specific hypotheses (Oberauer & Lewandowsky, 2019), such as, for example, about the form of specific non-linear dynamics. ILD studies could potentially help to refine these theories through a nuanced understanding of how the involved psychological variables interact over time. Such refined theories could, for instance, take the form of dynamic models, such as differential equation (Boker, 2012) or state-space models (Durbin & Koopman, 2012), that describe how a given process changes over time. However, in order to develop these types of theories it is first necessary to identify empirical phenomena in ILD, which can be thought of "stable and general features of the world that scientists seek to explain" (Borsboom et al., 2021). Examples of such phenomena could be the described state switching or regulatory oscillations, if they generalize beyond individual data sets and contexts. Formal theories about the underlying process should then be able to explain these phenomena and different candidate theories can be compared on their success to do so (Borsboom et al., 2021). While the study of non-linear phenomena in ILD is receiving increasingly more attention in psychology and different statistical techniques are developed to explore these phenomena (Cui et al., 2023; Humberg et al., 2024), researchers are currently still limited in their ability to infer non-linear phenomena from ILD. One reason for this is a lack of advanced statistical methods that are flexible enough to adequately capture and explore these processes, which hinders the development and evaluation of guiding theories.

Due to the absence of adequate statistical methods, non-linear trends in psychology are often addressed through polynomial or piecewise spline regression. Polynomial regression (Jebb et al., 2015) uses higher-order terms (e.g., squared or cubed time) as predictors in a standard multiple linear regression model. While effective for relatively simple non-linear relationships, particularly those that can be represented as polynomials, this method has significant limitations and likely leads to invalid results when applied to more complex processes, such as mean switching or (self-) regulatory systems (e.g., Figure 1 c & d). In these cases, polynomial

approximations require many higher-order terms to capture the process's complex trajectory, which raises the problem of over- or underfitting the data, causes model instability, and leads to nonsensical inferences (e.g., interpolating scores outside the scale range; Boyd and Xu (2009), Harrell (2001), and Jianan et al. (2023)).

An alternative approach is piecewise spline regression, which constructs a complex non-linear trend by joining multiple simple piecewise functions at specific points, called knots (e.g., combining multiple cubic functions into a growth curve with plateaus; Tsay and Chen (2019)). However, spline regression requires a careful, manual selection of the optimal piecewise functions and knot locations. This can be problematic in practice because, as mentioned, precise guiding theories about the functional form of most psychological processes are lacking (Tan et al., 2011). This absence of clear guidance can quickly lead to misspecified models and invalid results.

These limitations in the currently available methods underscore the need for more sophisticated statistical methods to study and explore non-linear processes. Various such advanced statistical methods like kernel regression, Gaussian processes, and smoothing splines are available outside of psychology. However, these methods have rarely been applied in psychology because they have not been reviewed for an applied audience, nor have their assumptions and inference possibilities been evaluated in the context of ILD. As a result, it is difficult for psychological researchers to select the most suitable method for a specific context. This challenge is further complicated by the fact that the ideal statistical method may depend on the characteristics of the underlying non-linear process, which are generally unknown. Especially, since the ideal smooth processes, as depicted in Figure 1, for which many of these methods were originally developed, are unlikely to occur in psychological research. This is because psychological constructs are (especially in EMA research) measured in an environment in which external influences affect and perturb the construct. These constant perturbations can quickly result in the non-smooth or rough processes that are typically seen in psychological time series. It is for instance easy to imagine that one's happiness may not decrease smooth and gradually, after stubbing one's toe.

To address this important gap, this article reviews three advanced non-linear analysis methods and evaluates their applicability to typical ILD scenarios (Section 1). The methods reviewed in this article are different semi- and non-parametric regression techniques, available in the open-source software R (R Core Team, 2024), which are able to infer non-linear functions from data while accommodating varying degrees of prior knowledge. In a simulation study, we compare how well each method recovers different non-linear processes under common ILD conditions (Section 2). Lastly, we demonstrate how the best-performing method can be applied to analyze an existing dataset (Section 3). Further, to introduce these methods accessibly and apply them under conditions where software implementations are available, this article focuses on the univariate single-subject design.

1 Non-linear analysis methods

In the following paragraphs the three selected semi- and non-parametric regression techniques will be introduced.

1.1 Local polynomial regression

The first technique is called local polynomial regression (LPR). Similarly to regular polynomial regression, LPR approximates the process using polynomial basis functions (e.g., squared or cubed time). However, instead of using one large polynomial function to approximate the entire process, LPR estimates smaller, local polynomials at any point in time. These local polynomials are then combined into a single non-linear function over the entire set of observations (Fan & Gijbels, 2018; Fan & Gijbels, 1995a; Ruppert & Wand, 1994).

To determine the value that the LPR predicts at a specific time point, the data is first centered around that point (by shifting the data along the time axis so that the chosen time point is at zero), and a low-order polynomial is fitted to data around it. Since polynomial approximation is more accurate for data points closer in time, a weighting function is applied during the polynomial estimation, which assigns weights to each data point based on its distance from the

point of interest. The value that the LPR predicts for the chosen time point is then given by the intercept of the locally weighted polynomial at this time point.

Formally this procedure can be expressed using the following set of equations:

$$\begin{aligned}
 y_t &= f(t) + \varepsilon_t \\
 \mathbf{X} &= \begin{bmatrix} 1 & (t_1 - t^*)^1 & \dots & (t_1 - t^*)^p \\ \vdots & \vdots & \ddots & \vdots \\ 1 & (t_n - t^*)^1 & \dots & (t_n - t^*)^p \end{bmatrix} \\
 \mathbf{W} &= \begin{bmatrix} w_{1,1} \\ & \ddots \\ & & w_{n,n} \end{bmatrix} \\
 \hat{f}(t^*) &= Intercept((\mathbf{X}^T \mathbf{W} \mathbf{X})^{-1} \mathbf{X}^T \mathbf{W} \mathbf{y})
 \end{aligned} \tag{1}$$

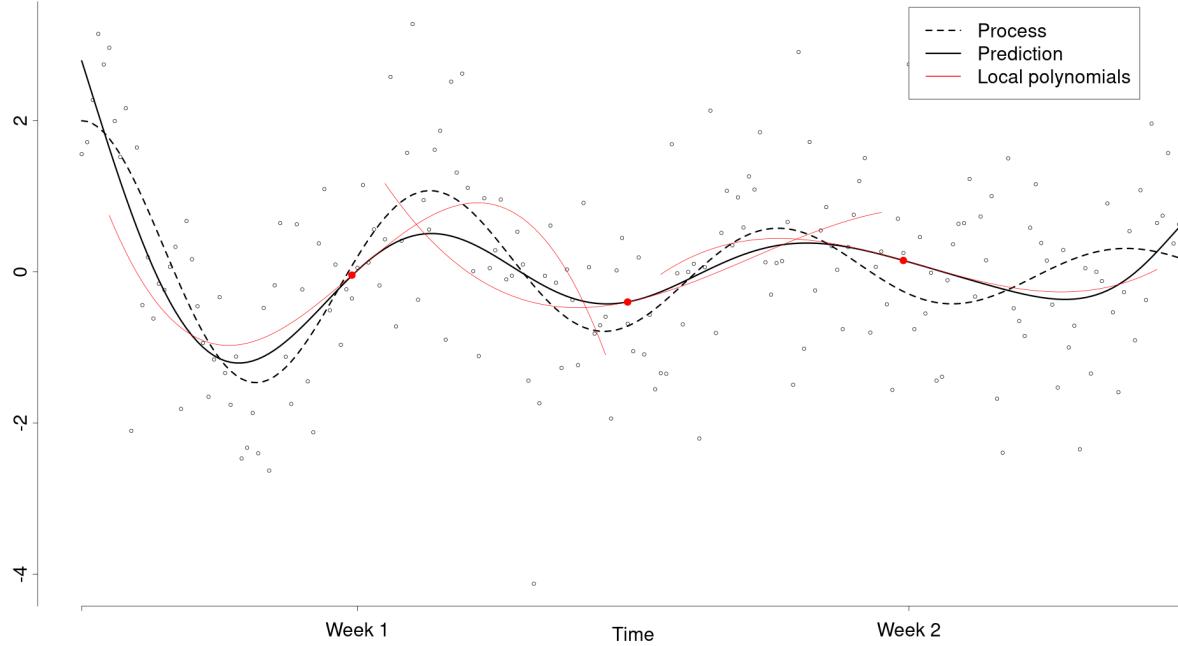
where a univariate process f is inferred at the chosen time point t^* . Then \mathbf{X} is the model matrix of a multiple linear regression, such that the columns correspond to polynomial transformations up to degree p of the data centered around t^* ¹. Specifically, \mathbf{X} may contain polynomial transformations of centered time (i.e., the first column is filled with ones, the second column contains the centered time points, the third column the centered time points squared, and so on). Further, \mathbf{W} is a diagonal matrix containing the weights associated with each datum. The last equation is a normal equation solving for the coefficients of a weighted multiple linear regression. The intercept of this regression gives the estimated value of the LPR at t^* .

The same procedure can be repeated to find the value that the LPR predicts for the process at a different time point by centering the data around the new point of interest. Since it is theoretically possible to repeat this process at infinitely many time points, LPR is a non-parametric technique. Figure 2 shows the estimated LPR for the damped oscillator example process introduced in Figure 1 (d). In this figure, three truncated examples of local cubic

¹ Note that the polynomial terms in this model matrix are derived from a Taylor series approximation around t^* with p derivative terms.

Figure 2

Demonstration of a local polynomial regression



Note. This figure shows how LPR (solid black) estimates the underlying process (dotted black). Here, three exemplary local cubic regressions (red) are shown that provide the values that the LPR predicts at the time points 50, 100, and 150. The complete LPR was plotted by evaluating such local cubic regressions at 200 points along the time axis.

regressions are shown in red, which contribute to the overarching LPR for this process.

When fitting an LPR, several decisions must be made regarding the optimal weighting of the data and the degree of the local polynomial. The data weighting in an LPR is achieved through a kernel function of the form

$$w_{i,i} = K\left(\frac{t_i - t^*}{h}\right) \quad (2)$$

which, in this case, is a mathematical equation that determines the influence of different data points during the local polynomial estimation. Because these kernels are usually symmetric around the origin, they assign weights to data points depending on how far away they are from the

point of interest t^* . Common choices for kernels include the Gaussian and Epanechnikov kernels. Both assign higher weights to data points in closer proximity. However, while the Gaussian kernel assigns small weights to all distant data points, the Epanechnikov kernel assigns zero weights beyond a certain distance. Additionally, the Epanechnikov kernel has also been shown to be optimal in many applications, particularly when estimating points where the entire width of the kernel lies within the data range (Fan et al., 1997). Each kernel is further defined by a bandwidth parameter h , which determines the kernel's width and effectively controls the influence of more distant data points. The bandwidth parameter represents the wigginess of the estimated process in practice. Several methods are available to find the optimal bandwidth by optimizing a data-dependent criterion function, such as the cross-validation error or the mean integrated squared error (Debruyne et al., 2008; Köhler et al., 2014). Most standard software packages offer automated default procedures for this task. However, the optimization criterion may also be selected based on specific research objectives (e.g., optimizing a cross-validation error may be most attractive, if the primary research interest is out of sample prediction).

In practice researchers also need to choose the degree of the local polynomials, which additionally reflects an assumption about how smooth the underlying process is. Specifically, for a first-degree LPR, the process should not exhibit any corners, discontinuities, or vertical sections. This ensures that the processes rate of change (i.e., derivative), approximated by the first-order polynomial term, is well behaved. Higher-order local polynomials require this smoothness for increasingly complex rates of change. For instance, a second-degree LPR requires that the rate of change itself is smooth, in turn ensuring that its rate of change is well behaved. This property should hold for all p rates of change of a process when using an LPR with p degrees (under Taylor's theorem). Typically, the degree of the local polynomials is chosen to be low and odd. This choice reflects a bias-variance tradeoff, where higher-order polynomials reduce bias but increase variance only when transitioning from an odd to an even power (Ruppert & Wand, 1994).

The second assumption made by the LPR is that the process has constant wigginess with respect to the chosen polynomial degree. This assumption is made, because a single bandwidth

parameter is used to capture the process at any point in time. For example, a linear LPR, estimates the linear polynomials using weights derived from a kernel with the same bandwidth at any point in time. This bandwidth should then be optimal across the entire process. This assumption is less strict for higher order polynomials, due to their increased flexibility and may be relaxed more by using a time-varying bandwidth (Fan & Gijbels, 1995b) or polynomial degree (Fan & Gijbels, 1995a), but this extension is beyond the scope of this paper.

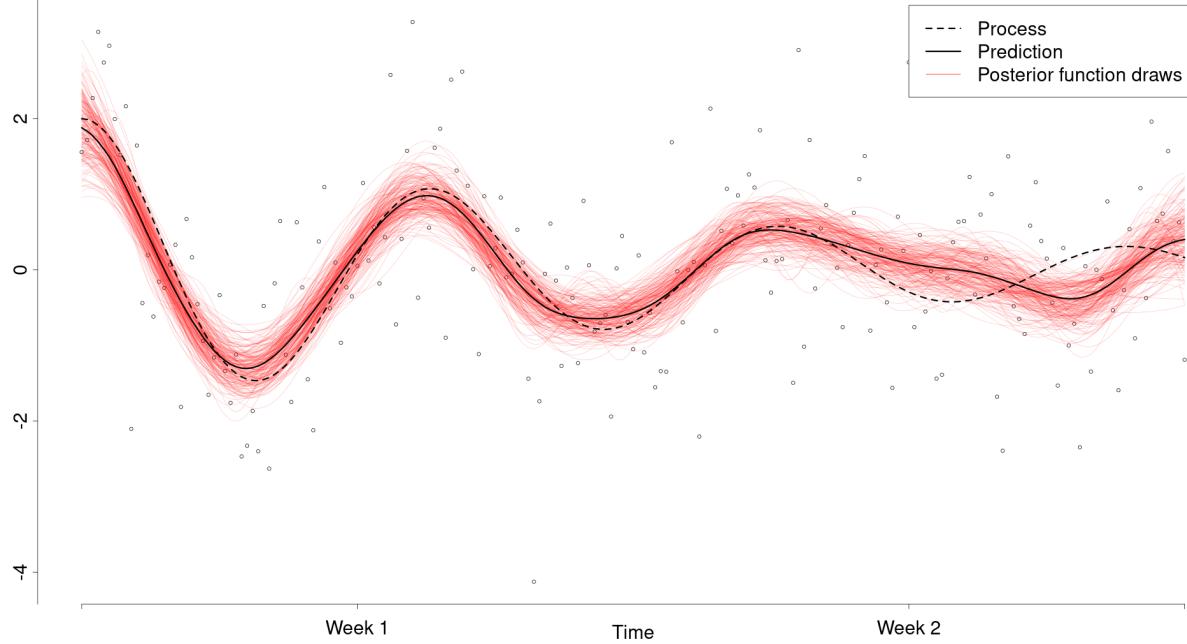
Lastly, it is noteworthy, that unless the underlying process follows a polynomial trajectory of at most degree p the approximation with local polynomials is biased. For instance, whereas a process that follows a quadratic trajectory can be accurately inferred at any point in time by a local quadratic regression (and all higher order LPRs), a process that follows an exponential trajectory cannot be inferred with perfect accuracy by any LPR with a finite degree. Instead, there will be a small bias in the estimate, that decreases for larger local polynomial degrees. However, this bias is usually negligible and there are methods available to correct for it (Calonico et al., 2019).

1.2 Gaussian process regression

The second non-parametric technique is Gaussian process (GP) regression, a Bayesian approach that directly defines a probability distribution over an entire family of non-linear functions, which is flexible enough to capture many complex processes effectively (Betancourt, 2020; Rasmussen & Williams, 2006; Roberts et al., 2013). Unlike regular probability distributions (e.g., normal distribution) that specify the likelihood of single values, Gaussian processes determine how likely entire (non-linear) functions are. A GP is defined indirectly, such that, if the functions it describes are evaluated at any finite set of time points, the resulting sample of function values would follow a multivariate normal distribution. In a Bayesian framework, one can use a GP to define a prior distribution for the latent process, as $P(f) \sim GP$. This prior is then combined with an appropriate likelihood for the observed data to obtain a posterior distribution for the latent process given the observed data.

Figure 3

Demonstration of a Gaussian process regression



Note. This figure shows how a Gaussian process regression estimates the underlying process (dotted black). Here, a sample of functions drawn from the posterior Gaussian process probability distribution with a squared exponential kernel is shown (red). The predicted value for the underlying process is then obtained by averaging the drawn functions.

$$P(f | \mathbf{y}) \propto P(\mathbf{y} | f)P(f) \quad (3)$$

This posterior distribution represents an updated belief about which functions describe the latent process well (Kruschke, 2011), allowing one to draw inferences about the process. Figure 3 illustrates such a posterior distribution for the running example process. The red lines represent a sample of non-linear functions drawn from the posterior distribution, such that the pointwise average of these functions provides a mean estimate for the underlying process.

The GP prior is parameterized by a mean function $m(t)$ and a covariance function $cov(t, t)$, which are continuous extensions of the mean vector and covariance matrix of a

multivariate normal distribution. These functions can be selected based on domain knowledge or through data-driven model selection (Abdessalem et al., 2017; Richardson et al., 2017). In practice, the mean function is often set to zero when no specific prior knowledge is available. This does not constrain the posterior mean to zero but instead indicates a lack of prior information about its deviations from zero. The covariance function is typically based on a kernel function, which assigns covariances between time points depending on their distance (e.g., quadratic exponential, Matern class kernels), such that

$$\text{cov}(t_i, t_j) = k(|t_i - t_j|) \quad (4)$$

Lastly, the mean and covariance functions also include parameters, called hyperparameters, which can be estimated by assigning appropriate priors to them. Most common covariance kernels build on a characteristic lengthscale and a marginal standard deviation parameter. The characteristic lengthscale effectively determines the wiggleness of the estimated process, by quantifying how quickly the covariance decreases with increasing distances between time points. The marginal standard deviation describes the spread of the functions which are described by the GP at any point in time. However, GP regression can also include additional hyperparameters, allowing for more specific theories to be tested through model comparison. Most standard software provides general default GP priors. However, it is also possible to choose or construct a specific GP prior to fit a specific context or research objective.

Whereas LPR is a mainly data driven procedure GP regression is more model based. The GP prior generates a family of functions to which the process is assumed to belong. This makes it possible to include theoretical domain knowledge and specific hypotheses of interest in the kinds of function that are modelled by a GP. If one for example expects that a non-linear process has a general upwards trend, it would be possible to model a linear mean function with non-linear deviations that are captured by a GP. However, this also means that to accurately capture a process, it is important that the functional family generated by a GP is similar to the actual process. Most common choices for covariance kernels for example result in smooth and

covariance-stationary GPs with constant wigginess. Another difference between GP regression and LPR is that the Bayesian estimation underlying the GP regression provides a natural approach to uncertainty quantification. While there is no direct way of quantifying the uncertainty associated with the bandwidth of the LPR, the uncertainty of the lengthscale of the GP is captured in its posterior distribution.

1.3 Generalized additive models

Generalized additive models (GAM) are a class of semi-parametric models that build on so called smooth terms, which are non-linear functions that are inferred from the data through smoothing splines (Hastie & Tibshirani, 1999; Wood, 2006, 2020). Smoothing splines extend regular spline regression to address the knot placement problem. Instead of placing knots at meaningful locations, smoothing splines use a very large number of knots to guarantee overfitting the data. One approach to this is to use cubic splines, which are piecewise cubic polynomials. By placing a knot at each data point these splines perfectly interpolate the data. Alternatively, thin-plate splines can be used, which entirely avoid knots and instead utilize increasingly wiggly basis functions that are defined over the entire range of the data (Wood, 2003). These basis functions are then combined in a regression model similar to polynomial regression (i.e., each basis function functions as predictors in a regression equation). Using as many basis functions as data points is analogous to placing a knot at each data point. To prevent the resulting overfitting, smoothing splines add an additional penalty term, similar to those used in a lasso or ridge regression, to control how closely the smooth term fits the data during the estimation (Gu, 2013; Wahba, 1980). This penalty balances the complexity and fit of the smooth term, ensuring the model captures the underlying process accurately. The optimal weight of the penalty is typically determined by minimizing a criterion function, such as the generalized cross-validation criterion or thorough likelihood maximization, which is performed automatically in standard software (Golub & von Matt, 1997; Wood, 2006).

A smoothing spline for a single smooth term β may then be written as

$$\begin{aligned}\hat{\beta}(t) &= \underset{\alpha}{\operatorname{argmin}} P(\mathbf{y} | \beta(t)) + \lambda \int (\beta(t)')^2 dt \\ \beta(t) &= \sum_{k=1}^K \alpha_k R_k(t)\end{aligned}\tag{5}$$

where the first part of the equation describes the likelihood of the data given the smooth term and the second part of the equation corresponds to the penalty term. This illustrates, how the smoothing spline balances data fit, in the form of the likelihood, and the complexity or wiggleness of the estimate, measured by the overall squared curvature of the curve. Here, λ denotes the weight assigned to the penalty term that is optimized over. Lastly, the smoothing spline is comprised of spline basis functions $R_k(t)$, such as the introduced cubic or thin-plate splines and their respective regression coefficients α_k .

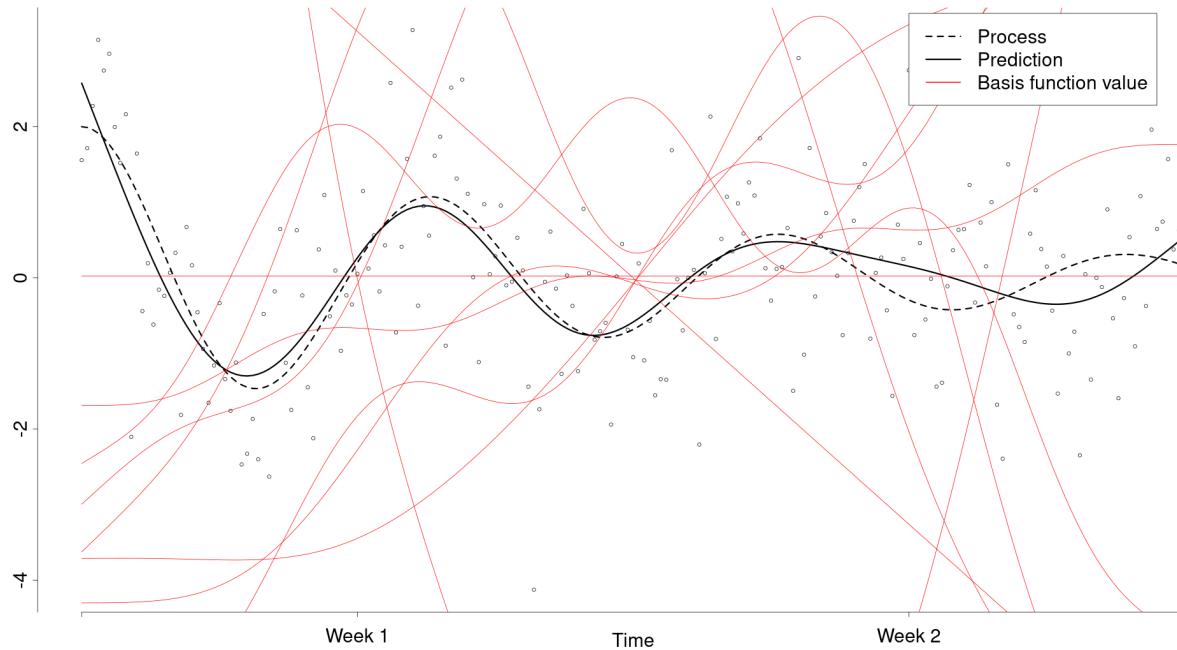
GAMs extend on the smoothing spline approach by making it possible to combine multiple smooth terms (of potentially different input variables) in an overarching additive regression model, where each smooth term essentially functions as a predictor within a regular regression analysis. This approach makes it possible to formulate models such as a time-varying autoregressive model, where the intercept and autoregressive parameters are smooth terms of time (Bringmann et al., 2015; Bringmann et al., 2017). By integrating non-parametric smooth terms into a broader parametric model, GAMs become semi-parametric models that are well-suited for testing specific hypotheses while keeping the flexibility needed to accurately capture the latent process. Figure 4 illustrates a simple GAM construction with a single smooth term for time, fitted to the example process. The first ten thin-plate smoothing spline basis functions of the nearly 200 basis functions that make up the smooth term are shown in red.

Similar to GPs, GAMs constitute a more model based approach to inferring non-linearity. Indeed, GAMs and GP regression are closely related techniques, as each GAM can be understood as a (unconventional) GP with a partially improper prior². These improper priors make the GAMs

² Any basis function regression with Gaussian priors for the coefficients is a GP in weight space. The smoothing spline underlying the GAMs is such a basis function regression in which the coefficient priors serve the same purpose as the penalty term. Due to the null space of the penalty term, some of these priors are generally going to be improper.

Figure 4

Demonstration of the construction of a GAM



Note. This figure shows how generalized additive models (solid black) estimate the underlying process (dotted black). Here, the predicted values for the process at any point in time correspond to the weighted average of the basis functions (red).

more flexible to model processes, which might be outside the functional family generated by a standard GP construction (Wahba, 1978). In addition to this and similar to GPs, GAMs offer an intuitive way to combine partial theories with data driven smooth terms. In this way, GAMs can also, for instance, model a linear function with non-linear deviations, which are captured by a smooth term. GAMs also provide an estimate of the wiggliness of the process through the effective degrees of freedom of the model. These are a measure of the models effective complexity, where an effective degree of freedom of one corresponds to a linear model. However, in contrast to LPR and the standard GP construction, GAMs do not assume constant wiggliness. Unfortunately, GAMs do not provide a measure of the uncertainty of the effective degrees of freedom (EDF). Lastly, it is possible to interpret the basis function coefficients in a GAM. The interpretation of these coefficients does however depend on the chosen basis functions and on the

other terms in the model. Table 1 summarizes the similarities and differences between the three introduced methods.

2 Simulation

2.1 Problem

A simulation study was conducted to assess the effectiveness of the introduced methods in recovering different non-linear processes, which may be encountered in EMA research (Figure 1). In this simulation, the three methods were not only compared against each other, but also to a polynomial regression model (which is the current most used method to model non-linear trends in psychology) and to parametric models that accurately specify the non-linear process. These (data generating) parametric models were added to serve as a benchmark for non-linear process recovery. To apply the introduced methods under the conditions described in the introduction, and within the constraints of available software implementations, the simulation focused on a univariate single-subject design. Hence, the simulated data represented repeated measurements of a single variable for one individual.

2.2 Design and Hypotheses

To conduct the simulation with processes that might be encountered in real EMA studies, we selected the exemplar processes illustrated in Figure 1 as a basis. These include two growth curves, modeled as an exponential and a logistic growth curve, a mean-level switching process, modeled as a cusp catastrophe, and a self-regulatory process, represented by a damped oscillator. For each of these processes, we varied the sampling period (i.e., how long each process is measured), the sampling frequency (i.e., how often each process is measured), and the smoothness of the process. Afterwards, each of the introduced non-parametric methods, as well as a polynomial regression and parametric models were used infer the underlying processes from the data.

For the LPR and the GP, we expect that both method, using the configurations in which

Table 1*A comparison of LPR, GP regression and GAMs*

	LPR	GP	GAM
Advantages	<ul style="list-style-type: none"> • Intuitive theory • Completely data driven 	<ul style="list-style-type: none"> • Most interpretable parameters • Natural uncertainty quantification • Flexible modelling framework can incorporate prior theory 	<ul style="list-style-type: none"> • Intuitive theory • Some interpretable parameters • Flexible modelling framework can incorporate prior theory
Disadvantages	<ul style="list-style-type: none"> • Least interpretable parameters • Biased for most processes • No uncertainty estimate for the wigginess 	<ul style="list-style-type: none"> • Unintuitive theory • Difficult to specify in practice 	<ul style="list-style-type: none"> • No uncertainty estimate for the wigginess
Key assumptions	<ul style="list-style-type: none"> • P-times differentiable process • Constant wigginess 	<ul style="list-style-type: none"> • Assumptions depend on chosen specifications 	<ul style="list-style-type: none"> • Smooth process • Homoscedasticity
Estimation	<ul style="list-style-type: none"> • OLS 	<ul style="list-style-type: none"> • Bayesian 	<ul style="list-style-type: none"> • OLS, MLE, Bayesian
Key sources of information	<ul style="list-style-type: none"> • Fan and Gijbels (2018) 	<ul style="list-style-type: none"> • Rasmussen and Williams (2006) 	<ul style="list-style-type: none"> • Wood (2006)

they are most often applied and implemented in standard software, will struggle to infer processes that are not continuous (i.e., with sudden jumps), have varying wiggleness, and are not smooth (i.e., differentiable). We expect this, because both methods by default produce continuous, smooth estimates with a single constant bandwidth or lengthscale. For the GAMs, we expect that only the continuity and smoothness of the process will influence the performance, since GAMs do not assume constant wiggleness. The parametric modeling approach is expected to provide the most accurate inferences, serving as a benchmark for comparison with the other methods.

First, we hypothesize that the cusp catastrophe model, which is the only process featuring jumps, will be least accurately inferred by all methods. Second, all four processes exhibit changes in wiggleness (as defined by each of the non-parametric methods respectively) over time. However, while the wiggleness of the exponential and logistic growth functions and the damped oscillator decreases monotonically, the cusp catastrophe's wiggleness changes cyclically (i.e., low wiggleness during the plateau phases and very high wiggleness during the jumps). Therefore, we hypothesize that longer sampling periods for the exponential and logistic growth curves and the damped oscillator will reduce the inference accuracy of the LPR and the GP, as the single bandwidth or lengthscale parameter becomes increasingly inadequate to capture the changing wiggleness over time. We do not expect this effect to occur for the cusp catastrophe process, or when using GAMs.

To manipulate the smoothness of the processes a dynamic error component was added to each process. These dynamic errors reflect external perturbations to the latent construct that are carried forward in time. For instance, if a participant experiences an unusually pleasant conversation that elevates their true positive affect, this change represents an error effect if it is not accounted for by the model. However, since the true positive affect level has increased, this will influence future measurements due to for example emotional inertia. To add these errors, each process was perturbed by a normally distributed error at each point in time, resulting in non-smooth (i.e., non-differentiable or rough) trajectories. The degree of roughness was controlled by the variance of these dynamic errors and we considered variances of 0.5, 1, and 2

Figure 5

Non-linear exemplar processes with dynamic errors



Note. This figure shows one possible realization of how the exemplar processes could unfold with dynamic errors. The processes have dynamic error standard deviations of 0.25 (left) and 0.5 (right).

reasonable relative to the process range. Figure 5 illustrates one possible realization of the exemplar processes with dynamic errors. Importantly, we intentionally omitted a condition without dynamic noise from this simulation, as dynamic noise is reasonably expected to be present in all psychological intensive longitudinal data (ILD).

Additionally, the sample size was varied during the simulation by manipulating both the sampling period and the sampling frequency, as these distinct methodological choices are expected to impact the performance of the analysis methods differently. Specifically, for the LPR and the GP, which rely only on data in local neighborhoods during the estimation, we expected that extending the sampling period beyond this neighborhood will not increase the inference accuracy. In fact, if the process exhibits changing wiggleness over the extended period, as previously discussed, increasing the sampling period might even negatively affect the inference

accuracy. In contrast to this, we expected the GAMs, which incorporate the entire dataset in their estimation, to perform better with a longer sampling period. Since within the simulation there is no inherent scaling to the time variable, we simulated either only the first half of each process or the entire process to represent different sampling periods. For the ease of reading and to correspond to typical EMA conditions, this will be referred to as sampling over either one or two weeks. However, this scaling is arbitrary and could be changed to any other time frame. Lastly, we expected that increasing the sampling frequency will generally improve the inference accuracy across all methods, since there is more information about the latent process available. Relative to the introduced weekly scale, we tested sampling frequencies of three, six, and nine measurements per day, to cover typical EMA sample sizes (Wrzus & Neubauer, 2023).

2.3 Outcome measures

To evaluate and compare the performance of the different analysis methods, we focused on three outcome measures. The first two assessed each method's accuracy in predicting the process values at or between the observed time points. These predictive accuracy measures indicate how well each method captures the underlying non-linear process. The third outcome measure evaluated the accuracy of the uncertainty estimates provided by each method. Specifically, whether the confidence or credible intervals produced by each method correctly included the true state value the expected proportion of times.

2.3.1 *Capturing the non-linear process*

To assess how effectively each method captured the non-linear process at the observed time points, we calculated the mean squared error (MSE) between the estimated and generated process values. Additionally, to evaluate how well each method would predict omitted process values within the process range, we computed the generalized cross-validation (GCV; Golub et al., 1979) criterion for each method and data set. The GCV is a more computationally efficient and rotation-invariant version of the ordinary leave-one-out cross-validation criterion, with the same interpretation. Both, the GCV and the leave-one-out cross-validation criterion, describe how

accurately the model predicts omitted data points within the design range, which provides information about how well the model interpolates the process.

The results of the mean MSE and GCV values in each condition will be presented in figures. Additionally, to efficiently summarize the high dimensional results and formalize the analysis, two separate ANOVAs were fit to the MSE and GCV values. Both ANOVAs included all possible main and interaction effects of the data generating processes, the analysis methods, and simulation conditions (i.e., sampling period, frequency, and dynamic error variance). Although, the parametric models were excluded from the ANOVAs as they were expected to outperform and hinder the comparison of the other methods. Since even very small effects can lead to statistical significance at the large sample sizes considered in this simulation, we focussed on the effects that showed at least a small effect size according to the partial η^2 (> 0.01). The assumptions of the ANOVAs were tested. However, given the large sample sizes in this simulation, the ANOVAs were assumed to be robust to moderate violations of normality (Blanca et al., 2017), and any potential violations of homoscedasticity were addressed using heteroscedasticity-consistent standard errors.

2.3.2 *Uncertainty quantification*

To evaluate the uncertainty estimates provided by each method, we recorded whether the true generated process was located within the confidence or credible intervals at each time point. Subsequently, the average confidence interval coverage proportion for each method and data set was obtained, by averaging over all time points. Given that all confidence or credible intervals were set at a 95% confidence level, the expected coverage proportion should ideally be close to 95%. Due to Monte Carlo error in the simulation ($\max(se_{CIC}) \approx 0.03$) individual average confidence interval coverages are expected to deviate from the ideal 95%. Because of this, average coverage proportions between 89% and 100% were also deemed acceptable. Average coverage proportions below 89% indicated either a poor approximation of the underlying process or underestimated uncertainty.

2.4 Data generation

Each process was represented as a generative stochastic differential equation model. These dynamic models describe the relationship between the process's current value and its instantaneous rate of change. This can be combined with information about the initial state of the process to infer the entire trajectory indirectly. For instance, the stochastic differential equation model used to represent the introduced logistic growth process can be expressed as follows:

$$dy = ry(1 - \frac{y}{k})dt + \sigma dW_t \quad (6)$$

The first half of this model defines the deterministic dynamics of the process. It relates the rate of change of y to its current value and to how far away the current value is from the asymptote k through a growth rate constant r . The second part of the model accounts for the dynamic errors in the form of a Wiener process. The Wiener process is a continuous non-differentiable stochastic process, which describes normally distributed dynamic errors over any given discrete time interval. These errors have a mean of zero and a variance depending on the length of the time interval and σ , making them optimal for this simulation. Importantly, these dynamic errors continuously influence the rate of change of the process and are propagated forward in time through the deterministic dynamics of the model.

All four processes were modeled as stochastic differential equations by substituting their respective deterministic dynamics into the first half of Equation 6, as detailed in the online supplementary material. Latent process data were simulated using the Euler-Maruyama method, which approximates stochastic differential equation systems with an arbitrarily high accuracy by linearizing them over small discrete time intervals. The resulting high-resolution data were then subsampled to achieve the desired sampling frequency. Finally, measurement errors were added to the latent process data at each time point from a standard normal distribution, generating the final sets of observations,

Based on an initial pilot sample of 30 data sets per condition, we determined the number of replications needed to achieve a monte carlo standard error of less than 0.03 for the confidence

interval coverage (Siepe et al., 2023). These Monte Carlo standard errors reflect the expected variation in the outcome statistics due to random processes within the simulation. This analysis showed that 100 replications per condition would result in maximal Monte Carlo standard errors of approximately $se_{MSE} \approx 0.05$, $se_{GCV} \approx 0.38$, and $se_{CIC} \approx 0.03$.

2.5 Model estimation

After simulating the data, all introduced methods were applied to each data set using the statistical software R (R Core Team, 2024). First, the LPR was estimated using the nprobust package (Calonico et al., 2019), which allows to correction for the bias inherent in LPRs. Second, GPs were estimated in STAN (Gabry et al., 2024) with a zero mean and a squared exponential covariance function, following common practice. Third, GAMs with a single smooth term for time were fitted using the mgcv package (Wood, 2011). The polynomial regressions were estimated using base R, with correlated (i.e., standard) polynomial terms. Finally, the parametric differential equation models corresponding to the true data-generating models were estimated using the Dynr package (Ou et al., 2019). After fitting each model to the data, they were used to obtain point and interval estimates (i.e., 95% confidence and credible intervals) for the latent process at each time point. A detailed description of each model fitting procedure is provided in the online supplementary material.

To ensure reliable model fit and reasonable inferences, the fitting procedures for each method were validated on pilot samples within each condition. After this, an initial run of the simulation was performed, which revealed that the GAMs over- or linearly underfit some data sets and that the parametric models overfit some data sets. To prevent this, the fitting procedures of both methods were adjusted and the simulation rerun. Further, if any models failed to converge during the simulation, the corresponding outcome measures were excluded from the following analyses.

2.6 Results

In the simulation some of the GP regressions and parametric models did not converge. This is most likely due to the small sample sizes considered and the automated model fitting in the simulation. Most notably, the parametric models were not able to infer the cusp catastrophe from the small simulated data sets due to the complexity of the model. The performance measures of the methods that did not converge were removed from the following analysis. Additionally, the parametric models did overfit for a small number of data sets. The complete simulation results and data are available in the online supplementary material.

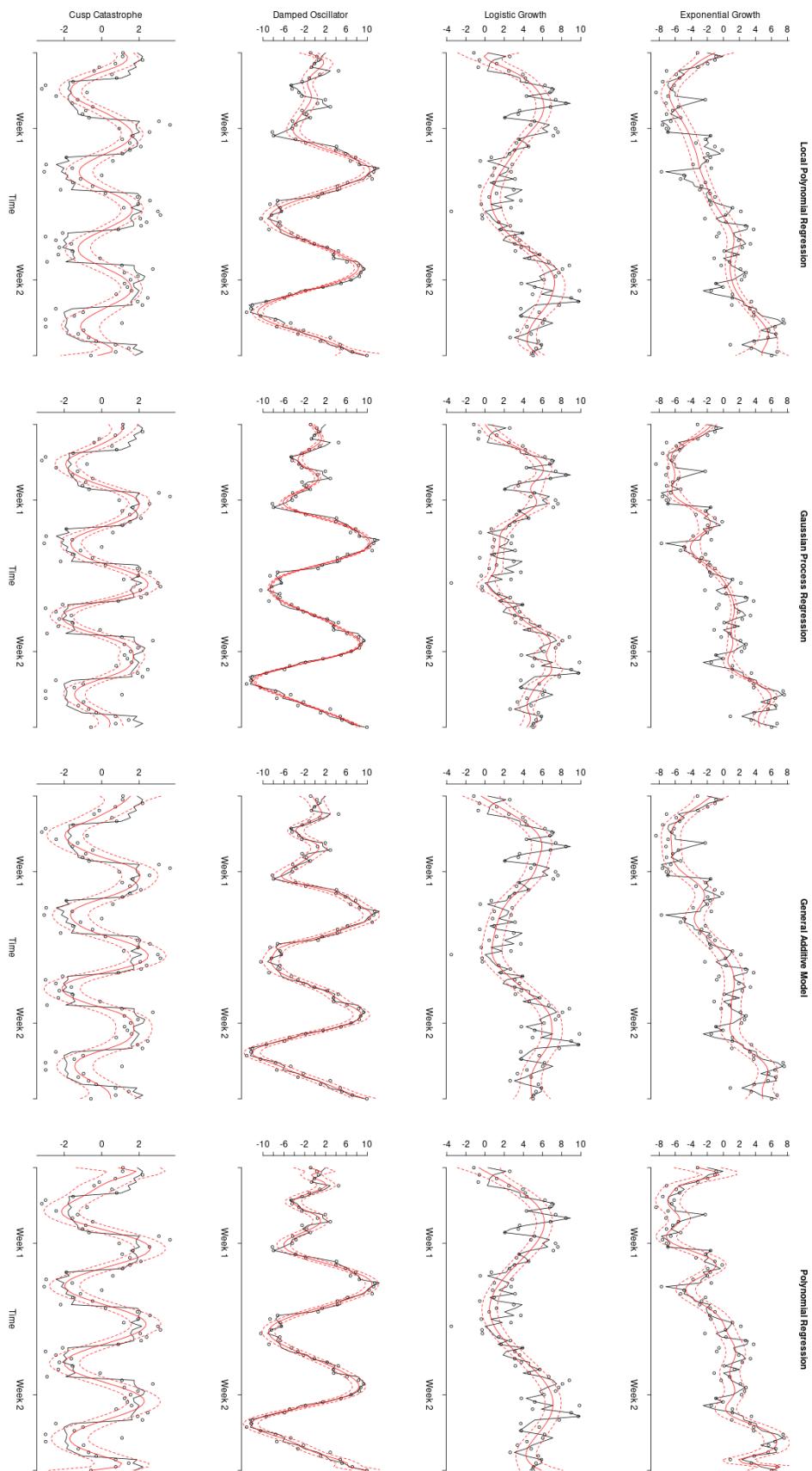
2.6.1 *Capturing the non-linear process*

First, it is noteworthy that all considered methods visually performed well in mean predicting the simulated processes. Figure 6 illustrates an example from each process being inferred by all methods. The predicted means produced by each method closely followed the simulated processes, although the LPR appears to underfit for some data sets. Additionally, near the boundaries (i.e., the ends) of the simulated time series, the GP regression sometimes tends towards zero, which is a characteristic of the squared exponential kernel (this can be seen by comparing the GP inference in Figure 3 to the estimates in Figure 4 or 2). *Further, the polynomial regression appears to overfit near the boundary, resulting in excessive uncertainty, which is a known behavior of polynomial regressions.* However, there is considerable variation and overlap in the accuracy of the different methods across the various data sets, which highlights the need for a more formal and objective analysis of the performance of each method.

To achieve this, two separate ANOVAs were fitted to the MSE and GCV values, including all possible main and interaction effects. Although the residuals for both models showed considerable deviations from normality, which were mainly characterized by being leptokurtic, the residuals were unimodal and approximately symmetric. Given the large sample sizes in this simulation, we thus assumed that the ANOVAs were robust to these deviations. Further, a Breusch-Pagan test indicated heteroscedasticity in the residuals for both outcome measures,

Figure 6

Example processes inferred by each of the introduced methods



Note. This figure shows how each of the introduced methods inferred an example of each of the processes from the simulation.

which was corrected for using heteroscedasticity-consistent standard errors. Lastly, Bonferroni corrections were applied to adjust the p-values for conducting two separate ANOVAs.

The type-III ANOVAs for both outcome measures indicated that all main and first order interaction effects were significant. Additionally, some higher order interaction terms were significant, which differed between the two outcome measures. However, due to the large sample size, even very small effects can lead to statistical significance. Therefore, Table 2 presents all effects for which the partial- η^2 , which gives the proportion of variance explained by an effect after partialling all other effects out, indicates at least a small effect size for either the MSE or the GCV. The following sections will focus on describing some of these effects and a comprehensive overview of all effects in the models can be found in the online supplementary material.

Figure 7 (a) illustrates the mean MSE values with which each method inferred each process, averaged across sampling periods, frequencies, and dynamic error variances. This figure highlights the main effect of the analysis method, as there are clear differences in the mean MSE with which each method inferred all processes. Specifically, the parametric modelling showed the lowest average MSE, followed closely by the GAMs, whereas the GP regression, LPR, and the polynomial regression had larger average MSEs. Additionally, Figure 7 (a) illustrates the main effect of the processes, as each process was inferred by all the methods with different mean MSE values. Most notably, the cusp catastrophe was inferred with lower MSE values than the other processes by all methods. Further, one can see that there is an interaction between the analysis methods and the processes, since the differences in how accurately each process was inferred differ between the methods. For example, the difference between the MSEs for the cusp catastrophe and the MSEs for the other processes is larger for the LPR than for the other considered methods. Notably, the polynomial regression displayed a larger mean MSE for all processes except the cusp catastrophe in comparison to the more advanced statistical methods.

Figure 7 (b) shows the average MSE over different measurement periods for each analysis method and process, averaged over measurement frequencies and dynamic error variances. The results indicate that sampling over the entire process, rather than just the first half, led to higher

Table 2*Effect sizes from the MSE and GCV ANOVAs*

Effect	partial- η^2 MSE	partial- η^2 GCV
Method	0.40	0.26
Process	0.55	0.41
SP	0.18	0.09
SF	0.15	0.24
DEV	0.56	0.49
Method:Process	0.23	0.13
Method:SP	0.18	0.11
Process:SP	0.07	0.05
Method:SF	0.05	0.01
Process:SF	0.01	0.05
Method:DEV	0.16	0.09
Process:DEV	0.27	0.24
SP:DEV	0.02	0.03
SF:DEV	0.01	0.05
Method:Process:SP	0.08	0.07
Method:Process:SF	0.02	0.01
Method:Process:DEV	0.07	0.05
Method:SP:DEV	0.02	0.03
Process:SP:DEV	0.01	0.01
Process:SF:DEV	0.01	0.02
Method:Process:SP:DEV	0.02	0.02

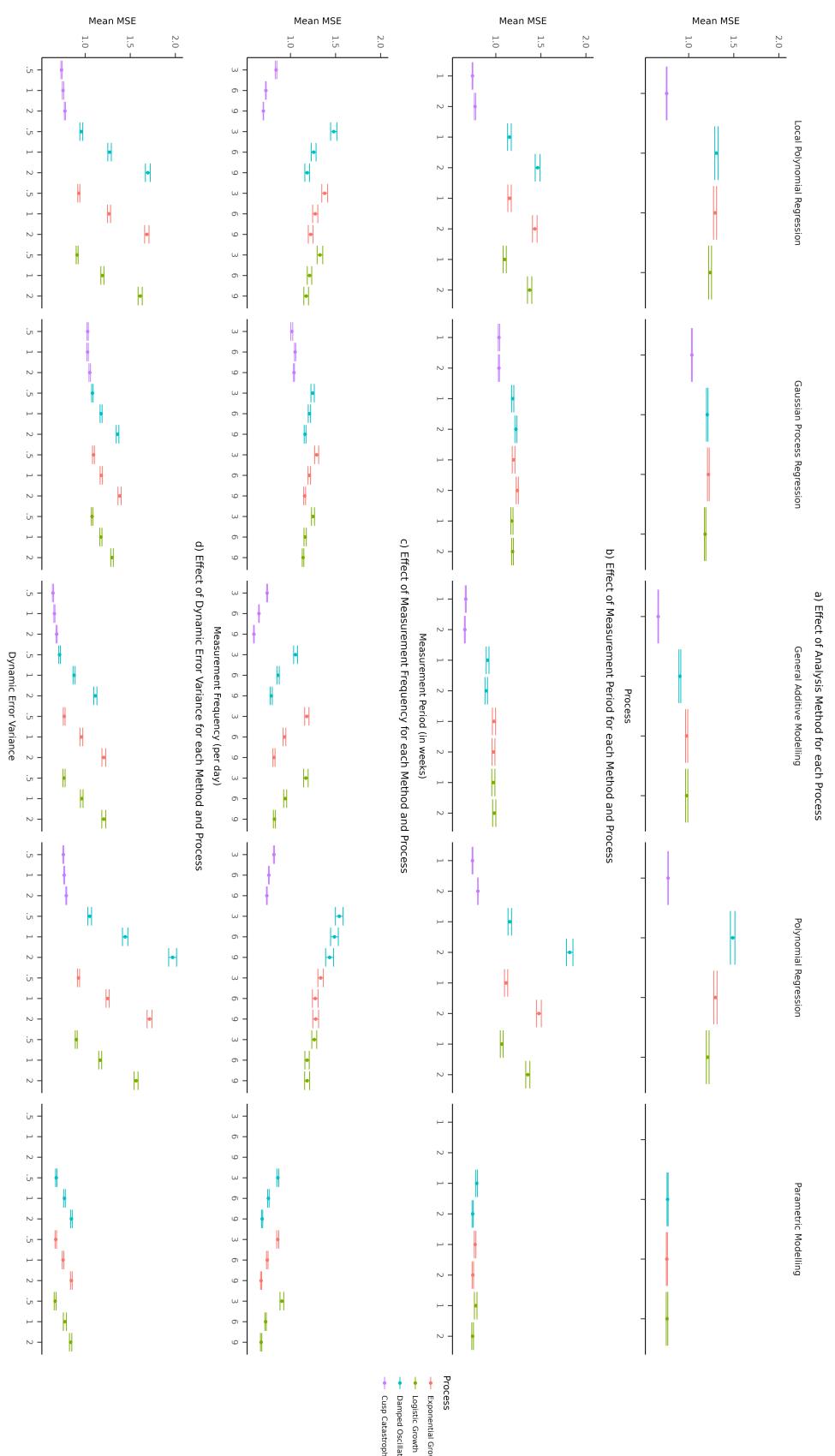
Note. This table shows all effects from the MSE and GCV ANOVA

that had at least a small effect partial- $\eta^2 \geq 0.01$ on either outcome.

SP: Sampling period; SF: Sampling frequency; DEV: Dynamic error variance.

Figure 7

Mean MSE effects across all processes, analysis methods, and simulation conditions



Note. Panel (a) shows the effect of the analysis method for each process. The other three panels show the effects of measurement period (b), measurement frequency (c), and dynamic error variance (d) for each analysis method and latent process.

average MSE values for both local and global polynomial regression across all processes. This effect was much less pronounced for the GP regression, absent for the GAMs, and reversed for the parametric models. Further, Figure 7 (c) illustrates that the mean MSE generally decreased with larger sampling frequencies for each method and process, while averaging over the sampling periods and dynamic error variances. Lastly, Figure 7 (d) demonstrates that larger dynamic error variances increased the mean MSE values across all methods and processes, when averaged over sampling periods and frequencies. However, this effect was least pronounced for the cusp catastrophe.

Figure 8 displays the corresponding effects for the mean GCV values. Similar to the MSE results, the GAMs show a mean GCV value closest to the benchmark parametric models. However, the GPs also display comparable performance to the GAMs in terms of the GCV. The local and global polynomial regressions show considerably larger mean GCV values for all processes except the cusp catastrophe. The effects of measurement period, frequency, and dynamic error variance on the mean GCV appear to follow largely the same patterns that were observed for the mean MSE.

2.6.2 *Uncertainty quantification*

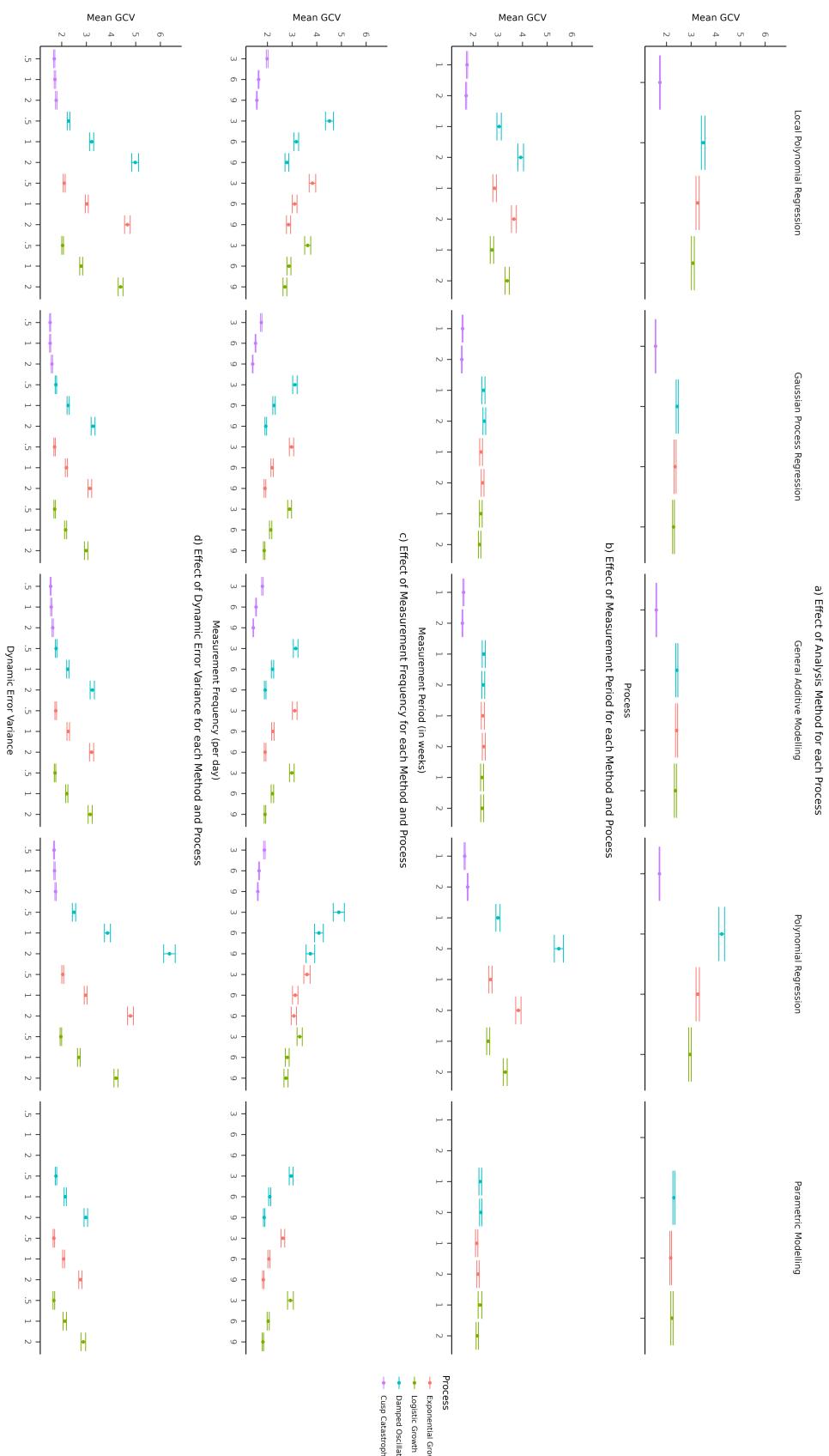
Figure 9 shows the average confidence interval coverage proportion for the conditions described above. The grey area represents an average confidence interval coverage between 89% and 100%, which indicates no considerable deviation from the ideal 95% given the Monte Carlo error of the simulation. Only the parametric models produced mean confidence interval coverages that fell within this area. Among the other methods, the GAMs, produced the largest average confidence interval coverage, followed by the global and then local polynomial regression. The GP regression appears to result in the smallest confidence interval coverage.

2.7 Conclusion

This simulation showed that among the considered methods the GAMs inferred all processes with the most accuracy as indicated by the mean MS, GCV and confidence interval

Figure 8

Mean GCV effects across all processes, analysis methods, and simulation conditions

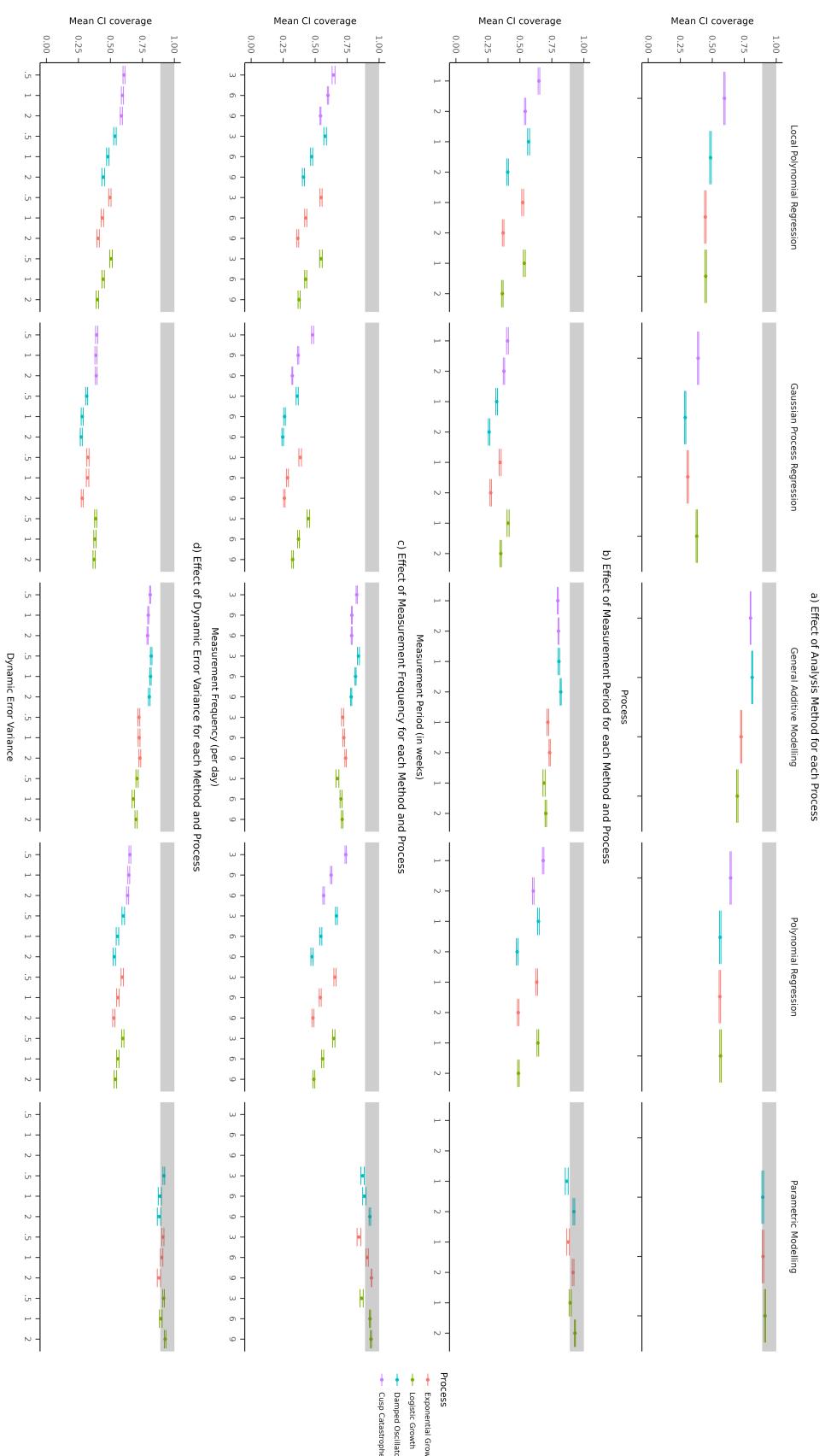


Note. Panel (a) shows the effect of the analysis methods for each latent process. The other three panels show the effects of measurement period (b), measurement frequency (c), and dynamic error variance (d) for each analysis method and latent process.

MODELLING NON-LINEAR PSYCHOLOGICAL PROCESSES

Figure 9

Average confidence interval coverage across all processes, analysis methods, and simulation conditions



Note. Panel (a) shows the effect of the analysis methods for each latent process. The other three panels show the effects of measurement period (b), measurement frequency (c), and dynamic error variance (d) for each analysis method and latent process.

coverage. The GAMs performed closest to the true data generating models, followed by the GP (with regards to the MSE and GCV, since the GP had the lost confidence interval coverage), and then the local and global polynomial regression. This result is unexpected, as we anticipated that the smooth inferences produced by GAMs may be ill-suited for inferring the rough (i.e., non-differentiable) processes in the simulation. However, as Figure 6 shows, all methods except the parametric models produced smooth inferences. This suggests that the additional flexibility that the GAMs provide over the LPR, by not assuming constant wigginess, and over the GP, by effectively relying on improper priors, enabled the GAMs to infer the processes more accurately in the simulation. From this follows, that GAMs are an attractive starting point for modelling unknown processes in practice. Especially, when there is little prior theory about the functional form of the process.

However, it is important to note that the observed results may be most attributable to the specific configurations that were used rather than the general modelling approaches underlying each method. The specific configurations of each method were chosen to reflect how each method is most commonly applied in practice and not to optimally infer the simulated processes. Consequently, different configurations and extensions are likely to improve the performance of the LPR and GP. Especially, since the GAMs used in this simulation correspond to a GP with a linear mean function (with improper priors) and a non-stationary covariance function (Rasmussen & Williams, 2006; Wahba, 1978). This implies that there exists a GP configuration which performs at least as well as the GAMs. While such a configuration would heavily deviate from how GPs are most commonly applied, common changes like using a Matern class kernel, which makes less strict smoothness assumption about the data, could also be expected to improve the accuracy of the GPs. Therefore, if there is already some prior knowledge about the form of the process available, GPs are still a very interesting modelling approach in practice, as they yield more interpretable inferences. However, this likely still requires fine tuning the configurations of the GP to one's specific conditions.

Regarding the LPR, several optimality results have been found indicating that LPR should

be at least as accurate as GAMs and GPs for p -times differentiable processes (Fan et al., 1997). It is however unclear whether these results generalize to the non-differentiable processes that we suspect can be found most often in psychology. Lastly, the tested polynomial regression inferred the underlying processes least accurately. This appeared to be due to numerical model instability that resulted in underfitting, since not enough higher-order terms could be included in the models, and due to divergence towards the ends of the timeseries. The numerical model instability can be mitigated by using orthogonal polynomials, which are however more difficult to interpret. Generally, there appears to be no reason to use a global polynomial regression instead of a GAM, in any situation in which prior theory does not strongly suggest that the process follows a low order polynomial trajectory.

Contrary to our expectation, the simulation indicated that the cusp catastrophe process was inferred most accurately by all methods. We had anticipated that the smooth, continuous estimates produced by these methods would struggle to adapt to the apparent jumps exhibited by this process. However, this effect seems to have been overshadowed by the cusp catastrophes strong resilience to external perturbations. This property is highlighted in Figure 5, where dynamic errors with the same variance have been applied to all four processes. Despite the perturbations being of equal variance, the cusp-catastrophe model appears to be the least affected and even closely resembles the unperturbed process (Figure 1). Further evidence of this can be seen in the simulation, where the effect of increasing the dynamic error variance was weakest for the cusp process. Due to this, the simulation was rerun with considerably smaller dynamic error variances, and under these conditions, the cusp model was indeed inferred with the least accuracy.

The results further indicate that measuring at a higher frequency increased the inference accuracy of all considered methods. Therefore, it is generally advantageous from a statistical point of view to measure as frequently as possible. However, in practice, this must be balanced against considerations such as participant burden and fatigue, which can adversely affect data quality if measurements are taken too often. Similarly, when selecting the sampling period, it is essential to use domain knowledge about the scale of the underlying dynamics to ensure that the

measurements capture sufficient variation in the latent process. Beyond this, the simulation showed that extending the sampling period improved the inference accuracy of the parametric models but may decrease the accuracy of the LPR and the polynomial regression. This reduction in accuracy could however possibly be mitigated by using extensions for a variable bandwidth, or polynomial degree in an LPR. Lastly, the simulation revealed that larger dynamic error variances decreased the accuracy of all methods. Therefore, reducing the magnitude of dynamic errors is advisable in practice. This could, for example, be achieved by measuring context variables and other sources of perturbations and incorporating them into the model.

3 An Empirical Example

In the following, we applied GAMs to depression data from the Leuven clinical study, which were obtained from the EMOTE database (Kalokerinos et al., n.d.). We selected the data for their heterogeneous sample, which contains momentary depression scores of participants who met the DSM criteria for mood disorders or borderline personality disorder during an intake, as well as depression scores for healthy controls. For a more thorough sample and data description, including screening protocols, see Heininga et al. (2019). The diversity in the study population makes it likely to find non-linear processes for at least some participants. For example, emotional inertia, the extent to which an emotional state carries over across time points, has been shown to vary within individuals over time (Koval & Kuppens, 2012), which makes it likely that the processes underlying these data are non-stationary and thus non-linear.

The dataset used for this application contained 77 participants in the clinical sample and 40 participants in the control sample, who were matched to the clinical sample by gender and age, resulting in a total sample size of 117³. The participants completed seven days of semi-random EMA assessments, with 10 equidistant assessments per day. Note that the starting date of the EMA measures varied between people. During each assessment, participants responded to the

³ The original published data set contained one additional participant who was removed for this analysis since they had a depression score of zero across all assessments.

item ‘How depressed do you feel at the moment?’ on a scale ranging from 0 to 100 to assess their momentary depression⁴.

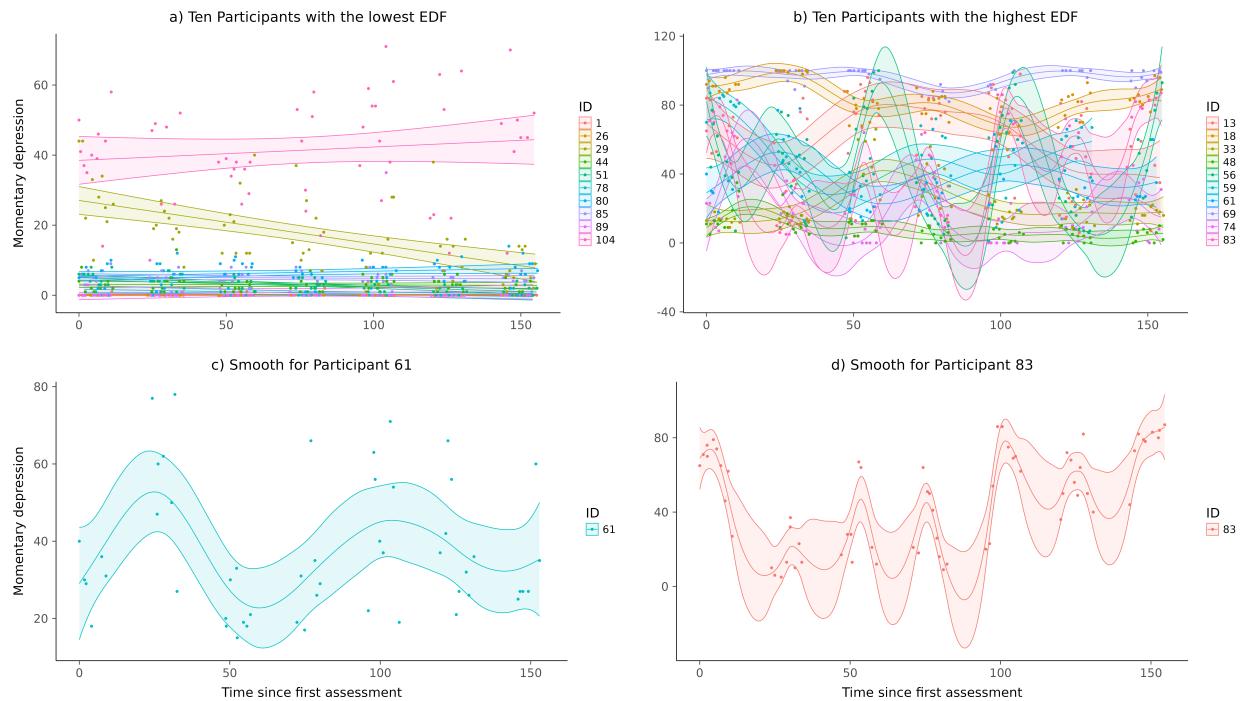
The initial study procedure was approved by the KU Leuven Social and Societal Ethics Committee and the KU Leuven Medical Ethics Committee. This secondary data analysis was approved by the Ethics Review Board of the Tilburg School of Social and Behavioral Sciences (TSB RP FT16).

3.1 Analysis and Results

To correspond to how the GAMs were introduced in this article, we applied them to explore the idiographic processes underlying the data of each individual. Each GAM was fit using the same specifications as in the simulation, with a single smooth term for time. Inspecting the estimated processes revealed a clear picture of the heterogeneity in the processes underlying the data. On one hand, Figure 10 (a) shows the ten estimated processes with the lowest EDF, which appear to be essentially linear. Indeed, for 72 out of 117 participants, the process inferred by the idiographic GAMs is effectively a linear trend ($EDF < 1.001$). For some of these participants, the linear estimates appear to be due to strong floor effects where participants repeatedly indicate depression scores close to zero. However, this explanation does not hold for all participants, as some participants also received linear estimates without displaying floor effects. Interestingly, the linear estimates also indicate an effective absence of dynamic errors, which would be expected to result in deviations from the linear trend, for these participants.

Figure 10 (b) shows the ten participants with the wiggliest estimated processes, indicated by the largest EDF. For these participants, the estimated processes clearly deviates from a linear trend, which may either be due to the presence of non-linearity or the presence of stronger dynamic errors for these participants. Unfortunately, it is not possible to distinguish which of

⁴ Note that this was only one of 27 items that were assessed at every measurement occasion. The other items (questions about emotions, social expectancies, emotion regulation, context, and psychiatric symptoms) are, however, not relevant to this application and, therefore, not further described.

Figure 10*GAM smooths*

these possibilities is correct with this analysis. Additionally, visual inspection of the non-linearly estimated processes, did not reveal any common behaviours across the processes of different participants. Nevertheless, the idiographic processes still displayed interesting behaviours. With respect to the processes introduced in this article, two participants show particularly interesting trajectories. First, Figure 10 (c) shows an estimated process that closely resembles the decreasing oscillations exhibited by a damped oscillator. Second, the estimated process of another participant illustrated in Figure 10 (d) could be seen as switching between a state low and a state of high depression, with considerable oscillations within each state.

3.2 Conclusion

4 Discussion

This results presented in this article have several important limitations. Since, only a limited selection of processes was used it is possible that the presented results may not generalize to other processes. However, the used processes already constitute violations to the smoothness assumption made by the LPR, GP, and GAM to which these methods demonstrated some robustness. In addition to this, as explained earlier, using different configurations for the LPR, GP, and GAM is likely going to change their performances. Further, we focused on introducing these methods by inferring time dependent non-linear processes from univariate, single subject data with independent normally distributed measurement errors. This is a statistically idealized setting, which does not address many of the goals and challenges researchers are facing when working with ILD in practice. Frequently, ILD contains measurements for many individuals on several psychological constructs. Classically, this enables researchers to study how these constructs vary and interact over time and how these dynamics differ between people. In addition to that, each construct is frequently measured using multiple indicators with (ordinal) measurement errors, which need to be modelled using different psychometric models (e.g., factor models, item response models).

There are fortunately many ways in which the presented methods could be adapted to these more complex data structures. The GP and GAM can be naturally adapted to incorporate multilevel data without substantially extending the statistical theory underlying both methods. For the GAM this extension is already implemented in some software. Another approach may be to study between person differences in the latent processes using functional data analysis. In this analysis the individual latent processes are first estimated using one of the presented data driven techniques. Subsequently, the inferred processes are treated as function valued data, which can be analyzed to find for example group differences in a functional ANOVA (Kaufman & Sain, 2010) or to find the functions which account for the maximum between person variation in a functional principal component analysis (Aue et al., 2015).

Similarly, the GP and by extension the GAMs can model latent variables underlying for

example a factor model, which naturally extends these methods to a setting with multiple indicator variables. This raises the possibility of extending these models to typical psychometric measurement models, to accurately capture more complex measurement error distributions. However, it is unclear how well these extensions work in practice, beyond the Gaussian process factor models, which are already implemented in software (Clark & Wells, 2023). The parametric models introduced already include a factor model for the observed variables, which can incorporate multiple indicators, which may also be non-normally distributed.

Lastly, there are many ways in which the presented methods can be used to study multivariate data. This is because, even though all methods were used to infer time dependent non-linear processes in this paper, they can in theory be used to estimate many smooth and continuous functions (and even non-smooth and non-continuous functions to the degree presented in this paper). This makes it possible to for example infer non-linear cross- and autoregressive relationships from data in discrete time (Eleftheriadis et al., 2017; Rasmussen & Williams, 2006; Wood, 2006) and non-linear differential equation models in continuous time (Yildiz et al., 2018). For the presented processes in particular this should even be more appropriate, since they were generated using autoregressive relations. These models also present the exciting possibility to combine partial parametric models with non-linear data driven functions, estimated by the presented methods. Lastly, the presented methods could be used to infer unobserved input variables to parametric dynamic models (Álvarez et al., 2009; Nayek et al., 2019), which could for example take the form of seasonal and cyclic influences to indicator variables (Clark & Wells, 2023). While these extensions and possibilities exist in theory and are in isolation implemented in software, most are currently not available and implemented in a form which would allow applied researchers to flexibly adapt these methods to the characteristics of specific ILD. Therefore it is important that future research, combines and extends the presented methods and implements these extensions in software that makes them accessible for applied modelling.

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