Non-linear Methods for Intensive Longitudinal Data

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Abstract

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Psychological constructs are increasingly understood as being part of dynamic systems (Gelfand & Engelhart, 2012; Nesselroade & Ram, 2004; Wang et al., 2012). As such, they naturally vary not only between but also within individuals. Thus, researchers are becoming more aware of the importance of studying how these constructs change over time to gain valuable insights into the underlying processes (Molenaar, 2004). This has led to a sharp increase in experience sampling (ESM) studies, in which individuals complete short questionnaires assessing psychological constructs over a longer period of time (Miller, 2012; Scollon et al., 2003). The resulting intensive longitudinal data (ILD) is then used to study how psychological constructs change over time within individuals in their natural context and how these dynamics vary between individuals.

Many of the phenomena that are interesting to study using ESM follow non-linear trajectories through time. Typical examples of this are second language acquisition (Hohenberger & Peltzer-Karpf, 2009; Murakami, 2016; Reid, 2019), the onset (Nelson et al., 2017; Van De Leemput et al., 2014), and treatment of psychopathologies (Hayes et al., 2007; Hosenfeld et al., 2015; Schiepek, 2009), attitude change (van der Maas et al., 2003), mood instability in bipolar disorder (Bonsall et al., 2012), and flow (Ceja & Navarro, 2009; Ceja & Navarro, 2012). Various methods for modeling non-linearity in time series exist, covering the entire spectrum from completely data-driven exploratory tools to confirmatory modeling and theory-testing frameworks. However, research extensively examining and comparing these methods in the context of ILD is scarce. Since many of these methods can capture different types of non-linearity, rely on different sets of assumptions, and allow researchers to draw distinct kinds of inference, it is oftentimes unclear which method is most suited in a specific situation. Because of this, in the following, we will review common non-linear time series methods, discuss the specific types of non-linearity that are described by each method, and show how these methods may be applied to ILD.

In ILD, researchers are usually measuring at least one construct over time, using

imperfect indicators, typically in the form of questionnaires. Hence, it is assumed that the observations, which are the specific values that the indicators take at a given time point, are caused by the underlying state or values of the construct (McNeish et al., 2021; Vogelsmeier et al., 2023). The difference between the observations and the state constitutes the measurement error. Further, it is important to note that this error is qualitatively distinct from dynamic error, which relates to disturbances or innovations to the state and carries forward into subsequent time points (Schuurman et al., 2015).

When dealing with such data, researchers typically aim to infer the system's state from the observations and examine the process of how the state evolves over time. This process can be described using a global model that establishes a relationship between time and the state values or a local model that links the extent of the state's change to its present value. Local or dynamic models take the form of difference equations in discrete time (Durbin & Koopman, 2012; Galor, 2007) and differential equations in continuous time (Van Montfort et al., 2018). Lastly, non-parametric methods can be used to approximate these models in a data-driven way, whereas semi- or fully parametric models make it possible to test specific theories about the data.

Kernel Regression

Kernel regression is a non-parametric regression technique that can be used to approximate non-linear functions. In time series analysis, this is used to infer a global function relating the underlying state to time (Tsay & Chen, 2019). This is achieved by averaging observations across time, thus removing the measurement error. However, to accommodate the fact that the state value changes over time, observations that are closer in time are given a higher weight when averaging than observations that are more distant (Nadaraya-Watson estimator; Bierens, 1994; Nadaraya, 1964; Watson, 1964). This means that the averages are calculated locally, for each point in time, with a unique set of weights. Here, the precise weight that is given to each of the observations is determined using a so-called kernel function.

Kernel functions are usually centered, symmetric probability density functions. As such, they associate larger weights to closer observations and smaller weights to more distant observations (Nadaraya, 1964). Common choices for kernel functions are the Gaussian or the symmetric Beta density. The width of the kernel function is referred to as the bandwidth. It modulates the extent by which the weights decrease as the observations get more distant. In practice, this means that the bandwidth controls the smoothness of the estimated state function. Many methods, such as mean integrated squared error minimization or generalized cross-validation, exist to determine the optimal bandwidth based on the data (Debruyne et al., 2008; Köhler et al., 2014).

One common extension of this estimator is local polynomial regression. Instead of calculating local weighted averages, the underlying non-linear regression function is here approximated using locally fitted weighted polynomial regressions of a low, odd order (Avery, n.d.; Fan & Gijbels, 2018; Fan & Gijbels, 1995; Ruppert & Wand, 1994). Alternatively, it is possible to approximate non-linear auto-regressive functions, which relate the state value at a certain time point to the previous observations. Here one may either choose to estimate the non-linear function directly, by including the lagged observations as inputs in a multivariate kernel regression (Häardie & Müller, 2000; Tsay & Chen, 2019), or to model the auto-regressive parameters in a classical autoregressive model as varying over time and approximate this function instead (Haslbeck et al., 2021; Shim & Lee, 2009).

Kernel regression is the most flexible method considered in this review and relies on the least assumptions. Hence, it can accommodate most forms of non-linearity that could be found in ILD. Since the Nadaraya-Watson estimator uses one constant bandwidth, which may be understood as a measure of wigglyness of the approximated function. This means that wigglyness has to be constant for the estimator to perform well. However, this limitation is overcome by the additional freedom that local polynomial regression provides. This additional flexibility makes it possible to approximate functions with varying degrees

of wigglyness as well as discontinuities relatively well. Considering its nonparametric nature, kernel regression is suitable for situations in which no assumptions on the form of the underlying function can be made based on theory. As a consequence, it also provides very little amount of information about the process and beyond visual inspection and provides only the bandwidth estimate as an interpretable parameter of the functional behavior.

Generalized Additive Models

Generalized additive models (GAM) choose a different approach to approximating the global non-linear state function, relying on smoothing splines (Hastie & Tibshirani, 1999; Wood, 2006, 2020). Spline methods approximate a complex function piecewise by joining simpler functions, usually low-order polynomials, together at so-called knots. For this, it is important to constrain the basis functions to fit together smoothly and to choose the knot locations optimally (Tsay & Chen, 2019). To circumvent this, smoothing splines approximate the complex function as a weighted sum of a collection of basis functions that range over the entire domain of the underlying function. In order to do this, all basis functions are evaluated at each time point, and the optimal weights for each basis function are found using penalized regression methods. The penalty term here controls the smoothness of the approximation and prevents over-fitting (Gu, 2013; Wahba, 1980). Subsequently, the weight of the penalty term is efficiently optimized using generalized cross-validation (Golub & von Matt, 1997; Wood, 2006). Common sets of basis functions are natural cubic splines (Tsay & Chen, 2019) or thin plate splines (Wood, 2003).

GAMs build upon smoothing splines and combine them in the form of a generalized linear model. Meaning that the parameters of the general linear model smooth functions instead (Hastie & Tibshirani, 1999; Wood, 2006). This makes it possible to formulate time-varying auto-regressive models within the GAM framework (Bringmann et al., 2015, 2017). In these models, any of the terms, intercept, or auto-regressive, is modeled as varying smoothly over time, simultaneously describing non-linear trends and changes in the

auto-regressive effect.

GAMs rely on the assumption that the underlying state function is smooth. One example of a process that may satisfy this assumption is emotional valence (Bringmann et al., 2015). Then, GAMs provide information about the underlying process, in the form of interpretable time-varying intercept and autocorrelation parameters. However, the interpretation of these parameters is complicated by the interaction between trend and autocorrelation effects in non-stationary models (Bauldry & Bollen, 2018; Bollen & Curran, 2004). Additionally, it is possible to interpret the coefficients for the basis functions, as the degree to which these basic trends are evident in the smooth functions describing the time-varying parameters. However, this interpretation is again complicated by the presence of many other terms in the model and the penalization.

Gaussian Process Regression

Gaussian process (GP) regression follows yet another approach. Here, a probability distribution is directly imposed on the set of all possible state functions. This is done by imposing a Bayesian GP prior on the data. The behavior of this prior is then defined by a continuous mean and covariance function (Betancourt, 2020; Rasmussen & Williams, 2006; Roberts et al., 2013). To develop an intuition for this, it makes sense to imagine this probability distribution pointwise, such that for any set of specific time points, the functions described by the GP take values, which are in turn described by a multivariate normal distribution. The mean and covariance of this multivariate normal distribution are respectively defined by the mean and covariance function of the GP. Since this relation holds for any arbitrary set of time points, the probability distribution is defined for functions over all time points (Betancourt, 2020; Rasmussen & Williams, 2006; Roberts et al., 2013).

For practical purposes, the mean function is oftentimes defined to be zero and the covariance function is chosen to be stationary, such that the covariance between two time points only depends on their distance. Nevertheless, these conventions can be broken, and

it is possible to include a specific mean function to model for example polynomial trends (Blight & Ott, 1975; Hwang et al., 2023; O'Hagan, 1978). It is also possible to combine multiple covariance functions or design custom covariance and mean functions to accommodate desired types of non-linear behavior within the Gaussian process. Typical choices of covariance functions usually result in smooth approximations of constant wigglyness, making them most appropriate for these types of processes. Covariance functions are typically selected through domain knowledge or model selection (Abdessalem et al., 2017; Richardson et al., 2017). Subsequently, the model is estimated using a Bayesian approach by defining hyper-priors for the parameters of the mean and the covariance function.

State Space Models

When constructing a parametric model for time-series data, one is faced with the decision to model the state function globally, locally, or mixed. Global models, that try to capture the entire behavior of a non-linear state function, are oftentimes too simplistic and generalize poorly across time and individuals (Hunter et al., 2022). Because of this, in many situations, it is easier to define a model in terms of a local dynamic. State space models (SSM) are a flexible modeling framework that combines these local dynamic models with measurement models, making them suitable for modeling psychological data (Durbin & Koopman, 2012). While SSMs formally only include lag one relationships, it is possible to include higher order lags by extending the state space (Hunter, 2018). Further, it is possible to redefine the model in continuous time using differential equations. This approach is especially beneficial when observations are unequally spaced in time (Van Montfort et al., 2018).

Latent Change Score Models

Importantly, when all the dynamic equations in an SSM are linear, this modeling approach is very similar to estimating structural equation models with lagged variables (Asparouhov et al., 2018; Usami et al., 2019). One such model is the latent change score

model (LCS), which models the amount by which the state changes as an additional latent variable. This latent change variable is then linearly predicted from the state value at the previous time point (Cáncer & Estrada, 2023; Cáncer et al., 2021). Importantly, this model highlights how a local linear dynamic results in a global exponential trend. To describe this exponential trend meaningfully, the model can be parameterized in terms of an asymptote and a growth rate. Such that, if the growth rate is smaller than one, the state converges to the asymptote and if the growth rate is larger than one, it diverges. One example of a process that could follow such a trajectory is the developmental trajectory of intelligence, where one might suspect large increases in early childhood that level off towards an asymptote (Savi et al., 2021).

Regime Switching Models

Another class of commonly used SSMs in psychology are regime switching models (Hamaker et al., 2010). Here one assumes that the system that underlies the time series has distinct regimes and that each regime is governed by a different local dynamic. To accommodate this, regime switching models combine multiple local models with a model for how the system switches between the regimes. In this way, regime switching models describe both the behavior within each regime as well as the regime switching simultaneously. The dynamic equations are usually chosen as part of the ARIMA model family (Box & Jenkins, 1970), such that only the parameter values of the dynamic equations change between the regimes. The regime switching is commonly modeled using one of two methods. First, threshold autoregressive models estimate a threshold value for one of the observed variables to model the regime switching (Tong & Lim, 1980). Second, Markov switching autoregressive models use a discrete latent variable that follows a Markov chain to capture the regime switching (Hamilton, 1989). One example of a process, in which one might reasonably suspect distinct regimes is flow, where individuals appear to suddenly change in and out of a flow state (Ceja & Navarro, 2012).

Research Questions

While all of these methods are well established in the time series literature, the differences between time series data and ILD make it unclear to what degree these methods are applicable to the latter and which method is best suited in a given situation. Importantly, ILD is often composed of time series data from multiple individuals, creating a multilevel structure, with several time points collected per individual. Additionally, observations may be spaced unevenly across time and include time-varying and invariant confounders. Lastly, when choosing a method to analyze ILD, researchers also need to take into account the available a-priori theory and what type of inference is desired. To clarify this, we will investigate the following:

- To what degree can each of these methods accommodate different types of non-linearity that can be suspected in ILD?
- How well do each of the methods perform at sample sizes common in ILD, when specified correctly or misspecified?
- To what degree can each method accommodate hierarchical data structures, unevenly spaced time points, and time-invariant confounders?
- What degree of theory is required by each method and what type of inference does it yield?

To elucidate these issues, we will first compare and evaluate the described methods on simulated ILD. Afterwards, we will show how these methods can be used to gain insight into non-linear processes in real ILD.

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