



ECE-QE AC2-2015 - Rhea

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AC-2 P1.

$$\begin{aligned}
 \text{a)} \quad e^{At} &= \begin{bmatrix} -1 & -1 & 1 \\ -1 & 1 & 2 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} e^t & 0 & 0 \\ 0 & e^{-t} & 0 \\ 0 & 0 & e^0 \end{bmatrix} \begin{bmatrix} -2 & -\frac{1}{2} & -\frac{3}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ -1 & 0 & -1 \end{bmatrix} \\
 &= \begin{bmatrix} -e^t & -e^{-t} & 1 \\ -e^t & e^{-t} & 2 \\ e^t & e^{-t} & -2 \end{bmatrix} \begin{bmatrix} -2 & -\frac{1}{2} & -\frac{3}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ -1 & 0 & -1 \end{bmatrix} \\
 &= \begin{bmatrix} 2e^t - 1 & \frac{1}{2}e^t - \frac{1}{2}e^{-t} & \frac{3}{2}e^t - \frac{1}{2}e^{-t} - 1 \\ 2e^t - 2 & \frac{1}{2}e^t + \frac{1}{2}e^{-t} & \frac{1}{2}e^t + \frac{1}{2}e^{-t} - 2 \\ -2e^t + 2 & -\frac{1}{2}e^t + \frac{1}{2}e^{-t} & -\frac{3}{2}e^t + \frac{1}{2}e^{-t} + 2 \end{bmatrix}
 \end{aligned}$$

b) $\lambda_1 = 1, \lambda_2 = -1, \lambda_3 = 0 \therefore \lambda_1 > 0 \therefore \text{unstable.}$

c) If we want $t \rightarrow \infty, X(t) \rightarrow 0$

Model 1 and Model 3 need to be zero.

$$\omega_1^T X[0] = 0$$

$$\omega_3^T X[0] = 0$$

$$\begin{bmatrix} -2 & -\frac{1}{2} & -\frac{3}{2} \\ -1 & 0 & -1 \end{bmatrix} X[0] = 0$$

$$\therefore X_1 = -X_3, X_2 = X_3$$

$$\therefore X[0] = X_3 \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

The set is $\left\{ \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \right\}$

If we want to remain bounded

Model 2 and 3 are already bounded when $t \geq 0$

$$\therefore \omega_1^T X[0] = 0$$

$$\begin{bmatrix} -2 & -\frac{1}{2} & -\frac{3}{2} \end{bmatrix} X[0] = 0$$

$$X_1 = -\frac{1}{4}X_2 - \frac{3}{4}X_3$$

$$X = \begin{bmatrix} -\frac{1}{4}X_2 - \frac{3}{4}X_3 \\ X_2 \\ X_3 \end{bmatrix} = X_2 \begin{bmatrix} -\frac{1}{4} \\ 1 \\ 0 \end{bmatrix} + X_3 \begin{bmatrix} -\frac{3}{4} \\ 0 \\ 1 \end{bmatrix}$$

$$\therefore \left\{ \begin{bmatrix} -\frac{1}{4} \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -\frac{3}{4} \\ 0 \\ 1 \end{bmatrix} \right\}$$

$$d) \quad C = [B \ AB \ A^2B] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ -1 & -1 & -1 \end{bmatrix}$$

rank = 1 ≠ 3, not controllable

The reachable subspace is $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \right\}$

$$e) \quad O = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

rank = 1 ≠ 3, not observable

unobservable subspace is $\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$

(f ~ j) Can't resolve.

$$P2. \Phi(t) = e^{\int_0^t \begin{bmatrix} -t & 1 \\ 0 & -1 \end{bmatrix} dt} = e^{\begin{bmatrix} -\frac{1}{2}t^2 & 0 \\ 0 & -t \end{bmatrix}} e^{\begin{bmatrix} 0 & t \\ 0 & 0 \end{bmatrix}} = \begin{bmatrix} e^{-\frac{1}{2}t^2} & 0 \\ 0 & e^{-t} \end{bmatrix} \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}$$

$$\Phi(t) = \begin{bmatrix} e^{-\frac{1}{2}t^2} & te^{-\frac{1}{2}t^2} \\ 0 & e^{-t} \end{bmatrix}$$

P3. Let $X[3] = [1 \ 1]^t$, $X[0] = [0 \ 0]^t$

$$C_3 = [B \ AB \ A^2B] = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}^t$$

$$U = \begin{bmatrix} u(2) \\ u(1) \\ u(0) \end{bmatrix} = C_3^t (C_3 C_3^t)^{-1} X[3] = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \\ \frac{1}{3} \end{bmatrix}$$

$$\therefore U[0] = \frac{1}{3}, \quad U[1] = \frac{2}{3}, \quad U[2] = \frac{1}{3}$$

$$\text{Energy is } U^2[0] + U^2[1] + U^2[2] = \frac{6}{9} = \frac{2}{3}$$

P4. $(X_1^2 - 1)(X_2 - 2) = 0$

$$-X_2(X_1^2 + 1) = 0$$

$$\text{soll: } X_1 = \pm 1, \quad X_2 = 0$$

$$X_{e1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad X_{e2} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$Df(X) = \begin{bmatrix} 2X_1X_2 - 4X_1 & X_1^2 - 1 \\ -2X_1X_2 & -X_1^2 - 1 \end{bmatrix}$$

$$Df(X_{e1}) = \begin{bmatrix} -4 & 0 \\ 0 & -2 \end{bmatrix} \text{ all } \lambda_i \text{ negative asy stable}$$

$$Df(X_{e2}) = \begin{bmatrix} 4 & 0 \\ 0 & -2 \end{bmatrix} \text{ has a positive } \lambda, \text{ unstable}$$

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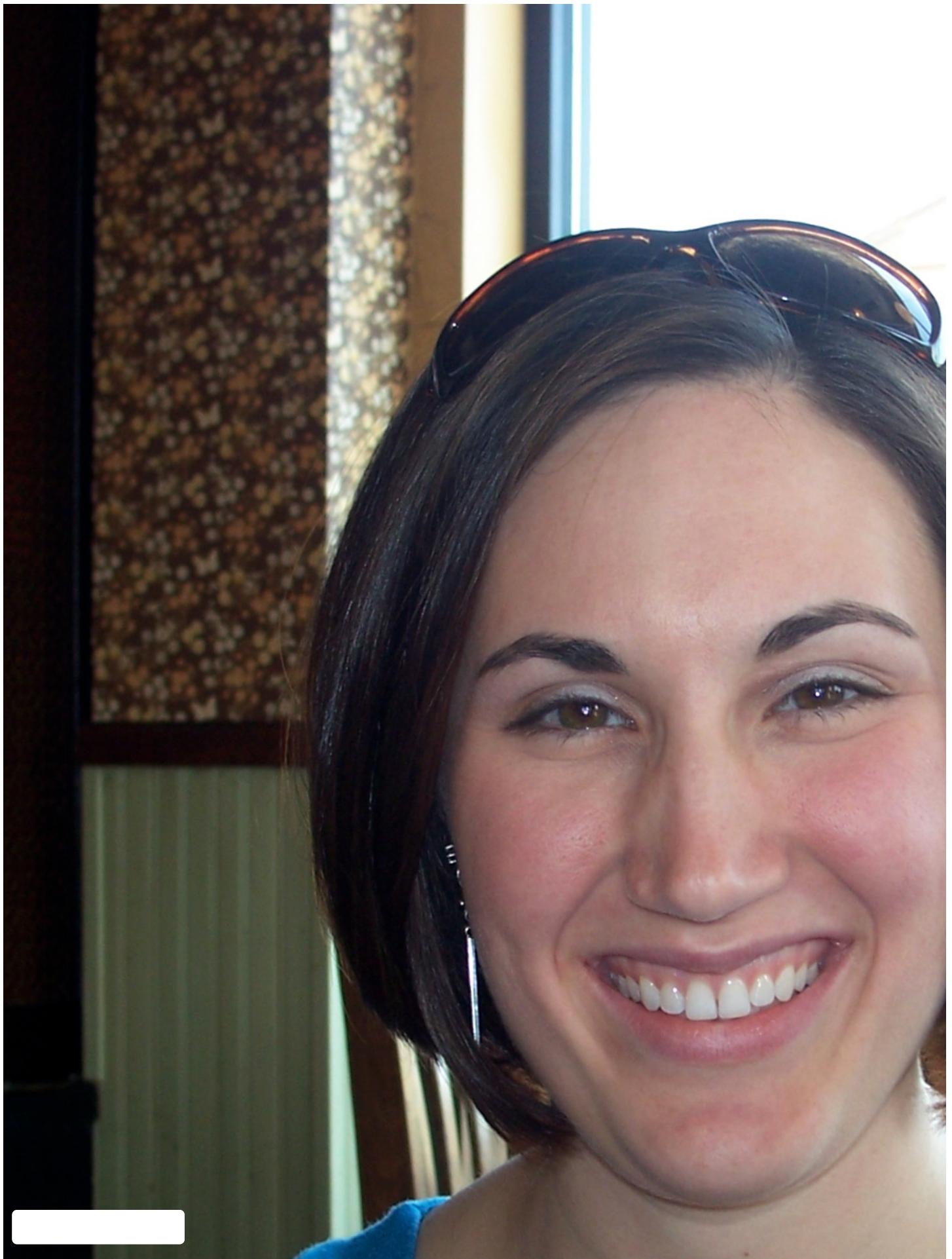
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