Problem 1.(50pt)
Consider the following discrete space system with input $x(m,n)$ and output $y(m,n)$.

$$y(m,n) = x(m,n) + ay(m-1,n) + ay(m,n-1) - a^2y(m-1,n-1)$$

Furthermore, assume that $x(m,n)$ is formed by independent and identically distributed Gaussian random variables with mean 0 and variance σ^2.

a) Calculate the transfer function of the system

$$H(z_1,z_2) = \frac{Y(z_1,z_2)}{X(z_1,z_2)}$$

b) Calculate the impulse response $h(m,n)$ of the system.

c) For which values of a is the system's output bounded when the input $x(m,n)$ is bounded?

d) Compute the autocovariance function $R_x(k,l) = E\left[x(m,n)x(m+k,n+l)\right]$ and the power spectral density $S_x(e^{j\mu},e^{j\nu})$ for the random process $x(m,n)$.

e) Compute the power spectral density $S_y(e^{j\mu},e^{j\nu})$ for the random process $y(m,n)$.

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Problem 2.(30pt)

Consider a color imaging device that takes input values of (r, g, b) and produces output (X, Y, Z) values given by

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = A \begin{bmatrix} r^\alpha \\ g^\alpha \\ b^\alpha \end{bmatrix} .$$

where

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} .$$

a) Calculate the white point of the device in chromaticity coordinates.

b) What are the primaries associated with the r, g, and b components respectively? Again, use chromaticity coordinates to specify your answer.

c) What is the gamma of the device?

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Problem 3.(20pt)
Let $x(m,n)$ be a gray scale image in the range of 0 to 255 that is gamma corrected with $\gamma_0 = 1.8$. You would like to view the image on a display with $\gamma_d = 2.2$.

a) Derive a formula for conversion of the image $x(m,n)$ to a new image $y(m,n)$ which will accurately represent the gray values of the image when used as input to the display.

b) If the image $x(m,n)$ is erroneously used as input to the display in place of $y(m,n)$, then how will the displayed image be distorted as compared to how it should have appeared?

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