ECE-QE CS1-2011 - Rhea

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ECE Ph.D. Qualifying Exam

Communication, Networking, Signal and Image Processing (CS)

Question 1: Probability and Random Processes

August 2011

Question

Part 1. 25 pts

Let $\mathbf{X}$, $\mathbf{Y}$, and $\mathbf{Z}$ be three jointly distributed random variables with joint pdf $f_{XYZ}(x,y,z) = \frac{3z^2}{7\sqrt{2\pi}}\, e^{-zy}$

(a) Find the joint probability density function $f_{YZ}(y,z)$.

(b) Find $f_x(x|y,z)$.

(c) Find $f_Z(z)$.

(d) Find $f_Y(y|z)$.

(e) Find $f_{XY}(x,y|z)$.

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Part 2. 25 pts

Show that if a continuous-time Gaussian random process $\mathbf{X}(t)$ is wide-sense stationary, it is also strict-sense stat

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Part 3. 25 pts

Show that the sum of two jointly distributed Gaussian random variables that are not necessarily statistically independent is a Gaussian random variable.

wers and discussions

Part 4. 25 pts

Assume that $\mathbf{X}(t)$ is a zero-mean continuous-time Gaussian white noise process with autocorrelation function

$$R_{\mathbf{XX}}(t_1, t_2) = \delta(t_1 - t_2).$$

Let $\mathbf{Y}(t)$ be a new random process ontained by passing $\mathbf{X}(t)$ through a linear time-invariant system with impulse response $h(t)$ whose Fourier transform $H(\omega)$ has the ideal low-pass characteristic

$$H(\omega) = \begin{cases} 1, & \text{if } |\omega| \leq \Omega, \\ 0, & \text{elsewhere,} \end{cases}$$

where $\Omega > 0$.

a) Find the mean of $\mathbf{Y}(t)$.

b) Find the autocorrelation function of $\mathbf{Y}(t)$.

c) Find the joint pdf of $\mathbf{Y}(t_1)$ and $\mathbf{Y}(t_2)$ for any two arbitrary sample time t_1 and t_2.

d) What is the minimum time difference $t_1 - t_2$ such that $\mathbf{Y}(t_1)$ and $\mathbf{Y}(t_2)$ are statistically independent?

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