CS-1

August 2010 QE

1. If X_1, X_2, X_3 are independent and identically distributed exponential random variables with parameter λ, compute
   (a) (10 points) $P(\min(X_1, X_2, X_3) \leq a)$.
   (b) (10 points) $P(\max(X_1, X_2, X_3) \leq a)$.

2. Consider the sum of two complex sinusoids with random coefficients:

$$X(t) = X_1 e^{j\omega_1 t} + X_2 e^{j\omega_2 t}$$

   where $\omega_1 \neq \omega_2$, and X_1 and X_2 are complex-valued random variables.
   (a) (15 points) Find the autocorrelation function of $X(t)$.
   (b) (15 points) Find conditions on X_1 and X_2 that make $X(t)$ a wide-sense stationary process.

3. Let A_n be a real-valued wide-sense stationary zero-mean discrete-time random process that has autocorrelation function

$$R_A(k) = \sigma_1^2 \rho_1^{|k|}$$

   A decimator takes every other sample to form the random process V_m, $m = 1, 2, \ldots$:

$$A_1 A_3 A_5 A_7 A_9 A_{11} \ldots$$

   (a) (10 points) Find the autocorrelation function of V_m.
   (b) (10 points) An interpolator takes the sequence V_m and inserts zeros between samples to form the sequence W_k:

$$A_1 0 A_3 0 A_5 0 A_7 0 A_9 0 A_{11} \ldots$$

   Find the autocorrelation function of W_k.
   (c) (10 points) Is V_m wide-sense stationary? Is W_k wide-sense stationary?

4. (20 points) Let X_n converge in distribution to a constant a and let Y_n converge in distribution to a constant b. If X_n and Y_n are independent sequences, does $X_n + Y_n$ converge in distribution to $a + b$? You must justify your answer.

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