ECE-QE AC2-2013 - Rhea

Print

p1 a) $A = \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix}$

$X(t) = \begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix}$

$\begin{cases} \dot{x}_1(t) = -X_1(t) + X_2(t) \\ \dot{x}_2(t) = -2X_2(t) \end{cases}$

$\Phi(t) = [\, \Phi_1(t) \quad \Phi_2(t) \,]$

For $\Phi_1(t) assume X_{(0)} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\begin{cases} \dot{x}_1(t) = e^{\int_t^0 -1 \, \mathrm{dt}} X_1(0) = e^{-t} \\ \dot{x}_2(t) = e^{\int_t^0 -2 \, \mathrm{dt}} X_2(0) = 0 \end{cases}$

$\therefore \Phi_1(t) = \begin{bmatrix} e^{-t} \\ 0 \end{bmatrix}$

$For \quad \Phi_2(t) assume X_(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$X_2(t) = e^{\int_t^0 -2 \, \mathrm{dt}} X_2(0) = e^{-2t}$

$\Phi_(t) = e^{\int_t^0 A \, \mathrm{dt}} = e^{\begin{bmatrix} -t & t \\ 0 & -2t \end{bmatrix}} = \begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-2t} \end{bmatrix} \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} e^{-t} & te^{-t} \\ 0 & e^{-2t} \end{bmatrix}$

$\Phi_(t, \iota) = \Phi_(t) \Phi_(\iota)^- 1 = \begin{bmatrix} e^{-t} & te^{-t} \\ 0 & e^{-2t} \end{bmatrix} \begin{bmatrix} e^{-\iota} & \iota e^{-\iota} \\ 0 & e^{-2\iota} \end{bmatrix}$

b) $A = \begin{bmatrix} -\cos t & \cos t \\ 0 & -2\cos t \end{bmatrix}$

$\Phi_(t) = e^{\int_t^0 A \, \mathrm{dt}} = \begin{bmatrix} e^{\sin t} & 0 \\ 0 & e^{2 \sin t} \end{bmatrix} \begin{bmatrix} 1 & -\sin t \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} e^{\sin t} & -\sin t e^{\sin t} \\ 0 & e^{2 \sin t} \end{bmatrix}$

$\Phi_(t, \iota) = \Phi_(t) \cdot \Phi_(\iota)^- 1 = \begin{bmatrix} e^{\sin t} & -sinte^{\sin t} \\ 0 & e^{2 \sin t} \end{bmatrix} \begin{bmatrix} e^{\sin \iota} & -sin\iota e^{\sin \iota} \\ 0 & e^{2 \sin \iota} \end{bmatrix}^{-1}$

p2 a) $|\lambda \mathrm{I} - A| = \begin{bmatrix} \lambda + 2 & -2 \\ 1 & \lambda - 1 \end{bmatrix}$

$\lambda_1 = 0 \quad \lambda_2 = -1$

Marginally stable ,not asy , stable.

b) $c = [\, B \quad AB \,] = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

$rank = 1 \neq 2$

not observable, unobservable subspace $\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$ c) $0 = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$rank = 1 \neq 2$

not observable, unobservable subspace$\left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$

d) $$C(SI-A)^{-1}B = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \begin{bmatrix} \frac{S-1}{2S} & \frac{S+2}{2S} \\ \frac{S+1}{-2S} & \frac{S-2}{2S} \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 2$$

e) i) false

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ii) false
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f) $A-BK = \begin{bmatrix} -2-k_1 & 2-k_2 \\ -1-k_1 & 1-k_2 \end{bmatrix} |\lambda - A + BK| = \lambda^2 + (a+b+1)\lambda + 3a + 3 - ab = 0$

$$\lambda_1 = -3 \quad and \quad \lambda_2 = -1 \begin{cases} -3a+9-ab=0 \\ 2a-b+3-ab=0 \end{cases}$$

$a = 0, b = 3k = \begin{bmatrix} 0 & 3 \end{bmatrix}$

g) $\begin{bmatrix} \lambda \mathrm{I} - A \\ C \end{bmatrix} = \begin{bmatrix} \lambda+2 & -2 \\ 1 & \lambda-1 \\ 1 & -1 \end{bmatrix}$ must contain $\lambda = 0, no$

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Ph.D. on Applied Mathematics in Aug 2007. Involved on applications of image super-resolution to electron microscopy

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