ECE-QE AC2-2016 - Rhea

Print

2016 AC-2 P1. (a) $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} y \\ \dot{y} \end{bmatrix}$

$$\begin{cases} \dot{x} = \begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} x_2 \\ -2x_1x_2 - ux_1 + 2u \end{bmatrix} \\ y = x_1 \end{cases}$$

(b) $u \equiv 2$

$$\dot{x} = \begin{bmatrix} x_2 \\ -2x_1x_2 - 2x_1 + 4 \end{bmatrix} = \begin{bmatrix} x_2 \\ -2x_1(x_2+1) + 4 \end{bmatrix}$$

let $\begin{cases} -2x_1(x_2+1) + 4 = 0 \\ x_2 = 0 \end{cases} \Rightarrow \begin{cases} -2x_1 + 4 = 0 \\ x_2 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = 2 \\ x_2 = 0 \end{cases}$

$\therefore The\ equilibrum\ point\ is\ x_e = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

(c) $u \equiv 2 \quad x_e = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \quad let\ x = f(x)$

The Jacobin of $\dot{x}$ is: $Df(x) = \begin{bmatrix} 0 & 1 \\ -2x_1 - 2 & -2x_1 \end{bmatrix}$

The linear dynamics around x_e is $\frac{d}{dt} f(x) = \begin{bmatrix} 0 & 1 \\ -2 & -4 \end{bmatrix} f(x)$

which is stable, locally stable at x_e.

P2. i) $x[k+1] = Ax[k]$

$y[k] = [-1 \quad 1]x[k]$

let $x_{[}k] = \begin{bmatrix} a \\ b \end{bmatrix} \quad y[0] = [-1 \quad 1]\begin{bmatrix} a \\ b \end{bmatrix} = 1$

$-a + b = 1$

$y[1] = [-1 \quad 1]x[1] = [-1 \quad 1]\begin{bmatrix} -2 & 4 \\ -1 & 2 \end{bmatrix}x[0] = 0$

$x[1] = Ax[0]$

$3a + 2b = 0$

$\therefore a = -\frac{2}{5} \quad b = \frac{3}{5}$

$$x[0] = \begin{bmatrix} -\frac{2}{5} \\ \frac{3}{5} \end{bmatrix}$$

ii)$x[0] = \begin{bmatrix} a \\ b \end{bmatrix} \quad x[2] = Ax[1] = A^2x[0]$

$A^2 = 0 \quad x[2] = 0$

$y[2] = [-1 \quad 1] \quad x[2] = 0 \quad y[1] = [-1 \quad 1] \quad x[1] = 1$

$[-1 \quad 1] \quad \begin{bmatrix} 2 & 4 \\ -1 & 2 \end{bmatrix} \quad \begin{bmatrix} a \\ b \end{bmatrix} = 1$

we only have -3a-2b=1,so we can't uniquely determine a,b.

P:

$\lambda_1 = \lambda_2 = 0$

$$\begin{bmatrix} -2-\lambda_1 & 4 \\ -1 & 2-\lambda_1 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} -2u_1 - \lambda_1 u_1 + 4u_2 = 0 \\ -u_1 + 2u_2 - \lambda_1 u_2 = 0 \end{cases}$$

$u_1 = 2u_2$

$\therefore eigenvector \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$J = MAM^{-1}$

(b)$e^{At} = L^{-1}\left[(SI - A)^{-1}\right] = L^{-1}\begin{bmatrix} \frac{s-2}{s^2} & \frac{4}{s^2} \\ \frac{-1}{s^2} & \frac{s+2}{s^2} \end{bmatrix}$

$$L^{-1}\left[\frac{s-2}{s^2}\right] = L^{-1}\left[\frac{1}{s} - \frac{2}{s^2}\right] = 1 - 2t$$

$$L^{-1}\left[\frac{4}{s^2}\right] = 4t$$

$$L^{-1}\left[\frac{-1}{s^2}\right] = -t$$

$$L^{-1}\left[\frac{s+2}{s^2}\right] = L^{-1}\left[\frac{1}{s} + \frac{2}{s^2}\right] = 1 + 2t$$

$$e^{At} = \begin{bmatrix} 1-2t & 4t \\ -t & 1+2t \end{bmatrix}$$

(c)T(s)=$C(SI - A)^{-1}B$

$=\begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} \frac{s-2}{s^2} & \frac{4}{s^2} \\ \frac{-1}{s^2} & \frac{s+2}{s^2} \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$$= \frac{\begin{bmatrix} -1 & 1 \end{bmatrix}}{s^2}\begin{bmatrix} 2s - 4 + 4 \\ -2 + s + 2 \end{bmatrix}$$

$$T(s) = \begin{bmatrix} -1 & 1 \end{bmatrix}\begin{bmatrix} \frac{2}{s} \\ \frac{1}{s} \end{bmatrix} = \frac{-2}{s} + \frac{1}{s}$$

$$T(s) = \frac{-1}{s}$$

pole is at s=0.

$\therefore marginally\ stable.$

(d) Given $\dot{x} = Ax + Bu$

$$Ax = \begin{bmatrix} -2 & u \\ -1 & 2 \end{bmatrix}x, \quad Bu = \begin{bmatrix} 2 \\ 1 \end{bmatrix}u$$

$$C = \begin{bmatrix} B & AB & A^2B \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 1 & 0 \end{bmatrix}$$

$\therefore not\ controllable$

To make $x(1) = \begin{bmatrix} -u & -2 \end{bmatrix}$

corres ponding characteristic equation.

$(s+u)(s+2) = 0$

$s^2 + 6s + 8 = 0$

$u = -kx$

$\dot{x} = (A - Bk)x$

$|\,SI - (A - Bk)\,| = 0$

$$A - BK = \begin{bmatrix} -2 & 4 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 2K_1 \\ 1K_2 \end{bmatrix}$$

$$\begin{vmatrix} S + 2 + 2k_1 & -4 \\ 1 + k_2 & S - 2 \end{vmatrix} = 0 \qquad \begin{bmatrix} A - Bk \end{bmatrix} = \begin{bmatrix} -2-2k_1 & 4 \\ -1-k_2 & 2 \end{bmatrix}$$

$($ [illegible] $k_2) = 0$

$2k_1 = 6$

$k_1 = 3$

$uk_2 = 20$

$k_2 = 5$

$\therefore$ control $u_{(t)} = -[\,k_1 \quad k_2\,]x$

or $u_{(t)} = -[\,3 \quad 5\,]x$

our goal is to have $x_{(1)} = -[\,1 \quad 2\,]$

$(s-1)(s+2) = 0$

$2k_1 = -3$

$k_1 = -\frac{3}{2}$

$uk_2 = -4$

$2k_1 = -3$

$k_2 = -1$

$\therefore u_{(t)} = -[\,k_1 \quad k_2\,]x = \begin{bmatrix} -\frac{3}{2} & -1 \end{bmatrix} x$

e) consider $\dot{x} = A_{C1}X$

$A_{C1} = A - Bk$

$s^2 + 2k_1 s - uk_1 + uk_2 = 0$

$1 + \frac{C_t(s)}{t(s)} = 0$

$\frac{C_t(s)}{t(s)} = s^2 + 2k_1 s - uk_1 + uk_2 - 1$

$\lim_{s \to 0} s \frac{C_t(s)}{t(s)} = \lim_{s \to 0} s(s^2 + 2k_1 s - uk_1 + uk_2 - 1)$

Final value is zero and no such controller is possible to design.

But for Bounded time period,it is possible to have solution of x(t). As we have already seen in previous problem.

$\alpha_1 + \alpha_2 = 2k_1, \quad \alpha_1\alpha_2 = -uk_1 + uk_2.$

$(s + \alpha_1)(s + \alpha_2) = 0$

$\begin{cases} \alpha^2 + (\alpha_1 + \alpha_2)s + \alpha_1\alpha_2 = 0 \\ s^2 + 2k_1 s - uk_1 + uk_2 = 0 \end{cases}$

(f) $\dot{x} = \begin{bmatrix} -2 & 4 \\ -1 & 2 \end{bmatrix} x + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u$

$y = [\,-1 \quad 1\,]x \quad D = [0]$

$N = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 1 & -2 \end{bmatrix}$

The system is observable.

(g) $\frac{Y(S)}{U(S)} = C[SI - A]^{-1}B + D$

$SI - A = \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} -2 & 4 \\ -1 & 2 \end{bmatrix}$

$[SI - A]^{-1} = \frac{1}{S^2 - 4 + 4} \begin{bmatrix} S-2 & 4 \\ -1 & S+2 \end{bmatrix}$

$\frac{Y(s)}{u(s)} == [\,-1 \quad 1\,] \begin{bmatrix} \frac{s-2}{s^2} & \frac{4}{s^2} \\ \frac{-1}{s^2} & \frac{s+2}{s^2} \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$H(s) = \frac{-1}{s}$ marginally stable ,not BIBO.

(h) Cant not resolve.

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Basic linear algebra uncovers and clarifies very important geometry and algebra.

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