ECE PhD QE CNSIP 2008 Problem1 - Rhea

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ECE Ph.D. Qualifying Exam

Communication, Networking, Signal and Image Processing (CS)

Question 1: Probability and Random Processes

August 2008

Question

1

The Weak Law of Large Numbers states that if $\mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3, \cdots$ is a sequence of i.i.d. random variables with finite mean $E[\mathbf{X}_i] = \mu$ for every i , then the sample mean $\mathbf{Y}_n = \frac{1}{n}\sum_{i=1}^{n} \mathbf{X}_i$ converges to μ in probability. Suppose that instead of being i.i.d , $\mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3, \cdots$ each have finite mean μ , and the covariance function of the sequence $\mathbf{X}_n$ is $Cov\big(\mathbf{X}_i, \mathbf{X}_j\big) = \sigma^2 \rho^{|i-j|}$, where $|\rho| < 1$ and $\sigma^2 > 0$.

a. (13 points)

Find the mean and variance of $\mathbf{Y}_n$.

b. (12 points)

Does the sample mean still converge to μ in probability? You must justify your answer.

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2.

Let $\mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3, \cdots$ be a sequence of i.i.d Bernoulli random variables with $p = 1/2$, and let $\mathbf{Y}_n = 2^n \mathbf{X}_1 \mathbf{X}_2 \cdots \mathbf{X}_n$.

a. (15 points)

Does the sequence $\mathbf{Y}_n$ converge to 0 almost everywhere?

b. (15 points)

Does the sequence $\mathbf{Y}_n$ converge to 0 in the mean square sense?

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3.

Consider a random process $\mathbf{X}(t)$ that assumes values ± 1 . Suppose that $\mathbf{X}(0) = \pm 1$ with probability $1/2$, and suppose that $\mathbf{X}(t)$ then changes polarity with each occurrence of an event in a Poisson process of rate λ .

Note:

You might find the equations $\frac{1}{2}\left(e^{x}+e^{-x}\right)=\sum_{j=0}^{\infty}\frac{x^{2j}}{(2j)!}$ and $\frac{1}{2}\left(e^{x}-e^{-x}\right)=\sum_{j=0}^{\infty}\frac{x^{2j+1}}{(2j+1)!}$ helpful.

a. (15 points)

Find the probability mass function of $\mathbf{X}(t)$.

b. (15 points)

Find the autocovariance function of the random process $\mathbf{X}(t)$.

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4 (15 points)

Messages arrive at a message center according to a Poisson process of rate λ messages per hour. Every hour the messages that have arrived during the previous hour are forwarded to their destination. Find the expected value of the total time waited by all messages that arrive during the hour.

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