



ECE PhD QE CNSIP 2006 Problem1 - Rhea

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ECE Ph.D. Qualifying Exam

Communication, Networking, Signal and Image Processing (CS)

Question 1: Probability and Random Processes

August 2006

Question

1

Let \mathbf{U}_n be a sequence of independent, identically distributed zero-mean, unit-variance Gaussian random variables. The sequence \mathbf{X}_n , $n \geq 1$, is given by

$$\mathbf{X}_n = \frac{1}{2} \mathbf{U}_n + \left(\frac{1}{2}\right)^2 \mathbf{U}_{n-1} + \cdots + \left(\frac{1}{2}\right)^n \mathbf{U}_1.$$

(a) (15 points)

Find the mean and variance of \mathbf{X}_n .

(b) (15 points)

Find the characteristic function of \mathbf{X}_n .

(c) (10 points)

Does the sequence \mathbf{X}_n converge in distribution? A simple yes or no answer is not sufficient. You must justify your answer.

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2

Let Φ be the standard normal distribution, i.e., the distribution function of a zero-mean, unit-variance Gaussian random variable. Let \mathbf{X} be a normal random variable with mean μ and variance 1. We want to find $E[\Phi(\mathbf{X})]$.

(a) (10 points)

First show that $E[\Phi(\mathbf{X})] = P(\mathbf{Z} \leq \mathbf{X})$, where \mathbf{Z} is a standard normal random variable independent of \mathbf{X} . Hint: Use an intermediate random variable \mathbf{I} defined as

(b) (10 points)

Now use the result from Part (a) to show that $E[\Phi(\mathbf{X})] = \Phi\left(\frac{\mu}{\sqrt{2}}\right)$.

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3 (15 points)

Let $\mathbf{Y}(t)$ be the output of linear system with impulse response $h(t)$ and input $\mathbf{X}(t) + \mathbf{N}(t)$, where $\mathbf{X}(t)$ and $\mathbf{N}(t)$ are jointly wide-sense stationary independent random processes. If $\mathbf{Z}(t) = \mathbf{X}(t) - \mathbf{Y}(t)$, find the power spectral density $S_{\mathbf{Z}}(\omega)$ in terms of $S_{\mathbf{X}}(\omega)$, $S_{\mathbf{N}}(\omega)$, $m_{\mathbf{X}} = E[\mathbf{X}]$, and $m_{\mathbf{Y}} = E[\mathbf{Y}]$.

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4

Suppose customer orders arrive according to an i.i.d. Bernoulli random process \mathbf{X}_n with parameter p . Thus, an order arrives at time index n (i.e., $\mathbf{X}_n = 1$) with probability p ; if an order does not arrive at time index n , then $\mathbf{X}_n = 0$. When an order arrives, its size is an exponential random variable with parameter λ . Let \mathbf{S}_n be the total size of all orders up to time n .

(a) (20 points)

Find the mean and autocorrelation function of \mathbf{S}_n .

(b) (5 points)

Is \mathbf{S}_n a stationary random process? Explain.

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