ECE-QE AC2-2014 - Rhea

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AC-2 2014

P1.（a）i）$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -1 & -\frac{1}{2} \\ \frac{1}{2} & -1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} \frac{x_0(t)}{2} \\ \frac{x_3(t)}{2} \end{bmatrix} = \begin{bmatrix} -1 & -\frac{1}{2} \\ \frac{1}{2} & -1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} x_0(t) \\ x_3(t) \end{bmatrix}$

ii) $A = \begin{bmatrix} -1 & \frac{1}{2} \\ \frac{1}{2} & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

$e^A = \begin{bmatrix} e^{-1} & 0 \\ 0 & e^{-1} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} e^{-\frac{1}{2}} & 0 \\ 0 & e^{\frac{1}{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} e^{-\frac{3}{2}} + e^{-\frac{1}{2}} & -e^{-\frac{3}{2}} + e^{-\frac{1}{2}} \\ -e^{-\frac{3}{2}} + e^{-\frac{1}{2}} & e^{-\frac{3}{2}} + e^{-\frac{1}{2}} \end{bmatrix}$

iii) $\lambda_1 = -\frac{1}{2}\ \lambda_2 = -\frac{3}{2}$

$$\begin{aligned} &stable \\ &X(t) \rightarrow X(\infty) \\ &as \\ &t \rightarrow \infty \end{aligned}$$

$$e^{At} = \frac{1}{2} \begin{bmatrix} e^{-\frac{3}{2}t} + e^{-\frac{1}{2}t} & e^{-\frac{3}{2}t} + e^{-\frac{1}{2}t} \\ e^{-\frac{3}{2}t} + e^{-\frac{1}{2}t} & e^{-\frac{3}{2}t} + e^{-\frac{1}{2}t} \end{bmatrix} t \rightarrow \infty e^{At} \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$X(t) = e^{At}X(0) + \int_0^t e^{A(t-I)}BUdI \ = e^{At}X(0) + \int_0^t e^{A(t-I)}dIBU$$

$$X(\infty) = e^(Atrightarrow\infty)X(0) + 0Bu = X(0) = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

(b)$X(t) = \begin{bmatrix} 0 & 0 & 0 \\ \frac{1}{2} & -1 & \frac{1}{2} \\ 0 & \frac{1}{2} & -1 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} U(t)$

ii）Can't resolve the rest of questions

P2 $\begin{aligned} &\lambda_1 = 1 \\ &\lambda_2 = -1 \\ &\lambda_3 = 2 \\ &notstable \end{aligned}$

$$C = \begin{bmatrix} B & AB & A^2B \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 2 & 5 & 11 \end{bmatrix}$$

$rank = 2$

$$not \quad controllable0 = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$\begin{aligned} &rank = 1 \\ &notobservable \end{aligned}$

For$\lambda_1 = 1$

$$rank\begin{bmatrix} \lambda I - A & B \end{bmatrix} = rank \begin{bmatrix} 0 & 0 & 0 & 1 \\ -2 & 2 & 0 & 1 \\ -5 & 4 & -1 & 2 \end{bmatrix} = 3$$

$$rank\begin{bmatrix} \lambda I - A \\ C \end{bmatrix} = rank \begin{bmatrix} 0 & 0 & 0 \\ -2 & 2 & 0 \\ -5 & 4 & -1 \\ 1 & 0 & 0 \end{bmatrix} = 3$$

For$\lambda_2 = -1$

$$rank[\,\lambda I - A \quad B\,] = rank\begin{bmatrix} -2 & 0 & 0 & 1 \\ -2 & 0 & 0 & 1 \\ -5 & 4 & -3 & 2 \end{bmatrix} = 2$$

$uncontrollable$

$$rank\begin{bmatrix} \lambda I - A \\ C \end{bmatrix} = rank\begin{bmatrix} -2 & 0 & 0 \\ -2 & 0 & 0 \\ -5 & 4 & -3 \\ 1 & 0 & 0 \end{bmatrix} = 2$$

$unobservable$

For$\lambda_3 = 2$

$$rank[\,\lambda I - A \quad B\,] = rank\begin{bmatrix} 1 & 0 & 0 & 1 \\ -2 & 3 & 0 & 1 \\ -5 & 4 & 0 & 2 \end{bmatrix} = 3$$

$$rank\begin{bmatrix} \lambda I - A \\ C \end{bmatrix} = rank\begin{bmatrix} 1 & 0 & 0 \\ -2 & 3 & 0 \\ -5 & 4 & 0 \\ 1 & 0 & 0 \end{bmatrix} = 3$$

$The \quad only \quad uncontrollable \quad and \quad unobservable \quad \lambda \quad has \quad negative \quad real \quad part$

$Stablizable \quad and \quad detectable$

$$H(S) = C(SI - A)^{-1}B + D = (\frac{1}{S-1} - \frac{2}{(S+1)(S-1)} - \frac{8}{(S-1)(S+1)(S-2)}) = \frac{S^2 - 3S - 6}{(S-1)(S+1)(S-2)}$$

$P_1 = 1$
$P_2 = -1$
$P_3 = 2$
$not\ all\ poles\ have\ neqative\ real\ parts$
$not\ BIBO\ stable$

P3 $X(t) = \begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix} \dot{x}_1(t) = -tx_1(t)\dot{x}_2(t) = -costx_1(t) - tx_2(t)$

$\Phi(t) = \begin{bmatrix} \Phi_1(t) & \Phi_2(t) \end{bmatrix} for \Phi_1(t)$
$assume$
$X(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$
$X_1(0) = 1 X_2(0) = 0$

$X_1(t) = e^{-\int_0^t IdI}$
$X_1(0) = e^{-\frac{1}{2}t^2}\dot{x}_2(t) = -coste^{-\frac{1}{2}t^2} - tx_2(t) for \Phi_2(t)$
$assume$
$X(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$X_1(t) = 0\dot{x}_2(t) = -tx_2(t)x_2(t) = e^{-\frac{1}{2}t^2}x_2(0) = e^{-\frac{1}{2}t^2}\Phi_2(t) = \begin{bmatrix} 0 \\ e^{-\frac{1}{2}t^2} \end{bmatrix}$

$$\Phi(t) = e^{-\int_0^t AdI} = e^{-\int_0^t \begin{bmatrix} -t & 0 \\ -cost & -t \end{bmatrix} dI} = e^{\begin{bmatrix} -\frac{1}{2}t^2 & 0 \\ sint & -\frac{1}{2}t^2 \end{bmatrix}} = e^{\begin{bmatrix} -\frac{1}{2}t^2 & 0 \\ 0 & -\frac{1}{2}t^2 \end{bmatrix}} e^{\begin{bmatrix} 0 & 0 \\ sint & 0 \end{bmatrix}} = \begin{bmatrix} e^{-\frac{1}{2}t^2} & 0 \\ 0 & e^{-\frac{1}{2}t^2} \end{bmatrix}\begin{bmatrix} 1 & 0 \\ sint & 1 \end{bmatrix} = \begin{bmatrix} e^{-\frac{1}{2}t^2} & 0 \\ sinte^{-\frac{1}{2}t^2} & e^{-\frac{1}{2}t^2} \end{bmatrix}$$

$$\Phi(tI) = \Phi(t)\Phi^{-1}(I) = \begin{bmatrix} e^{-\frac{1}{2}t^2} & 0 \\ sinte^{-\frac{1}{2}t^2} & e^{-\frac{1}{2}t^2} \end{bmatrix}\begin{bmatrix} e^{-\frac{1}{2}I^2} & 0 \\ sinIe^{-\frac{1}{2}I^2} & e^{-\frac{1}{2}I^2} \end{bmatrix} = \begin{bmatrix} e^{-\frac{1}{2}(t^2-I^2)} & 0 \\ (sint - sinI)e^{-\frac{1}{2}(t^2-I^2)} & e^{-\frac{1}{2}(t^2-I^2)} \end{bmatrix}$$

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