ECE-QE CS1-2013 - Rhea

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ECE Ph.D. Qualifying Exam

Communication, Networking, Signal and Image Processing (CS)

Question 1: Probability and Random Processes

August 2013

Question

Part 1.

Consider n independent flips of a coin having probability p of landing on heads. Say that a changeover occurs whenever an outcome differs from the one preceding it. For instance, if $n = 5$ and the sequence $HHTHT$ is observed, then there are 3 changeovers. Find the expected number of changeovers for n flips. *Hint*: Express the number of changeovers as a sum of Bernoulli random variables.

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Part 2.

Let $X_1, X_2, \ldots$ be a sequence of jointly Gaussian random variables with covariance

$$Cov(X_i, X_j) = \begin{cases} \sigma^2, & i = j \\ \rho\sigma^2, & |i - j| = 1 \\ 0, & otherwise \end{cases}$$

Suppose we take 2 consecutive samples from this sequence to form a vector X, which is then linearly transformed to form a 2-dimensional random vector $Y = AX$. Find a matrix A so that the components of Y are independent random variables You must justify your answer.

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Part 3.

Let X be an exponential random variable with parameter λ, so that $f_X(x) = \lambda exp(-\lambda x)u(x)$. Find the variance of X. You must show all of your work.

> **wers and discussions**

Part 4.

Consider a sequence of independent random variables $X_1, X_2, \ldots$, where X_n has pdf

$$f_n(x) = (1 - \frac{1}{n}) \frac{1}{\sqrt{2\pi}\sigma} exp[-\frac{1}{2\sigma^2}(x - \frac{n-1}{n}\sigma)^2]$$
$$+ \frac{1}{n}\sigma exp(-\sigma x)u(x)$$

Does this sequence converge in the mean-square sense? *Hint:* Use the Cauchy criterion for mean-square convergence, which states that a sequence of random variables $X_1, X_2, \ldots$ converges in mean-square if and only if $E[|X_n - X_{n+m}|] \to 0$ as $n \to \infty$, for every $m > 0$.

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