ECE-QE AC2-2012 - Rhea

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AC-2 P1.

a) $C = \begin{bmatrix} B & AB & A^2B \end{bmatrix} = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad rank = 2 \neq 3$

$Not \quad controllable$

b) $Subspace \quad is \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} \right\}.$

c) $0 = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad Not \quad observable$

d) $X_1 = r, X_2 = -s, X_3 = s \quad X = r \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} Subspace \quad is \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \right\}.$ **e)** $\lambda I - A = \begin{bmatrix} \lambda - 1 & -1 & -1 \\ 0 & \lambda & -1 \\ 0 & 0 & \lambda + 1 \end{bmatrix} \quad \lambda_1 = 1, \lambda_2 = 0, \lambda_3 = -1$

$for \ \lambda_1 = 1 \quad \begin{bmatrix} \lambda I - A & B \end{bmatrix} = \begin{bmatrix} 0 & -1 & -1 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 2 & 0 \end{bmatrix} \qquad for \ \lambda_2 = 0 \quad \begin{bmatrix} \lambda I - A & B \end{bmatrix} = \begin{bmatrix} -1 & -1 & -1 & 1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

$for \ \lambda_3 = -1 \quad \begin{bmatrix} \lambda I - A & B \end{bmatrix} = \begin{bmatrix} -2 & -1 & -1 & 1 \\ 0 & -1 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad rank < 3 \quad must \ contain \ \lambda = -1 \quad \therefore \ No.$

f) $\begin{bmatrix} \lambda I - A \\ C \end{bmatrix} = \begin{bmatrix} \lambda - 1 & -1 & -1 \\ 0 & \lambda & -1 \\ 0 & 0 & \lambda + 1 \\ 0 & 1 & 1 \end{bmatrix}$

$for \ \lambda_1 = 1 \quad \begin{bmatrix} \lambda I - A \\ C \end{bmatrix} = \begin{bmatrix} 0 & -1 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \\ 0 & 1 & 1 \end{bmatrix} \quad rank < 3 \quad must \ have \ \lambda_1 = 1$

$for \ \lambda_2 = 0 \quad \begin{bmatrix} \lambda I - A \\ C \end{bmatrix} = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & -1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

$for \lambda - 3 = -1 \quad \begin{bmatrix} \lambda I - A \\ C \end{bmatrix} = \begin{bmatrix} -2 & -1 & -1 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \quad rank < 3 \quad must \ have \ \lambda_3 = -1$

$\because \ eigenvalues \{1, -1, 2\} \quad \therefore \ Yes.$

$A - LC = < math > \begin{bmatrix} 1 & 1 - L_1 & 1 - L_1 \\ 0 & -L_2 & 1 - L_2 \\ 0 & -L_3 & -1 - L_3 \end{bmatrix} \quad LC = \begin{bmatrix} 0 & L_1 & L_1 \\ 0 & L_2 & L_2 \\ 0 & L_3 & L_3 \end{bmatrix}$

$\lambda I - (A - LC) = \begin{bmatrix} \lambda - 1 & L_1 - 1 & L_1 - 1 \\ 0 & \lambda + L_2 & L_2 - 1 \\ 0 & L_3 & \lambda + 1 + L_3 \end{bmatrix}$

For conditions $\quad \lambda_1 = 1, \quad \lambda_2 = -1, \quad \lambda_3 = -2$

$\begin{cases} 3L_2 + 6L_3 = 9 \\ L_2 + 2L_3 = 3 \\ L_3 = 1 \end{cases} \qquad \begin{bmatrix} 2 \end{bmatrix}$

g) $\because\ \lambda_1 = 1, \quad Not\ stable.$

h) $AU_1 = \lambda_1 U_1 \quad AU_2 = \lambda_2 U_2 \quad AU_3 = \lambda_3 U_3$

$$y = CX_(t) = C[U_1 e^t(\omega_1^T X_{(0)}) + U_2 e^0(\omega_2^T X_{(0)}) + U_3 e^{-t}(\omega_3^T X_{(0)})]$$
$$= -\omega_2^T \omega_{(0)}$$

$\therefore\ bounded$

:) $\because\ \frac{1}{s}\ has\ pole = 0 \quad \therefore\ Not\ BIBO\ Stable.$

P2.

(a) $A = \frac{1}{2}\begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}, \quad \lambda = 0$

$(0)^k = \beta_0,\ k \to \infty \quad \beta_0 = 0$

$$\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A^k = \begin{cases} I_2, k = 0 \\ A, & k = 1 \\ 0, & k > 1 \end{cases}$$

(b) $C_3 = \begin{bmatrix} B & AB & A^2B \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \end{bmatrix}$

$$X_{[3]} = A^3 X_{[0]} + \begin{bmatrix} A^2B & AB & B \end{bmatrix}\begin{bmatrix} u_{[0]} \\ u_{[1]} \\ u_{[2]} \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} u_{[0]} \\ u_{[1]} \\ u_{[2]} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$u_1 = 0, \quad u_2 = 1, \quad u_0 = 0$

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Ryne Rayburn

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