CS-1 2010

$\boxed{1(a)}$ $P(\min(X_1, X_2, X_3) \le a)$

$= 1 - P(\min(X_1, X_2, X_3) > a)$

$= 1 - P(\{X_1 > a\} \cap \{X_2 > a\} \cap \{X_3 > a\})$

using independence of $X_i$

$= 1 - P(X_1 > a)\, P(X_2 > a)\, P(X_3 > a)$

$= 1 - \left( \int_0^a \lambda e^{-\lambda x} dx \right)^3$

$= 1 - \left(1 - e^{-\lambda a}\right)^3 1_{[0,\infty)}(a)$

$\boxed{1(b)}$ $P(\max(X_1, X_2, X_3) \le a)$

$= P(\{X_1 \le a\} \cap \{X_2 \le a\} \cap \{X_3 \le a\})$

using independence of $X_i$

$= P(\{X_1 \le a\})\, P(\{X_2 \le a\})\, P(\{X_3 \le a\})$

$= (1 - e^{-\lambda a})^3 1_{[0,\infty)}(a)$

$\boxed{2(a)}$ $R_{XX}(t_1, t_2) = E[X(t_1) X^*(t_2)]$

$= E[(X_1 e^{j\omega_1 t_1} + X_2 e^{j\omega_2 t_1})(X_1^* e^{-j\omega_1 t_2} + X_2^* e^{-j\omega_2 t_2})]$

$= E[X_1 X_1^*]\, e^{j\omega_1 (t_1 - t_2)}$

$+ E[X_2 X_2^*]\, e^{j\omega_2 (t_1 - t_2)}$

$+ E[X_1 X_2^*]\, e^{j\omega_1 t_1} . e^{-j\omega_2 t_2}$

$+ E[X_1^* X_2]\, e^{j\omega_2 t_1} . e^{-j\omega_1 t_2}$

$\boxed{2(b)}$ For $X(t)$ to be WSS, $R_{XX}(t_1, t_2)$ has to be function of time difference $(t_1 - t_2)$.

Thus, $E[X_1 X_2^*] = (E[X_1^* X_2])^* = 0$ is a condition that make $R_{XX}(t_1, t_2) = f(t_1 - t_2)$.

$\boxed{3(a)}$ $R_V(k) = R_A(2k) = q^2 \rho_1^{2|k|}$

$\boxed{3(b)}$ Suppose $k = k_1 - k_2$.

If $k_1$ is even number and $k_2$ is odd, $R_W(k) = 0$. Similarly, $R_W(k) = 0$ if $k_1$ is odd and $k_2$ is even or $k_1$ is even and $k_2$ is even.

If $k_1$ is odd and $k_2$ is odd.

$R_W(k) = \sigma_1^2 \rho_1^{|k|} = \sigma_1^2 \rho_1^{|k_1 - k_2|}$

Thus, $R_W(k_1 - k_2) = \begin{cases} \sigma_1^2 \rho_1^{|k_1 - k_2|} & \text{if } k_1, k_2 \text{ are odd} \\ 0 & \text{otherwise} \end{cases}$

$\boxed{3(c)}$ $V_m$ is WSS but $W_k$ is not.

$\boxed{4}$ Given that $P(\{|X_n - a| \ge \varepsilon\}) \to 0$ and $P(\{|Y_n - b| \ge \varepsilon\}) \to 0$ when $n \to \infty$

From triangular inequality,

$|(X_n - a) + (Y_n - b)| \le |(X_n - a)| + |(Y_n - b)|$

If $|(X_n - a) + (Y_n - b)| \ge \varepsilon$, at least one of $|(X_n - a)|$ and $|(Y_n - b)|$ have to be greater than $\frac{\varepsilon}{2}$

on the back $\longrightarrow$

$\boxed{4}$ continue...

$$P(\{|(X_n - a) + (Y_n - b)| \geq \varepsilon\})$$

$$\leq Pr(\{|(X_n - a)| \geq \tfrac{\varepsilon}{2}\} \cup$$

$$\{|(Y_n - b)| \geq \tfrac{\varepsilon}{2}\})$$

$$\leq P(\{|X_n - a| \geq \tfrac{\varepsilon}{2}\})$$

$$+ P(\{|Y_n - b| \geq \tfrac{\varepsilon}{2}\})$$

$\leq 0$ when $n \longrightarrow \infty$