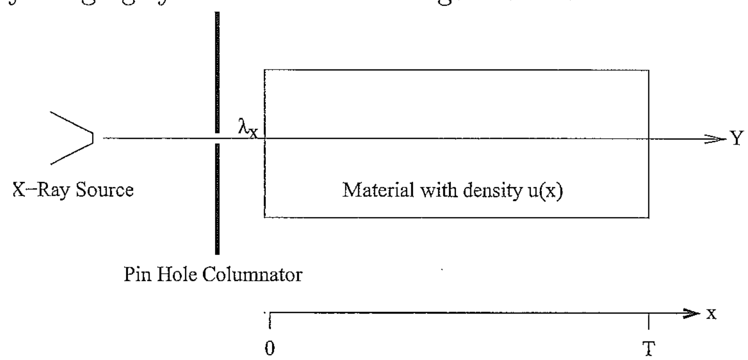
CS-5
August 2014 QE

Problem 1.(50pt)

Consider an X-ray imaging system shown in the figure below.



Photons are emitted from an X-ray source and columnated by a pin hole in a lead shield. The columated X-rays then pass in a straight line through an object of length T with density $u(x)$ where x is the depth into the object. The number of photons in the beam at depth x is denoted by the random variable Y_x with Poisson density given by

$$P\{Y_x = k\} = \frac{e^{-\lambda_x}\lambda_x^k}{k!} \, .$$

where x is measured in units of cm and $\mu(x)$ is measured in units of cm^{-1}.

a)(10pt) Calculate the $E\left[Y_x\right]$.

b)(10pt) Write a differential equation which describes the behavior of λ_x as a function of x.

c)(10pt) Calculate an expression for λ_x in terms of $u(x)$ and λ_0 by solving the differential equation.

d)(10pt) Calculate an expression for the integral of the density, $\int_0^T u(x)dx$, in terms of λ_0 and λ_T.

e)(10pt) Give an estimate for the integral of the density, $\int_0^T u(x)dx$, in terms of the measured values of Y_0 and Y_T.

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Problem 2.(50pt)
Consider the 2D difference equation

$$y(m,n) = bx(m,n) + ay(m-1,n) + ay(m,n-1) - a^2 y(m-1,n-1)$$

where $b \in \Re$ and $a \in (-1,1)$ are two constants, and $Y(z_1,z_2)$ and $X(z_1,z_2)$ are the 2D Z-transforms of $y(m,n)$ and $x(m,n)$ respectively.

a)(10pt) Calculate $H(z_1,z_2) = \frac{Y(z_1,z_2)}{X(z_1,z_2)}$, the 2D transfer function of the causal system. Make sure to express your result in factored form.

b)(10pt) Calculate, $h(m,n)$, the impulse response of the system with transfer function $H(z_1,z_2)$.

c)(10pt) In an application, $x(m,n)$ is an input image, and $y(m,n)$ is an output filtered image. Specify a relationship between a and b so that the average values of the input and output images remain the same.

d)(10pt) For parts d) and e), assume the input, $x(m,n)$, are i.i.d. Gaussian random variables with mean zero and variance 1. Calculate the autocovariance given by

$$R_x(k,l) = E\left[x(m,n)x(m+k,n+l)\right]$$

and its associated power spectral density $S_x(e^{j\mu}, e^{j\nu})$.

e)(10pt) Calculate $S_y(e^{j\mu}, e^{j\nu})$, the power spectral density of $y(m,n)$.

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