QE remediation critique AC-3

CNC

July 2019

Problem 1

Figure 1 is Liu's solution. I use KKT condition to solve the problem and her use three case to exclude one of the parameter to see the effect. Her solution is another point of view to solve this problem.

Problem 2

Figure 2 is Liu's solution. In first part, her answer for x is the same as mine, but it seemed that she didn't solve the answer for λ. The second part of the dimension answer is the same as mine.

Problem 3

Figure 3 is Liu's solution. We use the same concept and method to solve this problem.

Problem 4

Figure 4 is Liu's solution. Her transformation from primal to dual problem seems having some problem especially in the dimension part. As a result, there is further impact on the positive sign and negative sign for her final answer for λ.

Problem 5

Figure 5 is Liu's solution. We use the same concept and method to solve this problem.

Problem 1

minimize $x_1 + 2x_2 + 4x_3$, subject to $x_1 + 2x_2 + x_3 = 5$, $2x_1 + 3x_2 + x_3 = 6$

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

1. if $x_1 x_2$,

$$\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 1/(3-4) \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$

$f = -3 + 8 = 5$ minimum.

2. if $x_1 x_3$,

$$\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_3 \end{bmatrix} = 1/(1-2) \begin{bmatrix} 1 & -1 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$f = 1 + 4 \times 4 = 16$

3. if $x_2 x_3$,

$$\begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} x_2 \\ x_3 \end{bmatrix} = 1/(2-3) \begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$f = 2 + 12 = 14$.

Figure 1: JL's solution

Problem 2

1. $l = \frac{1}{2}x^T Qx - b^T x + \lambda(Ax - c)$
$$Dl = Qx - b^T + A\lambda^T = 0$$
$$x = Q^{-1}(b^T - A\lambda^T)$$
$$Ax = c$$
$$AQ^{-1}(b^T - A\lambda^T) = c$$
$$b^T - A\lambda^T = QA^{-1}c$$
$$x = A^{-1}c$$

2. A should be m by n, the rank of A is m.

Figure 2: JL's solution

Problem 3

1. minimize $x_1^2 - x_1 + x_2 + x_1x_2$,

$$Df = \begin{bmatrix} 2x_1 - 1 + x_2 \\ 1 + x_1 \end{bmatrix}$$

Basic feasible direction is

$$\begin{bmatrix} \frac{1}{2} \\ 0 \end{bmatrix} + \alpha \begin{bmatrix} d^1 \\ d^2 \end{bmatrix} \geq 0$$

$$\begin{cases} \frac{1}{2} + \alpha d^1 \geq 0 \\ \alpha d^2 \geq 0. \end{cases} \tag{1}$$

$$\begin{cases} d^1 \geq -\frac{1}{2\alpha} \\ d^2 \geq 0. \end{cases} \tag{2}$$

is feasible direction

2. For SONC,

$$Df(x_0) = \begin{bmatrix} 1-1 \\ \frac{3}{2} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{3}{2} \end{bmatrix}$$

is feasible direction.

$$D^2f(x_0) = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$$

$$d^T D^2 f(x_0) d = \begin{bmatrix} d^1 & d^2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} d^1 \\ d^2 \end{bmatrix} = \begin{bmatrix} 2d^1 + d^2 \\ d^1 \end{bmatrix} \begin{bmatrix} d^1 \\ d^2 \end{bmatrix}$$

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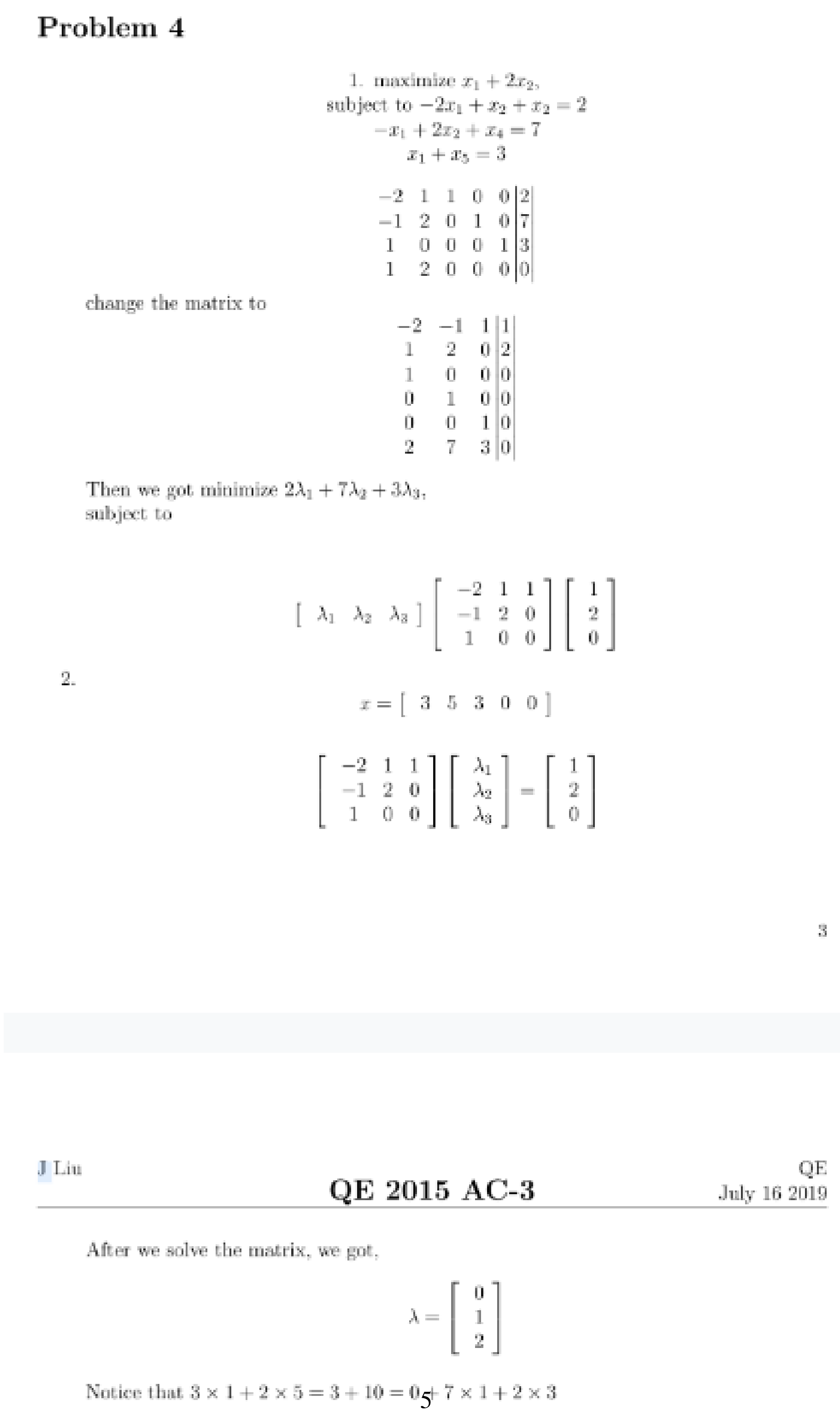
$$= 2(d^1)^2 + d^1d^2 + d^1d^2$$
$$= 2((d^1)^2 + d^1d^2)$$

since $d^1, d^2 \geq 0$, so $d^T D^2 f(x_0) d \geq 0$ is satisfied.

Figure 3: JL's solution



Problem 4

$$\begin{gathered}\text{1. maximize } x_1 + 2x_2,\\ \text{subject to } -2x_1 + x_2 + x_2 = 2\\ -x_1 + 2x_2 + x_4 = 7\\ x_1 + x_5 = 3\end{gathered}$$

$$\left.\begin{array}{rrrrr|r} -2 & 1 & 1 & 0 & 0 & 2\\ -1 & 2 & 0 & 1 & 0 & 7\\ 1 & 0 & 0 & 0 & 1 & 3\\ 1 & 2 & 0 & 0 & 0 & 0 \end{array}\right|$$

change the matrix to

$$\left.\begin{array}{rrr|r} -2 & -1 & 1 & 1\\ 1 & 2 & 0 & 2\\ 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 2 & 7 & 3 & 0 \end{array}\right|$$

Then we got minimize $2\lambda_1 + 7\lambda_2 + 3\lambda_3$,
subject to

$$\begin{bmatrix} \lambda_1 & \lambda_2 & \lambda_3 \end{bmatrix} \begin{bmatrix} -2 & 1 & 1\\ -1 & 2 & 0\\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1\\ 2\\ 0 \end{bmatrix}$$

2.

$$x = \begin{bmatrix} 3 & 5 & 3 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 1 & 1\\ -1 & 2 & 0\\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1\\ \lambda_2\\ \lambda_3 \end{bmatrix} = \begin{bmatrix} 1\\ 2\\ 0 \end{bmatrix}$$

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After we solve the matrix, we got,

$$\lambda = \begin{bmatrix} 0\\ 1\\ 2 \end{bmatrix}$$

Notice that $3 \times 1 + 2 \times 5 = 3 + 10 = 0 + 7 \times 1 + 2 \times 3$

Figure 4: JL's solution

Problem 5

$$f = \frac{1}{2}x^T \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} x + \frac{1}{2}\begin{bmatrix} 2 \\ 1 \end{bmatrix} x + 3$$

$$x^0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$g^0 = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$d^0 = -g^0 = -\frac{1}{2}\begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\alpha^0 = \frac{-g^{0T}d^0}{d^{0T}Qd^0} = \frac{-\begin{bmatrix} -2 & -1 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix}}{\begin{bmatrix} 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix}} = \frac{5}{\begin{bmatrix} 2 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix}} = \frac{5}{6}$$

$$x^1 = x^0 + \alpha^0 d^0 = \frac{5}{6}\frac{-1}{2}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = -\frac{5}{12}\begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$g^1 = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}(-\frac{5}{12})\begin{bmatrix} 2 \\ 1 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$\beta_k = \frac{\frac{1}{6}\begin{bmatrix} 1 & -2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} - \frac{1}{2}\begin{bmatrix} 2 \\ 1 \end{bmatrix}}{-\frac{1}{2}\begin{bmatrix} 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} - \frac{1}{2}\begin{bmatrix} 2 \\ 1 \end{bmatrix}} = -\frac{1}{3}\frac{\begin{bmatrix} 1 & -4 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix}}{\begin{bmatrix} 2 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix}} = \frac{1}{9}$$

$$d^1 = -g^1 + \beta d^0 = \frac{1}{18}\begin{bmatrix} -5 \\ 5 \end{bmatrix}$$

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$$\alpha^1 = \frac{-g^{1T}d^1}{d^{1T}Qd^1} = \frac{-\frac{1}{6}\begin{bmatrix} 1 & -2 \end{bmatrix}\frac{1}{18}\begin{bmatrix} -5 \\ 5 \end{bmatrix}}{\frac{1}{18}\begin{bmatrix} -5 & 5 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\frac{1}{18}\begin{bmatrix} -5 \\ 5 \end{bmatrix}} = -3\frac{-5-10}{\begin{bmatrix} -5 & 10 \end{bmatrix}\begin{bmatrix} -5 \\ 5 \end{bmatrix}} = \frac{3}{5}$$

$$x^2 = x^1 + \alpha^1 d^1 = \frac{1}{4}\begin{bmatrix} -4 \\ -1 \end{bmatrix}$$

$$g^2 = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\frac{1}{4}\begin{bmatrix} -4 \\ -1 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

x^2 is optimal.

Figure 5: JL's solution