

ECE-QE AC2-2011 - Rhea

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AC-2 2011

P1.

a)
$$C = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -3 & 3 \end{bmatrix}$$

$$\Rightarrow$$
 Not controllable. Subspace $\begin{bmatrix} 1 \\ -3 \end{bmatrix}$

$$\mathbf{b)} \qquad 0 = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 6 & 2 \end{bmatrix}$$

Not observable.

$$3x_1+r=0 \qquad x_1=-rac{1}{3}\,r \qquad spanegin{bmatrix}1\-3\end{bmatrix}$$

$$\mathbf{c)} \qquad \lambda I - A = \begin{vmatrix} \lambda - 2 & -1 \\ 0 & \lambda + 1 \end{vmatrix} \qquad (\lambda - 2)(\lambda + 1) = 0 \qquad \lambda_1 = 2 \qquad \lambda_2 = -1$$

$$\lambda = 2 \qquad \begin{bmatrix} 0 & -1 & 1 \\ 0 & 3 & -3 \end{bmatrix}$$

$$rank < 2 \qquad associated \qquad with \ \lambda = 2 > 0$$

 $\Rightarrow \qquad not \qquad stablizable$

d)
$$\lambda = -1$$
 $\begin{vmatrix} -3 & -1 & 1 \\ 0 & 0 & -3 \end{vmatrix}$ $rank = 2$

$$\Rightarrow$$
 $\lambda=2$ has to be eigenvalue

$$\Rightarrow$$
 [1 -1] can not be eigenvalues

$$which \hspace{0.5cm} is \hspace{0.5cm} stable \hspace{0.5cm} \longrightarrow \hspace{0.5cm} detectable$$

$$\mathbf{f)} \qquad \left[\frac{\lambda I - A}{C} \right] = \begin{bmatrix} -3 & -1 \\ 0 & 0 \\ 3 & 1 \end{bmatrix} \qquad \lambda = -1 \qquad rank < 2$$

$$must \quad contains \quad eigenvalue \quad = -1$$

$$let \qquad l = \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} \qquad A - lC = \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix} - \begin{bmatrix} 3l_1 & l_1 \\ 3l_2 & l_2 \end{bmatrix}$$

$$\bar{\mathbf{A}} = \begin{bmatrix} 2 - 3l_1 & 1 - l_1 \\ -3l_2 & -1 - l_2 \end{bmatrix}$$

$$\lambda I - \bar{\mathbf{A}} = \begin{bmatrix} -3 + 3l_1 & l_1 - 1 \\ 3l_2 & l_2 \end{bmatrix} \qquad \lambda = -1$$

$$\lambda I - ar{\mathtt{A}} = egin{bmatrix} -3 + 3l_1 & l_1 - 1 \ 3l_2 & l_2 \end{bmatrix} \quad \lambda = -1 \ (-3 + 3l_1)l_2 - 3l_2(l_1 - 1) = 0 \ 3(l_1 - 1)l_2 - 3l_2(l_1 - 1) = 0 \ Yes$$

$$\begin{split} \mathbf{g}) & y(t) = CAX + Cbu & u = 0 \\ y(t) = Ce^{At}X(0) \\ (SI - A)^{-1} &= \frac{1}{(S-2)(S+1)} \begin{vmatrix} S+1 & 1\\ 0 & S-2 \end{vmatrix} \\ &= \begin{bmatrix} \frac{1}{S-2} & \frac{\frac{1}{3}}{S-2} - \frac{\frac{1}{3}}{S+1}\\ 0 & \frac{1}{S+1} \end{bmatrix} \\ \xi^{-1} &= (SI - A)^{-1} = \begin{bmatrix} e^{2t} & \frac{1}{3}e^{2t} - \frac{1}{3}e^{-t}\\ 0 & e^{-t} \end{bmatrix} = e^{At} \\ y(t) &= \begin{bmatrix} 3 & 1 \end{bmatrix} \begin{bmatrix} e^{At} \end{bmatrix} \begin{bmatrix} 1\\ 0 \end{bmatrix} = \begin{bmatrix} 3e^{2t} & e^{2t} \end{bmatrix} \begin{bmatrix} 1\\ 0 \end{bmatrix} = 3e^{2t} \end{split}$$

P2.

$$C(ZI - A)^{-1}B = C(ZI - A)^{-1}\dot{\mathbf{B}}$$
 $C_1A_1^KB_1 = C_2A_2^KB_2$
 $C_1 = C_2 = C$
 $A_1 = A_2 = A$
observable
 $CA \neq 0$
 $B_1 = B_2$
 $\Rightarrow B = \dot{\mathbf{B}}$

P3.

$$\begin{bmatrix} \frac{2}{7} & \frac{3}{7} & \frac{2}{7} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{2}{7} & \frac{3}{7} & \frac{2}{7} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

a)
$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ & 1 \\ & & 1 \end{bmatrix}$$

$$\begin{array}{l} \text{b) } (A)^k = \mathbf{B}_o I + A \mathbf{B}_1 + A^2 \mathbf{B}_2 \\ \lambda I - A = \begin{bmatrix} \lambda - \frac{1}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{3} & \lambda - \frac{1}{3} & -\frac{1}{3} \\ 0 & -\frac{1}{2} & \lambda - \frac{1}{2} \end{bmatrix} \end{array}$$

$$\begin{split} &(\lambda - \frac{1}{2})^2 (\lambda - \frac{1}{3}) - (\lambda - \frac{1}{2}) \, \frac{1}{6} - (\lambda - \frac{1}{2}) \, \frac{1}{6} = 0 \\ & \lambda^2 - \frac{1}{3} \, \lambda - \frac{1}{2} \, \lambda + \frac{1}{6} - \frac{1}{6} - \frac{1}{6} = o \\ & \lambda^2 - \frac{5}{6} \, \lambda - \frac{1}{6} = 0 \\ & (\lambda - 1)(\lambda + \frac{1}{6}) = 0 \\ & \lambda_1 = 1 \quad \lambda_2 = \frac{1}{2} \quad \lambda_3 = -\frac{1}{6} \\ & \left[\frac{2}{7} \quad \frac{3}{7} \quad \frac{2}{7} \, \right] A = \left[\frac{2}{7} \quad \frac{3}{7} \quad \frac{2}{7} \right] \\ & \left[\frac{2}{7} \quad \frac{3}{7} \quad \frac{2}{7} \, \right] A^K = \left[\frac{2}{7} \quad \frac{3}{7} \quad \frac{2}{7} \right] \end{split}$$

c)
$$(1)^k = \beta_o + \beta_1 + \beta_2$$

 $(\frac{1}{2})^k = \beta_o + \frac{1}{2}\beta_1 + \frac{1}{4}\beta_2$
 $(-\frac{1}{6})^k = \beta_o - \frac{1}{6}\beta_1 + (\frac{1}{6})^2\beta_2$
 $\Rightarrow \beta_o = -\frac{1}{7} \qquad \beta_1 = -\frac{4}{7} \qquad \beta_2 = \frac{12}{7}$

$$\begin{split} A^k &= \mathbf{B}_o + \mathbf{B}_1 A + \mathbf{B}_2 A^2 \\ &= \begin{bmatrix} \frac{2}{7} & \frac{3}{7} & \frac{2}{7} \\ \frac{2}{7} & \frac{3}{7} & \frac{2}{7} \\ \frac{2}{7} & \frac{3}{7} & \frac{2}{7} \end{bmatrix} \\ X[K] &= A^K X[0] = [\overline{X} \qquad \overline{X} \qquad \overline{X}]^T \end{split}$$

$$e)$$
 $False$

$$\begin{split} a) \mathbf{\Phi}(t) &= e^{\int_0^t A \, \mathrm{d} \boldsymbol{\tau}} = e^{\left[\frac{\ln(t+1)}{\ln(t+1)} \quad o \right]} = \left[e^{-\ln(t+1)} \quad o \right] \left[\begin{array}{c} 1 & o \\ 0 & 1 \end{array} \right] \left[\begin{array}{c} 1 & o \\ -\ln(t+1) & 1 \end{array} \right] \\ &= \left[\begin{array}{c} e^{-\ln(t+1)} \quad o \\ -\ln(t+1) & 1 \end{array} \right] \\ \mathbf{\Phi}(t, \boldsymbol{\tau}) &= \mathbf{\Phi}(\boldsymbol{\tau}) \mathbf{\Phi}^{-1}(t, \boldsymbol{\tau}) = \left[\begin{array}{c} e^{-\ln(t+1)} & o \\ -\ln(t+1) & 1 \end{array} \right] \left[\begin{array}{c} e^{-\ln(\tau+1)} & o \\ -\ln(\tau+1) & 1 \end{array} \right]^{-1} \end{split}$$

b)
$$t o \infty$$
 $e^{-\ln(t+1)} o 0$ $-\ln(t+1) o 0$ $\Phi(t) o 0$ not asy $stable$

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