CS-3
August 2010 QE

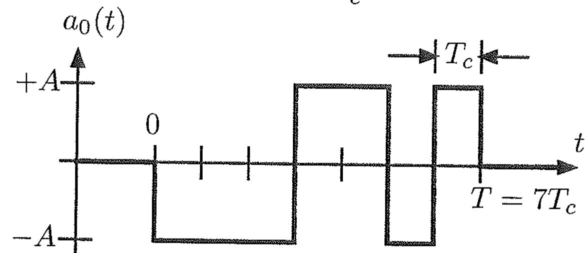
Problem 1. [50 pts. total] Consider the coherent binary detection problem for deciding between

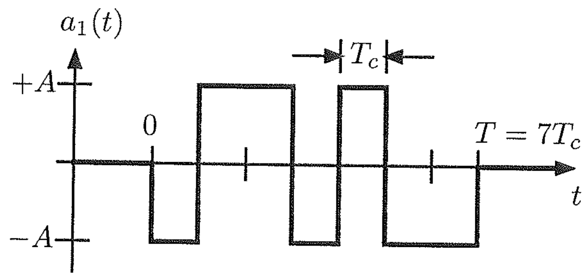
$$H_0: \ s_0(t) = \sqrt{2}a_0(t)\cos(2\pi f_c t + \phi) \quad \text{is transmitted}$$

and

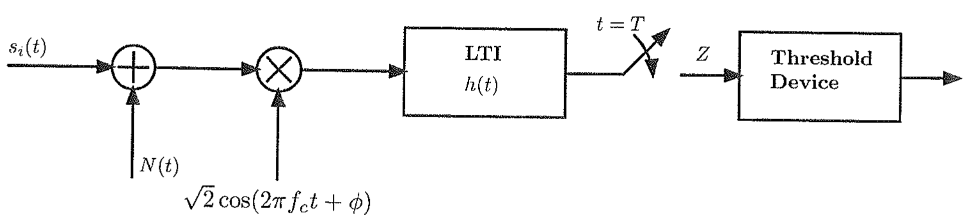
$$H_1: \ s_1(t) = \sqrt{2}a_1(t)\cos(2\pi f_c t + \phi) \quad \text{is transmitted}$$

where $a_0(t)$ and $a_1(t)$ are as indicated below. Note that the tic marks are spaced by T_c and that the signals start at $t = 0$ and end at $T = 7T_c$.





The receiver architecture for this problem consists of a mixer followed by an LTI filter with impulse response $h(t)$, a sampler, and a threshold device as drawn below. The noise $N(t)$ is AWGN with power spectral density height equal to $N_0/2$.



(a) How should the filter $h(t)$ be chosen for optimal performance?

(b) Given your choice for $h(t)$ characterize the decision statistic Z by finding its pdfs under each of the two hypotheses H_0 and H_1. Express the parameters of the pdfs in terms of A, T_c, and N_0.

(c) Assuming the two hypotheses are equally likely how should the threshold in the threshold device be chosen for minimum average probability of error and what is the exact form of the threshold test? Show all work.

(d) Find the minimum average probability of error for this communication system. Show all work.

Problem 2. [50 pts. total] This problem concerns a calculation related to the demodulation of differentially encoded BPSK. Assume that over a time interval $-T < t < T$ the signal transmitted is

$$s(t) = \begin{cases} \sqrt{2}d_{-1}\cos(\omega_c t + \phi) & -T < t < 0 \\ \sqrt{2}d_0\cos(\omega_c t + \phi) & 0 < t < T \end{cases}.$$

Where the data symbol d_k is either -1 or +1, $\omega_c T$ is a multiple of 2π, the channel is an additive white Gaussian noise channel, and a phase reference

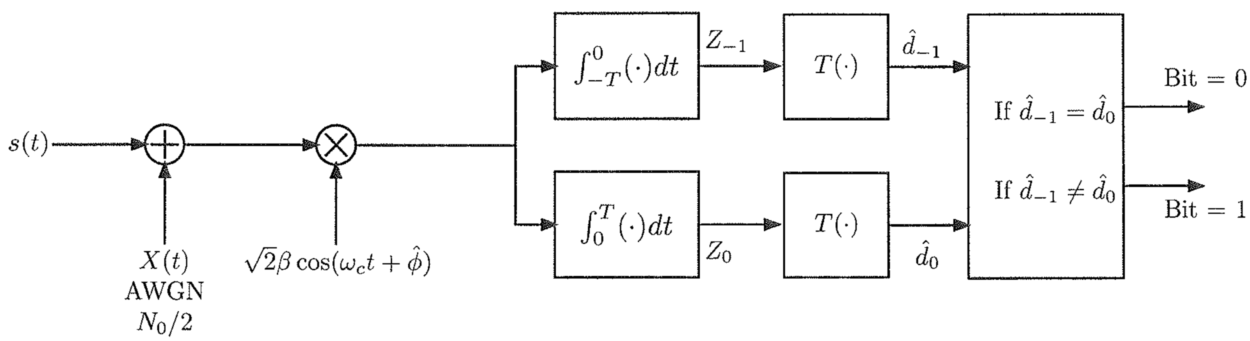
$$r(t) = \cos(\omega_c t + \hat{\phi})$$

is available where $\hat{\phi} = \phi$ or $\hat{\phi} = \phi + \pi$ (but it is constant over the length of the observation window $-T < t < T$).

The information to be transmitted is differentially encoded in the following way:

- If 0 is transmitted, then $d_0 = d_{-1}$.
- If 1 is transmitted, then $d_0 = -d_{-1}$.

The following receiver architecture is used:



where $T(\cdot)$ is a quantizer with input-output characteristic

$$T(\text{input}) = \begin{cases} +1 & \text{input } \geq 0 \\ -1 & \text{input } < 0 \end{cases}$$

and the final bit decision is made depending on whether or not the two quantizer outputs are equal as shown.

(a) Explain why the receiver works for either possible choice of reference phase $\hat{\phi} = \phi$ or $\hat{\phi} = \phi + \pi$. Therefore, what common issue in synchronization does this differential encoder / differential decoder address?

(b) Show that the conditional probability of error given that bit 0 is transmitted is of the form $P_{e,0} = 2p(1-p)$ and find the value of p in terms of the complementary distribution function for the Gaussian

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-\lambda^2/2\pi} d\lambda.$$