

## ECE-QE AC2-2013 - Rhea

pl a) 
$$A = \left[egin{array}{cc} -1 & 1 \ 0 & -2 \end{array}
ight]$$

$$X(t) = \left[egin{array}{c} X_1(t) \ X_2(t) \end{array}
ight]$$

$$\left\{ egin{aligned} \dot{x}_1(t) &= -X_1(t) + X_2(t) \ \dot{x}_2(t) &= -2X_2(t) \end{aligned} 
ight.$$

$$\Phi(t) = egin{bmatrix} \Phi_{\,1}(t) & \Phi_{\,2}(t) \end{bmatrix}$$

For 
$$\Phi_1(t)assumeX_{(0)}=egin{bmatrix}1\0\end{bmatrix}$$

$$\left\{ egin{aligned} \dot{x}_1(t) &= e^{\int_t^0 - 1 \, \mathrm{d}t} \, X_1(0) = e^{-t} \ \dot{x}_2(t) &= e^{\int_t^0 - 2 \, \mathrm{d}t} \, X_2(0) = 0 \end{aligned} 
ight.$$

$$(x_2(t) \equiv e^{jt})^{-2} \stackrel{\sim}{=} X_2(0)$$

$$\therefore \Phi_1(t) = \begin{bmatrix} e^{-t} \\ 0 \end{bmatrix}$$

$$egin{aligned} For & \Phi_2(t) assume X_(0) = egin{bmatrix} 0 \ 1 \end{bmatrix} \end{aligned}$$

$$X_2(t) = e^{\int_t^0 -2\,\mathrm{d}t}\, X_2(0) = e^{-2t}$$

$$\Phi_(t) = e^{\int_t^0 A \, \mathrm{d}t} = e^{\begin{bmatrix} -t & t \\ 0 & -2t \end{bmatrix}} = \begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-2t} \end{bmatrix} \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} e^{-t} & te^{-t} \\ 0 & e^{-2t} \end{bmatrix}$$

$$\Phi(t,\iota) = \Phi(t)\Phi(\iota)^- 1 = \left[egin{array}{cc} e^{-t} & te^{-t} \ 0 & e^{-2t} \end{array}
ight] \left[egin{array}{cc} e^{-\iota} & \iota e^{-\iota} \ 0 & e^{-2\iota} \end{array}
ight]$$

b) 
$$A = egin{bmatrix} -\cos t & \cos t \ 0 & -2\cos t \end{bmatrix}$$

$$\Phi_(t) = e^{\int_t^0 A \,\mathrm{d}t} = egin{bmatrix} e^{\sin t} & 0 \ 0 & e^{2\sin t} \end{bmatrix} egin{bmatrix} 1 & -\sin t \ 0 & 1 \end{bmatrix} = egin{bmatrix} e^{\sin t} & -\sin t e^{\sin t} \ 0 & e^{2\sin t} \end{bmatrix}$$

$$\Phi(t,\iota) = \Phi(t) \cdot \Phi(\iota)^- 1 = \begin{bmatrix} e^{\sin t} & -sinte^{\sin t} \\ 0 & e^{2\sin t} \end{bmatrix} \begin{bmatrix} e^{\sin \iota} & -sin\iotae^{\sin \iota} \\ 0 & e^{2\sin \iota} \end{bmatrix}^{-1}$$

p2 a) 
$$|\lambda {
m I}-A|=\left[egin{array}{cc} \lambda+2 & -2 \ 1 & \lambda-1 \end{array}
ight]$$

$$\lambda_1=0$$
  $\lambda_2=-1$ 

Marginally stable ,not asy , stable

b) 
$$c = egin{bmatrix} B & AB \end{bmatrix} = egin{bmatrix} 1 & 0 \ 1 & 0 \end{bmatrix}$$

$$rank = 1 \neq 2$$

not observable, unobservable subspace 
$$\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$$
 c)  $0 = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ 

$$rank=1 
eq 2$$

not observable, unobservable subspace 
$$\left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$$

$$\begin{split} C(SI-A)^{-1}B &= \begin{bmatrix} 1\\-1 \end{bmatrix} \begin{bmatrix} \frac{S-1}{2S} & \frac{S+2}{2S}\\ \frac{S+1}{-2S} & \frac{S-2}{2S} \end{bmatrix} \begin{bmatrix} 1\\-1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 1\\1 \end{bmatrix} = 2 \end{split}$$

e) i) false

ii) false

f) 
$$A-BK = egin{bmatrix} -2-k_1 & 2-k_2 \ -1-k_1 & 1-k_2 \end{bmatrix} |\lambda-A+BK| = \lambda^2 + (a+b+1)\lambda + 3a+3-ab = 0$$

$$\lambda_1 = -3 \quad and \quad \lambda_2 = -1 \left\{ egin{array}{ll} -3a + 9 - ab = 0 \ 2a - b + 3 - ab = 0 \end{array} 
ight.$$

$$a = 0, b = 3k = [0 \ 3]$$

$$\mathbf{g})\begin{bmatrix}\lambda\mathbf{I}-A\\C\end{bmatrix}=\begin{bmatrix}\lambda+2&-2\\1&\lambda-1\\1&-1\end{bmatrix}\text{ must contain }\lambda=0, no$$

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