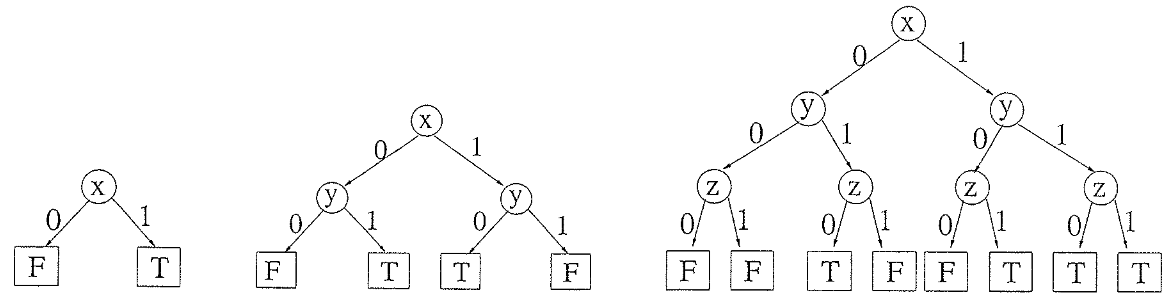
1 Logic Function and Static CMOS Implementation (32 points)

A *binary decision diagram* (BDD) is a representation of a logic function. A BDD has *internal nodes* and *leaves*. Each internal node is shown as a circle, and represents an input of the logic function. Each leaf is shown as a square, representing one of the two possible logic values: *true* (T) or *false* (F). An edge may connect (i) two internal nodes or (ii) one internal node and one leaf. Each internal node has exactly two outgoing edges, which are labeled 0 and 1, representing an assignment of 0 and 1 to the corresponding input of the function.

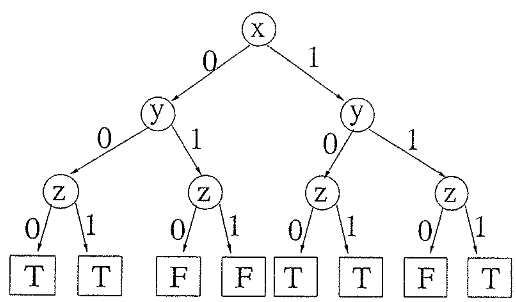
The following are three examples of BDDs: f is the output; x, y, and z are the inputs.



(a) $f = x$ (b) $f = \bar{x}y + x\bar{y}$ (c) $f = \bar{x}y\bar{z} + x\bar{y}z + xy\bar{z} + xyz$

1.1 Binary Decision Diagram and Truth Table (8 points)

Write the truth table of the following BDD.



1.2 Logic Minimization (12 points)

Minimize the logic expression. You should use the minimum number of literals. Write down your derivation.

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1.3 CMOS Gates (12 points)

Based on your answer in 1.2, draw a transistor-level implementation of the function using static CMOS gates. The inputs to your implementation should include only x, y, and z.

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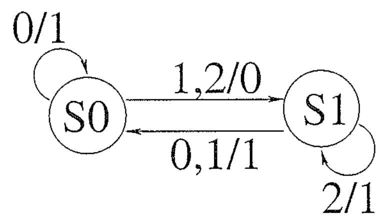
2 Minimizing Finite State Machines (28 points)

In this question, all numbers use **decimal representations**.

In a *Mealy* state machine, the output depends on both the current state and the input. The input and output are annotated to each state transition, and represented as "input / output". The following is an example of a Mealy state machine with two states $S0$ and $S1$.



- If the current state is $S0$:
  - If the input is 0, the machine stays at state $S0$ and generates ouput 1.
  - If the input is 1 or 2, the machine goes to state $S1$ and generates ouput 0.
- If the current state is $S1$:
  - If the input is 0 or 1, the machine goes to state $S0$ and generates ouput 1.
  - If the input is 2, the machine stays at state $S1$ and generates ouput 1.

It is often desirable to design a state machine with the minimum number of states. This is usually achieved by *merging equivalent states.* **Definition:** Two states are equivalent if and only if, for any input, the two states have identical outputs and the corresponding next states are equivalent. State equivalence is symmetric, reflexive, and transitive.

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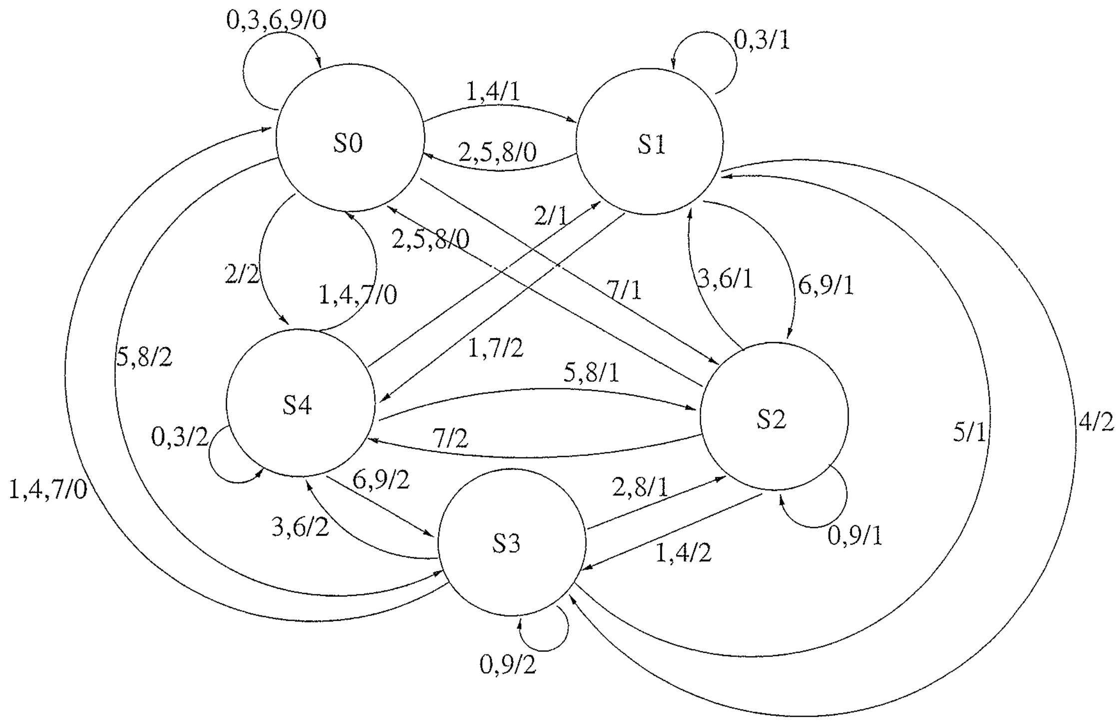
Consider the following state machine.



2.1 Equivalent States 1 (6 points)

Is it possible that $S0$ and $S2$ are equivalent? Justify your answer (*i.e.*, state the conditions under which they may be equivalent, or why they are not equivalent).

2.2 Equivalent States 2 (6 points)

Is it possible that $S1$ and $S4$ are equivalent? Justify your answer (*i.e.*, state the conditions under which they may be equivalent, or why they are not equivalent).

2.3 Equivalent States 3 (6 points)

Is it possible that $S1$ and $S2$ are equivalent? Justify your answer (*i.e.*, state the conditions under which they may be equivalent, or why they are not equivalent).

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2.4 Minimizing Finite State Machines (10 points)

Based on your reasoning, minimize the state machine. How many states does the minimized machine have?

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3 Sequential Circuits (32 points)

Consider the output of the following sequential circuit with positive-edge-triggered D flip flops. Suppose the clock cycle is 10 ns with 50% duty cycle. **Assumption:** the setup time of the flip flops, the delay of the inverter, the delay of the AND gate, and the delay of the wires can be ignored. **Answer the following questions and justify your answers.**

3.1 Y Rise and Fall (16 points)

Between 39ns and 121ns, how many times does Y change from Low to High? At what times?

Between 39ns and 121ns, how many times does Y change from High to Low? At what times?

3.2 Z Rise and Fall (10 points)

Between 39ns and 121ns, how many times does Z change from Low to High? At what times?

Between 39ns and 121ns, how many times does Z change from High to Low? At what times?

3.3 F Rise and Fall (6 points)

Between 39ns and 121ns, how many times does F change from Low to High? At what times?

Between 39ns and 121ns, how many times does F change from High to Low? At what times?

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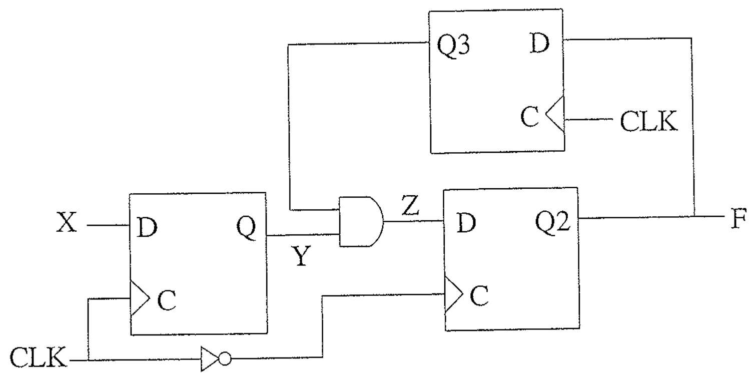

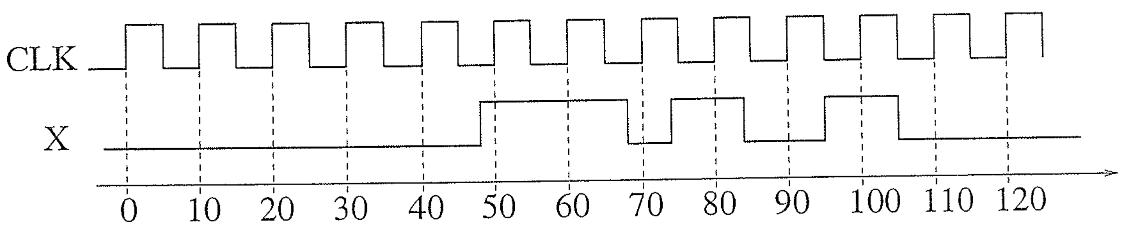

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