ECE-QE CS1-2012 - Rhea

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ECE Ph.D. Qualifying Exam

Communication, Networking, Signal and Image Processing (CS)

Question 1: Probability and Random Processes

August 2012

Question

Problem 1. 25 pts Consider a random experiment in which a point is selected at random from the unit square (sample space $\mathcal{S} = [0,1] \times [0,1]$). Assume that all points in $\mathcal{S}$ are equally likely to be selected. Let the random variable $\mathbf{X}(\omega)$ be the distance from the outcome ω to the nearest edge (i.e. the nearest point on one of the four sides) of the unit square.

(a) Find the c.d.f. of $\mathbf{X}$. Draw a graph of the c.d.f..

(b) Find the p.d.f. of $\mathbf{X}$. Draw a graph of the p.d.f..

(c) What is the probability that $\mathbf{X}$ is less than 1/8?

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Problem 2. 25 pts

State and prove the Chebyshev inequality for random variable $\mathbf{X}$ with mean μ and variance σ^2. In constructing your proof, keep in mind that $\mathbf{X}$ may be either a discrete or continuous random variable.

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Problem 3. 25 pts

Let $\mathbf{X}_1 \ldots \mathbf{X}_n \ldots$ be a sequence of independent, identical distributed random variables, each uniformly distributed on the interval [0, 1], an hence having pdf

$$f_X(x) = \begin{cases} 1, & \text{for } 0 \le x \le 1 \\ 0, & \text{elsewhere.} \end{cases}$$

Let $\mathbf{Y}_n$ be a new random variable defined by

$$\mathbf{Y}_n = min\{\mathbf{X}_1, \mathbf{X}_2, \ldots \mathbf{X}_n\}$$

(a) Find the pdf of $\mathbf{Y}_n$.

(b) Does the sequence $\mathbf{Y}_n$ converge in probability?

(c) Does the sequence $\mathbf{Y}_n$ converge in distribution? If yes, specify the cumulative function of the random variable it converges to.

wers and discussions

Back to ECE Qualifying Exams (QE) page

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