ECE-QE AC2-2011 - Rhea

Print

AC-2 2011

P1.

a) $C = [B \quad AB] = \begin{bmatrix} 1 & -1 \\ -3 & 3 \end{bmatrix}$

$\Rightarrow \quad Not \quad controllable. \quad Subspace \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

b) $0 = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 6 & 2 \end{bmatrix}$

Not observable.

$3x_1 + r = 0 \quad x_1 = -\frac{1}{3}r \quad span \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

c) $\lambda I - A = \begin{vmatrix} \lambda - 2 & -1 \\ 0 & \lambda + 1 \end{vmatrix} \quad (\lambda - 2)(\lambda + 1) = 0 \quad \lambda_1 = 2 \quad \lambda_2 = -1$

$\lambda = 2 \quad \begin{bmatrix} 0 & -1 & 1 \\ 0 & 3 & -3 \end{bmatrix}$

$rank < 2 \quad associated \quad with$
$\lambda = 2 > 0$

$\Rightarrow \quad not \quad stablizable$

d) $\lambda = -1 \quad \begin{vmatrix} -3 & -1 & 1 \\ 0 & 0 & -3 \end{vmatrix} \quad rank = 2$

$\Rightarrow \quad \lambda = 2 \quad has \quad to \quad be \quad eigeavalue$

$\Rightarrow \quad [1 \quad -1\ ] \quad can \quad not \quad be \quad eigeavalues$

e) $\lambda = 2 \quad \begin{bmatrix} 0 & -1 \\ 0 & 3 \\ 3 & 1 \end{bmatrix} \quad \lambda = -1 \quad \begin{bmatrix} -3 & -1 \\ 0 & 0 \\ 3 & 1 \end{bmatrix}$

$unobservable \quad modes \quad associated \quad to \quad \lambda = -1$
$which \quad is \quad stable \quad \longrightarrow \quad detectable$

f) $\left[\frac{\lambda I - A}{C} \right] = \begin{bmatrix} -3 & -1 \\ 0 & 0 \\ 3 & 1 \end{bmatrix} \quad \lambda = -1 \quad rank < 2$

$must \quad contains \quad eigenvalue \quad = -1$

$let \quad l = \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} \quad A - lC = \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix} - \begin{bmatrix} 3l_1 & l_1 \\ 3l_2 & l_2 \end{bmatrix}$

$\bar{A} = \begin{bmatrix} 2 - 3l_1 & 1 - l_1 \\ -3l_2 & -1 - l_2 \end{bmatrix}$

$\lambda I - \bar{A} = \begin{bmatrix} -3 + 3l_1 & l_1 - 1 \\ 3l_2 & l_2 \end{bmatrix} \quad \lambda = -1$

$(-3 + 3l_1)l_2 - 3l_2(l_1 - 1) = 0$

$3(l_1 - 1)l_2 - 3l_2(l_1 - 1) = 0$

Yes

g) $y(t) = CAX + Cbu \qquad u = 0$

$y(t) = Ce^{At}X(0)$

$(SI-A)^{-1} = \frac{1}{(S-2)(S+1)} \begin{vmatrix} S+1 & 1 \\ 0 & S-2 \end{vmatrix}$

$= \begin{bmatrix} \frac{1}{S-2} & \frac{\frac{1}{3}}{S-2} - \frac{\frac{1}{3}}{S+1} \\ 0 & \frac{1}{S+1} \end{bmatrix}$

$\xi^{-1} = (SI-A)^{-1} = \begin{bmatrix} e^{2t} & \frac{1}{3}e^{2t} - \frac{1}{3}e^{-t} \\ 0 & e^{-t} \end{bmatrix} = e^{At}$

$y(t) = [3 \quad 1][e^{At}]\begin{bmatrix} 1 \\ 0 \end{bmatrix} = [3e^{2t} \quad e^{2t}]\begin{bmatrix} 1 \\ 0 \end{bmatrix} = 3e^{2t}$

P2.

$C(ZI-A)^{-1}B = C(ZI-A)^{-1}\dot{B}$

$C_1A_1^KB_1 = C_2A_2^KB_2$

$C_1 = C_2 = C$

$A_1 = A_2 = A$

observable

$CA \neq 0$

$B_1 = B_2$

$\Rightarrow B = \dot{B}$

P3.

$$\begin{bmatrix} \frac{2}{7} & \frac{3}{7} & \frac{2}{7} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{2}{7} & \frac{3}{7} & \frac{2}{7} \end{bmatrix} \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}$$

a) $$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}$$

b) $(A)^k = ß_oI + Aß_1 + A^2ß_2$

$\lambda I - A = \begin{bmatrix} \lambda - \frac{1}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{3} & \lambda - \frac{1}{3} & -\frac{1}{3} \\ 0 & -\frac{1}{2} & \lambda - \frac{1}{2} \end{bmatrix}$

$(\lambda - \frac{1}{2})^2(\lambda - \frac{1}{3}) - (\lambda - \frac{1}{2})\frac{1}{6} - (\lambda - \frac{1}{2})\frac{1}{6} = 0$

$\lambda^2 - \frac{1}{3}\lambda - \frac{1}{2}\lambda + \frac{1}{6} - \frac{1}{6} - \frac{1}{6} = o$

$\lambda^2 - \frac{5}{6}\lambda - \frac{1}{6} = 0$

$(\lambda - 1)(\lambda + \frac{1}{6}) = 0$

$\lambda_1 = 1 \qquad \lambda_2 = \frac{1}{2} \qquad \lambda_3 = -\frac{1}{6}$

$\begin{bmatrix} \frac{2}{7} & \frac{3}{7} & \frac{2}{7} \end{bmatrix} A = \begin{bmatrix} \frac{2}{7} & \frac{3}{7} & \frac{2}{7} \end{bmatrix}$

$\begin{bmatrix} \frac{2}{7} & \frac{3}{7} & \frac{2}{7} \end{bmatrix} A^K = \begin{bmatrix} \frac{2}{7} & \frac{3}{7} & \frac{2}{7} \end{bmatrix}$

c) $(1)^k = ß_o + ß_1 + ß_2$

$(\frac{1}{2})^k = ß_o + \frac{1}{2}ß_1 + \frac{1}{4}ß_2$

$(-\frac{1}{6})^k = ß_o - \frac{1}{6}ß_1 + (\frac{1}{6})^2ß_2$

$\Rightarrow \qquad ß_o = -\frac{1}{7} \qquad ß_1 = -\frac{4}{7} \qquad ß_2 = \frac{12}{7}$

$A^k = ß_o + ß_1A + ß_2A^2$

$= \begin{bmatrix} \frac{2}{7} & \frac{3}{7} & \frac{2}{7} \\ \frac{2}{7} & \frac{3}{7} & \frac{2}{7} \\ \frac{2}{7} & \frac{3}{7} & \frac{2}{7} \end{bmatrix}$

$X[K] = A^KX[0] = [\overline{X} \qquad \overline{X} \qquad \overline{X}]^T$

P4. $a)\qquad False$

$b)\qquad False$

$c)\qquad Ture$

$d)\qquad Ture$

$e)\qquad False$

P5.

$$a)\boldsymbol{\Phi}(t)=e^{\int_0^t A\,\mathrm{d}\boldsymbol{\tau}}=e^{\begin{bmatrix}\ln(t+1) & o\\ \ln(t+1) & o\end{bmatrix}}=\begin{bmatrix}e^{-\ln(t+1)} & o\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & o\\ -\ln(t+1) & 1\end{bmatrix}$$

$$=\begin{bmatrix}e^{-\ln(t+1)} & o\\ -\ln(t+1) & 1\end{bmatrix}$$

$$\boldsymbol{\Phi}(t,\boldsymbol{\tau})=\boldsymbol{\Phi}(\boldsymbol{\tau})\boldsymbol{\Phi}^{-1}(t,\boldsymbol{\tau})=\begin{bmatrix}e^{-\ln(t+1)} & o\\ -\ln(t+1) & 1\end{bmatrix}\begin{bmatrix}e^{-\ln(\boldsymbol{\tau}+1)} & o\\ -\ln(\boldsymbol{\tau}+1) & 1\end{bmatrix}^{-1}$$

b) $\quad t\to\infty \qquad e^{-\ln(t+1)}\to 0 \qquad -\ln(t+1)\nrightarrow 0$

$\boldsymbol{\Phi}(t)\nrightarrow 0 \qquad not \qquad asy \qquad stable$