ECE PhD QE CNSIP 2006 Problem1 - Rhea

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ECE Ph.D. Qualifying Exam

Communication, Networking, Signal and Image Processing (CS)

Question 1: Probability and Random Processes

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Question

1

Let $\mathbf{U}_n$ be a sequence of independent, identically distributed zero-mean, unit-variance Gaussian random variables. The sequence $\mathbf{X}_n$, $n \geq 1$, is given by $\mathbf{X}_n = \frac{1}{2}\,\mathbf{U}_n + \left(\frac{1}{2}\right)^2 \mathbf{U}_{n-1} + \cdots + \left(\frac{1}{2}\right)^n \mathbf{U}_1.$

(a) (15 points)

Find the mean and variance of $\mathbf{X}_n$.

(b) (15 points)

Find the characteristic function of $\mathbf{X}_n$.

(c) (10 points)

Does the sequence $\mathbf{X}_n$ converge in distribution? A simple yes or no answer is not sufficient. You must justify your answer.

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2

Let Φ be the standard normal distribution, i.e., the distribution function of a zero-mean, unit-variance Gaussian random variable. Let $\mathbf{X}$ be a normal random variable with mean μ and variance 1 . We want to find $E[\Phi(\mathbf{X})]$.

(a) (10 points)

First show that $E[\Phi(\mathbf{X})] = P(\mathbf{Z} \leq \mathbf{X})$, where $\mathbf{Z}$ is a standard normal random variable independent of $\mathbf{X}$. Hint: Use an intermediate random variable $\mathbf{I}$ defined as

(b) (10 points)

Now use the result from Part (a) to show that $E[\Phi(\mathbf{X})] = \Phi\left(\frac{\mu}{\sqrt{2}}\right)$.

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3 (15 points)

Let $\mathbf{Y}(t)$ be the output of linear system with impulse response $h(t)$ and input $\mathbf{X}(t)+\mathbf{N}(t)$, where $\mathbf{X}(t)$ and $\mathbf{N}(t)$ are jointly wide-sense stationary independent random processes. If $\mathbf{Z}(t)=\mathbf{X}(t)-\mathbf{Y}(t)$, find the power spectral density $S_{\mathbf{Z}}(\omega)$ in terms of $S_{\mathbf{X}}(\omega), S_{\mathbf{N}}(\omega), m_{\mathbf{X}}=E[\mathbf{X}]$, and $m_{\mathbf{Y}}=E[\mathbf{Y}]$.

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4

Suppose customer orders arrive according to an i.i.d. Bernoulli random process $\mathbf{X}_n$ with parameter p . Thus, an order arrives at time index n (i.e., $\mathbf{X}_n=1$) with probability p ; if an order does not arrive at time index n , then $\mathbf{X}_n=0$. When an order arrives, its size is an exponential random variable with parameter λ . Let $\mathbf{S}_n$ be the total size of all orders up to time n .

(a) (20 points)

Find the mean and autocorrelation function of $\mathbf{S}_n$.

(b) (5 points)

Is $\mathbf{S}_n$ a stationary random process? Explain.

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