1. (35 pts.) Consider the image

$$f(x,y) = \mathrm{rect}\left(\frac{(x-1)}{2}, 2y\right),$$

where $\mathrm{rect}(x,y) \triangleq \begin{cases} 1, & |x| \text{ and } |y| < 1/2 \\ 0, & \text{else} \end{cases}$.

a. (7) Sketch $f(x,y)$ accurately enough to show that you know what it looks like. Be sure to dimension each important aspect of the image.

The continuous-space Fourier transform (CSFT) of an image $f(x,y)$ is defined as $F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-j2\pi(ux+vy)}dxdy$.

b. (14) Find a simple expression for the CSFT $F(u,v)$ of the image $f(x,y)$ given above.

The transform relations

(i)
$$rect(x) \overset{\text{1-D CSFT}}{\leftrightarrow} \mathrm{sinc}(u),$$

where
$$\mathrm{rect}(x) \triangleq \begin{cases} 1, & |x| < 1/2 \\ 0, & \text{else} \end{cases} \text{ and } \mathrm{sinc}(u) \triangleq \frac{\sin(\pi u)}{\pi u};$$

and (ii)
$$f\left(\frac{x-x_0}{a}\right) \overset{\text{1-D CSFT}}{\leftrightarrow} |a| \mathrm{F}(au) e^{-j2\pi u x_0}$$

may be helpful for solving this problem.

Now consider the image $g(x,y) = f(x,y)\left[1+\cos(2\pi(x+y))\right]/2$.

c. (14) Find a simple expression for the CSFT $G(u,v)$ of the image $g(x,y)$ given above.

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2. (35 pts) Consider a spatial filter with point spread function or 2-D impulse response $h[k,l]$ given below

$$
h[k,l] \qquad
\begin{array}{c|ccc}
k \backslash l & -1 & 0 & 1 \\
\hline
-1 & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\
0 & 1 & 1 & 1 \\
1 & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3}
\end{array}
$$

a. (20) Find the output $g[m,n] = \sum_k \sum_l h[m-k, n-l] f[k,l]$ when this filter is applied to the following input image $f[k,l]$. You may assume that the boundary pixel values are extended beyond the boundary. You need only calculate the output over the original 6×6 set of pixels in the input image.

$$
\begin{array}{cccccc}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 1 & 0 \\
0 & 1 & 1 & 1 & 1 & 0 \\
0 & 1 & 1 & 1 & 1 & 0 \\
0 & 1 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{array}
$$

b. (10) Find a simple expression for the frequency response $H(e^{j\mu}, e^{j\nu})$ of this filter, and sketch the magnitude $|H(e^{j\mu}, e^{j\nu})|$ along the μ axis and the ν axis. Here the 2-D Discrete-space Fourier transform (DSFT) is defined according to: $H(e^{j\mu}, e^{j\nu}) = \sum_m \sum_n h[m,n] e^{j(\mu m + \nu n)}$.

c. (5) Using your results from parts a) and b), explain what this filter does. Relate spatial domain properties to frequency domain properties. Be sure to examine what happens at each edge of the region of 1's above, and how this relates to the frequency domain, as well as what happens in the center of the region of 1's and in the border of 0's that surrounds the non-zero portion of the image.

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3. (30 pts) The quantity known as *luminance* is a visually weighted measure of the number of photons incident on the human retina. It is commonly defined as $Y = k\int \bar{y}(\lambda)S(\lambda)d\lambda$, where $S(\lambda)$ is the spectral power distribution of the light incident on the retina as a function of wavelength λ, and $\bar{y}(\lambda) = V_\lambda$ is the CIE color matching function for the Y component of the CIE XYZ color coordinates. The function $\bar{y}(\lambda)$ is also known as the relative luminous efficiency function V_λ.

   a. (6) Provide an intuitive interpretation of what $\bar{y}(\lambda) = V_\lambda$ represents in terms of the human visual system.

   Suppose that you have a monochrome display monitor that takes as its input an 8-bit gray value g between 0 and 255. Suppose also that you have an instrument that can measure the luminance emitted by the display. You measure the luminance Y for each gray value g and determine that it obeys the relationship $\bar{Y} = (g/255)^{2.5}$, where the over-bar signifies that the luminance is now normalized to lie between 0 and 1.

   Suppose that you also have a monochrome digital image $f[m,n]$ with pixel values between 0 and 255 that are proportional to luminance.

   b. (18) Determine the transformation $g = \varphi(f)$ needed between the values f of the image pixels and the gray level g input to your monitor so that the output light $\bar{Y}$ displayed by the monitor is linearly proportional to the input pixel values f.

   It is common practice to store monochrome images in a form, such that the pixel values can be directly input to the monitor without any transformation to yield the correct values in terms of luminance.

   c. (6) State the two major advantages to doing this.

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