Outline

1. Fourier series: sum of sines and cosines

2. Discrete Fourier transform

3. Power spectrum using Matlab

4. Power spectrum using Chronux

4. Sampling frequency, aliasing

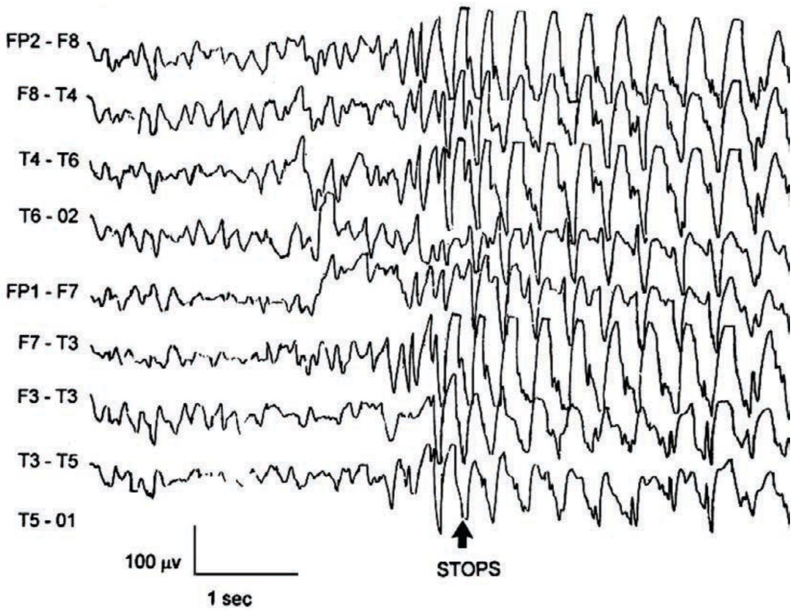
5. Spectrogram

6. Autocorrelation, crosscorrelation, coherence

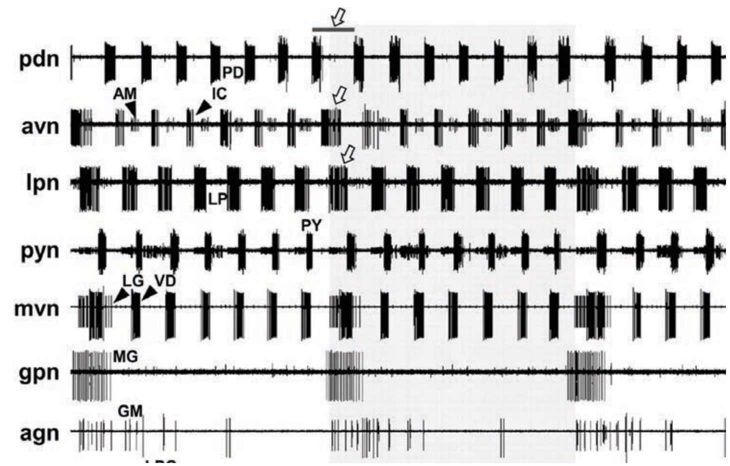
7. Noise and filters

Periodicity in time

EEG during seizures



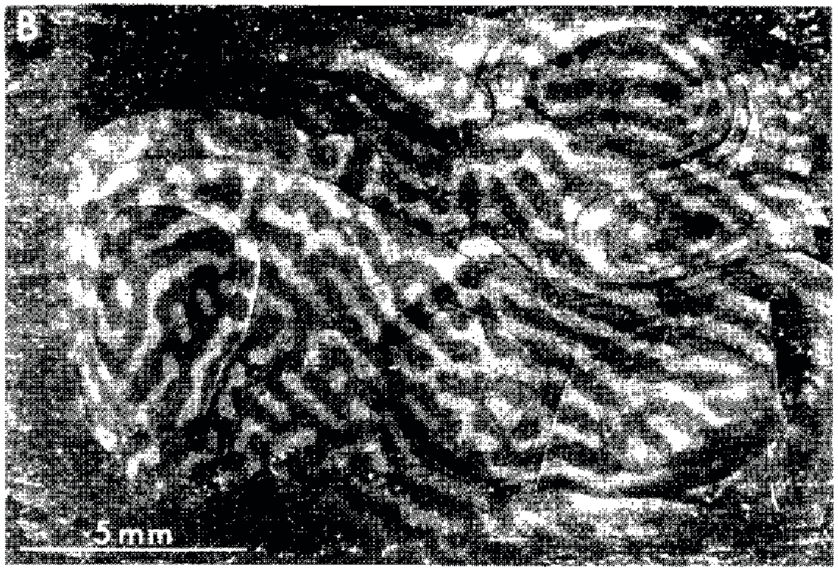
Rhythmic firing in STG



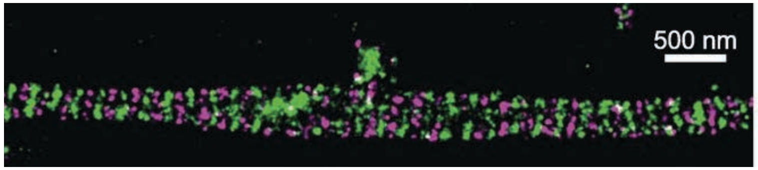
California spiny lobster

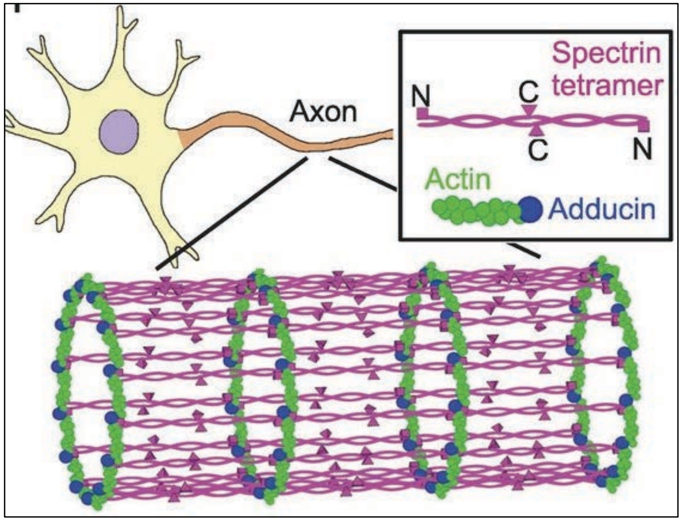
Periodicity in space

Ocular dominance columns in V1



Axons





Fourier series ~ sum of sines and cosines

$$\boxed{x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{+\infty} a_k e^{jk(2\pi/T)t}}$$

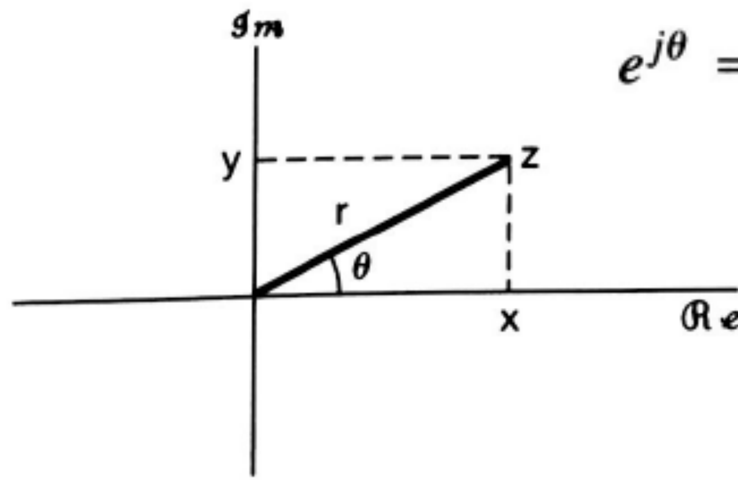
Infinite sum

Coefficients

Basis function

Euler's relation,

$$e^{j\theta} = \cos\theta + j\sin\theta$$



Fourier series

How to find the coefficients:

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} \longleftrightarrow a_k = \frac{1}{T}\int_T x(t) e^{-jk\omega_0 t}\,dt$$

a_k is a complex-valued coefficient

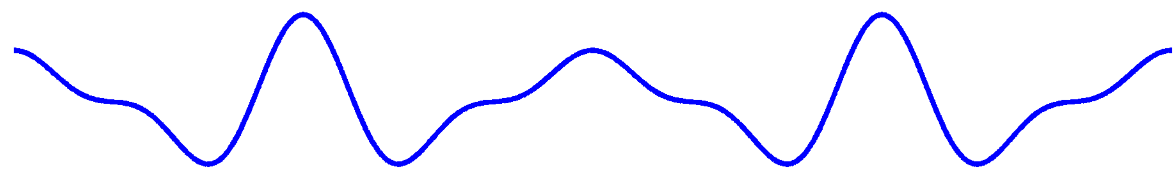
Magnitude of a_k: Content at that frequency

- The sequence of squared magnitude $|a_k|^2$ is the power spectrum

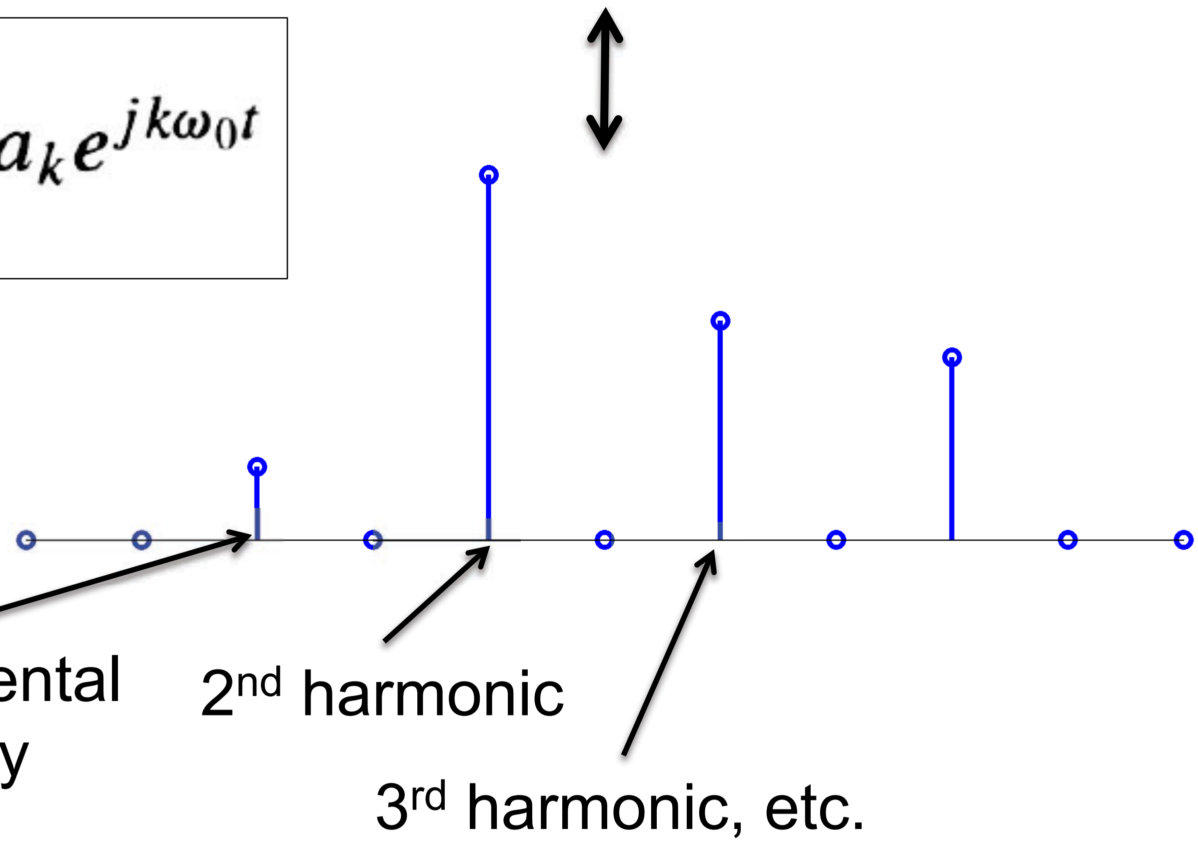
Phase of a_k: Time-shifts of the sinusoidal

Harmonic frequencies

cos(2 π t) + 0.75*cos(4 π t) - cos(π t) + 0.5*cos(3 π t)



$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$$



Fourier transform

Recall Fourier series at harmonic frequencies:

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} \qquad a_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}\,dt$$

More general is the Fourier transform:

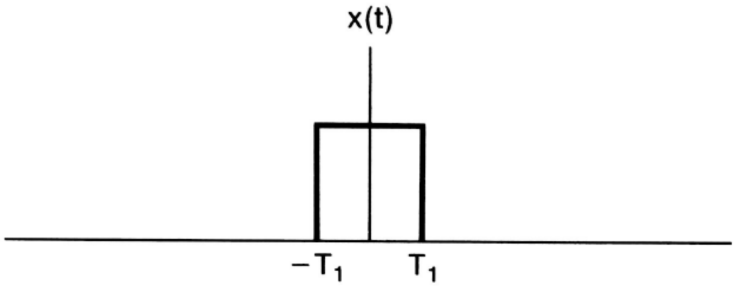
$$\boxed{x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)e^{j\omega t}\,d\omega \qquad X(j\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}\,dt.}$$

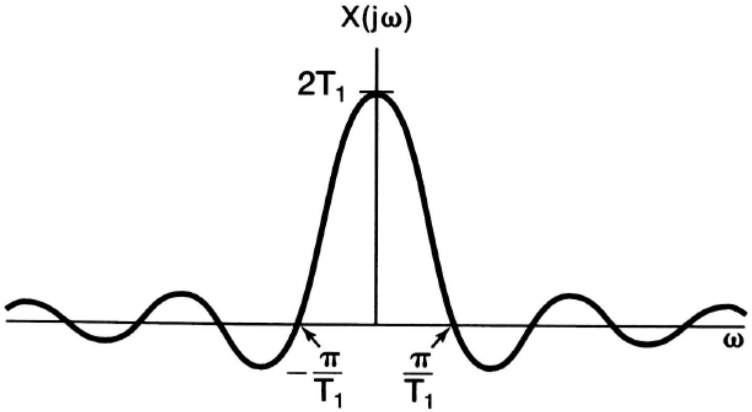
There are many types of transform (e.g. Hilbert, Laplace).

The idea is that some signals are easier to work with in the transformed space.

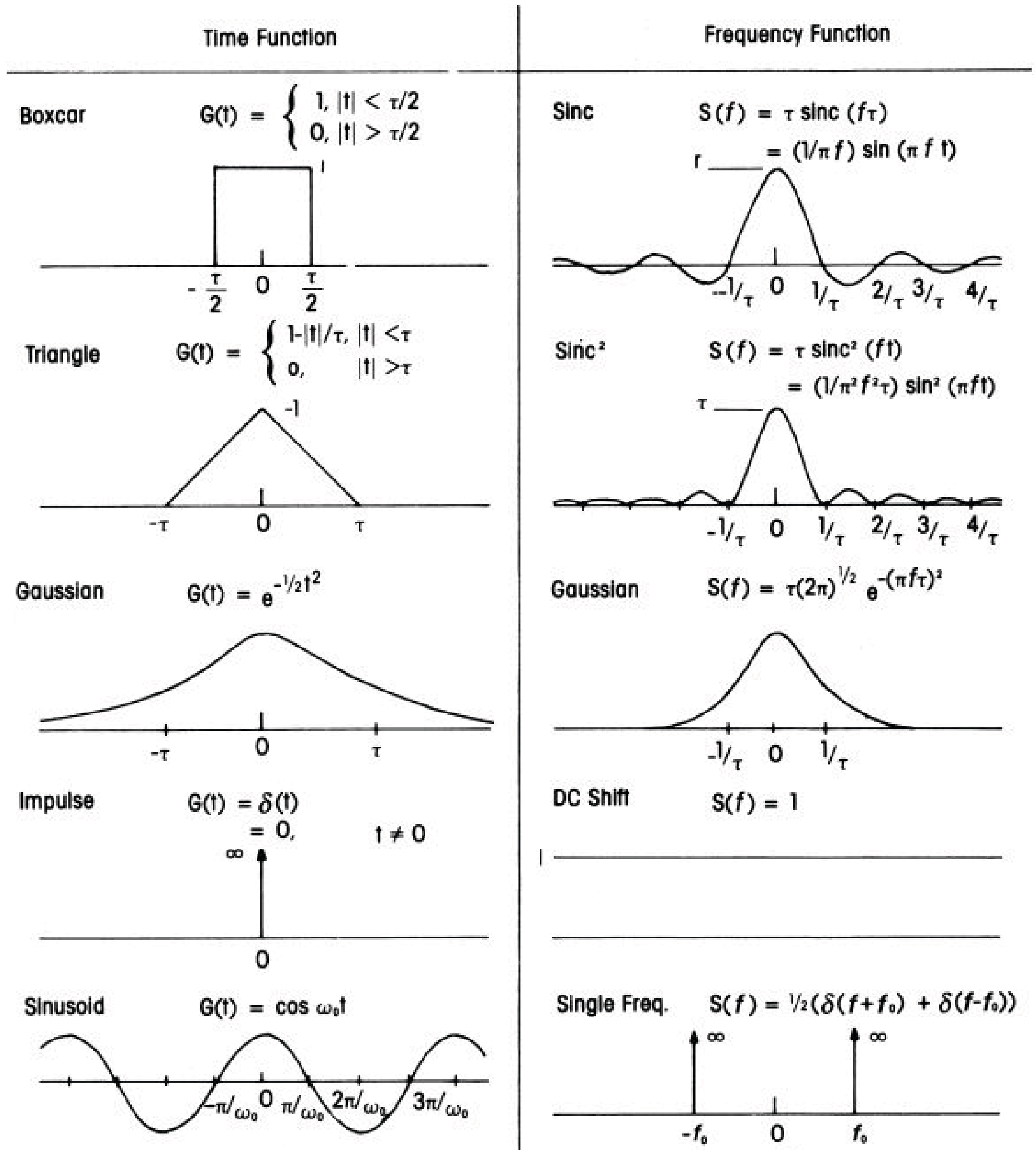
Example of Fourier transform pairs

Time domain ⟷ Frequency domain



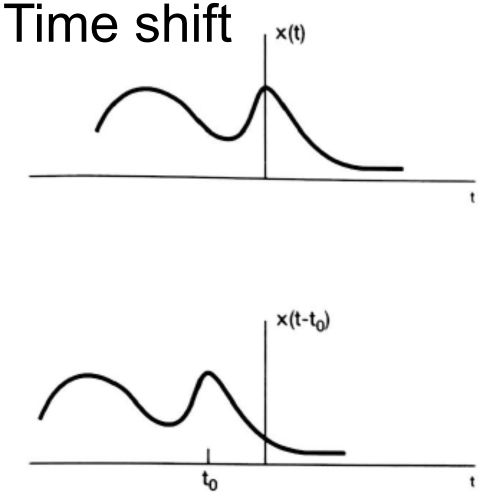


Examples of Fourier transform pairs

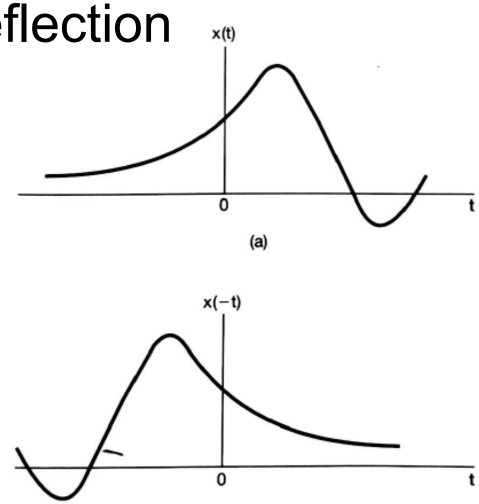


Examples of operations

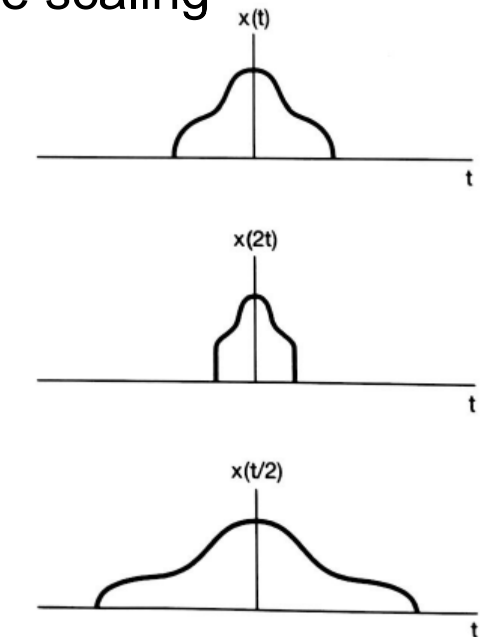
Time shift



Reflection



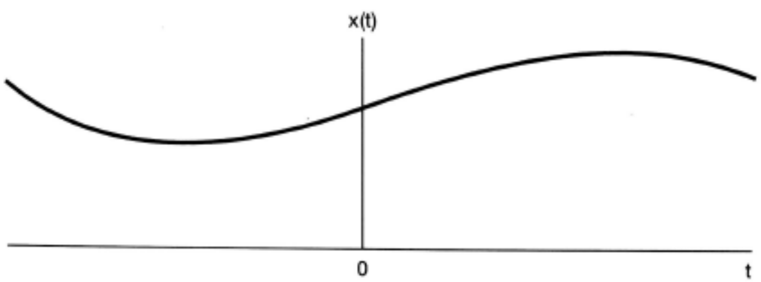
Time scaling

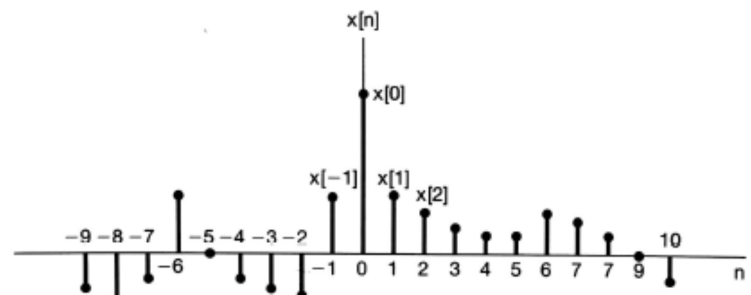


Property	Aperiodic signal	Fourier transform
	$x(t)$	$X(j\omega)$
	$y(t)$	$Y(j\omega)$
Linearity	$ax(t) + by(t)$	$aX(j\omega) + bY(j\omega)$
Time Shifting	$x(t - t_0)$	$e^{-j\omega t_0}X(j\omega)$
Frequency Shifting	$e^{j\omega_0 t}x(t)$	$X(j(\omega - \omega_0))$
Conjugation	$x^*(t)$	$X^*(-j\omega)$
Time Reversal	$x(-t)$	$X(-j\omega)$
Time and Frequency Scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{j\omega}{a}\right)$
Convolution	$x(t) * y(t)$	$X(j\omega)Y(j\omega)$
Multiplication	$x(t)y(t)$	$\frac{1}{2\pi}X(j\omega) * Y(j\omega)$
Differentiation in Time	$\frac{d}{dt}x(t)$	$j\omega X(j\omega)$

Discrete Fourier transform

<u>Problem</u>: Fourier transform works on continuous functions, but real data are measured at discrete time points.





(b)

Discrete Fourier transform
(finite sequence of samples x[n])

$$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x[n]e^{-j\omega n}.$$

Interim summary

- Fourier series
    - Sum of sines and cosines

- Fourier transform
    - For continuous functions

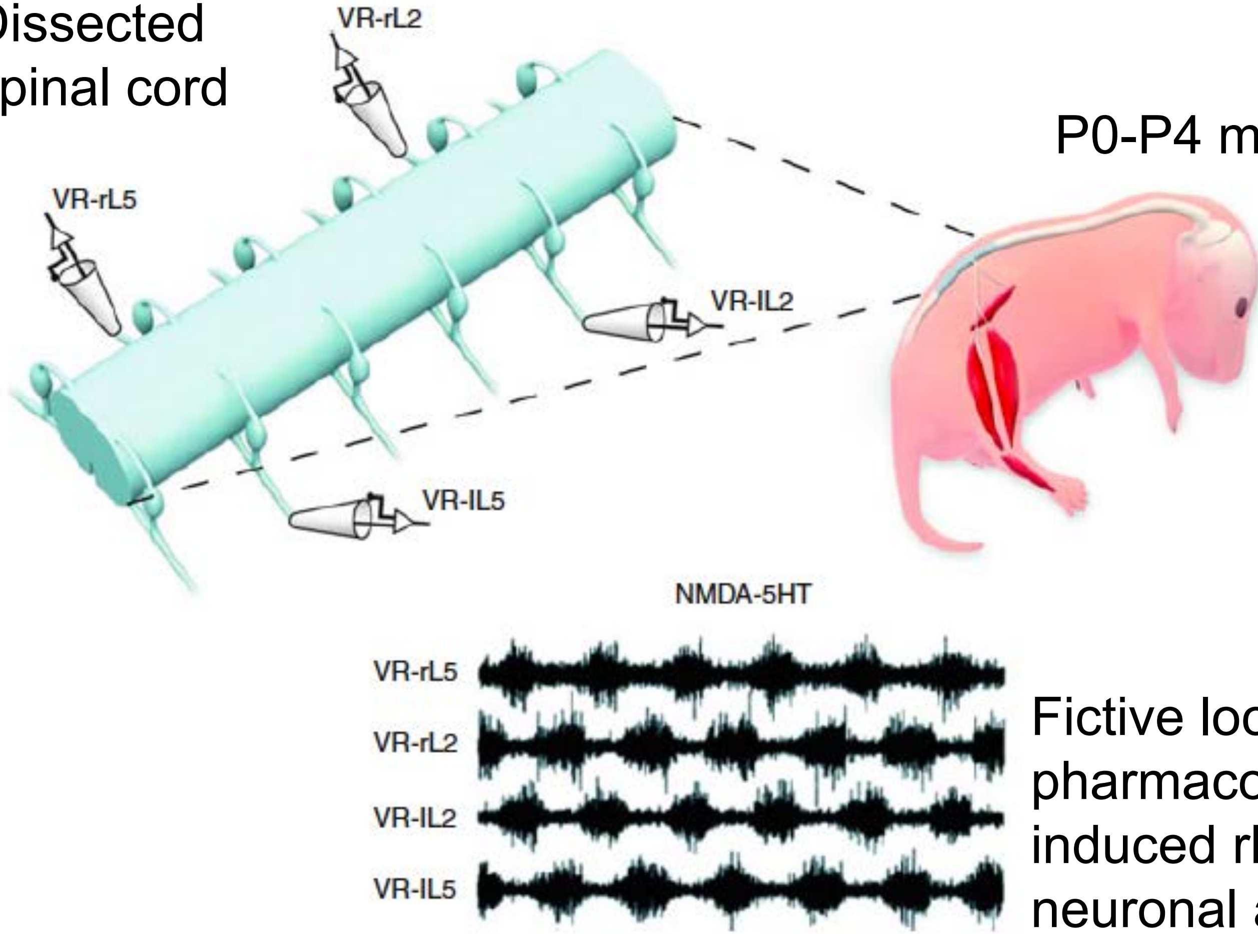
- Discrete Fourier transform
    - For discretized sequence of samples
    - `fft` in MATLAB = Fast Fourier transform, an efficient algorithm for calculating DFT

    - Chronux – a Matlab toolbox for spectral analysis (www.chronux.org)

The neonatal spinal cord preparation

Dissected spinal cord

P0-P4 mice



Fictive locomotion: pharmacologically-induced rhythmic neuronal activity

Kiehn and Dougherty, 2013

Coherence

The association between two signals in frequency domain.

$$\boxed{C_{xy}(\omega) := \frac{P_{xy}(\omega)}{\sqrt{P_{xx}(\omega)P_{yy}(\omega)}},}$$

$$P_{xx}(\omega) := |\hat{x}(\omega)|^2 = \hat{x}(\omega)\overline{\hat{x}(\omega)},$$

$$P_{xy}(\omega) := \hat{x}(\omega)\overline{\hat{y}(\omega)},$$

$$\hat{x}(\omega) := \int_{-\infty}^{\infty} x(t)e^{-i\omega t}dt$$

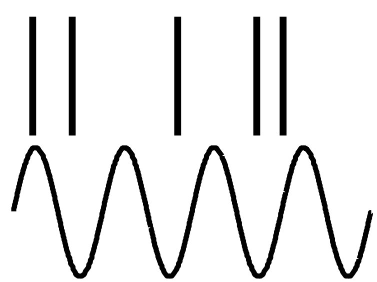
Power spectrum and
cross-power spectrum

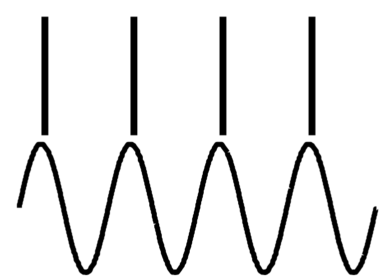
Intuition:

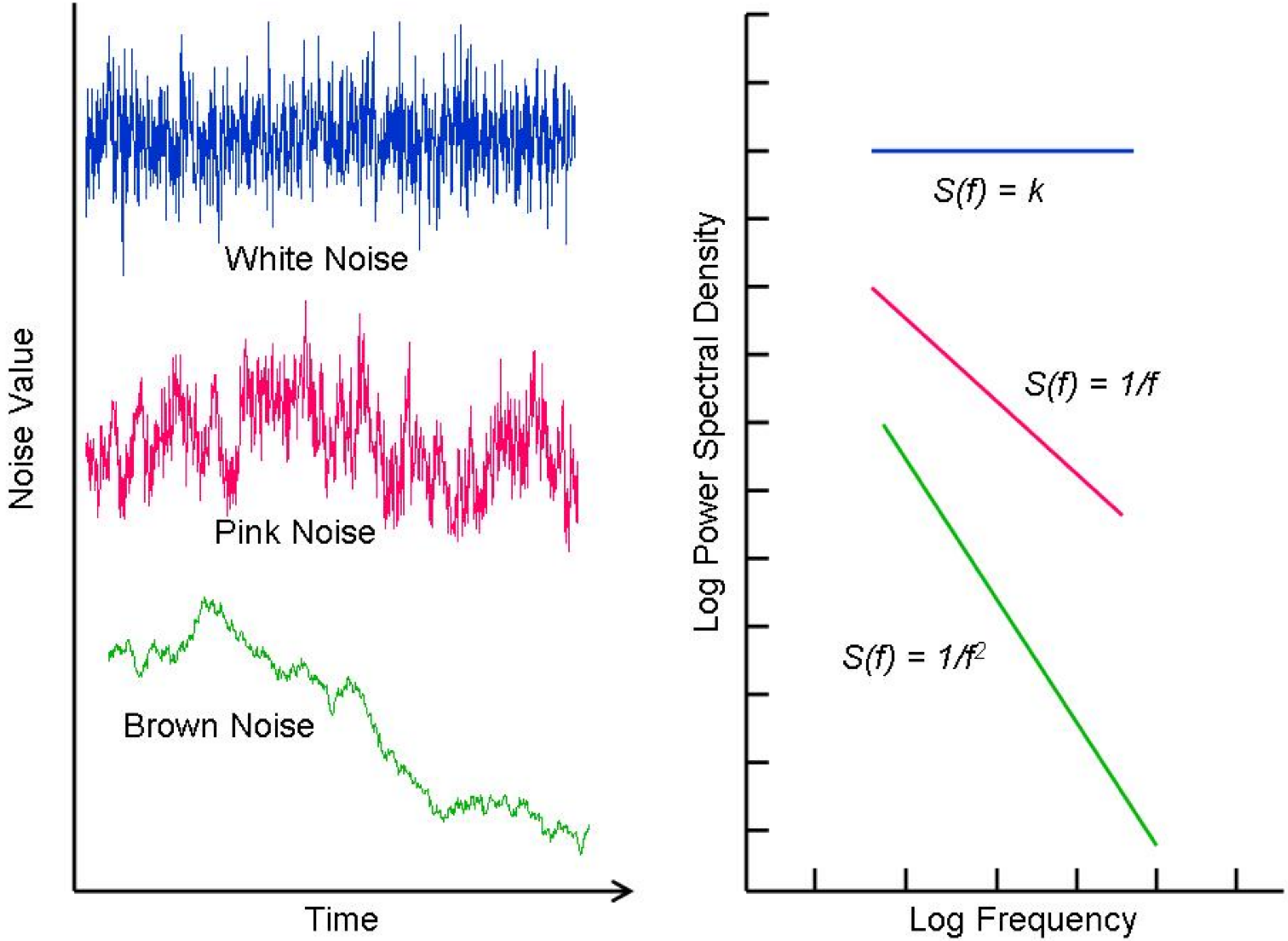
Coherency — Low — High $\phi = 0°$

Spikes

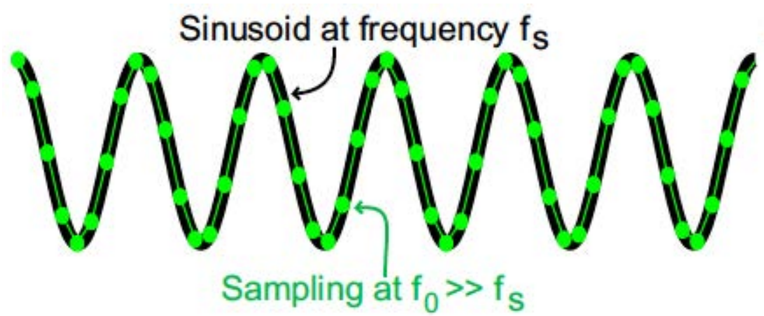
Field



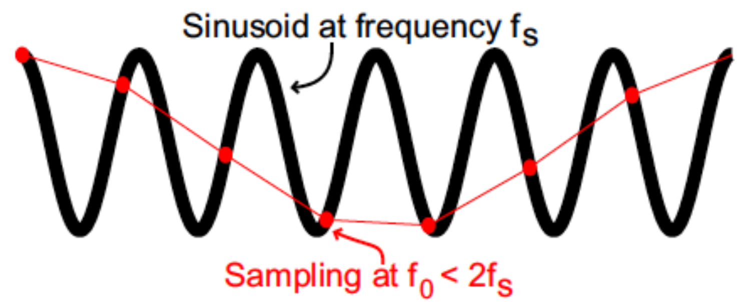



White Noise
Pink Noise
Brown Noise
Noise Value
Time
S(f) = k
S(f) = 1/f
S(f) = 1/f2
Log Power Spectral Density
Log Frequency

Sampling frequency



Aliasing



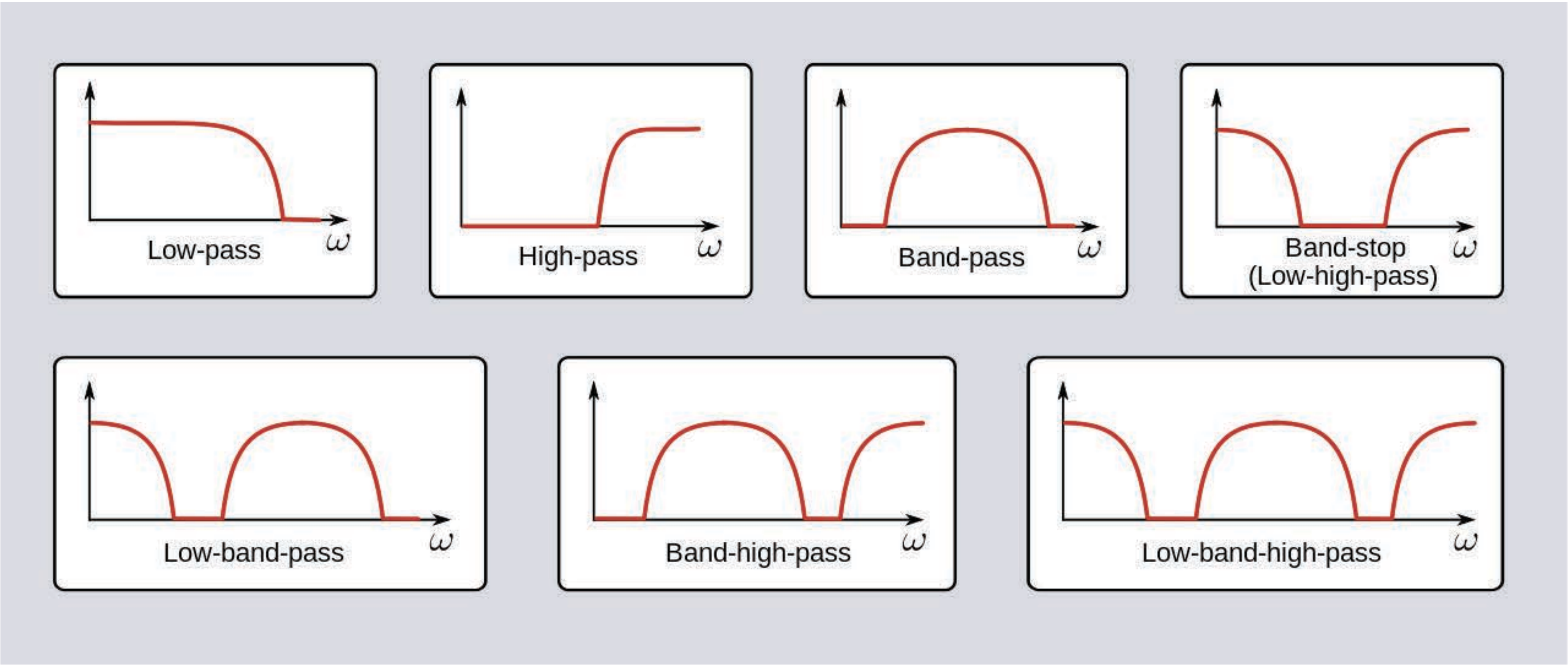
Nyquist frequency

= f_o / 2 (half of the sampling frequency)

e.g. want to record axon potential width ~ 2 ms

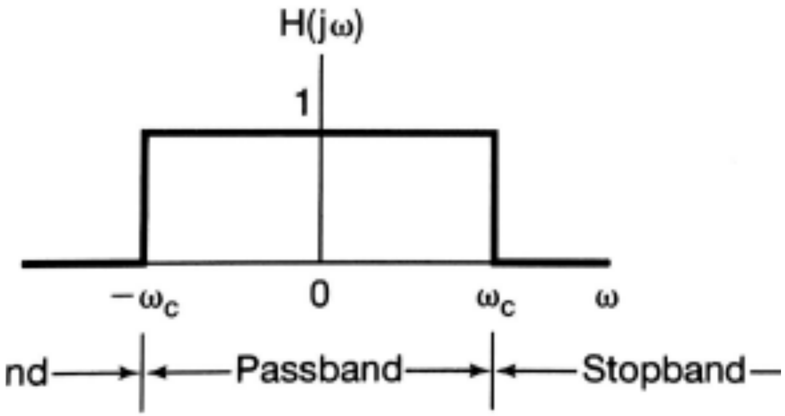
set patch-clamp amplifier sampling rate > 1 kHz

Filters (in frequency domain)

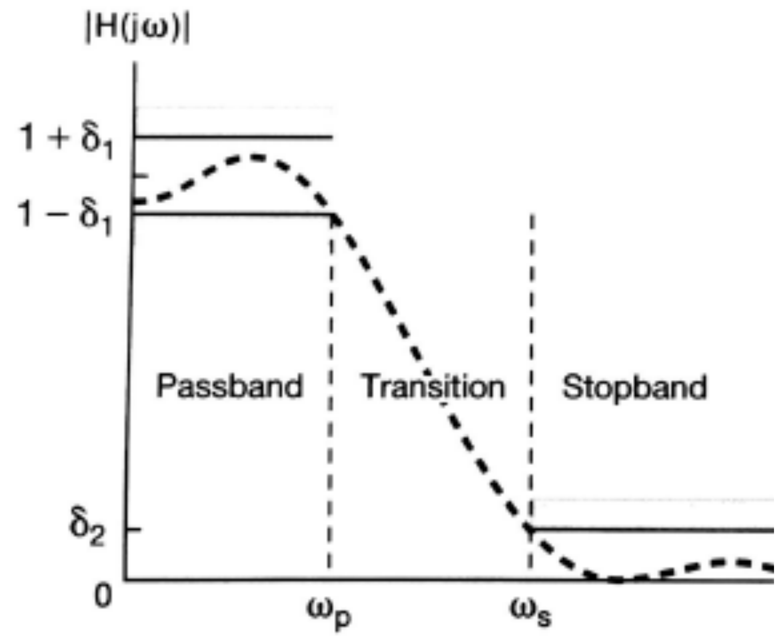


Filtering characteristics

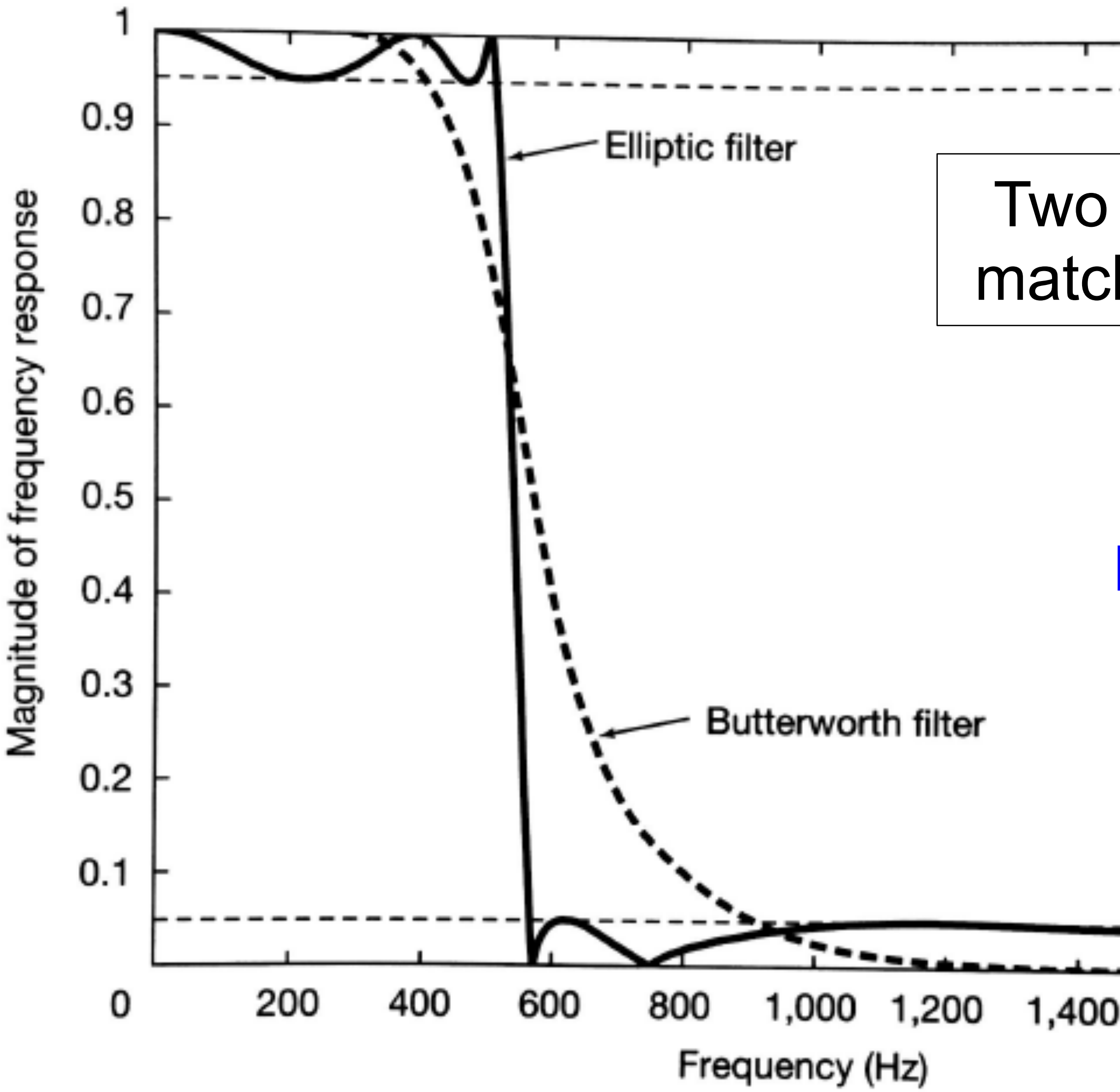
An <u>ideal</u>
low-pass filter



An <u>non-ideal</u>
low-pass filter



Filtering characteristics



Two example filters with matched cutoff frequency

<u>Tradeoffs</u>

Elliptic filter is sharper, but has poorer rejection and transmission ratios