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A test-suite of non-convex constrained optimization problems from the real-world and some baseline results

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ABSTRACT

Real-world optimization problems have been comparatively difficult to solve due to the complex nature of the objective function with a substantial number of constraints. To deal with such problems, several metaheuristics as well as constraint handling approaches have been suggested. To validate the effectiveness and strength, performance of a newly designed approach should be benchmarked by using some complex real-world problems, instead of only the toy problems with synthetic objective functions, mostly arising from the area of numerical analysis. A list of standard real-life problems appears to be the need of the time for benchmarking new algorithms in an efficient and unbiased manner. In this study, a set of 57 real-world Constrained Optimization Problems (COPs) are described and presented as a benchmark suite to validate the COPs. These problems are shown to capture a wide range of difficulties and challenges that arise from the real life optimization scenarios. Three state-of-the-art constrained optimization methods are exhaustively tested on these problems to analyze their hardness. The experimental outcomes reveal that the selected problems are indeed challenging to these algorithms, which have been shown to solve many synthetic benchmark problems easily.

1. Introduction

Optimization is a numerical process used to determine the decision variables for minimizing or maximizing an objective function while satisfying the linear and/or non-linear constraints imposed on the decision-space. In a plethora of real-world applications, the problems contain non-linear objective function and constraints with multiple local optima with a smaller feasible region [1]. Solving these problems using classical algorithms is hard and tedious due to the presence of many local optima and a small feasible region.

Since the last three decades, Swarm-based Algorithms (SAs) and Evolutionary Algorithms (EAs) have drawn considerable attention and have become a common choice for solving challenging real-world optimization problems. One of the main advantages of these algorithms over the classical mathematical programming is that these algorithms only evaluate the objective function and constraints (if available) to solve the optimization problems and do not require any explicit gradient information. Therefore, these algorithms can deal with non-linear as well as non-convex problems with discrete decision-space. Also, the stochastic behaviors of the individuals (candidate solutions) of these algorithms provide global search capability.

Due to the absence of a solid theoretical backbone for these algorithms, the performance analysis and design of new algorithmic variants generally depend on the benchmarking procedures. To validate the effectiveness of the new algorithms, it is essential that the performance is evaluated on a good set of benchmark problems and is also compared against the popular existing algorithms. Benchmark problems can be categorized into two groups: test problems or functions and real-world problems. Artificial or synthetic problems are called test problems (functions) and the behavior of optimization algorithms are generally evaluated on these test problems. Several benchmark suites of test problems have been proposed for the validation of unconstrained

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and constrained optimization algorithms. For unconstrained optimization algorithms, a diverse set of test problems with different levels of difficulty are proposed in Ref. [2,3,4,5,6,7,8]. Similarly, several collections of test problems are proposed for constrained optimization algorithms in Refs. [9,10,11,12].

On the other hand, hard optimization problems originating from the real-life applications are called real-world problems. For performance analysis of the unconstrained heuristic optimizers, a set of real-world problems is reviewed and compiled in Ref. [13]. However, the majority of real-world optimization problems involve non-linear and non-convex constraints and the lower volume of the feasible region in the decision variable space may degrade the robustness and effectiveness of any optimization algorithm. Moreover, most of the algorithms are inherently designed for unconstrained optimization problems. Accordingly, an added mechanism called Constraint Handling Technique (CHT) is required to account for the constraints of real-world problems. Several CHTs have been suggested in the literature. Historically, the most common practice to handle constraints has been by penalizing the fitness value of the infeasible solutions. But, penalty functions have numerous limitations pertaining to different levels of emphasis on the objective reduction versus the extent of constraint violations. Due to limitations of the penalty functions, CHTs of other flavors have been proposed in the last two decades. Some popular CHTs are: superiority of feasible solutions [14], self-adaptive penalty function [15], epsilon constraint handling [16], stochastic ranking [17] and ensemble of CHTs [18].

The role of various search methods that form the components of a constrained optimizer also needs to be analyzed on COPs to better understand the contribution of each component. In recent years, benchmarking and ablation studies have been undertaken on the newly developed algorithms on several engineering optimization problems (for example, see Ref. [19,20], and [21]). However, in most of these studies, the chosen problems and algorithms have been complementary to each other i.e. algorithms perform well on selected problems, while may not perform well on other sets of problems. Therefore, evaluation of the algorithms needs to be done on a variety of real-world problems in a systematic manner. However, a benchmark suite having real-world COPs has not been available in the literature that provides systematic steps to evaluate and compare the performance of algorithms for the class of real-world problems.

Benchmarking suites are required to compare and evaluate the performance of algorithms on given problem classes. Ideally, these test suites are assumed to assist in the selection of the best suitable one from a group of available algorithms for the given real-world applications. Furthermore, the benchmark suites can also qualify to confirm analytical predictions of the behavior of the algorithm. Yet, most of the available benchmark suites contain artificial unconstrained test problems. Consequently, an algorithm that performs very well on these benchmark suites may not provide robust performance on the given real-world problems.

The above reasons motivated us to construct a benchmark suite containing real-world optimization problems for constrained optimization algorithms. The major contributions of this work are as follows.

- 57 real-world constrained problems selected from different real-world applications are reviewed to create a benchmark suite.
- Three recently proposed state-of-the-art constrained algorithms viz. IUDE [22], ϵMAgES [23], and iLSHADE$_\epsilon$ [24], are used to demonstrate the difficulty level of these real-world optimization problems.
- A ranking scheme is proposed to grade the performance of the compared algorithms on the proposed benchmark suite.

2. Real-world constrained optimization problems

The real-world COPs can be represented as follows.

$$\text{Minimize}, f(\overline{x}), \overline{x} = (x_1, x_2, .., x_n) \tag{1}$$

$$\text{Subject to}: g_i(\overline{x}) \le 0, i = 1, .., n$$
$$h_j(\overline{x}) = 0, j = n+1, .., m$$

Generally, an equality constraint can be transformed into two inequality constraints using the following equation.

$$|h_j(\overline{x})| - \epsilon \le 0, j = n+1, .., m \tag{2}$$

where ϵ is set to a small value (10^{-4}).

2.1. Industrial chemical process problems

Chemical engineering practice involves several non-linear COPs [25]. Design relations of process equipment and equations of mass and heat balance introduce non-linearities in the problems. Many chemical process problems have been proposed which are highly complex and non-linear due to many non-linear inequality and equality constraints. The following problems are considered in this work.

2.1.1. Heat exchanger network design (case 1) [26].

The optimal shape of the heat exchanger structure is considered in this problem. In three hot currents, one cold current is heated to reduce the comprehensive area of heat exchange structure. The mathematical model can be described in the following way.

Minimize:

$$f(\overline{x}) = 35x_1^{0.6} + 35x_2^{0.6} \tag{3}$$

subject to:

$$h_1(\overline{x}) = 200x_1x_4 - x_3 = 0,$$

$$h_2(\overline{x}) = 200x_2x_6 - x_5 = 0,$$

$$h_3(\overline{x}) = x_3 - 10000(x_7 - 100) = 0,$$

$$h_4(\overline{x}) = x_5 - 10000(300 - x_7) = 0,$$

$$h_5(\overline{x}) = x_3 - 10000(600 - x_8) = 0,$$

$$h_6(\overline{x}) = x_5 - 10000(900 - x_9) = 0,$$

$$h_7(\overline{x}) = x_4\ln(x_8 - 100) - x_4\ln(600 - x_7) - x_8 + x_7 + 500 = 0,$$

$$h_8(\overline{x}) = x_6\ln(x_9 - x_7) - x_6\ln(600) - x_9 + x_7 + 600 = 0$$

with bounds:

$$0 \le x_1 \le 10, 0 \le x_2 \le 200, 0 \le x_3 \le 100, 0 \le x_4 \le 200,$$

$$1000 \le x_5 \le 2000000, 0 \le x_6 \le 600, 100 \le x_7 \le 600, 100 \le x_8 \le 600,$$

$$100 \le x_9 \le 900.$$

2.1.2. Heat exchanger network design (case 2) [27].

This is the second case of heat exchange network design problem. In this case, three nonlinear equality constraints and six linear equality constraints with a nonlinear objective function are involved in the problem. Moreover, seven additional linear inequality constraints are included due to bounds on the temperatures.

Minimize:

$$f(\overline{x}) = \left(\frac{x_1}{120x_4}\right)^{0.6} + \left(\frac{x_2}{80x_5}\right)^{0.6} + \left(\frac{x_3}{40x_6}\right)^{0.6} \tag{4}$$

subject to:

$$h_1(\overline{x}) = x_1 - 10^4(x_7 - 100) = 0,$$

$$h_2(\overline{x}) = x_2 - 10^4(x_8 - x_7) = 0,$$

$$h_3(\overline{x}) = x_3 - 10^4(500 - x_8) = 0,$$

$$h_4(\overline{x}) = x_1 - 10^4(300 - x_9) = 0,$$

$$h_5(\overline{x}) = x_2 - 10^4(400 - x_{10}) = 0,$$

$$h_6(\overline{x}) = x_3 - 10^4(600 - x_{11}) = 0,$$

$$h_7(\overline{x}) = x_4\ln(x_9 - 100) - x_4\ln(300 - x_7) - x_9 - x_7 + 400 = 0,$$

$$h_8(\overline{x}) = x_5\ln(x_{10} - x_7) - x_5\ln(400 - x_8) - x_{10} + x_7 - x_8 + 400 = 0,$$

$$h_9(\overline{x}) = x_6\ln(x_{11} - x_8) - x_6\ln(100) - x_{11} + x_8 + 100 = 0,$$

with bounds:

$$10^4 \le x_1 \le 81.9\times10^4, 10^4 \le x_2 \le 113.1\times10^4, 10^4 \le x_3 \le 205\times10^4,$$

$$0 \le x_4, x_5, x_6 \le 5.074\times10^{-2}, 100 \le x_7 \le 200, 100 \le x_8, x_9, x_{10} \le 300,$$

$$100 \le x_{11} \le 400.$$

2.1.3. Haverly's pooling problem [28].

Haverly's Pooling problem is a linear-objective non-linear COP and has the following form.

Maximize:

$$f(\overline{x}) = 9x_1 + 15x_2 - 6x_3 - 16x_4 - 10(x_5 + x_6) \tag{5}$$

subject to:

$$h_1(\overline{x}) = x_7 + x_8 - x_4 - x_3 = 0,$$

$$h_2(\overline{x}) = x_1 - x_5 - x_7 = 0,$$

$$h_3(\overline{x}) = x_2 - x_6 - x_8 = 0,$$

$$h_4(\overline{x}) = x_9x_7 + x_9x_8 - 3x_3 - x_4 = 0,$$

$$g_1(\overline{x}) = x_9x_7 + 2x_5 - 2.5x_1 \le 0,$$

$$g_2(\overline{x}) = x_9x_8 + 2x_6 - 1.5x_2 \le 0,$$

with bounds:

$$0 \le x_1, x_3, x_4, x_5, x_6, x_8 \le 100, 0 \le x_2, x_7, x_9 \le 200.$$

2.1.4. Blending-pooling-separation problem [29].

This problem contains a feed mixture having three components that are utilized to separate out into two multi-component outputs by employing separators and splitting/blending/pooling. The operating cost of each separator depends linearly on the flow-rate of the separator and the constraints based on mass balances relation around the individual separators, splitters, and mixers.

Minimize:

$$f(\overline{x}) = 0.9979 + 0.00432x_5 + 0.01517x_{13} \tag{6}$$

subject to:

$$h_1(\overline{x}) = x_4 + x_3 + x_2 + x_1 = 300,$$

$$h_2(\overline{x}) = x_6 - x_8 - x_7 = 0,$$

$$h_3(\overline{x}) = x_9 - x_{11} - x_{10} - x_{12} = 0,$$

$$h_4(\overline{x}) = x_{14} - x_{16} - x_{17} - x_{15} = 0,$$

$$h_5(\overline{x}) = x_{18} - x_{20} - x_{19} = 0,$$

$$h_6(\overline{x}) = x_5x_{21} - x_6x_{22} - x_9x_{23} = 0,$$

$$h_7(\overline{x}) = x_5x_{24} - x_6x_{25} - x_9x_{26} = 0,$$

$$h_8(\overline{x}) = x_5x_{27} - x_6x_{28} - x_9x_{29} = 0,$$

$$h_9(\overline{x}) = x_{13}x_{30} - x_{14}x_{31} - x_{18}x_{32} = 0,$$

$$h_{10}(\overline{x}) = x_{13}x_{33} - x_{14}x_{34} - x_{18}x_{35} = 0,$$

$$h_{11}(\overline{x}) = x_{13}x_{36} - x_{14}x_{37} - x_{18}x_{35} = 0,$$

$$h_{12}(\overline{x}) = 0.333x_1 + x_{15}x_{31} - x_5x_{21} = 0,$$

$$h_{13}(\overline{x}) = 0.333x_1 + x_{15}x_{34} - x_5x_{24} = 0,$$

$$h_{14}(\overline{x}) = 0.333x_1 + x_{15}x_{37} - x_5x_{27} = 0,$$

$$h_{15}(\overline{x}) = 0.333x_2 + x_{10}x_{23} - x_{13}x_{30} = 0,$$

$$h_{16}(\overline{x}) = 0.333x_2 + x_{10}x_{26} - x_{13}x_{33} = 0,$$

$$h_{17}(\overline{x}) = 0.333x_2 + x_{10}x_{29} - x_{13}x_{36} = 0,$$

$$h_{18}(\overline{x}) = 0.333x_3 + x_7x_{22} + x_{11}x_{23} + x_{16}x_{31} + x_{19}x_{32} = 30,$$

$$h_{19}(\overline{x}) = 0.333x_3 + x_7x_{25} + x_{11}x_{26} + x_{16}x_{34} + x_{19}x_{35} = 50,$$

$$h_{20}(\overline{x}) = 0.333x_3 + x_7x_{28} + x_{11}x_{29} + x_{16}x_{37} + x_{19}x_{38} = 30,$$

$$h_{21}(\overline{x}) = x_{21} + x_{24} + x_{27} = 1,$$

$$h_{22}(\overline{x}) = x_{22} + x_{25} + x_{28} = 1,$$

$$h_{23}(\overline{x}) = x_{23} + x_{26} + x_{29} = 1,$$

$h_{24}(\overline{x}) = x_{30} + x_{33} + x_{36} = 1,$

$h_{25}(\overline{x}) = x_{31} + x_{34} + x_{37} = 1,$

$h_{26}(\overline{x}) = x_{32} + x_{35} + x_{38} = 1,$

$h_{27}(\overline{x}) = x_{25} = 0,$

$h_{28}(\overline{x}) = x_{28} = 0,$

$h_{29}(\overline{x}) = x_{23} = 0,$

$h_{30}(\overline{x}) = x_{37} = 0,$

$h_{31}(\overline{x}) = x_{32} = 0,$

$h_{32}(\overline{x}) = x_{35} = 0,$

with bounds:

$0 \le x_1, x_3, x_8, x_9, x_5, x_6, x_{14}, x_{18}, x_{10}, x_{16}, x_{13}, x_{20} \le 90,$

$0 \le x_2, x_4, x_7, x_{11}, x_{12}, x_{15}, x_{17}, x_{19} \le 150,$

$0 \le x_{21}, x_{23}, x_{24}, x_{25}, x_{27}, x_{28} \le 1,$

$0 \le x_{22}, x_{32}, x_{34}, x_{35}, x_{37}, x_{38} \le 1.2,$

$0 \le x_{26}, x_{29}, x_{30}, x_{31}, x_{33}, x_{36} \le 0.5.$

2.1.5. Propane, isobutane, n-butane nonsharp separation [30].

This test problem contains a three-component feed mixture that is required to separate products into two three-component products. The problem is defined as a non-linear constrained optimization problem and has a following form.

Minimize:

$$f(\overline{x}) = c_{11} + (c_{21} + c_{31}x_{24} + c_{41}x_{28} + c_{51}x_{33} + c_{61}x_{34})x_5 + c_{12} + (c_{22} + c_{32}x_{26} + c_{42}x_{31} + c_{52}x_{38} + c_{62}x_{39})x_{13}, \quad (7)$$

where,

c	$i = 1$	$i = 2$
c_{1i}	0.23947	0.75835
c_{2i}	−0.0139904	−0.0661588
c_{3i}	0.0093514	0.0338147
c_{4i}	0.0077308	0.0373349
c_{5i}	−0.0005719	0.0016371
c_{6i}	0.0042656	0.0288996

subject to:

$h_1(\overline{x}) = x_4 + x_3 + x_2 + x_1 = 300,$

$h_2(\overline{x}) = x_6 - x_8 - x_7 = 0,$

$h_3(\overline{x}) = x_9 - x_{12} - x_{10} - x_{11} = 0,$

$h_4(\overline{x}) = x_{14} - x_{17} - x_{15} - x_{16} = 0,$

$h_5(\overline{x}) = x_{18} - x_{20} - x_{19} = 0,$

$h_6(\overline{x}) = x_6x_{21} - x_{24}x_{25} = 0,$

$h_7(\overline{x}) = x_{14}x_{22} - x_{26}x_{27} = 0,$

$h_8(\overline{x}) = x_9x_{23} - x_{28}x_{29} = 0,$

$h_9(\overline{x}) = x_{18}x_{30} - x_{31}x_{32} = 0,$

$h_{10}(\overline{x}) = x_{25} - x_5x_{33} = 0,$

$h_{11}(\overline{x}) = x_{29} - x_5x_{34} = 0,$

$h_{12}(\overline{x}) = x_{35} - x_5x_{36} = 0,$

$h_{13}(\overline{x}) = x_{37} - x_{13}x_{38} = 0,$

$h_{14}(\overline{x}) = x_{27} - x_{13}x_{39} = 0,$

$h_{15}(\overline{x}) = x_{32} - x_{13}x_{40} = 0,$

$h_{16}(\overline{x}) = x_{25} - x_6x_{21} - x_9x_{41} = 0,$

$h_{17}(\overline{x}) = x_{29} - x_6x_{42} - x_9x_{23} = 0,$

$h_{18}(\overline{x}) = x_{35} - x_6x_{43} - x_9x_{44} = 0,$

$h_{19}(\overline{x}) = x_{37} - x_{14}x_{45} - x_{18}x_{46} = 0,$

$h_{20}(\overline{x}) = x_{27} - x_{14}x_{22} - x_{18}x_{47} = 0,$

$h_{21}(\overline{x}) = x_{32} - x_{14}x_{48} - x_{18}x_{30} = 0,$

$h_{22}(\overline{x}) = 0.333x_1 + x_{15}x_{45} - x_{25} = 0,$

$h_{23}(\overline{x}) = 0.333x_1 + x_{15}x_{22} - x_{29} = 0,$

$h_{24}(\overline{x}) = 0.333x_1 + x_{15}x_{48} - x_{35} = 0,$

$h_{25}(\overline{x}) = 0.333x_2 + x_{10}x_{41} - x_{37} = 0,$

$h_{26}(\overline{x}) = 0.333x_2 + x_{10}x_{23} - x_{27} = 0,$

$h_{27}(\overline{x}) = 0.333x_2 + x_{10}x_{44} - x_{32} = 0,$

$h_{28}(\overline{x}) = 0.333x_3 + x_7x_{21} + x_{11}x_{41} + x_{16}x_{45} + x_{19}x_{46} = 30,$

$h_{29}(\overline{x}) = 0.333x_3 + x_7x_{42} + x_{11}x_{23} + x_{16}x_{22} + x_{19}x_{47} = 50,$

$h_{30}(\overline{x}) = 0.333x_3 + x_7x_{43} + x_{11}x_{44} + x_{16}x_{48} + x_{19}x_{30} = 30,$

$h_{31}(\overline{x}) = x_{33} + x_{34} + x_{36} = 1,$

$h_{32}(\overline{x}) = x_{21} + x_{42} + x_{43} = 1,$

$$h_{33}(\overline{x}) = x_{41} + x_{23} + x_{44} = 1,$$

$$h_{34}(\overline{x}) = x_{38} + x_{39} + x_{40} = 1,$$

$$h_{35}(\overline{x}) = x_{45} + x_{22} + x_{48} = 1,$$

$$h_{36}(\overline{x}) = x_{46} + x_{47} + x_{30} = 1,$$

$$h_{37}(\overline{x}) = x_{43} = 0,$$

$$h_{38}(\overline{x}) = x_{46} = 0,$$

with bounds:

$$0 \le x_1, \ldots, x_{20} \le 150; 0 \le x_{25}, x_{27}, x_{32}, x_{35}, x_{37}, x_{29} \le 30;$$

$$0 \le x_{21}, x_{22}, x_{23}, x_{30}, x_{33}, x_{34}, x_{36}, x_{38}, x_{39}, x_{40}, x_{42}, x_{43}, x_{44}, x_{45} \le 1,$$

$$0 \le x_{46}, x_{47}, x_{48} \le 1,$$

$$0.85 \le x_{24}, x_{26}, x_{28}, x_{31} \le 1$$

2.1.6. Optimal operation of alkylation unit [31].

The main aim of this problem is to maximize the octane number of olefin feed in the presence of acid. The objective function is defined as an alkylating product. The problem is formulated as follows.

Maximize:

$$f(\overline{x}) = 0.035x_1x_6 + 1.715x_1 + 10.0x_2 + 4.0565x_3 - 0.063x_3x_5 \tag{8}$$

subject to:

$$g_1(\overline{x}) = 0.0059553571x_6^2x_1 + 0.88392857x_3 - 0.1175625x_6x_1 - x_1 \le 0,$$

$$g_2(\overline{x}) = 1.1088x_1 + 0.1303533x_1x_6 - 0.0066033x_1x_6^2 - x_3 \le 0,$$

$$g_3(\overline{x}) = 6.66173269x_6^2 - 56.596669x_4 + 172.39878x_5 - 10000 - 191.20592x_6 \le 0,$$

$$g_4(\overline{x}) = 1.08702x_6 - 0.03762x_6^2 + 0.32175x_4 + 56.85075 - x_5 \le 0,$$

$$g_5(\overline{x}) = 0.006198x_7x_4x_3 + 2462.3121x_2 - 25.125634x_2x_4 - x_3x_4 \le 0,$$

$$g_6(\overline{x}) = 161.18996x_3x_4 + 5000.0x_2x_4 - 489510.0x_2 - x_3x_4x_7 \le 0,$$

$$g_7(\overline{x}) = 0.33x_7 + 44.333333 - x_5 \le 0,$$

$$g_8(\overline{x}) = 0.022556x_5 - 1.0 - 0.007595x_7 \le 0,$$

$$g_9(\overline{x}) = 0.00061x_3 - 1.0 - 0.0005x_1 \le 0,$$

$$g_{10}(\overline{x}) = 0.819672x_1 - x_3 + 0.819672 \le 0,$$

$$g_{11}(\overline{x}) = 24500.0x_2 - 250.0x_2x_4 - x_3x_4 \le 0,$$

$$g_{12}(\overline{x}) = 1020.4082x_4x_2 + 1.2244898x_3x_4 - 100000x_2 \le 0,$$

$$g_{13}(\overline{x}) = 6.25x_1x_6 + 6.25x_1 - 7.625x_3 - 100000 \le 0,$$

$$g_{14}(\overline{x}) = 1.22x_3 - x_6x_1 - x_1 + 1.0 \le 0.$$

with bounds:

$$1000 \le x_1 \le 2000, 0 \le x_2 \le 100$$

$$2000 \le x_3 \le 4000, 0 \le x_4 \le 100$$

$$0 \le x_5 \le 100, 0 \le x_6 \le 20$$

$$0 \le x_7 \le 200.$$

2.1.7. Reactor network design [32].

A reactor network design problem is solved to design a sequence of two Continuously Stirred Tank Reactors (CSTRs). The main aim of this problem is to optimize the concentration of the product. This problem is formulated as follows.

Maximize:

$$f(\overline{x}) = x_4 \tag{9}$$

subject to:

$$h_1(\overline{x}) = k_1x_5x_2 + x_1 - 1 = 0,$$

$$h_2(\overline{x}) = k_3x_5x_3 + x_3 + x_1 - 1 = 0,$$

$$h_3(\overline{x}) = k_2x_6x_2 - x_1 + x_2 = 0,$$

$$h_4(\overline{x}) = k_4x_6x_4 + x_2 - x_1 + x_4 - x_3 = 0,$$

$$g_1(\overline{x}) = x_5^{0.5} + x_6^{0.5} \le 4$$

with bounds:

$$0 \le x_4, x_3, x_2, x_1 \le 1,$$

$$0.00001 \le x_6, x_5 \le 16.$$

where, $k_3 = 0.0391908$, $k_4 = 0.9k_3$, $k_1 = 0.09755988$, and $k_2 = 0.99k_1$.

2.2. Process design and synthesis problems [33,34].

Process design and synthesis problems in chemical engineering can be defined as a mixed-integer nonlinear constrained optimization problem.

2.2.1. Process synthesis problem [35].

This problem incorporates a non-linear constraint and is defined as follows.

Minimize:

$$f(\overline{x}) = x_2 + 2x_1 \tag{10}$$

subject to:

$$g_1(\overline{x}) = -x_1^2 - x_2 + 1.25 \le 0,$$

$$g_2(\overline{x}) = x_1 + x_2 \le 1.6.$$

with bounds:

$$0 \le x_1 \le 1.6$$

$$x_2 \in \{0, 1\}$$

2.2.2. Process synthesis and design problem [36].

This problem incorporates a non-linear constraint and is represented as follows.

Minimize:

$$f(\overline{x}) = -x_3 + x_2 + 2x_1 \tag{11}$$

subject to:

$$h_1(\overline{x}) = -2\exp(-x_2) + x_1 = 0,$$

$$g_1(\overline{x}) = x_2 - x_1 + x_3 \le 0.$$

with bounds:

$$0.5 \le x_1, x_2 \le 1.4,$$

$$x_3 \in \{0, 1\}.$$

2.2.3. Process flow sheeting problem [25].

This problem can be formulated as a non-convex constrained optimization problem, which is expressed as follows.

Minimize:

$$f(\overline{x}) = -0.7x_3 + 0.8 + 5(0.5 - x_1)^2 \tag{12}$$

subject to:

$$g_1(\overline{x}) = -exp(x_1 - 0.2) - x_2 \le 0,$$

$$g_2(\overline{x}) = x_2 + 1.1x_3 \le -1.0,$$

$$g_3(\overline{x}) = x_1 - x_3 \le 0.2.$$

with bounds:

$$-2.22554 \le x_2 \le -1, 0.2 \le x_1 \le 1, x_3 \in \{0, 1\}.$$

2.2.4. Two-reactor problem [35].

The essential purpose of this problem is to choose one among two reactors to optimize the production cost.

Minimize:

$$f(\overline{x}) = 7.5x_7 + 5.5x_8 + 7x_5 + 6x_6 + 5(x_1 + x_2) \tag{13}$$

subject to:

$$h_1(\overline{x}) = x_7 + x_8 - 1 = 0,$$

$$h_2(\overline{x}) = x_3 - 0.9(1 - \exp(0.5x_5))x_1 = 0,$$

$$h_3(\overline{x}) = x_4 - 0.8(1 - \exp(0.4x_6))x_2 = 0,$$

$$h_4(\overline{x}) = x_3 + x_4 - 10 = 0,$$

$$h_5(\overline{x}) = x_3x_7 + x_4x_8 - 10 = 0,$$

$$g_1(\overline{x}) = x_5 - 10x_7 \le 0,$$

$$g_2(\overline{x}) = x_6 - 10x_8 \le 0,$$

$$g_3(\overline{x}) = x_1 - 20x_7 \le 0,$$

$$g_4(\overline{x}) = x_2 - 20x_8 \le 0$$

with bounds:

$$0 \le x_6, x_5, x_4, x_3, x_2, x_1 \le 100$$

$$x_8, x_7 \in \{0, 1\}.$$

2.2.5. Process synthesis problem [35].

This problem has non-linearities in all real variables and binary variables.

Minimize:

$$f(\overline{x}) = (1 - x_4)^2 + (1 - x_5)^2 + (1 - x_6)^2 - \ln(1 + x_7) + (1 - x_1)^2 + (2 - x_2)^2 + (3 - x_3)^2 \tag{14}$$

subject to:

$$g_1(\overline{x}) = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 - 5 \le 0,$$

$$g_2(\overline{x}) = x_6^3 + x_1^2 + x_2^2 + x_3^2 = 5.5 \le 0,$$

$$g_3(\overline{x}) = x_1 + x_4 - 1.2 \le 0,$$

$$g_4(\overline{x}) = x_2 + x_5 - 1.8 \le 0,$$

$$g_5(\overline{x}) = x_3 + x_6 - 2.5 \le 0,$$

$$g_6(\overline{x}) = x_1 + x_7 - 1.2 \le 0,$$

$$g_7(\overline{x}) = x_5^2 + x_2^2 - 1.64 \le 0,$$

$$g_8(\overline{x}) = x_6^2 + x_3^2 - 4.25 \le 0,$$

$$g_9(\overline{x}) = x_5^2 + x_3^2 - 4.64 \le 0,$$

with bounds:

$$0 \le x_2, x_3, x_1 \le 100,$$

$$x_7, x_6, x_5, x_4 \in \{0, 1\}.$$

2.2.6. Process design problem [37].

It is a minimization problem which can be formulated in the following way.

Minimize:

$$f(\overline{x}) = 5.357854x_1^2 + 40792.141 - 37.29329x_4 + 0.835689x_4x_3 \tag{15}$$

subject to:

$$g_1(\overline{x}) = -92 + a_3x_4x_2 + a_1 + a_2x_4x_3 - a_4x_4x_3 \le 0,$$

$$g_2(\overline{x}) = -110 + a_7x_4x_2 + a_5 + a_6x_5x_3 + a_8x_1^2 \le 0,$$

$$g_3(\overline{x}) = a_9 + a_{11}x_4x_1 + a_{10}x_4x_3 - 25 + a_{12}x_1x_2 \le 0$$

with bounds:

$$27 \le x_3, x_1, x_2 \le 45,$$

$$x_4 \in \{78, 79, \dots, 102\},$$

$$x_5 \in \{33, 34, \dots, 45\}.$$

where, parameters a_1 to a_{12} are constant and values of these constants are shown in Table 1.

Table 1
Constants for process design problem.

a_1 = 85.334407	a_5 = 80.51249	a_9 = 9.300961
a_2 = 0.0056858	a_6 = 0.0071317	a_{10} = 0.0047026
a_3 = 0.0006262	a_7 = 0.0029955	a_{11} = 0.0012547
a_4 = 0.0022053	a_8 = 0.0021813	a_{12} = 0.0019085

Table 2
Values of S_{ij} and t_{ij}.

S_{ij}			t_{ij}		
2	3	4	8	20	8
4	6	3	16	4	4

2.2.7. Multi-product batch plant [38].

This is a multi-product batch plant problem with M serial processing stages, where fixed amounts Q_i from N products must be produced. The problem is formulated as follows.

Minimize:

$$f(\bar{x}) = \sum_{j=1}^{M} \alpha_j N_j V_j^{\beta_j} \tag{16}$$

subject to:

$$g_1(\bar{x}) = S_{ij}B_i - V_j \leq 0,$$

$$g_2(\bar{x}) = -H + \sum_{i=1}^{N} \frac{Q_i T_{Li}}{B_i} \leq 0,$$

$$g_3(\bar{x}) = t_{ij} - N_j T_{Li} \leq 0,$$

with bounds:

$$1 \leq N_i \leq N_j^u,$$

$$V_j^l \leq V_j \leq V_j^u,$$

$$T_{Li}^l \leq T_{Li} \leq T_{Li}^u,$$

$$B_j^l \leq B_j \leq B_j^u.$$

where, $N = 2$, $M = 3$, $\alpha_j = 250$, $H = 6000$, $\beta_j = 0.6$, $N_j^u = 3$, $V_j^l = 250$, and $V_j^u = 2500$. The value of other parameters are calculated by

$$T_{Li}^l = \max\left(\frac{t_{ij}}{N_j^u}\right), \tag{17}$$

$$T_{Li}^u = \max\left(t_{ij}\right), \tag{18}$$

$$B_j^l = \frac{Q_i^* T_{Li}}{H}, \tag{19}$$

$$B_j^u = \min\left(Q_i, \min_j\left(\frac{V_j^u}{S_{ij}}\right)\right) \tag{20}$$

Parameters S_{ij} and t_{ij} are given in Table 2.

2.3. Mechanical design problems

In this section, several mechanical element design problems are considered. A brief description of different mechanical design problems is provided in the following subsections.

2.3.1. Weight minimization of a speed reducer [39].

It involves the design of a speed reducer for a small aircraft engine. The resulting optimization problem has the following form.

Minimize:

$$\begin{aligned} f(\bar{x}) = {} & 0.7854x_2^2x_1(14.9334x_3 - 43.0934 + 3.3333x_3^2) \\ & +0.7854(x_5x_7^2 + x_4x_6^2) - 1.508x_1(x_7^2 + x_6^2) + 7.477(x_7^3 + x_6^3) \end{aligned} \tag{21}$$

subject to:

$$g_1(\bar{x}) = -x_1x_2^2x_3 + 27 \leq 0,$$

$$g_2(\bar{x}) = -x_1x_2^2x_3^2 + 397.5 \leq 0,$$

$$g_3(\bar{x}) = -x_2x_6^4x_3x_4^{-3} + 1.93 \leq 0,$$

$$g_4(\bar{x}) = -x_2x_7^4x_3x_5^{-3} + 1.93 \leq 0,$$

$$g_5(\bar{x}) = 10x_6^{-3}\sqrt{16.91 \times 10^6 + (745x_4x_2^{-1}x_3^{-1})^2} - 1100 \leq 0,$$

$$g_6(\bar{x}) = 10x_7^{-3}\sqrt{157.5 \times 10^6 + (745x_5x_2^{-1}x_3^{-1})^2} - 850 \leq 0,$$

$$g_7(\bar{x}) = x_2x_3 - 40 \leq 0,$$

$$g_8(\bar{x}) = -x_1x_2^{-1} + 5 \leq 0,$$

$$g_9(\bar{x}) = x_1x_2^{-1} - 12 \leq 0,$$

$$g_{10}(\bar{x}) = 1.5x_6 - x_4 + 1.9 \leq 0,$$

$$g_{11}(\bar{x}) = 1.1x_7 - x_5 + 1.9 \leq 0,$$

with bounds:

$$0.7 \leq x_2 \leq 0.8, 17 \leq x_3 \leq 28, 2.6 \leq x_1 \leq 3.6,$$

$$5 \leq x_7 \leq 5.5, 7.3 \leq x_5, x_4 \leq 8.3, 2.9 \leq x_6 \leq 3.9.$$

2.3.2. Optimal design of industrial refrigeration system [40].

The mathematical model of this problem is described in Ref. [40,41]. This problem can be formulated as a non-linear inequality COP and has the following form:

Minimize:

$$\begin{aligned} f(\bar{x}) = {} & 63098.88x_2x_4x_{12} + 5441.5x_2^2x_{12} + 115055.5x_2^{1.664}x_6 \\ & + 6172.27x_2^2x_6 + 63098.88x_1x_3x_{11} + 5441.5x_1^2x_{11} \\ & + 115055.5x_1^{1.664}x_5 + 6172.27x_1^2x_5 + 140.53x_1x_{11} \\ & + 281.29x_3x_{111} + 70.26x_1^2 + 281.29x_1x_3 + 281.29x_3^2 \\ & + 14437x_8^{1.8812}x_{12}^{0.3424}x_{10}x_{14}^{-1}x_1^2x_7x_9^{-1} + 20470.2x_7^{2.893}x_{11}^{0.316}x_1^2 \end{aligned} \tag{22}$$

subject to:

$$g_1(\bar{x}) = 1.524x_7^{-1} \leq 1,$$

$$g_2(\bar{x}) = 1.524x_8^{-1} \leq 1,$$

$$g_3(\bar{x}) = 0.07789x_1 - 2x_7^{-1}x_9 - 1 \leq 0,$$

$$g_4(\bar{x}) = 7.05305x_9^{-1}x_1^2x_{10}x_8^{-1}x_2^{-1}x_{14}^{-1} - 1 \leq 0,$$

$$g_5(\bar{x}) = 0.0833x_{13}^{-1}x_{14} - 1 \leq 0,$$

$$\begin{aligned} g_6(\bar{x}) = {} & 47.136x_2^{0.333}x_{10}^{-1}x_{12} - 1.333x_8x_{13}^{2.1195} \\ & + 62.08x_{13}^{2.1195}x_{12}^{-1}x_8^{0.2}x_{10}^{-1} - 1 \leq 0, \end{aligned}$$

$$g_7(\overline{x}) = 0.04771x_{10}x_8^{1.8812}x_{12}^{0.3424} - 1 \le 0,$$

$$g_8(\overline{x}) = 0.0488x_9x_7^{1.893}x_{11}^{0.316} - 1 \le 0,$$

$$g_9(\overline{x}) = 0.0099x_1x_3^{-1} - 1 \le 0,$$

$$g_{10}(\overline{x}) = 0.0193x_2x_4^{-1} - 1 \le 0,$$

$$g_{11}(\overline{x}) = 0.0298x_1x_5^{-1} - 1 \le 0,$$

$$g_{12}(\overline{x}) = 0.056x_2x_6^{-1} - 1 \le 0,$$

$$g_{13}(\overline{x}) = 2x_9^{-1} - 1 \le 0,$$

$$g_{14}(\overline{x}) = 2x_{10}^{-1} - 1 \le 0,$$

$$g_{15}(\overline{x}) = x_{12}x_{11}^{-1} - 1 \le 0,$$

with bounds:

$$0.001 \le x_i \le 5, i = 1, \dots, 14.$$

2.3.3. Tension/compression spring design [42].

The main objective of this problem is to optimize the weight of a tension or compression spring. This problem contains four constraints and three variables are utilized to calculate the weight: the diameter of the wire (x_1), the mean of the diameter of coil (x_2), and the number of active coils (x_3). This problem is defined in the following way.

Minimize:

$$f(\overline{x}) = x_1^2x_2(2 + x_3) \tag{23}$$

subject to:

$$g_1(\overline{x}) = 1 - \frac{x_2^3x_3}{71785x_1^4} \le 0$$

$$g_2(\overline{x}) = \frac{4x_2^2 - x_1x_2}{12566(x_2x_1^3 - x_1^4)} + \frac{1}{5108x_1^2} - 1 \le 0$$

$$g_3(\overline{x}) = 1 - \frac{140.45x_1}{x_2^2x_3} \le 0$$

$$g_4(\overline{x}) = \frac{x_1 + x_2}{1.5} - 1 \le 0$$

with bounds:

$$0.05 \le x_1 \le 2.00$$

$$0.25 \le x_2 \le 1.30$$

$$2.00 \le x_3 \le 15.0$$

2.3.4. Pressure vessel design [43].

The main objective of this problem is to optimize the welding cost, material, and forming of a vessel. This problem contains four constraints which are needed to be satisfied, and four variables are used to calculate the objective function: shell thickness (z_1), head thickness (z_2), inner radius (x_3), and length of the vessel without including the head (x_4). This problem can be stated as.

Minimize:

$$f(\overline{x}) = 1.7781z_2x_3^2 + 0.6224z_1x_3x_4 + 3.1661z_1^2x_4 + 19.84z_1^2x_3 \tag{24}$$

subject to:

$$g_1(\overline{x}) = 0.00954x_3 \le z_2,$$

$$g_2(\overline{x}) = 0.0193x_3 \le z_1,$$

$$g_3(\overline{x}) = x_4 \le 240,$$

$$g_4(\overline{x}) = -\pi x_3^2x_4 - \frac{4}{3}\pi x_3^3 \le -1296000.$$

where:

$$z_1 = 0.0625x_1,$$

$$z_2 = 0.0625x_2.$$

with bounds:

$$10 \le x_4, x_3 \le 200$$

$$1 \le x_2, x_1 \le 99 \ \text{(integer variables)}.$$

2.3.5. Welded beam design [44].

The main objective of this problem is to design a welded beam with minimum cost. This problem contains five constraints, and four variables are used to develop a welded beam. The mathematical description of this problem can be defined as follows.

Minimize:

$$f(\overline{x}) = 0.04811x_3x_4(x_2 + 14) + 1.10471x_1^2x_2 \tag{25}$$

subject to:

$$g_1(\overline{x}) = x_1 - x_4 \le 0,$$

$$g_2(\overline{x}) = \delta(\overline{x}) - \delta_{max} \le 0,$$

$$g_3(\overline{x}) = P \le P_c(\overline{x})$$

$$g_4(\overline{x}) = \tau_{max} \ge \tau(\overline{x}),$$

$$g_5(\overline{x}) = \sigma(\overline{x}) - \sigma_{max} \le 0,$$

where,

$$\tau = \sqrt{\tau'^2 + \tau''^2 + 2\tau'\tau''\frac{x_2}{2R}}, \tau'' = \frac{RM}{J}, \tau' = \frac{P}{\sqrt{2}x_2x_1},$$

$$M = p\left(\frac{x_2}{2} + L\right),$$

$$R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2}, J = 2\left(\left(\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2\right)\sqrt{2}x_1x_2\right),$$

$$\sigma(\overline{x}) = \frac{6PL}{x_4x_3^2}, \delta(\overline{x}) = \frac{6PL^3}{Ex_3^2X_4}, P_c(\overline{x}) = \frac{4.013Ex_3x_4^3}{6L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right)$$

$$L = 14\text{in}, P = 6000\text{lb}, E = 30.10^6\text{psi}, \sigma_{max} = 30,000\text{psi},$$

$$\tau_{max} = 13,600\text{psi}, G = 12.10^6\text{psi}, \delta_{max} = 0.25\text{in},.$$

with bounds:

$0.1 \leq x_3, x_2 \leq 10$

$0.1 \leq x_4 \leq 2$

$0.125 \leq x_1 \leq 2$

2.3.6. *Three-bar truss design problem [45]*

This optimization problem is taken from civil engineering, and has an accidented constrained space. The main objective of this problem is to minimize the weight of the bar structures. The constraints of this problem are based on the stress constraints of each bar. The resultant problem has linear objective function with three non-linear constraints. The mathematical description of this problem is given below.

Minimize:

$$f(\overline{x}) = l\left(x_2 + 2\sqrt{2}x_1\right) \tag{26}$$

subject to:

$$g_1(\overline{x}) = \frac{x_2}{2x_2x_1 + \sqrt{2}x_1^2}P - \sigma \leq 0,$$

$$g_2(\overline{x}) = \frac{x_2 + \sqrt{2}x_1}{2x_2x_1 + \sqrt{2}x_1^2}P - \sigma \leq 0,$$

$$g_3(\overline{x}) = \frac{1}{x_1 + \sqrt{2}x_2}P - \sigma \leq 0,$$

where,

$l = 100, P = 2,$ and $\sigma = 2.$

with bounds:

$0 \leq x_1, x_2 \leq 1.$

2.3.7. *Multiple disk clutch brake design problem [46].*

The main objective of this problem is to minimize the mass of a multiple disk clutch brake. In this problem, five integer decision variables are used which are inner radius (x_1), outer radius (x_2), disk thickness (x_3), force of actuators (x_4), and number of frictional surfaces (x_5). This problem contains nine non-linear constraints. The problem can be defined as follows.

Minimize:

$$f(\overline{x}) = \pi(x_2^2 - x_1^2)x_3(x_5 + 1)\rho \tag{27}$$

subject to:

$$g_1(\overline{x}) = -p_{max} + p_{rz} \leq 0,$$

$$g_2(\overline{x}) = p_{rz}V_{sr} - V_{sr,max}p_{max} \leq 0,$$

$$g_3(\overline{x}) = \triangle R + x_1 - x_2 \leq 0,$$

$$g_4(\overline{x}) = -L_{max} + (x_5 + 1)(x_3 + \delta) \leq 0,$$

$$g_5(\overline{x}) = sM_s - M_h \leq 0,$$

$$g_6(\overline{x}) = T \geq 0,$$

$$g_7(\overline{x}) = -V_{sr,max} + V_{sr} \leq 0,$$

$$g_7(\overline{x}) = T - T_{max} \leq 0,$$

where,

$$M_h = \frac{2}{3}\mu x_4 x_5 \frac{x_2^3 - x_1^3}{x_2^2 - x_1^2}\text{N.mm},$$

$$\omega = \frac{\pi n}{30}\text{rad/s},$$

$$A = \pi(x_2^2 - x_1^2)\text{mm}^2,$$

$$p_{rz} = \frac{x_4}{A}\text{N/mm}^2,$$

$$V_{sr} = \frac{\pi R_{sr} n}{30}\text{mm/s},$$

$$R_{sr} = \frac{2}{3}\frac{x_2^3 - x_1^3}{x_2^2 x_1^2}\text{mm},$$

$$T = \frac{I_z\omega}{M_h + M_f},$$

$$\triangle R = 20\text{mm}, L_{max} = 30\text{mm}, \mu = 0.6,$$

$$V_{sr,max} = 10\text{m/s}, \delta = 0.5\text{mm}, s = 1.5,$$

$$T_{max} = 15\text{s}, n = 250\text{rpm}, I_z = 55\text{Kg.m}^2,$$

$$M_s = 40\text{Nm}, M_f = 3\text{Nm}, \text{and } p_{max} = 1.$$

with bounds:

$60 \leq x_1 \leq 80, 90 \leq x_2 \leq 110, 1 \leq x_3 \leq 3,$

$0 \leq x_4 \leq 1000, 2 \leq x_5 \leq 9.$

2.3.8. *Planetary gear train design optimization problem [47].*

The main objective of this problem is to minimize the maximum errors in the gear ratio, which is used in automobiles. To minimize the maximum error, the total number of gear-teeth is calculated for an automatic planetary transmission system. This problem contains six integer variables and 11 constraints of different geometric and assembly restrictions. The problem can be defined as follows.

Minimize:

$$f(\overline{x}) = \max|i_k - i_{0k}|, k = \{1, 2, .., R\}, \tag{28}$$

where,

$$i_1 = \frac{N_6}{N_4}, i_{01} = 3.11, i_2 = \frac{N_6(N_1N_3 + N_2N_4)}{N_1N_3(N_6 - N_4)}, \quad i_{0R} = -3.11,$$

$$I_R = -\frac{N_2N_6}{N_1N_3}, i_{02} = 1.84, \quad \overline{x} = (p, N_6, N_5, N_4, N_3, N_2, N_1, m_2, m_1)$$

subject to:

$$g_1(\overline{x}) = m_3(N_6 + 2.5) - D_{max} \leq 0,$$

$$g_2(\overline{x}) = m_1(N_1 + N_2) + m_1(N_2 + 2) - D_{max} \leq 0,$$

$$g_3(\overline{x}) = m_3(N_4 + N_5) + m_3(N_5 + 2) - D_{max} \leq 0,$$

$$g_4(\overline{x}) = |m_1(N_1 + N_2) - m_3(N_6 - N_3)| - m_1 - m_3 \leq 0,$$

$$g_5(\overline{x}) = -(N_1 + N_2)\sin(\pi/p) + N_2 + 2 + \delta_{22} \leq 0,$$

$$g_6(\overline{x}) = -(N_6 - N_3)\sin(\pi/p) + N_3 + 2 + \delta_{33} \leq 0,$$

$$g_7(\overline{x}) = -(N_4 + N_5)\sin(\pi/p) + N_5 + 2 + \delta_{55} \le 0,$$

$$g_8(\overline{x}) = (N_3 + N_5 + 2 + \delta_{35})^2 - (N_6 - N_3)^2 - (N_4 + N_5)^2 + 2(N_6 - N_3)(N_4 + N_5)\cos\left(\frac{2\pi}{p} - \beta\right) \le 0,$$

$$g_9(\overline{x}) = N_4 - N_6 + 2N_5 + 2\delta_{56} + 4 \le 0,$$

$$g_{10}(\overline{x}) = 2N_3 - N_6 + N_4 + 2\delta_{34} + 4 \le 0,$$

$$h_1(\overline{x}) = \frac{N_6 - N_4}{p} = \text{integer},$$

where,

$$\delta_{22} = \delta_{33} = \delta_{55} = \delta_{35} = \delta_{56} = 0.5.$$

$$\beta = \frac{cos^{-1}((N_4 + N_5)^2 + (N_6 - N_3)^2 - (N_3 + N_5)^2)}{2(N_6 - N_3)(N_4 + N_5)}, D_{max} = 220,$$

with bounds:

$$p = (3, 4, 5),$$

$$m_1 = (1.75, 2.0, 2.25, 2.5, 2.75, 3.0),$$

$$m_3 = (1.75, 2.0, 2.25, 2.5, 2.75, 3.0),$$

$$17 \le N_1 \le 96, \quad 14 \le N_2 \le 54, \quad 14 \le N_3 \le 51$$
$$17 \le N_4 \le 46, \quad 14 \le N_5 \le 51, \quad 48 \le N_6 \le 124,$$

and N_i = integer.

2.3.9. Step-cone pulley problem [48].

The main objective of this problem is to minimize weight of 4 step-cone pulley using five variables in which four variables are the diameters of each step of the pulley and the last one is the width of the pulley. This problem contains 11 non-linear constraints to assure that the transmit power must be at 0.75 hp. The mathematical formulation of this problem can be defined as follows.

Minimize:

$$f(\overline{x}) = \rho\omega\left[d_1^2\left\{11 + \left(\frac{N_1}{N}\right)^2\right\} + d_2^2\left\{1 + \left(\frac{N_2}{N}\right)^2\right\} + d_3^2\left\{1 + \left(\frac{N_3}{N}\right)^2\right\} + d_4^2\left\{1 + \left(\frac{N_4}{N}\right)^2\right\}\right] \tag{29}$$

subject to:

$$h_1(\overline{x}) = C_1 - C_2 = 0,$$

$$h_2(\overline{x}) = C_1 - C_3 = 0,$$

$$h_3(\overline{x}) = C_1 - C_4 = 0,$$

$$g_{i=1,2,3,4}(\overline{x}) = -R_i \le 2,$$

$$g_{i=5,6,7,8}(\overline{x}) = (0.75 \times 745.6998) - P_i \le 0$$

where,

$$C_i = \frac{\pi d_i}{2}\left(1 + \frac{N_i}{N}\right) + \frac{\left(\frac{N_i}{N} - 1\right)^2}{4a} + 2a, i = (1, 2, 3, 4),$$

$$R_i = \exp\left(\mu\left\{\pi - 2sin^{-1}\{\left(\frac{N_i}{N} - 1\right)\frac{d_i}{2a}\right\}\}\right), i = (1, 2, 3, 4)$$

$$P_i = st\omega(1 - R_i)\frac{\pi d_i N_i}{60}, i = (1, 2, 3, 4)$$

$$t = 8\text{mm}, \ s = 1.75\text{MPa}, \quad \mu = 0.35, \ \rho = 7200\text{kg/m}^3, \quad a = 3\text{mm}.$$

2.3.10. Robot gripper problem [49].

In this problem, the difference between the minimum and maximum force generated by the robot gripper is used as an objective function. This problem contains seven design variables and six non-linear design constraints associated with the robot. Mathematically, this problem is defined as follows.

Minimize:

$$f(\overline{x}) = -\min_z F_k(x, z) + \max_z F_k(x, z) \tag{30}$$

subject to:

$$g_1(\overline{x}) = -Y_{min} + y(\overline{x}, Z_{max}) \le 0,$$

$$g_2(\overline{x}) = -y(x, Z_{max}) \le 0,$$

$$g_3(\overline{x}) = Y_{max} - y(\overline{x}, 0) \le 0,$$

$$g_4(\overline{x}) = y(\overline{x}, 0) - Y_G \le 0,$$

$$g_5(\overline{x}) = l^2 + e^2 - (a + b)^2 \le 0,$$

$$g_6(\overline{x}) = b^2 - (a - e)^2 - (l - Z_{max})^2 \le 0,$$

$$g_7(\overline{x}) = Z_{max} - l \le 0$$

where,

$$\alpha = cos^{-1}\left(\frac{a^2 + g^2 - b^2}{2ag}\right) + \phi, g = \sqrt{e^2 + (z - l)^2},$$

$$\beta = cos^{-1}\left(\frac{b^2 + g^2 - a^2}{2bg}\right) - \phi, \phi = tan^{-1}\left(\frac{e}{l - z}\right),$$

$$y(x, z) = 2(f + e + c\sin(\beta + \delta)), \ F_k = \frac{Pb\sin(\alpha + \beta)}{2c\cos(\alpha)}, \quad Y_{min} = 50,$$

$$Y_{max} = 100, Y_G = 150, Z_{max} = 100, P = 100.$$

with bounds:

$$0 \le e \le 50, 100 \le c \le 200, 10 \le f, a, b \le 150,$$

$$1 \le \delta \le 3.14, 100 \le l \le 300.$$

2.3.11. Hydro-staticbThrust bearing design problem [50].

The main objective of this design problem is to optimize bearing power loss using four design variables. These design variables are oil viscosity μ, bearing radius R, flow rate Q, and recess radius R_o. This problem contains seven non-linear constraints associated with inlet oil pressure, load-carrying capacity, oil film thickness, and inlet oil pressure. The problem is defined as follows.

Minimize:

$$f(\overline{x}) = \frac{QP_0}{0.7} + E_f \tag{31}$$

subject to:

$$g_1(\overline{x}) = 1000 - P_0 \le 0,$$

$$g_2(\overline{x}) = W - 101000 \le 0,$$

$$g_3(\overline{x}) = 5000 - \frac{W}{\pi(R^2 - R_0^2)} \le 0,$$

$$g_4(\overline{x}) = 50 - P_0 \le 0,$$

$$g_5(\overline{x}) = 0.001 - \frac{0.0307}{386.4P_0}\left(\frac{Q}{2\pi Rh}\right) \le 0,$$

$$g_6(\overline{x}) = R - R_0 \le 0,$$

$$g_7(\overline{x}) = h - 0.001 \le 0,$$

where,

$$W = \frac{\pi P_0}{2}\frac{R^2 - R_0^2}{ln\left(\frac{R}{R_0}\right)}, P_0 = \frac{6\mu Q}{\pi h^3} ln\left(\frac{R}{R_0}\right),$$

$$E_f = 9336Q \times 0.0307 \times 0.5 \triangle T, \triangle T = 2(10^P - 559.7),$$

$$P = \frac{log_{10}log_{10}\left(8.122 \times 10^6\mu + 0.8\right) + 3.55}{10.04},$$

$$h = \left(\frac{2\pi \times 750}{60}\right)^2 \frac{2\pi\mu}{E_f}\left(\frac{R^4}{4} - \frac{R_0^4}{4}\right)$$

with bounds:

$$1 \le R \le 16, 1 \le R_0 \le 16,$$

$$1 \times 10^{-6} \le \mu \le 16 \times 10^{-6}, 1 \le Q \le 16.$$

$$g_{9-12}(\overline{x}) = -N_{pi}\sqrt{\frac{\sin^2(\phi)}{4} - \frac{1}{N_{pi}} + \left(\frac{1}{N_{pi}}\right)^2} + N_{gi}\sqrt{\frac{\sin^2(\phi)}{4} + \frac{1}{N_{gi}}\left(\frac{1}{N_{gi}}\right)^2} + \frac{\sin(\phi)\left(N_{pi} + N_{gi}\right)}{2} + CR_{min}\pi\cos(\phi) \le 0,$$

2.3.12. Four-stage gear box problem [51].

In this problem, the minimization of gearbox weight is considered as an objective where 22 design variables are used. These design variables are discrete in nature which include positions of the gear, positions of pinion, blank thickness, and number of teeth. This problem contains 86 non-linear design constraints associated with the pitch, kinematics, contact ratio, strength of the gears, assembly of gears, and size of gears. The feasible search-space of this problem is in ratio less than 0.0001 with many local solutions. The problem is defined as.

Minimize:

$$f(\overline{x}) = \left(\frac{\pi}{1000}\right)\sum_{i=1}^{4}\frac{b_i c_i^2(N_{pi}^2 + N_{gi}^2)}{(N_{pi} + N_{gi})^2}, \text{ where, } i = (1, 2, 3, 4) \tag{32}$$

subject to:

$$g_1(\overline{x}) = \left(\frac{366000}{\pi\omega_1} + \frac{2c_1N_{p1}}{N_{pi} + N_{g1}}\right)\left(\frac{\left(N_{p1} + N_{g1}\right)^2}{4b_1c_1^2N_{p1}}\right) - \frac{\sigma_N J_R}{0.0167WK_oK_m} \le 0,$$

$$g_2(\overline{x}) = \left(\frac{366000N_{g1}}{\pi\omega_1N_{p1}} + \frac{2c_2N_{p2}}{N_{p2} + N_{g2}}\right)\left(\frac{\left(N_{p2} + N_{g2}\right)^2}{4b_2c_2^2N_{p2}}\right) - \frac{\sigma_N J_R}{0.0167WK_oK_m} \le 0,$$

$$g_3(\overline{x}) = \left(\frac{366000N_{g1}N_{g2}}{\pi\omega_1N_{p1}N_{p2}} + \frac{2c_3N_{p3}}{N_{p3} + N_{g3}}\right)\left(\frac{\left(N_{p3} + N_{g3}\right)^2}{4b_3c_3^2N_{p3}}\right) - \frac{\sigma_N J_R}{0.0167WK_oK_m} \le 0,$$

$$g_4(\overline{x}) = \left(\frac{366000N_{g1}N_{g2}N_{g3}}{\pi\omega_1N_{p1}N_{p2}N_{p3}} + \frac{2c_4N_{p4}}{N_{p4} + N_{g4}}\right)\left(\frac{\left(N_{p4} + N_{g4}\right)^2}{4b_4c_4^2N_{p4}}\right) - \frac{\sigma_N J_R}{0.0167WK_oK_m} \le 0,$$

$$g_5(\overline{x}) = \left(\frac{366000}{\pi\omega_1} + \frac{2c_1N_{p1}}{N_{p1} + N_{g1}}\right)\left(\frac{\left(N_{p1} + N_{g1}\right)^3}{4b_1c_1^2N_{g1}N_{p1}^2}\right) - \left(\frac{\sigma_H}{C_p}\right)^2\left(\frac{\sin(\phi)\cos(\phi)}{0.0334WK_oK_m}\right) \le 0,$$

$$g_6(\overline{x}) = \left(\frac{366000N_{g1}}{\pi\omega_1N_{p1}} + \frac{2c_2N_{p2}}{N_{p2} + N_{g2}}\right)\left(\frac{\left(N_{p2} + N_{g2}\right)^3}{4b_2c_2^2N_{g2}N_{p2}^2}\right) - \left(\frac{\sigma_H}{C_p}\right)^2\left(\frac{\sin(\phi)\cos(\phi)}{0.0334WK_oK_m}\right) \le 0,$$

$$g_7(\overline{x}) = \left(\frac{366000N_{g1}N_{g2}}{\pi\omega_1N_{p1}N_{p2}} + \frac{2c_3N_{p3}}{N_{p3} + N_{g3}}\right)\left(\frac{\left(N_{p3} + N_{g3}\right)^3}{4b_3c_3^2N_{g3}N_{p3}^2}\right) - \left(\frac{\sigma_H}{C_p}\right)^2\left(\frac{\sin(\phi)\cos(\phi)}{0.0334WK_oK_m}\right) \le 0,$$

$$g_8(\overline{x}) = \left(\frac{366000N_{g1}N_{g2}N_{g3}}{\pi\omega_1N_{p1}N_{p2}N_{p3}} + \frac{2c_4N_{p4}}{N_{p4} + N_{g4}}\right)\left(\frac{\left(N_{p4} + N_{g4}\right)^3}{4b_4c_4^2N_{g4}N_{p4}^2}\right) - \left(\frac{\sigma_H}{C_p}\right)^2\left(\frac{\sin(\phi)\cos(\phi)}{0.0334WK_oK_m}\right) \le 0,$$

$$g_{13-16}(\overline{x}) = d_{min} - \frac{2c_iN_{pi}}{N_{pi} + N_{gi}} \le 0,$$

$$g_{17-20}(\overline{x}) = d_{min} - \frac{2c_iN_{gi}}{N_{pi} + N_{gi}} \le 0,$$

$$g_{21}(\overline{x}) = x_{p1} + \left(\frac{\left(N_{p1} + 2\right)c_1}{N_{p1} + N_{g1}}\right) - L_{max} \le 0,$$

$$g_{22-24}(\overline{x}) = -L_{max} + \left(\frac{\left(N_{pi} + 2\right)c_i}{N_{gi} + N_{pi}}\right)_{i=2,3,4} + x_{g(i-1)} \le 0,$$

$$g_{25}(\overline{x}) = -x_{p1} + \frac{\left(N_{p1} + 2\right)c_1}{N_{p1} + N_{g1}} \le 0,$$

$$g_{26-28}(\overline{x}) = \left(\frac{\left(N_{pi} + 2\right)c_i}{N_{pi} + N_{gi}} - x_{g(i-1)}\right)_{i=2,3,4} \le 0,$$

$$g_{29}(\overline{x}) = y_{p1} + \frac{\left(N_{p1} + 2\right)c_1}{N_{p1} + N_{g1}} - L_{max} \le 0,$$

$$g_{30-32}(\overline{x}) = -L_{max} + \left(\frac{c_i\left(2 + N_{pi}\right)}{N_{pi} + N_{gi}} + y_{g(i-1)}\right)_{i=2,3,4} \le 0,$$

$$g_{33}(\overline{x}) = \frac{\left(2 + N_{p1}\right)c_1}{N_{p1} + N_{g1}} - y_{p1} \le 0,$$

$$g_{34-36}(\overline{x}) = \left(\frac{c_i\left(2 + N_{pi}\right)}{N_{pi} + N_{gi}} - y_{g(i-1)}\right)_{i=2,3,4} \le 0,$$

$$g_{37-40}(\overline{x}) = -L_{max} + \frac{c_i\left(2+N_{gi}\right)}{N_{pi}+N_{gi}} + x_{gi} \le 0,$$

$$g_{41-44}(\overline{x}) = -x_{gi} + \left(\frac{\left(N_{gi}+2\right)c_i}{N_{pi}+N_{gi}}\right) \le 0,$$

$$g_{45-48}(\overline{x}) = y_{gi} + \left(\frac{\left(N_{gi}+2\right)c_i}{N_{pi}+N_{gi}}\right) - L_{max} \le 0,$$

$$g_{49-52}(\overline{x}) = -y_{gi} + \left(\frac{\left(N_{gi}+2\right)ci}{N_{pi}+N_{gi}}\right) \le 0,$$

$$g_{53-56}(\overline{x}) = (b_i-8.255)(b_i-5.715)(b_i-12.70)\left(-N_{pi}+0.945c_i-N_{gi}\right)(-1) \le 0,$$

$$g_{57-60}(\overline{x}) = (b_i-8.255)(b_i-3.175)(b_i-12.70)\left(-N_{pi}+0.646c_i-N_{gi}\right) \le 0,$$

$$g_{61-64}(\overline{x}) = (b_i-5.715)(b_i-3.175)(b_i-12.70)\left(-N_{pi}+0.504c_i-N_{gi}\right) \le 0,$$

$$g_{65-68}(\overline{x}) = (b_i-5.715)(b_i-3.175)(b_i-8.255)\left(0c_i-N_{gi}-N_{pi}\right) \le 0,$$

$$g_{69-72}(\overline{x}) = (b_i-8.255)(b_i-5.715)(b_i-12.70)\left(N_{gi}+N_{pi}-1.812c_i\right)(-1) \le 0,$$

$$g_{73-76}(\overline{x}) = (b_i-8.255)(b_i-3.175)(b_i-12.70)\left(-0.945c_i+N_{pi}+N_{gi}\right) \le 0,$$

$$g_{77-80}(\overline{x}) = (b_i-5.715)(b_i-3.175)(b_i-12.70)\left(-0.646c_i+N_{pi}+N_{gi}\right)(-1) \le 0,$$

$$g_{81-84}(\overline{x}) = (b_i-5.715)(b_i-3.175)(b_i-8.255)\left(N_{pi}+N_{gi}-0.504c_i\right) \le 0,$$

$$g_{85} = \omega_{min} - \frac{\omega_1\left(N_{p1}N_{p2}N_{p3}N_{p4}\right)}{\left(N_{g1}N_{g2}N_{g3}N_{g4}\right)} \le 0,$$

$$g_{86} = \frac{\omega_1\left(N_{p1}N_{p2}N_{p3}N_{p4}\right)}{\left(N_{g1}N_{g2}N_{g3}N_{g4}\right)} - \omega_{max} \le 0,$$

where,

$$\overline{x} = \{N_{p1}, N_{g1}, N_{p2}, N_{g2} \dots .b_1, b_2 \dots .x_{p1}, x_{g1}, x_{g2} \dots .y_{p1}, y_{g1}, y_{g2} \dots .y_{g4}\},$$

$$c_i = \sqrt{(y_{gi}-y_{pi})^2 + (x_{gi}-x_{pi})^2}, K_0 = 1.5, d_{min} = 25, J_R = 0.2,$$

$$\phi = 120^o, W = 55.9, K_M = 1.6, CR_{min} = 1.4,$$

$$L_{max} = 127, C_p = 464, \sigma_H = 3290, \omega_{max} = 255,$$

$$\omega_1 = 5000, \sigma_N = 2090, \omega_{min} = 245.$$

with bounds:

$$b_i \in \{3.175, 12.7, 8.255, 5.715\},$$

$$y_{p1}, x_{p1}, y_{gi}, x_{gi} \in \{12.7, 38.1, 25.4, 50.8, 76.2, 63.5, 88.9, 114.3, 101.6\},$$

$$7 \le N_{gi}, N_{pi} \le 76 \in \text{integer}.$$

2.3.13. 10-Bar truss optimization with frequency constraints [52].

The main aim of this problem is to minimize the weight of the truss structure with satisfying frequency constraints. The mathematical formulation of this problem can be defined as follows.

Minimize:

$$f(\overline{x}) = \sum_{i=1}^{10} L_i(x_i)\rho_i A_i \tag{33}$$

subject to:

$$g_1(\overline{x}) = \frac{7}{\omega_1(\overline{x})} - 1 \le 0,$$

$$g_2(\overline{x}) = \frac{15}{\omega_2(\overline{x})} - 1 \le 0,$$

$$g_3(\overline{x}) = \frac{20}{\omega_3(\overline{x})} - 1 \le 0,$$

with bounds:

$$6.45 \times 10^{-5} \le A_i \le 5 \times 10^{-3}, i = 1, 2, \dots, 10.$$

where,

$$\overline{x} = \{A_1, A_2, \dots ., A_{10}\}, \rho = 2770.$$

2.3.14. Rolling element bearing [53].

This problem is formulated to optimize the load-carrying capacity of a rolling element bearing using five design variables and five design parameters. These design variables are pitch diameter (D_m), ball diameter (D_b), outer and inner raceway curvature coefficients (f_o and f_i) and total number of balls (Z). The design parameters are e, ϵ, ζ, K_{Dmax}, and K_{Dmin} appeared only in the constraints. These all are considered as variables i.e. five design variables and five design parameters. This problem contains nine non-linear constraints based on manufacturing and kinematic factors.

Maximize:

$$f(\overline{x}) = \begin{cases} f_c Z^{2/3} D_b^{1.8} & \text{, if } D_b \le 25.4 \text{ mm} \\ 3.647 f_c Z^{2/3} D_b^{1.4} & \text{, otherwise} \end{cases} \tag{34}$$

subject to:

$$g_1(\overline{x}) = Z - \frac{\phi_0}{2\sin^{-1}(D_b/D_m)} - 1 \le 0,$$

$$g_2(\overline{x}) = K_{Dmin}(D-d) - 2D_b \le 0,$$

$$g_3(\overline{x}) = 2D_b - K_{Dmax}(D-d) \le 0,$$

$$g_4(\overline{x}) = D_b - \zeta B_w \le 0,$$

$$g_5(\overline{x}) = 0.5(D+d) - D_m \le 0,$$

$$g_6(\overline{x}) = D_m - (0.5+e)(D+d) \le 0,$$

$$g_7(\overline{x}) = \epsilon D_b - 0.5(D - D_m - D_b) \le 0,$$

$$g_8(\overline{x}) = 0.515 - f_i \le 0,$$

$$g_9(\overline{x}) = 0.515 - f_0 \le 0,$$

where,

$$f_c = 37.91\left\{1 + \left\{1.04\left(\frac{1-\gamma}{1+\gamma}\right)^{1.72}\left(\frac{f_i\,(2f_0-1)}{f_0\,(2f_i-1)}\right)^{0.41}\right\}^{10/3}\right\}^{-0.3},$$

$$\gamma = \frac{D_b\cos(\alpha)}{D_m}, f_i = \frac{r_i}{D_b}, f_0 = \frac{r_0}{D_b},$$

$$\phi_0 = 2\pi - 2\cos^{-1}\left(\frac{\{(D-d)/2 - 3(T/4)\}^2 + \{D/2 - (T/4) - D_b\}^2 - \{d/2 + (T/4)\}^2}{2\{(D-d)/2 - 3(T/4)\}\{D/2 - (T/4) - D_b\}}\right)$$

$T = D - d - 2D_b, D = 160, d = 90, B_w = 30.$

with bounds:

$0.5(D+d) \le D_m \le 0.6(D+d),$

$0.15(D-d) \le D_b \le 0.45(D-d),$

$4 \le Z \le 50,$

$0.515 \le f_i \le 0.6,$

$0.515 \le f_0 \le 0.6,$

$0.4 \le K_{Dmin} \le 0.5,$

$0.6 \le K_{Dmax} \le 0.7,$

$0.3 \le \epsilon \le 0.4,$

$0.02 \le e \le 0.1,$

$0.6 \le \zeta \le 0.85,$

2.3.15. Gas transmission compressor design [54].

The mathematical formulation of this problem can be defined as follows.

Minimize:

$$f(\overline{x}) = 8.61\times 10^5 x_1^{1/2} x_2 x_3^{-2/3} x_4^{-1/2} + 3.69\times 10^4 x_3 + 7.72 \times 10^8 x_1^{-1} x_2^{0.219} - 765.43\times 10^6 x_1^{-1} \tag{35}$$

subject to:

$x_4 x_2^{-2} + x_2^{-2} - 1 \le 0,$

with bounds:

$20 \le x_1 \le 50,$

$1 \le x_2 \le 10,$

$20 \le x_3 \le 50,$

$0.1 \le x_4 \le 60.$

2.3.16. Tension/compression string design problem (case 2) [55].

The main objective of this problem is to optimize the required volume of steel wire used to build the helical compression spring. There are three design variables in this problem which are the outside diameter (D), a number of spring coils (N), and the diameter of the spring wire (d). This problem contains eight non-linear inequality constraints and contains a discreate (d), an integer (N), and a continuous variable (D). This problem can be stated as follows.

Minimize:

$$f(\overline{x}) = \frac{\pi^2 x_2 x_3^2 (x_1+2)}{4} \tag{36}$$

subject to:

$g_1(\overline{x}) = \dfrac{8000 C_f x_2}{\pi x_3^3} - 189000 \le 0,$

$g_2(\overline{x}) = l_f - 14 \le 0,$

$g_3(\overline{x}) = 0.2 - x_3 \le 0,$

$g_4(\overline{x}) = x_2 - 3 \le 0,$

$g_5(\overline{x}) = 3 - \dfrac{x_2}{x_3} \le 0,$

$g_6(\overline{x}) = \sigma_p - 6 \le 0,$

$g_7(\overline{x}) = \sigma_p + \dfrac{700}{K} + 1.05(x_1+2)x_3 - l_f \le 0,$

$g_8(\overline{x}) = 1.25 - \dfrac{700}{K} \le 0,$

where,

$$C_f = \frac{4\frac{x_2}{x_3} - 1}{4\frac{x_2}{x_3} - 4} + \frac{0.615 x_3}{x_2}, K = \frac{11.5\times 10^6 x_3^4}{8 x_1 x_2^3}, \sigma_p = \frac{300}{K},$$

$$l_f = \frac{1000}{K} + 1.05(x_1+2)x_3.$$

with bounds:

$1 \le x_1(\text{integer}) \le 70,$

$x_3(\text{discreate}) \in \{0.009, 0.0095, 0.0104, 0.0118, 0.0128, 0.0132, 0.014, 0.015, 0.0162, 0.0173, 0.018, 0.020, 0.023, 0.025, 0.028, 0.032, 0.035, 0.041, 0.047, 0.054, 0.063, 0.072, 0.080, 0.092, 0.0105, 0.120, 0.135, 0.148, 0.162, 0.177, 0.192, 0.207, 0.225, 0.244, 0.263, 0.283, 0.307, 0.0331, 0.362, 0.394, 0.4375, 0.500\}$

$0.6 \le x_2(\text{continuous}) \le 3.$

2.3.17. Gear train design problem [43].

The main objective of this problem is to minimize the ratio of gears for the arrangement of the compound gear train. The ratio of gear train is described as the ratio of the angular velocities of the output and input shaft. To generate the desired overall ratio of gears, the compound gear train is assembled using two pairs of gearwheels, $b - f$ and $d - a$. The overall ratio of gears, i_{tot} is defined by the following equation.

$$i_{tot} = \frac{\omega_o}{\omega_i} = \frac{z_d z_b}{z_a z_f} \tag{37}$$

where, z is the total number of teeth on every gearwheel and variables ω_i and ω_o represent angular velocities of the input and output shafts, respectively. The main aim of this problem is to calculate the total number of teeth for every gearwheel to generate an optimum ratio of gears closer to the desired ratio ($i_{trg} = 1/6.931$). For every gearwheel, the maximum required number of teeth is 60 and the minimum is 12. The mathematical formulation of this problem is shown as follows.

Minimize:

$$f(\overline{x}) = (i_{trg} - i_{tot})^2 = \left(\frac{1}{6.931} - \frac{x_1 x_2}{x_3 x_4}\right)^2 \tag{38}$$

subject to:

$g_{1-4}(\overline{x}) = 12 - x_i \le 0,$

$g_{5-8}(\overline{x}) = (60 - \overline{x}) \le 0$

2.3.18. Himmelblau's function [56].

Proctor and Gamble Corporation proposes this problem to simulate the Process Design Problems and was cited by D. M. Himmelblau in Ref. [56] which is used as common benchmark to analyze non-linear constrained optimization algorithms. This problem contains six nonlinear constraints and five variables. The description of this problem is shown as follows.

Minimize:

$$f(\overline{x}) = 5.3578547x_3^2 + 0.8356891x_1x_5 + 37.293239x_1 - 40792.141 \tag{39}$$

subject to:

$$g_1(\overline{x}) = -G1 \le 0,$$

$$g_2(\overline{x}) = G1 - 92 \le 0,$$

$$g_3(\overline{x}) = 90 - G2 \le 0,$$

$$g_4(\overline{x}) = G2 - 110 \le 0,$$

$$g_5(\overline{x}) = 20 - G3 \le 0,$$

$$g_6(\overline{x}) = G3 - 25 \le 0,$$

where,

$$G1 = 85.334407 + 0.0056858x_2x_5 + 0.0006262x_1x_4 - 0.0022053x_3x_5,$$

$$G2 = 80.51249 + 0.0071317_2x_5 + 0.0029955x_1x_2 + 0.0021813x_3^2,$$

$$G3 = 9.300961 + 0.0047026x_3x_5 + 0.00125447x_1x_3 + 0.0019085x_3x_4.$$

with bounds:

$$78 \le x_1 \le 102,$$

$$33 \le x_2 \le 45,$$

$$27 \le x_3 \le 45,$$

$$27 \le x_4 \le 45,$$

$$27 \le x_5 \le 45.$$

2.3.19. Topology optimization [57].

The main aim of this problem is to optimize the material layout for a provided set of loads, within given design search-space, and constraints associated with the performance of the system. This problem is based on the power-law approach and mathematically, it can be defined as follows.

Minimize:

$$f(\overline{x}) = \mathbf{U}^T\mathbf{KU} = \sum_{e=1}^{N} (x_e)^p u_e^T k_0 u_0 \tag{40}$$

subject to:

$$h_1(\overline{x}) = \frac{V(\overline{x})}{V_0} - f = 0,$$

$$h_2(\overline{x}) = \mathbf{KU} - \mathbf{F} = 0,$$

with bounds:

$$0 < \overline{x}_{min} \le x \le 1.$$

where $\mathbf{F}$ and $\mathbf{U}$ represent the force and global displacement vectors, respectively, $\mathbf{K}$ represents the global stiffness matrix, k_e and u_e represent the stiffness matrix and element displacement vector, respectively, $\overline{x}$ represents the vector of design variables, $\overline{x}_{min}$ represents a vector of non-zero minimum relative densities to avoid singularity, N represents the number of elements used to discretize the design domain, p ($p = 3$) represents the penalization power, $V(\overline{x})$ and V_0 represent the material volume and design domain volume, respectively and f represents the prescribed volume fraction.

2.4. Power system problems

2.4.1. Optimal sizing of single phase distributed generation with reactive power support for phase balancing at main transformer/grid [58].

Unbalance in a practical distribution system is a natural phenomenon. The unbalance in a distribution system creates negative and zero sequence currents leading to the inefficient working of a rotating machine along with losses in neutral conductors. In a balanced system when there are no negative or zero sequences current the neutral current flow is zero. Balanced or assumption of balance in the phases of distribution systems the neutral conductor is designed to carry a smaller current. Apart from the above-said problem due to unbalanced phase currents, one of the major concerns is of overloading of the main substation transformer. Due to unbalance the phase having the maximum loading decides the capacity of the substation transformer. Thus, even if the transformer may be underloaded on the other two phases, it cannot be further loaded to take any extra load. In the present scenario, the Distribution Generators (DGs) have been employed in the distribution system in good numbers. A DG, in general, is a suitable generation and therefore no special arrangement is required to switch the DG feeding a phase to another one. The problem of phase balancing can be easily addressed if there are single-phase DGs to redistribute the phase currents such that the unbalance is minimized. Single-phase DG can be sized to mitigate phase unbalance, thereby reducing non-positive sequence currents at the root node. This problem can be formulated as a COP, which is as follows.

Minimize:

$$\begin{aligned} f = {} & (I_{r,1}^a + I_{r,1}^b + I_{r,1}^c)^2 + (I_{m,1}^a + I_{m,1}^b + I_{m,1}^c)^2 \\ & + (I_{r,1}^a - 0.5(I_{r,1}^b + I_{r,1}^c) - 0.5\sqrt{3}(I_{m,1}^b - I_{m,1}^c))^2 \\ & + (I_{m,1}^a - 0.5(I_{m,1}^b + I_{m,1}^c) + 0.5\sqrt{3}(I_{r,1}^b - I_{r,1}^c))^2, \end{aligned} \tag{41}$$

where,

$$I_{r,1}^s = \sum_{k\in\{a,b,c\}} \sum_{i=1}^{N} \left(G_{1,i}^{sk} V_{r,i}^k - B_{1i}^{sk} V_{m,i}^k \right)$$

$$I_{m,1}^s = \sum_{k\in\{a,b,c\}} \sum_{i=1}^{N} \left(B_{1,i}^{sk} V_{r,i}^k + G_{1i}^{sk} V_{m,i}^k \right)$$

subject to:

$$h_k = \sum_{s\in(a,b,c)} \sum_{i=1}^{N} (G_{k,i}^{js} V_{r,i}^s - B_{ki}^{js} V_{m,i}^s) - \frac{P_k^j V_{r,k}^j + Q_k^j V_{m,k}^j}{(V_{r,k}^j)^2 + (V_{m,k}^j)^2} = 0,$$

$$h_{N+k} = \sum_{s\in(a,b,c)} \sum_{i=1}^{N} (B_{ki}^{js} V_{r,i}^s + G_{ki}^{js} V_{m,i}^s) - \frac{P_k^j V_{m,k}^j - Q_k^j V_{r,k}^j}{(V_{r,k}^j)^2 + (V_{m,k}^j)^2} = 0,$$

$$h_{2N+k} = P_k^j - P_{dg,k}^j + P_{l,k}^j = 0,$$

$$h_{3N+k} = Q_k^j - Q_{dg,k}^j + Q_{l,k}^j = 0,$$

with bounds:

$$V_{min} \le V_{r,k}^j, V_{m,k}^j \le V_{max}$$

$$P_{min} \le P_k^j \le P_{max}$$

$$Q_{min} \le Q^j_k \le Q_{max}$$

$$P_{dg,min} \le P^j_{dg,k} \le P_{dg,max}$$

$$Q_{dg,min} \le Q^j_{dg,k} \le Q_{dg,max}$$

where $k = 1, 2, \ldots N$; $j = \{a, b, c\}$; P^j_i and Q^j_i represent the active and reactive injected power, respectively, at the i-th bus in the j-th phase; $Ybus^{st}_{ij}(= G^{st}_{ij} + 1jB^{st}_{ij})$ is the ij-th element of the st-th block of the admittance matrix; $V^j_i(= V^j_{r,i} + 1jV^j_{m,i})$ is the bus voltage at the i-th bus in the j-th phase; $P^j_{dg,k}$ and $Q^j_{dg,k}$ represent the active and reactive power generation, respectively, at the k-th DG in the j-th phase; N represents the total number of buses in the system.

2.4.2. Optimal sizing of distributed generation for active power loss minimization

Optimal sizing of both DG is of significance in improving system performance. The aim of this problem is to decide proper size of DG in the network that will offer minimum power loss. This problem can be formulated as a COP, which is as follows.

Minimize:

$$f = \sum_{i=1}^{N} P_i \tag{42}$$

subject to:

$$h_k = \sum_{i=1}^{N} (G_{k,i}V_{r,i} - B_{ki}V_{m,i}) - \frac{P_kV_{r,k} + Q_kV_{m,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0,$$

$$h_{N+k} = \sum_{i=1}^{N} (B_{ki}V_{r,i} + G_{ki}V_{m,i}) - \frac{P_kV_{m,k} - Q_kV_{r,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0,$$

$$h_{2N+k} = P_k - P_{dg,k} + P_{l,k} = 0,$$

$$h_{3N+k} = Q_k + Q_{l,k} = 0,$$

with bounds:

$$V_{min} \le V_{r,k}, V_{m,k} \le V_{max}$$

$$P_{min} \le P_k \le P_{max}$$

$$Q_{min} \le Q_k \le Q_{max}$$

$$P_{min,dg} \le P_{dg,k} \le P_{max,dg}$$

where $k = 1, 2, \ldots N$; P_i and Q_i represent the active and reactive injected power, respectively, at i-th bus; $Ybus_{ij}(= G_{ij} + 1jB_{ij})$ is ij-th element of admittance matrix; $V_i(= V_{r,i} + 1jV_{m,i})$ is bus voltage at i-th bus; $P_{dg,k}$ represents the active power generation of DG at $k-$th bus; N represents the total number of buses in the system.

2.4.3. Optimal sizing of distributed generation (DG) and capacitors for reactive power loss minimization

System loads such as transformer, induction motors, cables, and transmission lines are usually inductive. These loads consume reactive power and introduce a lagging power factor. Consequently, poor performance and losses are introduced in the system. Shunt capacitors are used to deliver reactive power to improve lagging VAR of the system. Moreover, DGs are a better and efficient mean to reduce the active power loss of the system. Further, Shunt Capacitors (SC) integrated with DG can be utilized to cut down the active and reactive power loss of the distribution network. The optimal sizing of DGs and SCs can be developed as a COP.

Minimize:

$$f = 0.5\sum_{i=1}^{N} P_i + 0.5\sum_{i=1}^{N} Q_i \tag{43}$$

subject to:

$$h_k = \sum_{i=1}^{N} (G_{k,i}V_{r,i} - B_{ki}V_{m,i}) - \frac{P_kV_{r,k} + Q_kV_{m,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0,$$

$$h_{N+k} = \sum_{i=1}^{N} (B_{ki}V_{r,i} + G_{ki}V_{m,i}) - \frac{P_kV_{m,k} - Q_kV_{r,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0,$$

$$h_{2N+k} = P_k - P_{dg,k} + P_{l,k} = 0,$$

$$h_{3N+k} = Q_k - Q_{sc,k} + Q_{l,k} = 0,$$

with bounds:

$$V_{min} \le V_{r,k}, V_{m,k} \le V_{max}$$

$$P_{min} \le P_k \le P_{max}$$

$$Q_{min} \le Q_k \le Q_{max}$$

$$P_{min,dg} \le P_{dg,k} \le P_{max,dg}$$

$$Q_{min,sc} \le Q_{sc,k} \le Q_{max,sc}$$

where $k = 1, 2, \ldots N$; P_i and Q_i represent the active and reactive injected power, respectively, at the i-th bus; $Ybus_{ij}(= G_{ij} + 1jB_{ij})$ is the ij-th element of the admittance matrix; $V_i(= V_{r,i} + 1jV_{m,i})$ is the bus voltage at the i-th bus; $P_{dg,k}$ and $Q_{sc,k}$ represent the active power generation of DG and reactive power support from SC, respectively, at the $k-$th bus; N represents the total number of buses in the system.

2.4.4. Optimal power flow (minimization of active power loss)

The Optimal Power Flow (OPF) remains as a commonly developed topic among the researchers. The OPF can be posed as a single-objective COP of emission, voltage deviation, minimizing fuel cost, transmission loss, etc. with constraints based on line capacity, generator potential, power flow balance and bus voltage to be fulfilled. In this case, minimization of active power losses are considered as an objective function and this problem can be stated as.

Minimize:

$$f = \sum_{i=1}^{N} \left(P_{g,i} - P_{l,i}\right) \tag{44}$$

subject to:

$$h_i = P_{g,i} - P_{l,i} - V_i \sum_{j=1}^{N} V_j(G_{ij}cos(\delta_i - \delta_j) + B_{ij}sin(\delta_i - \delta_j)) = 0,$$

$$h_{N+i} = Q_{g,i} - Q_{l,i} - V_i \sum_{j=1}^{N} V_j(G_{ij}sin(\delta_i - \delta_j) - B_{ij}cos(\delta_i - \delta_j)) = 0,$$

with bounds:

$$V_{min} \le V_i \le V_{max},$$

$$\delta_{min} \le \delta_i \le \delta_{max},$$

$$P_{min} \le P_{g,i} \le P_{max}$$

$$Q_{min} \le Q_{g,i} \le Q_{max}$$

where $i = 1, 2, ..N$; $P_i(= P_{g,i} - P_{l,i})$ and $Q_i(= Q_{g,i} - Q_{l,i})$ represent the active and reactive injected power, respectively, at the i-th bus; $Ybus_{ij}(= G_{ij} + 1jB_{ij})$ is the ij-th element of the admittance matrix; $V_i(= V_i\angle\delta_i)$ is the bus voltage at the i-th bus; N represents the total number of buses in the system.

2.4.5. Optimal power flow (minimization of fuel cost)

In this case, the minimization of fuel cost is treated as an objective function. This problem can also be formulated as a COP.

Minimize:

$$f = \sum_{i=1}^{N} \left(a_i + b_i P_{g,i} + c_i P_{g,i}^2\right) \tag{45}$$

where a_i, b_i, and c_i are the cost coefficient of the i-th bus generator,

subject to:

$$h_i = P_{g,i} - P_{l,i} - V_i \sum_{j=1}^{N} V_j(G_{ij}cos(\delta_i - \delta_j) + B_{ij}sin(\delta_i - \delta_j)) = 0,$$

$$h_{N+i} = Q_{g,i} - Q_{l,i} - V_i \sum_{j=1}^{N} V_j(G_{ij}sin(\delta_i - \delta_j) - B_{ij}cos(\delta_i - \delta_j)) = 0,$$

with bounds:

$$V_{min} \leq V_i \leq V_{max},$$

$$\delta_{min} \leq \delta_i \leq \delta_{max},$$

$$P_{min} \leq P_{g,i} \leq P_{max}$$

$$Q_{min} \leq Q_{g,i} \leq Q_{max}$$

where $i = 1, 2, \ldots N$; $P_i(= P_{g,i} - P_{l,i})$ and $Q_i(= Q_{g,i} - Q_{l,i})$ represent the active and reactive injected power, respectively, at the i-th bus; $Ybus_{ij}(= G_{ij} + 1jB_{ij})$ is the ij-th element of the admittance matrix; $V_i(= V_i\angle\delta_i)$ is the bus voltage at the i-th bus; N represents the total number of buses in the system.

2.4.6. Optimal power flow (minimization of active power loss and fuel cost)

Minimization of fuel cost and loss are considered as objective functions in this case. By using weight factors, this problem is converted from multi-objective to a constrained single-objective problem. This problem is formulated as follows.

Minimize:

$$f = \sum_{i=1}^{N} \left(a_i + (b_i + \lambda_p) P_{g,i} + c_i P_{g,i}^2 - \lambda_p P_{l,i}\right) \tag{46}$$

where a_i, b_i, and c_i are the cost coefficient of i-th bus generator and λ_p represents the weight factors.

subject to:

$$h_i = P_{g,i} - P_{l,i} - V_i \sum_{j=1}^{N} V_j(G_{ij}cos(\delta_i - \delta_j) + B_{ij}sin(\delta_i - \delta_j)) = 0,$$

$$h_{N+i} = Q_{g,i} - Q_{l,i} - V_i \sum_{j=1}^{N} V_j(G_{ij}sin(\delta_i - \delta_j) - B_{ij}cos(\delta_i - \delta_j)) = 0,$$

with bounds:

$$V_{min} \leq V_i \leq V_{max},$$

$$\delta_{min} \leq \delta_i \leq \delta_{max},$$

$$P_{min} \leq P_{g,i} \leq P_{max}$$

$$Q_{min} \leq Q_{g,i} \leq Q_{max}$$

where $i = 1, 2, ..N$; $P_i(= P_{g,i} - P_{l,i})$ and $Q_i(= Q_{g,i} - Q_{l,i})$ represent the active and reactive injected power, respectively, at the i-th bus; $Ybus_{ij}(= G_{ij} + 1jB_{ij})$ is the ij-th element of the admittance matrix; $V_i(= V_i\angle\delta_i)$ is the bus voltage at the i-th bus; N represents the total number of buses in the system.

2.4.7. Microgrid power flow (islanded case)

Since the early 1960s, power flow analysis has been an essential research topic for power engineers. For the operational analysis of an islanded microgrid, there is a requirement of a suitable power flow tool. In the case of Droop-Based Islanded Microgrids (DBIMGs), the sharing of the reactive and active power between the Distribution Generations (DGs) is controlled by the droop controllers. In general, conventional power flow techniques consider four variables to be unknown, such as active power, reactive power, voltage magnitude, and voltage angle for different buses. In the case of the PQ bus, the value of voltage angle and voltage magnitude are unknown while reactive power and active power are known. On the contrary, in the case of the PV bus, voltage magnitude and reactive power are unknown while voltage angle and active power are known. But, in the case of a droop bus, all these variables are unknown. Conventional techniques cannot be applied to the power flow problem of islanded MGs as a frequency is not considered constant. In islanded MGs, the operating frequency is operated as an extra unknown variable of the power flow problem. To address this issue, this problem can be formulated as a COP, which is as follows.

Minimize:

$$\begin{aligned} f = &\sum_{i=1}^{N}\left(P_i - \left(\begin{array}{l} V_{r,i}\sum_{j=1}^{N}(G_{ij}V_{r,j} - B_{ij}V_{m,j}) \\ +V_{m,i}\sum_{i=1}^{N}(b_{ij}V_{r,j} + G_{ij}V_{m,j}) \end{array}\right)\right)^2 \\ &+\sum_{i=1}^{N}\left(Q_i - \left(\begin{array}{l} V_{m,i}\sum_{j=1}^{N}(G_{ij}V_{r,j} - B_{ij}V_{m,j}) \\ -V_{r,i}\sum_{i=1}^{N}(b_{ij}V_{r,j} + G_{ij}V_{m,j}) \end{array}\right)\right)^2 \end{aligned} \tag{47}$$

subject to:

$$h_k = \sum_{i=1}^{N}(G_{k,i}V_{r,i} - B_{ki}V_{m,i}) - \frac{P_kV_{r,k} + Q_kV_{m,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0,$$

$$h_{N+k} = \sum_{i=1}^{N}(B_{ki}V_{r,i} + G_{ki}V_{m,i}) - \frac{P_kV_{m,k} - Q_kV_{r,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0,$$

$$h_{2N+k} = P_k - Cp_k(w_k^* - w) + P_{l,k} = 0,$$

$$h_{3N+k} = Q_k - Cq_k(V_k^* - \sqrt{(V_{r,k})^2 + (V_{m,k})^2}) + Q_{l,k} = 0,$$

with bounds:

$$V_{min} \leq V_{r,k}, V_{m,k} \leq V_{max}$$

$$P_{min} \leq P_k \leq P_{max}$$

$$Q_{min} \leq Q_k \leq Q_{max}$$

$$w_{min} \leq w \leq w_{max}$$

where $k = 1, \ldots N$; P_i and Q_i represent the active and reactive injected power, respectively, at the i-th bus; $Ybus_{ij}(= G_{ij} + 1jB_{ij})$ is the ij-th element of the admittance matrix; $V_i(= V_{r,i} + 1jV_{m,i})$ is the bus voltage at the i-th bus; Cp_k and Cq_k represent the active and reactive power droop parameters of controllers, respectively; w is the operating frequency; N represents the total number of buses in the system.

2.4.8. Microgrid power flow (grid-connected case)

One of the key challenges in the steady-state power systems analysis is the Power Flow Problem (PFP) of grid-connected microgrid. Since the 1950s, different techniques have been utilized to solve the PFP issue of transmission systems. The developments of these techniques have been essentially done by utilizing numerical techniques that are used to solve non-linear simultaneous equations i.e. Newton's based Numerical Techniques (NNTs) and their variants. During the solution process of PFP, the Jacobian Matrix becomes near singular or singular in gird connected microgrids in the case of NNTs. Therefore, they cannot provide solutions in this case. To address this issue, PFP can be formulated as an alternative COP, which is as follows.

Minimize:

$$f = \sum_{i=2}^{N}\left(P_i - \left(\begin{array}{l} V_{r,i}\sum_{j=1}^{N}(G_{ij}V_{r,j} - B_{ij}V_{m,j}) \\ +V_{m,i}\sum_{i=1}^{N}(b_{ij}V_{r,j} + G_{ij}V_{m,j}) \end{array}\right)\right)^2 + \sum_{i=2}^{N}\left(Q_i - \left(\begin{array}{l} V_{m,i}\sum_{j=1}^{N}(G_{ij}V_{r,j} - B_{ij}V_{m,j}) \\ -V_{r,i}\sum_{i=1}^{N}(b_{ij}V_{r,j} + G_{ij}V_{m,j}) \end{array}\right)\right)^2 \tag{48}$$

subject to:

$$h_k = \sum_{i=2}^{N}(G_{k,i}V_{r,i} - B_{ki}V_{m,i}) - \frac{P_kV_{r,k} + Q_kV_{m,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0,$$

$$h_{N+k} = \sum_{i=2}^{N}(B_{ki}V_{r,i} + G_{ki}V_{m,i}) - \frac{P_kV_{m,k} - Q_kV_{r,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0,$$

with bounds:

$$V_{min} \le V_{r,k}, V_{m,k} \ldots k = 1, 2, \ldots N \le V_{max}$$

where $k = 2, \ldots N$; P_i and Q_i represent the active and reactive injected power, respectively, at the i-th bus; $Ybus_{ij}(= G_{ij} + 1jB_{ij})$ is the ij-th element of admittance matrix; $V_i(= V_{r,i} + 1jV_{m,i})$ is the bus voltage at the i-th bus; N represents the total number of buses in the system.

2.4.9. Optimal setting of droop controller for minimization of active power loss in islanded microgrids

For the application of an Islanded Microgrid (IMG), the Distributed Generations (DGs) can distribute the local loads accordingly without crossing acceptable limits of bus voltages and system frequency. Further, the flow of current in the lines must be within the bars. In an IMG, several droop control systems have been adopted for power-sharing among the DGs. It is essential to conduct the schemes not entirely stable, but also optimally. In an IMG, tuning of the droop parameters is required to reduce active losses. This problem can be developed as a challenging COP.

Minimize:

$$f = \sum_{i=1}^{N} P_i \tag{49}$$

subject to:

$$h_k = \sum_{i=1}^{N}(G_{k,i}V_{r,i} - B_{ki}V_{m,i}) - \frac{P_kV_{r,k} + Q_kV_{m,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0,$$

$$h_{N+k} = \sum_{i=1}^{N}(B_{ki}V_{r,i} + G_{ki}V_{m,i}) - \frac{P_kV_{m,k} - Q_kV_{r,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0,$$

$$h_{2N+k} = P_k - Cp_k(w_k^* - w) + P_{l,k} = 0,$$

$$h_{3N+k} = Q_k - Cq_k(V_k^* - \sqrt{(V_{r,k})^2 + (V_{m,k})^2}) + Q_{l,k} = 0,$$

with bounds:

$$V_{min} \le V_{r,k}, V_{m,k} \le V_{max}$$

$$P_{min} \le P_k \le P_{max}$$

$$Q_{min} \le Q_k \le Q_{max}$$

$$Cp_{min,k} \le Cp_k \le Cp_{max,k}$$

$$Cq_{min,k} \le Cq_k \le Cq_{max,k}$$

$$w_{min} \le w \le w_{max}$$

where $k = 1, \ldots N$; P_i and Q_i represent the active and reactive injected power, respectively, atthe i-th bus; $Ybus_{ij}(= G_{ij} + 1jB_{ij})$ is the ij-th element of the admittance matrix; $V_i(= V_{r,i} + 1jV_{m,i})$ is the bus voltage at the i-th bus; Cp_k and Cq_k represent the active and reactive power droop parameters of controllers, respectively; w is the operating frequency; N represents the total number of buses in the system.

2.4.10. Optimal setting of droop controller for minimization of reactive power loss in islanded microgrids

The main aim of this problem is to minimize reactive losses by tuning the droop parameters. The mathematical model of this problem can be expressed as a COP.

Minimize:

$$f = \sum_{i=1}^{N} Q_i \tag{50}$$

subject to:

$$h_k = \sum_{i=1}^{N}(G_{k,i}V_{r,i} - B_{ki}V_{m,i}) - \frac{P_kV_{r,k} + Q_kV_{m,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0,$$

$$h_{N+k} = \sum_{i=1}^{N}(B_{ki}V_{r,i} + G_{ki}V_{m,i}) - \frac{P_kV_{m,k} - Q_kV_{r,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0,$$

$$h_{2N+k} = P_k - Cp_k(w_k^* - w) + P_{l,k} = 0,$$

$$h_{3N+k} = Q_k - Cq_k(V_k^* - \sqrt{(V_{r,k})^2 + (V_{m,k})^2}) + Q_{l,k} = 0,$$

with bounds:

$$V_{min} \le V_{r,k}, V_{m,k} \le V_{max}$$

$$P_{min} \le P_k \le P_{max}$$

$$Q_{min} \le Q_k \le Q_{max}$$

$$Cp_{min,k} \le Cp_k \le Cp_{max,k}$$

$$Cq_{min,k} \le Cq_k \le Cq_{max,k}$$

$$w_{min} \le w \le w_{max}$$

where $k = 1, \ldots N$; P_i and Q_i represent the active and reactive injected power, respectively, at the i-th bus; $Ybus_{ij}(= G_{ij} + 1jB_{ij})$ is the ij-th element of the admittance matrix; $V_i(= V_{r,i} + 1jV_{m,i})$ is the bus voltage at the i-th bus; Cp_k and Cq_k represent the active and reactive power droop parameters of controllers, respectively; w is the operating frequency; N represents the total number of buses in the system.

2.4.11. *Wind farm layout problem [59].*

The wind farm layout is a key factor which determines the power output of a wind farm during its life cycle. A general target of wind farm layout is to maximize the total power output through optimizing the locations of wind turbines. The objective function can be described as.

Minimize:

$$f = \sum_{i=1}^{N} E(P_i) \tag{51}$$

subject to:

$$\underline{x} + R \le x_i \le \overline{x} - R,$$

$$\underline{y} + R \le y_i \le \overline{y} - R,$$

$$\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \ge 5R,\ j = 1, 2, \ldots, N \text{ and } j \ne i,$$

where,

$$E(P_i) = \sum_{n=1}^{h} \xi_n \left\{ P_r \left(e^{-(v_r/c'_i((\theta_{n-1}+\theta_n)/2))^{k_i((\theta_{n-1}+\theta_n)/2)}} - e^{-(v_{co}/(c'_i((\theta_{n-1}+\theta_n)/2))^{k_i((\theta_{n-1}+\theta_n)/2)}} \right) \right.$$

$$\left. + \sum_{j=1}^{s} \left(e^{-(v_{j-1}/(c'_i((\theta_{n-1}+\theta_n))/2))^{k_i((\theta_{n-1}+\theta_n)/2)}} - e^{-(v_j/(c'_i((\theta_{n-1}+\theta_n)/2))^{k_i((\theta_{n-1}+\theta_n)/2)}} \right) \frac{e^{(v_{j-1}+v_j)/2}}{\alpha + \beta e^{(v_{j-1}+v_j)/2}} \right\},$$

and, ξ_n is the frequency of the interval $[\theta_{n-1}, \theta_n)$.

2.5. *Power electronics: synchronous optimal pulse-width modulation*

Synchronous Optimal Pulse-Width Modulation (SOPWM) is a rising approach to regulate Medium-Voltage (MV) drives. It provides a significant reduction of switching frequency without raising the distortion. Consequently, it reduces the switching losses which enhances the performance of the inverter. Over a single fundamental period, switching angles are calculated while reducing the distortion of current. SOPWM can be cast as a scalable COP. For different level inverters, the SOPWM problem can be stated in the following way.

2.5.1. *SOPWM for 3-level inverters [60].*

Minimize:

$$f = \frac{\sqrt{\sum_k (k^{-4})(\sum_{i=1}^{N} s(i)cos(k\alpha_i))^2}}{\sqrt{\sum_k k^{-4}}} \tag{52}$$

where, $k = 5, 7, 11, 13 \ldots \ldots 97$, $N = \lfloor \frac{f_{s,max}}{f.m} \rfloor$, and $s(i) = (-1)^{i+1}$

subject to:

$$g_i = \alpha_{i+1} - \alpha_i - 10^{-5} > 0, i = 1, 2, \ldots N-1,$$

$$h_1 = m - \sum_{i=1}^{N} s(i)cos(\alpha_i) = 0,$$

with bounds:

$$0 < \alpha_i < \frac{\pi}{2}, i = 1, 2, \ldots N.$$

2.5.2. *SOPWM for 5-level inverters [61].*

Minimize:

$$f = \frac{\sqrt{\sum_k (k^{-4})(\sum_{i=1}^{N} s(i)cos(k\alpha_i))^2}}{2\sqrt{\sum_k k^{-4}}} \tag{53}$$

where, $k = 5, 7, 11, 13 \ldots \ldots 97$, $N = \lfloor \frac{2.f_{s,max}}{f.m} \rfloor$, and $s = [1, -1, 1, 1, -1, 1, -1, 1, -1, -1]$.

subject to:

$$g_i = \alpha_{i+1} - \alpha_i - 10^{-5} > 0, i = 1, 2, \ldots N-1,$$

$$h_1 = 2m - \sum_{i=1}^{N} s(i)cos(\alpha_i) = 0,$$

with bounds:

$$0 < \alpha_i < \frac{\pi}{2}, i = 1, 2, \ldots N.$$

2.5.3. *SOPWM for 7-level inverters [62].*

Minimize:

$$f = \frac{\sqrt{\sum_k (k^{-4})(\sum_{i=1}^{N} s(i)cos(k\alpha_i))^2}}{3\sqrt{\sum_k k^{-4}}} \tag{54}$$

where, $k = 5, 7, 11, 13 \ldots \ldots 97$, $N = \lfloor \frac{3.f_{s,max}}{f.m} \rfloor$, and $s = [1, -1, 1, 1, 1, -1, -1, -1, 1, 1, -1]$.

subject to:

$$g_i = \alpha_{i+1} - \alpha_i - 10^{-5} > 0, i = 1, 2, \ldots N-1,$$

$$h_1 = 3m - \sum_{i=1}^{N} s(i)cos(\alpha_i) = 0,$$

with bounds:

$$0 < \alpha_i < \frac{\pi}{2}, i = 1, 2, \ldots N.$$

2.5.4. *SOPWM for 9-level inverters [63].*

Minimize:

$$f = \frac{\sqrt{\sum_k (k^{-4})(\sum_{i=1}^{N} s(i)cos(k\alpha_i))^2}}{4\sqrt{\sum_k k^{-4}}} \tag{55}$$

where, $k = 5, 7, 11, 13 \ldots \ldots 97$, $N = \lfloor \frac{4.f_{s,max}}{f.m} \rfloor$, and $s = [1, 1, 1, 1, -1, 1, -1 -1, -1, 1, -1, -1]$.

subject to:

$$g_i = \alpha_{i+1} - \alpha_i - 10^{-5} > 0, i = 1, 2, \ldots N-1,$$

$$h_1 = 4m - \sum_{i=1}^{N} s(i)cos(\alpha_i) = 0,$$

with bounds:

$$0 < \alpha_i < \frac{\pi}{2}, i = 1, 2, \ldots N.$$

2.5.5. SOPWM for 11-level inverters [64].

Minimize:

$$f = \frac{\sqrt{\sum_k (k^{-4})(\sum_{i=1}^{N} s(i)cos(k\alpha_i))^2}}{5\sqrt{\sum_k k^{-4}}} \tag{56}$$

where, $k = 5, 7, 11, 13 \dots ..97$, $N = \lfloor \frac{5.f_{s,max}}{f.m} \rfloor$, and $s =[1,-1,1,1,1,-1,-1,-1,1,1,1,1]$.

subject to:

$$g_i = \alpha_{i+1} - \alpha_i - 10^{-5} > 0, i = 1, 2, \dots N-1,$$

$$h_1 = 5m - \sum_{i=1}^{N} s(i)cos(\alpha_i) = 0,$$

with bounds:

$$0 < \alpha_i < \frac{\pi}{2}, i = 1, 2, \dots N.$$

2.5.6. SOPWM for 13-level inverters [64].

Minimize:

$$f = \frac{\sqrt{\sum_k (k^{-4})(\sum_{i=1}^{N} s(i)cos(k\alpha_i))^2}}{6\sqrt{\sum_k k^{-4}}} \tag{57}$$

where, $k = 5, 7, 11, 13 \dots ..97$, $N = \lfloor \frac{6.f_{s,max}}{f.m} \rfloor$, and $s =[1,1,1,-1,1,-1,1,-1,1,1,1,1,1]$.

subject to:

$$g_i = \alpha_{i+1} - \alpha_i - 10^{-5} > 0, i = 1, 2, \dots N-1,$$

$$h_1 = 6m - \sum_{i=1}^{N} s(i)cos(\alpha_i) = 0,$$

with bounds:

$$0 < \alpha_i < \frac{\pi}{2}, i = 1, 2, \dots N.$$

2.6. Livestock feed ration optimization

In the livestock production industry, feeding represents a significant part as it accounts for about 60–80% of the product cost depending on stage and race of the animal [65,66]. Consequently, it is required to provide the most desirable diet at the lowest cost to prune down the operational cost for obtaining added profit. Moreover, the purpose of the feed mix formulation is to determine relevant ingredients and their cost for reducing feed cost while satisfying constraints based on various nutrient necessities [67,68]. Two type of cattle cases viz. beef cattle and dairy cattle are selected and are introduced in the following subsections.

2.6.1. Beef cattle case [69].

In case of beef cattle, the primary target is to achieve the allocation (x_i) of each available material Y_i with respect to their costs (C_i). The constraints are based on the amount of expected nutrients (in Kg) and weight (w_i). The formulation of this problem can be expressed as follows.

Minimize:

$$f(\overline{x}) = \sum_{i=1}^{n} x_i C_i \tag{58}$$

subject to:

$$h_1(\overline{x}) = \sum_{i=1}^{n} x_i \mathrm{DMI}_i - a = 0,$$

$$g_1(\overline{x}) = b - \sum_{i=1}^{n} x_i \mathrm{CP}_i \le 0,$$

$$g_2(\overline{x}) = \sum_{i=1}^{n} x_i \mathrm{CP}_i - c \le 0,$$

$$g_3(\overline{x}) = d - \sum_{i=1}^{n} x_i \mathrm{TDN}_i \le 0,$$

$$g_4(\overline{x}) = \sum_{i=1}^{n} x_i \mathrm{TDN}_i - e \le 0,$$

$$g_5(\overline{x}) = f - \sum_{i=0}^{n} x_i \mathrm{Ca}_i \le 0,$$

$$g_6(\overline{x}) = \sum_{i=0}^{n} x_i \mathrm{Ca}_i - g \le 0,$$

$$g_7(\overline{x}) = h - \sum_{i=0}^{n} x_i \mathrm{P}_i \le 0,$$

$$g_8(\overline{x}) = \sum_{i=0}^{n} x_i \mathrm{P}_i - j \le 0,$$

$$g_9(\overline{x}) = k - \sum_{i=0}^{n} x_i \mathrm{Rhage}_i \le 0,$$

$$g_{10}(\overline{x}) = \sum_{i=0}^{n} x_i \mathrm{Rhage}_i - l \le 0,$$

$$g_{11}(\overline{x}) = m - \sum_{i=0}^{n} x_i \mathrm{MC}_i \le 0,$$

$$g_{12}(\overline{x}) = \sum_{i=0}^{n} x_i \mathrm{MC}_i - o \le 0,$$

$$g_{13}(\overline{x}) = p - \sum_{i=1}^{n} x_i \mathrm{Conc}_i \le 0,$$

$$g_{14}(\overline{x}) = \sum_{i=1}^{n} x_i \mathrm{Conc}_i - q \le 0.$$

where DMI, CP, TDN, Ca, P, and Rhage represent Dry Matter Intake, Crude Protein, Total Digestible Nutrients, Calcium, Phosphorous, and Roughages, respectively, in Kg. Four different cases are defined using the above-mentioned problem in this paper.

2.6.2. Dairy cattle case [69].

In case of dairy cattle, the principal objective is defined as allocation (x_i) of the chosen materials (Y_i) multiplied by their costs (C_i). The purpose of this problem is to obtain x_i for reducing the production cost while meeting the problem constraints. The numerical formulation can be expressed as follows.

Minimize:

$$f(\overline{x}) = \sum_{i=1}^{n} x_i C_i \tag{59}$$

subject to:

$$h_1(\overline{x}) = \sum_{i=1}^{n} x_i \mathrm{MP}_i - r = 0,$$

$$h_2(\overline{x}) = \sum_{i=1}^{n} x_i \mathrm{Lys}_i - s = 0,$$

Table 3
Details of the 57 real-world COPs. D is the total number of decision variables of the problem, g is the number of inequality constraints and h is the number of equality constraints, $f(\overline{x}^*)$ is best known feasible objective function value.

Prob	Name	D	g	h	$f(\overline{x}^*)$
Industrial Chemical Processes					
RC01	Heat Exchanger Network Design (case 1)	9	0	8	1.8931162966E+02
RC02	Heat Exchanger Network Design (case 2)	11	0	9	7.0490369540E+03
RC03	Optimal Operation of Alkylation Unit	7	14	0	−4.5291197395E+03
RC04	Reactor Network Design (RND)	6	1	4	−3.8826043623E-01
RC05	Haverly's Pooling Problem	9	2	4	−4.0000560000E+02
RC06	Blending-Pooling-Separation problem	38	0	32	1.8638304088E+00
RC07	Propane, Isobutane, n-Butane Nonsharp Separation	48	0	38	2.1158627569E+00
Process Synthesis and Design Problems					
RC08	Process synthesis problem	2	2	0	2.0000000000E+00
RC09	Process synthesis and design problem	3	1	1	2.5576545740E+00
RC10	Process flow sheeting problem	3	3	0	1.0765430833E+00
RC11	Two-reactor Problem	7	4	4	9.9238463653E+01
RC12	Process synthesis problem	7	9	0	2.9248305537E+00
RC13	Process design Problem	5	3	0	2.6887000000E+04
RC14	Multi-product batch plant	10	10	0	5.3638942722E+04
Mechanical Engineering Problems					
RC15	Weight Minimization of a Speed Reducer	7	11	0	2.9944244658E+03
RC16	Optimal Design of Industrial refrigeration System	14	15	0	3.2213000814E-02
RC17	Tension/compression spring design (case 1)	3	3	0	1.2665232788E-02
RC18	Pressure vessel design	4	4	0	5.8853327736E+03
RC19	Welded beam design	4	5	0	1.6702177263E+00
RC20	Three-bar truss design problem	2	3	0	2.6389584338E+02
RC21	Multiple disk clutch brake design problem	5	7	0	2.3524245790E-01
RC22	Planetary gear train design optimization problem	9	10	1	5.2576870748E-01
RC23	Step-cone pulley problem	5	8	3	1.6069868725E+01
RC24	Robot gripper problem	7	7	0	2.5287918415E+00
RC25	Hydro-static thrust bearing design problem	4	7	0	1.6254428092E+03
RC26	Four-stage gear box problem	22	86	0	3.5359231973E+01
RC27	10-bar truss design	10	3	0	5.2445076066E+02
RC28	Rolling element bearing	10	9	0	1.4614135715E+04
RC29	Gas Transmission Compressor Design (GTCD)	4	1	0	2.9648954173E+06
RC30	Tension/compression spring design (case 2)	3	8	0	2.6138840583E+00
RC31	Gear train design Problem	4	1	1	0.0000000000E+00
RC32	Himmelblau's Function	5	6	0	−3.0665538672E+04
RC33	Topology Optimization	30	30	0	2.6393464970E+00
Power System Problems					
RC34	Optimal Sizing of Single Phase Distributed Generation with reactive power support for Phase Balancing at Main Transformer/Grid	118	0	108	0.0000000000E+00
RC35	Optimal Sizing of Distributed Generation for Active Power Loss Minimization	153	0	148	8.9093896456E-02
RC36	Optimal Sizing of Distributed Generation (DG) and Capacitors for Reactive Power Loss Minimization	158	0	148	7.2066551720E-02
RC37	Optimal Power flow (Minimization of Active Power Loss)	126	0	116	2.1962851478E-02
RC38	Optimal Power flow (Minimization of Fuel Cost)	126	0	116	2.7766131989E+00
RC39	Optimal Power flow (Minimization of Active Power Loss and Fuel Cost)	126	0	116	2.8677165770E+00
RC40	Microgrid Power flow (Islanded case)	76	0	76	0.0000000000E+00
RC41	Microgrid Power flow (Grid-connected case)	74	0	74	0.0000000000E+00
RC42	Optimal Setting of Droop Controller for Minimization of Active Power Loss in Islanded Microgrids	86	0	76	8.6241006360E-02
RC43	Optimal Setting of Droop Controller for Minimization of Reactive Power Loss in Islanded Microgrids	86	0	76	8.0420545897E-02
RC44	Wind Farm Layout Problem	30	91	0	−6.2607000000E+03
Power Electronic Problems					
RC45	SOPWM for 3-level Invereters	25	24	1	3.8029250566E-02
RC46	SOPWM for 5-level Inverters	25	24	1	2.1215000000E-02
RC47	SOPWM for 7-level Inverters	25	24	1	1.5164538375E-02
RC48	SOPWM for 9-level Inverters	30	29	1	1.6787535766E-02
RC49	SOPWM for 11-level Inverters	30	29	1	9.3118741800E-03
RC50	SOPWM for 13-level Inverters	30	29	1	1.5096451396E-02
Livestock Feed Ration Optimization					
RC51	Beef Cattle(case 1)	59	14	1	4.5508511497E+03
RC52	Beef Cattle (case 2)	59	14	1	3.3489821493E+03
RC53	Beef Cattle (case 3)	59	14	1	4.9976069290E+03
RC54	Beef Cattle (case 4)	59	14	1	4.2405482538E+03
RC55	Dairy Cattle (case 1)	64	0	6	6.6964145128E+03
RC56	Dairy Cattle (case 2)	64	0	6	1.4748932529E+04
RC57	Dairy Cattle (case 3)	64	0	6	3.2132917019E+03

$$h_3(\overline{x}) = \sum_{i=1}^{n} x_i \mathrm{Ca}_i - t = 0,$$

$$h_4(\overline{x}) = \sum_{i=1}^{n} x_i \mathrm{P}_i - u = 0,$$

$$h_5(\overline{x}) = \sum_{i=1}^{n} x_i \mathrm{ME}_i - v = 0,$$

$$h_6(\overline{x}) = \sum_{i=1}^{n} x_i \mathrm{Met}_i - z = 0,$$

where MP, Lys, Ca, P, ME, and Met represent Metabolizable Protein, Lysine, Calcium, Phosphorous, Metabolizable Energy, and Methionine, respectively in Kg. Three different cases are defined using the above-mentioned problem in this paper.

2.7. Benchmark suite

A benchmark suite is created using the above-mentioned real-world constrained problems. A total number of 57 problems is designed from the above-listed problems and is included in the benchmark suite. The details of these problems are reported in Table 3. As shown in Table 3, the number of decision variables vary from 2 to 158, number of equality constraints vary from 0 to 148, and number of equality constraints vary from 0 to 91. The source code of this benchmark suite in MATLAB is available in "https://github.com/P-N-Suganthan/2020-RW-Constrained-Optimization".

3. Evaluation of the proposed benchmark suite

In this section, the proposed benchmark suite is evaluated using three state-of-the-art algorithms viz. IUDE [22], ϵMAgES [23], and iLSHADE$_\epsilon$ [24].

3.1. Improved unified differential evolution algorithm

To solve COPs, an Improved variant of Unified Differential Evolution (IUDE) is proposed in Ref. [22]. This algorithm employs three mutation strategies viz. current-to-pbest, current-to-rand, and rand mutation strategy with binomial crossover operator to generate trial solutions. IUDE has been a dual population-based approach where the current population is divided into two sub-population at each generation. The mutation operation utilizes the ranking-based mutation and parameter self-adaptation procedure of SHADE [70]. The constraint handling technique employed in IUDE is a combined approach proposed in C^{2}oDE [71], where a combination of ϵ-constraint and feasibility based rule approach is applied. For selection, the traditional one-to-one replacement approach is applied in IUDE.

3.2. Matrix adaptation evolution Strategy

In [23], a state-of-the-art Evolution Strategy, Matrix Adaptation Evolution Strategy (MA-ES), is proposed to solve real-parameter COPs. MA-ES is a computationally efficient version of CMA-ES variant [72]. To handle the constraint, ϵ-constraint is incorporated in MA-ES. In addition, a repair step based on gradient approximation is also employed to deal with equality constraints. This algorithm is named as ϵMAgES.

3.3. LSHADE44 with an improved ϵ constraint-handling method

For solving COPs, a differential evolution based variant LSHADE44 is proposed in Ref. [24]. Additionally, an improved constrained handling technique, named IEpsilon, is also proposed to deal with complex constraints. In IEpsilon, the ϵ-level is adaptively adjusted according to the number of feasible solutions in the current population to provide a balance between infeasible regions and feasible regions during the optimization process. Moreover, a new trail vector generation strategy, DE/randr1∗/1, is proposed. This algorithm is named as iLSHADE$_\epsilon$.

3.4. Experimental setting

All the above-mentioned algorithms have been implemented in MATLAB. The evaluation of the proposed benchmark suite has been done on MATLAB r2017b in a PC having Microsoft Windows 10 operating system with INTEL Core i7 CPU and 8 Gb RAM. The parameters settings of all algorithms is directly taken from their respective papers viz [22,23,24].

To stop the optimization process, a stopping rule based on the number of decision variables is applied in all algorithms. A fixed amount of function evaluations is allotted during the optimization process and after the maximum function evaluations, the optimization process of the algorithms is stopped and the best solution is returned. The following criteria are used to decide the maximum function evaluation for each problem of the proposed benchmark suite.

$$Max_{FEs} = \begin{cases} 1\times 10^5, & \text{if } D \le 10 \\ 2\times 10^5, & \text{if } 10 < D \le 30 \\ 4\times 10^5, & \text{if } 30 < D \le 50 \\ 8\times 10^5, & \text{if } 50 < D \le 150 \\ 10^6, & \text{if } 150 < D \end{cases} \tag{60}$$

where Max_{FEs} is the maximum allowable number of function evaluations and D is the dimensionality (number of decision variables) of the problem.

3.5. Algorithmic complexity

The algorithmic complexity of all algorithms is also calculated using the proposed benchmark suite. The following procedures are adopted to calculate the algorithmic complexity.

1. $T_1 = \frac{\sum_{i=1}^{57} t_{1i}}{57}$, where t_{1i} is the computation time required to perform 100000 function evaluations for problem i.
2. $T_2 = \frac{\sum_{i=1}^{57} t_{2i}}{57}$, where t_{2i} is the computation time required by an algorithm to perform 100000 function evaluations for problem i.
3. The algorithmic complexity is evaluated using T_1, T_2, and $\frac{T_2-T_1}{T_1}$.

Complexity of all algorithms is reported in Table 4. It can be seen from Table 4 that IUDE has the lowest complexity and iLSHADE$_\epsilon$ has the highest computational complexity.

3.6. Performance evaluation procedure

The problems of the proposed benchmark suite are taken from different engineering applications. Therefore, the difficulty level and complexity of problems have been different from each other. In order to determine the relative difficulty level of each problem of the benchmark suite, the following procedures are adopted in this study.

Table 4
Computational complexity.

Algorithm	T_1 (sec)	T_2 (sec)	$\frac{T_2-T_1}{T_1}$
IUDE	8.57	9.93	0.16
ϵMAgES	8.57	12.67	0.48
iLSHADE$_\epsilon$	8.57	15.76	0.84

Table 5
Results of Industrial Chemical Processes problems (RC01 -RC07) using IUDE, ϵMAgES, and iLSHADE$_\epsilon$.

Problem	Algorithm	Best	Median	Mean	Worst	Std	FR	MV	SR
RC01	IUDE	1.89E+02	2.60E+02	2.29E+02	1.85E+02	8.06E+01	24	1.12E+05	4
	ϵMAgES	**1.89E+02**	**4.92E+02**	**4.55E+02**	**4.37E+02**	**2.23E+02**	**84**	**1.47E-04**	**20**
	iLSHADE$_\epsilon$	1.90E+02	1.94E+02	2.06E+02	2.29E+02	1.93E+01	28	1.36E-02	4
RC02	IUDE	7.05E+03	7.05E+03	7.15E+03	5.94E+03	7.54E+02	92	6.17E+03	92
	eMAgES	7.05E+03	7.80E+03	7.74E+03	7.48E+03	7.50E+02	96	3.71E+03	96
	iLSHADE$_\epsilon$	**7.05E+03**	**7.05E+03**	**7.05E+03**	**7.05E+03**	**2.13E-11**	**100**	**0.00E+00**	**100**
RC03	IUDE	−4.53E+03	−1.43E+02	−6.25E+03	−1.83E+04	6.75E+03	64	3.66E+00	12
	ϵMAgES	−1.43E+02	7.63E+01	−1.60E+02	4.95E+02	8.88E+02	92	1.61E-02	0
	iLSHADE$_\epsilon$	**−4.53E+03**	**−1.43E+02**	**−8.18E+02**	**5.34E+02**	**1.91E+03**	**100**	**0.00E+00**	**20**
RC04	IUDE	−2.86E-01	−5.92E-01	−4.96E-01	−1.00E+00	1.82E-01	0	7.90E-02	0
	ϵMAgES	**−3.88E-01**	**−3.75E-01**	**−5.50E-01**	**−1.00E+00**	**2.86E-01**	**72**	**3.50E-02**	**24**
	iLSHADE$_\epsilon$	−3.75E-01	−3.75E-01	−3.75E-01	−3.73E-01	4.61E-04	100	0.00E+00	0
RC05	IUDE	**−4.00E+02**	**−4.00E+02**	**−3.51E+02**	**−8.30E-03**	**1.32E+02**	**100**	**0.00E+00**	**80**
	ϵMAgES	−4.00E+02	−3.98E+02	−3.63E+02	−1.00E+02	7.55E+01	100	0.00E+00	36
	iLSHADE$_\epsilon$	−4.00E+02	−8.06E-03	−1.17E+02	1.57E+01	1.79E+02	100	0.00E+00	44
RC06	IUDE	**1.71E+00**	**9.98E-01**	**1.07E+00**	**9.98E-01**	**1.98E-01**	**0**	**2.10E+00**	**0**
	ϵMAgES	2.09E+00	1.83E+00	1.59E+00	1.20E+00	3.39E-01	0	7.07E-02	0
	iLSHADE$_\epsilon$	1.72E+00	1.24E+00	1.27E+00	1.11E+00	1.63E-01	0	3.94E+00	0
RC07	IUDE	**1.76E+00**	**1.34E+00**	**1.40E+00**	**9.98E-01**	**3.82E-01**	**0**	**2.82E-01**	**0**
	ϵMAgES	2.00E+00	1.71E+00	1.80E+00	2.01E+00	1.74E-01	0	1.98E-02	0
	iLSHADE$_\epsilon$	1.75E+00	1.73E+00	1.66E+00	1.67E+00	1.00E-01	0	2.99E+00	0

Table 6
Results of Process Synthesis and Design Problems (RC08 -RC14) using IUDE, ϵMAgES, and iLSHADE$_\epsilon$.

Problem	Algorithm	Best	Median	Mean	Worst	Std	FR	MV	SR
RC08	IUDE	**2.00E+00**	**2.00E+00**	**2.00E+00**	**2.00E+00**	**6.41E-17**	**100**	**0.00E+00**	**100**
	ϵMAgES	2.00E+00	2.00E+00	1.99E+00	1.29E+00	1.52E-01	96	4.58E-03	64
	iLSHADE$_\epsilon$	**2.00E+00**	**2.00E+00**	**2.00E+00**	**2.00E+00**	**0.00E+00**	**100**	**0.00E+00**	**100**
RC09	IUDE	**2.56E+00**	**2.56E+00**	**2.56E+00**	**2.56E+00**	**1.36E-15**	**100**	**0.00E+00**	**100**
	ϵMAgES	2.56E+00	2.56E+00	2.55E+00	1.93E+00	2.70E-01	92	1.15E-02	92
	iLSHADE$_\epsilon$	**2.56E+00**	**2.56E+00**	**2.56E+00**	**2.56E+00**	**1.46E-07**	**100**	**0.00E+00**	**100**
RC10	IUDE	1.08E+00	1.08E+00	1.11E+00	1.25E+00	7.08E-02	100	0.00E+00	88
	ϵMAgES	**1.08E+00**	**1.08E+00**	**1.08E+00**	**1.25E+00**	**3.47E-02**	**100**	**0.00E+00**	**92**
	iLSHADE$_\epsilon$	1.08E+00	1.25E+00	1.22E+00	1.25E+00	6.48E-02	100	0.00E+00	88
RC11	IUDE	**9.92E+01**	**9.92E+01**	**1.02E+02**	**1.07E+02**	**4.07E+00**	**100**	**0.00E+00**	**52**
	ϵMAgES	1.07E+02	9.92E+01	1.06E+02	1.15E+02	6.88E+00	0	2.20E-02	0
	iLSHADE$_\epsilon$	9.92E+01	1.07E+02	1.06E+02	1.32E+02	7.39E+00	96	4.00E-02	44
RC12	IUDE	2.92E+00	2.95E+00	3.00E+00	4.21E+00	2.54E-01	100	0.00E+00	16
	ϵMAgES	2.92E+00	3.64E+00	3.65E+00	4.69E+00	5.87E-01	100	0.00E+00	20
	iLSHADE$_\epsilon$	**2.92E+00**	**2.92E+00**	**2.92E+00**	**2.92E+00**	**8.14E-07**	**100**	**0.00E+00**	**100**
RC13	IUDE	**2.69E+04**	**2.69E+04**	**2.69E+04**	**2.69E+04**	**1.11E-11**	**100**	**0.00E+00**	**100**
	ϵMAgES	**2.69E+04**	**2.69E+04**	**2.69E+04**	**2.69E+04**	**1.11E-11**	**100**	**0.00E+00**	**100**
	iLSHADE$_\epsilon$	**2.69E+04**	**2.69E+04**	**2.69E+04**	**2.69E+04**	**1.11E-11**	**100**	**0.00E+00**	**100**
RC14	IUDE	5.85E+04	6.65E+04	6.60E+04	7.36E+04	5.14E+03	100	0.00E+00	0
	ϵMAgES	**5.36E+04**	**5.85E+04**	**5.81E+04**	**5.85E+04**	**1.35E+03**	**100**	**0.00E+00**	**0**
	iLSHADE$_\epsilon$	5.36E+04	5.92E+04	5.91E+04	6.36E+04	1.85E+03	100	0.00E+00	0

- The above-mentioned algorithms are run independently 25 times on each problem of the benchmark suite.
- The outcomes of algorithms for 25 runs are reported in terms of the mean objective function (Mean), mean constraint violation (MV), Feasibility Rate (FR), and Success Rate (SR).
  1. *Mean Constraint Violation:* Mean constraint violation, $\overline{v}$, is calculated using the following equation.

$$\overline{v} = \frac{\sum_{i=1}^{p} \max\left(g_i(\overline{x}), 0\right) + \sum_{j=p+1}^{m} \max\left(|h_i(\overline{x})| - \varepsilon, 0\right)}{m}, \tag{61}$$

     where ε is set to 0.0001.
  2. *Feasibility Rate:* The ratio of the number of runs in which at least one feasible solution is attained within Max_{FEs} and total number of runs.
  3. *Success Rate:* The ratio of the total number of runs in which an algorithm obtained a feasible solution $\overline{x}$ satisfying $f(\overline{x}) - f(\overline{x}^*) \leq 10^{-8}$ within Max_{FEs} and total runs.
- The difficulty level of problems is evaluated using the following criteria.
  1. Evaluation of problems based on SR,
  2. Then evaluation of problems based on FR, and
  3. At last, evaluation of problems based on MV.

3.7. Evaluation of problems of the benchmark suite

The above discussed procedures are adopted on each problem to evaluate the problems of the proposed benchmark suite. The outcomes of all three algorithms on each problem are reported in Tables 5–10. In Table 5, the outcomes of the Industrial Chemical Process problems are reported for all algorithms. By analyzing the outcomes of this Table, it can be concluded that four problems (RC06, RC07, RC01, and RC03) are the hardest problems and three problems (RC02, RC04, and RC05) are the easiest to solve using constrained optimization algorithms. The outcomes of Process Synthesis and Design problems for all algorithms are depicted in Table 6. From this Table, it can be seen that RC14 is the hardest problem, and RC13, RC08, RC10, and RC09 are the easiest problems of this group. The difficulty level of the rest of the prob-

Table 7
Results of Mechanical Engineering Problems (RC15 -RC33) using IUDE, ϵMAgES, and iLSHADE$_\epsilon$.

Problem	Algorithm	Best	Median	Mean	Worst	Std	FR	MV	SR
RC15	IUDE	**2.99E+03**	**2.99E+03**	**2.99E+03**	**2.99E+03**	**4.64E-13**	**100**	**0.00E+00**	**100**
	ϵMAgES	**2.99E+03**	**2.99E+03**	**2.99E+03**	**2.99E+03**	**4.64E-13**	**100**	**0.00E+00**	**100**
	iLSHADE$_\epsilon$	**2.99E+03**	**2.99E+03**	**2.99E+03**	**2.99E+03**	**4.64E-13**	**100**	**0.00E+00**	**100**
RC16	IUDE	3.22E-02	3.22E-02	6.24E+00	2.95E+00	2.86E+01	80	1.28E-02	80
	ϵMAgES	**3.22E-02**	**3.22E-02**	**3.22E-02**	**3.22E-02**	**2.78E-17**	**100**	**0.00E+00**	**100**
	iLSHADE$_\epsilon$	3.22E-02	3.23E-02	3.82E-01	2.95E+00	9.67E-01	88	1.16E-01	72
RC17	IUDE	**1.27E-02**	**1.27E-02**	**1.27E-02**	**1.27E-02**	**1.08E-05**	**100**	**0.00E+00**	**100**
	ϵMAgES	1.27E-02	1.27E-02	1.27E-02	1.37E-02	2.16E-04	100	0.00E+00	96
	iLSHADE$_\epsilon$	1.27E-02	1.27E-02	1.30E-02	1.78E-02	1.06E-03	100	0.00E+00	88
RC18	IUDE	**6.06E+03**	**6.06E+03**	**6.06E+03**	**6.09E+03**	**6.16E+00**	**100**	**0.00E+00**	**24**
	ϵMAgES	6.06E+03	6.41E+03	7.38E+03	1.19E+04	1.93E+03	100	0.00E+00	16
	iLSHADE$_\epsilon$	6.06E+03	6.11E+03	8.48E+03	1.49E+04	3.14E+03	100	0.00E+00	0
RC19	IUDE	**1.67E+00**	**1.67E+00**	**1.67E+00**	**1.67E+00**	**1.20E-16**	**100**	**0.00E+00**	**100**
	ϵMAgES	1.67E+00	1.67E+00	1.69E+00	1.85E+00	3.95E-02	100	0.00E+00	44
	iLSHADE$_\epsilon$	**1.67E+00**	**1.67E+00**	**1.67E+00**	**1.67E+00**	**7.59E-07**	**100**	**0.00E+00**	**100**
RC20	IUDE	**2.64E+02**	**2.64E+02**	**2.64E+02**	**2.64E+02**	**0.00E+00**	**100**	**0.00E+00**	**100**
	ϵMAgES	2.64E+02	2.64E+02	2.65E+02	2.74E+02	2.88E+00	100	0.00E+00	88
	iLSHADE$_\epsilon$	2.64E+02	2.64E+02	2.64E+02	2.64E+02	1.99E-02	100	0.00E+00	92
RC21	IUDE	**2.35E-01**	**2.35E-01**	**2.35E-01**	**2.35E-01**	**1.13E-16**	**100**	**0.00E+00**	**100**
	ϵMAgES	**2.35E-01**	**2.35E-01**	**2.35E-01**	**2.35E-01**	**1.13E-16**	**100**	**0.00E+00**	**100**
	iLSHADE$_\epsilon$	**2.35E-01**	**2.35E-01**	**2.35E-01**	**2.35E-01**	**1.13E-16**	**100**	**0.00E+00**	**100**
RC22	IUDE	**5.26E-01**	**5.26E-01**	**5.26E-01**	**5.27E-01**	**5.50E-04**	**100**	**0.00E+00**	**36**
	ϵMAgES	5.26E-01	5.46E-01	6.16E-01	1.12E+00	1.98E-01	76	1.31E+00	20
	iLSHADE$_\epsilon$	5.26E-01	5.26E-01	5.27E-01	5.31E-01	1.69E-03	100	0.00E+00	28
RC23	IUDE	**1.61E+01**	**1.61E+01**	**1.61E+01**	**1.61E+01**	**4.17E-15**	**100**	**0.00E+00**	**100**
	ϵMAgES	**1.61E+01**	**1.61E+01**	**1.61E+01**	**1.61E+01**	**1.78E-14**	**100**	**0.00E+00**	**100**
	iLSHADE$_\epsilon$	**1.61E+01**	**1.61E+01**	**1.61E+01**	**1.61E+01**	**8.62E-08**	**100**	**0.00E+00**	**100**
RC24	IUDE	**2.54E+00**	**2.54E+00**	**2.54E+00**	**2.54E+00**	**5.99E-14**	**100**	**0.00E+00**	**100**
	ϵMAgES	2.54E+00	2.54E+00	2.54E+00	2.55E+00	8.81E-04	100	0.00E+00	96
	iLSHADE$_\epsilon$	2.54E+00	2.54E+00	2.54E+00	2.55E+00	1.99E-03	100	0.00E+00	92
RC25	IUDE	1.86E+03	2.60E+02	1.93E+03	2.60E+02	2.37E+03	40	9.01E-05	0
	ϵMAgES	1.62E+03	2.26E+03	2.35E+03	6.34E+02	1.41E+03	88	1.84E-05	0
	iLSHADE$_\epsilon$	**1.66E+03**	**2.09E+03**	**1.76E+03**	**6.12E+02**	**1.28E+03**	**76**	**2.54E-04**	**0**
RC26	IUDE	4.54E+01	4.91E+01	6.21E+01	4.55E+01	2.93E+01	12	1.35E-01	0
	ϵMAgES	8.23E+01	2.13E+01	3.38E+01	7.26E+00	3.61E+01	8	3.77E-01	0
	iLSHADE$_\epsilon$	**3.54E+01**	**3.63E+01**	**3.63E+01**	**3.73E+01**	**6.59E-01**	**100**	**0.00E+00**	**24**
RC27	IUDE	**5.24E+02**	**5.24E+02**	**5.24E+02**	**5.24E+02**	**4.72E-04**	**100**	**0.00E+00**	**88**
	ϵMAgES	5.24E+02	5.31E+02	5.30E+02	5.31E+02	2.03E+00	100	0.00E+00	72
	iLSHADE$_\epsilon$	5.24E+02	5.25E+02	5.25E+02	5.25E+02	7.14E-02	100	0.00E+00	0
RC28	IUDE	**1.46E+04**	**1.46E+04**	**1.46E+04**	**1.46E+04**	**9.28E-12**	**100**	**0.00E+00**	**100**
	ϵMAgES	**1.46E+04**	**1.46E+04**	**1.46E+04**	**1.46E+04**	**9.28E-12**	**100**	**0.00E+00**	**100**
	iLSHADE$_\epsilon$	**1.46E+04**	**1.46E+04**	**1.46E+04**	**1.46E+04**	**9.28E-12**	**100**	**0.00E+00**	**100**
RC29	IUDE	**2.96E+06**	**2.96E+06**	**2.96E+06**	**2.96E+06**	**1.43E-09**	**100**	**0.00E+00**	**100**
	ϵMAgES	**2.96E+06**	**2.96E+06**	**2.96E+06**	**2.96E+06**	**1.43E-09**	**100**	**0.00E+00**	**100**
	iLSHADE$_\epsilon$	2.96E+06	2.96E+06	2.97E+06	2.97E+06	6.57E+02	100	0.00E+00	96
RC30	IUDE	**2.66E+00**	**4.54E+00**	**5.53E+00**	**2.67E+01**	**4.74E+00**	**92**	**1.81E-03**	**44**
	ϵMAgES	2.66E+00	3.11E+00	2.21E+00	4.34E-02	1.22E+00	68	1.04E+06	32
	iLSHADE$_\epsilon$	6.90E+00	2.87E+00	6.26E+00	2.05E+01	6.32E+00	20	7.95E-01	0
RC31	IUDE	**0.00E+00**	**1.74E-18**	**4.55E-16**	**8.41E-15**	**1.68E-15**	**100**	**0.00E+00**	**100**
	ϵMAgES	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**100**	**0.00E+00**	**100**
	iLSHADE$_\epsilon$	**0.00E+00**	**5.41E-18**	**5.56E-17**	**3.91E-16**	**1.17E-16**	**100**	**0.00E+00**	**100**
RC32	IUDE	**−3.07E+04**	**−3.07E+04**	**−3.07E+04**	**−3.07E+04**	**3.71E-12**	**100**	**0.00E+00**	**100**
	ϵMAgES	**−3.07E+04**	**−3.07E+04**	**−3.07E+04**	**−3.07E+04**	**3.56E-12**	**100**	**0.00E+00**	**100**
	iLSHADE$_\epsilon$	**−3.07E+04**	**−3.07E+04**	**−3.07E+04**	**−3.07E+04**	**3.64E-12**	**100**	**0.00E+00**	**100**
RC33	IUDE	**2.64E+00**	**2.64E+00**	**2.64E+00**	**2.64E+00**	**1.41E-15**	**100**	**0.00E+00**	**100**
	ϵMAgES	2.64E+00	2.65E+00	2.65E+00	2.68E+00	1.26E-02	100	0.00E+00	44
	iLSHADE$_\epsilon$	**2.64E+00**	**2.64E+00**	**2.64E+00**	**2.64E+00**	**8.11E-16**	**100**	**0.00E+00**	**100**

lems is at a modest level. The detailed outcomes of Mechanical Design problems are outlined in Table 7. Analyzing the results of this Table, it is easy to find out that most of problems (16 out of 19 problems) of this group are easier to solve. Problems RC25, RC26, and RC22 are the hardest problems of this group. The outcome of Power System problems, Power Electronics problems, and Livestock Feed Ration problems for all algorithms are reported in Tables 8–10, respectively. From these Tables, it can be concluded that all the problems of these groups are difficult to solve.

From the above analysis, we can conclude that most of the problems of the proposed benchmark suite are challenging to solve. State-of-the-art algorithms are not able to provide feasible solutions for 14 problems within Max_{FEs} function evaluation. This conclusion suggests that the proposed benchmark suite can be utilized to analyze the performance of constrained optimization algorithms.

3.8. Evaluation of the performance of algorithms

Alongside the illustration of the performance of an individual algorithm, the purpose of a benchmark suite is to determine which algorithm is the best performer in solving the benchmark problems. For comparative analysis of the performance of algorithms, an appropriate ranking procedure is required.

For the constrained benchmark, the comparative analysis of the performance of algorithms is generally done using an ordering approach or a quality indicator that can distinguish between feasible and infeasible

Table 8
Results of Power System Problems (RC34 -RC44) using IUDE, ϵMAgES, and iLSHADE$_\epsilon$.

Problem	Algorithm	Best	Median	Mean	Worst	Std	FR	MV	SR
RC34	IUDE	5.07E+00	1.17E+01	5.01E+00	2.38E+00	1.99E+00	0	4.27E-02	0
	ϵMAgES	**3.06E+00**	**3.75E+00**	**5.50E+00**	**6.53E+00**	**2.87E+00**	**0**	**1.43E-02**	**0**
	iLSHADE$_\epsilon$	1.04E+01	6.73E+00	8.85E+00	5.76E+00	2.49E+00	0	5.10E+00	0
RC35	IUDE	**9.33E+01**	**9.68E+01**	**1.01E+02**	**1.03E+02**	**1.32E+01**	**0**	**8.52E-01**	**0**
	ϵMAgES	8.80E+01	4.68E+01	7.77E+01	9.40E+01	2.09E+01	0	9.24E-01	0
	iLSHADE$_\epsilon$	1.88E+02	1.87E+02	1.57E+02	1.64E+02	2.49E+01	0	2.75E+01	0
RC36	IUDE	**8.08E+01**	**6.38E+01**	**8.16E+01**	**9.99E+01**	**1.82E+01**	**0**	**8.17E-01**	**0**
	ϵMAgES	7.69E+01	7.37E+01	7.70E+01	8.42E+01	8.96E+00	0	9.33E-01	0
	iLSHADE$_\epsilon$	1.18E+02	9.62E+01	1.29E+02	1.36E+02	1.84E+01	0	5.77E+01	0
RC37	IUDE	1.91E+00	−1.88E+00	3.42E-01	−6.31E+00	2.20E+00	0	1.23E-01	0
	ϵMAgES	**1.35E+00**	**1.57E+00**	**1.55E+00**	**5.37E-01**	**4.62E-01**	**0**	**1.94E-02**	**0**
	iLSHADE$_\epsilon$	3.02E+00	3.03E+00	3.48E+00	3.99E+00	4.53E-01	0	6.32E+00	0
RC38	IUDE	3.67E+00	−1.57E+01	−1.41E+01	−2.85E+01	1.02E+01	0	1.98E-01	0
	ϵMAgES	**6.09E+00**	**3.92E+00**	**4.17E+00**	**5.65E+00**	**1.69E+00**	**0**	**2.99E-02**	**0**
	iLSHADE$_\epsilon$	3.86E+00	4.17E+00	3.12E+00	1.44E+00	1.19E+00	0	6.67E+00	0
RC39	IUDE	−2.99E+00	−1.58E+01	−2.24E+01	−4.00E+01	1.41E+01	0	2.09E-01	0
	ϵMAgES	**1.81E+00**	**3.56E+00**	**3.92E+00**	**2.36E+00**	**1.50E+00**	**0**	**3.01E-02**	**0**
	iLSHADE$_\epsilon$	5.01E+00	1.07E+00	3.18E+00	4.07E-01	1.65E+00	0	6.91E+00	0
RC40	IUDE	8.61E+01	3.82E+01	1.16E+02	1.33E+02	6.66E+01	0	1.35E+00	0
	ϵMAgES	**3.05E-01**	**2.16E+01**	**3.02E+01**	**5.12E+01**	**2.38E+01**	**0**	**2.44E-01**	**0**
	iLSHADE$_\epsilon$	1.14E+02	1.11E+02	1.89E+02	6.00E+02	1.15E+02	0	1.50E+02	0
RC41	IUDE	5.06E+01	9.19E+01	9.14E+01	6.18E+02	1.20E+02	0	1.14E+00	0
	ϵMAgES	**4.08E-08**	**3.02E-03**	**6.60E+00**	**3.15E+01**	**1.45E+01**	**0**	**6.71E-02**	**0**
	iLSHADE$_\epsilon$	4.17E+01	2.26E+02	1.24E+02	4.75E+02	1.06E+02	0	1.13E+02	0
RC42	IUDE	−1.70E+00	2.25E+02	−3.64E+01	−1.01E+03	2.48E+02	0	5.25E+00	0
	ϵMAgES	**−1.33E+00**	**6.49E+01**	**9.24E+01**	**7.77E+01**	**3.89E+01**	**0**	**8.03E-01**	**0**
	iLSHADE$_\epsilon$	−1.00E+00	−1.01E+00	−1.06E+00	−1.17E+00	1.72E-01	0	1.65E+02	0
RC43	IUDE	3.25E+01	1.18E+00	1.34E+01	−1.73E+01	1.95E+01	0	2.53E+00	0
	ϵMAgES	**1.04E+02**	**1.18E+02**	**9.82E+01**	**9.85E+01**	**2.22E+01**	**0**	**8.21E-01**	**0**
	iLSHADE$_\epsilon$	2.72E+01	2.21E+01	3.80E+01	3.84E+01	7.86E+00	0	1.54E+02	0
RC44	IUDE	−6.09E+03	−6.02E+03	−6.00E+03	−5.90E+03	4.61E+01	100	0.00E+00	0
	ϵMAgES	−6.17E+03	−5.97E+03	−5.97E+03	−5.71E+03	1.05E+02	100	0.00E+00	0
	iLSHADE$_\epsilon$	**−6.34E+03**	**−6.20E+03**	**−6.20E+03**	**−6.14E+03**	**3.66E+01**	**100**	**0.00E+00**	**0**

Table 9
Results of Power Electronic Problems (RC45 -RC50) using IUDE, ϵMAgES, and iLSHADE$_\epsilon$.

Problem	Algorithm	Best	Median	Mean	Worst	Std	FR	MV	SR
RC45	IUDE	6.85E-02	1.13E-01	1.17E-01	2.49E-01	4.55E-02	100	0.00E+00	0
	ϵMAgES	**3.14E-02**	**4.94E-02**	**4.66E-02**	**5.71E-02**	**9.46E-03**	**100**	**0.00E+00**	**0**
	iLSHADE$_\epsilon$	7.08E-02	1.51E-01	1.48E-01	2.49E-01	4.50E-02	100	0.00E+00	0
RC46	IUDE	2.02E-02	5.85E-02	5.54E-02	7.70E-02	1.58E-02	100	0.00E+00	0
	ϵMAgES	**2.04E-02**	**5.10E-02**	**4.82E-02**	**8.36E-02**	**1.25E-02**	**100**	**0.00E+00**	**0**
	iLSHADE$_\epsilon$	5.17E-02	1.03E-01	1.05E-01	1.92E-01	3.75E-02	100	0.00E+00	0
RC47	IUDE	2.76E-02	5.37E-02	6.84E-02	2.06E-01	3.79E-02	100	0.00E+00	0
	ϵMAgES	**1.86E-02**	**3.19E-02**	**3.13E-02**	**5.16E-02**	**8.34E-03**	**100**	**0.00E+00**	**0**
	iLSHADE$_\epsilon$	2.44E-02	5.10E-02	5.45E-02	1.01E-01	2.36E-02	100	0.00E+00	0
RC48	IUDE	4.64E-02	7.62E-02	1.00E-01	3.05E-01	6.43E-02	96	5.61E-04	0
	ϵMAgES	**2.24E-02**	**5.21E-02**	**6.14E-02**	**1.46E-01**	**2.89E-02**	**100**	**0.00E+00**	**0**
	iLSHADE$_\epsilon$	4.64E-02	3.38E-01	2.92E-01	3.29E-01	1.50E-01	80	5.66E-02	0
RC49	IUDE	3.61E-02	6.96E-02	7.56E-02	2.84E-01	5.28E-02	100	0.00E+00	0
	ϵMAgES	**1.18E-02**	**2.47E-02**	**2.82E-02**	**5.51E-02**	**1.25E-02**	**100**	**0.00E+00**	**0**
	iLSHADE$_\epsilon$	6.74E-02	1.61E-01	1.85E-01	4.04E-01	7.26E-02	100	0.00E+00	0
RC50	IUDE	1.67E-01	3.34E-01	3.14E-01	1.70E-01	6.31E-02	16	6.05E-03	0
	ϵMAgES	**1.50E-02**	**1.62E-02**	**1.80E-02**	**3.51E-02**	**5.23E-03**	**100**	**0.00E+00**	**0**
	iLSHADE$_\epsilon$	2.66E-01	3.58E-01	3.32E-01	3.76E-01	4.49E-02	0	2.34E-01	0

solutions. A most popular ordering approach used in the field of constrained optimization is the *superiority of feasible solutions* [14]. In this approach, two solutions are compared by using the following criteria.

1. Any feasible solution is considered better than an infeasible solution.
2. Two feasible solutions are ranked based on objective function value (A solution with the lower value is the better one).
3. Among two infeasible solutions, the one with the lower constrained violation value is preferred.

Mathematically, this order is defined as

$$\overline{x}_1 \geq_{lex} \overline{x}_2 \Leftrightarrow \begin{cases} f(\overline{x}_1) \geq f(\overline{x}_2), & \text{if } v(\overline{x}_1) == v(\overline{x}_2), \\ v(\overline{x}_1) \geq v(\overline{x}_2), & \text{else.} \end{cases} \tag{62}$$

The above-mentioned approach is commonly utilized in the ranking schemes of CEC benchmarks [9,10,11,73]. Different quality indicators are proposed in CEC benchmarks. But, the question is which quality indicator will be used because it involves a certain degree of subjectivity. Due to that reason, it is recommended to use the quality indicator that comprises more than one order relation [74]. We believe that researchers can use any of the quality indicators available in the litera-

Table 10
Results of Livestock Feed Ration Optimization Problems (RC51 -RC57) using IUDE, ϵMAgES, and iLSHADE$_\epsilon$.

Problem	Algorithm	Best	Median	Mean	Worst	Std	FR	MV	SR
RC51	IUDE	**4.55E+03**	**4.55E+03**	**4.55E+03**	**4.55E+03**	**4.16E-02**	**0**	**2.82E-06**	**0**
	ϵMAgES	4.55E+03	4.55E+03	4.50E+03	3.05E+03	3.02E+02	0	1.67E-02	0
	iLSHADE$_\epsilon$	4.55E+03	4.57E+03	4.56E+03	4.58E+03	1.14E+01	0	1.97E-04	0
RC52	IUDE	**3.35E+03**	**3.39E+03**	**3.39E+03**	**3.43E+03**	**2.16E+01**	**100**	**0.00E+00**	**0**
	ϵMAgES	3.45E+03	3.76E+03	3.82E+03	4.38E+03	2.47E+02	100	0.00E+00	0
	iLSHADE$_\epsilon$	3.65E+03	3.96E+03	3.98E+03	4.28E+03	1.64E+02	100	0.00E+00	0
RC53	IUDE	**5.00E+03**	**5.07E+03**	**5.08E+03**	**5.31E+03**	**8.70E+01**	**100**	**0.00E+00**	**8**
	ϵMAgES	5.01E+03	5.25E+03	5.11E+03	2.57E+03	5.50E+02	92	3.56E-02	0
	iLSHADE$_\epsilon$	5.06E+03	5.24E+03	5.27E+03	5.53E+03	1.56E+02	100	0.00E+00	0
RC54	IUDE	**4.24E+03**	**4.24E+03**	**4.24E+03**	**4.24E+03**	**2.10E-01**	**100**	**0.00E+00**	**20**
	ϵMAgES	4.24E+03	4.07E+03	3.69E+03	1.38E+03	7.31E+02	40	4.02E-02	0
	iLSHADE$_\epsilon$	4.24E+03	4.24E+03	4.24E+03	4.25E+03	2.78E+00	100	0.00E+00	12
RC55	IUDE	**3.51E+03**	**2.06E+03**	**2.28E+03**	**2.23E+03**	**3.32E+02**	**0**	**9.71E-03**	**0**
	ϵMAgES	6.16E+03	2.67E+03	2.74E+03	3.61E+03	1.20E+03	0	6.84E-02	0
	iLSHADE$_\epsilon$	6.27E+03	6.39E+03	5.38E+03	2.76E+03	1.20E+03	0	2.29E-02	0
RC56	IUDE	**1.41E+04**	**1.15E+04**	**1.19E+04**	**7.37E+03**	**1.49E+03**	**0**	**9.85E-03**	**0**
	ϵMAgES	1.32E+04	9.36E+03	9.12E+03	6.57E+03	2.23E+03	0	1.70E-02	0
	iLSHADE$_\epsilon$	1.38E+04	1.44E+04	1.30E+04	1.12E+04	1.40E+03	0	5.16E-02	0
RC57	IUDE	**2.88E+03**	**2.72E+03**	**2.66E+03**	**2.39E+03**	**2.85E+02**	**0**	**1.98E-03**	**0**
	ϵMAgES	4.01E+03	2.61E+03	3.40E+03	1.06E+03	1.60E+03	0	2.34E-02	0
	iLSHADE$_\epsilon$	8.31E+03	6.41E+03	6.98E+03	1.26E+04	2.62E+03	0	1.12E-02	0

ture to rank the performance of algorithms on the proposed benchmark suite.

To determine whether a small difference in the performance of algorithms can be considered significant, the comparative analysis generally includes statistical hypothesis testings. Due to the requirement of smaller sample sizes and fewer restrictions than parametric approaches, non-parametric tests such as Freidman test and Wilcoxon tests are generally utilized when analyzing SAs and EAs for statistical significance [75]. Still, practical and statistical significance is not necessarily equivalent to each other.

To compare the performance of algorithms on the proposed benchmark suite, we have considered the ranking scheme proposed in CEC 2020's competition [73]. In Ref. [73], the performance measure (*PM*) for each algorithm is defined using following equation.

$$PM_i = 0.5 * \sum_{j=1}^{57} w_j * \widehat{Af}_{i,j}^{best} + 0.3 * \sum_{j=1}^{57} w_j * \widehat{Af}_{i,j}^{mean} + 0.2 * \sum_{j=1}^{57} w_j * \widehat{Af}_{i,j}^{medium}, \tag{63}$$

where, $\widehat{Af}_{i,j}^{best}$, $\widehat{Af}_{i,j}^{mean}$, and $\widehat{Af}_{i,j}^{median}$ are normalized adjusted objective function value of the best, mean and medium solution, respectively, of the j-th problem for the i-th algorithm and w_j is weight value of the j-th problem. The weight value of the $j-$th problem is set as follows.

$$w_j = \begin{cases} 0.008, & \text{if } D_j \leq 10 \\ 0.016, & \text{if } 10 < D_j \leq 30 \\ 0.024, & \text{if } 30 < D_j \leq 50 \\ 0.032, & \text{if } 50 < D_j \leq 150 \\ 0.040, & \text{if } 150 < D_j \end{cases} \tag{64}$$

To calculate the normalized adjusted objective function value of the best solution of an algorithm on a benchmark problem, the following procedure is adopted.

1. Select the worst feasible solution ($f_{worst,j}^{F,best}$) from the combined set of best solutions of all algorithms in the competition for the j-th problem. If there is no feasible solution in the combined set, then $f_{worst,j}^{F,best}$ is set to 0.

Table 11
PM value of all algorithms on real-world constrained benchmark suite.

Algorithm	Best	Mean	Median	Weighted	Rank
IUDE [22]	0.5827	0.5849	0.5832	0.5835	3
ϵMAgES [23]	**0.0902**	**0.2999**	**0.2191**	**0.1789**	**1**
ILSHADE$_\epsilon$ [24]	0.5713	0.4852	0.5664	0.5445	2

2. Then, calculate the adjusted objective function value of the best solution for each algorithm using the following equation.

$$Af_{i,j}^{best} = \begin{cases} f_{worst,j}^{F,best} + v_{i,j}^{best}, & \text{if } v_{i,j}^{best} > 0 \\ f_{i,j}^{best}, & \text{if } v_{i,j}^{best} \leq 0 \end{cases} \tag{65}$$

3. At last, normalize the adjusted objective function value of best solution for each algorithm using the following equation.

$$\widehat{Af}_{i,j}^{best} = \frac{Af_{i,j}^{best} - Af_{min,j}^{best}}{Af_{max,j}^{best} - Af_{min,j}^{best}}, \tag{66}$$

where,

$$Af_{min,j}^{best} = min\{Af_{1,j}^{best}, Af_{2,j}^{best}, \dots .Af_{i,j}^{best}, \dots ..\}, \tag{67}$$

$$Af_{max,j}^{best} = max\{Af_{1,j}^{best}, Af_{2,j}^{best}, \dots .Af_{i,j}^{best}, \dots ..\}. \tag{68}$$

A similar procedure is utilized to calculate the adjusted objective function value of the mean and median solution of the algorithms.

The *PM* value of IUDE, ϵMAgES, and iLSHADE$_\epsilon$ on the proposed benchmark suite are depicted in Table 11. From Table 11, it can be seen that *PM* of ϵMAgES is better than other algorithms i.e. ϵMAgES outperforms other algorithms on the proposed benchmark suite. It is worth to note that the performance of IUDE (winner of IEEE CEC 2018's competition) is the least efficient and robust from all algorithms tested. From this outcome, it can be assumed that the performance of the algorithms on artificial benchmark problems can be inappropriate to choose an algorithm from a group for real-world applications. Therefore, a benchmark suite of the real-world COPs can be the best tool for the benchmark of SAs and EAs.

4. Conclusion

In this work, a benchmark suite containing 57 real-world COPs is proposed for validating the performance and robustness of the population-based derivative-free optimization algorithms specifically tailor-made for handling constraints with a wide spectrum of difficulties. Three state-of-the-art constrained optimization algorithms have been tested to show the complexity of the problems included in the benchmark suite. The comparative analysis of the outcomes of these algorithms concludes that the problems are hard to solve for the recently developed constrained evolutionary optimizers. Furthermore, these challenging problems can motivate researchers to develop new approaches for handling even more challenging non-linear constraints and especially the equality constraints.

CRediT authorship contribution statement

Abhishek Kumar: Methodology, Resources, Software, Formal analysis, Investigation, Validation, Data curation, Writing - original draft, Visualization. **Guohua Wu:** Methodology, Resources, Software, Formal analysis, Data curation, Writing - review & editing, Visualization. **Mostafa Z. Ali:** Methodology, Resources, Software, Formal analysis, Data curation, Writing - review & editing, Visualization. **Rammohan Mallipeddi:** Methodology, Resources, Software, Formal analysis, Data curation, Writing - review & editing, Visualization. **Ponnuthurai Nagaratnam Suganthan:** Conceptualization, Methodology, Supervision, Writing - review & editing, Visualization, Project administration. **Swagatam Das:** Conceptualization, Methodology, Supervision, Writing - review & editing, Visualization, Project administration.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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