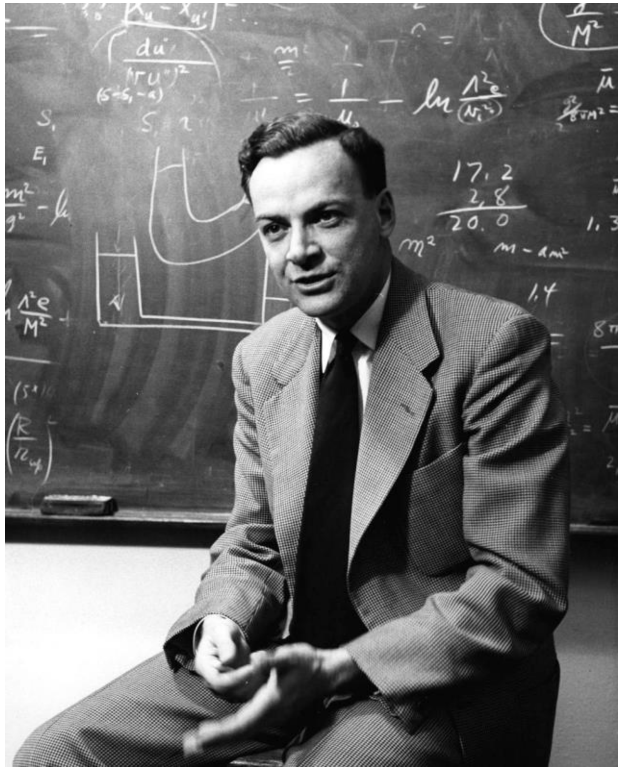
10.12.2024

The Effects of Casimir Interactions in Experiments on Gravitationally Induced Enganglement

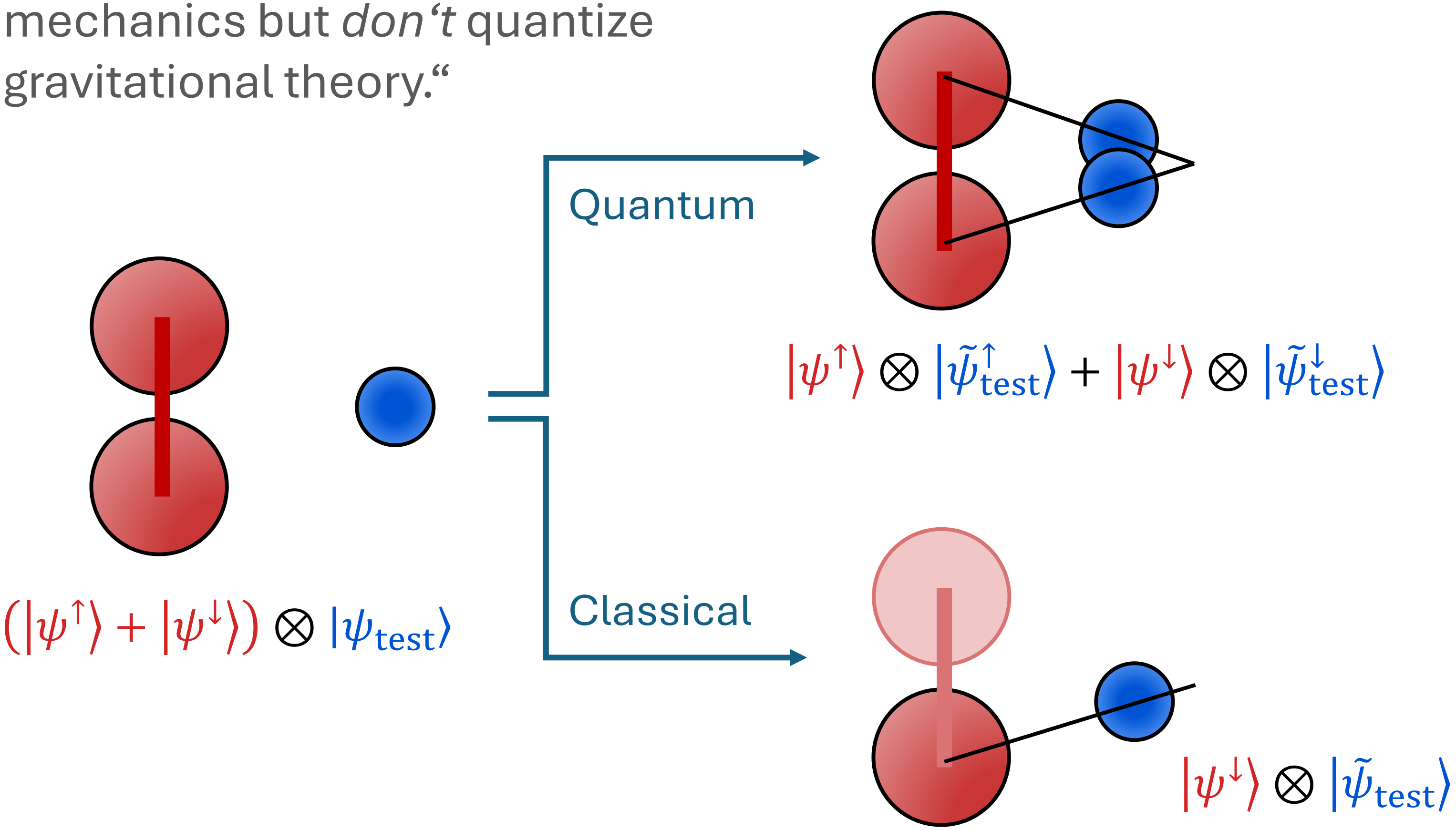
Bachelor Thesis final talk – Jan Bulling

Supervised by: Marit and Julen

1957 – Chapel Hill, North Carolina

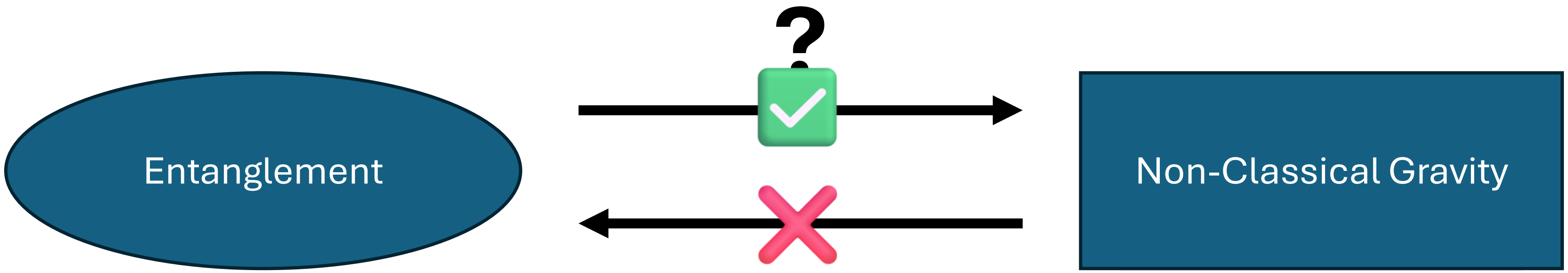


„[...] it seems clear to me that we‘re in trouble if we believe in quantum mechanics but *don‘t* quantize gravitational theory.“



[J. Pedernales et. al, , Contemporary Physics 64, 147–163 (2023)]

Gravitationally induced entanglement as a „proof“ of quantum gravity?



e.g. swap-operations
$|\psi\rangle_A |\phi\rangle_B \rightarrow |\phi\rangle_A |\psi\rangle_B$

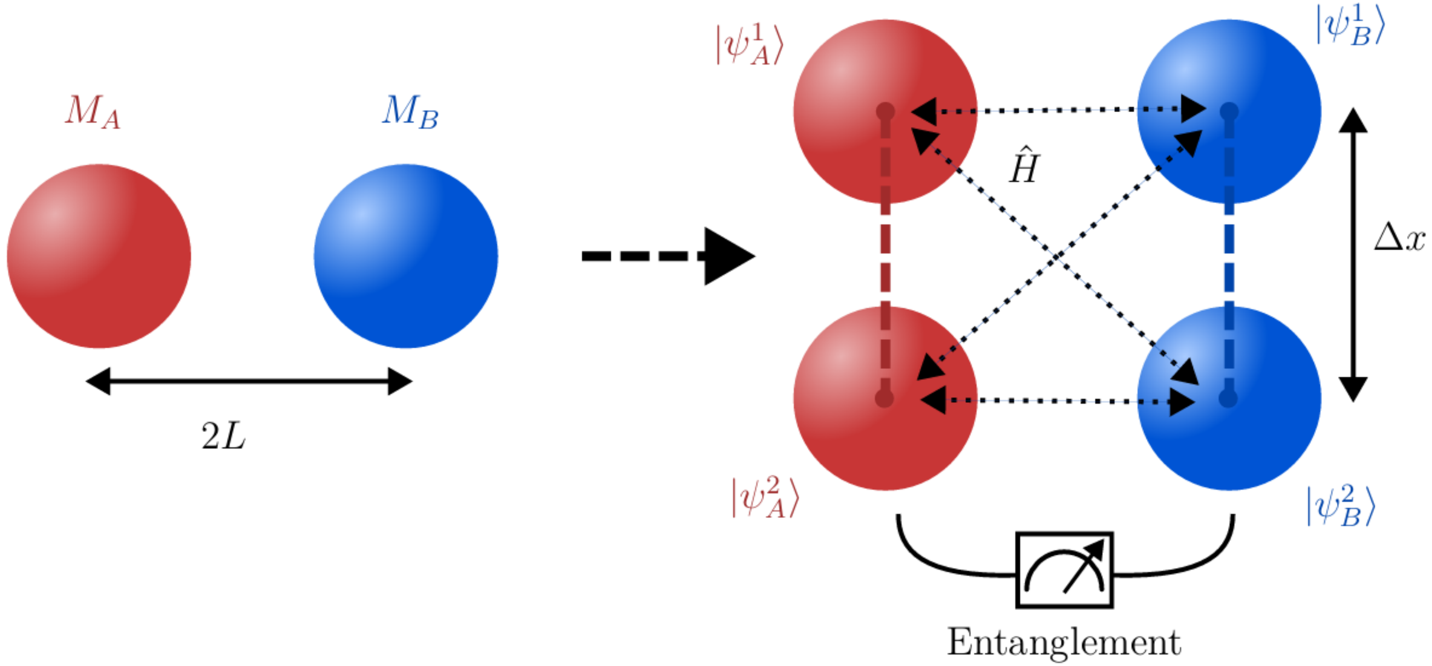
Non-Classical Gravity
= non-LOCC interactions mediated by gravity
[L. Lami et. al, Phys. Rev. X 14, 021022 (2023)]

Experimental setup

Distance between cat-states $A^{(i)}$ and $B^{(j)}$

Gravitational coupling: $H = -\frac{GM_A M_B}{|\hat{L}|}$ with $\hat{L}\left|\psi_A^{(i)}\psi_B^{(j)}\right\rangle = 2L^{(ij)}\left|\psi_A^{(i)}\psi_B^{(j)}\right\rangle$

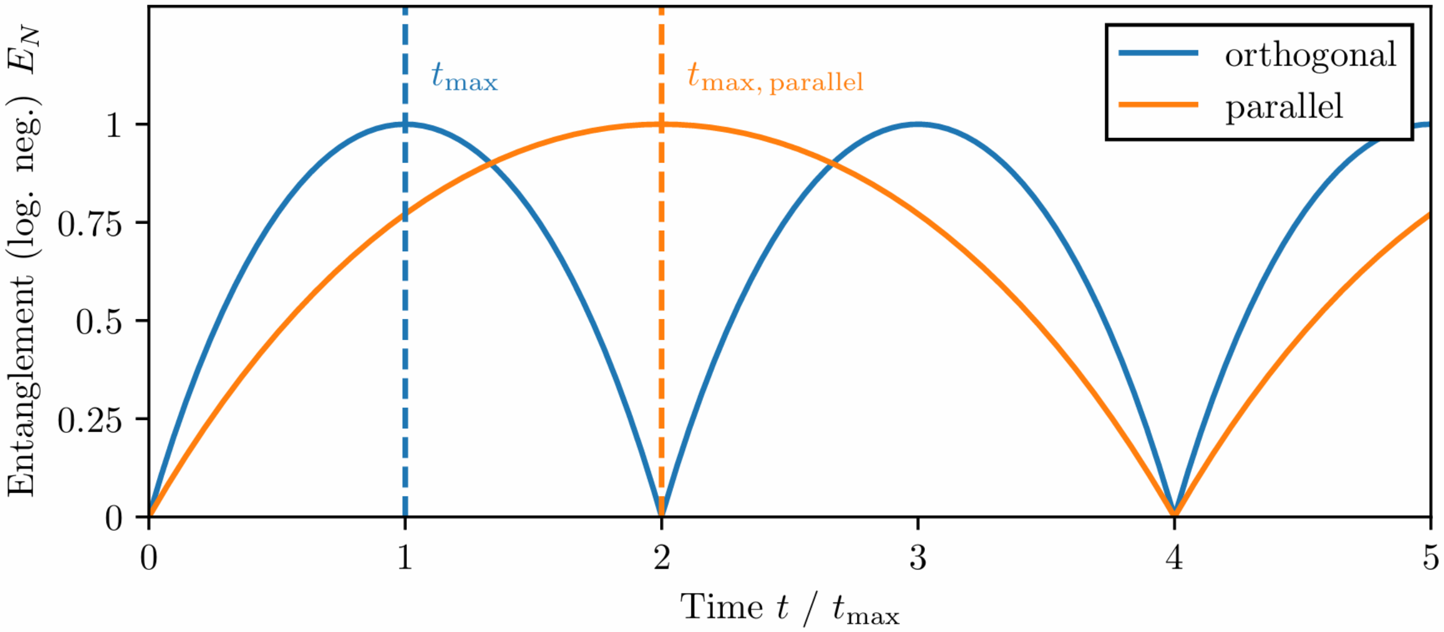
(low energy limit)



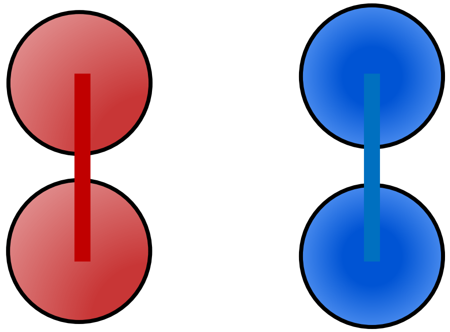
[S. Bose et. al, Phys. Rev. Lett. 119, 240401 (2017)]
[J. Pedernales et. al, Contemporary Physics 64, 147–163 (2023)]

Entanglement dynamics

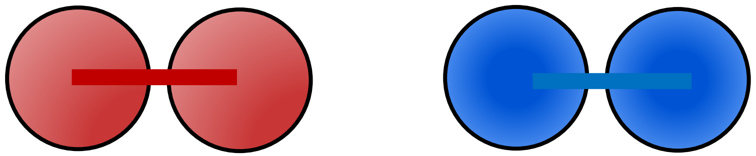
$$E_N \approx \log_2(1 + |\sin \Delta\phi|) \quad \text{where} \quad \Delta\phi = \frac{G M_A M_B (\Delta x)^2}{8\hbar L^3} t$$



„Parallel“ configuration:



„Orthogonal“ configuration:



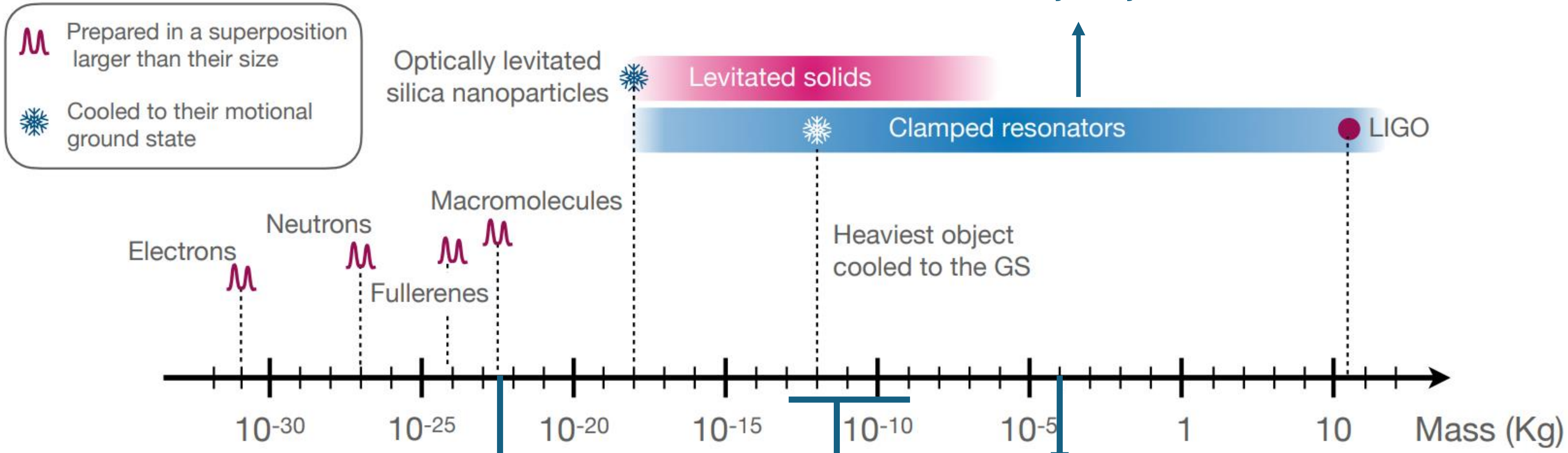
$$t_{\text{max,orthogonal}} = \frac{4\pi L^3 \hbar}{G M_A M_B (\Delta x)^2}$$

$$\frac{M^2 (\Delta x)^2}{L^3} t \gtrsim \frac{\hbar}{G}$$

$$\frac{M^2(\Delta x)^2}{L^3} t \gtrsim \frac{\hbar}{G}$$

What is possible today?

Usually very short coherence times



Gravitational force measured
[T. Westphal et.al, Nature 591, 225–228 (2021)]

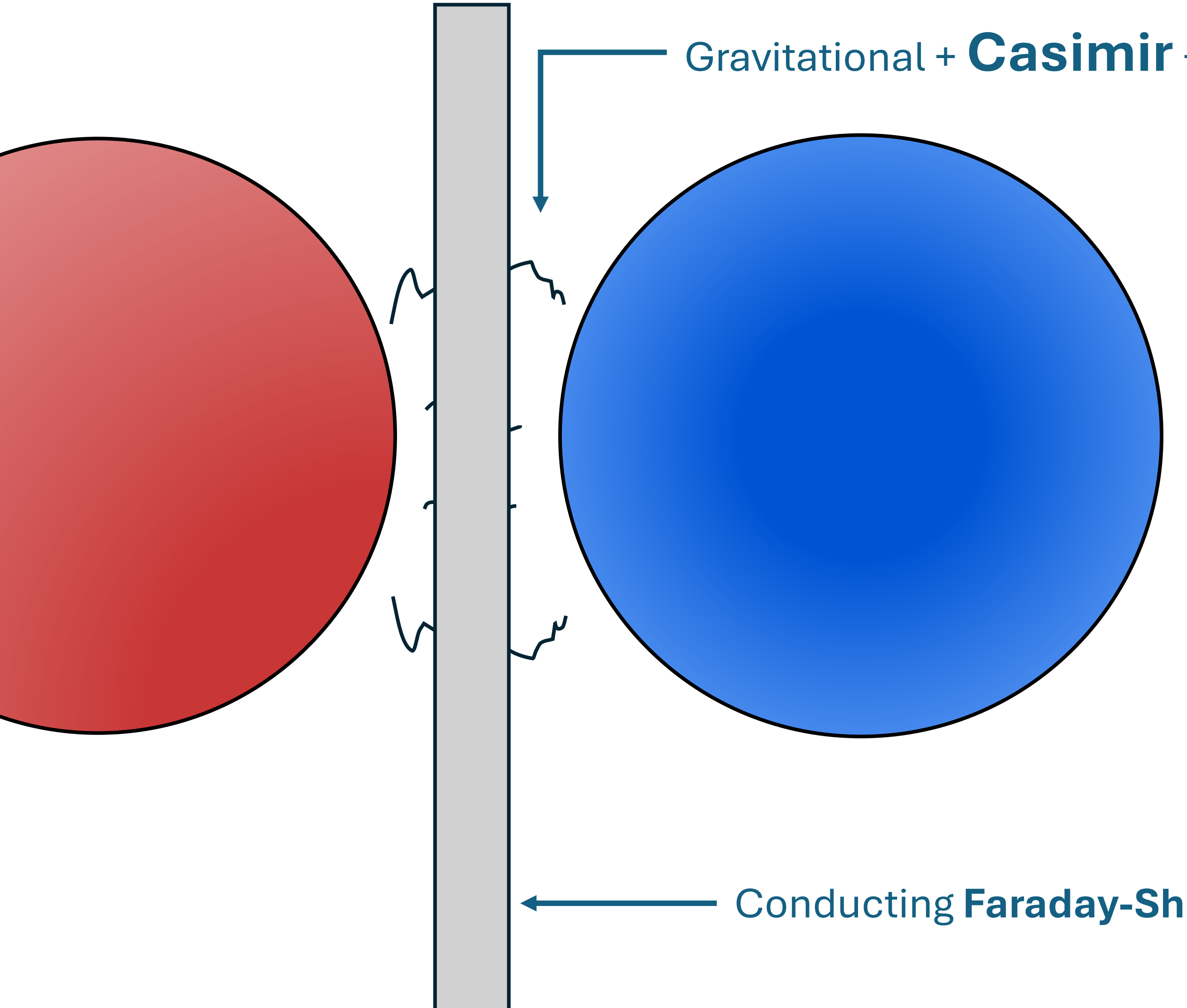
Proposed for entanglement experiments
[S. Rijavec et.al, New Journal of Physics 23, 043040 (2020)]

Molecules with 4×10^{-23} kg and $\Delta x = 500$ nm
[Y. Y. Fein et.al, Nature Physics 15, 1242–1245 (2019)]

$$\frac{M^2(\Delta x)^2}{L^3}t \gtrsim \frac{\hbar}{G}$$

How small can we make L?

Gravitational + **Casimir** + (Coulomb ??) interactions



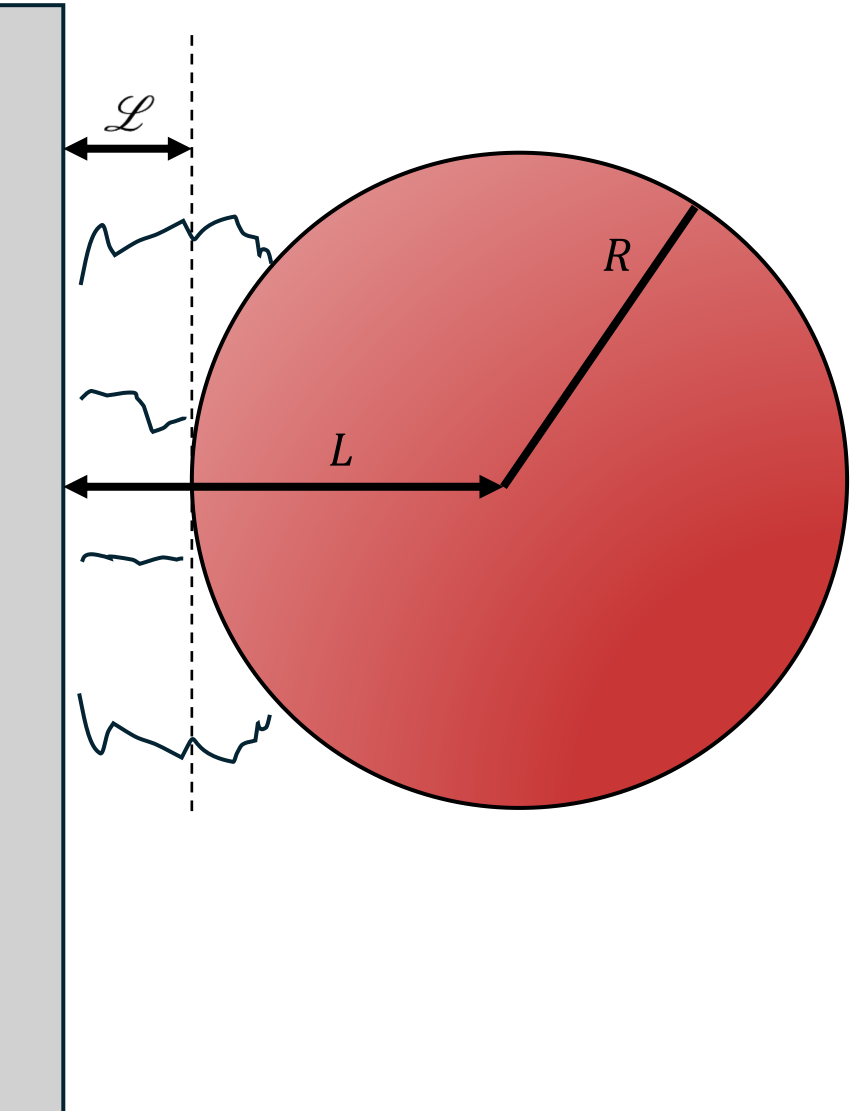
$$V_{\text{Gravity}} \sim -\frac{G\ M_A M_B}{L}$$

$$V_{\text{Casimir}} \sim -\frac{23\ \hbar c}{4\pi L^7}\left(\frac{\varepsilon_r - 1}{\varepsilon_r + 2}\right)^2 R^6$$

[T. Emig et.al, Phys. Rev. Lett. 99, 170403 (2007)]

Conducting **Faraday-Shield** required for $L \lesssim 100\ \mu\text{m}$

Casimir Interactions: Particles ↔ Shield



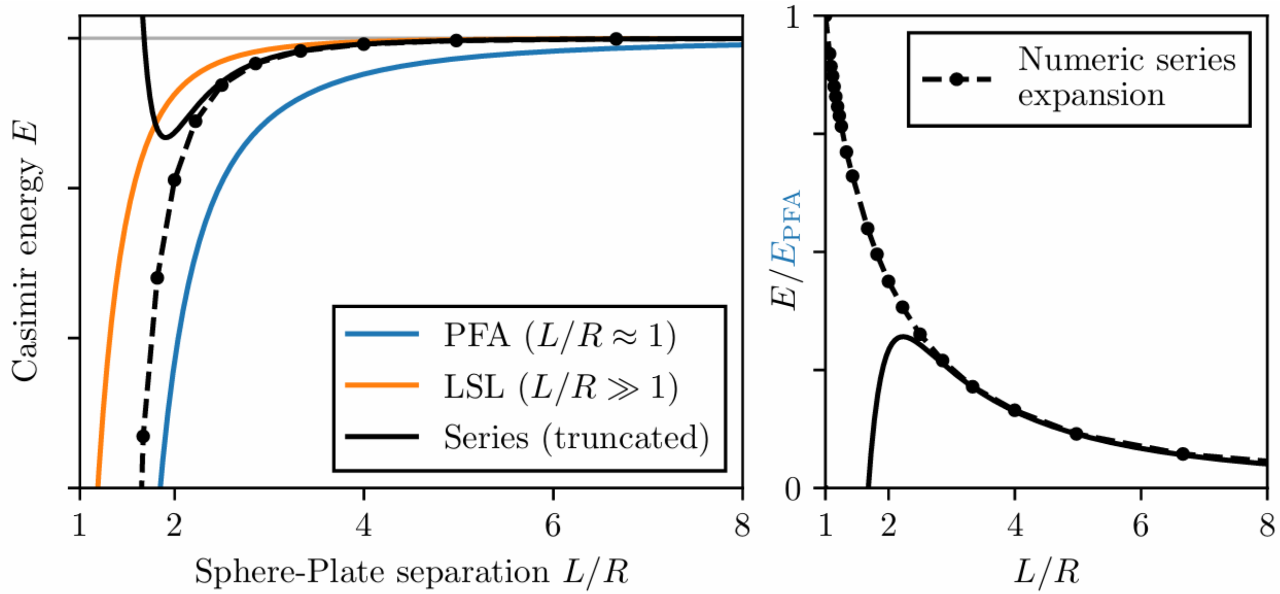
Proximity-Force-Approximation **(PFA)** for $L \approx R$:

$$E_{\text{PFA}} = -\frac{\hbar c \pi^3}{720}\left(\frac{\varepsilon_r - 1}{\varepsilon_r + 1}\right)\varphi(\varepsilon_r)\frac{R}{\mathscr{L}^2}$$

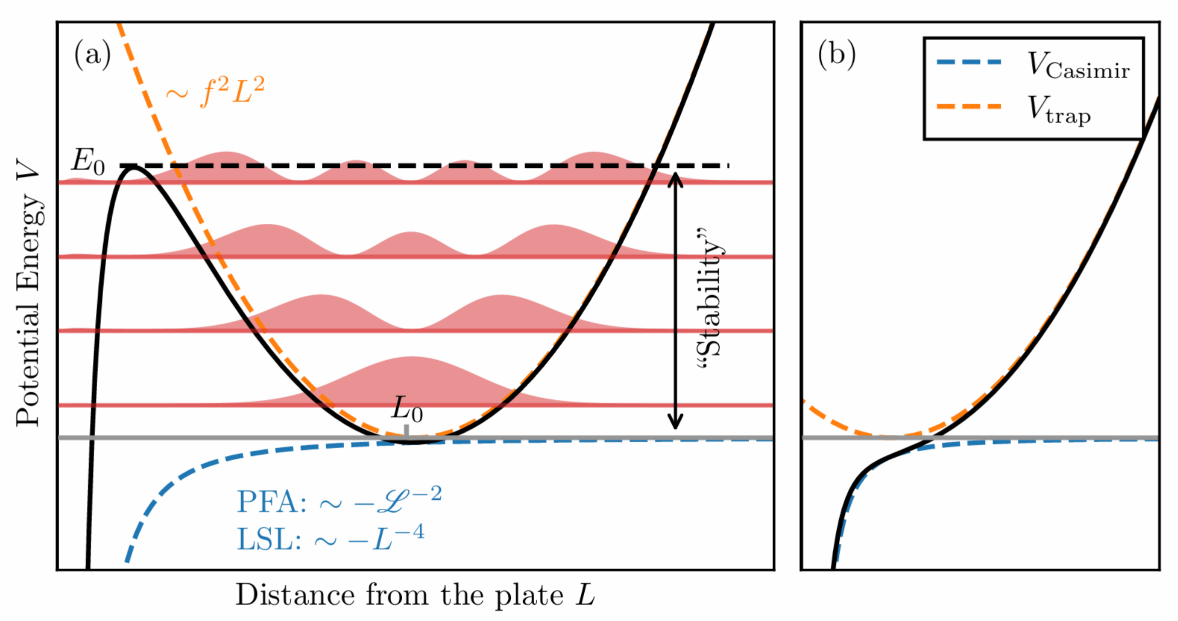
Large separation limit **(LSL)** for $L/R \gg 1$:

$$E_{\text{LSL}} = -\frac{3}{8}\frac{\hbar c}{\pi}\left(\frac{\varepsilon_r - 1}{\varepsilon_r + 2}\right)\frac{R^3}{L^4}$$

[T. Emig, J. Stat. Mech. P04007 (2008)]

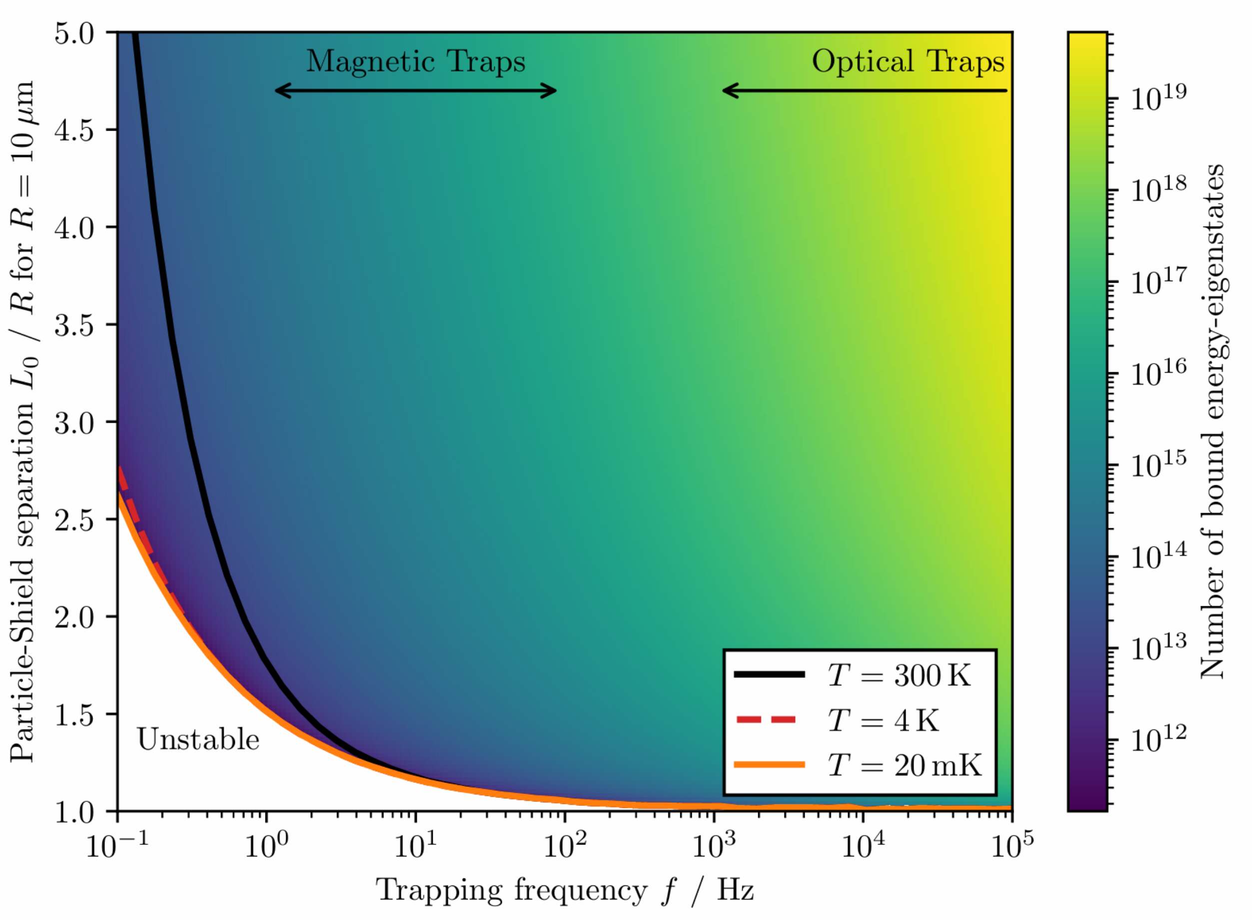


Trapping the particles close to the shield



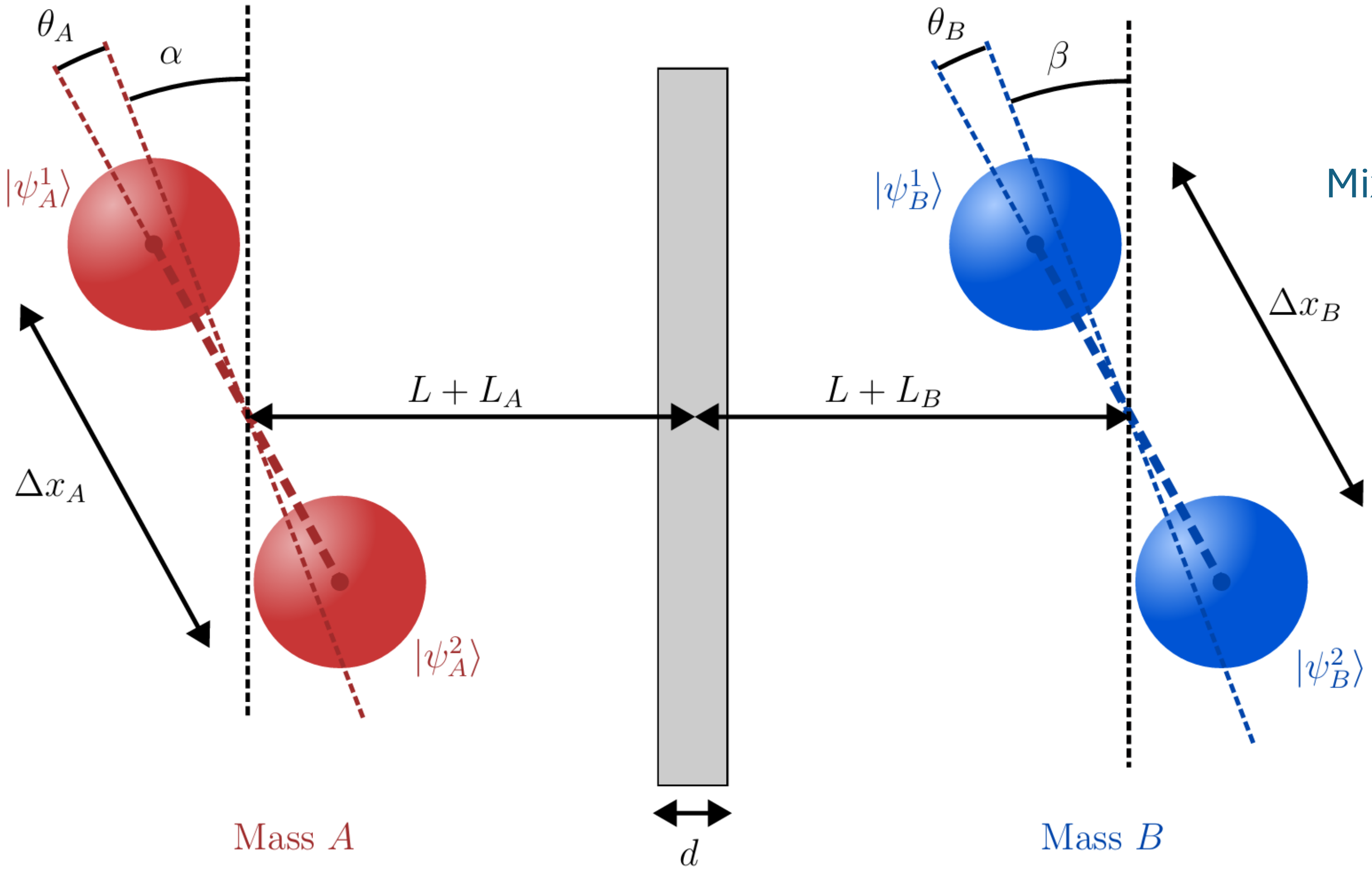
WKB-Approximation:

$$n(E_0) \approx \frac{1}{\hbar\pi} \int_{x_1}^{x_2} \mathrm{d}x \sqrt{2m\big(E_0 - V(x)\big)} \; < \; \bar{n}$$



→ Trapping close to the shield should not be a problem!

The general problem...



$$\langle \rho \rangle = \int_{-\infty}^{\infty} \mathrm{d}X \, \frac{1}{\sqrt{2\pi}\Delta X} e^{-\frac{X^2}{2\,(\Delta X)^2}} |\psi_X\rangle \langle \psi_X|$$

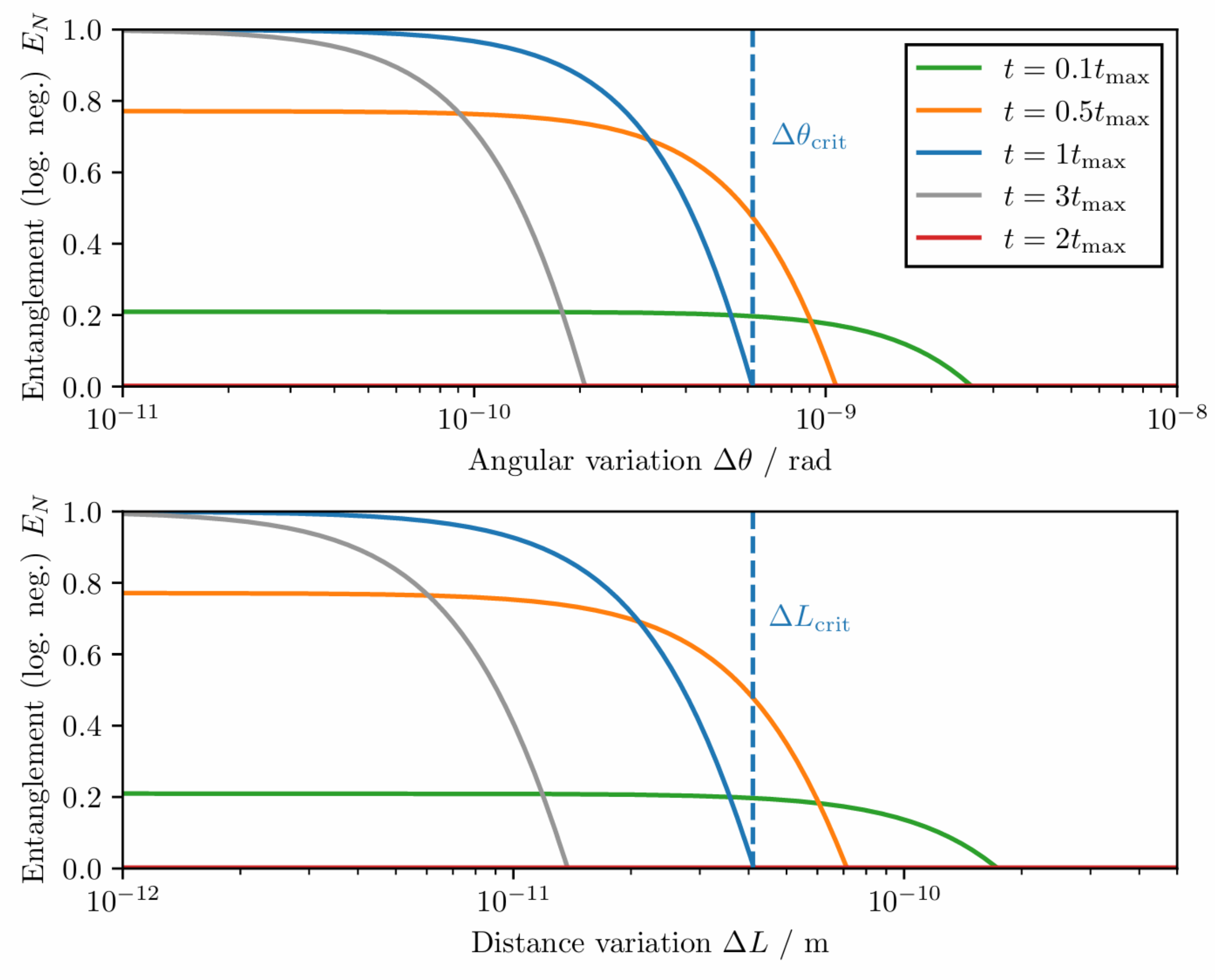
with $\quad X \in \{\theta_A, \theta_B, L_A, L_B\}$

Mixed state $\operatorname{tr} \rho^2 < 1$

Pure state of a single measurement

All variations are gaussian distributed around $\langle X \rangle = 0$ and standard deviation ΔX ,
where $X \in \{\theta_A, \theta_B, L_A, L_B\}$

Loss of entanglement



Parameters:

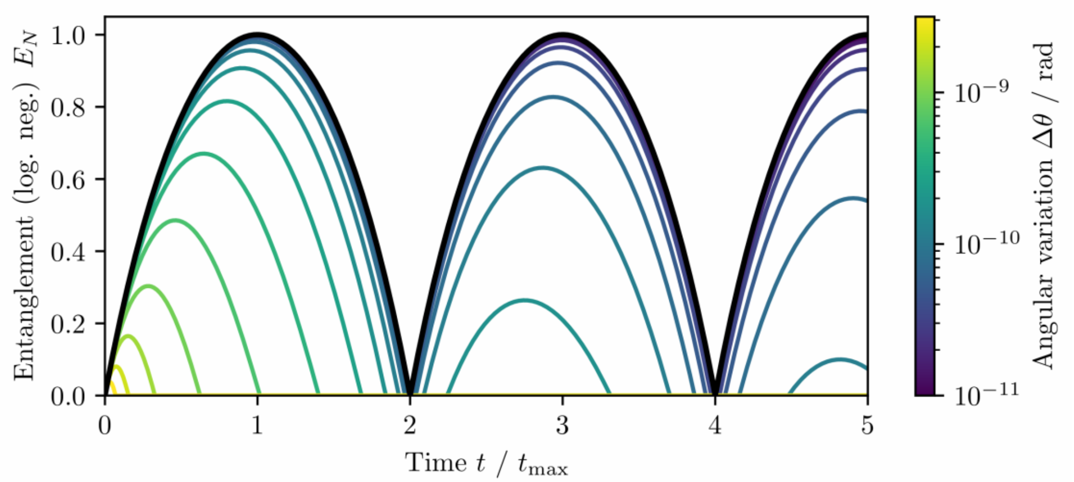
Orientation	Particle size: Radius R	Particle size: Mass M	L	Δx
Parallel ($\alpha = \beta = 0$)	$10\,\mu$m $= 10^{-5}$ m	$\approx 10^{-11}$ kg $= 5 \times 10^{-4}\, m_p$	$2R = 20\,\mu$m	100 nm

Logarithmic negativiy (analytical):

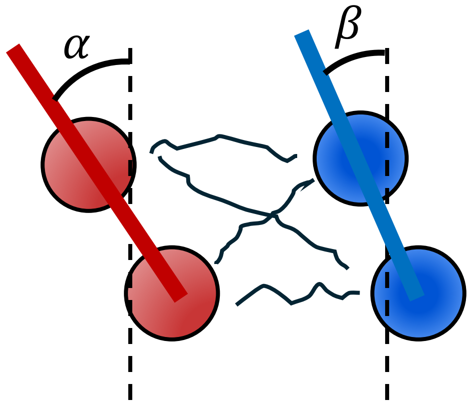
$$E_N = \max\{0, \log_2[e^{-\gamma}(\cosh\gamma + |\sin\phi|)]\}$$

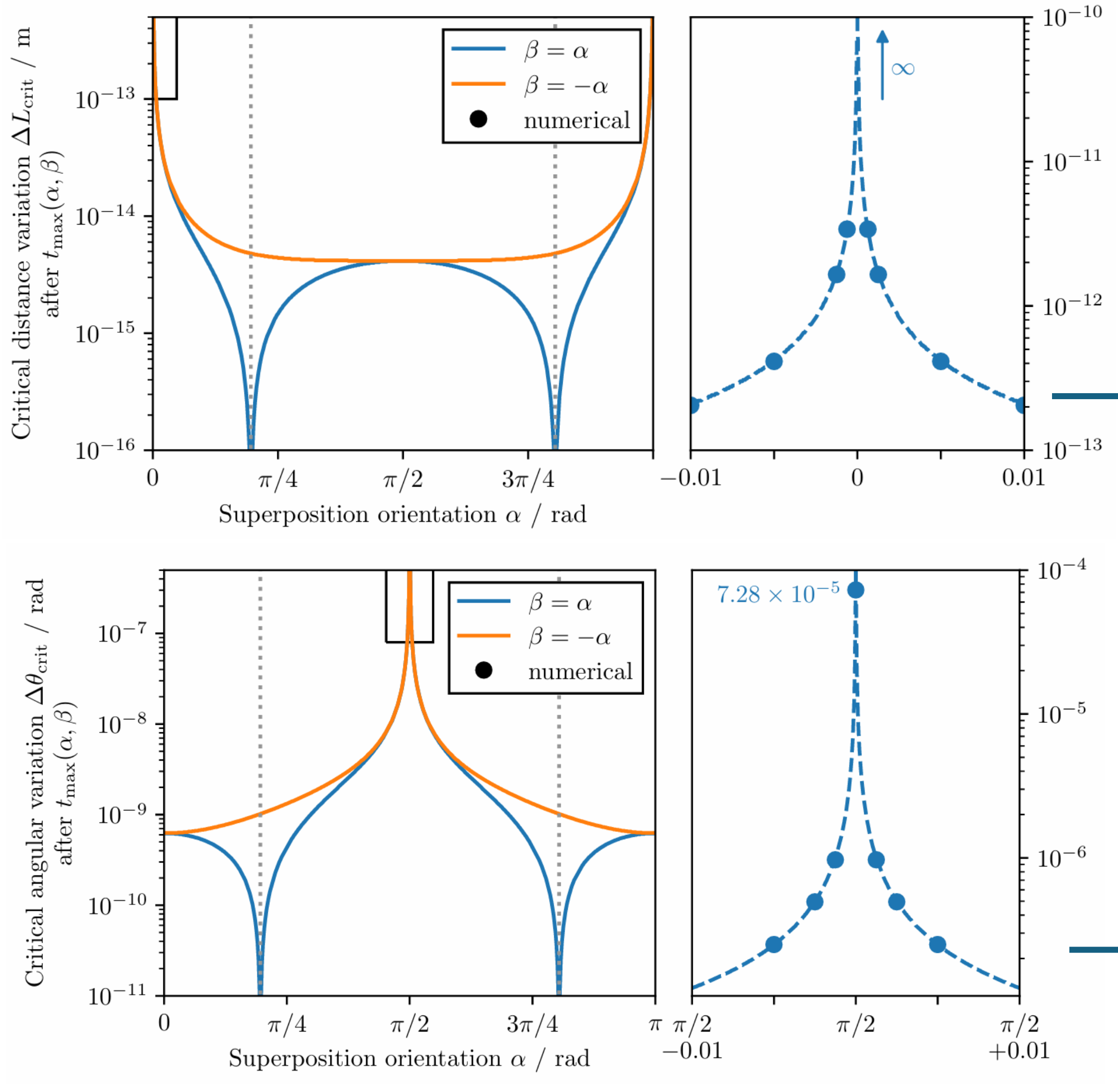
with $\gamma \sim [\xi\,(\Delta\theta)^2 + \zeta\,(\Delta L)^2]t^2$

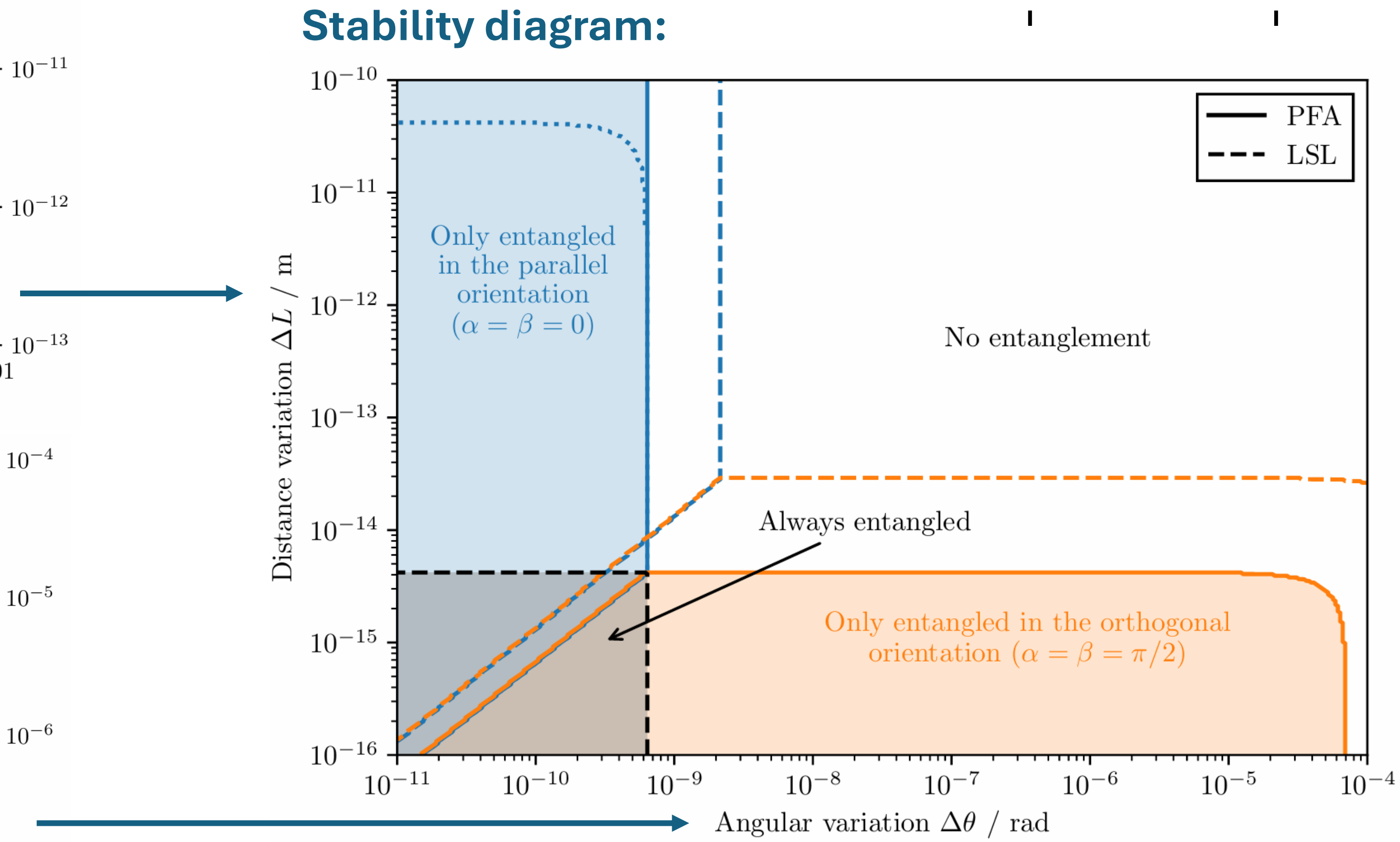
$$\phi = \frac{GM^2(\Delta x)^2 t}{8\hbar L^3}\left[\sin\alpha\sin\beta - \frac{1}{2}\cos\alpha\cos\beta\right]$$



Which orientation is the most stable?



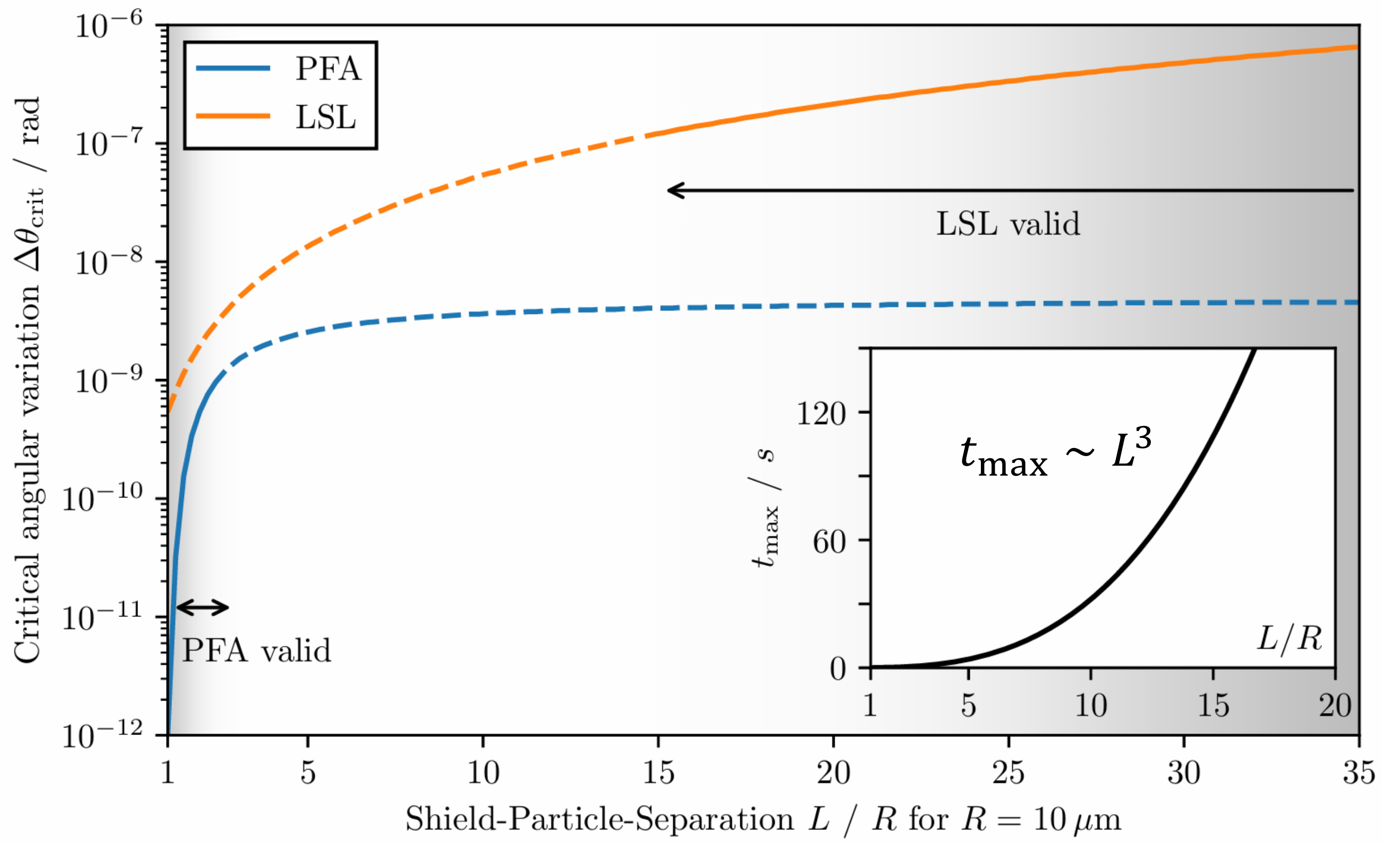




Stability improvements for other parameters

Orientation	Particle size		L	Δx
	Radius R	Mass M [a]		
Parallel ($\alpha = \beta = 0$)	$10\,\mu\text{m} = 10^{-5}\,\text{m}$	$\approx 10^{-11}\,\text{kg} = 5 \times 10^{-4}\, m_p$	$2R = 20\,\mu\text{m}$	$100\,\text{nm}$

Particle-shield separation:



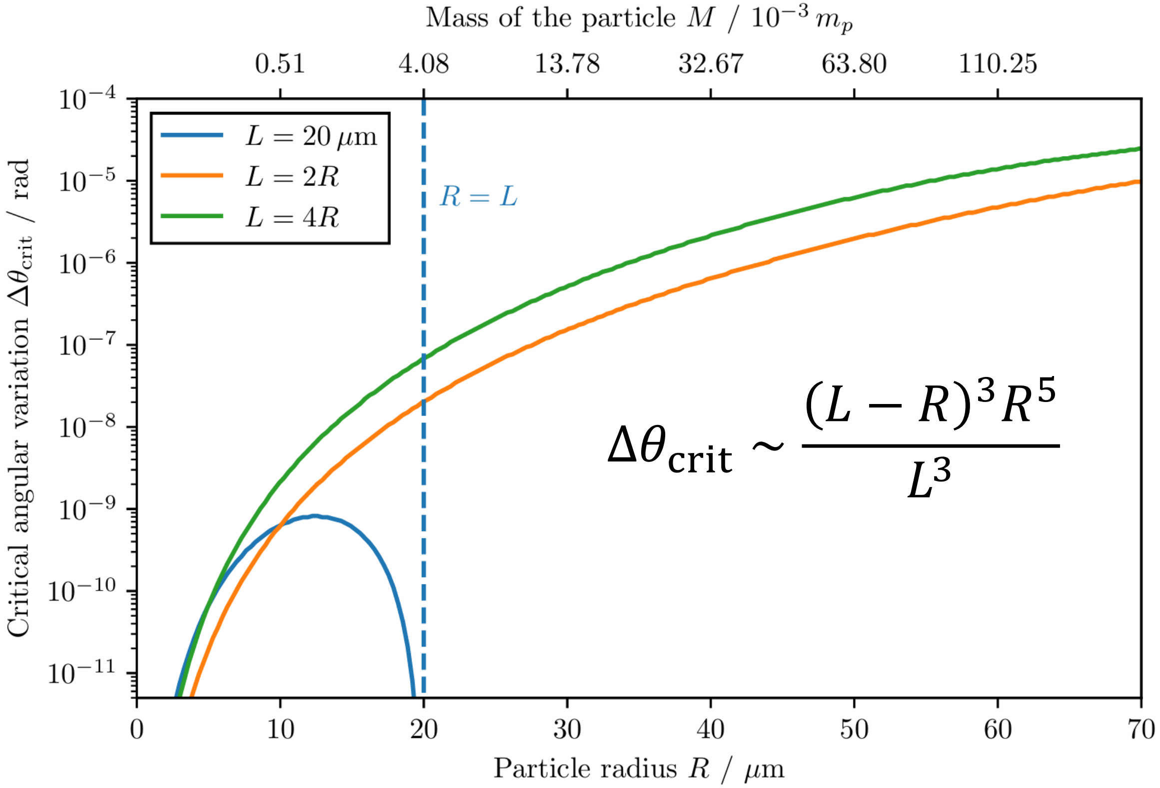
$$\Delta\theta_{crit} \sim L^2$$

$$\Delta\theta_{crit} \sim \frac{(L-R)^3}{L^3}$$

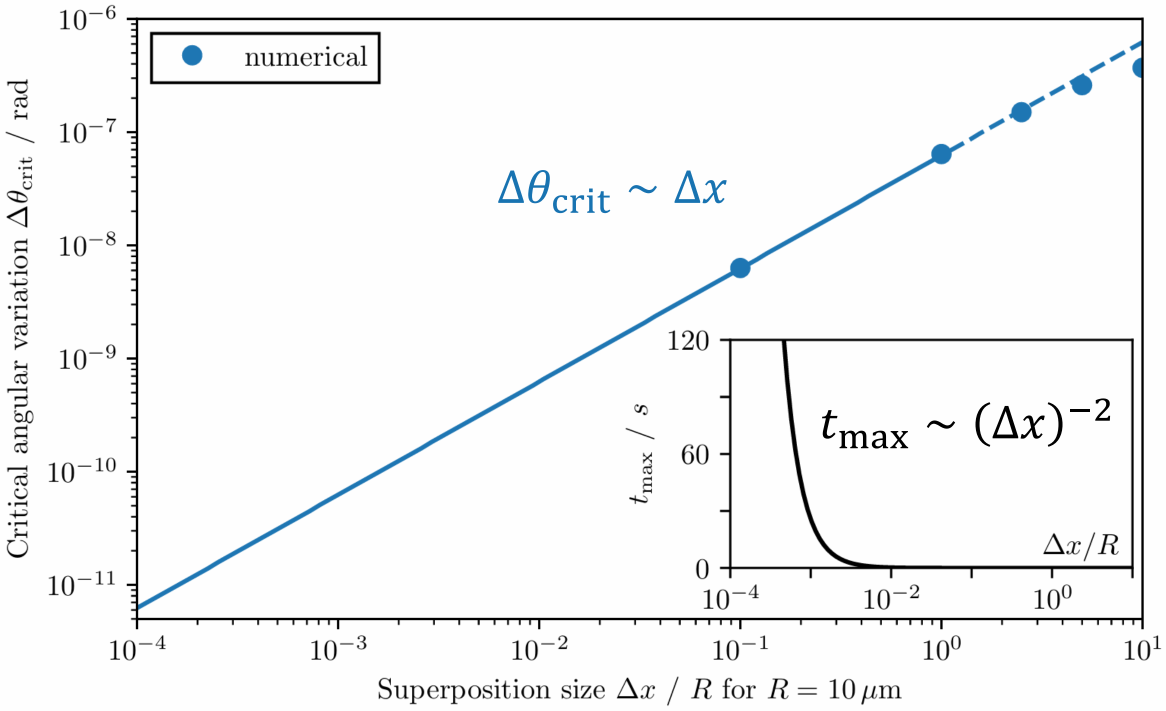
Stability improvements for other parameters

Orientation	Particle size		L	Δx
	Radius R	Mass M [a]		
Parallel ($\alpha = \beta = 0$)	$10\,\mu\text{m} = 10^{-5}\,\text{m}$	$\approx 10^{-11}\,\text{kg} = 5 \times 10^{-4}\, m_p$	$2R = 20\,\mu\text{m}$	$100\,\text{nm}$

Particle size / particle mass:



Spatial superposition extension:



Optimization?

$$\frac{M^2(\Delta x)^2}{L^3} t \gtrsim \frac{\hbar}{G}$$

The largest possible allowed setup variations $\max \Delta\theta_{\mathrm{crit}}$ and $\max \Delta L_{\mathrm{crit}}$

- **Increase** L as $\Delta\theta_{\mathrm{crit}} \propto L^2$
- Increase mass M
- Increase superposition size Δx
- Maybe parallel orientation?

Shortest coherence time $\min t_{\max}$
$\leftrightarrow$ fastest entanglement rate

- **Decrease** L as $t_{\max} \propto L^3$
- Increase mass M
- Increase superposition size Δx

Consider experimental limitations

$$\Gamma_{\mathrm{Entanglement}} > \Gamma_{\mathrm{Decoherence}}$$

- Limited generation of spatially delocalized states ($M \cdot \Delta x$)
- Entanglement rate larger than decoherence rates

The size of the shield

Orientation	Particle size		L	Δx
	Radius R	Mass M [a]		
Parallel $(\alpha = \beta = 0)$	$10\,\mu\mathrm{m} = 10^{-5}\,\mathrm{m}$	$\approx 10^{-11}\,\mathrm{kg} = 5 \times 10^{-4}\, m_p$	$2R = 20\,\mu\mathrm{m}$	$100\,\mathrm{nm}$

Gravitational entanglement rate:

$$\Gamma_{\text{Gravity}} = \frac{\mathrm{d}}{\mathrm{d}t} E_N \Big|_{t=0} = \frac{G\pi^2 R^6 \rho_{\text{Silica}}^2 (\Delta x)^2}{9\hbar L^3 \log 2} > \Gamma_{\text{All other interactions}}$$

$\Gamma_{\text{Gravity}} > \Gamma_{\text{Coulomb}}$

→ Required thickness: $d \geq 10$ nm at 4 K

→ Required radius: $r_s \geq 60$ cm!!

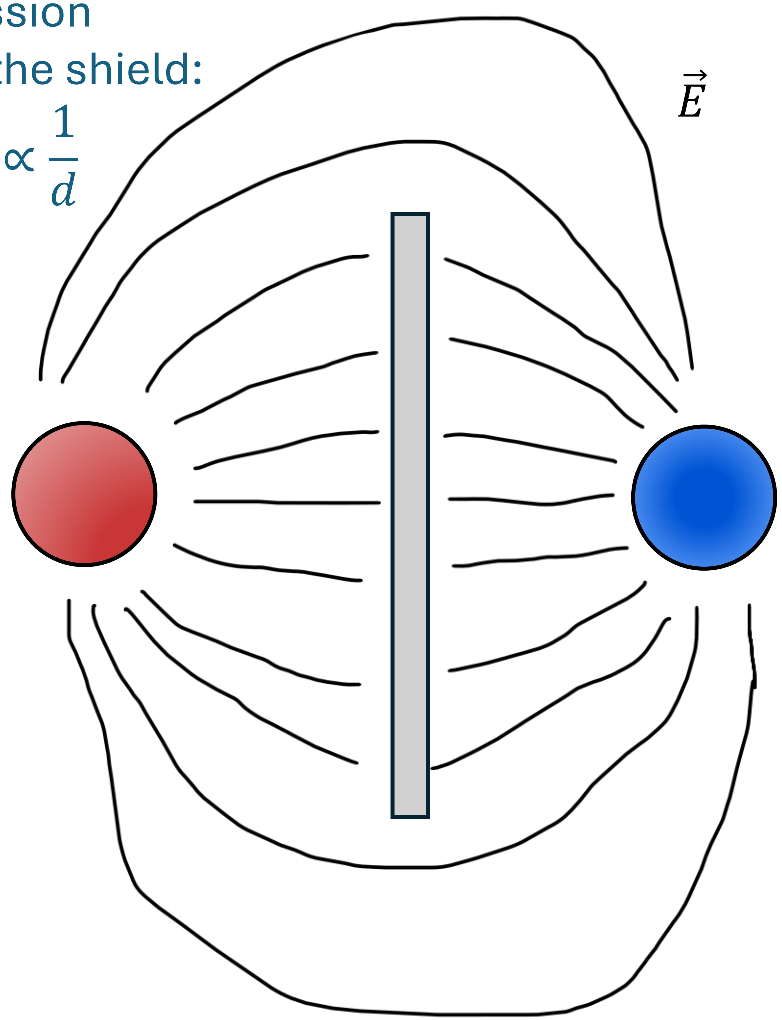
$\Gamma_{\text{Gravity}} > \Gamma_{\text{Casimir}}$

→ Required thickness: $d \geq 0.04$ nm at 4 K

→ Required radius: $r_s \gtrsim R + \frac{\Delta x}{2} \approx 10\ \mu\text{m}$

Transmission through the shield:

$T \propto \frac{1}{d}$



Vibrational modes

Physical round plate with thickness d and radius r_s

~ zeros of the Bessel functions J_ν and I_ν

Vibrational frequency: $\omega \propto \frac{d}{r_s^2}\sqrt{\frac{E}{12\,\rho(1-\nu^2)}}$

Parameters:

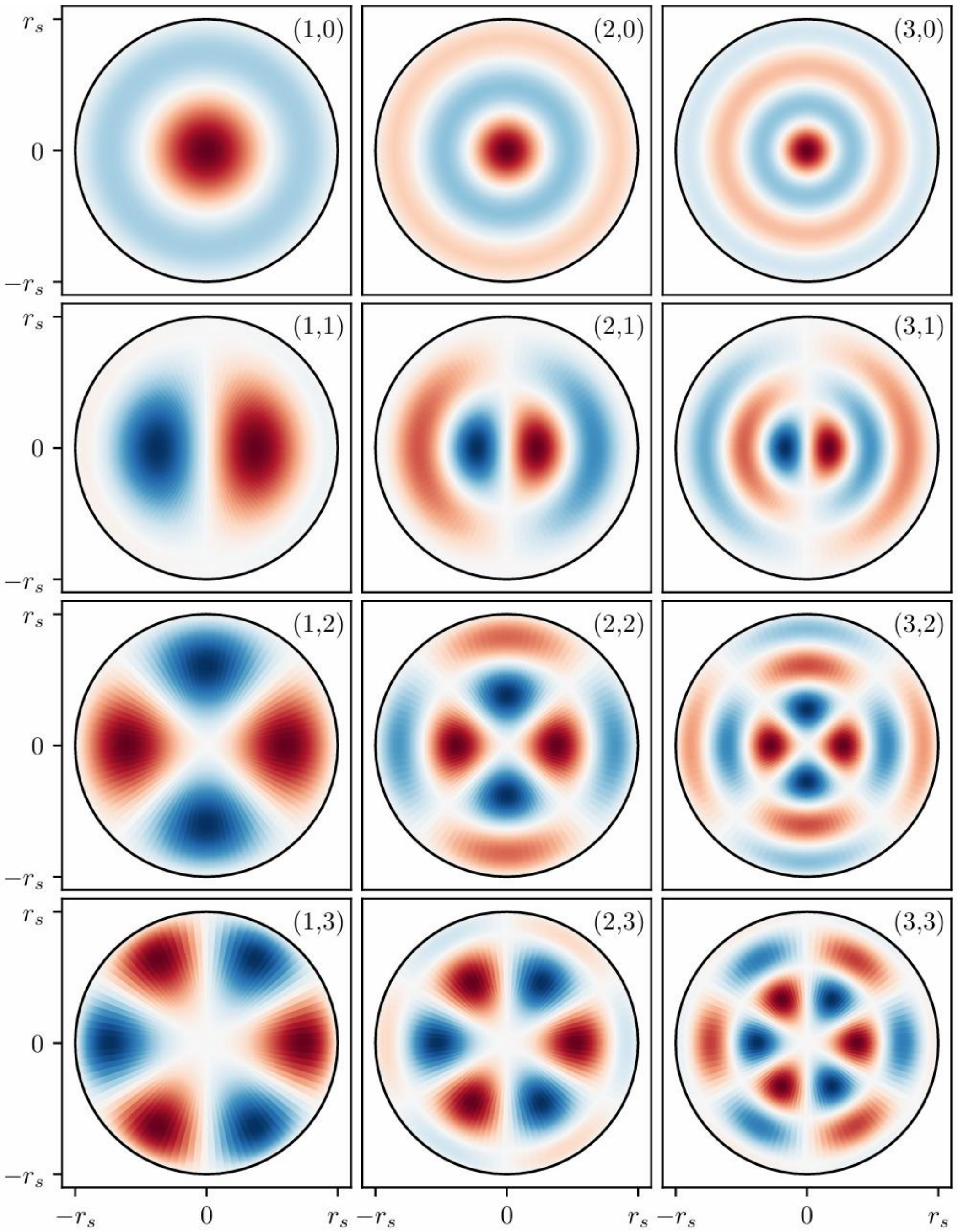
Shield size	
d	r_s
100 nm	1 cm

$\omega_{(1,0)} \approx 11.0\ \mathrm{s}^{-1}$

$\omega_{(7,6)} \approx 1018\ \mathrm{s}^{-1}$

Vibrational energy $\hbar\omega \ll k_B T$

→ Thousands of modes are all occupied simultaneously!



Entanglement dynamics with thermal shield

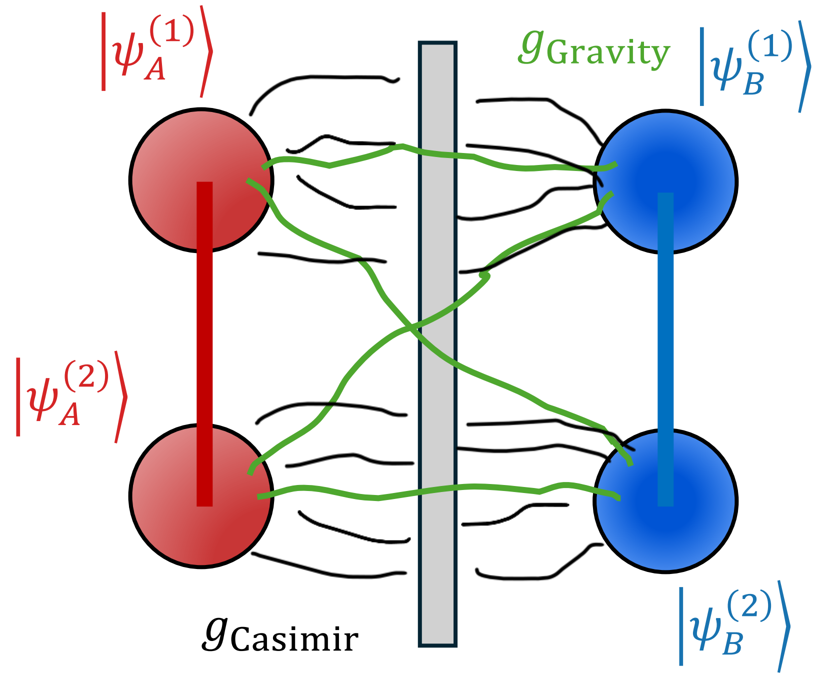
vibrational amplitude $= \sqrt{\hbar / 2m\omega}\left(a + a^{\dagger}\right)$

$$H = \sum_{\substack{m \in \{(k,l)\} \\ k\geq 1, l\geq 0}} \Big\{ \hbar\omega_m \left(a_m^{\dagger} a_m + \frac{1}{2} \right) + g^{(1)}_{A,m,\mathrm{Casimir}} \left(a_m + a_m^{\dagger} \right) \left| \psi_A^{(1)} \right\rangle \left\langle \psi_A^{(1)} \right| + g^{(2)}_{A,m,\mathrm{Casimir}} \left(a_m + a_m^{\dagger} \right) \left| \psi_A^{(2)} \right\rangle \left\langle \psi_A^{(2)} \right|$$
$$+ g^{(1)}_{B,m,\mathrm{Casimir}} \left(a_m + a_m^{\dagger} \right) \left| \psi_B^{(1)} \right\rangle \left\langle \psi_B^{(1)} \right| + g^{(2)}_{B,m,\mathrm{Casimir}} \left(a_m + a_m^{\dagger} \right) \left| \psi_B^{(2)} \right\rangle \left\langle \psi_B^{(2)} \right| \Big\}$$
$$+ g^{(1,1)}_{\mathrm{Gravity}} \left| \psi_A^{(1)} \psi_B^{(1)} \right\rangle \left\langle \psi_A^{(1)} \psi_B^{(1)} \right| + g^{(1,2)}_{\mathrm{Gravity}} \left| \psi_A^{(1)} \psi_B^{(2)} \right\rangle \left\langle \psi_A^{(1)} \psi_B^{(2)} \right|$$
$$+ g^{(2,1)}_{\mathrm{Gravity}} \left| \psi_A^{(2)} \psi_B^{(1)} \right\rangle \left\langle \psi_A^{(2)} \psi_B^{(1)} \right| + g^{(2,2)}_{\mathrm{Gravity}} \left| \psi_A^{(2)} \psi_B^{(2)} \right\rangle \left\langle \psi_A^{(2)} \psi_B^{(2)} \right|$$

Independent of the thermal shield

$$\left|\psi_{\mathrm{particle}}\right\rangle = \frac{1}{2}\left(\left|\psi_A^{(1)}\right\rangle + \left|\psi_A^{(2)}\right\rangle \right) \otimes \left(\left|\psi_B^{(1)}\right\rangle + \left|\psi_B^{(2)}\right\rangle \right)$$

Initial state: $\rho_0 = \rho_{\mathrm{particles}} \otimes \left(\bigotimes_{m \in \{(k,l)\}} \rho_{\mathrm{th},m} \right)$



Entanglement dynamics with thermal shield

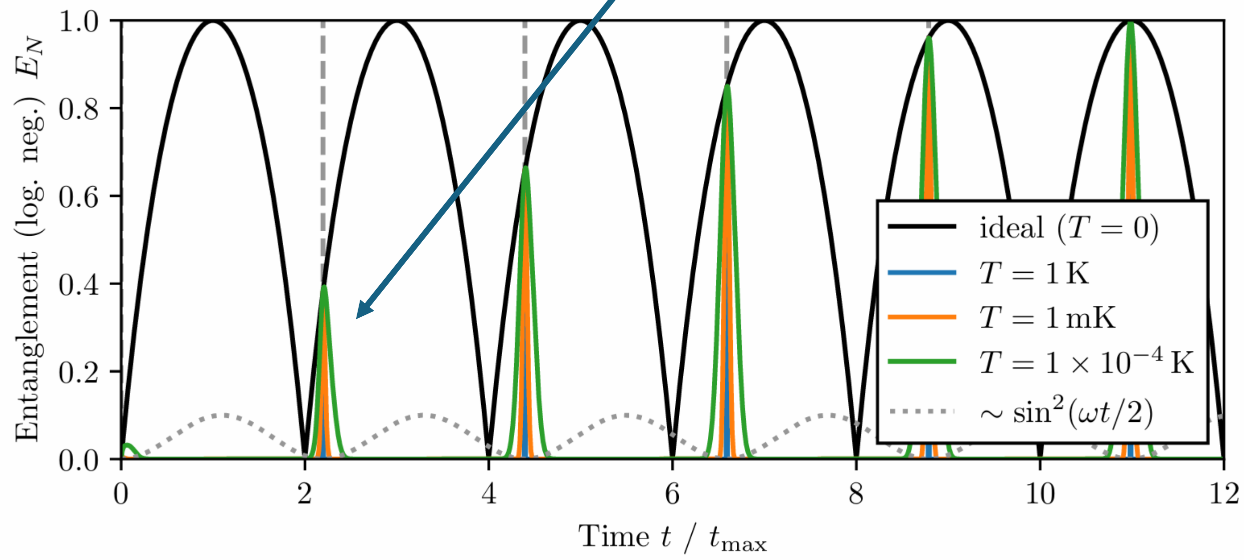
Decoherence due to interactions with the shield:

$$\gamma \sim \sum_m \frac{1}{\hbar^2 \omega_m^2} |g_{\text{Casimir}}|^2 \sin^2\left(\frac{\omega_m}{2} t\right) \left[\bar{n}_m + \frac{1}{2}\right]$$

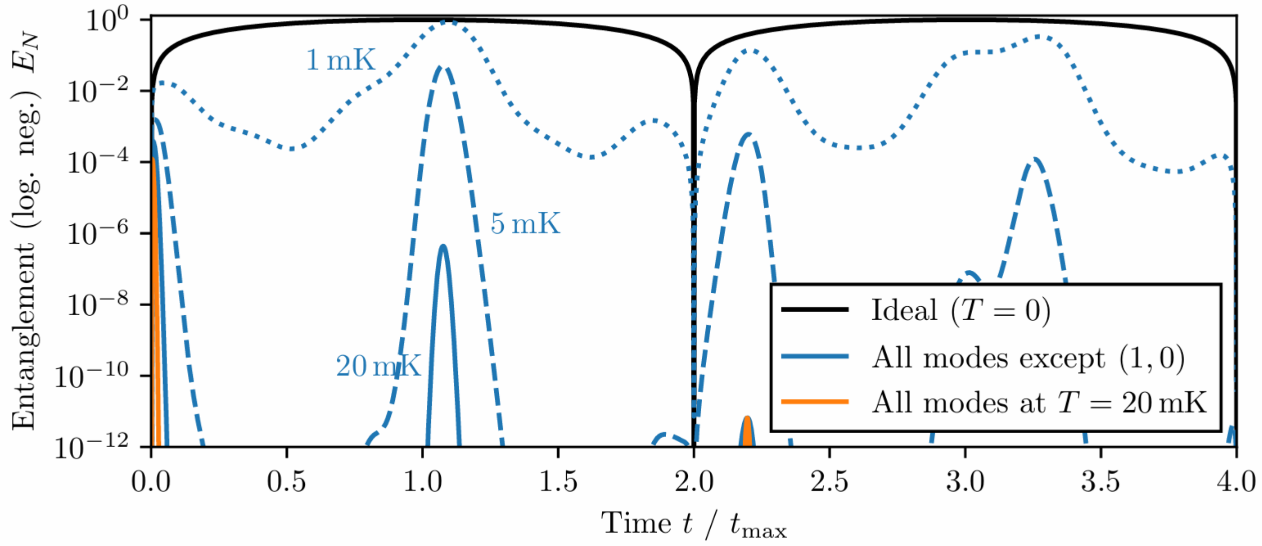
Effect of all infinitely many other modes:
$\sim 1.7 \times 10^{-11}\ \%$

$t = 2\pi/\omega_{(1,0)}$

For the (1,0) mode:



For the first 50 modes:

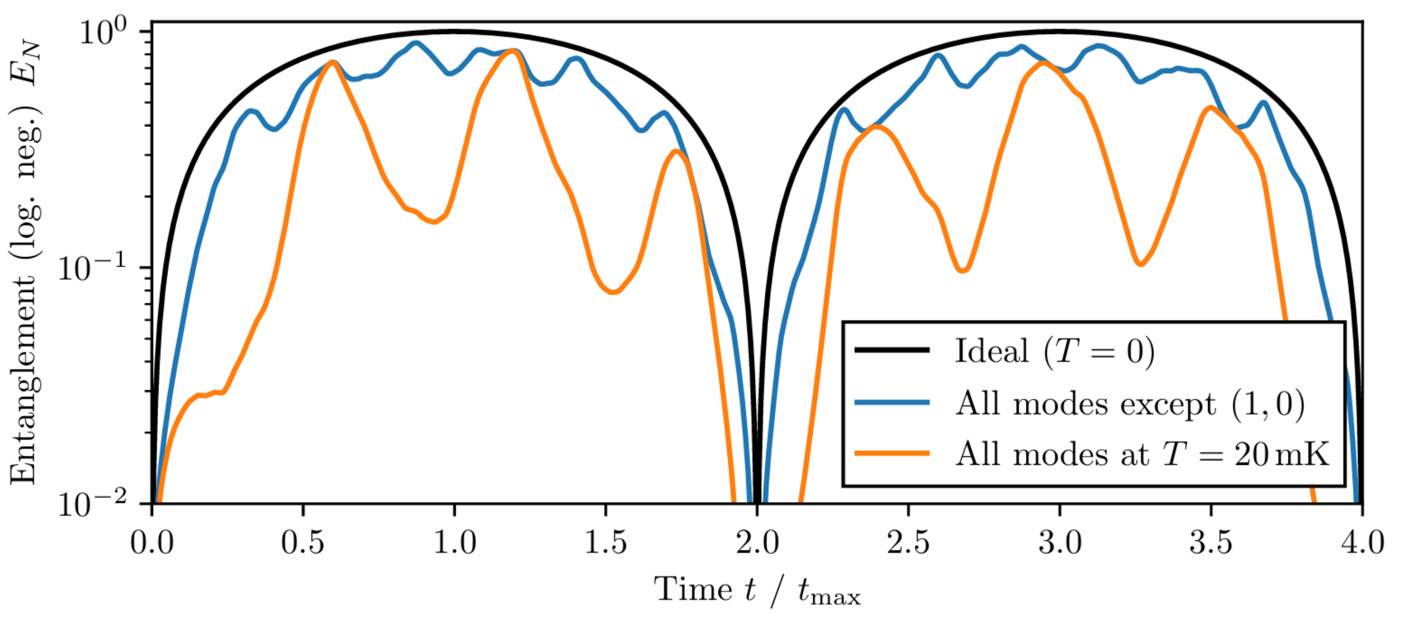


Effect of the shield radius

$$\gamma \sim \sum_m \frac{1}{\hbar^2 \omega_m^2} |g_{Casimir}|^2 \sin^2\left(\frac{\omega_m}{2} t\right)\left[\bar{n}_m + \frac{1}{2}\right]$$

since $\omega \propto \frac{1}{r_s^2}$ and $\gamma \propto \frac{1}{\omega^4}$: strong dependence on the shields radius

For the first 50 modes at $r_s = 5$ mm:



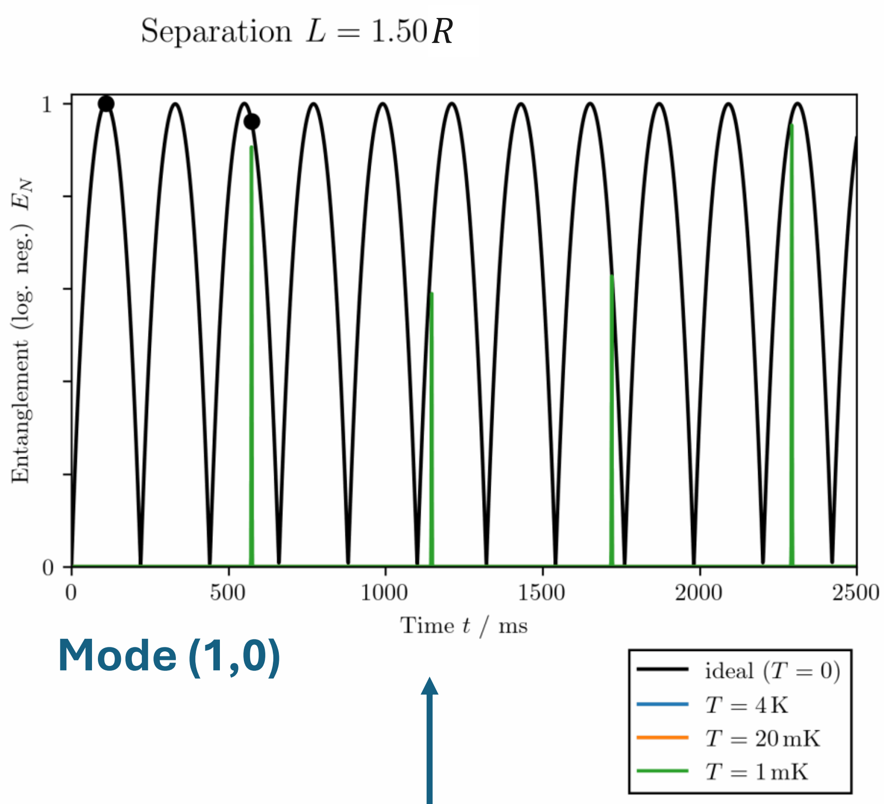
Advantages of **small shields** (and thus uncharged particles):

- Fast vibrations result in smaller amplitudes
- Fast vibrations average out over time → no effective vibrations

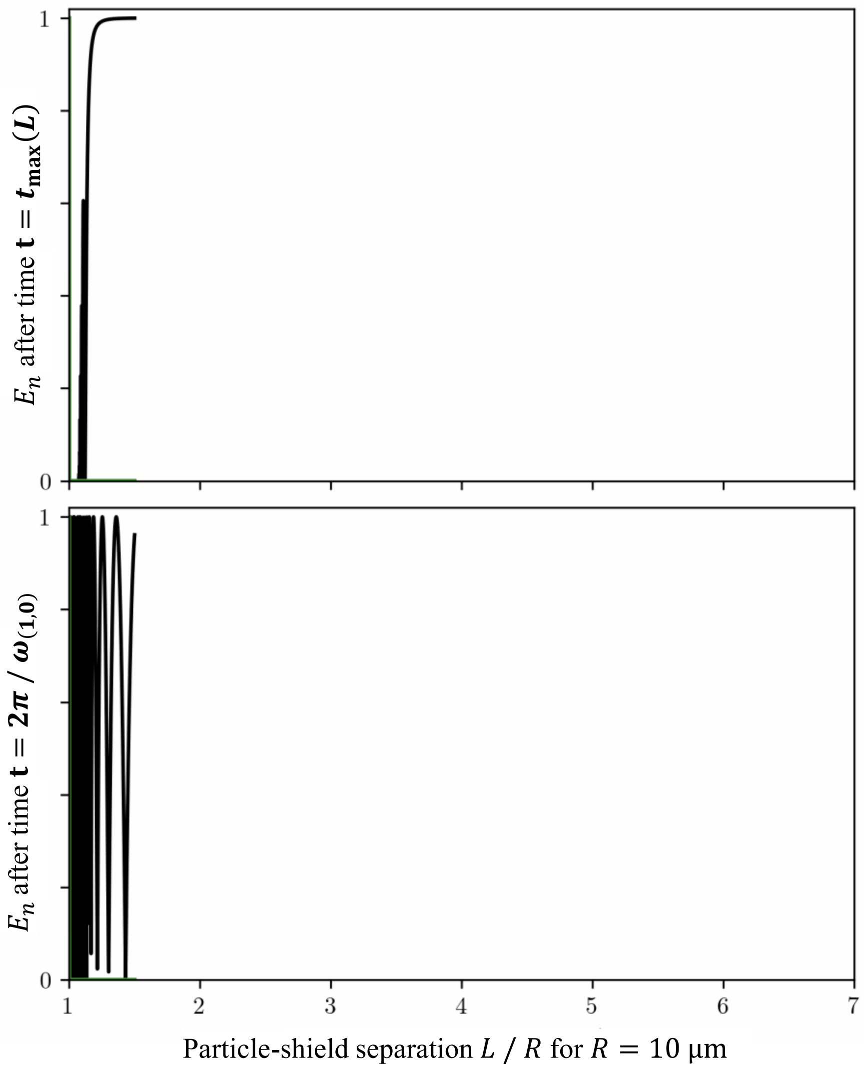
Entanglement for larger separations L

For the first 50 modes:

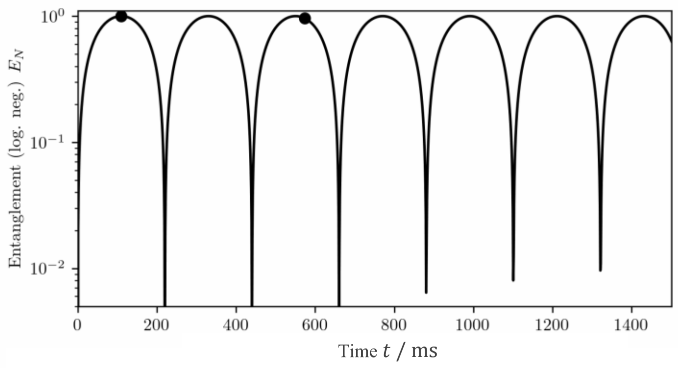
$t_{\max} \propto L^3$



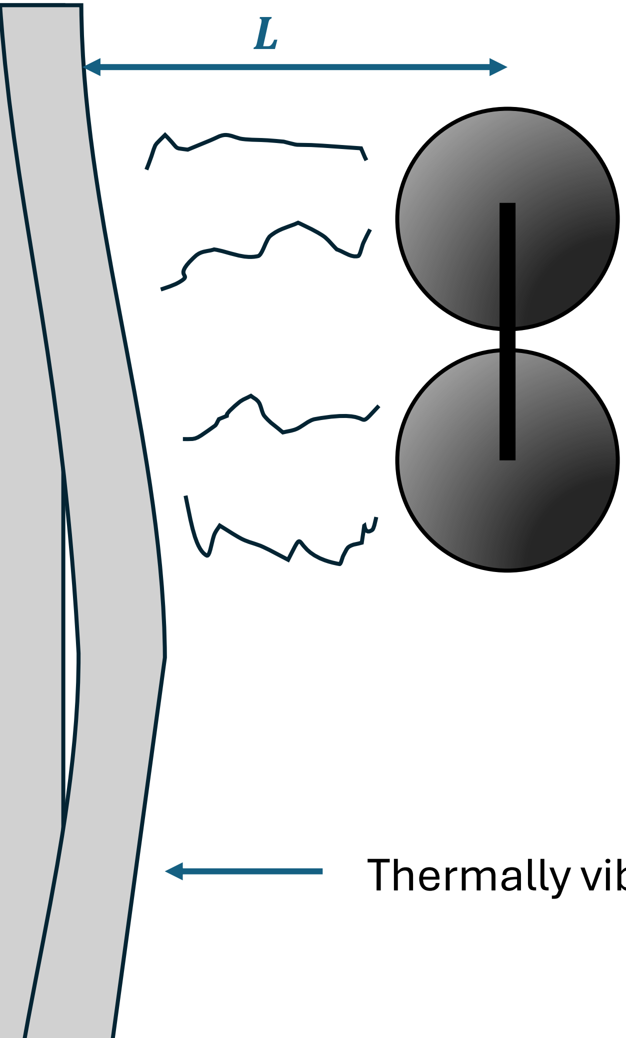
Mode (1,0)



First 50 modes:



Outlook – A new and precise method for measuring Casimir interactions



Levitated atoms/molecules, nano-particles, …

Thermally vibrating shield

- Measure dephasing of a single particle in spatial superposition
- Coupling strength dependent on the Casimir interaction
- Casimir interactions are already being studied with levitated particles [Z. Xu, arXiv:2403.06051, (2024)]

Differences to current methods:

- Easy change of materials (conductor, dielectric, …)
- Easily adjustable separation L
- Should be measurable for atoms or small molecules with current technologies

Thanks 😊