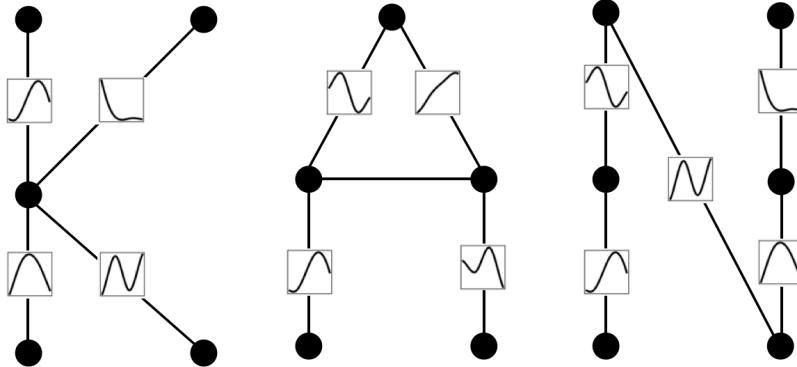


Kolmogorov-Arnold Networks



Mathematical

Accurate

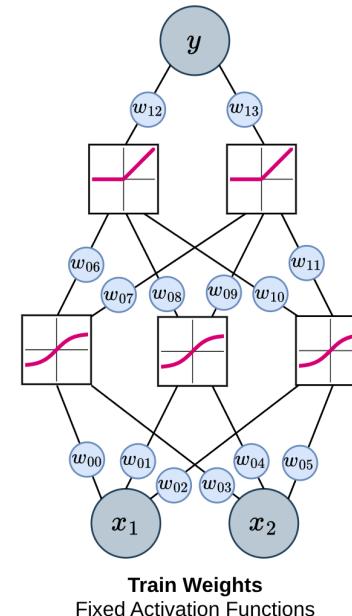
Interpretable

Jan Duchscherer, Felix Schlacht

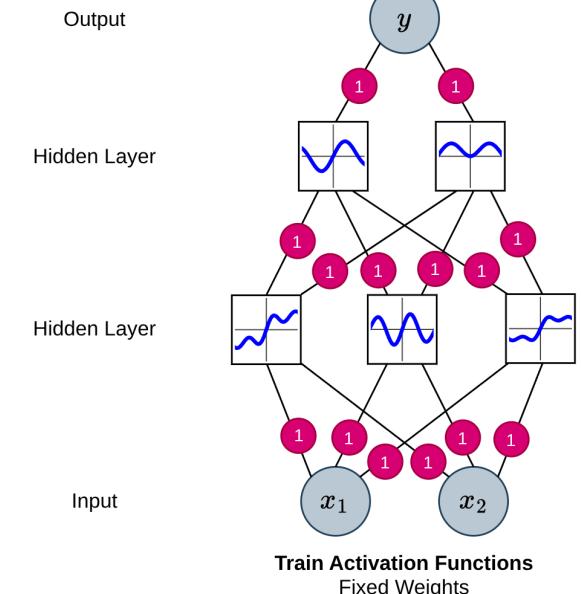
Advanced Deep Learning Deep Dive

Introduction to Kolmogorov-Arnold Networks

MLP (Multi-Layer Perceptron)



KAN (Kolmogorov-Arnold Network)



MLP vs KAN overview. [Liu+25]

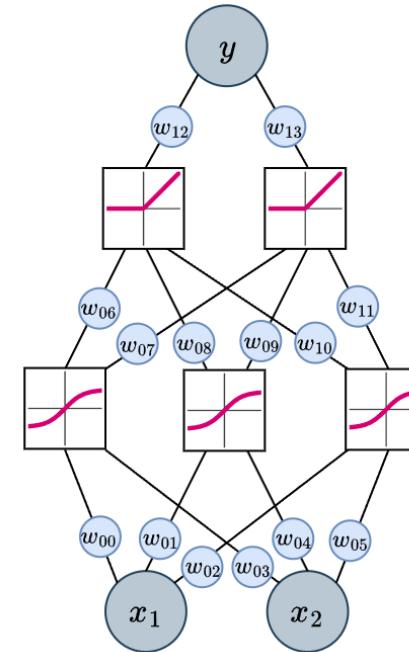
MLPs: strengths and structural limits

MLPs are universal approximators — but not structure-aware.

Structural limitations

- **Interpretability:** Knowledge is contained in distributed parametric pattern; individual components are **not** meaningful by design.
- **Entangled degrees of freedom:** the same parameters encode both structure and precision.

MLP (Multi-Layer Perceptron)



A small MLP (structure and precision are intertwined)

- 1 Can we separate **what** is computed from **how precisely** it is computed?
- 2 Can we design a network that is equally expressive but more interpretable?

Features of KANs

Where KANs work well

- Built for **smooth + compositional** relationships
- Matches structure common in physics / biology
- Allocate capacity where needed (local splines)

Efficient in science tasks

- Often similar error with fewer parameters than MLP baselines
- Function fitting and differential-equation solving (common in science)
- Precision scales independently from structure

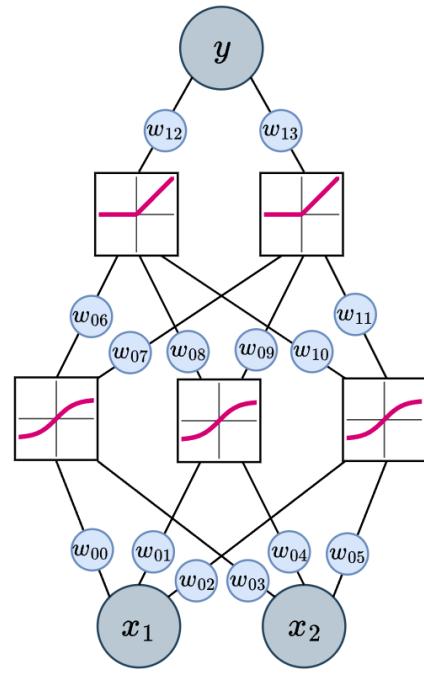
Interpretability

- Individual components / connections are meaningful.
- Can be inspected, pruned, simplified into compact relations
- Extract a symbolic equation from the trained model

[Liu+25]

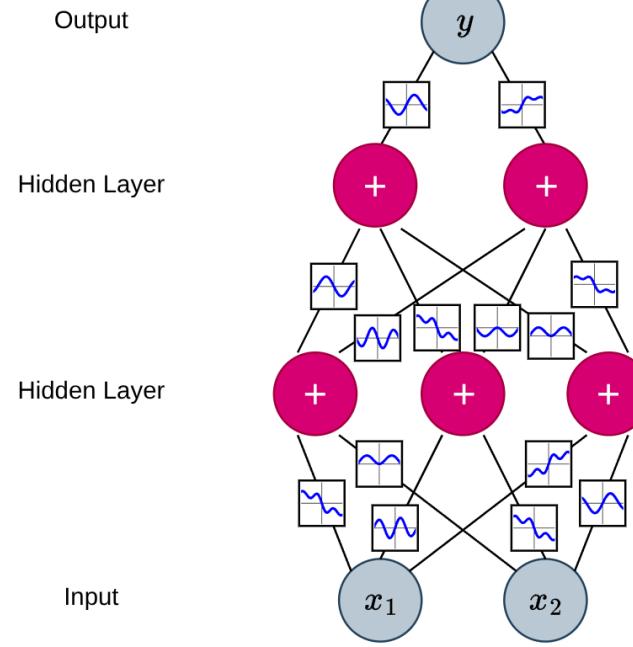
MLP vs KAN: Visual comparison

MLP (Multi-Layer Perceptron)



Train Weights
Fixed Activation Functions

KAN (Kolmogorov-Arnold Network)



Train Activation Functions
Add functions

MLP vs. KAN visualization (learnable parts in blue, fixed in pink) [Ser24]

From MLPs to KANs

MLP

- Scalar weights $w_{j,i}$ on edges.
- Fixed activation functions σ on nodes.

$$x_{l+1} = \sigma(\mathbf{W}_l x_l + \mathbf{b}_l)$$

Learnable weights + fixed activations

KAN

- Each edge is a **learnable 1D function** $\varphi_{j,i}(x)$
- Nodes add input → interpret learned functions.

$$x_{l+1,j} = \sum_{i=1}^{n_l} \varphi_{l,j,i}(x_{l,i})$$

Nodes add, function on edges are learned

Same dense bipartite wiring.
Activations become learnable & move from nodes to edges.

[Liu+25]

The Kolmogorov-Arnold theorem

- Any continuous $f(x_1, \dots, x_n)$ on $[0, 1]^n$ can be represented using **1D functions + addition**.
- The only **true** multivariate operation is **sum**; everything else can be composed from univariate transforms + additions.
- Splines are well-suited to approximate 1D functions efficiently.

[Liu+25]

$$f(\mathbf{x}) = f(x_1, \dots, x_n) = \sum_{q=1}^{2n+1} \Phi_q \left(\sum_{p=1}^n \varphi_{q,p}(x_p) \right)$$

Kolmogorov-Arnold Theorem [Liu+25]

$$xy = -\frac{x^2}{2} - \frac{y^2}{2} + \frac{(x+y)^2}{2}$$

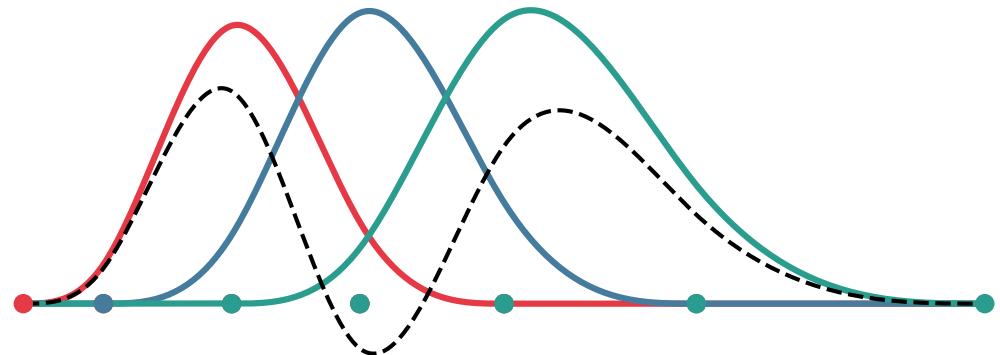
KAT representation example for xy
 $(\frac{(x+y)^2}{2})$ requires 1D summary of $t = x + y$

Splines

- Smooth, piecewise-polynomial & locally bounded functions of one variable x .
- Approximate 1D functions well with few parameters.
- Controlled by knots $B_m(x)$ + coefficients c_m .
- Each Spline is a combination of many cubic Basis Splines (B-Splines)

$$\varphi_{j,i}(x) = \sum_{m=0}^{G+k-1} c_m B_m(x)$$

[Liu+25, Ser24]



Edge function (dashed): linear combination of
 $G = 3$ B-Splines of degree $k = 3$

In KANs, each edge learns its own 1D spline
 $\varphi_{i,j}(x)$. [Liu+25]

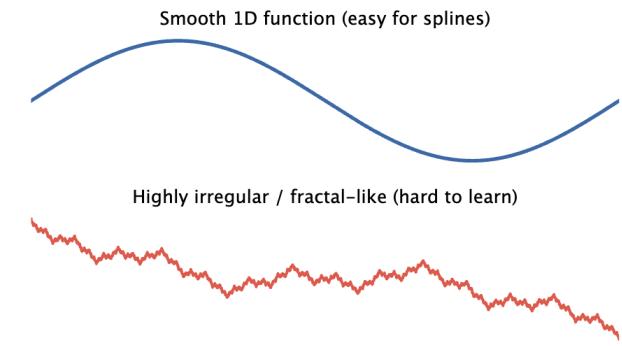
Can KAT represent any high-dimensional function?

Classical KAT is elegant, but resulting 1D inner functions can be non-smooth or fractal

- Hard to learn in the rigid depth-2 KART form in practice
- Earlier research described it as “theoretically sound but practically useless” [GP89, PBL19]

Mitigation

- Don’t stick to the rigid depth-2, width $(2n + 1)$ form
 - deeper/wider KANs for smoother representations
- In real tasks we often expect smooth + compositionally sparse structure
 - most typical cases allows smooth KA-like representations



Smooth vs Fractal Curves
visualization

Curse of Dimensionality

More input dimensions → combinations explode → exponential growth of parameters

MLPs

- Universal Approximation Theorem: 2-layer MLP can approximate any continuous function
- But no efficiency guarantee
 - width can grow exponentially with dimension (CoD in practice)

KANs

- Stack layers to learn compositional structure (feature learning)
- Replace weights with learnable 1D functions
- No high-D spline grid:
 - many 1D splines + sums, can beat CoD when the target is smooth + compositional

KAN Layer

Layer = matrix of 1D functions

- Each edge learns a univariate mapping

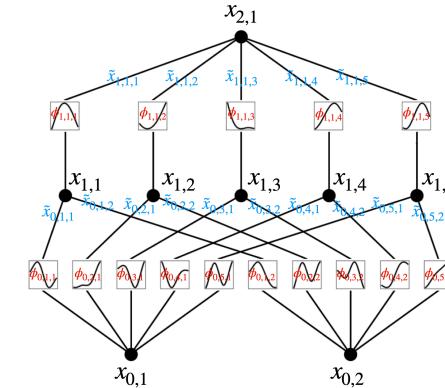
$$\varphi_{l,j,i} : \mathbb{R} \rightarrow \mathbb{R}.$$

- Activation of the j -th neuron in layer $l + 1$:

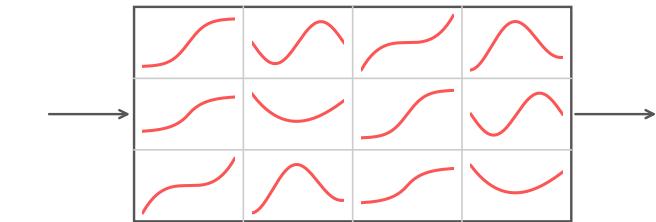
$$x_{l+1,j} = \sum_{i=1}^{n_l} \varphi_{l,j,i}(x_{l,i})$$

- A single KAN layer can be written in **matrix form**:

$$x_{l+1} = \underbrace{\begin{pmatrix} \varphi_{l,1,1}(\cdot) & \varphi_{l,1,2}(\cdot) & \dots & \varphi_{l,1,n_l}(\cdot) \\ \varphi_{l,2,1}(\cdot) & \varphi_{l,2,2}(\cdot) & \dots & \varphi_{l,2,n_l}(\cdot) \\ \vdots & \vdots & & \vdots \\ \varphi_{l,n_{l+1},1}(\cdot) & \varphi_{l,n_{l+1},2}(\cdot) & \dots & \varphi_{l,n_{l+1},n_l}(\cdot) \end{pmatrix}}_{\Phi_l \in (\mathbb{R} \rightarrow \mathbb{R})^{n_{l+1} \times n_l}} x_l$$



B-spline parametrization and grid refinement. [Liu+25]



“KAN layer in matrix form”

General KAN with L layers

$$\text{KAN}(\mathbf{x}) = (\Phi_{L-1} \circ \Phi_{L-2} \circ \dots \circ \Phi_0)(\mathbf{x})$$

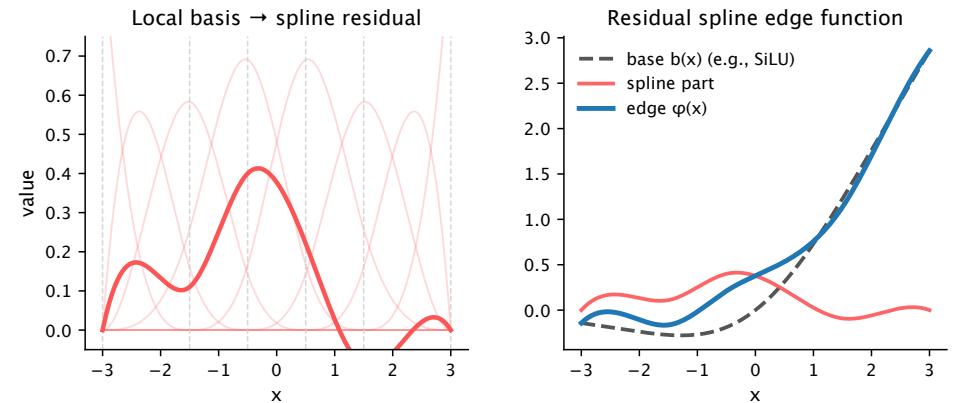
KART \rightsquigarrow KAN of shape $[n \rightarrow 2n + 1 \rightarrow 1]$

Edge Functions - Residual Splines

Edge - Residual Spline

$$\varphi(x) = w_b b(x) + w_s \sum_i c_i B_i(x)$$

- Trainable (per edge, backprop): c_i , w_b , w_s
- $B_i(x)$: B-spline basis functions, fixed given the current grid.
- $b(x)$: fixed **global** non-linearity (i.e. SiLU).
 - 1 Ensure φ is well-defined on \mathbb{R}
 - 2 Residual path eases optimization – learn deviation from $b(x)$ rather than full function



Residual spline edge function: local basis → spline → $\varphi(x)$.

Why B-Splines?

- **local, translation-invariant basis**
 - local capacity allocation
 - continual learning ↑, catastrophic forgetting ↓
- Allows for other orthogonal bases (Fourier, Chebyshev).
- Locality ↔ global efficiency.

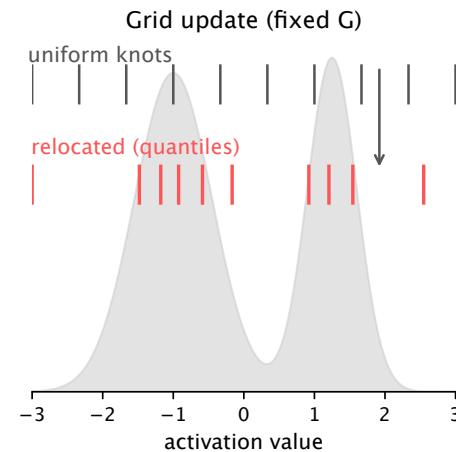
Grid Update vs Grid Extension

Two spline adaptation mechanisms

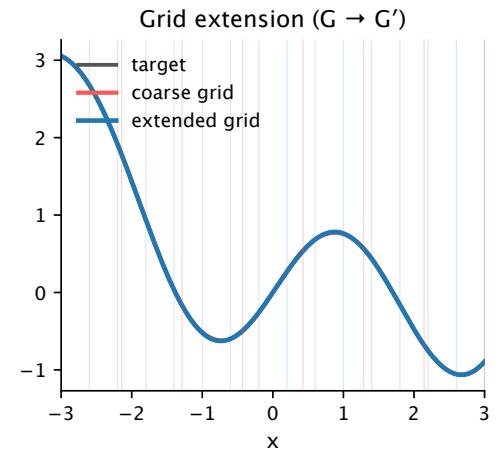
- **Grid update:** relocate existing knots to follow activations (fixed G).
- **Grid extension:** add knots to increase resolution ($G \rightarrow G'$).
- Update keeps the same number of knots; extension increases **internal DoF**.

Update answers “**where do we need resolution?**”

Extension answers “**how much resolution do we need?**”



Conceptual comparison: relocation (left) vs refinement (right).



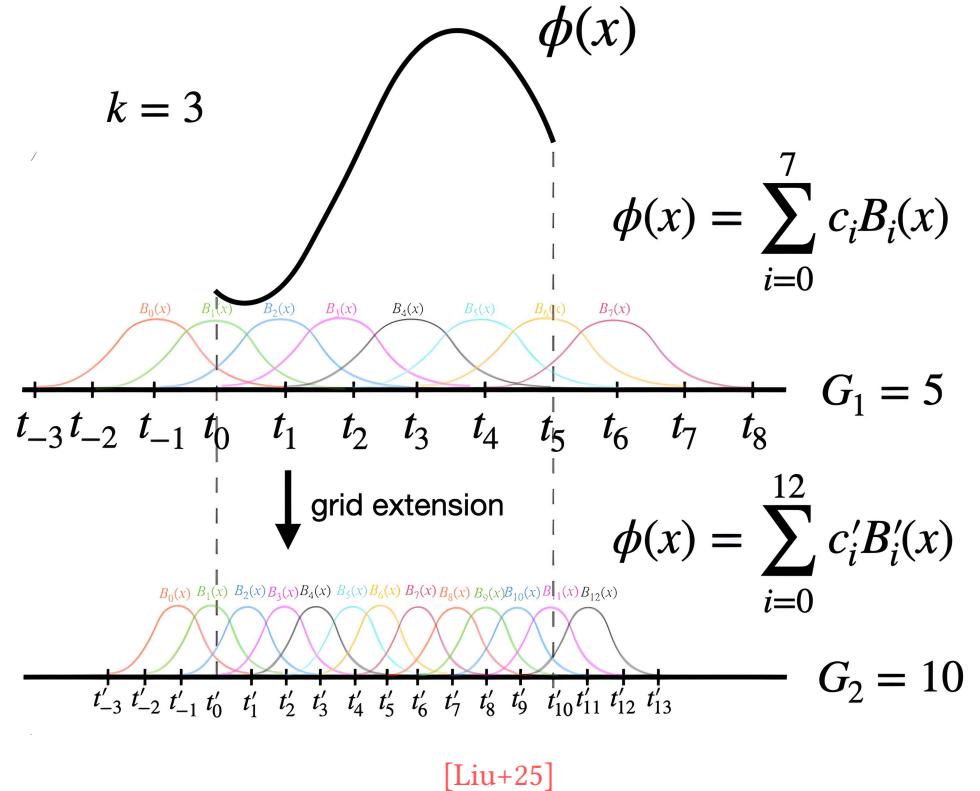
Grid Update - Knot Relocation

Keep knots where the data lives

- Activations are **non-stationary** during training, while splines live on a **bounded** knot grid.
- Goal: concentrate spline resolution where data is dense (activity-aware knot density).
- Grid update:** relocate knots (typically by quantiles); coefficients stay trainable by backprop.

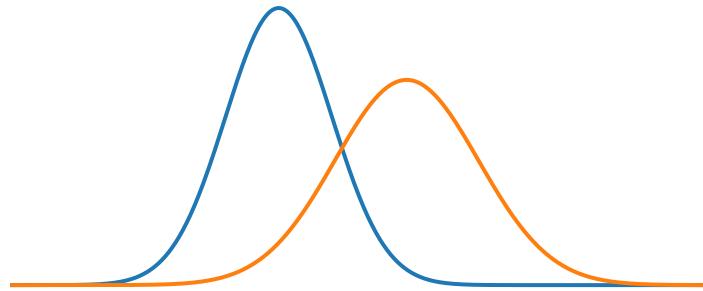
Example (quantile knots):

$$t_r := Q_{\frac{r}{G}}(\{x^s\}_{s=1..N}) \quad (r = 0..G)$$



Grid Update - Knot Relocation

Uniform: $t_r = t_{\min} + \left(\frac{r}{G}\right)(t_{\max} - t_{\min})$; variants learn t_r via GD.



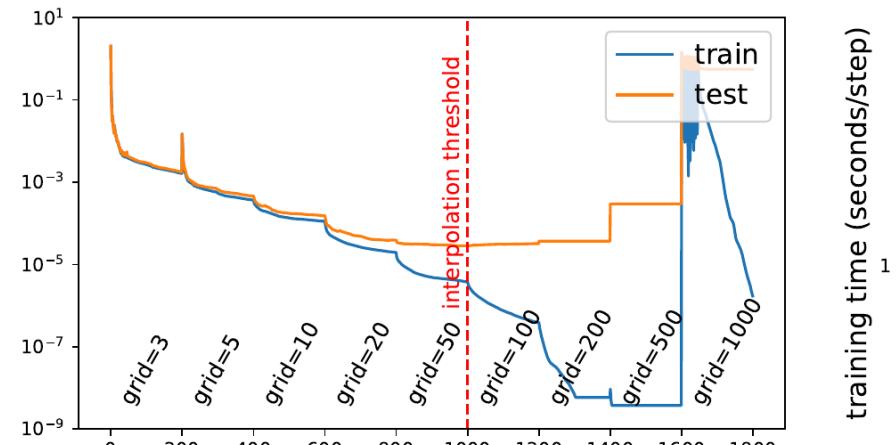
Fixed G : relocate knots, then re-express the same edge function on the new grid (function-preserving reprojection).

$$\{c'\} := \underset{\{c'\}}{\operatorname{argmin}}, E_{x \sim p(x)}, \left(\sum_j c'_j B'_{j(x)} - \sum_i c_i B_{i(x)} \right)^2$$

Grid Extension: Accuracy Scaling

Key idea

- **Grid extension:** add knots ($G \rightarrow G'$) \rightarrow higher spline resolution.
- Curriculum-style schedule:
 - 1 Start with coarse spatial resolution – fewer knots, global structure, simpler optimization.
 - 2 Gradually increase resolution, initialize finer splines via least-squares fit to coarse spline.
- Monitor validation error to stop grid extension once improvement ceases.
- RMSE $\propto G^{-4}$ (on test split)



Staircase-like loss drops after each grid refinement. [Liu+25]

$$\{c'_j\} = \operatorname*{argmin}_{\{c'_j\}} \mathbb{E}_{x \sim p(x)} \left(\sum_{j=0}^{G_2+k-1} c'_j B'_j(x) - \sum_{i=0}^{G_1+k-1} c_i B_i(x) \right)^2$$

External vs Internal DoF: Structure vs Precision

Two kinds of degrees of freedom (DoF)

- **External DoF:** width/depth/connectivity → **which interactions** among variables.
- **Internal DoF:** spline grid G + coefficients c_i → **how precisely** each interaction is represented.
- **Key shift:** KANs separate structure from precision, so each can scale independently.

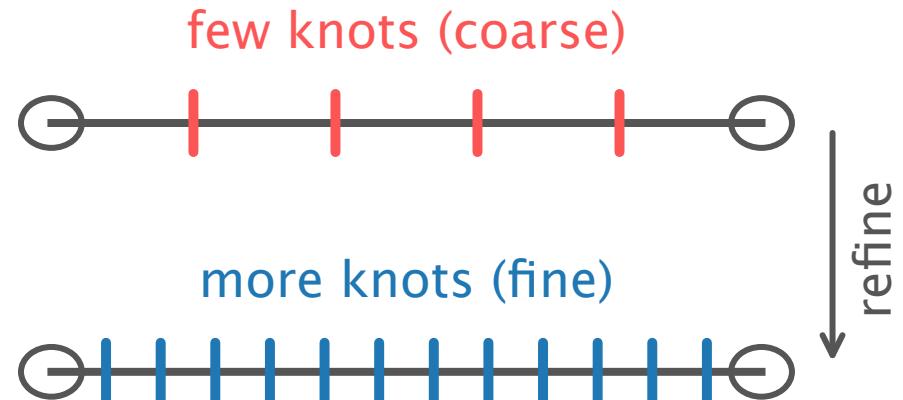
External vs Internal DoF: Structure vs Precision

MLPs

- Structure + precision are **entangled**.
- More accuracy → scale width/depth (external DoF).
- No dedicated control over internal DoF.
- $\mathcal{O}(N^2 L)$

KANs

- External DoF \leftrightarrow **compositional structure**.
- Internal DoF \leftrightarrow **local precision** on edges.
- Grid extension increases internal DoF while keeping structure fixed.
- $\mathcal{O}(N^2 L(G + k))$

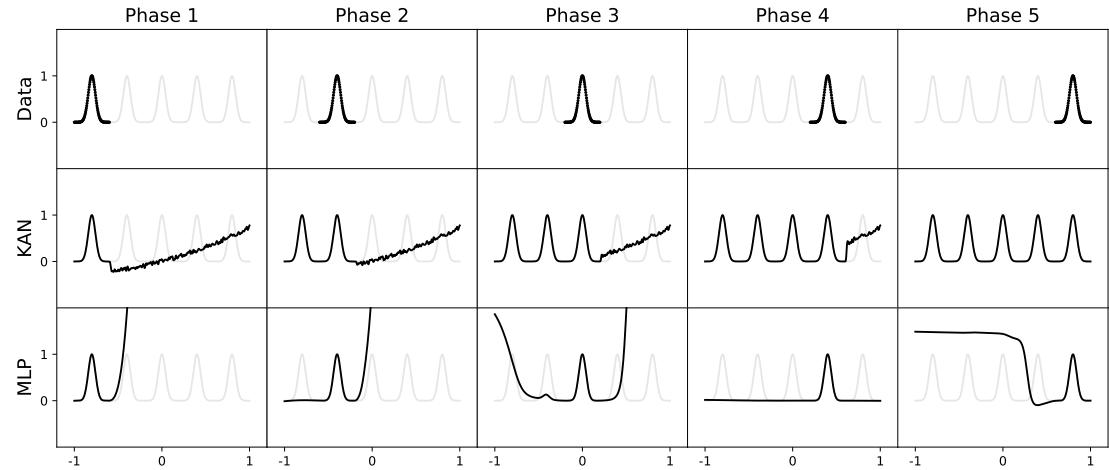


Conceptual: change the graph (external) vs refine each edge function (internal).

Continual Learning & Locality

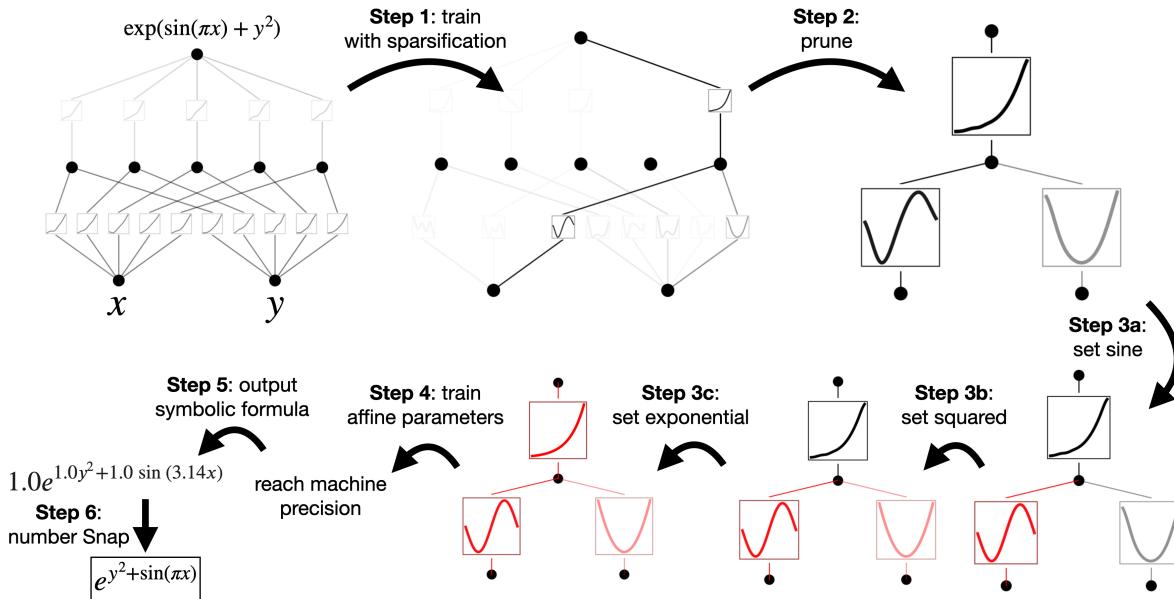
Why KANs forget less

- **Local plasticity:** Each spline basis is local:
 → only a few basis functions are non-zero per sample
 → gradients touch only a few nearby coefficients.
 → **updates are localized.**
- **Contrast (MLPs):** Any local update has global effects:
 → learning a new task may overwrite previous knowledge
 → **catastrophic forgetting.**
- Generalization to higher dimensions is unclear [YYW24].



Sequential learning (toy 1D): KAN preserves earlier regions; MLP forgets. [Liu+25]

Reductionist Symbolification



Symbolic regression workflow with KANs. [Liu+25]

KANs are **connectionist** throughout training & **symbolic** after **reductionist sparsification** & compression into symbolic forms.

Structure → equation

- 1 Sparsify:** L1 on activations + entropy regularization
→ few dominant edges.
- 2 Prune** low-importance nodes
→ minimal graph
→ external DoF ↓.
- 3 Symbolify:** replace splines with analytic forms + affine wrapper
$$y \approx cf(ax + b) + d$$

→ internal DoF ↓.
- 4 Further training** of the (affine) parameters.

Summary: Takeaways & Limitations

Takeaways

- KANs are most suited for **structured, compositional, small-scale scientific tasks.**
- **Decoupled internal and external DoF** improve fine-tuning, **continual learning** & scaling.
- **Symbolic interpretability** and discovery of symbolic formulas (**white-box ML**).

Limitations

- Baseline KANs are up to 10x **slower** (batching, spline updates, HW); faster follow-ups exist [Li24].
- Likely **not an MLP replacement** in large-scale tasks [YYW24].
- Sensitivity to **additional hyperparameters** (grid size, update schedule).

Trade-off: interpretability & controllable precision vs training efficiency.

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- [YYW24] R. Yu, W. Yu, and X. Wang, “KAN or MLP: A Fairer Comparison.” Accessed: Jan. 12, 2026. [Online]. Available: <http://arxiv.org/abs/2407.16674>
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