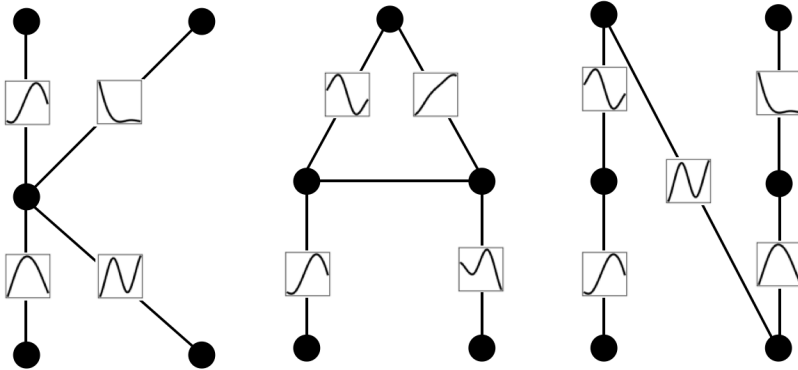# Kolmogorov-Arnold Networks

Mathematical

Accurate

Interpretable
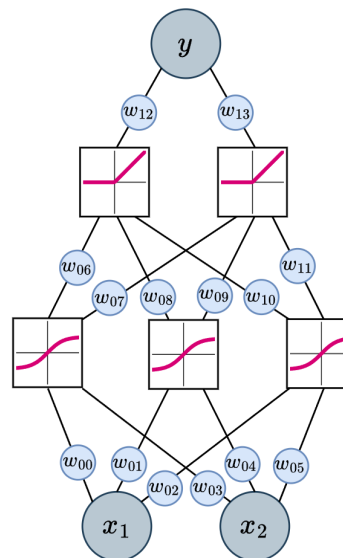
Jan Duchscherer, Felix Schladt

Advanced Deep Learning Deep Dive

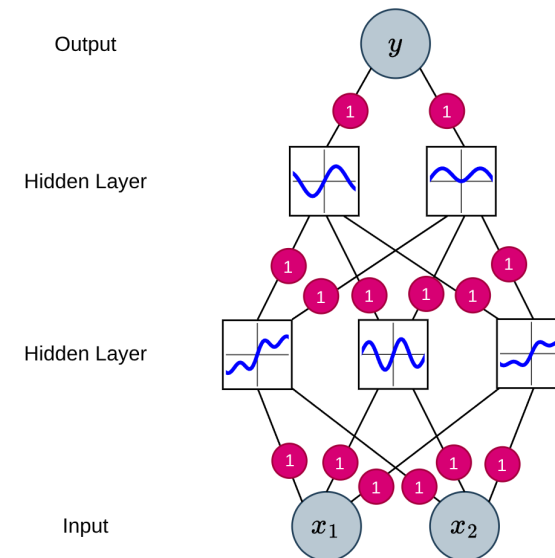> Munich University of Applied Sciences

Introduction to Kolmogorov-Arnold Networks

MLP (Multi-Layer Perceptron)

y

Output

w_{12} w_{13}

Hidden Layer

w_{06} w_{11}

w_{07} w_{08} w_{09} w_{10}

Hidden Layer

w_{00} w_{01} w_{04} w_{05}

w_{02} w_{03}

x_1 x_2

Input

Train Weights
Fixed Activation Functions

KAN (Kolmorogov-Arnold Network)

y

1   1

1   1

1   1   1   1

1   1

1   1   1   1

1   1

x_1 x_2

Train Activation Functions
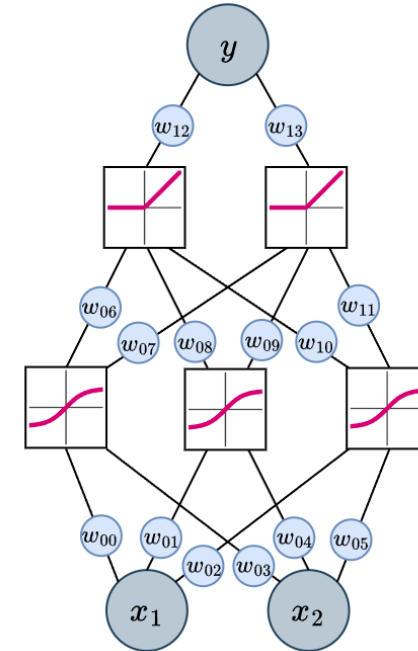Fixed Weights

MLP vs KAN overview. [Liu+25]

MLPs: strengths and structural limits

MLPs are universal approximators — but not structure-aware.

Structural limitations

- **Interpretability**: Knowledge is contained in distributed parametric pattern; individual components are not meaningful by design.

- **Entangled degrees of freedom**: the same parameters encode both structure and precision.

MLP (Multi-Layer Perceptron)

A small MLP (structure and precision are intertwined)

MLPs: strengths and structural limits

1. Can we separate **what** is computed from **how precisely** it is computed?
2. Can we design a network that is equally expressive but more interpretable?

Features of KANs

Where KANs work well

- Built for smooth + compositional relationships

- Matches structure common in physics / biology

- Allocate capacity where needed (local splines)

[Liu+25]

Efficient in science tasks

- Often similar error with fewer parameters than MLP baselines

- Function fitting and differential-equation solving (common in science)

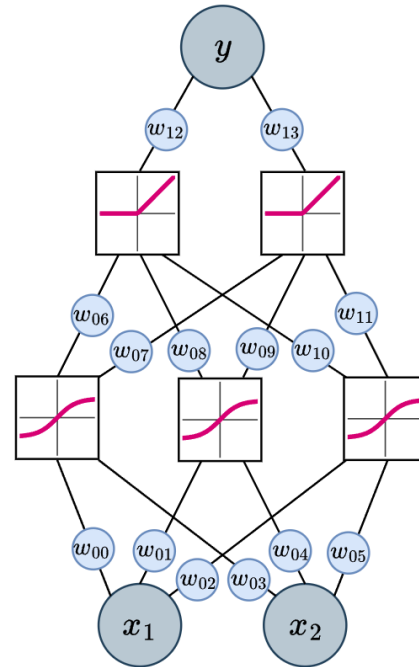- Precision scales independently from structure

Interpretability

- Individual components / connections are meaningful.

- Can be inspected, pruned, simplified into compact relations
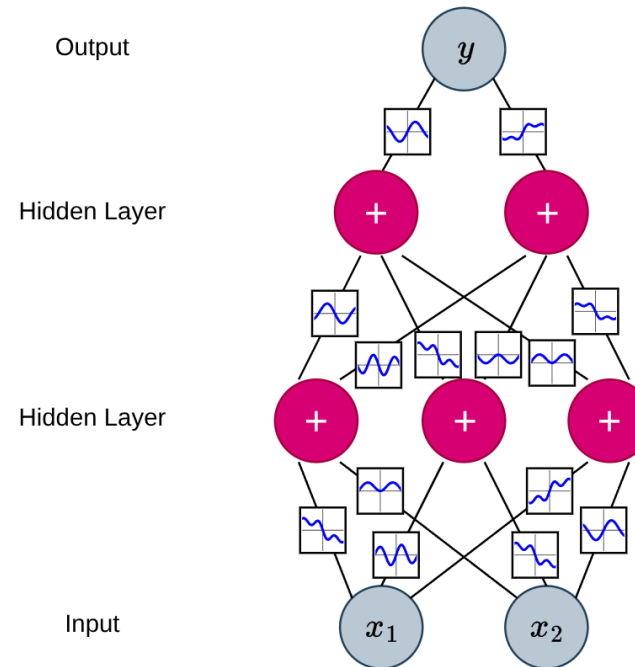
- Extract a symbolic equation from the trained model

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MLP vs KAN: Visual comparison

MLP vs. KAN visualization (learnable parts in blue, fixed in pink) [Ser24]

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From MLPs to KANs

MLP

- Scalar weights $w_{j,i}$ on edges.

- Fixed activation functions σ on nodes.

$$x_{l+1} = \sigma(W_l x_l + b_l)$$

Learnable weights + fixed activations

KAN

- Each edge is a **learnable 1D function** $\varphi_{j,i}(x)$

- Nodes add input $\rightarrow$ interpret learned functions.

$$x_{l+1,j} = \sum_{i=1}^{n_l} \varphi_{l,j,i}(x_{l,i})$$

Nodes add, function on edges are learned

Same dense bipartite wiring.
Activations become learnable & move from nodes to edges.

[Liu+25]

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The Kolmogorov-Arnold theorem

- Any continuous $f(x_1, ..., x_n)$ on $[0,1]^n$ can be represented using **1D functions + addition**.

- The only true multivariate operation is **sum**; everything else can be composed from univariate transforms + additions.

- Splines are well-suited to approximate 1D functions efficiently.

[Liu+25]

$$f(\boldsymbol{x}) = f(x_1, ..., x_n) = \sum_{q=1}^{2n+1} \Phi_q \left(\sum_{p=1}^{n} \varphi_{q,p}(x_p) \right)$$

Kolmogorov-Arnold Theorem[Liu+25]

$$xy = -\frac{x^2}{2} - \frac{y^2}{2} + \frac{(x+y)^2}{2}$$

KAT representation example for xy
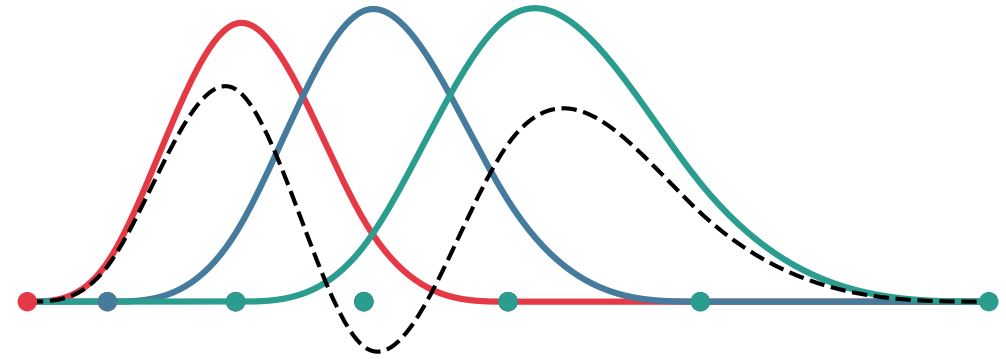($\frac{(x+y)^2}{2}$ requires 1D summary of $t = x + y$)

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Splines

- **Smooth**, **piecewise-polynomial** & **locally bounded** functions of one variable x.

- Approximate 1D functions well with few parameters.

- Controlled by knots $B_m(x)$ + coefficients c_m.

- Each Spline is a combination of many cubic Basis Splines (B-Splines)

$$\varphi_{j,i}(x) = \sum_{m=0}^{G+k-1} c_m B_m(x)$$

[Liu+25, Ser24]

Edge function (dashed): linear combination of $G = 3$ B-Splines of degree $k = 3$

In KANs, each edge learns its own 1D spline $\varphi_{i,j}(x)$. [Liu+25]

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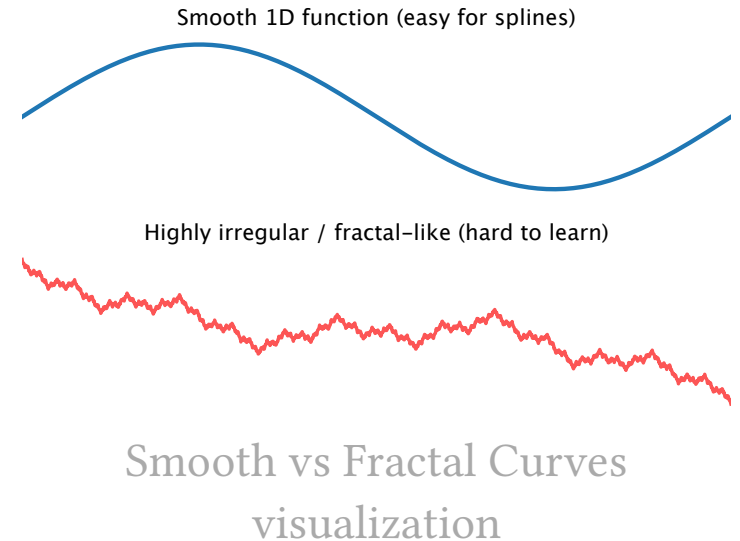
Can KAT represent any high-dimensional function?

Classical KAT is elegant, but resulting 1D inner functions can be non-smooth or fractal

- Hard to learn in the rigid depth-2 KART form in practice
- Earlier research described it as "theoretically sound but practically useless"[GP89, PBL19]

Mitigation

- Don't stick to the rigid depth-2, width $(2n + 1)$ form
  - deeper/wider KANs for smoother representations
- In real tasks we often expect smooth + compositionally sparse structure
  - most typical cases allows smooth KA-like representations

Smooth 1D function (easy for splines)

Highly irregular / fractal–like (hard to learn)

Smooth vs Fractal Curves visualization

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Curse of Dimensionality

More input dimensions → combinations explode → exponential growth of parameters

MLPs

- Universal Approximation Theorem: 2-layer MLP can approximate any continuous f
- But no efficiency guarantee
  - width can grow exponentially with dimension (CoD in practice)

KANs

- Stack layers to learn compositional structure (feature learning)
- Replace weights with learnable 1D functions
- No high-D spline grid:
  - many 1D splines + sums, can beat CoD when the target is smooth + compositional

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KAN Layer

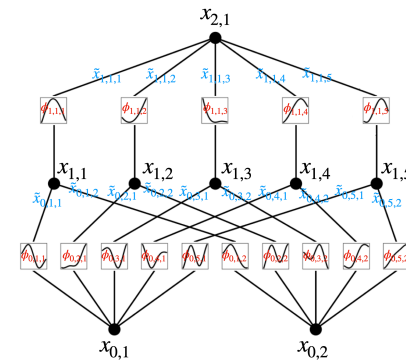## Layer = matrix of 1D functions

- Each edge learns a univariate mapping
$$\varphi_{l,j,i} : \mathbb{R} \to \mathbb{R}.$$

- Activation of the j-th neuron in layer $l+1$:
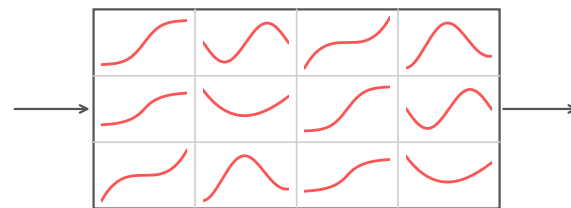$$x_{l+1,j} = \sum_{i=1}^{n_l} \varphi_{l,j,i}(x_{l,i})$$

- A single KAN layer can be written in **matrix form**:
$$\boldsymbol{x}_{l+1} = \underbrace{\begin{pmatrix} \varphi_{l,1,1}(\cdot) & \varphi_{l,1,2}(\cdot) & \cdots & \varphi_{l,1,n_l}(\cdot) \\ \varphi_{l,2,1}(\cdot) & \varphi_{l,2,2}(\cdot) & \cdots & \varphi_{l,2,n_l}(\cdot) \\ \vdots & \vdots & & \vdots \\ \varphi_{l,n_{l+1},1}(\cdot) & \varphi_{l,n_{l+1},2}(\cdot) & \cdots & \varphi_{l,n_{l+1},n_l}(\cdot) \end{pmatrix}}_{\boldsymbol{\Phi}_l \in (\mathbb{R}\to\mathbb{R})^{n_{l+1}\times n_l}} \boldsymbol{x}_l$$

B-spline parametrization and grid refinement. [Liu+25]

KAN layer in matrix form

General KAN with L layers

$$\mathrm{KAN}(\boldsymbol{x}) = (\boldsymbol{\Phi}_{L-1} \circ \boldsymbol{\Phi}_{L-2} \circ \ldots \circ \boldsymbol{\Phi}_0)(\boldsymbol{x})$$

KART $\hookleftarrow$ KAN of shape $[n \to 2n+1 \to 1]$

Edge Functions - Residual Splines

Edge - Residual Spline

$$\varphi(x) = w_b b(x) + w_s \sum_i c_i B_i(x)$$

- Trainable (per edge, backprop): c_i, w_b, w_s

- $B_i(x)$: B-spline basis functions, fixed given the current grid.

- $b(x)$: fixed **global** non-linearity (i.e. SiLU).

  1 Ensure φ is well-defined on $\mathbb{R}$

  2 Residual path eases optimization – learn deviation from $b(x)$ rather than full function

Residual spline edge function: local basis $\rightarrow$ spline $\rightarrow \varphi(x)$.
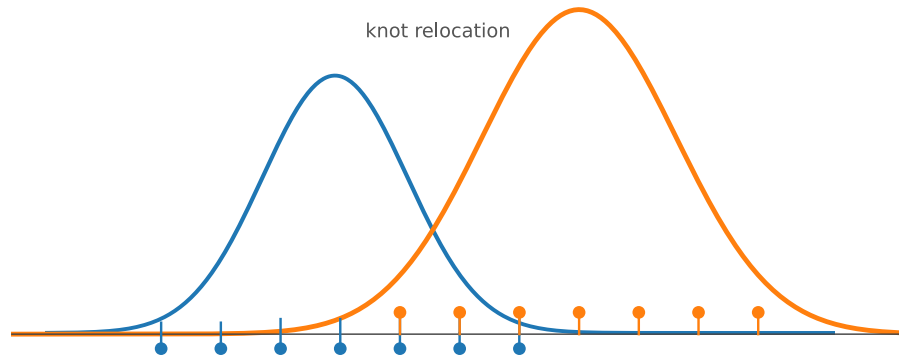
Why B-Splines?

- **local**, **translation-invariant** basis

  - local capacity allocation

  - continual learning ↑, catastrophic forgetting ↓

- Allows for other orthogonal bases (Fourier, Chebyshev).

- Locality ↔ global efficiency.

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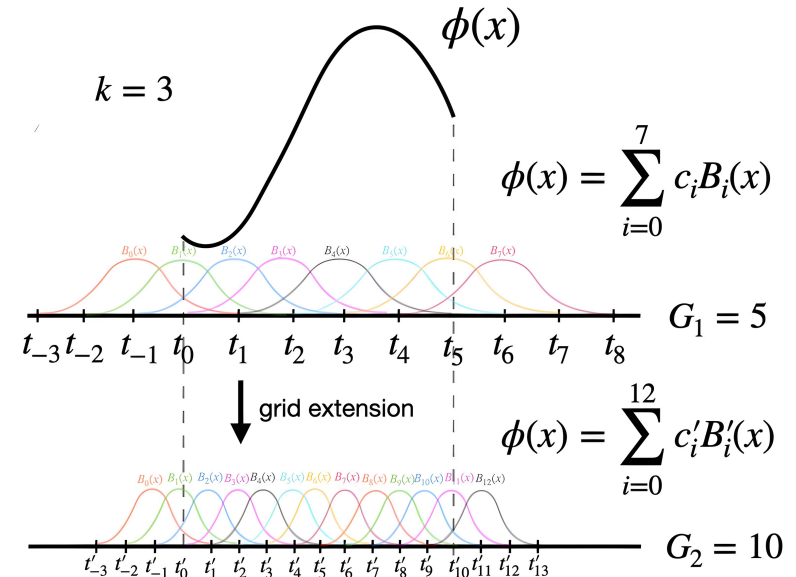
Grid Update - Knot Relocation

Keep knots where the data lives

- **Non-stationary** activations in training, but splines live on bounded grid

- **Grid update:** periodically estimate activation distributions; **move knots** to maintain coverage.

- **non-differentiable** reparameterization

$\phi(x)$

$k = 3$

$\phi(x) = \sum_{i=0}^{7} c_i B_i(x)$

$B_0(x)$ $B_1(x)$ $B_2(x)$ $B_3(x)$ $B_4(x)$ $B_5(x)$ $B_6(x)$ $B_7(x)$

t_{-3} t_{-2} t_{-1} t_0 t_1 t_2 t_3 t_4 t_5 t_6 t_7 t_8

$G_1 = 5$

↓ grid extension

$\phi(x) = \sum_{i=0}^{12} c_i' B_i'(x)$

$B_0(x)$ $B_1(x)$ $B_2(x)$ $B_3(x)$ $B_4(x)$ $B_5(x)$ $B_6(x)$ $B_7(x)$ $B_8(x)$ $B_9(x)$ $B_{10}(x)$ $B_{11}(x)$ $B_{12}(x)$

t_{-3}' t_{-2}' t_{-1}' t_0' t_1' t_2' t_3' t_4' t_5' t_6' t_7' t_8' t_9' t_{10}' t_{11}' t_{12}' t_{13}'

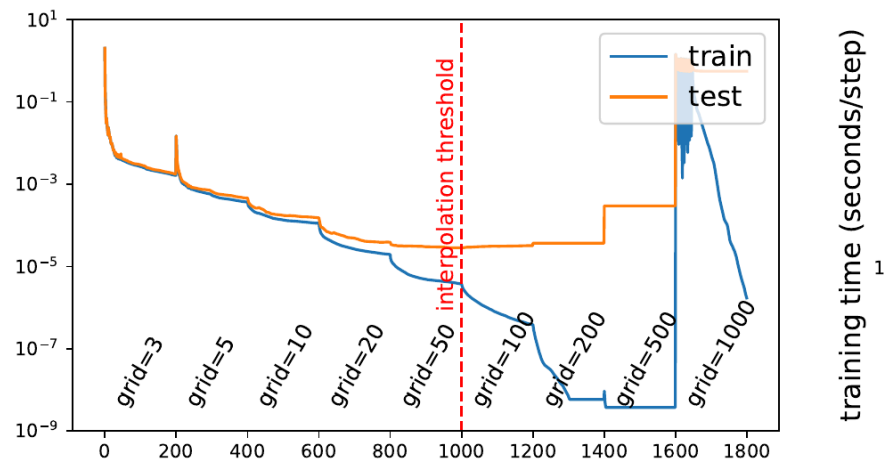$G_2 = 10$

[Liu+25]

knot relocation

Non-stationarity motivates knot relocation.

Grid updates reallocate representational capacity at **fixed number of knots** (contrast: grid extension adds knots).

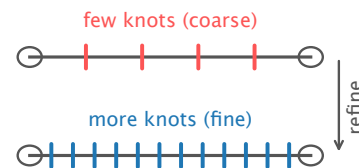Felix Schladt · Jan Duchscherer   Advanced Deep Learning   20. Jan 2026

Grid Extension: Accuracy Scaling

Key idea

- **Grid extension**: add knots $(G \rightarrow G') \rightarrow$ higher spline resolution.

- Curriculum-style schedule:
  1. Start with coarse spatial resolution – fewer knots, global structure, simpler optimization.
  2. Gradually increase resolution, initialize finer splines via least-squares fit to coarse spline.

- Monitor validation error to stop grid extension once improvement ceases.

- RMSE $\propto G^{-4}$ (on test split)

Staircase-like loss drops after each grid refinement. [Liu+25]

$$\{c'_j\} = \underset{\{c'_j\}}{\arg\min}\, \mathbb{E}_{x \sim p(x)} \left(\sum_{j=0}^{G_2+k-1} c'_j B'_j(x) - \sum_{i=0}^{G_1+k-1} c_i B_i(x) \right)^2$$

External vs Internal DoF: Structure vs Precision

Two kinds of degrees of freedom (DoF)

- **External DoF**: width/depth/connectivity $\rightarrow$ **which interactions** among variables.

- **Internal DoF**: spline grid G + coefficients c_i $\rightarrow$ **how precisely** each interaction is represented.

- **Key shift**: KANs separate structure from precision, so each can scale independently.

MLPs

- Structure + precision are **entangled**.

- More accuracy $\rightarrow$ scale width/depth (external DoF).

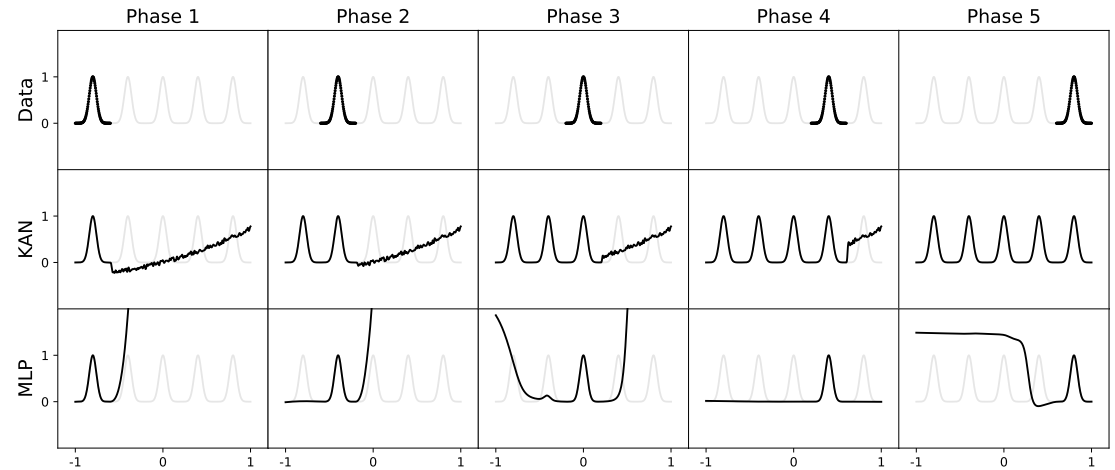- No dedicated control over internal DoF.

- $\mathcal{O}(N^2 L)$

KANs

- External DoF $\leftrightarrow$ **compositional structure**.

- Internal DoF $\leftrightarrow$ **local precision** on edges.

- Grid extension increases internal DoF while keeping structure fixed.

- $\mathcal{O}(N^2 L(G + k))$
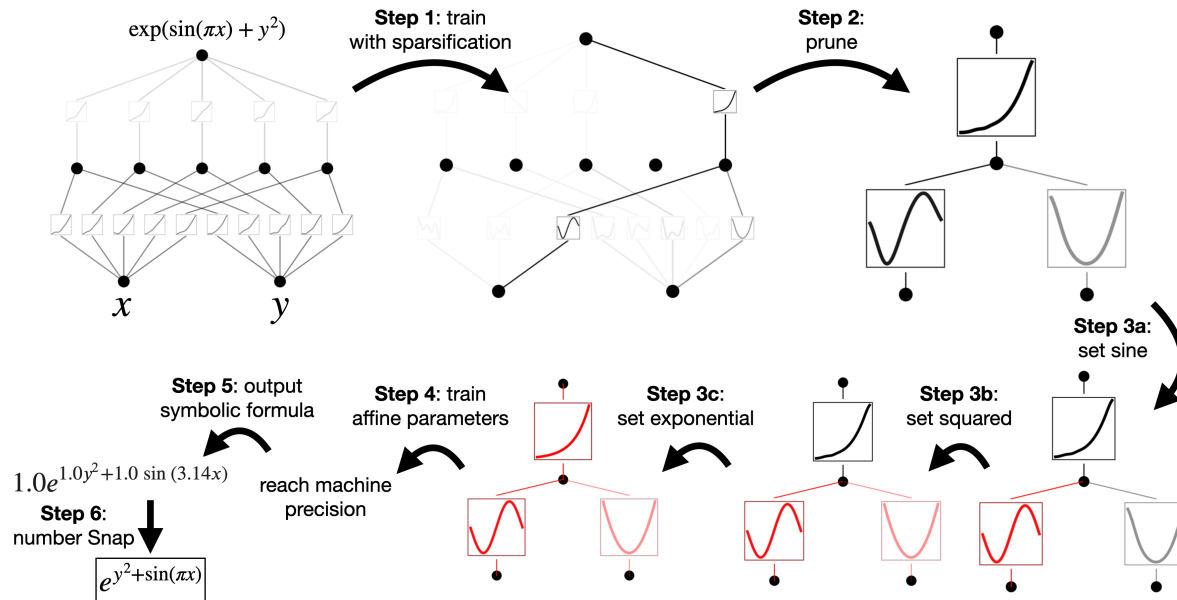
Continual Learning & Locality

Why KANs forget less

- **Local plasticity:** Each spline basis is local:
  → only a few basis functions are non-zero per sample
  → gradients touch only a few nearby coefficients.
  → updates are localized.

- **Contrast (MLPs):** Any local update has global effects:
  → learning a new task may overwrite previous knowledge
  → catastrophic forgetting.

- Generalization to higher dimensions is unclear [YYW24].

Sequential learning (toy 1D): KAN preserves earlier regions; MLP forgets. [Liu+25]

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Reductionist Symbolification

Symbolic regression workflow with KANs. [Liu+25]

KANs are connectionist throughout training & symbolic after reductionist sparsification & compression into symbolic forms.

Structure → equation

1. **Sparsify**: L1 on activations + entropy regularization → few dominant edges.
2. **Prune** low-importance nodes → minimal graph → external DoF ↓.
3. **Symbolify**: replace splines with analytic forms + affine wrapper
$$y \approx cf(ax + b) + d$$
→ internal DoF ↓.
4. **Further training** of the (affine) parameters.

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Summary: Takeaways & Limitations

Takeaways

- KANs are most suited for **structured, compositional, small-scale scientific tasks**.

- **Decoupled internal and external DoF** improve fine-tuning, **continual learning** & scaling.

- **Symbolic interpretability** and discovery of symbolic formulas (**white-box ML**).

- Rapid **follow-up ecosystem**: efficiency variants, hybrids, theory, and broad applications.

Limitations

- Baseline KANs are up to 10x **slower** (batching, spline updates, HW); faster follow-ups exist [Li24].

- Likely **not an MLP replacement** in large-scale tasks [YYW24].

- Sensitivity to **additional hyperparameters** (grid size, update schedule).

Trade-off: interpretability & controllable precision vs training efficiency.

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