Parameter estimation with correlated photon pairs

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Motivation

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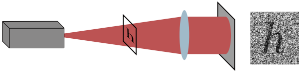

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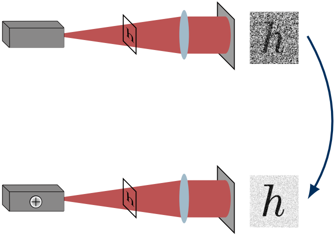
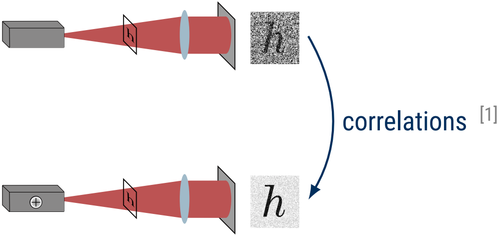


correlations [1]

[1] Brida, Genovese, and Ruo Berchera, “Experimental Realization of Sub-Shot-Noise Quantum Imaging”, Apr. 2010

Motivation



Objective: Can correlated photons provide advantages in terms of precision in noisy regimes for parameter estimation?

[1] Brida, Genovese, and Ruo Berchera, “Experimental Realization of Sub-Shot-Noise Quantum Imaging”, Apr. 2010

Spontaneous parametric down-conversion

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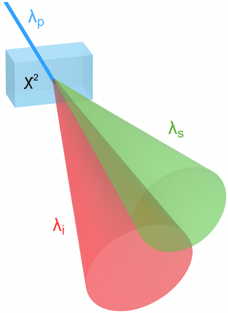
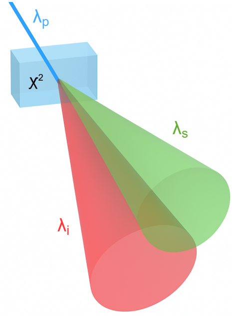
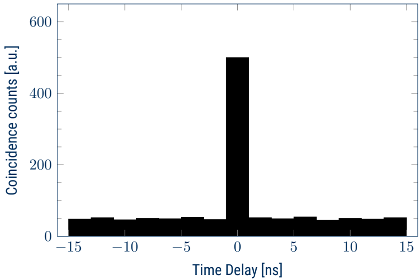


Spontaneous parametric down-conversion





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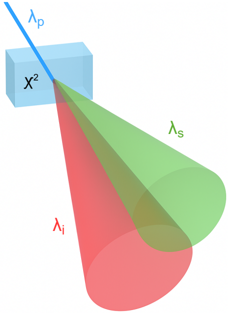
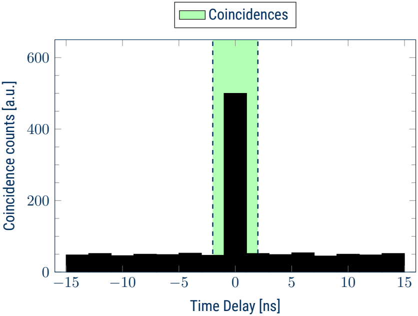
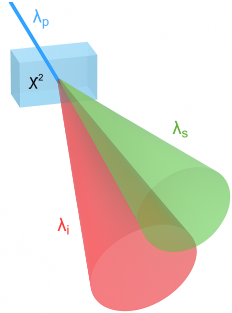
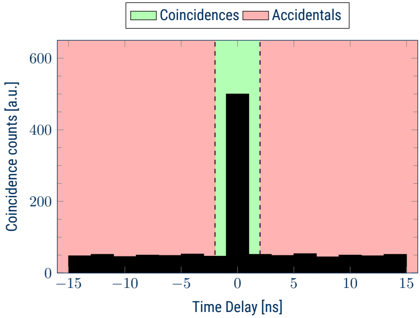




Spontaneous parametric down-conversion





Parameter estimation

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Setup: Transmission setup

Parameter estimation

Setup: Transmission setup
Parameter: Transmittance T

Parameter estimation

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Precision: $\mathrm{Var}(T)$

Parameter estimation

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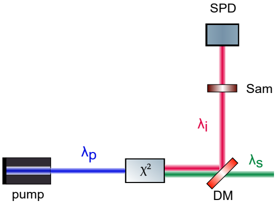
Parameter: Transmittance T
Precision: Var(T)

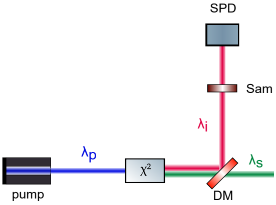
Conventional approach:



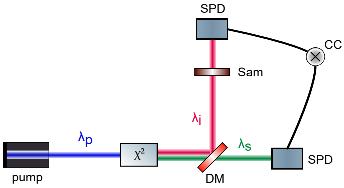
Parameter estimation

Setup: Transmission setup
Parameter: Transmittance T
Precision: Var(T)

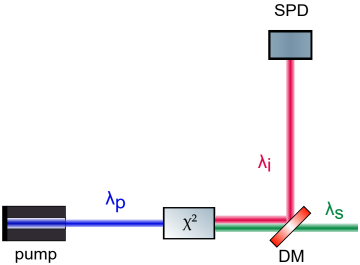
Conventional approach:



Coincidence approach:

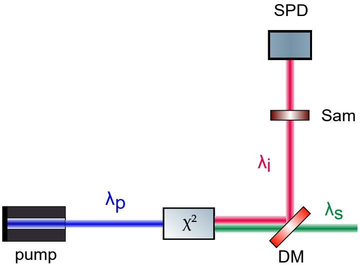


Conventional approach



$$N_{\text{sing}}^{\text{ref}} = \eta_{\text{idl}} N_{\text{g}} + N_{\text{noise}}^{\text{ref}}$$

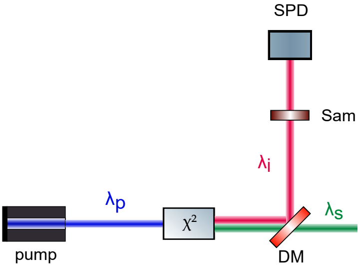
Conventional approach



$$N_{\mathrm{sing}}^{\mathrm{ref}} = \eta_{\mathrm{idl}} N_{\mathrm{g}} + N_{\mathrm{noise}}^{\mathrm{ref}}$$

$$N_{\mathrm{sing}}^{\mathrm{sam}} = T\, \eta_{\mathrm{idl}} N_{\mathrm{g}} + N_{\mathrm{noise}}^{\mathrm{sam}}$$

Conventional approach

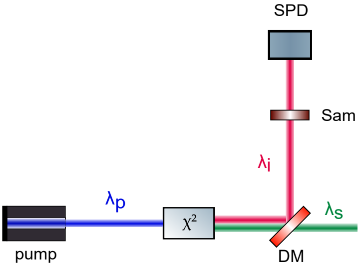


$$N_{\text{sing}}^{\text{ref}} = \eta_{\text{idl}} N_{\text{g}} + N_{\text{noise}}^{\text{ref}}$$

$$N_{\text{sing}}^{\text{sam}} = T\, \eta_{\text{idl}} N_{\text{g}} + N_{\text{noise}}^{\text{sam}}$$

$$\Rightarrow T = \frac{N_{\text{sing}}^{\text{sam}} - N_{\text{noise}}^{\text{sam}}}{N_{\text{sing}}^{\text{ref}} - N_{\text{noise}}^{\text{ref}}}$$

Conventional approach



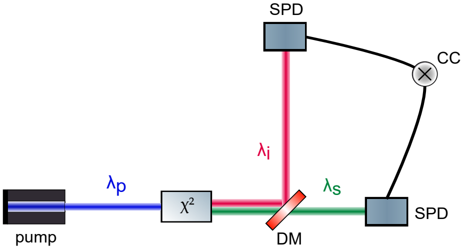
$$N^{\text{ref}}_{\text{sing}} = \eta_{\text{idl}} N_{\text{g}} + N^{\text{ref}}_{\text{noise}}$$

$$N^{\text{sam}}_{\text{sing}} = T\, \eta_{\text{idl}} N_{\text{g}} + N^{\text{sam}}_{\text{noise}}$$

$$\Rightarrow T = \frac{N^{\text{sam}}_{\text{sing}} - N^{\text{sam}}_{\text{noise}}}{N^{\text{ref}}_{\text{sing}} - N^{\text{ref}}_{\text{noise}}}$$

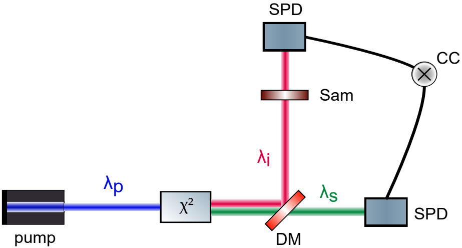
$$\text{Var}(T) = (\eta_{\text{idl}} N_{\text{g}})^{-2} \left[\text{Var}\left(N^{\text{sam}}_{\text{sing}}\right) + \text{Var}\left(N^{\text{sam}}_{\text{noise}}\right) + T^2 \left[\text{Var}\left(N^{\text{ref}}_{\text{sing}}\right) + \text{Var}\left(N^{\text{ref}}_{\text{noise}}\right) \right] \right]$$

Coincidence approach



$$N_{\mathrm{coin}}^{\mathrm{ref}} = \eta_{\mathrm{idl}}\,\eta_{\mathrm{sig}}\,N_{\mathrm{g}} + N_{\mathrm{ac}}^{\mathrm{ref}}$$

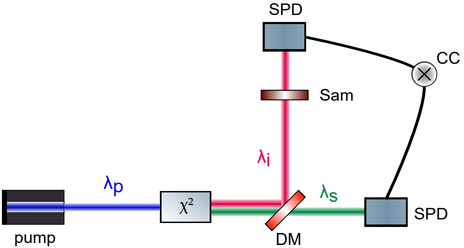
Coincidence approach



$$N_{\mathrm{coin}}^{\mathrm{ref}} = \eta_{\mathrm{idl}}\,\eta_{\mathrm{sig}}\,N_{\mathrm{g}} + N_{\mathrm{ac}}^{\mathrm{ref}}$$

$$N_{\mathrm{coin}}^{\mathrm{sam}} = T\,\eta_{\mathrm{idl}}\,\eta_{\mathrm{sig}}\,N_{\mathrm{g}} + N_{\mathrm{ac}}^{\mathrm{sam}}$$

Coincidence approach



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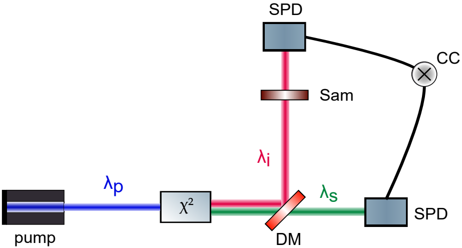
Coincidence approach



$$N_{\mathrm{coin}}^{\mathrm{ref}} = \eta_{\mathrm{idl}}\,\eta_{\mathrm{sig}}\,N_{\mathrm{g}} + N_{\mathrm{ac}}^{\mathrm{ref}}$$

$$N_{\mathrm{coin}}^{\mathrm{sam}} = T\,\eta_{\mathrm{idl}}\,\eta_{\mathrm{sig}}\,N_{\mathrm{g}} + N_{\mathrm{ac}}^{\mathrm{sam}}$$

$$\Rightarrow T = \frac{N_{\mathrm{coin}}^{\mathrm{sam}} - N_{\mathrm{ac}}^{\mathrm{sam}}}{N_{\mathrm{coin}}^{\mathrm{ref}} - N_{\mathrm{ac}}^{\mathrm{ref}}}$$

$$\mathrm{Var}(T) = \left(\eta_{\mathrm{sig}}\,\eta_{\mathrm{idl}}\,N_{\mathrm{g}}\right)^{-2}\left[\mathrm{Var}\left(N_{\mathrm{coin}}^{\mathrm{sam}}\right) + \mathrm{Var}\left(N_{\mathrm{ac}}^{\mathrm{sam}}\right) + T^2\left[\mathrm{Var}\left(N_{\mathrm{coin}}^{\mathrm{ref}}\right) + \mathrm{Var}\left(N_{\mathrm{ac}}^{\mathrm{ref}}\right)\right]\right]$$

Transmittance model

Conventional approach:

$$\mathrm{Var}(T) = (\eta_{\mathrm{idl}}\, N_{\mathrm{g}})^{-2} \left[\mathrm{Var}\left(N_{\mathrm{sing}}^{\mathrm{sam}}\right) + \mathrm{Var}\left(N_{\mathrm{noise}}^{\mathrm{sam}}\right) + T^2 \left[\mathrm{Var}\left(N_{\mathrm{sing}}^{\mathrm{ref}}\right) + \mathrm{Var}\left(N_{\mathrm{noise}}^{\mathrm{ref}}\right) \right] \right]$$

Coincidence approach:

$$\mathrm{Var}(T) = (\eta_{\mathrm{sig}}\, \eta_{\mathrm{idl}}\, N_{\mathrm{g}})^{-2} \left[\mathrm{Var}\left(N_{\mathrm{coin}}^{\mathrm{sam}}\right) + \mathrm{Var}\left(N_{\mathrm{ac}}^{\mathrm{sam}}\right) + T^2 \left[\mathrm{Var}\left(N_{\mathrm{coin}}^{\mathrm{ref}}\right) + \mathrm{Var}\left(N_{\mathrm{ac}}^{\mathrm{ref}}\right) \right] \right]$$

Transmittance model

Conventional approach:

$$\mathrm{Var}(T) = \left(\eta_{\mathrm{idl}}\, N_{\mathrm{g}}\right)^{-2} \left[\mathrm{Var}\left(N^{\mathrm{sam}}_{\mathrm{sing}}\right) + \mathrm{Var}\left(N^{\mathrm{sam}}_{\mathrm{noise}}\right) + T^2 \left[\mathrm{Var}\left(N^{\mathrm{ref}}_{\mathrm{sing}}\right) + \mathrm{Var}\left(N^{\mathrm{ref}}_{\mathrm{noise}}\right)\right]\right]$$

Coincidence approach:

$$\mathrm{Var}(T) = \left(\eta_{\mathrm{sig}}\, \eta_{\mathrm{idl}}\, N_{\mathrm{g}}\right)^{-2} \left[\mathrm{Var}\left(N^{\mathrm{sam}}_{\mathrm{coin}}\right) + \mathrm{Var}\left(N^{\mathrm{sam}}_{\mathrm{ac}}\right) + T^2 \left[\mathrm{Var}\left(N^{\mathrm{ref}}_{\mathrm{coin}}\right) + \mathrm{Var}\left(N^{\mathrm{ref}}_{\mathrm{ac}}\right)\right]\right]$$

Photon statistics

Photon statistics

Poisson distribution (coherent light):

$$\mathcal{P}(n) = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}$$

$$\mathrm{Var}(n) = \langle n \rangle$$

Photon statistics

Poisson distribution (coherent light): [2][3]

$$\mathcal{P}(n) = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}$$
$$\mathrm{Var}(n) = \langle n \rangle$$

[2] Kim et al., "Photon-Counting Statistics-Based Support Vector Machine with Multi-Mode Photon Illumination for Quantum Imaging", Oct. 2022

[3] Fouche, "Detection and False-Alarm Probabilities for Laser Radars That Use Geiger-mode Detectors", Sept. 2003

Photon statistics

Poisson distribution (coherent light): [2][3]

$$\mathcal{P}(n) = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}$$
$$\mathrm{Var}(n) = \langle n \rangle$$

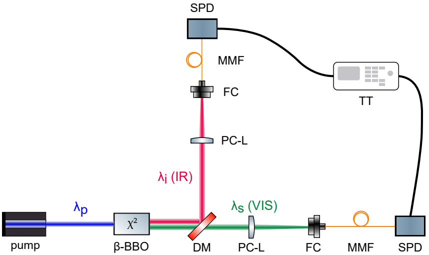
Multi-mode Bose-Einstein distribution (thermal light): [2]

$$\mathcal{P}_m(n) = \frac{(n+m-1)!}{(m-1)!\, n!} \frac{m^m \langle n \rangle^n}{(m + \langle n \rangle)^{n+m}}$$
$$\mathrm{Var}(n) = \langle n \rangle \left(1 + \frac{\langle n \rangle}{m}\right)$$

[2] Kim et al., "Photon-Counting Statistics-Based Support Vector Machine with Multi-Mode Photon Illumination for Quantum Imaging", Oct. 2022

[3] Fouche, "Detection and False-Alarm Probabilities for Laser Radars That Use Geiger-mode Detectors", Sept. 2003

Experimental setup



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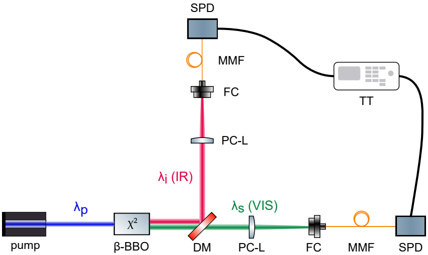

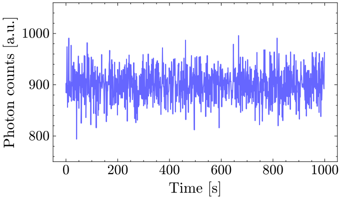

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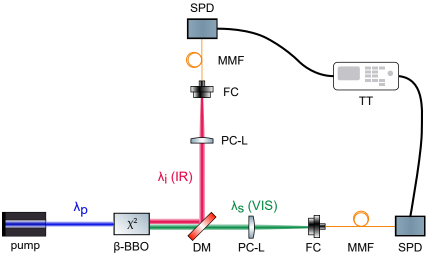
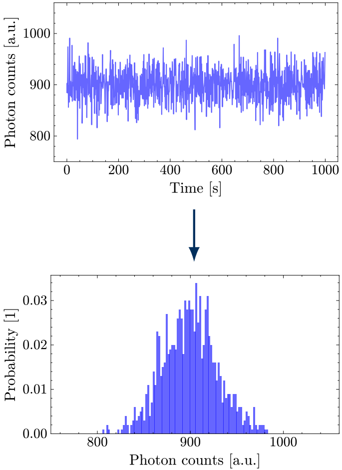
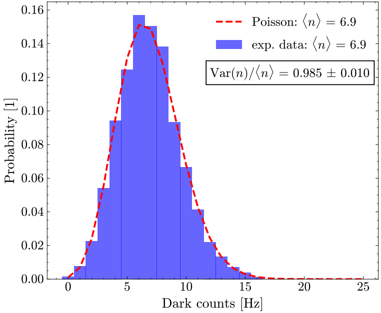




Dark counts, $\mathrm{Var}(N_{\mathrm{noise}})$

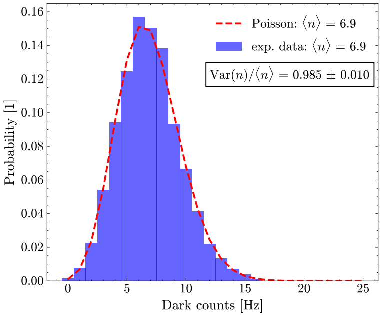
Dark counts, $\mathrm{Var}(N_{\mathrm{noise}})$

Signal arm

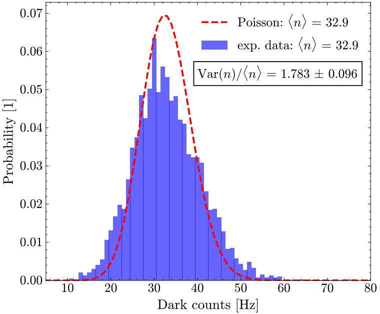


Dark counts, Var(N_{noise})

Signal arm

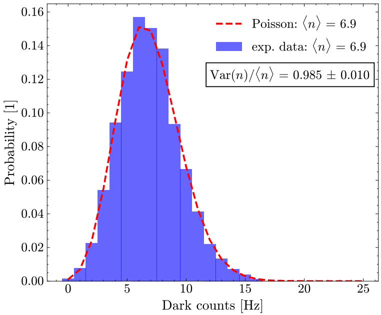


Idler arm

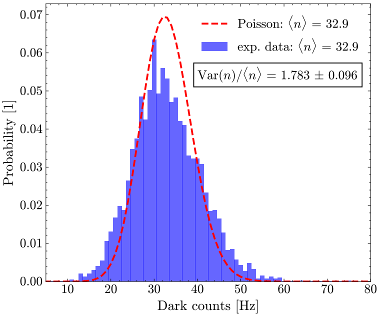


Dark counts, $\mathrm{Var}(N_{\text{noise}})$

Signal arm



Idler arm

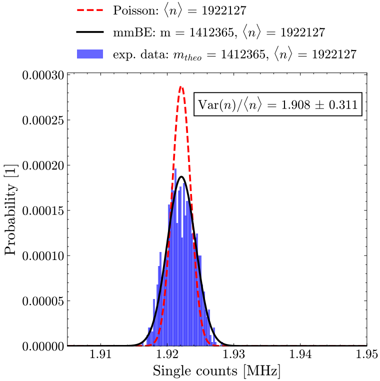


$$\mathrm{Var}(N_{\text{noise}}) = 1.8 \cdot \langle N_{\text{noise}} \rangle$$

Single counts, $\mathrm{Var}(N_{\mathrm{sing}})$

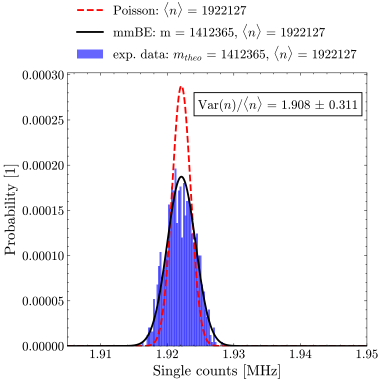
Single counts, $\mathrm{Var}(N_{\mathrm{sing}})$

Signal arm

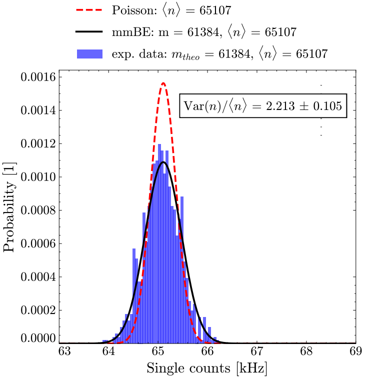


Single counts, $\mathrm{Var}(N_{\mathrm{sing}})$

Signal arm

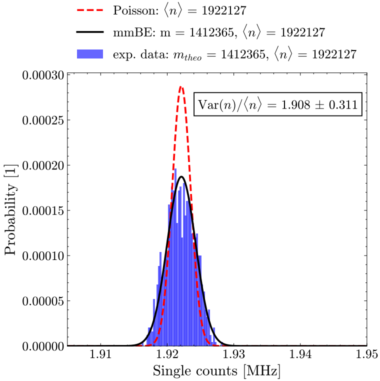


Idler arm

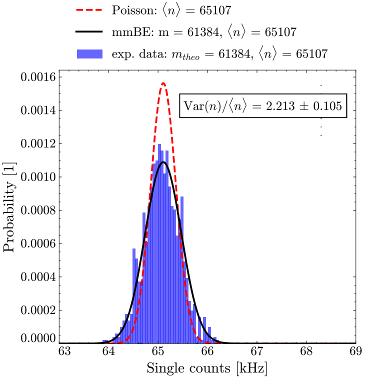


Single counts, $\mathrm{Var}(N_{\text{sing}})$

Signal arm



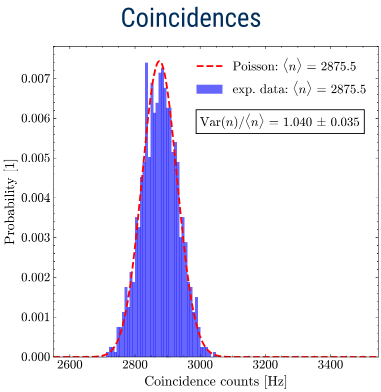
Idler arm



$$\mathrm{Var}(N_{\text{sing}}) = 2.2 \cdot \langle N_{\text{sing}} \rangle$$

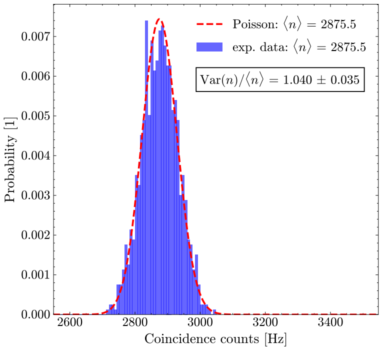
Coincidence counts, $\mathrm{Var}(N_{\mathrm{coin}})$

Coincidence counts, $\mathrm{Var}(N_{\mathrm{coin}})$



Coincidence counts, $\mathrm{Var}(N_{\mathrm{coin}})$

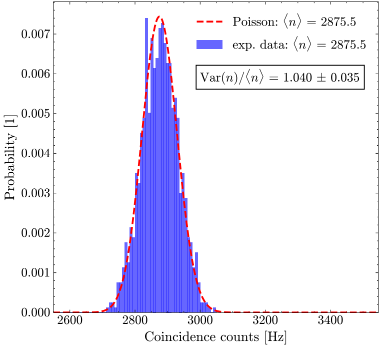
Coincidences



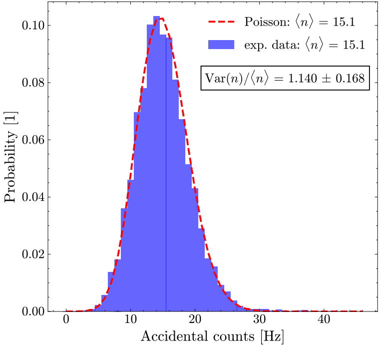
$$\mathrm{Var}(N_{\mathrm{coin}}) = \langle N_{\mathrm{coin}} \rangle$$

Coincidence counts, $\mathrm{Var}(N_{\mathrm{coin}})$

Coincidences



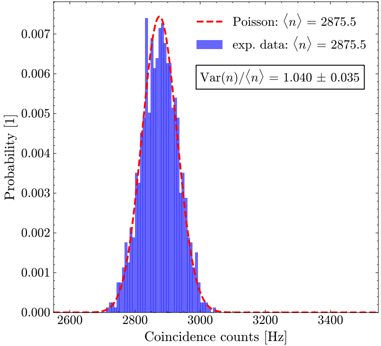
Accidentals



$$\mathrm{Var}(N_{\mathrm{coin}}) = \langle N_{\mathrm{coin}} \rangle$$

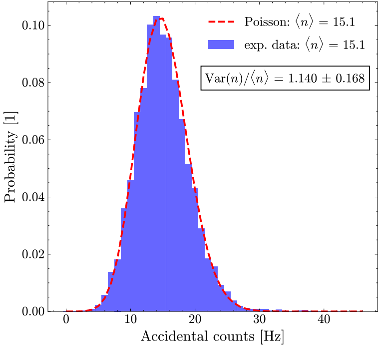
Coincidence counts, $\mathrm{Var}(N_{\mathrm{coin}})$

Coincidences



$$\mathrm{Var}(N_{\mathrm{coin}}) = \langle N_{\mathrm{coin}} \rangle$$

Accidentals



$$\mathrm{Var}(N_{\mathrm{ac}}) = \langle N_{\mathrm{ac}} \rangle$$

Simulation

Conventional approach:

$$\mathrm{Var}(T) = (\eta_{\mathrm{idl}}\, N_{\mathrm{g}})^{-2} \left[\mathrm{Var}\left(N_{\mathrm{sing}}^{\mathrm{sam}}\right) + \mathrm{Var}\left(N_{\mathrm{noise}}^{\mathrm{sam}}\right) + T^2 \left[\mathrm{Var}\left(N_{\mathrm{sing}}^{\mathrm{ref}}\right) + \mathrm{Var}\left(N_{\mathrm{noise}}^{\mathrm{ref}}\right) \right] \right]$$

Simulation

Conventional approach:

$$\mathrm{Var}(T) = (\eta_{\mathrm{idl}}\, N_{\mathrm{g}})^{-2}\left[2.2 \cdot \langle N_{\mathrm{sing}}^{\mathrm{sam}}\rangle + 1.8 \cdot \langle N_{\mathrm{noise}}^{\mathrm{sam}}\rangle + T^2\left[2.2 \cdot \langle N_{\mathrm{sing}}^{\mathrm{ref}}\rangle + 1.8 \cdot \langle N_{\mathrm{noise}}^{\mathrm{ref}}\rangle\right]\right]$$

Simulation

Conventional approach:

$$\mathrm{Var}(T) = (\eta_{\mathrm{idl}}\, N_{\mathrm{g}})^{-2}\left[2.2\cdot\langle N_{\mathrm{sing}}^{\mathrm{sam}}\rangle + 1.8\cdot\langle N_{\mathrm{noise}}^{\mathrm{sam}}\rangle + T^2\left[2.2\cdot\langle N_{\mathrm{sing}}^{\mathrm{ref}}\rangle + 1.8\cdot\langle N_{\mathrm{noise}}^{\mathrm{ref}}\rangle\right]\right]$$

Coincidence approach:

$$\mathrm{Var}(T) = \left(\eta_{\mathrm{sig}}\,\eta_{\mathrm{idl}}\, N_{\mathrm{g}}\right)^{-2}\left[\mathrm{Var}\left(N_{\mathrm{coin}}^{\mathrm{sam}}\right) + \mathrm{Var}\left(N_{\mathrm{ac}}^{\mathrm{sam}}\right) + T^2\left[\mathrm{Var}\left(N_{\mathrm{coin}}^{\mathrm{ref}}\right) + \mathrm{Var}\left(N_{\mathrm{ac}}^{\mathrm{ref}}\right)\right]\right]$$

Simulation

Conventional approach:

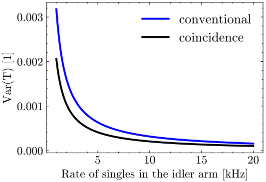
$$\mathrm{Var}(T) = (\eta_{\mathrm{idl}}\, N_{\mathrm{g}})^{-2} \left[2.2 \cdot \langle N^{\mathrm{sam}}_{\mathrm{sing}} \rangle + 1.8 \cdot \langle N^{\mathrm{sam}}_{\mathrm{noise}} \rangle + T^2 \left[2.2 \cdot \langle N^{\mathrm{ref}}_{\mathrm{sing}} \rangle + 1.8 \cdot \langle N^{\mathrm{ref}}_{\mathrm{noise}} \rangle \right] \right]$$

Coincidence approach:

$$\mathrm{Var}(T) = \left(\eta_{\mathrm{sig}}\, \eta_{\mathrm{idl}}\, N_{\mathrm{g}}\right)^{-2} \left[\langle N^{\mathrm{sam}}_{\mathrm{coin}} \rangle + \langle N^{\mathrm{sam}}_{\mathrm{ac}} \rangle + T^2 \left[\langle N^{\mathrm{ref}}_{\mathrm{coin}} \rangle + \langle N^{\mathrm{ref}}_{\mathrm{ac}} \rangle \right] \right]$$

Simulation

Simulation

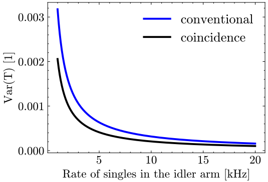


$\eta_{\text{idl}} = 70\%$

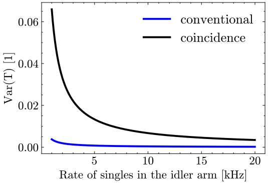
$\eta_{\text{sig}} = 70\%$

$R_{\text{noise,idl}} = 0\text{ Hz}$

Simulation

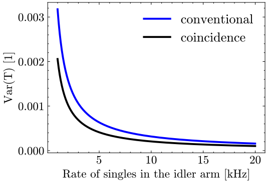


$$\eta_{\mathrm{idl}} = 70\%$$
$$\eta_{\mathrm{sig}} = 70\%$$
$$R_{\mathrm{noise,idl}} = 0\ \mathrm{Hz}$$



$$\eta_{\mathrm{idl}} = 0.09\%$$
$$\eta_{\mathrm{sig}} = 2.6\%$$
$$R_{\mathrm{noise,idl}} = 0\ \mathrm{Hz}$$

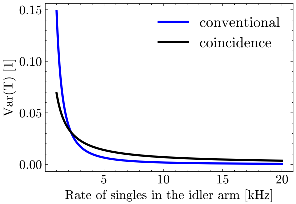
Simulation



$\eta_{\mathrm{idl}} = 70\%$

$\eta_{\mathrm{sig}} = 70\%$

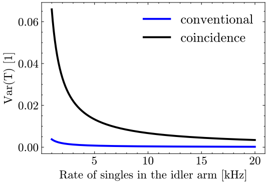
$R_{\mathrm{noise,idl}} = 0\ \mathrm{Hz}$



$\eta_{\mathrm{idl}} = 0.09\%$

$\eta_{\mathrm{sig}} = 2.6\%$

$R_{\mathrm{noise,idl}} = 20\ \mathrm{kHz}$

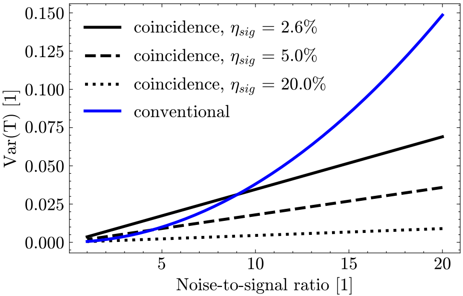


$\eta_{\mathrm{idl}} = 0.09\%$

$\eta_{\mathrm{sig}} = 2.6\%$

$R_{\mathrm{noise,idl}} = 0\ \mathrm{Hz}$

Noise-to-signal ratio



Parameter	Value
η_{idl} (%)	0.09
η_{sig} (%)	2.6, 5, 20
R_{idl} (kHz)	1 - 20
$R_{noise,idl}$ (kHz)	20
$R_{noise,sig}$ (Hz)	7
T (1)	0.9

Summary and Outlook

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- Established a model for the variance of transmittance
- Experimental verification of photon statistics
- Found regions where coincidence approach offers higher precision

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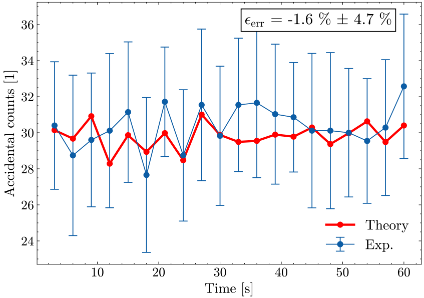
Outlook:

- Experimental verification of the found parameter regions
- Determine the experimental limiations for the variance measurement

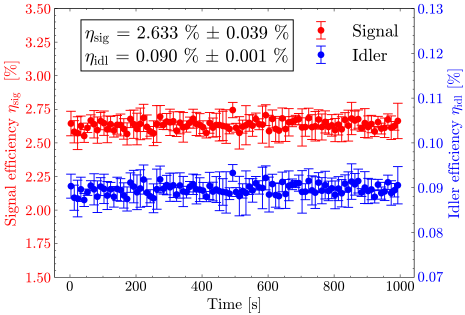
Accidental counts



$$R_{\mathrm{ac}}^{\mathrm{sam}} = \left(T\,\eta_{\mathrm{idl}} R_{\mathrm{g}} + R_{\mathrm{dc,idl}} - R_{\mathrm{cc,pure}}^{\mathrm{sam}}\right) \cdot \left(\eta_{\mathrm{sig}} R_{\mathrm{g}} + R_{\mathrm{dc,sig}} - R_{\mathrm{cc,pure}}^{\mathrm{sam}}\right) \cdot \tau_{\mathrm{cw}}$$

$$R_{\mathrm{ac}}^{\mathrm{ref}} = \left(\eta_{\mathrm{idl}} R_{\mathrm{g}} + R_{\mathrm{dc,idl}} - R_{\mathrm{cc,pure}}^{\mathrm{ref}}\right) \cdot \left(\eta_{\mathrm{sig}} R_{\mathrm{g}} + R_{\mathrm{dc,sig}} - R_{\mathrm{cc,pure}}^{\mathrm{ref}}\right) \cdot \tau_{\mathrm{cw}}$$

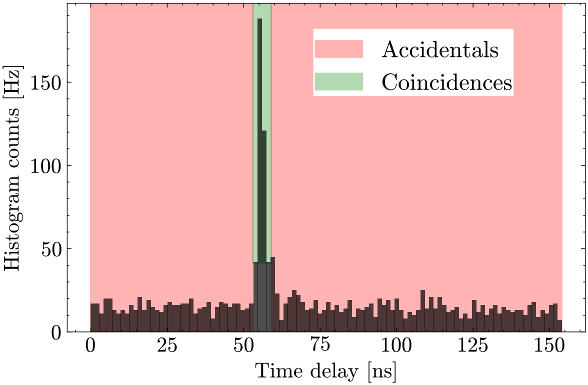
Heralding efficiencies



$$\eta_{sig} = \frac{N_{coin} - N_{ac}}{N_{sing,idl} - N_{noise}} = \frac{\eta_{sig}\eta_{idl} N_g}{\eta_{idl} N_g}$$

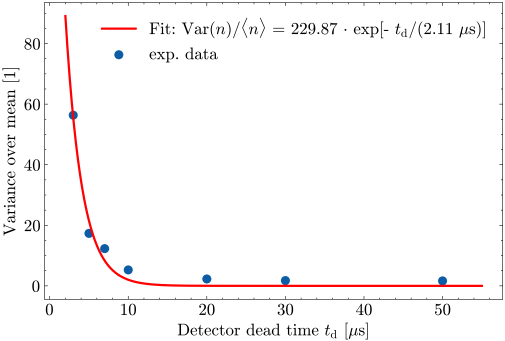
$$\eta_{idl} = \frac{N_{coin} - N_{ac}}{N_{sing,sig} - N_{noise}} = \frac{\eta_{sig}\eta_{idl} N_g}{\eta_{sig} N_g}$$

Histogram



Accidentals
Coincidences
Histogram counts [Hz]
0
50
100
150
0
25
50
75
100
125
150
Time delay [ns]

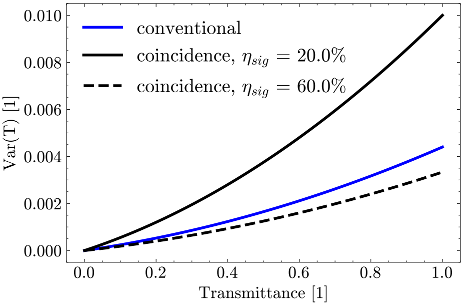
Afterpulsing



$$R_{aft} \propto e^{-\frac{t_d}{\tau_{de}}}$$ [4]

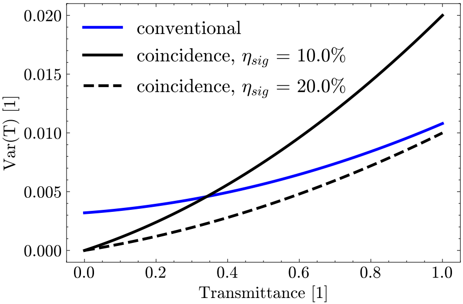
[4] Humer et al., "A Simple and Robust Method for Estimating Afterpulsing in Single Photon Detectors", July 2015

Noiseless case



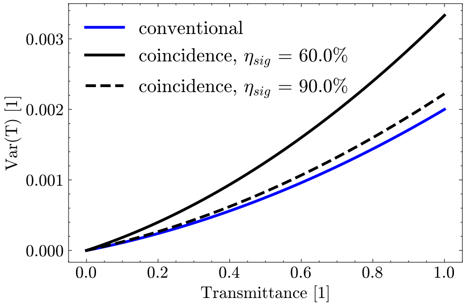
Parameter	**Value**
η_{idl} (%)	60
η_{sig} (%)	20, 60
R_{idl} (kHz)	1
$R_{\mathrm{noise,idl}}$ (kHz)	0
$R_{\mathrm{noise,sig}}$ (Hz)	7
T (1)	0 - 1

In presence of noise



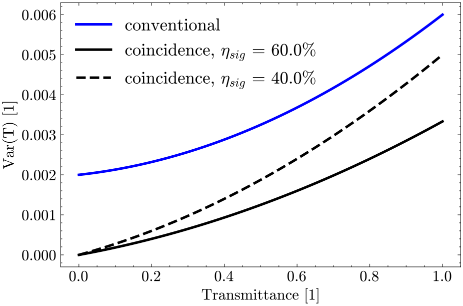
Parameter	**Value**
η_{idl} (%)	60
η_{sig} (%)	10, 20
R_{idl} (kHz)	1
$R_{\mathrm{noise,idl}}$ (kHz)	1
$R_{\mathrm{noise,sig}}$ (Hz)	7
T (1)	0 - 1

Coherent Illumination, noiseless



Parameter	Value
η_{idl} (%)	60
η_{sig} (%)	60, 90
R_{idl} (kHz)	1
$R_{\mathrm{noise,idl}}$ (kHz)	0
$R_{\mathrm{noise,sig}}$ (Hz)	7
T (1)	0 - 1

Coherent Illumination, with noise



Parameter	Value
η_{idl} (%)	60
η_{sig} (%)	40, 60
R_{idl} (kHz)	1
$R_{\mathrm{noise,idl}}$ (kHz)	1
$R_{\mathrm{noise,sig}}$ (Hz)	7
T (1)	0 - 1