

Parameter estimation with correlated photon pairs

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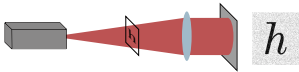
Jena, November 9, 2025

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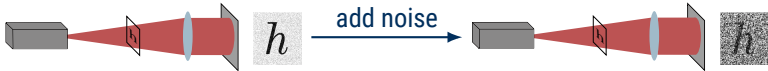
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- 4 Results
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Motivation

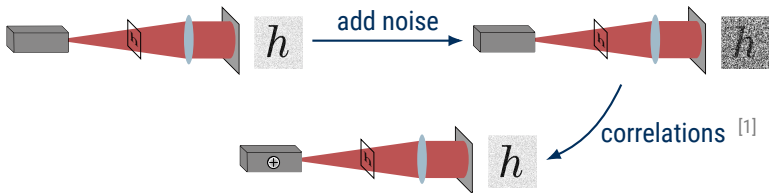
Motivation



Motivation

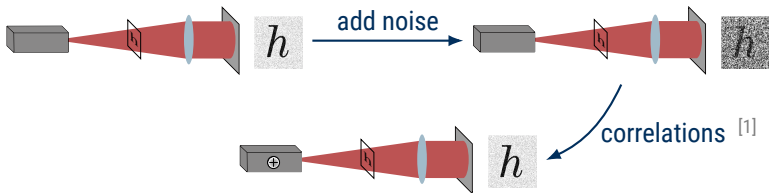


Motivation



[1] Brida, Genovese, and Ruo Berchera, "Experimental Realization of Sub-Shot-Noise Quantum Imaging"

Motivation

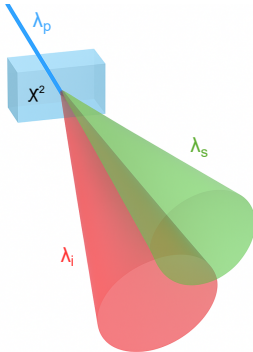


Objective: Can correlated photons provide advantages in terms of precision in noisy regimes for parameter estimation?

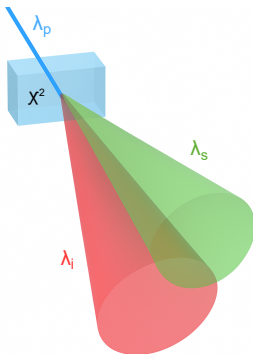
[1] Brida, Genovese, and Ruo Berchera, "Experimental Realization of Sub-Shot-Noise Quantum Imaging"



SPDC



SPDC

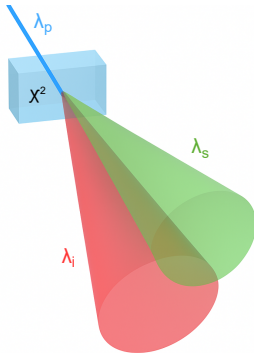


Energy conservation



$$\omega_p = \omega_s + \omega_i$$

SPDC

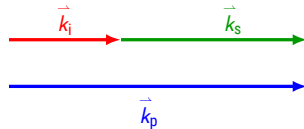


Energy conservation



$$\omega_p = \omega_s + \omega_i$$

Momentum conservation



$$\vec{k}_p = \vec{k}_s + \vec{k}_i$$

Parameter estimation

Parameter estimation

Setup: Transmission setup

Parameter estimation

Setup: Transmission setup

Parameter: Transmittance T

Parameter estimation

Setup: Transmission setup

Parameter: Transmittance T

Estimator: $\text{Var}(T)$

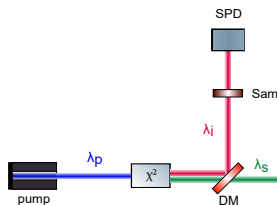
Parameter estimation

Setup: Transmission setup

Parameter: Transmittance T

Estimator: $\text{Var}(T)$

Conventional approach:



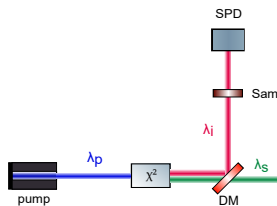
Parameter estimation

Setup: Transmission setup

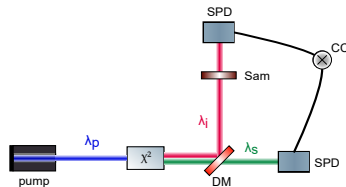
Parameter: Transmittance T

Estimator: $\text{Var}(T)$

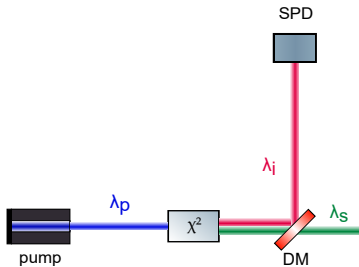
Conventional approach:



Coincidence approach:

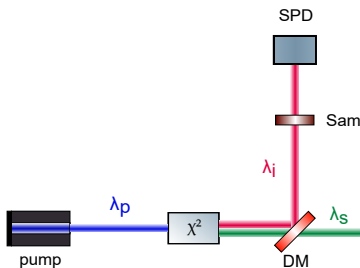


Conventional approach



$$N_{\text{sing}}^{\text{ref}} = \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{ref}}$$

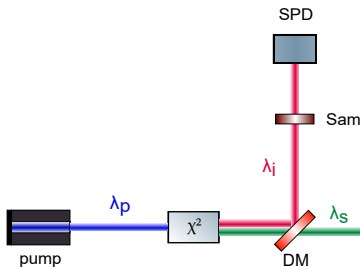
Conventional approach



$$N_{\text{sing}}^{\text{ref}} = \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{ref}}$$

$$N_{\text{sing}}^{\text{sam}} = T \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{sam}}$$

Conventional approach

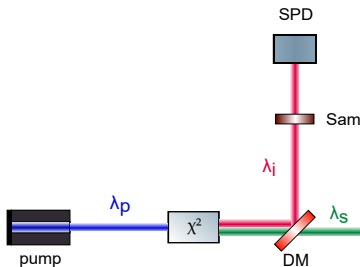


$$N_{\text{sing}}^{\text{ref}} = \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{ref}}$$

$$N_{\text{sing}}^{\text{sam}} = T \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{sam}}$$

$$\Rightarrow T = \frac{N_{\text{sing}}^{\text{sam}} - N_{\text{noise}}^{\text{sam}}}{N_{\text{sing}}^{\text{ref}} - N_{\text{noise}}^{\text{ref}}}$$

Conventional approach



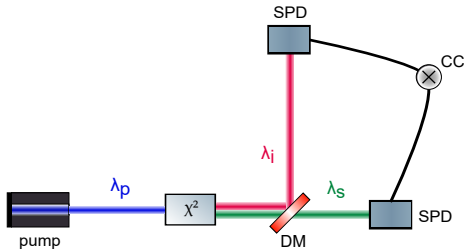
$$N_{\text{sing}}^{\text{ref}} = \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{ref}}$$

$$N_{\text{sing}}^{\text{sam}} = T \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{sam}}$$

$$\Rightarrow T = \frac{N_{\text{sing}}^{\text{sam}} - N_{\text{noise}}^{\text{sam}}}{N_{\text{sing}}^{\text{ref}} - N_{\text{noise}}^{\text{ref}}}$$

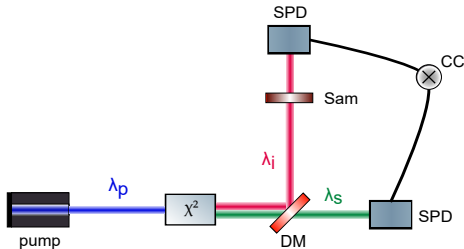
$$\text{Var}(T) = (\eta_{\text{idl}} N_g)^{-2} \left[\text{Var}(N_{\text{sing}}^{\text{sam}}) + \text{Var}(N_{\text{noise}}^{\text{sam}}) + T^2 \left[\text{Var}(N_{\text{sing}}^{\text{ref}}) + \text{Var}(N_{\text{noise}}^{\text{ref}}) \right] \right]$$

Coincidence approach



$$N_{\text{coin}}^{\text{ref}} = \eta_{\text{idl}} \eta_{\text{sig}} N_g + N_{\text{ac}}^{\text{ref}}$$

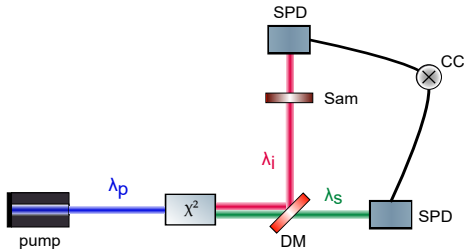
Coincidence approach



$$N_{\text{coin}}^{\text{ref}} = \eta_{\text{idl}} \eta_{\text{sig}} N_g + N_{\text{ac}}^{\text{ref}}$$

$$N_{\text{coin}}^{\text{sam}} = T \eta_{\text{idl}} \eta_{\text{sig}} N_g + N_{\text{ac}}^{\text{sam}}$$

Coincidence approach

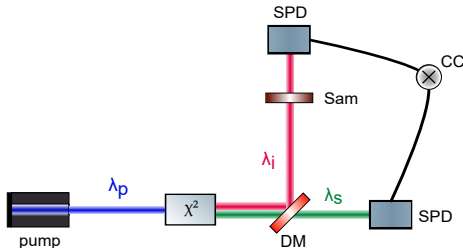


$$N_{\text{coin}}^{\text{ref}} = \eta_{\text{idl}} \eta_{\text{sig}} N_g + N_{\text{ac}}^{\text{ref}}$$

$$N_{\text{coin}}^{\text{sam}} = T \eta_{\text{idl}} \eta_{\text{sig}} N_g + N_{\text{ac}}^{\text{sam}}$$

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Coincidence approach



$$N_{\text{coin}}^{\text{ref}} = \eta_{\text{idl}} \eta_{\text{sig}} N_g + N_{\text{ac}}^{\text{ref}}$$

$$N_{\text{coin}}^{\text{sam}} = T \eta_{\text{idl}} \eta_{\text{sig}} N_g + N_{\text{ac}}^{\text{sam}}$$

$$\Rightarrow T = \frac{N_{\text{coin}}^{\text{sam}} - N_{\text{ac}}^{\text{sam}}}{N_{\text{coin}}^{\text{ref}} - N_{\text{ac}}^{\text{ref}}}$$

$$\text{Var}(T) = (\eta_{\text{sig}} \eta_{\text{idl}} N_g)^{-2} \left[\text{Var}(N_{\text{coin}}^{\text{sam}}) + \text{Var}(N_{\text{ac}}^{\text{sam}}) + T^2 \left[\text{Var}(N_{\text{coin}}^{\text{ref}}) + \text{Var}(N_{\text{ac}}^{\text{ref}}) \right] \right]$$

Transmittance model

Conventional approach:

$$\text{Var}(T) = (\eta_{\text{idl}} N_g)^{-2} \left[\text{Var}(N_{\text{sing}}^{\text{sam}}) + \text{Var}(N_{\text{noise}}^{\text{sam}}) + T^2 \left[\text{Var}(N_{\text{sing}}^{\text{ref}}) + \text{Var}(N_{\text{noise}}^{\text{ref}}) \right] \right]$$

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Transmittance model

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Coincidence approach:

$$\text{Var}(T) = (\eta_{\text{sig}} \eta_{\text{idl}} N_g)^{-2} \left[\text{Var}(N_{\text{coin}}^{\text{sam}}) + \text{Var}(N_{\text{ac}}^{\text{sam}}) + T^2 \left[\text{Var}(N_{\text{coin}}^{\text{ref}}) + \text{Var}(N_{\text{ac}}^{\text{ref}}) \right] \right]$$

Photon statistics

Photon statistics

Poisson distribution (coherent light):

$$\mathcal{P}(n) = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}$$

$$\text{Var}(n) = \langle n \rangle$$

Photon statistics

Poisson distribution (coherent light): ^[2]^[3]

$$\mathcal{P}(n) = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}$$

$$\text{Var}(n) = \langle n \rangle$$

^[2]Kim et al., "Photon-Counting Statistics-Based Support Vector Machine with Multi-Mode Photon Illumination for Quantum Imaging"

^[3]Fouche, "Detection and False-Alarm Probabilities for Laser Radars That Use Geiger-mode Detectors"

Photon statistics

Poisson distribution (coherent light): ^[2]^[3]

$$\mathcal{P}(n) = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}$$
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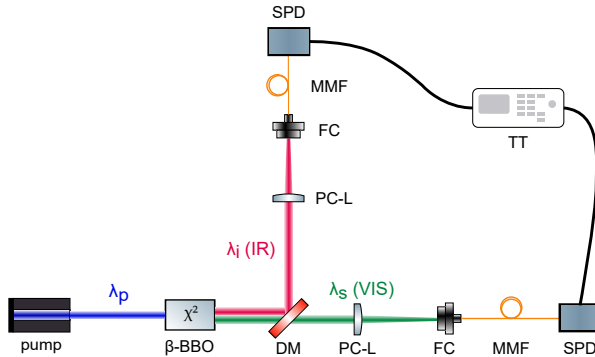
Multi-mode Bose-Einstein distribution (thermal light): ^[2]

$$\mathcal{P}_m(n) = \frac{(n+m-1)!}{(m-1)!n!} \frac{m^m \langle n \rangle^n}{(m + \langle n \rangle)^{n+m}}$$
$$\text{Var}(n) = \langle n \rangle \left(1 + \frac{\langle n \rangle}{m} \right)$$

^[2]Kim et al., "Photon-Counting Statistics-Based Support Vector Machine with Multi-Mode Photon Illumination for Quantum Imaging"

^[3]Fouche, "Detection and False-Alarm Probabilities for Laser Radars That Use Geiger-mode Detectors"

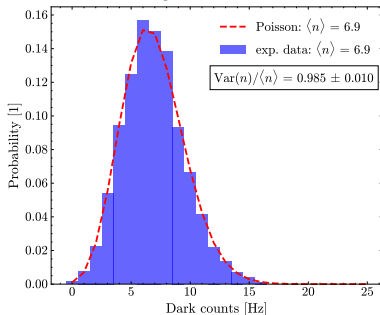
Experimental setup



Dark counts

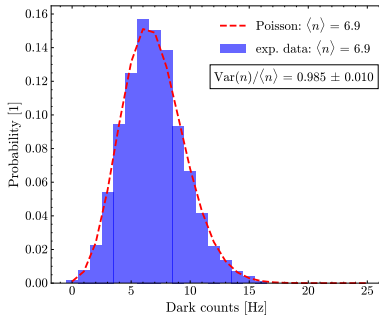
Dark counts

Signal arm

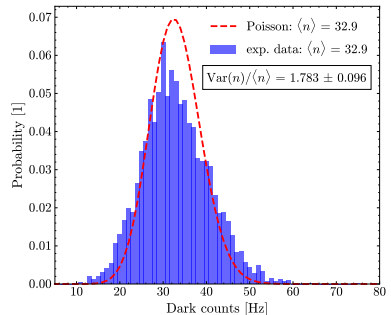


Dark counts

Signal arm

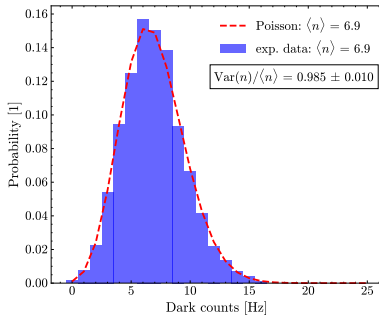


Idler arm

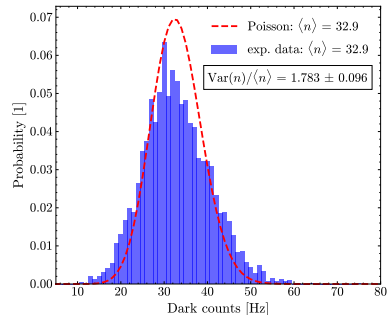


Dark counts

Signal arm



Idler arm



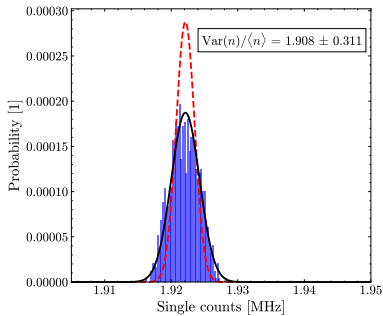
$$\text{Var}(N_{\text{noise}}) = 1.8 \cdot \langle N_{\text{noise}} \rangle$$

Single counts

Single counts

Signal arm

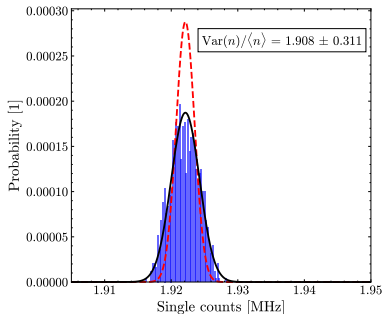
- Poisson: $\langle n \rangle = 1922127$
- mmBE: $m = 1412365$, $\langle n \rangle = 1922127$
- exp. data: $m_{theo} = 1412365$, $\langle n \rangle = 1922127$



Single counts

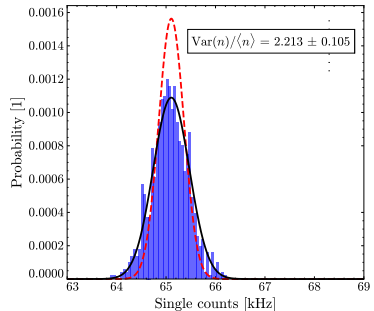
Signal arm

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Idler arm

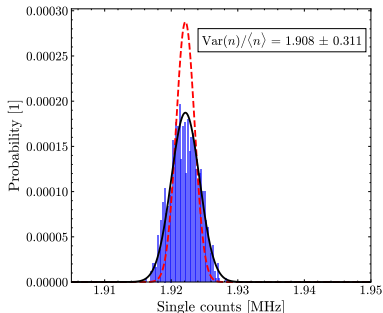
- Poisson: $\langle n \rangle = 65107$
- mmBE: $m = 61384$, $\langle n \rangle = 65107$
- exp. data: $m_{theo} = 61384$, $\langle n \rangle = 65107$



Single counts

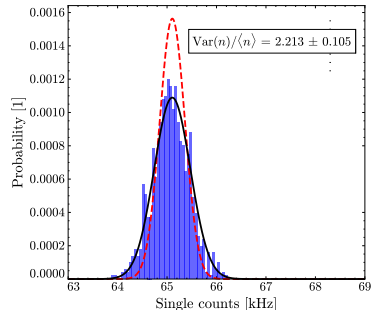
Signal arm

- Poisson: $\langle n \rangle = 1922127$
- mmBE: $m = 1412365$, $\langle n \rangle = 1922127$
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Idler arm

- Poisson: $\langle n \rangle = 65107$
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- exp. data: $m_{theo} = 61384$, $\langle n \rangle = 65107$

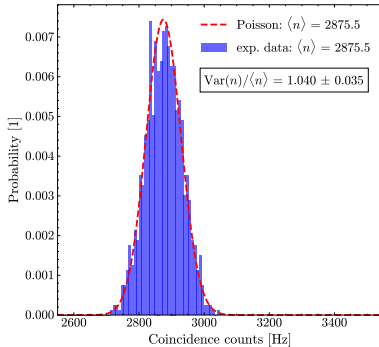


$$\text{Var}(N_{\text{sing}}) = 2.2 \cdot \langle N_{\text{sing}} \rangle$$

Coincidence counts

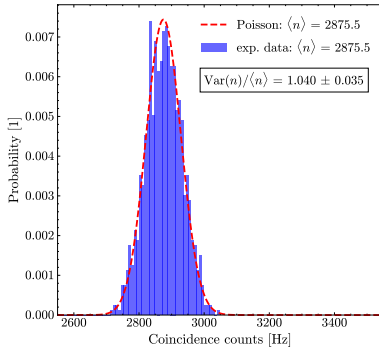
Coincidence counts

Coincidences



Coincidence counts

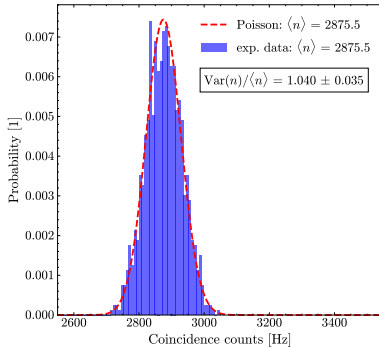
Coincidences



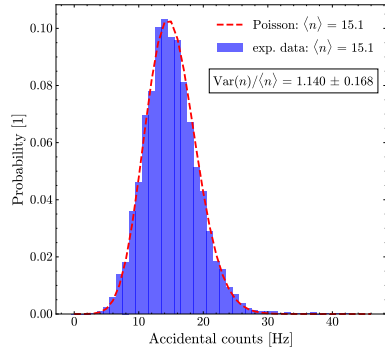
$$\text{Var}(N_{\text{cc}}) = \langle N_{\text{cc}} \rangle$$

Coincidence counts

Coincidences



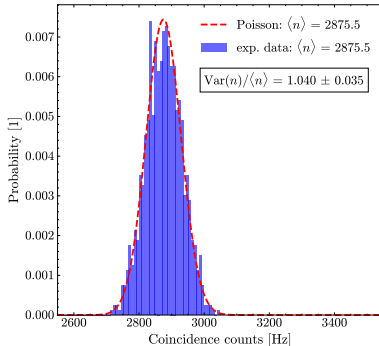
Accidentals



$$\text{Var}(N_{\text{cc}}) = \langle N_{\text{cc}} \rangle$$

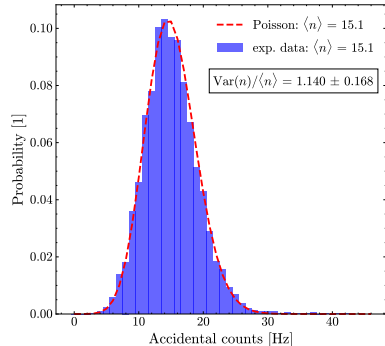
Coincidence counts

Coincidences



$$\text{Var}(N_{\text{cc}}) = \langle N_{\text{cc}} \rangle$$

Accidentals



$$\text{Var}(N_{\text{ac}}) = \langle N_{\text{ac}} \rangle$$

Simulation

Conventional approach:

$$\text{Var}(\mathcal{T}) = (\eta_{\text{idl}} N_g)^{-2} \left[\text{Var}(N_{\text{sing}}^{\text{sam}}) + \text{Var}(N_{\text{noise}}^{\text{sam}}) + \mathcal{T}^2 \left[\text{Var}(N_{\text{sing}}^{\text{ref}}) + \text{Var}(N_{\text{noise}}^{\text{ref}}) \right] \right]$$

Simulation

Conventional approach:

$$\text{Var}(T) = (\eta_{\text{idl}} N_g)^{-2} \left[2.2 \cdot \langle N_{\text{sing}}^{\text{sam}} \rangle + 1.8 \cdot \langle N_{\text{noise}}^{\text{sam}} \rangle + T^2 \left[2.2 \cdot \langle N_{\text{sing}}^{\text{ref}} \rangle + 1.8 \cdot \langle N_{\text{noise}}^{\text{ref}} \rangle \right] \right]$$

Simulation

Conventional approach:

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Coincidence approach:

$$\text{Var}(T) = (\eta_{\text{sig}} \eta_{\text{idl}} N_g)^{-2} \left[\text{Var}(N_{\text{coin}}^{\text{sam}}) + \text{Var}(N_{\text{ac}}^{\text{sam}}) + T^2 \left[\text{Var}(N_{\text{coin}}^{\text{ref}}) + \text{Var}(N_{\text{ac}}^{\text{ref}}) \right] \right]$$

Simulation

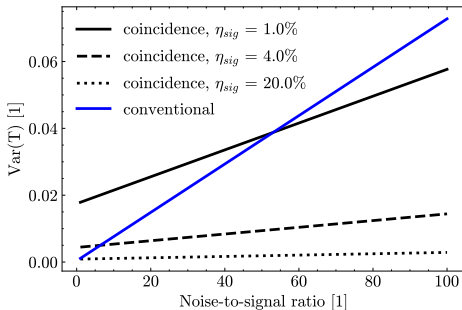
Conventional approach:

$$\text{Var}(T) = (\eta_{\text{idl}} N_g)^{-2} \left[2.2 \cdot \langle N_{\text{sing}}^{\text{sam}} \rangle + 1.8 \cdot \langle N_{\text{noise}}^{\text{sam}} \rangle + T^2 \left[2.2 \cdot \langle N_{\text{sing}}^{\text{ref}} \rangle + 1.8 \cdot \langle N_{\text{noise}}^{\text{ref}} \rangle \right] \right]$$

Coincidence approach:

$$\text{Var}(T) = (\eta_{\text{sig}} \eta_{\text{idl}} N_g)^{-2} \left[\langle N_{\text{coin}}^{\text{sam}} \rangle + \langle N_{\text{ac}}^{\text{sam}} \rangle + T^2 \left[\langle N_{\text{coin}}^{\text{ref}} \rangle + \langle N_{\text{ac}}^{\text{ref}} \rangle \right] \right]$$

Noise-to-signal ratio



Parameter	Value
η_{idl} (%)	0.09
η_{sig} (%)	1, 4, 20
R_{idl} (kHz)	10
$R_{\text{noise,idl}}$ (kHz)	10 - 1000
$R_{\text{noise,sig}}$ (Hz)	7
T (1)	0.9

Summary and Outlook

Summary and Outlook

Summary:

- Established a model for the variance of transmittance
- Experimental verification of photon statistics
- Found regions where coincidence approach offers higher precision

Summary and Outlook

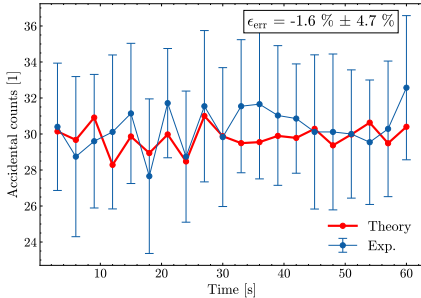
Summary:

- Established a model for the variance of transmittance
- Experimental verification of photon statistics
- Found regions where coincidence approach offers higher precision

Outlook:

- Experimental verification of the found parameter regions
- Determine the experimental limitations for the variance measurement

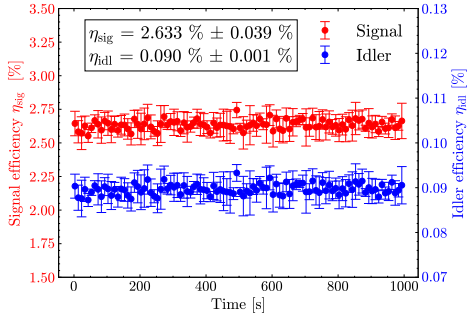
Accidental counts



$$R_{\text{ac}}^{\text{sam}} = \left(T \eta_{\text{idl}} R_{\text{g}} + R_{\text{dc,idl}} - R_{\text{cc,pure}}^{\text{sam}} \right) \cdot \left(\eta_{\text{sig}} R_{\text{g}} + R_{\text{dc,sig}} - R_{\text{cc,pure}}^{\text{sam}} \right) \cdot \tau_{\text{cw}}$$

$$R_{\text{ac}}^{\text{ref}} = \left(\eta_{\text{idl}} R_{\text{g}} + R_{\text{dc,idl}} - R_{\text{cc,pure}}^{\text{ref}} \right) \cdot \left(\eta_{\text{sig}} R_{\text{g}} + R_{\text{dc,sig}} - R_{\text{cc,pure}}^{\text{ref}} \right) \cdot \tau_{\text{cw}}$$

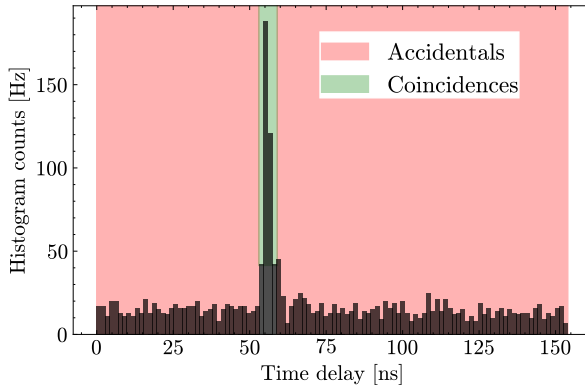
Heralding efficiencies



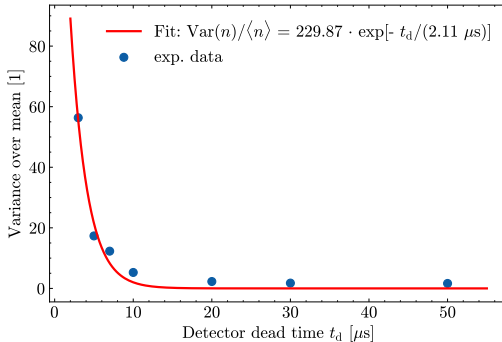
$$\eta_{\text{sig}} = \frac{N_{\text{coin}} - N_{\text{ac}}}{N_{\text{sing, idl}} - N_{\text{noise}}} = \frac{\eta_{\text{sig}} \eta_{\text{idl}} N_{\text{g}}}{\eta_{\text{idl}} N_{\text{g}}}$$

$$\eta_{\text{idl}} = \frac{N_{\text{coin}} - N_{\text{ac}}}{N_{\text{sing, sig}} - N_{\text{noise}}} = \frac{\eta_{\text{sig}} \eta_{\text{idl}} N_{\text{g}}}{\eta_{\text{sig}} N_{\text{g}}}$$

Histogram

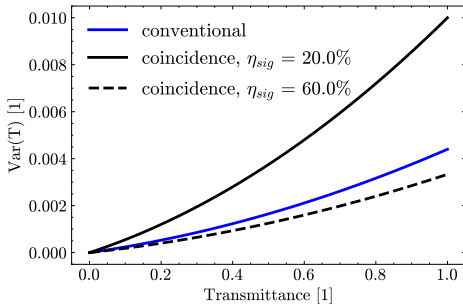


Afterpulsing



$$R_{aft} \propto e^{-\frac{t_d}{\tau_{de}}}$$

Noiseless case

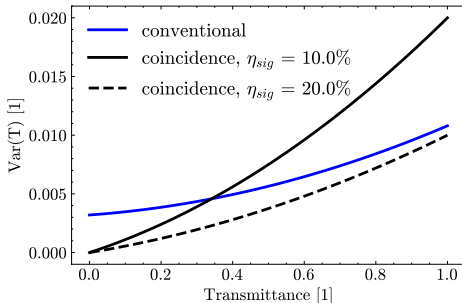


Parameter

Value

η_{idl} (%)	60
η_{sig} (%)	20, 60
R_{idl} (kHz)	1
$R_{noise,idl}$ (kHz)	0
$R_{noise,sig}$ (Hz)	7
T (1)	0 - 1

In presence of noise



Parameter

Value

η_{idl} (%)

60

η_{sig} (%)

10, 20

R_{idl} (kHz)

1

$R_{noise,idl}$ (kHz)

1

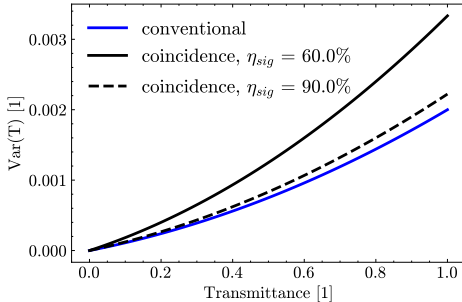
$R_{noise,sig}$ (Hz)

7

T (1)

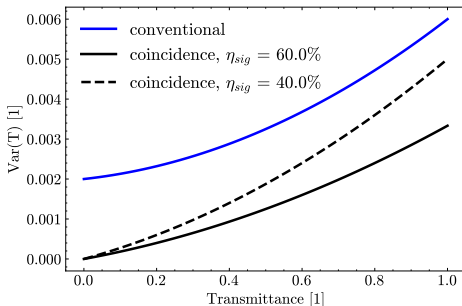
0 - 1

Coherent Illumination, noiseless



Parameter	Value
η_{idl} (%)	60
η_{sig} (%)	60, 90
R_{idl} (kHz)	1
$R_{\text{noise,idl}}$ (kHz)	0
$R_{\text{noise,sig}}$ (Hz)	7
T (1)	0 - 1

Coherent Illumination, with noise



Parameter	Value
η_{idl} (%)	60
η_{sig} (%)	40, 60
R_{idl} (kHz)	1
$R_{\text{noise,idl}}$ (kHz)	1
$R_{\text{noise,sig}}$ (Hz)	7
T (1)	0 - 1