Parameter estimation with correlated photon pairs

Jan Gößwein

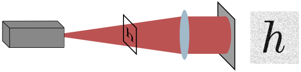
Institute of Applied Physics

Jena, November 4, 2025

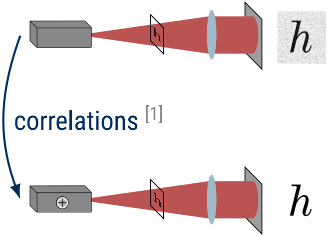
Table of content

Motivation

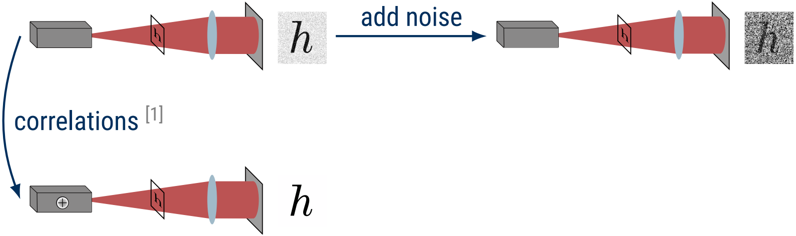
Motivation



Motivation

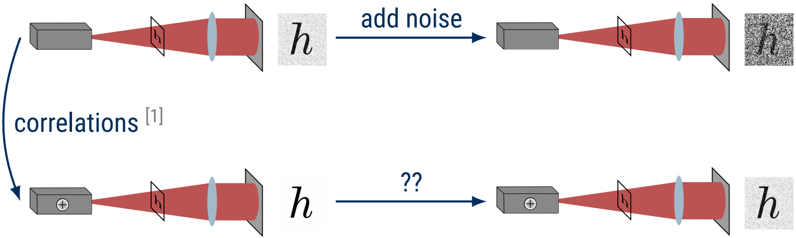


Motivation



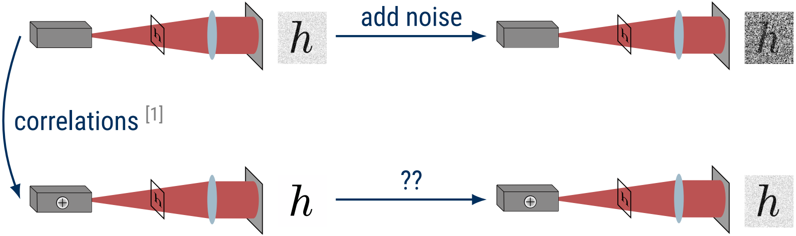
[1] Brida, Genovese, and Ruo Berchera, "Experimental Realization of Sub-Shot-Noise Quantum Imaging"

Motivation



[1] Brida, Genovese, and Ruo Berchera, "Experimental Realization of Sub-Shot-Noise Quantum Imaging"

Motivation

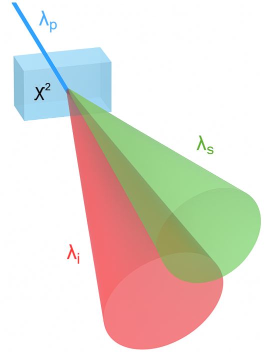


Objective: Can correlated photons provide advantages in terms of precision in noisy regimes for parameter estimation?

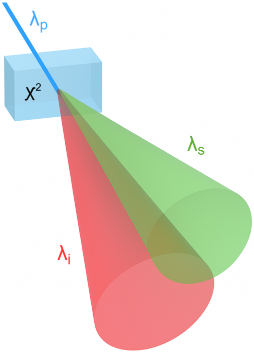
[1] Brida, Genovese, and Ruo Berchera, "Experimental Realization of Sub-Shot-Noise Quantum Imaging"

SPDC

SPDC



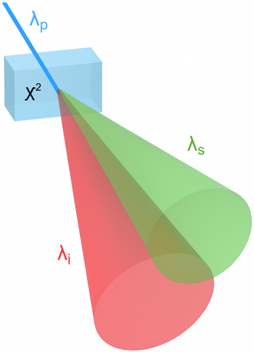
SPDC



Energy conservation

$$\omega_p = \omega_s + \omega_i$$

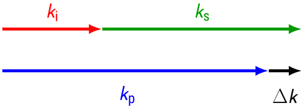
SPDC



Energy conservation

$$\omega_p = \omega_s + \omega_i$$

Momentum conservation



$$\vec{k}_p = \vec{k}_s + \vec{k}_i - \Delta\vec{k}$$

Parameter estimation

Setup: Transmission setup

Parameter estimation

Setup: Transmission setup
Parameter: Transmittance T

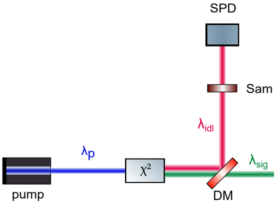
Parameter estimation

Setup: Transmission setup
Parameter: Transmittance T
Estimator: $\mathrm{Var}(T)$

Parameter estimation

Setup: Transmission setup
Parameter: Transmittance T
Estimator: $\mathrm{Var}(T)$

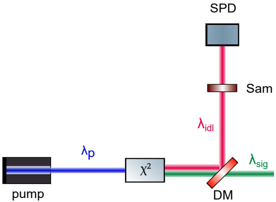
Conventional approach:



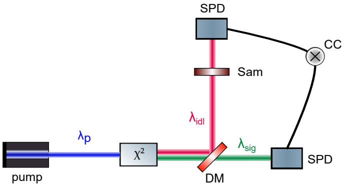
Parameter estimation

Setup: Transmission setup
Parameter: Transmittance T
Estimator: Var(T)

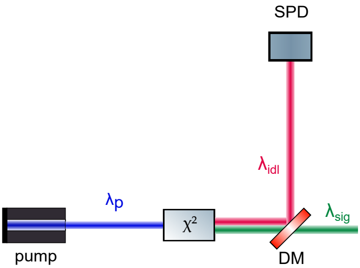
Conventional approach:



Coincidence approach:

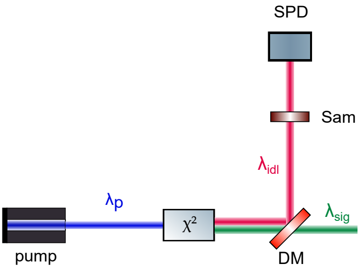


Conventional approach



$$N_{\mathrm{tot}}^{\mathrm{ref}} = \eta_{\mathrm{idl}} N_{\mathrm{g}} + N_{\mathrm{noise}}^{\mathrm{ref}}$$

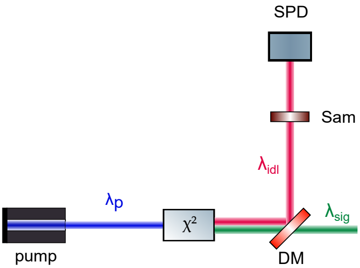
Conventional approach



$$N_{\mathrm{tot}}^{\mathrm{ref}} = \eta_{\mathrm{idl}} N_{\mathrm{g}} + N_{\mathrm{noise}}^{\mathrm{ref}}$$

$$N_{\mathrm{tot}}^{\mathrm{sam}} = T\, \eta_{\mathrm{idl}} N_{\mathrm{g}} + N_{\mathrm{noise}}^{\mathrm{sam}}$$

Conventional approach

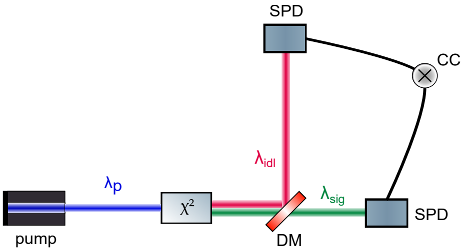


$$N_{\mathrm{tot}}^{\mathrm{ref}} = \eta_{\mathrm{idl}} N_{\mathrm{g}} + N_{\mathrm{noise}}^{\mathrm{ref}}$$

$$N_{\mathrm{tot}}^{\mathrm{sam}} = T\,\eta_{\mathrm{idl}} N_{\mathrm{g}} + N_{\mathrm{noise}}^{\mathrm{sam}}$$

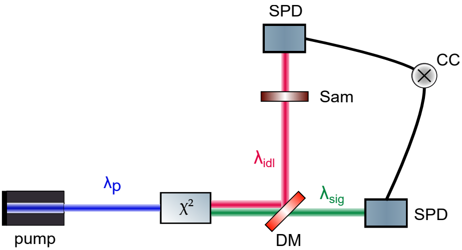
$$\Rightarrow T = \frac{N_{\mathrm{tot}}^{\mathrm{sam}} - N_{\mathrm{noise}}^{\mathrm{sam}}}{N_{\mathrm{tot}}^{\mathrm{ref}} - N_{\mathrm{noise}}^{\mathrm{ref}}}$$

Coincidence approach



$$N_{\mathrm{cc,tot}}^{\mathrm{ref}} = \eta_{\mathrm{idl}}\, \eta_{\mathrm{sig}}\, N_{\mathrm{g}} + N_{\mathrm{ac}}^{\mathrm{ref}}$$

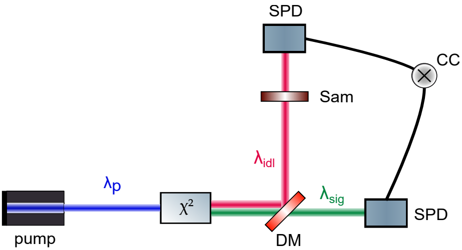
Coincidence approach



$$N_{\text{cc,tot}}^{\text{ref}} = \eta_{\text{idl}}\,\eta_{\text{sig}}\,N_{\text{g}} + N_{\text{ac}}^{\text{ref}}$$

$$N_{\text{cc,tot}}^{\text{sam}} = T\,\eta_{\text{idl}}\,\eta_{\text{sig}}\,N_{\text{g}} + N_{\text{ac}}^{\text{sam}}$$

Coincidence approach



$$N^{ref}_{cc,tot} = \eta_{idl}\,\eta_{sig}\,N_g + N^{ref}_{ac}$$

$$N^{sam}_{cc,tot} = T\,\eta_{idl}\,\eta_{sig}\,N_g + N^{sam}_{ac}$$

$$\Rightarrow T = \frac{N^{sam}_{tot,cc} - N^{sam}_{ac}}{N^{ref}_{tot,cc} - N^{ref}_{ac}}$$

Transmittance model

Conventional approach:

$$\mathrm{Var}(T) = (\eta_{\mathrm{idl}}\, N_{\mathrm{g}})^{-2} \left[\mathrm{Var}(N^{\mathrm{sam}}_{\mathrm{tot}}) + \mathrm{Var}\left(N^{\mathrm{sam}}_{\mathrm{noise}}\right) + T^2 \left[\mathrm{Var}\left(N^{\mathrm{ref}}_{\mathrm{tot}}\right) + \mathrm{Var}\left(N^{\mathrm{ref}}_{\mathrm{noise}}\right) \right] \right]$$

Coincidence approach:

$$\mathrm{Var}(T) = \left(\eta_{\mathrm{sig}}\, \eta_{\mathrm{idl}}\, N_{\mathrm{g}}\right)^{-2} \left[\mathrm{Var}\left(N^{\mathrm{sam}}_{\mathrm{tot,cc}}\right) + \mathrm{Var}(N^{\mathrm{sam}}_{\mathrm{ac}}) + T^2 \left[\mathrm{Var}\left(N^{\mathrm{ref}}_{\mathrm{tot,cc}}\right) + \mathrm{Var}\left(N^{\mathrm{ref}}_{\mathrm{ac}}\right) \right] \right]$$

Transmittance model

Conventional approach:

$$\mathrm{Var}(T) = (\eta_{\mathrm{idl}}\, N_{\mathrm{g}})^{-2} \left[\mathrm{Var}(N^{\mathrm{sam}}_{\mathrm{tot}}) + \mathrm{Var}(N^{\mathrm{sam}}_{\mathrm{noise}}) + T^2 \left[\mathrm{Var}(N^{\mathrm{ref}}_{\mathrm{tot}}) + \mathrm{Var}(N^{\mathrm{ref}}_{\mathrm{noise}}) \right] \right]$$

Coincidence approach:

$$\mathrm{Var}(T) = (\eta_{\mathrm{sig}}\, \eta_{\mathrm{idl}}\, N_{\mathrm{g}})^{-2} \left[\mathrm{Var}(N^{\mathrm{sam}}_{\mathrm{tot,cc}}) + \mathrm{Var}(N^{\mathrm{sam}}_{\mathrm{ac}}) + T^2 \left[\mathrm{Var}(N^{\mathrm{ref}}_{\mathrm{tot,cc}}) + \mathrm{Var}(N^{\mathrm{ref}}_{\mathrm{ac}}) \right] \right]$$

Photon statistics

Photon statistics

Poisson distribution (coherent light): [2][3]

$$\mathcal{P}(n) = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}$$
$$\mathrm{Var}(n) = \langle n \rangle$$

Photon statistics

Poisson distribution (coherent light): [2][3]

$$\mathcal{P}(n) = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}$$
$$\mathrm{Var}(n) = \langle n \rangle$$

[2] Kim et al., “Photon-Counting Statistics-Based Support Vector Machine with Multi-Mode Photon Illumination for Quantum Imaging”
[3] Fouche, “Detection and False-Alarm Probabilities for Laser Radars That Use Geiger-mode Detectors”

Photon statistics

Poisson distribution (coherent light): [2][3]

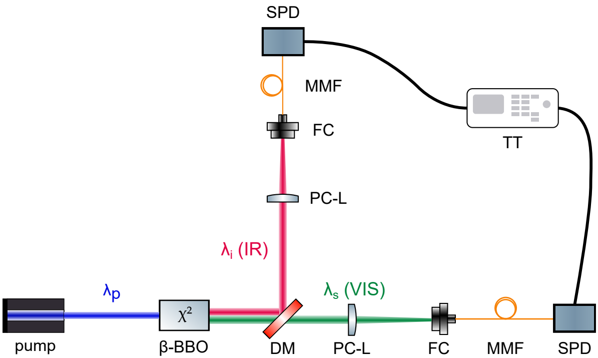
$$\mathcal{P}(n) = \frac{\langle n \rangle^n}{n!}\, e^{-\langle n \rangle}$$
$$\mathrm{Var}(n) = \langle n \rangle$$

Multi-mode Bose-Einstein distribution (thermal light): [2]

$$\mathcal{P}_m(n) = \frac{(n+m-1)!}{(m-1)!\, n!}\, \frac{m^m \langle n \rangle^n}{(m+\langle n \rangle)^{n+m}}$$
$$\mathrm{Var}(n) = \langle n \rangle \left(1 + \frac{\langle n \rangle}{m}\right)$$

[2] Kim et al., "Photon-Counting Statistics-Based Support Vector Machine with Multi-Mode Photon Illumination for Quantum Imaging"
[3] Fouche, "Detection and False-Alarm Probabilities for Laser Radars That Use Geiger-mode Detectors"

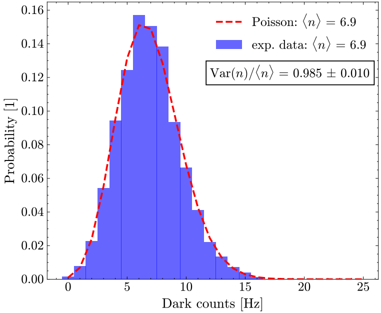
Experimental setup



Dark counts

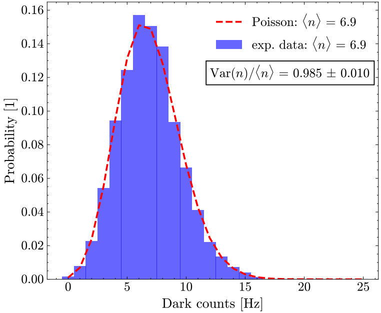
Dark counts

Signal arm

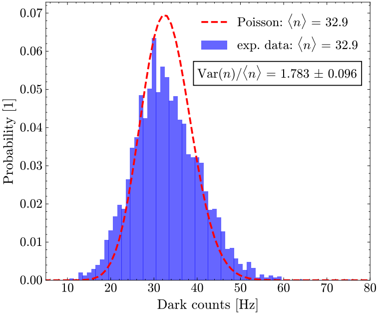


Dark counts

Signal arm

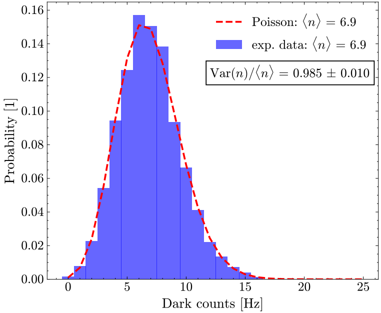


Idler arm

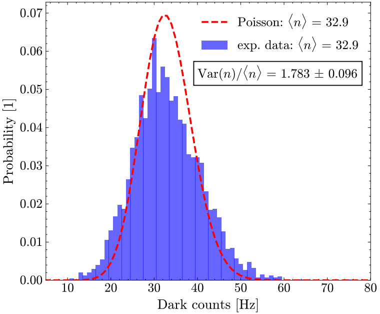


Dark counts

Signal arm



Idler arm

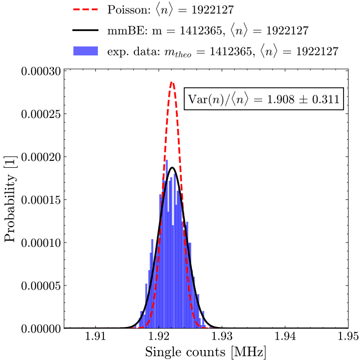


$$\mathrm{Var}(N_{\mathrm{noise}}) = 1.8 \cdot \langle N_{\mathrm{noise}} \rangle$$

Single counts

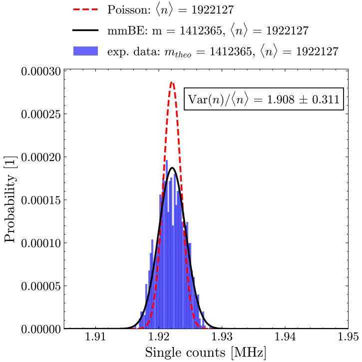
Single counts

Signal arm

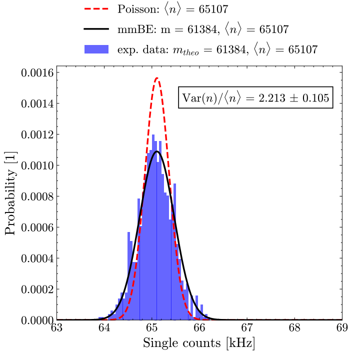


Single counts

Signal arm

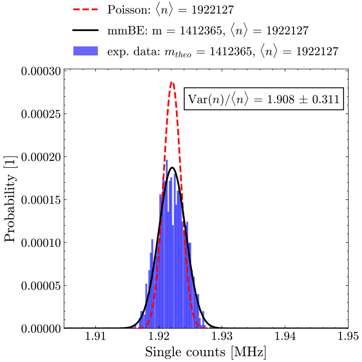


Idler arm

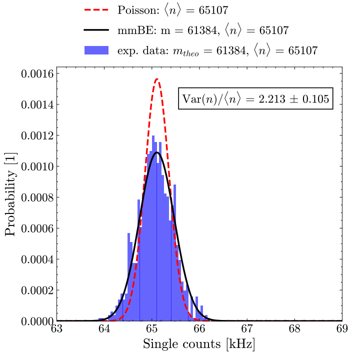


Single counts

Signal arm



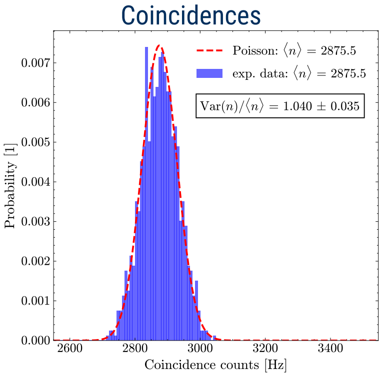
Idler arm



$$\mathrm{Var}(N_{\mathrm{noise}}) = 2.2 \cdot \langle N_{\mathrm{noise}} \rangle$$

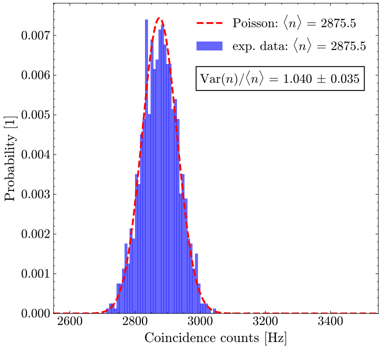
Coincidence counts

Coincidence counts



Coincidence counts

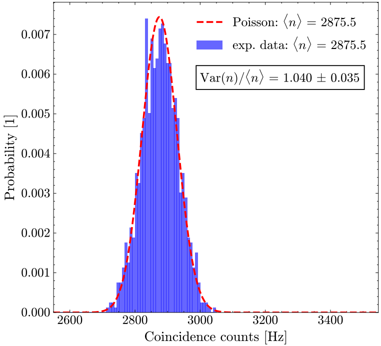
Coincidences



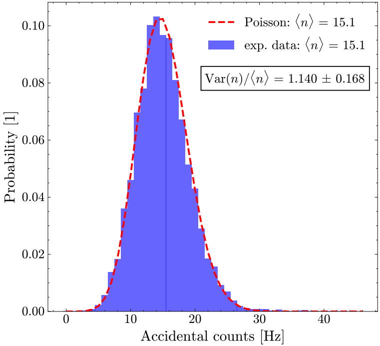
$$\mathrm{Var}(N_{\mathrm{cc}}) = \langle N_{\mathrm{cc}} \rangle$$

Coincidence counts

Coincidences

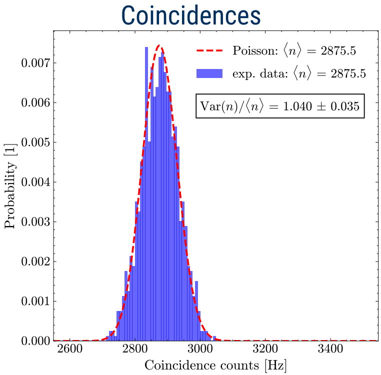


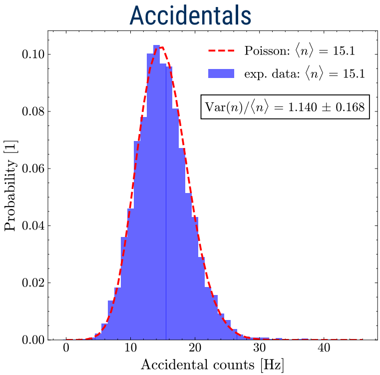
Accidentals



$$\mathrm{Var}(N_{\mathrm{cc}}) = \langle N_{\mathrm{cc}} \rangle$$

Coincidence counts





$$\mathrm{Var}(N_{cc}) = \langle N_{cc} \rangle$$

$$\mathrm{Var}(N_{ac}) = \langle N_{ac} \rangle$$

Simulation

Conventional approach:

$$\mathrm{Var}(T) = (\eta_{\mathrm{idl}}\, N_{\mathrm{g}})^{-2} \left[\mathrm{Var}(N_{\mathrm{tot}}^{\mathrm{sam}}) + \mathrm{Var}(N_{\mathrm{noise}}^{\mathrm{sam}}) + T^2 \left[\mathrm{Var}(N_{\mathrm{tot}}^{\mathrm{ref}}) + \mathrm{Var}(N_{\mathrm{noise}}^{\mathrm{ref}}) \right] \right]$$

Simulation

Conventional approach:

$$\mathrm{Var}(T) = (\eta_{\mathrm{idl}}\, N_{\mathrm{g}})^{-2} \left[2.2 \cdot \langle N_{\mathrm{tot}}^{\mathrm{sam}} \rangle + 1.8 \cdot \langle N_{\mathrm{noise}}^{\mathrm{sam}} \rangle + T^2 \left[2.2 \cdot \langle N_{\mathrm{tot}}^{\mathrm{ref}} \rangle + 1.8 \cdot \langle N_{\mathrm{noise}}^{\mathrm{ref}} \rangle \right] \right]$$

Simulation

Conventional approach:

$$\mathrm{Var}(T) = (\eta_{\mathrm{idl}}\, N_{\mathrm{g}})^{-2} \left[2.2 \cdot \langle N_{\mathrm{tot}}^{\mathrm{sam}} \rangle + 1.8 \cdot \langle N_{\mathrm{noise}}^{\mathrm{sam}} \rangle + T^2 \left[2.2 \cdot \langle N_{\mathrm{tot}}^{\mathrm{ref}} \rangle + 1.8 \cdot \langle N_{\mathrm{noise}}^{\mathrm{ref}} \rangle \right] \right]$$

Coincidence approach:

$$\mathrm{Var}(T) = (\eta_{\mathrm{sig}}\, \eta_{\mathrm{idl}}\, N_{\mathrm{g}})^{-2} \left[\mathrm{Var}(N_{\mathrm{tot,cc}}^{\mathrm{sam}}) + \mathrm{Var}(N_{\mathrm{ac}}^{\mathrm{sam}}) + T^2 \left[\mathrm{Var}(N_{\mathrm{tot,cc}}^{\mathrm{ref}}) + \mathrm{Var}(N_{\mathrm{ac}}^{\mathrm{ref}}) \right] \right]$$

Simulation

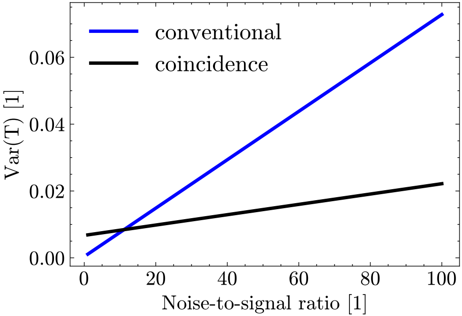
Conventional approach:

$$\mathrm{Var}(T) = (\eta_{\mathrm{idl}}\, N_{\mathrm{g}})^{-2} \left[2.2 \cdot \langle N_{\mathrm{tot}}^{\mathrm{sam}} \rangle + 1.8 \cdot \langle N_{\mathrm{noise}}^{\mathrm{sam}} \rangle + T^2 \left[2.2 \cdot \langle N_{\mathrm{tot}}^{\mathrm{ref}} \rangle + 1.8 \cdot \langle N_{\mathrm{noise}}^{\mathrm{ref}} \rangle \right] \right]$$

Coincidence approach:

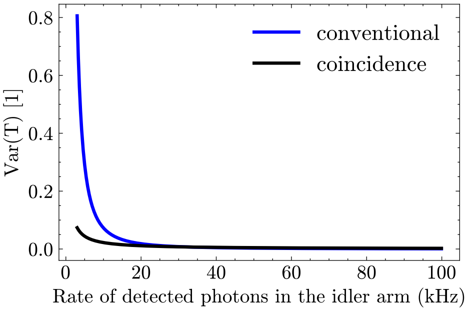
$$\mathrm{Var}(T) = \left(\eta_{\mathrm{sig}}\, \eta_{\mathrm{idl}}\, N_{\mathrm{g}}\right)^{-2} \left[\langle N_{\mathrm{tot,cc}}^{\mathrm{sam}} \rangle + \langle N_{\mathrm{ac}}^{\mathrm{sam}} \rangle + T^2 \left[\langle N_{\mathrm{tot,cc}}^{\mathrm{ref}} \rangle + \langle N_{\mathrm{ac}}^{\mathrm{ref}} \rangle \right] \right]$$

Simulation



Parameter	Value
η_{idl} (%)	0.09
η_{sig} (%)	2.6
R_{idl} (kHz)	10
$R_{\mathrm{noise,idl}}$ (kHz)	10 - 1000
$R_{\mathrm{noise,sig}}$ (Hz)	7
T (1)	0.9

Simulation



Parameter	Value
η_{idl} (%)	0.09
η_{sig} (%)	2.6
R_{idl} (kHz)	3 - 100
$R_{\mathrm{noise,idl}}$ (kHz)	1000
$R_{\mathrm{noise,sig}}$ (Hz)	7
T (1)	0.9

Summary and Outlook

Summary and Outlook

Achievements:

- Model for variance of transmittance
- Experimental verification of photon statistics
- Found regions where coincidence approach offers advantages

Summary and Outlook

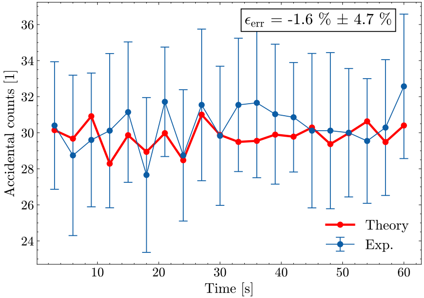
Achievements:

- Model for variance of transmittance
- Experimental verification of photon statistics
- Found regions where coincidence approach offers advantages

Outlook:

- Experimental verification of found parameter regions
- Determine the experimental limiations of the variance resolution

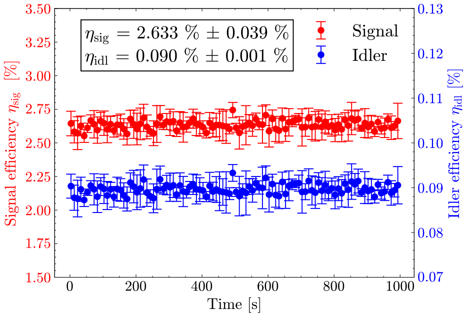
Accidental counts



$$R_{\mathrm{ac}}^{\mathrm{sam}} = \left(T\,\eta_{\mathrm{idl}}\,R_{\mathrm{g}} + R_{\mathrm{dc,idl}} - R_{\mathrm{cc,pure}}^{\mathrm{sam}}\right)\cdot \left(\eta_{\mathrm{sig}}\,R_{\mathrm{g}} + R_{\mathrm{dc,sig}} - R_{\mathrm{cc,pure}}^{\mathrm{sam}}\right)\cdot \tau_{\mathrm{cw}}$$

$$R_{\mathrm{ac}}^{\mathrm{ref}} = \left(\eta_{\mathrm{idl}}\,R_{\mathrm{g}} + R_{\mathrm{dc,idl}} - R_{\mathrm{cc,pure}}^{\mathrm{ref}}\right)\cdot \left(\eta_{\mathrm{sig}}\,R_{\mathrm{g}} + R_{\mathrm{dc,sig}} - R_{\mathrm{cc,pure}}^{\mathrm{ref}}\right)\cdot \tau_{\mathrm{cw}}$$

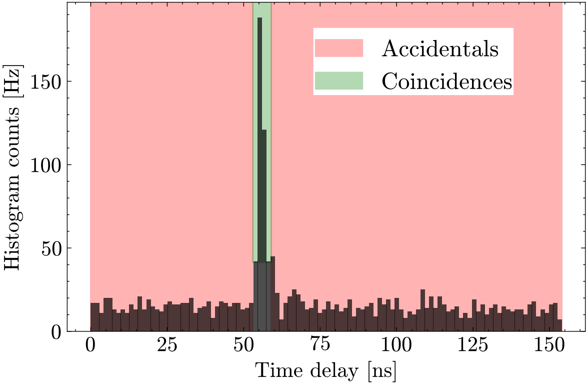
Heralding efficiencies



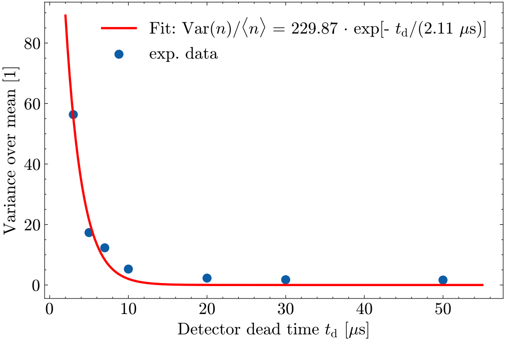
$$\eta_{sig} = \frac{N_{tot,cc} - N_{ac}}{N_{tot,idl} - N_{noise}} = \frac{\eta_{sig}\eta_{idl} N_g}{\eta_{idl} N_g}$$

$$\eta_{idl} = \frac{N_{tot,cc} - N_{ac}}{N_{tot,sig} - N_{noise}} = \frac{\eta_{sig}\eta_{idl} N_g}{\eta_{sig} N_g}$$

Histogram


Accidentals
Coincidences
Histogram counts [Hz]
0
50
100
150
0
25
50
75
100
125
150
Time delay [ns]

Afterpulsing



$$R_{aft} \propto e^{-t_d}$$