

# Parameter estimation with correlated photon pairs

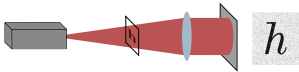
Jan Gößwein

Institute of Applied Physics

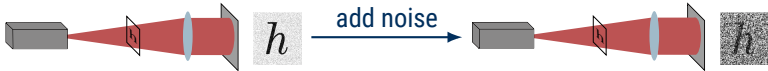
Jena, November 13, 2025

# Motivation

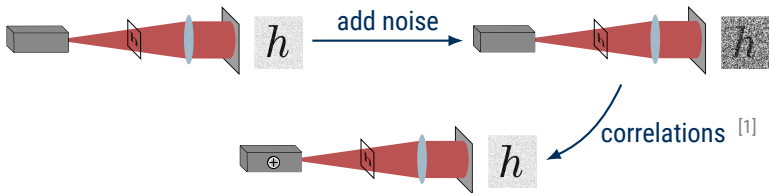
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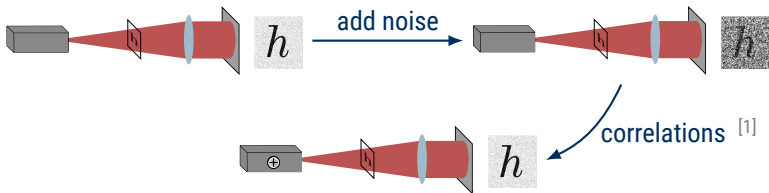


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[1] Brida, Genovese, and Ruo Berchera, "Experimental Realization of Sub-Shot-Noise Quantum Imaging"

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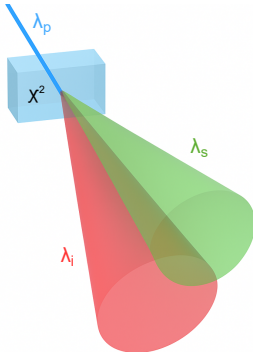


**Objective:** Can correlated photons provide advantages in terms of precision in noisy regimes for parameter estimation?

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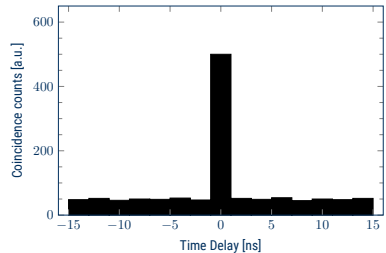
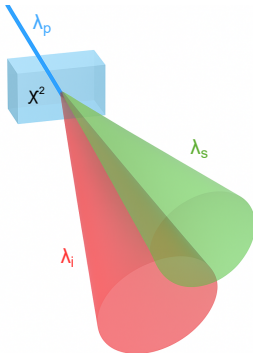
# Spontaneous parametric down-conversion

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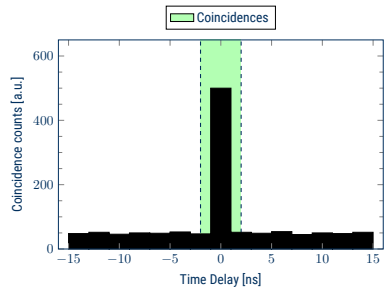
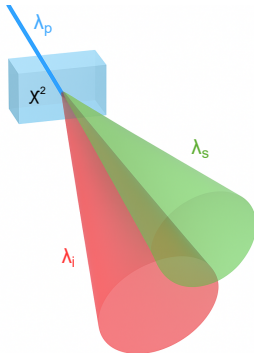




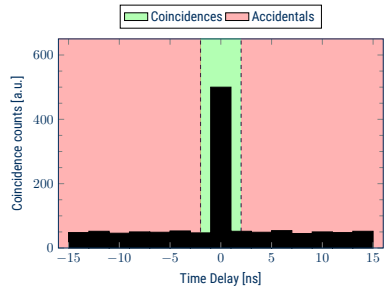
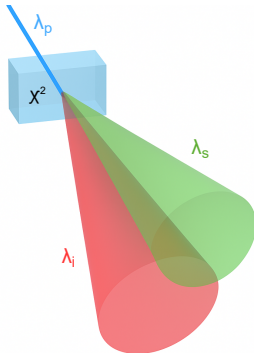
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# Parameter estimation

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**Setup:** Transmission setup

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**Precision:**  $\text{Var}(T)$

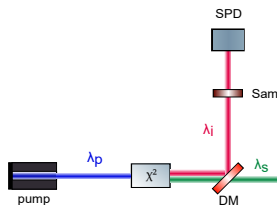
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**Setup:** Transmission setup

**Parameter:** Transmittance  $T$

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Conventional approach:





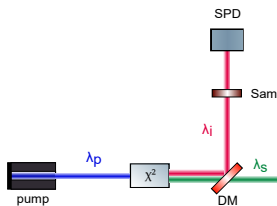
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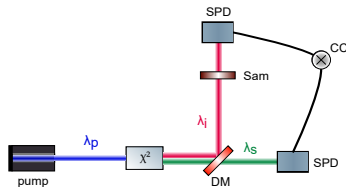
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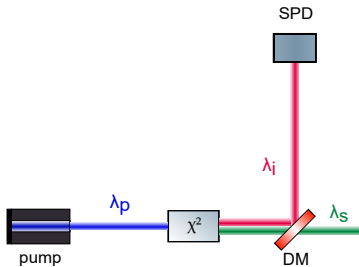
Conventional approach:



Coincidence approach:

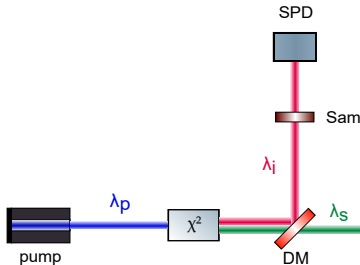


# Conventional approach



$$N_{\text{sing}}^{\text{ref}} = \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{ref}}$$

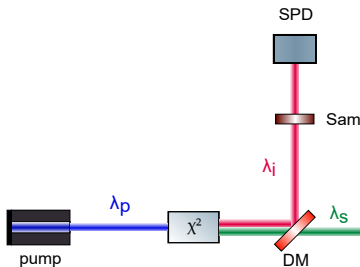
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$$N_{\text{sing}}^{\text{ref}} = \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{ref}}$$

$$N_{\text{sing}}^{\text{sam}} = T \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{sam}}$$

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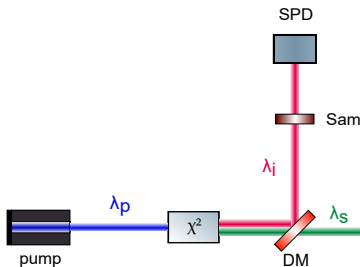


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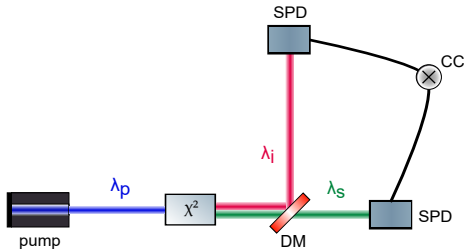
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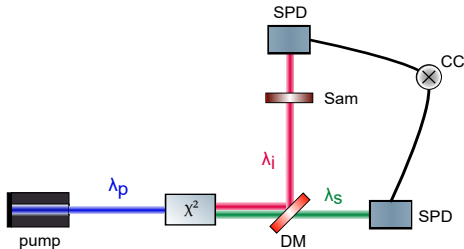
$$\text{Var}(T) = (\eta_{\text{idl}} N_g)^{-2} \left[ \text{Var}(N_{\text{sing}}^{\text{sam}}) + \text{Var}(N_{\text{noise}}^{\text{sam}}) + T^2 \left[ \text{Var}(N_{\text{sing}}^{\text{ref}}) + \text{Var}(N_{\text{noise}}^{\text{ref}}) \right] \right]$$

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$$N_{\text{coin}}^{\text{ref}} = \eta_{\text{idl}} \eta_{\text{sig}} N_g + N_{\text{ac}}^{\text{ref}}$$

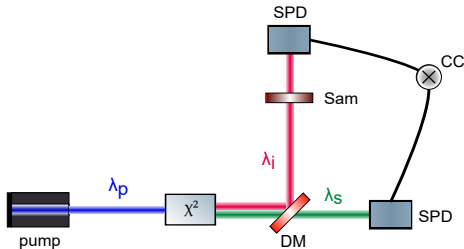
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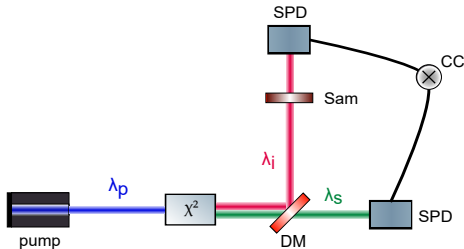
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# Transmittance model

Conventional approach:

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# Photon statistics

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Poisson distribution (coherent light):

$$\mathcal{P}(n) = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}$$

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<sup>[2]</sup>Kim et al., "Photon-Counting Statistics-Based Support Vector Machine with Multi-Mode Photon Illumination for Quantum Imaging"

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Multi-mode Bose-Einstein distribution (thermal light): <sup>[2]</sup>

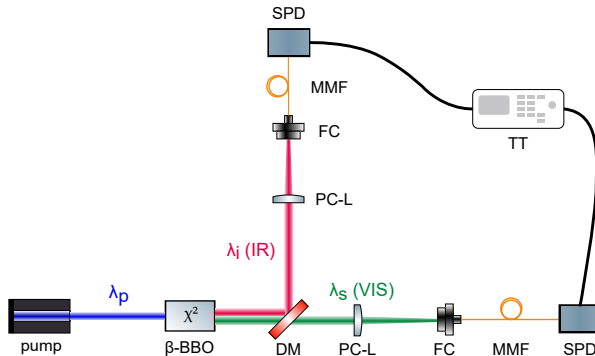
$$\mathcal{P}_m(n) = \frac{(n+m-1)!}{(m-1)!n!} \frac{m^m \langle n \rangle^n}{(m + \langle n \rangle)^{n+m}}$$
$$\text{Var}(n) = \langle n \rangle \left( 1 + \frac{\langle n \rangle}{m} \right)$$

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# Experimental setup

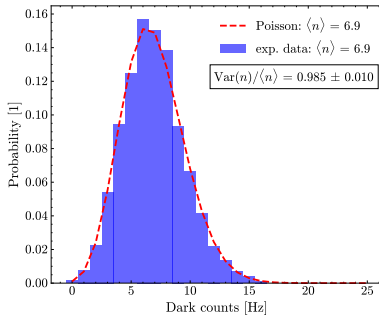




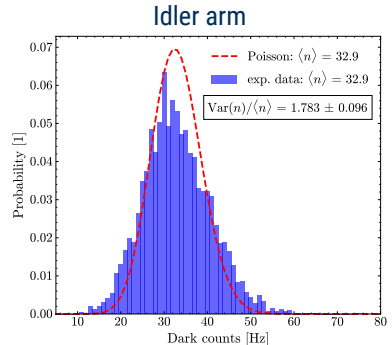
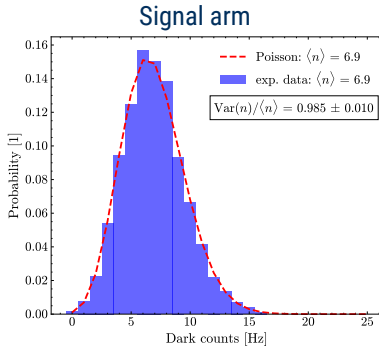
# Dark counts, $\text{Var}(N_{\text{noise}})$

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## Signal arm

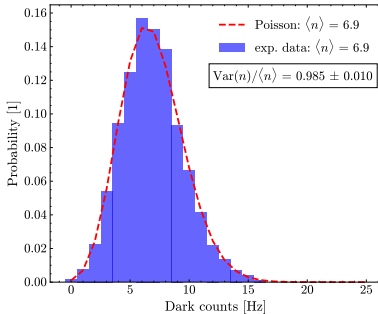


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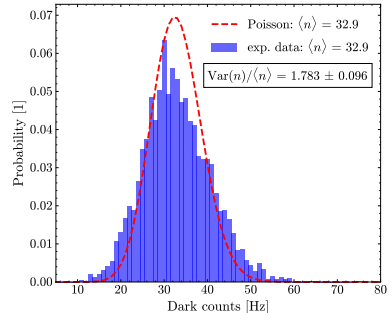


# Dark counts, $\text{Var}(N_{\text{noise}})$

## Signal arm



## Idler arm



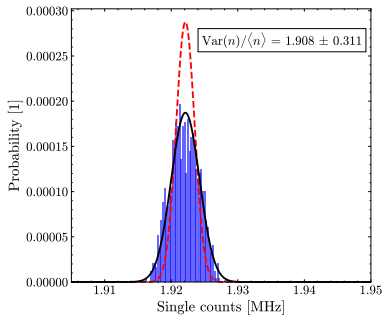
$$\text{Var}(N_{\text{noise}}) = 1.8 \cdot \langle N_{\text{noise}} \rangle$$

# Single counts, $\text{Var}(N_{\text{sing}})$

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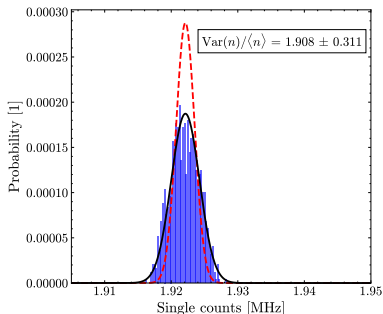
- Poisson:  $\langle n \rangle = 1922127$
- mmBE:  $m = 1412365$ ,  $\langle n \rangle = 1922127$
- exp. data:  $m_{\text{theo}} = 1412365$ ,  $\langle n \rangle = 1922127$



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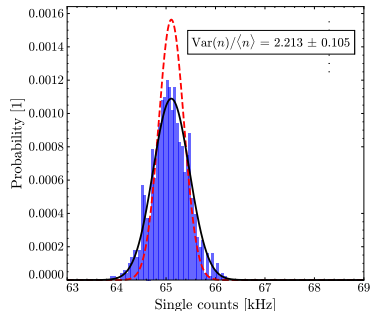
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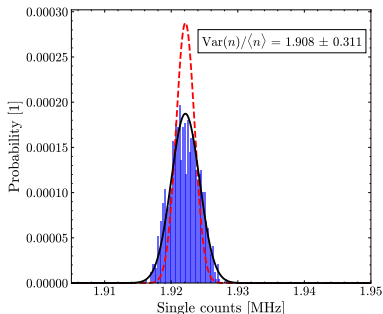
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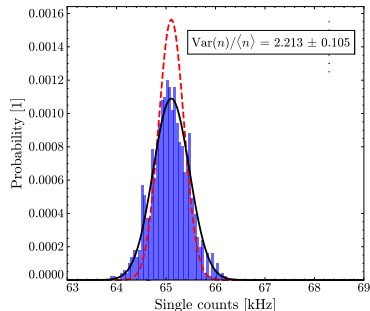
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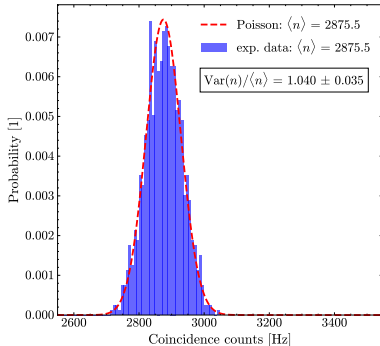
$$\text{Var}(N_{\text{sing}}) = 2.2 \cdot \langle N_{\text{sing}} \rangle$$



# Coincidence counts, $\text{Var}(N_{\text{coin}})$

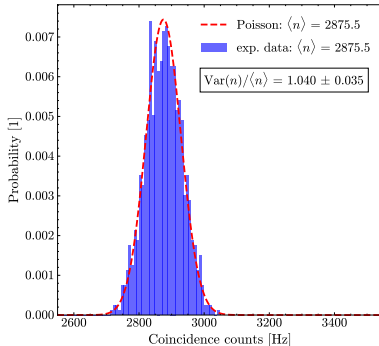
# Coincidence counts, $\text{Var}(N_{\text{coin}})$

## Coincidences



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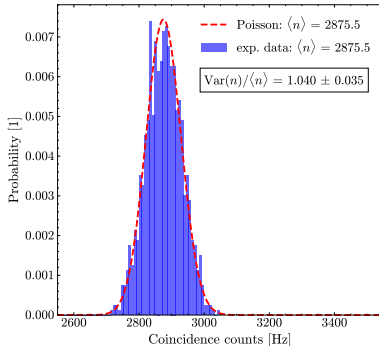
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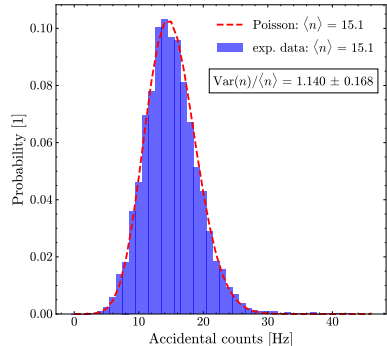
$$\text{Var}(N_{\text{coin}}) = \langle N_{\text{coin}} \rangle$$

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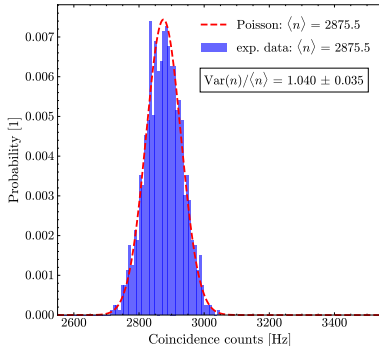
## Accidentals



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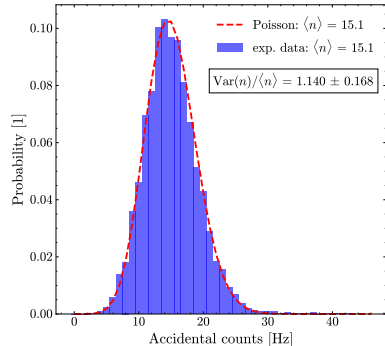
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## Coincidences



$$\text{Var}(N_{\text{coin}}) = \langle N_{\text{coin}} \rangle$$

## Accidentals



$$\text{Var}(N_{\text{ac}}) = \langle N_{\text{ac}} \rangle$$

# Simulation

## Conventional approach:

$$\text{Var}(\mathcal{T}) = (\eta_{\text{idl}} N_g)^{-2} \left[ \text{Var}(N_{\text{sing}}^{\text{sam}}) + \text{Var}(N_{\text{noise}}^{\text{sam}}) + \mathcal{T}^2 \left[ \text{Var}(N_{\text{sing}}^{\text{ref}}) + \text{Var}(N_{\text{noise}}^{\text{ref}}) \right] \right]$$

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$$\text{Var}(T) = (\eta_{\text{idl}} N_g)^{-2} \left[ 2.2 \cdot \langle N_{\text{sing}}^{\text{sam}} \rangle + 1.8 \cdot \langle N_{\text{noise}}^{\text{sam}} \rangle + T^2 \left[ 2.2 \cdot \langle N_{\text{sing}}^{\text{ref}} \rangle + 1.8 \cdot \langle N_{\text{noise}}^{\text{ref}} \rangle \right] \right]$$

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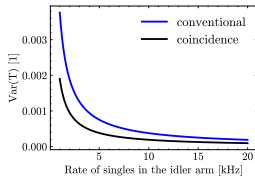
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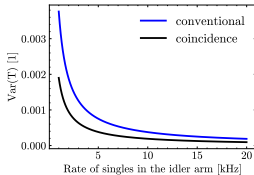


$$\eta_{\text{idl}} = 90\%$$

$$\eta_{\text{sig}} = 90\%$$

$$R_{\text{noise,idl}} = 0 \text{ Hz}$$

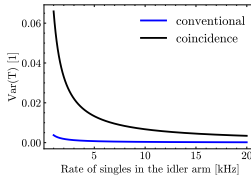
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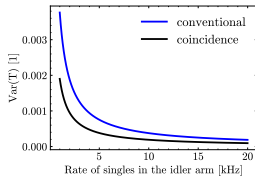


$$\eta_{\text{idl}} = 0.09\%$$

$$\eta_{\text{sig}} = 2.6\%$$

$$R_{\text{noise,idl}} = 0 \text{ Hz}$$

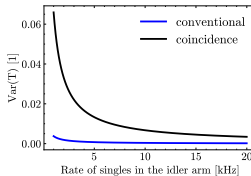
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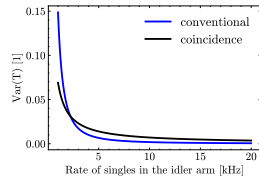
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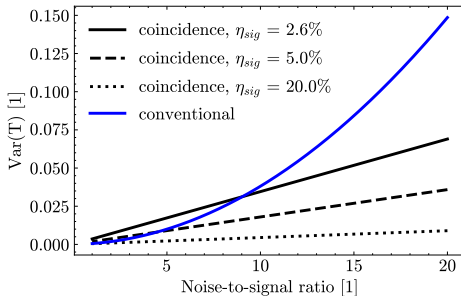


$$\eta_{\text{idl}} = 0.09\%$$

$$\eta_{\text{sig}} = 2.6\%$$

$$R_{\text{noise,idl}} = 20 \text{ kHz}$$

# Noise-to-signal ratio



## Parameter

## Value

$\eta_{idl}$ (%)	0.09
$\eta_{sig}$ (%)	2.6, 5, 20
$R_{idl}$ (kHz)	1 - 20
$R_{noise,idl}$ (kHz)	20
$R_{noise,sig}$ (Hz)	7
$T$ (1)	0.9

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# Summary and Outlook

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## Summary:

- Established a model for the variance of transmittance
- Experimental verification of photon statistics
- Found regions where coincidence approach offers higher precision



# Summary and Outlook

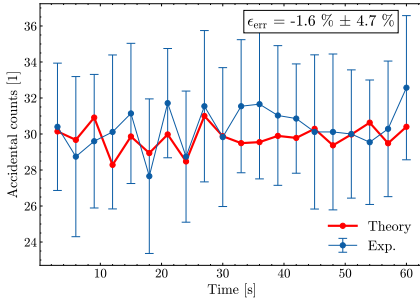
## Summary:

- Established a model for the variance of transmittance
- Experimental verification of photon statistics
- Found regions where coincidence approach offers higher precision

## Outlook:

- Experimental verification of the found parameter regions
- Determine the experimental limitations for the variance measurement

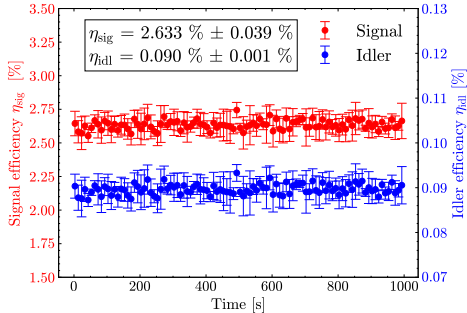
# Accidental counts



$$R_{\text{ac}}^{\text{sam}} = \left( T \eta_{\text{idl}} R_{\text{g}} + R_{\text{dc,idl}} - R_{\text{cc,pure}}^{\text{sam}} \right) \cdot \left( \eta_{\text{sig}} R_{\text{g}} + R_{\text{dc,sig}} - R_{\text{cc,pure}}^{\text{sam}} \right) \cdot \tau_{\text{cw}}$$

$$R_{\text{ac}}^{\text{ref}} = \left( \eta_{\text{idl}} R_{\text{g}} + R_{\text{dc,idl}} - R_{\text{cc,pure}}^{\text{ref}} \right) \cdot \left( \eta_{\text{sig}} R_{\text{g}} + R_{\text{dc,sig}} - R_{\text{cc,pure}}^{\text{ref}} \right) \cdot \tau_{\text{cw}}$$

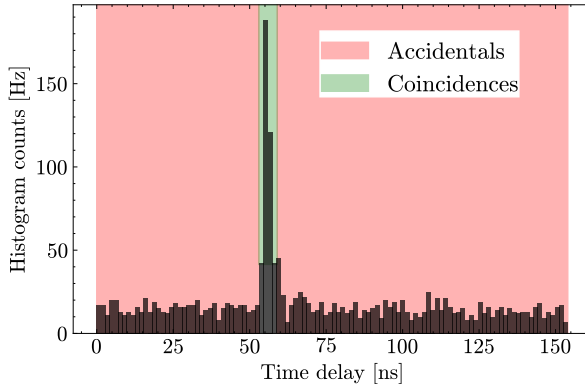
# Heralding efficiencies



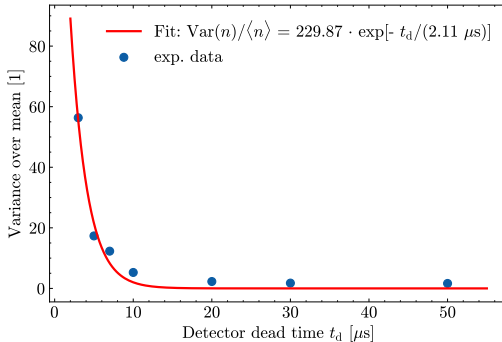
$$\eta_{\text{sig}} = \frac{N_{\text{coin}} - N_{\text{ac}}}{N_{\text{sing,idl}} - N_{\text{noise}}} = \frac{\eta_{\text{sig}} \eta_{\text{idl}} N_{\text{g}}}{\eta_{\text{idl}} N_{\text{g}}}$$

$$\eta_{\text{idl}} = \frac{N_{\text{coin}} - N_{\text{ac}}}{N_{\text{sing,sig}} - N_{\text{noise}}} = \frac{\eta_{\text{sig}} \eta_{\text{idl}} N_{\text{g}}}{\eta_{\text{sig}} N_{\text{g}}}$$

# Histogram

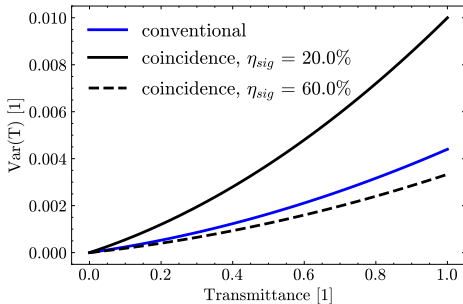


# Afterpulsing



$$R_{aft} \propto e^{-\frac{t_d}{\tau_{de}}}$$

# Noiseless case

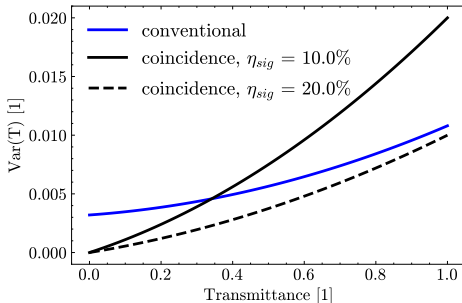


## Parameter

## Value

$\eta_{idl}$ (%)	60
$\eta_{sig}$ (%)	20, 60
$R_{idl}$ (kHz)	1
$R_{noise,idl}$ (kHz)	0
$R_{noise,sig}$ (Hz)	7
$T$ (1)	0 - 1

# In presence of noise

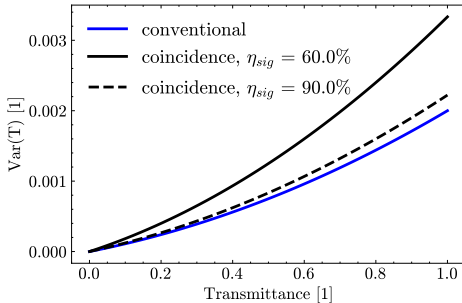


## Parameter

## Value

$\eta_{idl}$ (%)	60
$\eta_{sig}$ (%)	10, 20
$R_{idl}$ (kHz)	1
$R_{noise, idl}$ (kHz)	1
$R_{noise, sig}$ (Hz)	7
$T$ (1)	0 - 1

# Coherent Illumination, noiseless



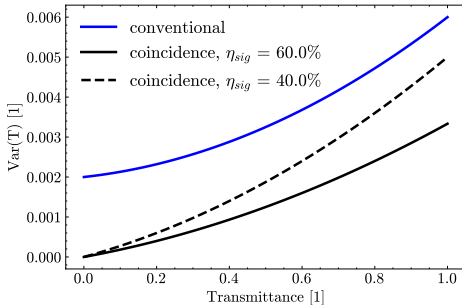
## Parameter

## Value

$\eta_{\text{idl}}$ (%)	60
$\eta_{\text{sig}}$ (%)	60, 90
$R_{\text{idl}}$ (kHz)	1
$R_{\text{noise,idl}}$ (kHz)	0
$R_{\text{noise,sig}}$ (Hz)	7
$T$ (1)	0 - 1



# Coherent Illumination, with noise



## Parameter

## Value

$\eta_{\text{idl}}$ (%)	60
$\eta_{\text{sig}}$ (%)	40, 60
$R_{\text{idl}}$ (kHz)	1
$R_{\text{noise,idl}}$ (kHz)	1
$R_{\text{noise,sig}}$ (Hz)	7
$T$ (1)	0 - 1