

# Parameter estimation with correlated photon pairs

Jan Gößwein

Institute of Applied Physics

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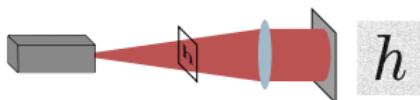
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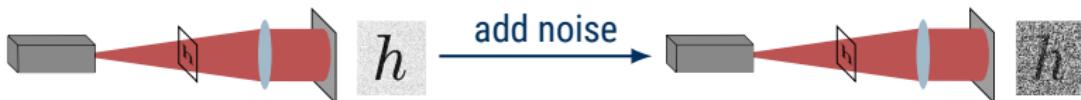
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# Motivation

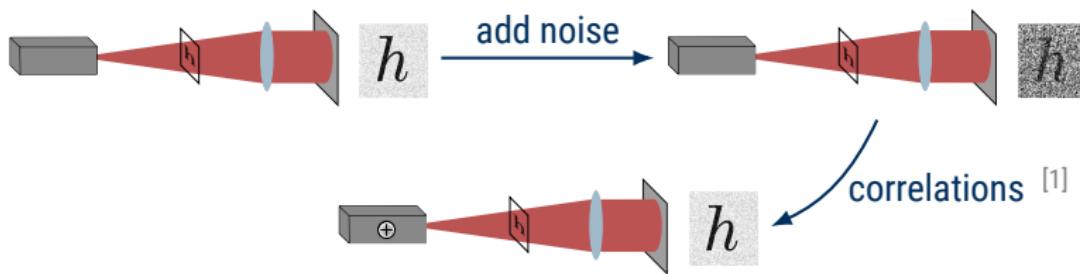
# Motivation



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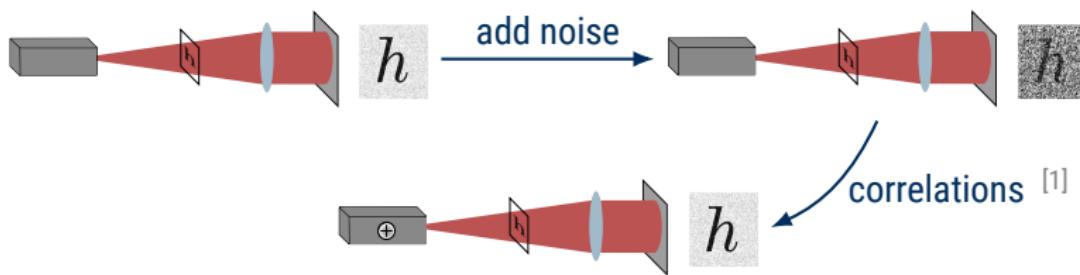


# Motivation



[1] Brida, Genovese, and Ruo Berchera, "Experimental Realization of Sub-Shot-Noise Quantum Imaging"

# Motivation

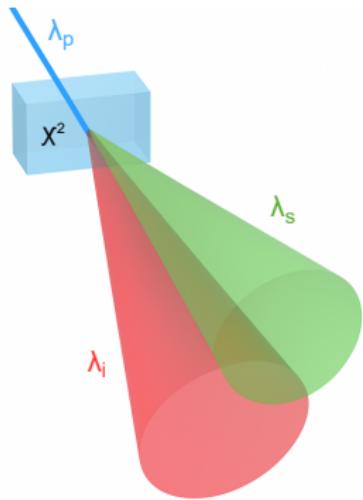


**Objective:** Can correlated photons provide advantages in terms of precision in noisy regimes for parameter estimation?

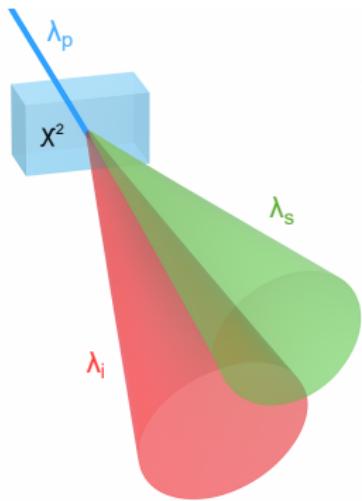
[1] Brida, Genovese, and Ruo Berchera, "Experimental Realization of Sub-Shot-Noise Quantum Imaging"

# SPDC

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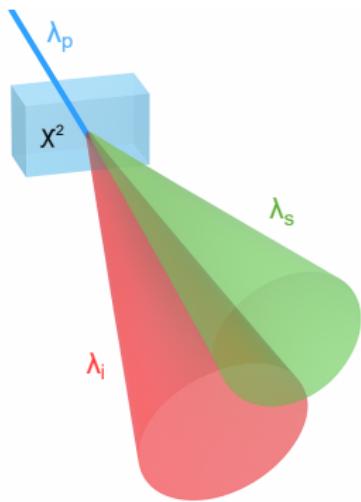


Energy conservation



$$\omega_p = \omega_s + \omega_i$$

# SPDC

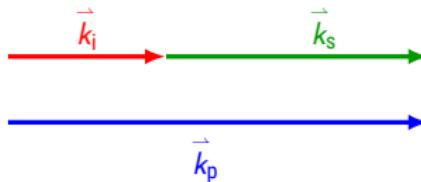


Energy conservation



$$\omega_p = \omega_s + \omega_i$$

Momentum conservation



$$\vec{k}_p = \vec{k}_s + \vec{k}_i$$

# Parameter estimation

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**Setup:** Transmission setup

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**Parameter:** Transmittance  $T$

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**Estimator:**  $\text{Var}(T)$

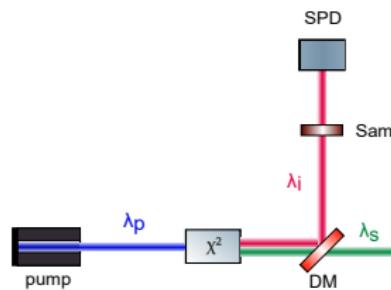
# Parameter estimation

**Setup:** Transmission setup

**Parameter:** Transmittance  $T$

**Estimator:**  $\text{Var}(T)$

Conventional approach:



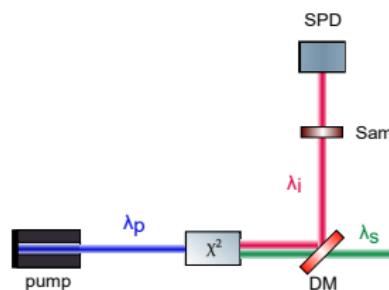
# Parameter estimation

**Setup:** Transmission setup

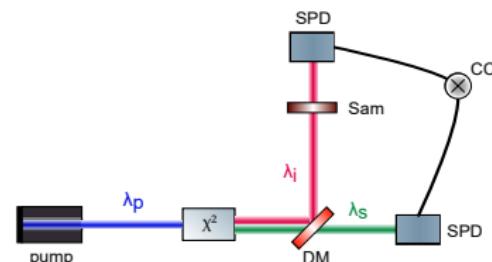
**Parameter:** Transmittance  $T$

**Estimator:**  $\text{Var}(T)$

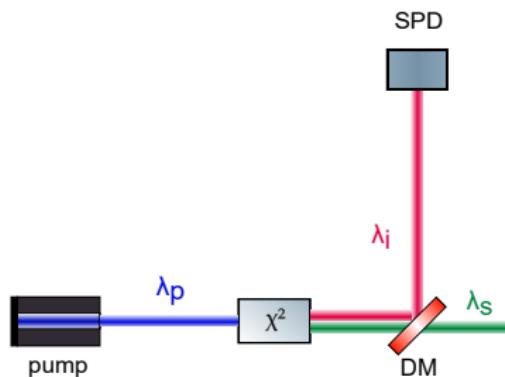
Conventional approach:



Coincidence approach:

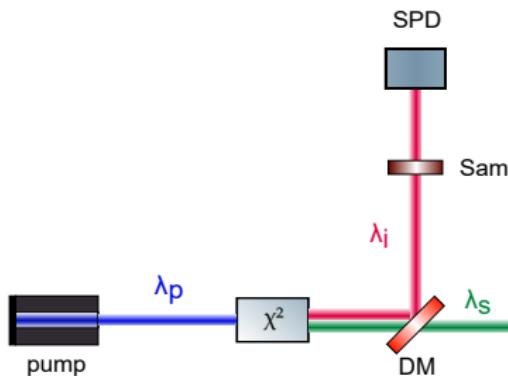


# Conventional approach



$$N_{\text{sing}}^{\text{ref}} = \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{ref}}$$

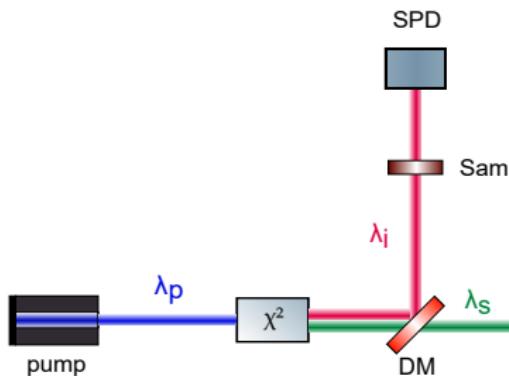
# Conventional approach



$$N_{\text{sing}}^{\text{ref}} = \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{ref}}$$

$$N_{\text{sing}}^{\text{sam}} = T \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{sam}}$$

# Conventional approach

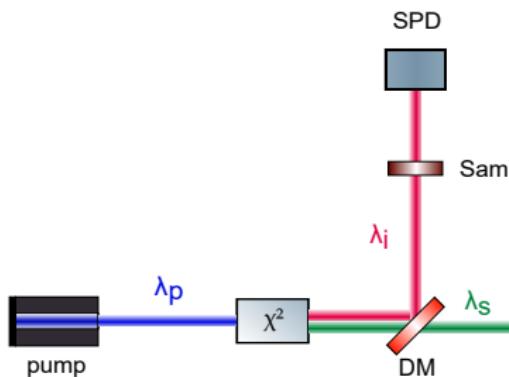


$$N_{\text{sing}}^{\text{ref}} = \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{ref}}$$

$$N_{\text{sing}}^{\text{sam}} = T \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{sam}}$$

$$\Rightarrow T = \frac{N_{\text{sing}}^{\text{sam}} - N_{\text{noise}}^{\text{sam}}}{N_{\text{sing}}^{\text{ref}} - N_{\text{noise}}^{\text{ref}}}$$

# Conventional approach



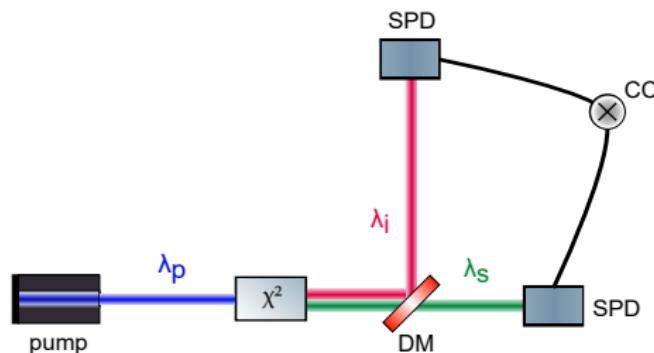
$$N_{\text{sing}}^{\text{ref}} = \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{ref}}$$

$$N_{\text{sing}}^{\text{sam}} = T \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{sam}}$$

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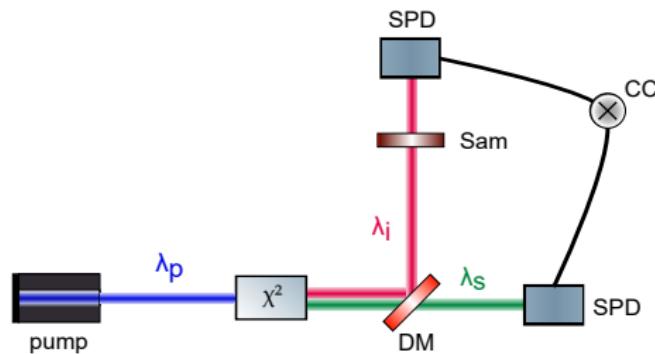
$$\text{Var}(T) = (\eta_{\text{idl}} N_g)^{-2} \left[ \text{Var}(N_{\text{sing}}^{\text{sam}}) + \text{Var}(N_{\text{noise}}^{\text{sam}}) + T^2 \left[ \text{Var}(N_{\text{sing}}^{\text{ref}}) + \text{Var}(N_{\text{noise}}^{\text{ref}}) \right] \right]$$

# Coincidence approach



$$N_{\text{coin}}^{\text{ref}} = \eta_{\text{idr}} \eta_{\text{sig}} N_g + N_{\text{ac}}^{\text{ref}}$$

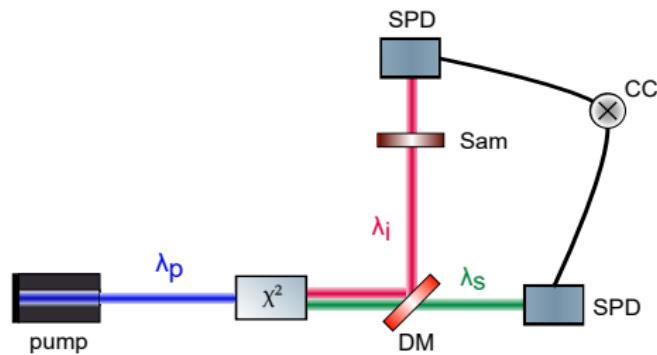
# Coincidence approach



$$N_{\text{coin}}^{\text{ref}} = \eta_{\text{idl}} \eta_{\text{sig}} N_g + N_{\text{ac}}^{\text{ref}}$$

$$N_{\text{coin}}^{\text{sam}} = T \eta_{\text{idl}} \eta_{\text{sig}} N_g + N_{\text{ac}}^{\text{sam}}$$

# Coincidence approach

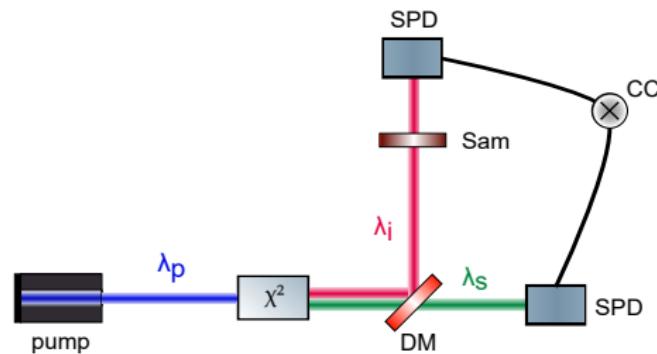


$$N_{\text{coin}}^{\text{ref}} = \eta_{\text{idl}} \eta_{\text{sig}} N_g + N_{\text{ac}}^{\text{ref}}$$

$$N_{\text{coin}}^{\text{sam}} = T \eta_{\text{idl}} \eta_{\text{sig}} N_g + N_{\text{ac}}^{\text{sam}}$$

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# Coincidence approach



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$$\Rightarrow T = \frac{N_{\text{coin}}^{\text{sam}} - N_{\text{ac}}^{\text{sam}}}{N_{\text{coin}}^{\text{ref}} - N_{\text{ac}}^{\text{ref}}}$$

$$\text{Var}(T) = (\eta_{\text{sig}} \eta_{\text{idl}} N_g)^{-2} \left[ \text{Var}(N_{\text{coin}}^{\text{sam}}) + \text{Var}(N_{\text{ac}}^{\text{sam}}) + T^2 \left[ \text{Var}(N_{\text{coin}}^{\text{ref}}) + \text{Var}(N_{\text{ac}}^{\text{ref}}) \right] \right]$$

# Transmittance model

Conventional approach:

$$\text{Var}(T) = (\eta_{\text{idl}} N_g)^{-2} \left[ \text{Var}(N_{\text{sing}}^{\text{sam}}) + \text{Var}(N_{\text{noise}}^{\text{sam}}) + T^2 [\text{Var}(N_{\text{sing}}^{\text{ref}}) + \text{Var}(N_{\text{noise}}^{\text{ref}})] \right]$$

Coincidence approach:

$$\text{Var}(T) = (\eta_{\text{sig}} \eta_{\text{idl}} N_g)^{-2} \left[ \text{Var}(N_{\text{coin}}^{\text{sam}}) + \text{Var}(N_{\text{ac}}^{\text{sam}}) + T^2 [\text{Var}(N_{\text{coin}}^{\text{ref}}) + \text{Var}(N_{\text{ac}}^{\text{ref}})] \right]$$

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Coincidence approach:

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# Photon statistics

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Poisson distribution (coherent light):

$$\mathcal{P}(n) = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}$$

$$\text{Var}(n) = \langle n \rangle$$

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Poisson distribution (coherent light): [2][3]

$$\mathcal{P}(n) = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}$$

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---

[2] Kim et al., "Photon-Counting Statistics-Based Support Vector Machine with Multi-Mode Photon Illumination for Quantum Imaging"

[3] Fouche, "Detection and False-Alarm Probabilities for Laser Radars That Use Geiger-mode Detectors"

# Photon statistics

Poisson distribution (coherent light): [2][3]

$$\mathcal{P}(n) = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}$$
$$\text{Var}(n) = \langle n \rangle$$

Multi-mode Bose-Einstein distribution (thermal light): [2]

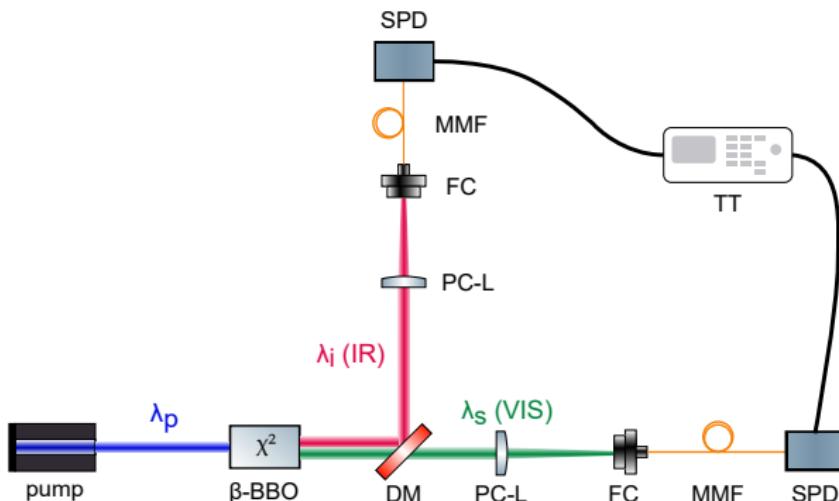
$$\mathcal{P}_m(n) = \frac{(n+m-1)!}{(m-1)! n!} \frac{m^m \langle n \rangle^n}{(m + \langle n \rangle)^{n+m}}$$
$$\text{Var}(n) = \langle n \rangle \left( 1 + \frac{\langle n \rangle}{m} \right)$$

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[2] Kim et al., "Photon-Counting Statistics-Based Support Vector Machine with Multi-Mode Photon Illumination for Quantum Imaging"

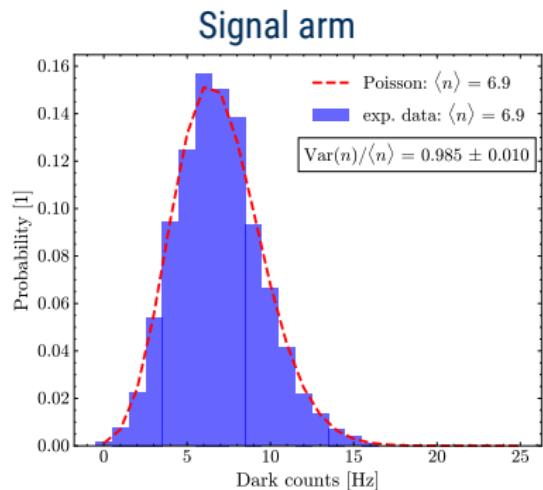
[3] Fouche, "Detection and False-Alarm Probabilities for Laser Radars That Use Geiger-mode Detectors"

# Experimental setup

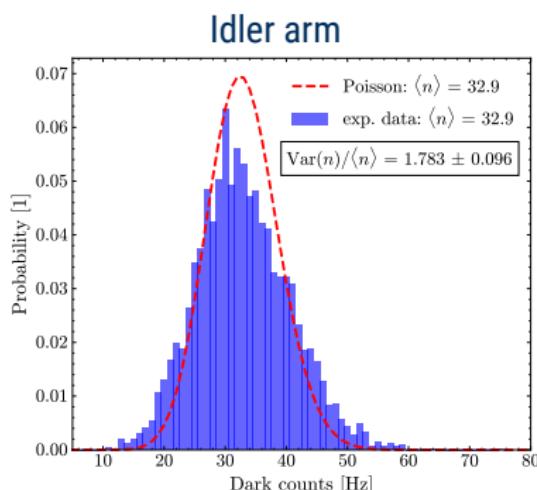
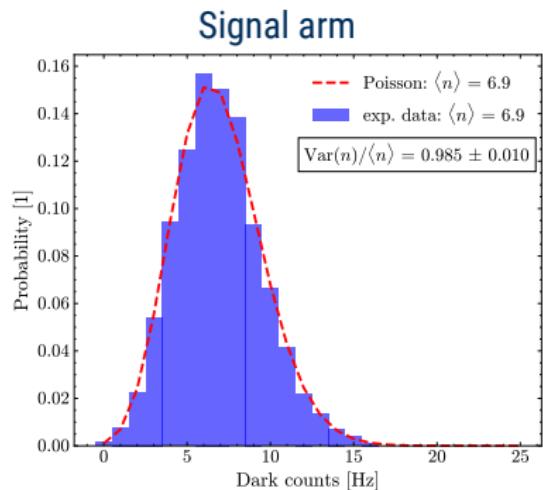


# Dark counts

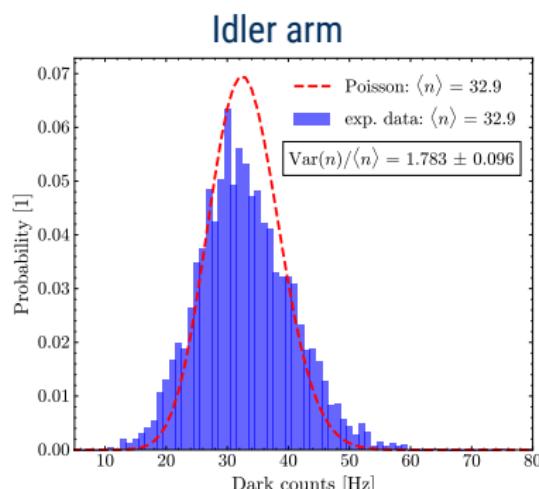
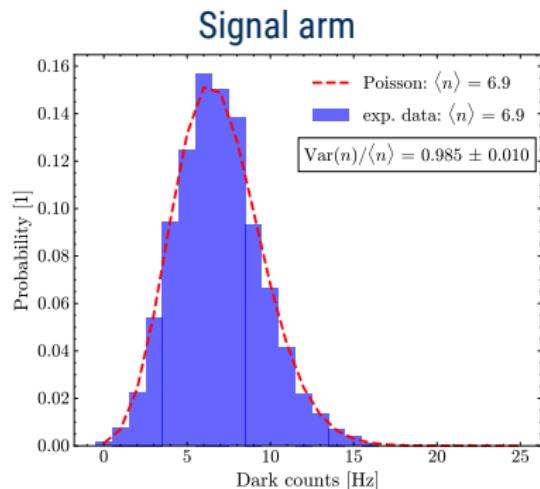
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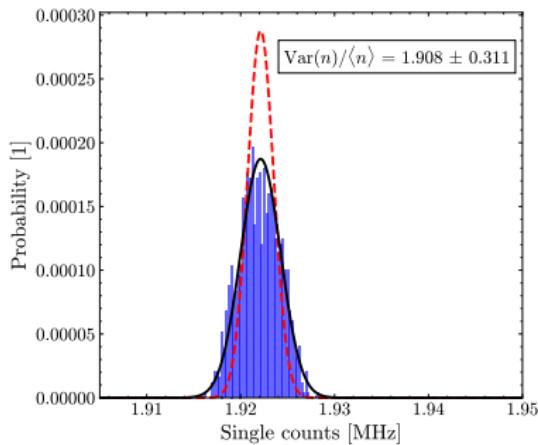
$$\text{Var}(N_{\text{noise}}) = 1.8 \cdot \langle N_{\text{noise}} \rangle$$

# Single counts

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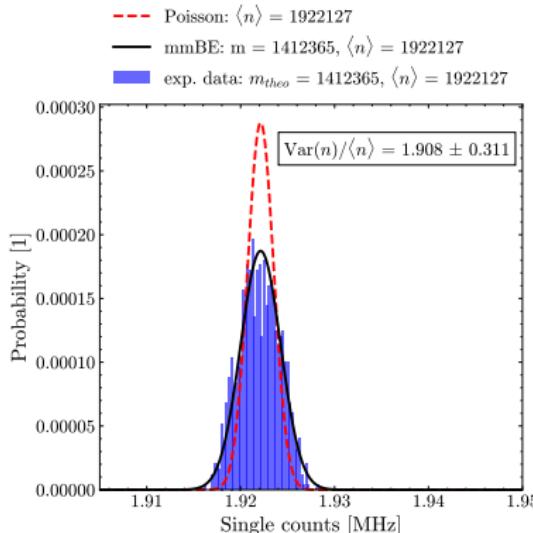
## Signal arm

- Poisson:  $\langle n \rangle = 1922127$
- mmBE:  $m = 1412365$ ,  $\langle n \rangle = 1922127$
- exp. data:  $m_{\text{theo}} = 1412365$ ,  $\langle n \rangle = 1922127$

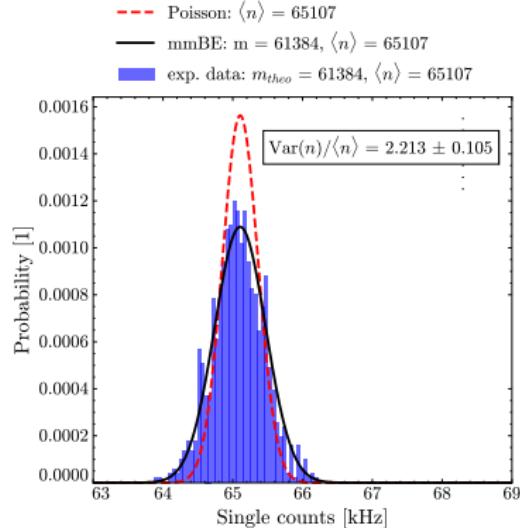


# Single counts

## Signal arm

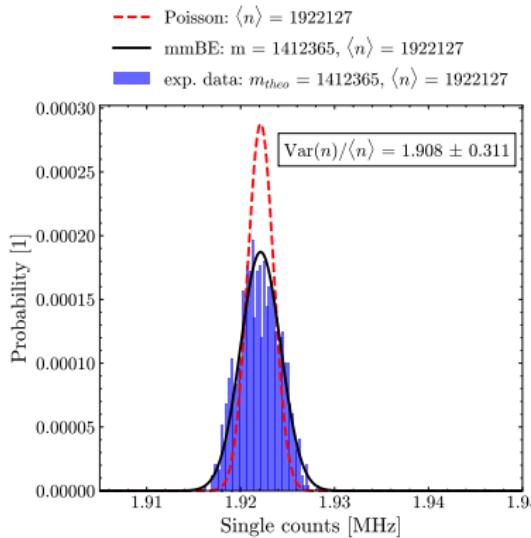


## Idler arm

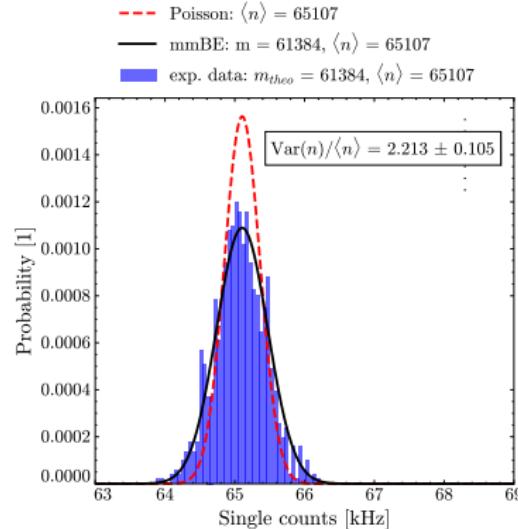


# Single counts

## Signal arm



## Idler arm

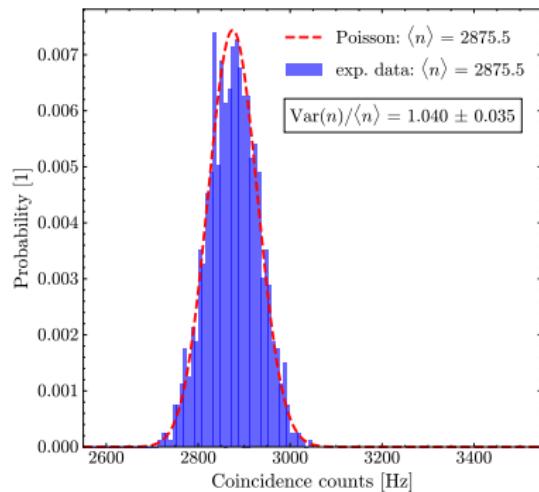


$$\text{Var}(N_{\text{sing}}) = 2.2 \cdot \langle N_{\text{sing}} \rangle$$

# Coincidence counts

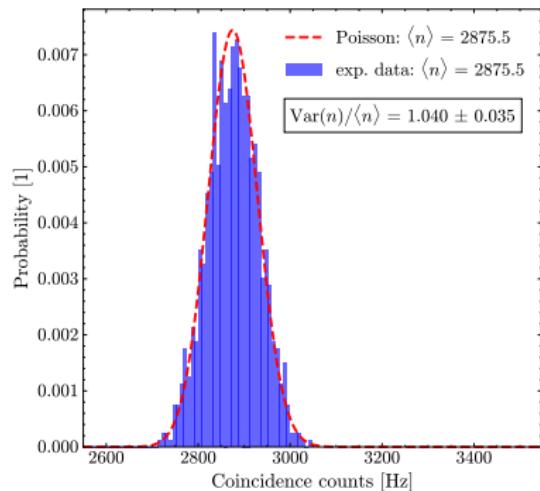
# Coincidence counts

## Coincidences



# Coincidence counts

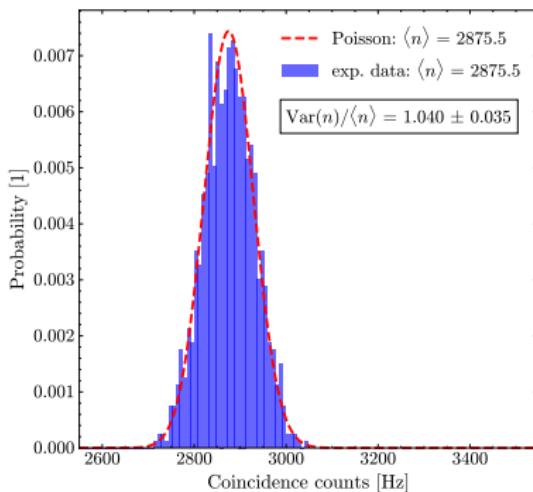
## Coincidences



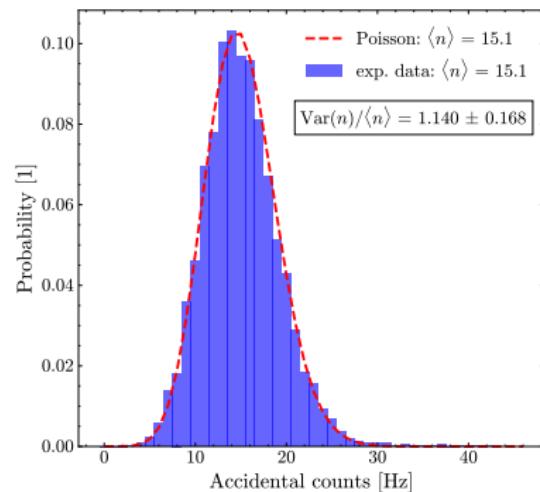
$$\text{Var}(N_{cc}) = \langle N_{cc} \rangle$$

# Coincidence counts

## Coincidences



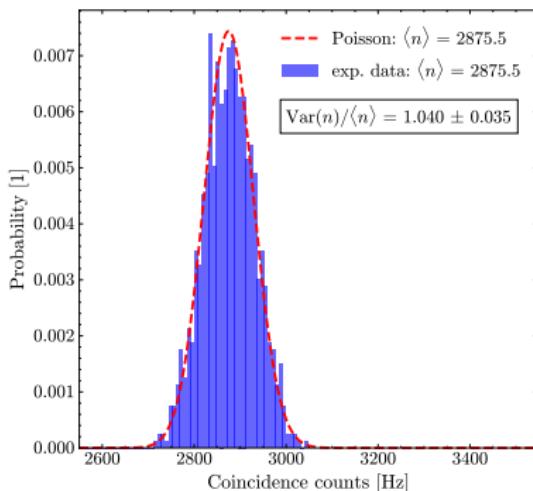
## Accidentals



$$\text{Var}(N_{cc}) = \langle N_{cc} \rangle$$

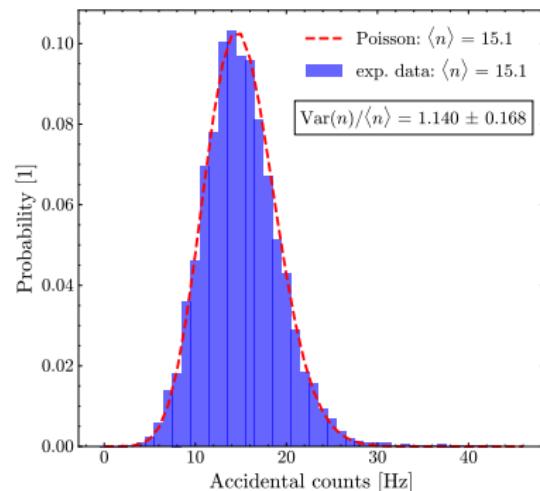
# Coincidence counts

## Coincidences



$$\text{Var}(N_{cc}) = \langle N_{cc} \rangle$$

## Accidentals



$$\text{Var}(N_{ac}) = \langle N_{ac} \rangle$$

# Simulation

Conventional approach:

$$\text{Var}(T) = (\eta_{\text{idl}} N_g)^{-2} \left[ \text{Var}(N_{\text{sing}}^{\text{sam}}) + \text{Var}(N_{\text{noise}}^{\text{sam}}) + T^2 \left[ \text{Var}(N_{\text{sing}}^{\text{ref}}) + \text{Var}(N_{\text{noise}}^{\text{ref}}) \right] \right]$$

# Simulation

Conventional approach:

$$\text{Var}(T) = (\eta_{\text{idl}} N_g)^{-2} \left[ 2.2 \cdot \langle N_{\text{sing}}^{\text{sam}} \rangle + 1.8 \cdot \langle N_{\text{noise}}^{\text{sam}} \rangle + T^2 [2.2 \cdot \langle N_{\text{sing}}^{\text{ref}} \rangle + 1.8 \cdot \langle N_{\text{noise}}^{\text{ref}} \rangle] \right]$$

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Coincidence approach:

$$\text{Var}(T) = (\eta_{\text{sig}} \eta_{\text{idl}} N_g)^{-2} \left[ \text{Var}(N_{\text{coin}}^{\text{sam}}) + \text{Var}(N_{\text{ac}}^{\text{sam}}) + T^2 [\text{Var}(N_{\text{coin}}^{\text{ref}}) + \text{Var}(N_{\text{ac}}^{\text{ref}})] \right]$$

# Simulation

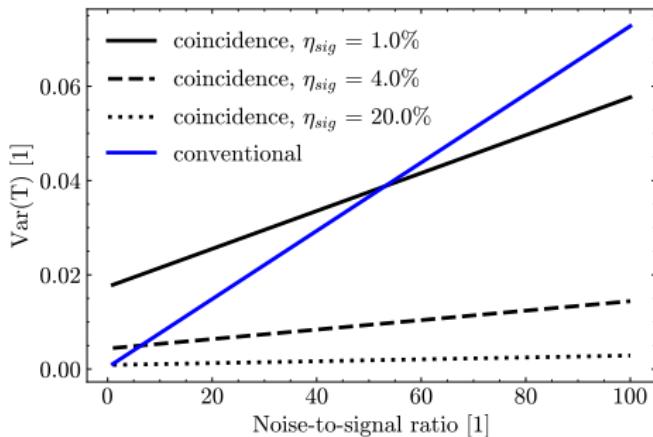
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## Coincidence approach:

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# Noise-to-signal ratio



Parameter	Value
$\eta_{\text{idl}}$ (%)	0.09
$\eta_{\text{sig}}$ (%)	1, 4, 20
$R_{\text{idl}}$ (kHz)	10
$R_{\text{noise,idl}}$ (kHz)	10 - 1000
$R_{\text{noise,sig}}$ (Hz)	7
$T$ (1)	0.9

# Summary and Outlook

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## Summary:

- Established a model for the variance of transmittance
- Experimental verification of photon statistics
- Found regions where coincidence approach offers higher precision

# Summary and Outlook

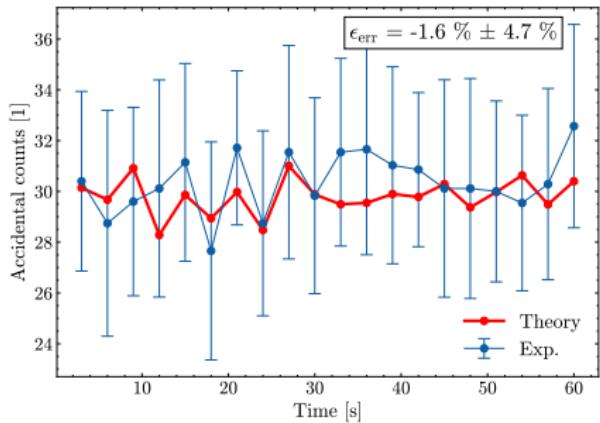
## Summary:

- Established a model for the variance of transmittance
- Experimental verification of photon statistics
- Found regions where coincidence approach offers higher precision

## Outlook:

- Experimental verification of the found parameter regions
- Determine the experimental limitations for the variance measurement

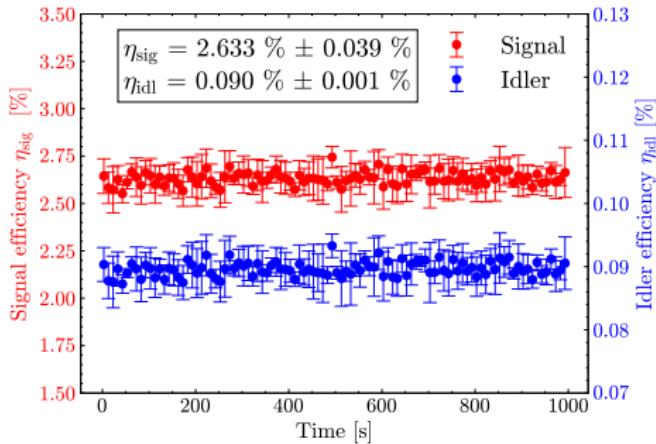
# Accidental counts



$$R_{\text{ac}}^{\text{sam}} = \left( T \eta_{\text{idl}} R_g + R_{\text{dc,idl}} - R_{\text{cc,pure}}^{\text{sam}} \right) \cdot \\ \left( \eta_{\text{sig}} R_g + R_{\text{dc,sig}} - R_{\text{cc,pure}}^{\text{sam}} \right) \cdot \tau_{\text{cw}}$$

$$R_{\text{ac}}^{\text{ref}} = \left( \eta_{\text{idl}} R_g + R_{\text{dc,idl}} - R_{\text{cc,pure}}^{\text{ref}} \right) \cdot \\ \left( \eta_{\text{sig}} R_g + R_{\text{dc,sig}} - R_{\text{cc,pure}}^{\text{ref}} \right) \cdot \tau_{\text{cw}}$$

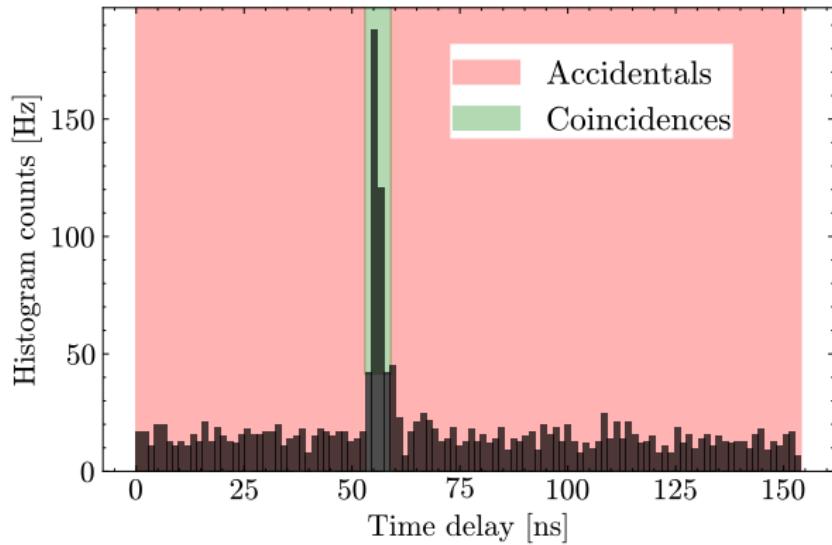
# Heralding efficiencies



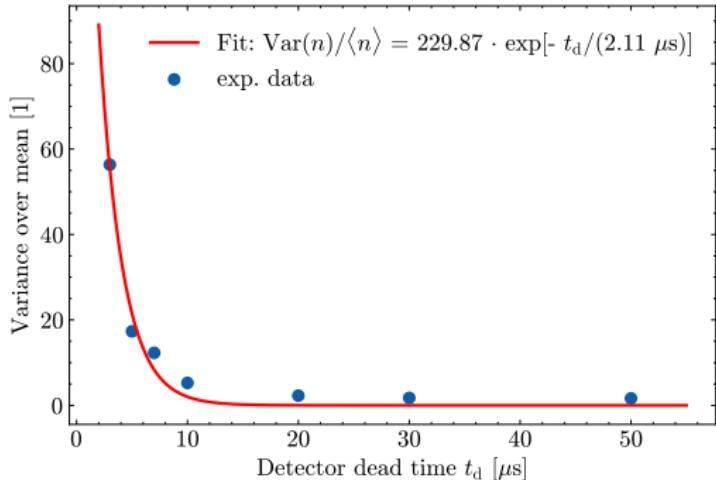
$$\eta_{\text{sig}} = \frac{N_{\text{coin}} - N_{\text{ac}}}{N_{\text{sing,idl}} - N_{\text{noise}}} = \frac{\eta_{\text{sig}} \eta_{\text{idl}} N_g}{\eta_{\text{idl}} N_g}$$

$$\eta_{\text{idl}} = \frac{N_{\text{coin}} - N_{\text{ac}}}{N_{\text{sing,sig}} - N_{\text{noise}}} = \frac{\eta_{\text{sig}} \eta_{\text{idl}} N_g}{\eta_{\text{sig}} N_g}$$

# Histogram

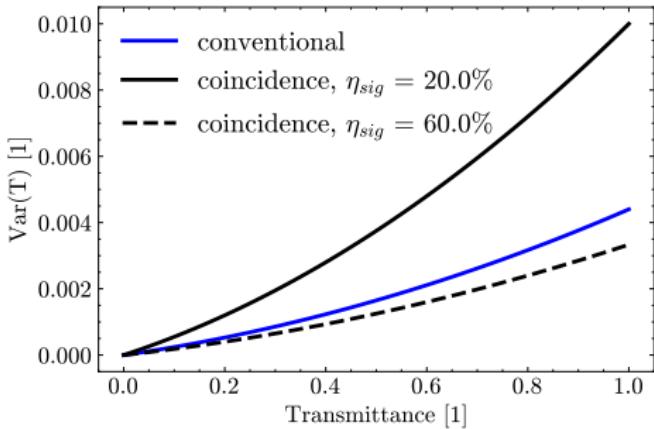


# Afterpulsing



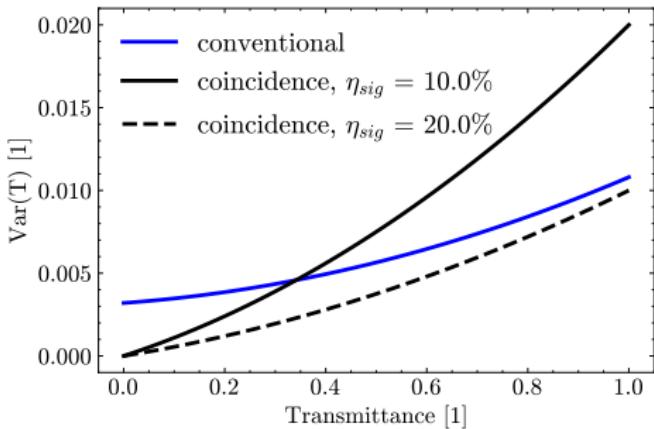
$$R_{aft} \propto e^{-\frac{t_d}{\tau_{de}}}$$

# Noiseless case



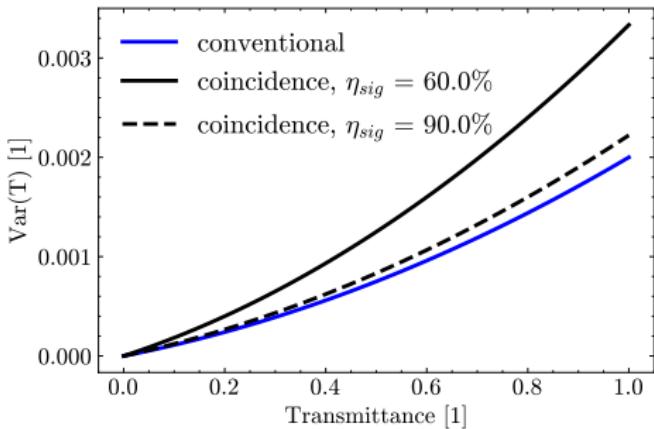
Parameter	Value
$\eta_{\text{idl}}$ (%)	60
$\eta_{\text{sig}}$ (%)	20, 60
$R_{\text{idl}}$ (kHz)	1
$R_{\text{noise,idl}}$ (kHz)	0
$R_{\text{noise,sig}}$ (Hz)	7
$T(1)$	0 - 1

# In presence of noise



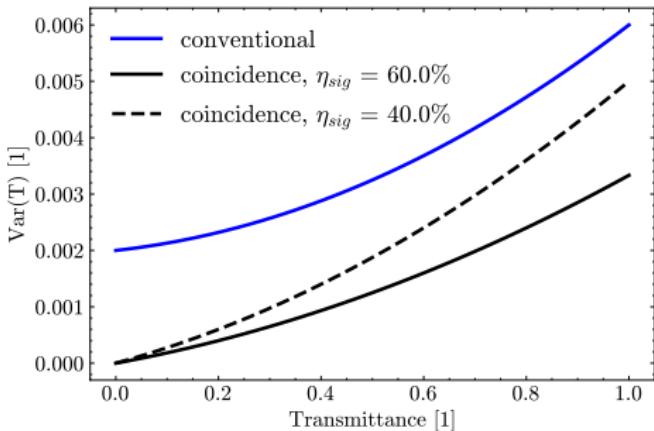
Parameter	Value
$\eta_{idl}$ (%)	60
$\eta_{sig}$ (%)	10, 20
$R_{idl}$ (kHz)	1
$R_{noise,idl}$ (kHz)	1
$R_{noise,sig}$ (Hz)	7
$T(1)$	0 - 1

# Coherent Illumination, noiseless



Parameter	Value
$\eta_{idl}$ (%)	60
$\eta_{sig}$ (%)	60, 90
$R_{idl}$ (kHz)	1
$R_{noise,idl}$ (kHz)	0
$R_{noise,sig}$ (Hz)	7
$T$ (1)	0 - 1

# Coherent Illumination, with noise



Parameter	Value
$\eta_{idl}$ (%)	60
$\eta_{sig}$ (%)	40, 60
$R_{idl}$ (kHz)	1
$R_{noise,idl}$ (kHz)	1
$R_{noise,sig}$ (Hz)	7
$T(1)$	0 - 1