

Parameter estimation with correlated photon pairs

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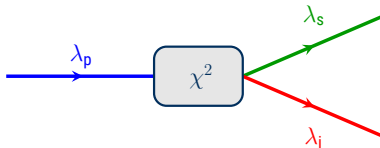
Jena, October 24, 2025

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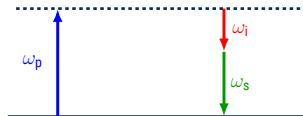
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Motivation

SPDC

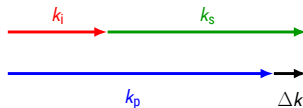


Energy conservation



$$\omega_p = \omega_s + \omega_i$$

Momentum conservation



$$\vec{k}_p = \vec{k}_s + \vec{k}_i - \Delta \vec{k}$$

Transmittance model

Conventional approach:

$$N_{\text{tot}}^{\text{ref}} = \eta_{\text{idl}} N_{\text{g}} + N_{\text{noise}}^{\text{ref}}$$

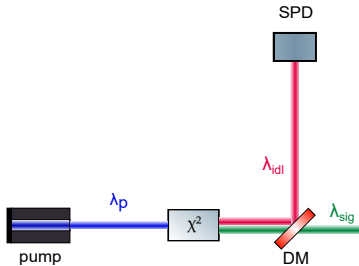
$$N_{\text{tot}}^{\text{sam}} = T \eta_{\text{idl}} N_{\text{g}} + N_{\text{noise}}^{\text{sam}}$$

Coincidence approach:

$$N_{\text{cc}}^{\text{pure,sam}} = T \eta_{\text{idl}} \eta_{\text{sig}} N_{\text{g}},$$

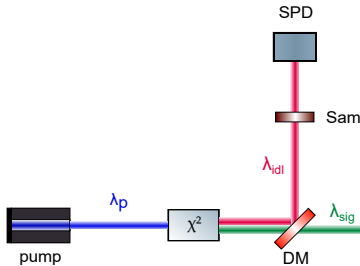
$$N_{\text{cc}}^{\text{pure,ref}} = \eta_{\text{idl}} \eta_{\text{sig}} N_{\text{g}}$$

Conventional approach



$$N_{\text{tot}}^{\text{ref}} = \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{ref}}$$

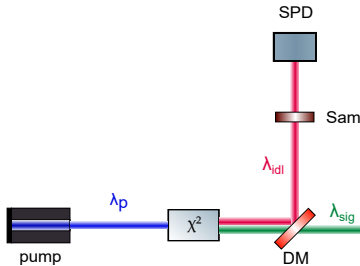
Conventional approach



$$N_{\text{tot}}^{\text{ref}} = \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{ref}}$$

$$N_{\text{tot}}^{\text{sam}} = T \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{sam}}$$

Conventional approach

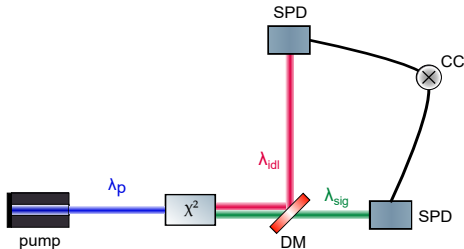


$$N_{\text{tot}}^{\text{ref}} = \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{ref}}$$

$$N_{\text{tot}}^{\text{sam}} = T \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{sam}}$$

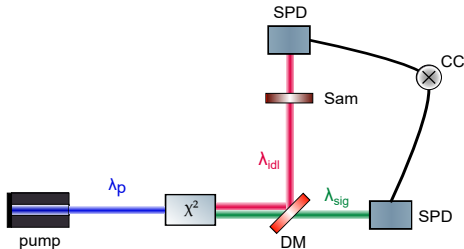
$$\Rightarrow T = \frac{N_{\text{tot}}^{\text{sam}} - N_{\text{noise}}^{\text{sam}}}{N_{\text{tot}}^{\text{ref}} - N_{\text{noise}}^{\text{ref}}}$$

Coincidence approach



$$N_{cc,tot}^{ref} = \eta_{idl} \eta_{sig} N_g + N_{ac}^{ref}$$

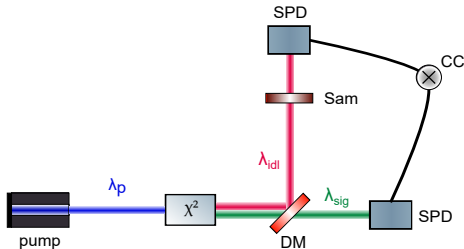
Coincidence approach



$$N_{cc,tot}^{ref} = \eta_{idl} \eta_{sig} N_g + N_{ac}^{ref}$$

$$N_{cc,tot}^{sam} = T \eta_{idl} \eta_{sig} N_g + N_{ac}^{sam}$$

Coincidence approach



$$N_{cc,tot}^{ref} = \eta_{idl} \eta_{sig} N_g + N_{ac}^{ref}$$

$$N_{cc,tot}^{sam} = T \eta_{idl} \eta_{sig} N_g + N_{ac}^{sam}$$

$$\Rightarrow T = \frac{N_{tot,cc}^{sam} - N_{ac}^{sam}}{N_{tot,cc}^{ref} - N_{ac}^{ref}}$$

Transmittance model

Conventional approach:

$$\text{Var}(T) = (\eta_{\text{idl}} N_g)^{-2} \left[\text{Var}(N_{\text{tot}}^{\text{sam}}) + \text{Var}(N_{\text{noise}}^{\text{sam}}) + T^2 \left[\text{Var}(N_{\text{tot}}^{\text{ref}}) + \text{Var}(N_{\text{noise}}^{\text{ref}}) \right] \right]$$

Coincidence approach:

$$\text{Var}(T) = (\eta_{\text{sig}} \eta_{\text{idl}} N_g)^{-2} \left[\text{Var}(N_{\text{tot,cc}}^{\text{sam}}) + \text{Var}(N_{\text{ac}}^{\text{sam}}) + T^2 \left[\text{Var}(N_{\text{tot,cc}}^{\text{ref}}) + \text{Var}(N_{\text{ac}}^{\text{ref}}) \right] \right]$$

Transmittance model

Conventional approach:

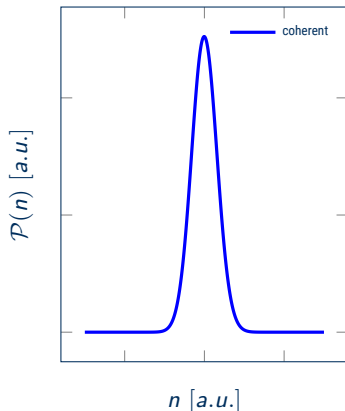
$$\text{Var}(T) = (\eta_{\text{idl}} N_{\text{g}})^{-2} \left[\text{Var}(N_{\text{tot}}^{\text{sam}}) + \text{Var}(N_{\text{noise}}^{\text{sam}}) + T^2 \left[\text{Var}(N_{\text{tot}}^{\text{ref}}) + \text{Var}(N_{\text{noise}}^{\text{ref}}) \right] \right]$$

Coincidence approach:

$$\text{Var}(T) = (\eta_{\text{sig}} \eta_{\text{idl}} N_{\text{g}})^{-2} \left[\text{Var}(N_{\text{tot,cc}}^{\text{sam}}) + \text{Var}(N_{\text{ac}}^{\text{sam}}) + T^2 \left[\text{Var}(N_{\text{tot,cc}}^{\text{ref}}) + \text{Var}(N_{\text{ac}}^{\text{ref}}) \right] \right]$$

Photon statistics

Photon statistics

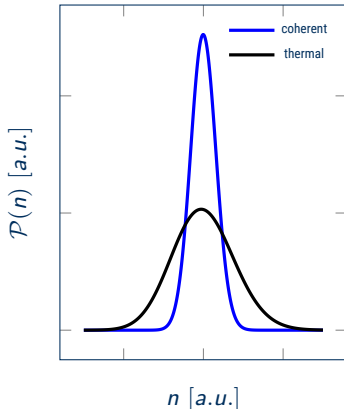


Coherent light:

$$\mathcal{P}(n) = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}$$

$$\text{Var}(n) = \langle n \rangle$$

Photon statistics



Coherent light:

$$\mathcal{P}(n) = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}$$

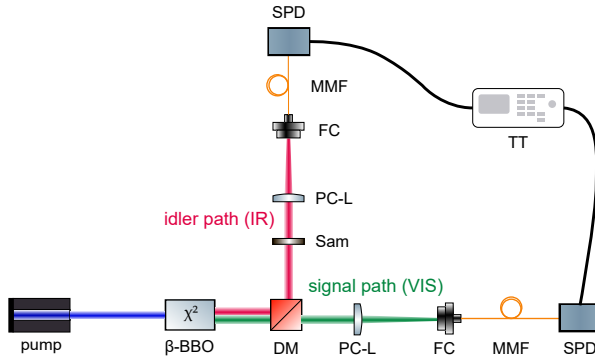
$$\text{Var}(n) = \langle n \rangle$$

Thermal light:

$$\mathcal{P}_m(n) = \frac{(n+m-1)!}{(m-1)! n!} \frac{m^m \langle n \rangle^n}{(m + \langle n \rangle)^{n+m}}$$

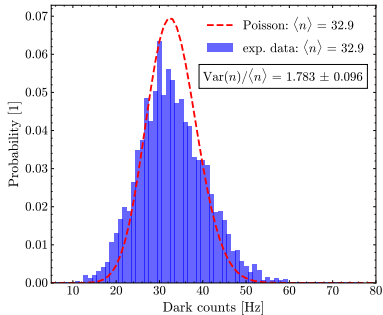
$$\text{Var}(n) = \langle n \rangle \left(1 + \frac{\langle n \rangle}{m} \right)$$

Experimental setup

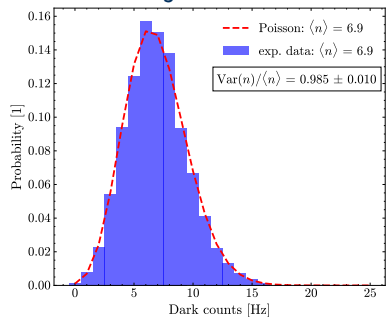


Dark counts

Idler arm

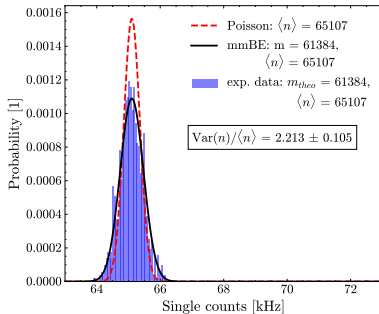


Signal arm

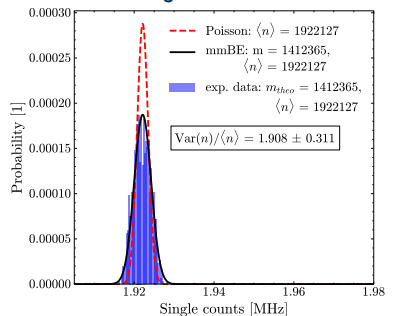


Single counts

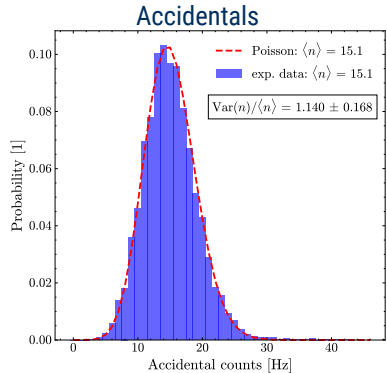
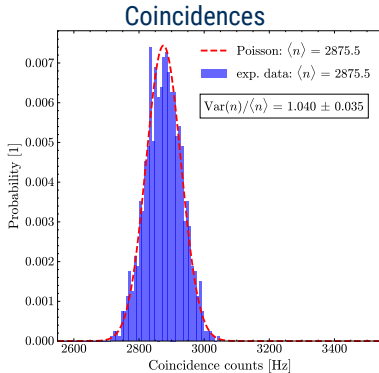
Idler arm



Signal arm



Coincidence counts



Simulation

Conventional approach:

$$\text{Var}(\mathcal{T}) = (\eta_{\text{idl}} N_{\text{g}})^{-2} \left[\text{Var}(N_{\text{tot}}^{\text{sam}}) + \text{Var}(N_{\text{noise}}^{\text{sam}}) + T^2 \left[\text{Var}(N_{\text{tot}}^{\text{ref}}) + \text{Var}(N_{\text{noise}}^{\text{ref}}) \right] \right]$$

Simulation

Conventional approach:

$$\text{Var}(T) = (\eta_{\text{idl}} N_{\text{g}})^{-2} \left[2.2 \cdot \langle N_{\text{tot}}^{\text{sam}} \rangle + 1.8 \cdot \langle N_{\text{noise}}^{\text{sam}} \rangle + T^2 \left[2.2 \cdot \langle N_{\text{tot}}^{\text{ref}} \rangle + 1.8 \cdot \langle N_{\text{noise}}^{\text{ref}} \rangle \right] \right]$$

Simulation

Conventional approach:

$$\text{Var}(T) = (\eta_{\text{idl}} N_g)^{-2} \left[2.2 \cdot \langle N_{\text{tot}}^{\text{sam}} \rangle + 1.8 \cdot \langle N_{\text{noise}}^{\text{sam}} \rangle + T^2 \left[2.2 \cdot \langle N_{\text{tot}}^{\text{ref}} \rangle + 1.8 \cdot \langle N_{\text{noise}}^{\text{ref}} \rangle \right] \right]$$

Coincidence approach:

$$\text{Var}(T) = (\eta_{\text{sig}} \eta_{\text{idl}} N_g)^{-2} \left[\text{Var}(N_{\text{tot,cc}}^{\text{sam}}) + \text{Var}(N_{\text{ac}}^{\text{sam}}) + T^2 \left[\text{Var}(N_{\text{tot,cc}}^{\text{ref}}) + \text{Var}(N_{\text{ac}}^{\text{ref}}) \right] \right]$$

Simulation

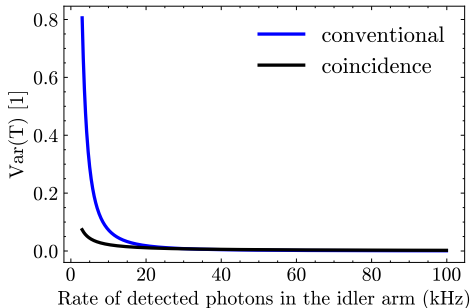
Conventional approach:

$$\text{Var}(T) = (\eta_{\text{idl}} N_g)^{-2} \left[2.2 \cdot \langle N_{\text{tot}}^{\text{sam}} \rangle + 1.8 \cdot \langle N_{\text{noise}}^{\text{sam}} \rangle + T^2 \left[2.2 \cdot \langle N_{\text{tot}}^{\text{ref}} \rangle + 1.8 \cdot \langle N_{\text{noise}}^{\text{ref}} \rangle \right] \right]$$

Coincidence approach:

$$\text{Var}(T) = (\eta_{\text{sig}} \eta_{\text{idl}} N_g)^{-2} \left[\langle N_{\text{tot,cc}}^{\text{sam}} \rangle + \langle N_{\text{ac}}^{\text{sam}} \rangle + T^2 \left[\langle N_{\text{tot,cc}}^{\text{ref}} \rangle + \langle N_{\text{ac}}^{\text{ref}} \rangle \right] \right]$$

Simulation



Parameter	Value
η_{idl} (%)	0.09
η_{sig} (%)	2.6
R_{idl} (kHz)	3 - 100
$R_{\text{noise,idl}}$ (kHz)	1000
$R_{\text{noise,sig}}$ (Hz)	7
T (1)	0.9

Summary and Outlook

Git repository

public accessible:

https://git.tpi.uni-jena.de/mstnhsr/latexbeamer_corporatedesign

Feedback

marc.steinhauser@uni-jena.de

Slide title in Palatino Linotype Font

block environment (lower-case b)

itemize:

- First Level
 - Second Level

Third Level has no item mark

Block environment (upper-case B)

enumerate:

1. First Level
 - 1.1 Second Level
 - 1.1.1 Third Level