

# Parameter estimation with correlated photon pairs

Jan Gößwein

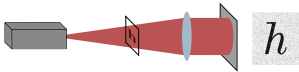
Institute of Applied Physics

Jena, November 10, 2025

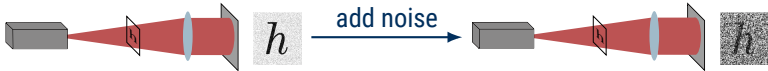
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# Motivation

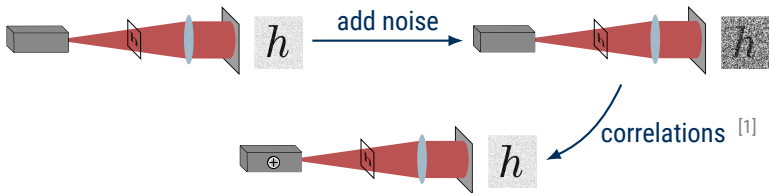
# Motivation



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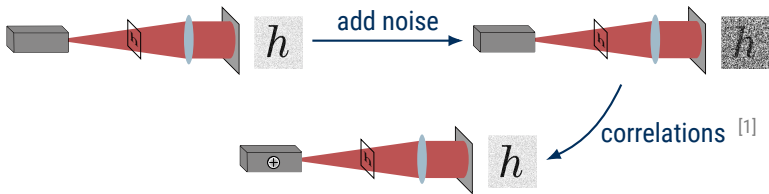


# Motivation



[1] Brida, Genovese, and Ruo Berchera, "Experimental Realization of Sub-Shot-Noise Quantum Imaging"

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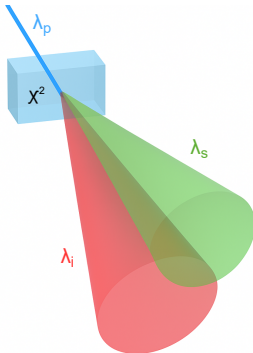


**Objective:** Can correlated photons provide advantages in terms of precision in noisy regimes for parameter estimation?

[1] Brida, Genovese, and Ruo Berchera, "Experimental Realization of Sub-Shot-Noise Quantum Imaging"

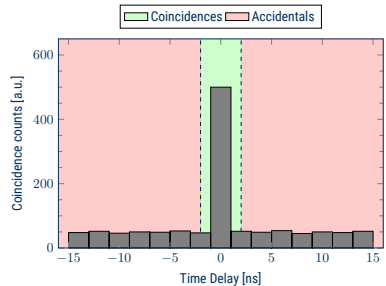
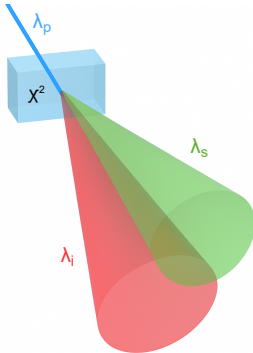
# Spontaneous parametric down-conversion

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# Parameter estimation

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**Setup:** Transmission setup

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**Precision:**  $\text{Var}(T)$

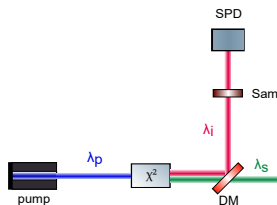
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**Setup:** Transmission setup

**Parameter:** Transmittance  $T$

**Precision:**  $\text{Var}(T)$

Conventional approach:



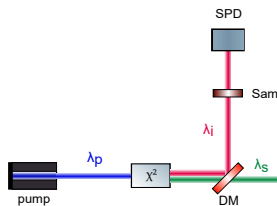
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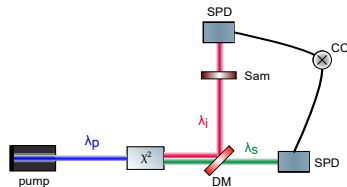
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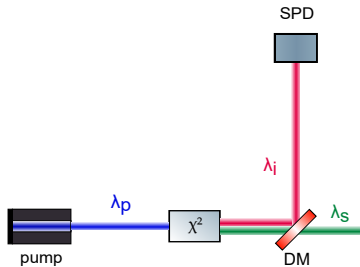
Conventional approach:



Coincidence approach:



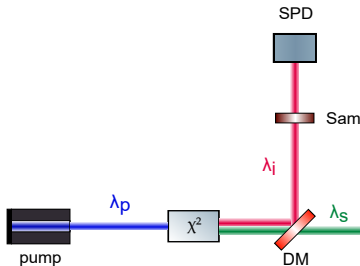
# Conventional approach



$$N_{\text{sing}}^{\text{ref}} = \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{ref}}$$



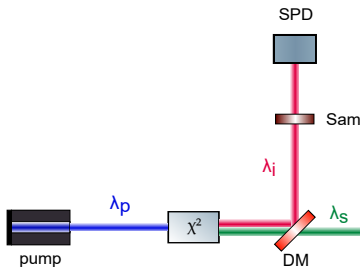
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$$N_{\text{sing}}^{\text{ref}} = \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{ref}}$$

$$N_{\text{sing}}^{\text{sam}} = T \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{sam}}$$

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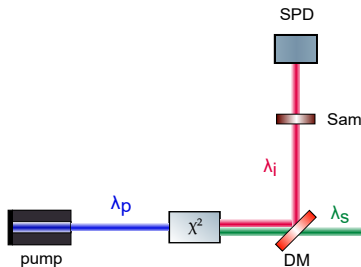


$$N_{\text{sing}}^{\text{ref}} = \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{ref}}$$

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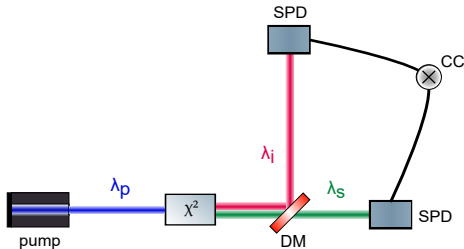
$$N_{\text{sing}}^{\text{ref}} = \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{ref}}$$

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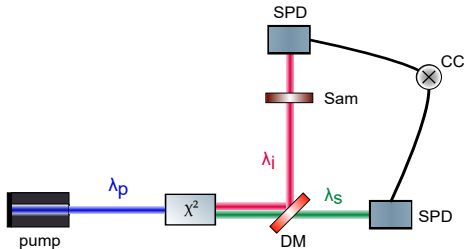
$$\text{Var}(T) = (\eta_{\text{idl}} N_g)^{-2} \left[ \text{Var}(N_{\text{sing}}^{\text{sam}}) + \text{Var}(N_{\text{noise}}^{\text{sam}}) + T^2 \left[ \text{Var}(N_{\text{sing}}^{\text{ref}}) + \text{Var}(N_{\text{noise}}^{\text{ref}}) \right] \right]$$

# Coincidence approach



$$N_{\text{coin}}^{\text{ref}} = \eta_{\text{idl}} \eta_{\text{sig}} N_g + N_{\text{ac}}^{\text{ref}}$$

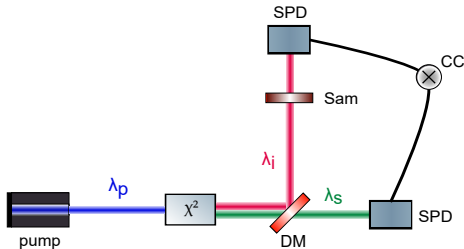
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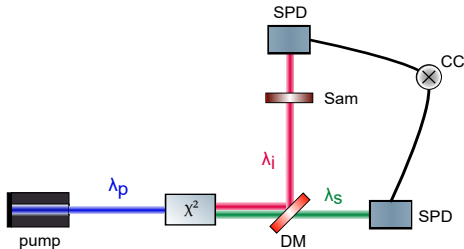


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$$\text{Var}(T) = (\eta_{\text{sig}} \eta_{\text{idl}} N_{\text{g}})^{-2} \left[ \text{Var}(N_{\text{coin}}^{\text{sam}}) + \text{Var}(N_{\text{ac}}^{\text{sam}}) + T^2 \left[ \text{Var}(N_{\text{coin}}^{\text{ref}}) + \text{Var}(N_{\text{ac}}^{\text{ref}}) \right] \right]$$

# Transmittance model

Conventional approach:

$$\text{Var}(T) = (\eta_{\text{idl}} N_g)^{-2} \left[ \text{Var}(N_{\text{sing}}^{\text{sam}}) + \text{Var}(N_{\text{noise}}^{\text{sam}}) + T^2 \left[ \text{Var}(N_{\text{sing}}^{\text{ref}}) + \text{Var}(N_{\text{noise}}^{\text{ref}}) \right] \right]$$

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# Photon statistics

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$$\mathcal{P}(n) = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}$$

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<sup>[2]</sup>Kim et al., "Photon-Counting Statistics-Based Support Vector Machine with Multi-Mode Photon Illumination for Quantum Imaging"

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Multi-mode Bose-Einstein distribution (thermal light): <sup>[2]</sup>

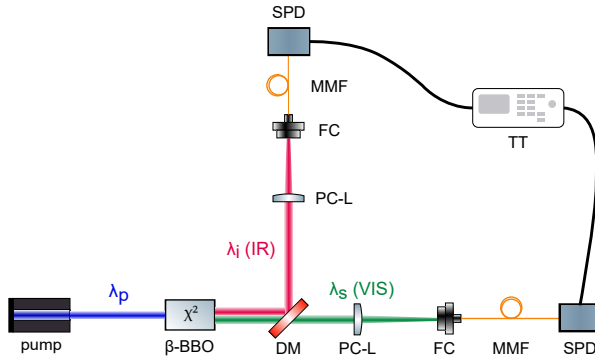
$$\mathcal{P}_m(n) = \frac{(n+m-1)!}{(m-1)! n!} \frac{m^m \langle n \rangle^n}{(m + \langle n \rangle)^{n+m}}$$
$$\text{Var}(n) = \langle n \rangle \left( 1 + \frac{\langle n \rangle}{m} \right)$$

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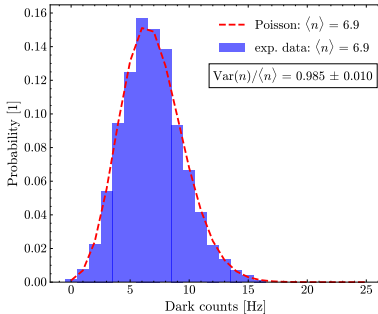
# Experimental setup



# Dark counts, $\text{Var}(N_{\text{noise}})$

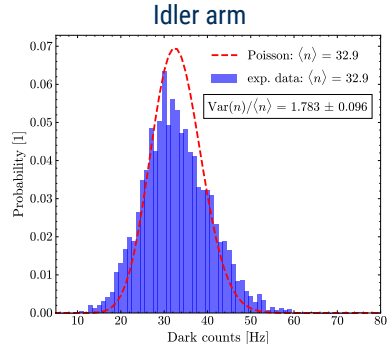
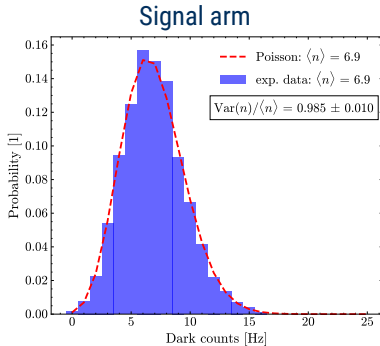
# Dark counts, $\text{Var}(N_{\text{noise}})$

## Signal arm

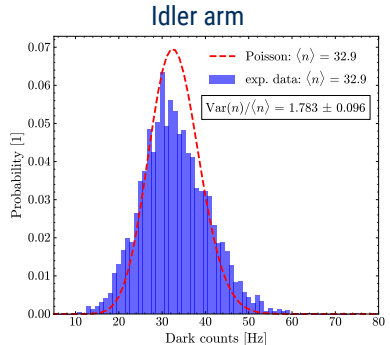
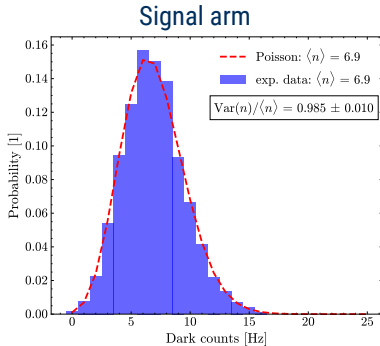




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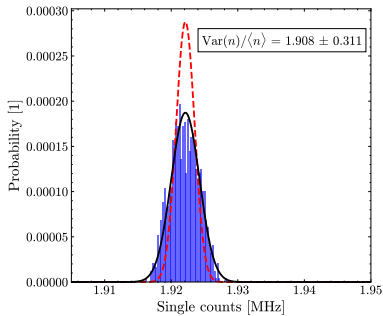
$$\text{Var}(N_{\text{noise}}) = 1.8 \cdot \langle N_{\text{noise}} \rangle$$

# Single counts, $\text{Var}(N_{\text{sing}})$

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## Signal arm

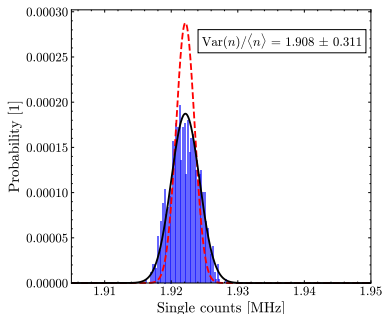
- Poisson:  $\langle n \rangle = 1922127$
- mmBE:  $m = 1412365$ ,  $\langle n \rangle = 1922127$
- exp. data:  $m_{\text{theo}} = 1412365$ ,  $\langle n \rangle = 1922127$



# Single counts, $\text{Var}(N_{\text{sing}})$

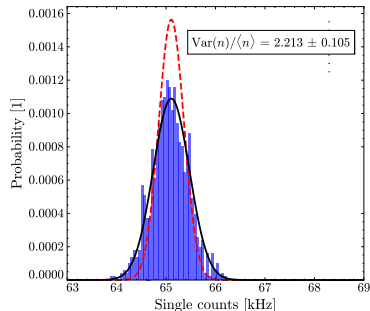
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## Idler arm

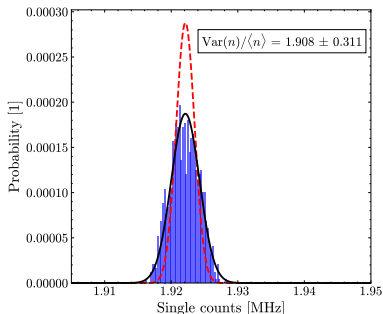
- Poisson:  $\langle n \rangle = 65107$
- mmBE:  $m = 61384$ ,  $\langle n \rangle = 65107$
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# Single counts, $\text{Var}(N_{\text{sing}})$

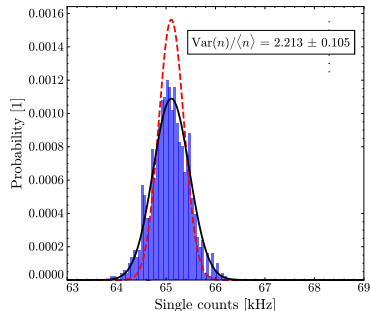
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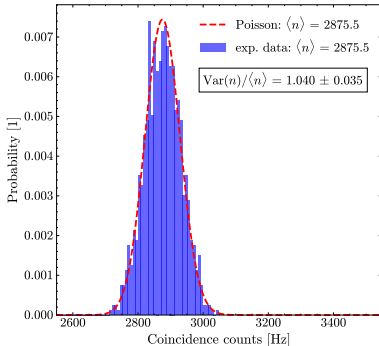


$$\text{Var}(N_{\text{sing}}) = 2.2 \cdot \langle N_{\text{sing}} \rangle$$

# Coincidence counts, $\text{Var}(N_{\text{coin}})$

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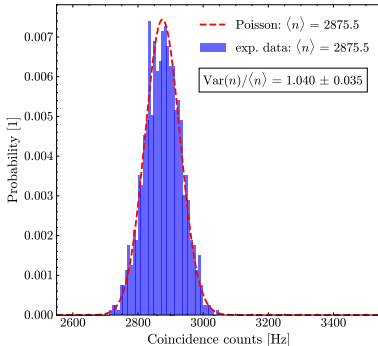
## Coincidences





# Coincidence counts, $\text{Var}(N_{\text{coin}})$

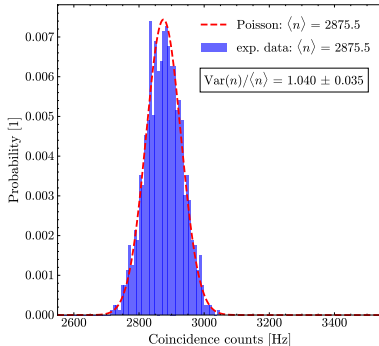
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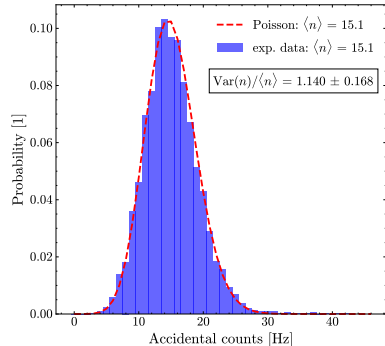
$$\text{Var}(N_{\text{coin}}) = \langle N_{\text{coin}} \rangle$$

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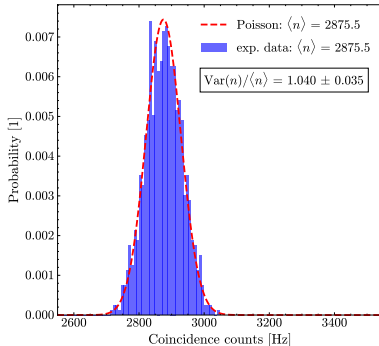
## Accidentals



$$\text{Var}(N_{\text{coin}}) = \langle N_{\text{coin}} \rangle$$

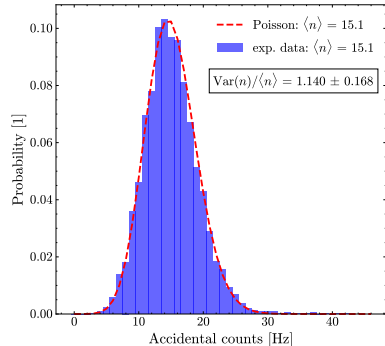
# Coincidence counts, $\text{Var}(N_{\text{coin}})$

## Coincidences



$$\text{Var}(N_{\text{coin}}) = \langle N_{\text{coin}} \rangle$$

## Accidentals



$$\text{Var}(N_{\text{ac}}) = \langle N_{\text{ac}} \rangle$$

# Simulation

## Conventional approach:

$$\text{Var}(\mathcal{T}) = (\eta_{\text{idl}} N_g)^{-2} \left[ \text{Var}(N_{\text{sing}}^{\text{sam}}) + \text{Var}(N_{\text{noise}}^{\text{sam}}) + \mathcal{T}^2 \left[ \text{Var}(N_{\text{sing}}^{\text{ref}}) + \text{Var}(N_{\text{noise}}^{\text{ref}}) \right] \right]$$

# Simulation

## Conventional approach:

$$\text{Var}(T) = (\eta_{\text{idl}} N_g)^{-2} \left[ 2.2 \cdot \langle N_{\text{sing}}^{\text{sam}} \rangle + 1.8 \cdot \langle N_{\text{noise}}^{\text{sam}} \rangle + T^2 \left[ 2.2 \cdot \langle N_{\text{sing}}^{\text{ref}} \rangle + 1.8 \cdot \langle N_{\text{noise}}^{\text{ref}} \rangle \right] \right]$$

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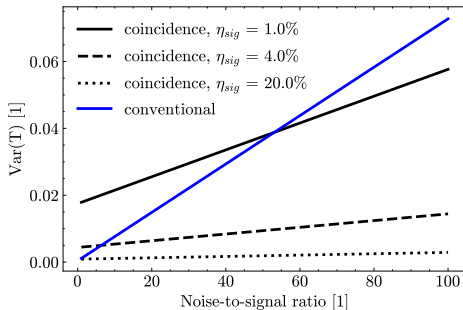
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# Noise-to-signal ratio



## Parameter

## Value

|                              |           |
|------------------------------|-----------|
| $\eta_{\text{idl}}$ (%)      | 0.09      |
| $\eta_{\text{sig}}$ (%)      | 1, 4, 20  |
| $R_{\text{idl}}$ (kHz)       | 10        |
| $R_{\text{noise,idl}}$ (kHz) | 10 - 1000 |
| $R_{\text{noise,sig}}$ (Hz)  | 7         |
| $T$ (1)                      | 0.9       |



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# Summary and Outlook

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- Established a model for the variance of transmittance
- Experimental verification of photon statistics
- Found regions where coincidence approach offers higher precision

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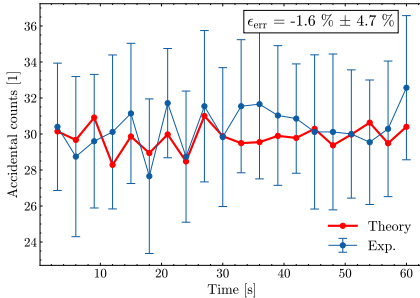
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- Experimental verification of photon statistics
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## Outlook:

- Experimental verification of the found parameter regions
- Determine the experimental limitations for the variance measurement

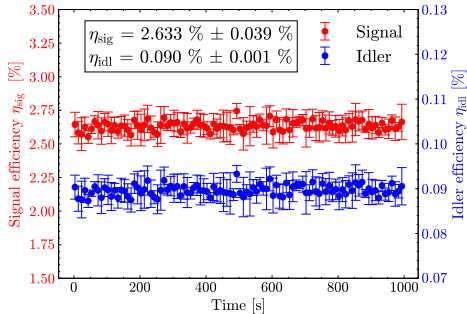
# Accidental counts



$$R_{\text{ac}}^{\text{sam}} = \left( T \eta_{\text{idl}} R_{\text{g}} + R_{\text{dc,idl}} - R_{\text{cc,pure}}^{\text{sam}} \right) \cdot \left( \eta_{\text{sig}} R_{\text{g}} + R_{\text{dc,sig}} - R_{\text{cc,pure}}^{\text{sam}} \right) \cdot \tau_{\text{cw}}$$

$$R_{\text{ac}}^{\text{ref}} = \left( \eta_{\text{idl}} R_{\text{g}} + R_{\text{dc,idl}} - R_{\text{cc,pure}}^{\text{ref}} \right) \cdot \left( \eta_{\text{sig}} R_{\text{g}} + R_{\text{dc,sig}} - R_{\text{cc,pure}}^{\text{ref}} \right) \cdot \tau_{\text{cw}}$$

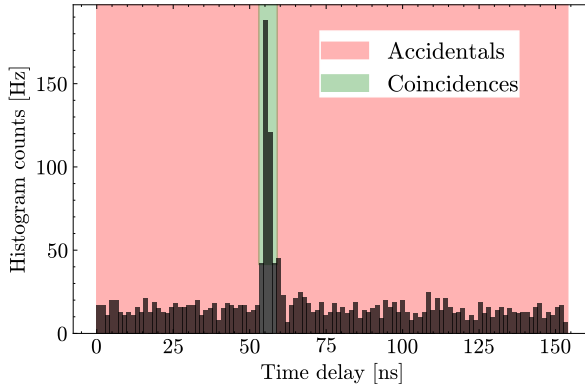
# Heralding efficiencies



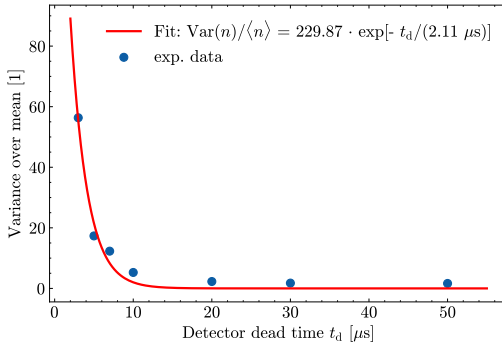
$$\eta_{\text{sig}} = \frac{N_{\text{coin}} - N_{\text{ac}}}{N_{\text{sing, idl}} - N_{\text{noise}}} = \frac{\eta_{\text{sig}} \eta_{\text{idl}} N_{\text{g}}}{\eta_{\text{idl}} N_{\text{g}}}$$

$$\eta_{\text{idl}} = \frac{N_{\text{coin}} - N_{\text{ac}}}{N_{\text{sing, sig}} - N_{\text{noise}}} = \frac{\eta_{\text{sig}} \eta_{\text{idl}} N_{\text{g}}}{\eta_{\text{sig}} N_{\text{g}}}$$

# Histogram

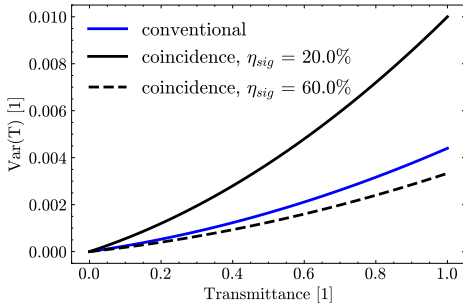


# Afterpulsing



$$R_{aft} \propto e^{-\frac{t_d}{\tau_{de}}}$$

# Noiseless case



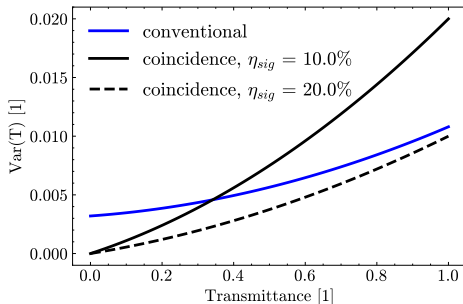
## Parameter

## Value

|                        |        |
|------------------------|--------|
| $\eta_{idl}$ (%)       | 60     |
| $\eta_{sig}$ (%)       | 20, 60 |
| $R_{idl}$ (kHz)        | 1      |
| $R_{noise, idl}$ (kHz) | 0      |
| $R_{noise, sig}$ (Hz)  | 7      |
| $T$ (1)                | 0 - 1  |



# In presence of noise

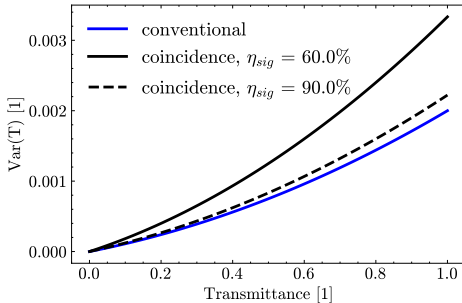


## Parameter

## Value

|                        |        |
|------------------------|--------|
| $\eta_{idl}$ (%)       | 60     |
| $\eta_{sig}$ (%)       | 10, 20 |
| $R_{idl}$ (kHz)        | 1      |
| $R_{noise, idl}$ (kHz) | 1      |
| $R_{noise, sig}$ (Hz)  | 7      |
| $T$ (1)                | 0 - 1  |

# Coherent Illumination, noiseless

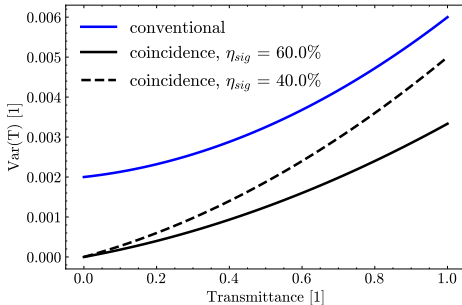


## Parameter

## Value

|                              |        |
|------------------------------|--------|
| $\eta_{\text{idl}}$ (%)      | 60     |
| $\eta_{\text{sig}}$ (%)      | 60, 90 |
| $R_{\text{idl}}$ (kHz)       | 1      |
| $R_{\text{noise,idl}}$ (kHz) | 0      |
| $R_{\text{noise,sig}}$ (Hz)  | 7      |
| $T$ (1)                      | 0 - 1  |

# Coherent Illumination, with noise



## Parameter

## Value

|                              |        |
|------------------------------|--------|
| $\eta_{\text{idl}}$ (%)      | 60     |
| $\eta_{\text{sig}}$ (%)      | 40, 60 |
| $R_{\text{idl}}$ (kHz)       | 1      |
| $R_{\text{noise,idl}}$ (kHz) | 1      |
| $R_{\text{noise,sig}}$ (Hz)  | 7      |
| $T$ (1)                      | 0 - 1  |