

Parameter estimation with correlated photon pairs

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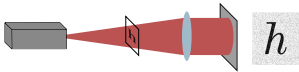
Jena, November 7, 2025

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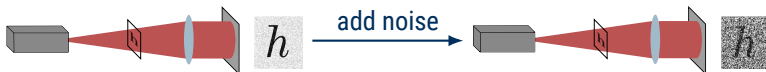
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Motivation

Motivation

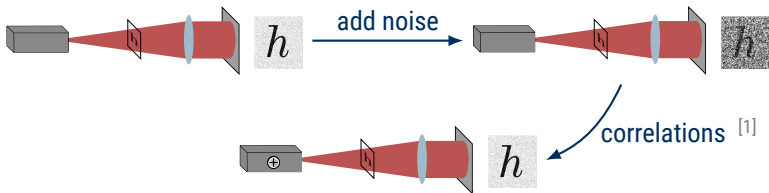


Motivation



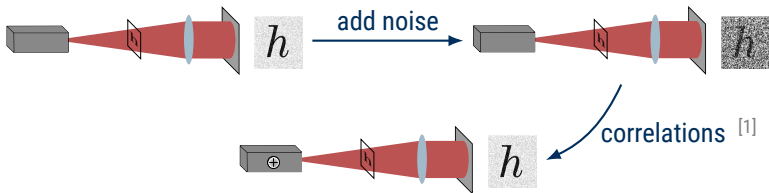
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Motivation



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Motivation

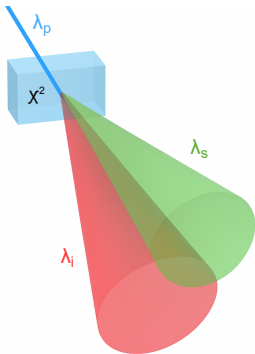


Objective: Can correlated photons provide advantages in terms of precision in noisy regimes for parameter estimation?

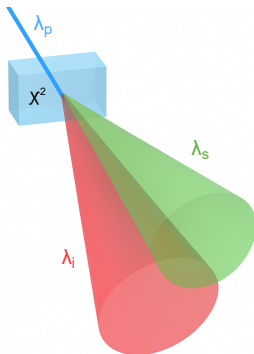
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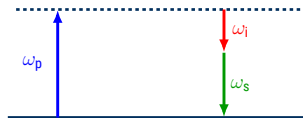
SPDC



SPDC

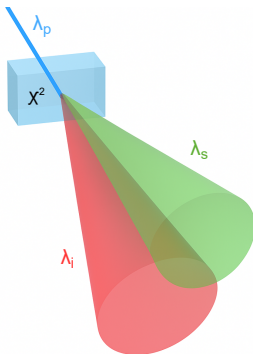


Energy conservation

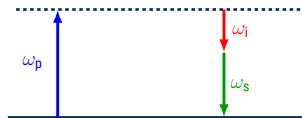


$$\omega_p = \omega_s + \omega_i$$

SPDC

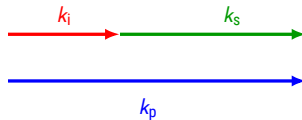


Energy conservation



$$\omega_p = \omega_s + \omega_i$$

Momentum conservation



$$\vec{k}_p = \vec{k}_s + \vec{k}_i$$

Parameter estimation

Parameter estimation

Setup: Transmission setup

Parameter estimation

Setup: Transmission setup

Parameter: Transmittance T

Parameter estimation

Setup: Transmission setup

Parameter: Transmittance T

Estimator: $\text{Var}(T)$

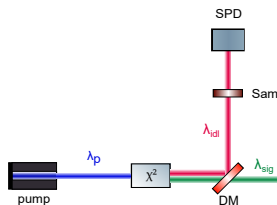
Parameter estimation

Setup: Transmission setup

Parameter: Transmittance T

Estimator: $\text{Var}(T)$

Conventional approach:



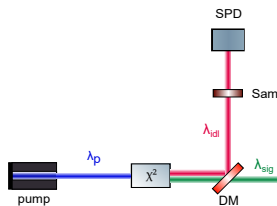
Parameter estimation

Setup: Transmission setup

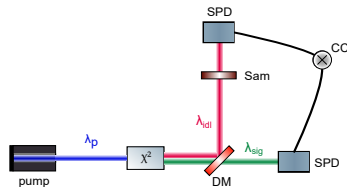
Parameter: Transmittance T

Estimator: $\text{Var}(T)$

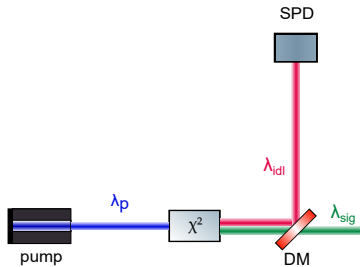
Conventional approach:



Coincidence approach:

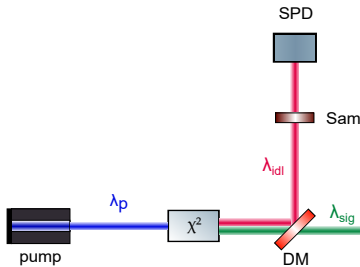


Conventional approach



$$N_{\text{tot}}^{\text{ref}} = \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{ref}}$$

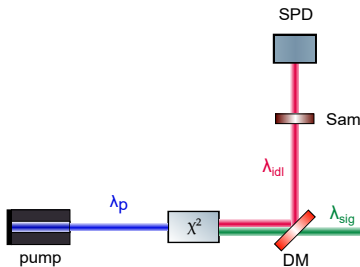
Conventional approach



$$N_{\text{tot}}^{\text{ref}} = \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{ref}}$$

$$N_{\text{tot}}^{\text{sam}} = T \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{sam}}$$

Conventional approach

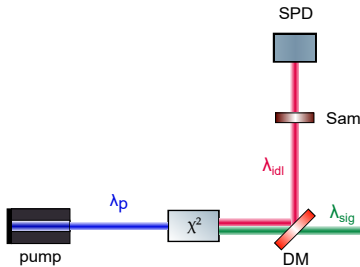


$$N_{\text{tot}}^{\text{ref}} = \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{ref}}$$

$$N_{\text{tot}}^{\text{sam}} = T \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{sam}}$$

$$\Rightarrow T = \frac{N_{\text{tot}}^{\text{sam}} - N_{\text{noise}}^{\text{sam}}}{N_{\text{tot}}^{\text{ref}} - N_{\text{noise}}^{\text{ref}}}$$

Conventional approach



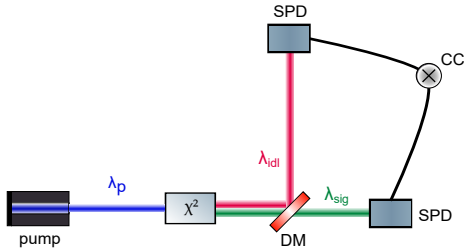
$$N_{\text{tot}}^{\text{ref}} = \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{ref}}$$

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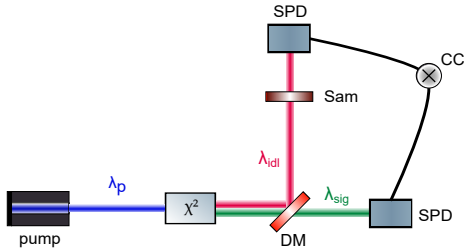
$$\text{Var}(T) = (\eta_{\text{idl}} N_g)^{-2} \left[\text{Var}(N_{\text{tot}}^{\text{sam}}) + \text{Var}(N_{\text{noise}}^{\text{sam}}) + T^2 \left[\text{Var}(N_{\text{tot}}^{\text{ref}}) + \text{Var}(N_{\text{noise}}^{\text{ref}}) \right] \right]$$

Coincidence approach



$$N_{cc,tot}^{ref} = \eta_{idl} \eta_{sig} N_g + N_{ac}^{ref}$$

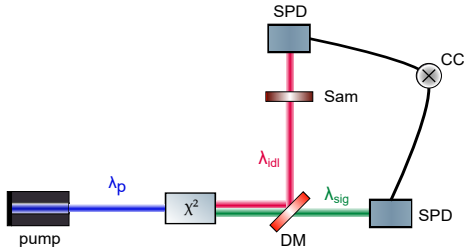
Coincidence approach



$$N_{cc,tot}^{ref} = \eta_{idl} \eta_{sig} N_g + N_{ac}^{ref}$$

$$N_{cc,tot}^{sam} = T \eta_{idl} \eta_{sig} N_g + N_{ac}^{sam}$$

Coincidence approach

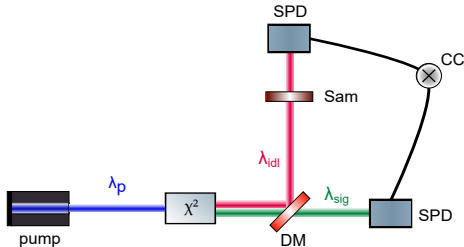


$$N_{\text{cc,tot}}^{\text{ref}} = \eta_{\text{idl}} \eta_{\text{sig}} N_{\text{g}} + N_{\text{ac}}^{\text{ref}}$$

$$N_{\text{cc,tot}}^{\text{sam}} = T \eta_{\text{idl}} \eta_{\text{sig}} N_{\text{g}} + N_{\text{ac}}^{\text{sam}}$$

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Coincidence approach



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$$N_{\text{cc,tot}}^{\text{sam}} = T \eta_{\text{idl}} \eta_{\text{sig}} N_{\text{g}} + N_{\text{ac}}^{\text{sam}}$$

$$\Rightarrow T = \frac{N_{\text{tot,cc}}^{\text{sam}} - N_{\text{ac}}^{\text{sam}}}{N_{\text{tot,cc}}^{\text{ref}} - N_{\text{ac}}^{\text{ref}}}$$

$$\text{Var}(T) = (\eta_{\text{sig}} \eta_{\text{idl}} N_{\text{g}})^{-2} \left[\text{Var}(N_{\text{tot,cc}}^{\text{sam}}) + \text{Var}(N_{\text{ac}}^{\text{sam}}) + T^2 \left[\text{Var}(N_{\text{tot,cc}}^{\text{ref}}) + \text{Var}(N_{\text{ac}}^{\text{ref}}) \right] \right]$$

Transmittance model

Conventional approach:

$$\text{Var}(T) = (\eta_{\text{idl}} N_g)^{-2} \left[\text{Var}(N_{\text{tot}}^{\text{sam}}) + \text{Var}(N_{\text{noise}}^{\text{sam}}) + T^2 \left[\text{Var}(N_{\text{tot}}^{\text{ref}}) + \text{Var}(N_{\text{noise}}^{\text{ref}}) \right] \right]$$

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Photon statistics

Photon statistics

Poisson distribution (coherent light):

$$\mathcal{P}(n) = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}$$

$$\text{Var}(n) = \langle n \rangle$$

Photon statistics

Poisson distribution (coherent light): ^[2]^[3]

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^[2]Kim et al., "Photon-Counting Statistics-Based Support Vector Machine with Multi-Mode Photon Illumination for Quantum Imaging"

^[3]Fouche, "Detection and False-Alarm Probabilities for Laser Radars That Use Geiger-mode Detectors"

Photon statistics

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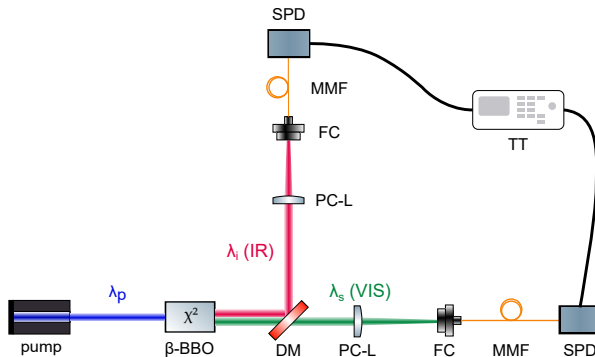
Multi-mode Bose-Einstein distribution (thermal light): ^[2]

$$\mathcal{P}_m(n) = \frac{(n+m-1)!}{(m-1)! n!} \frac{m^m \langle n \rangle^n}{(m + \langle n \rangle)^{n+m}}$$
$$\text{Var}(n) = \langle n \rangle \left(1 + \frac{\langle n \rangle}{m} \right)$$

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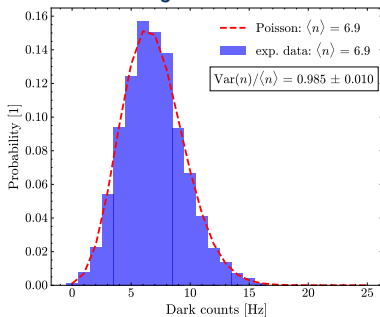
Experimental setup



Dark counts

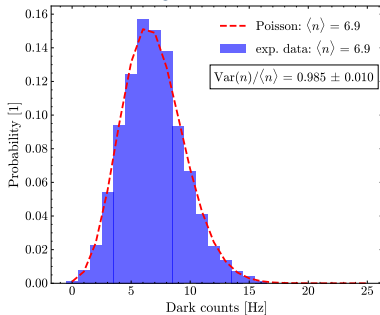
Dark counts

Signal arm

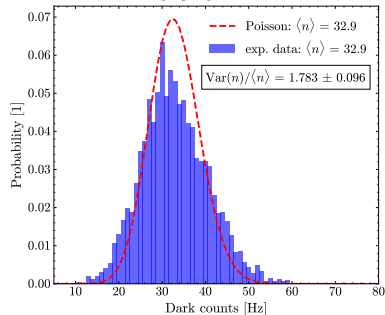


Dark counts

Signal arm

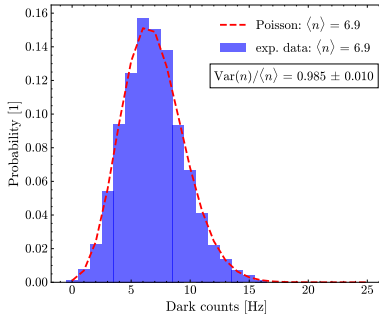


Idler arm

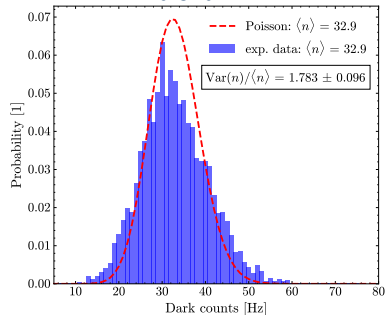


Dark counts

Signal arm



Idler arm



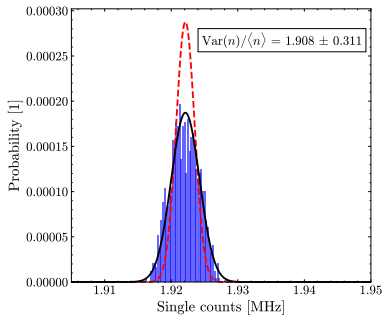
$$\text{Var}(N_{\text{noise}}) = 1.8 \cdot \langle N_{\text{noise}} \rangle$$

Single counts

Single counts

Signal arm

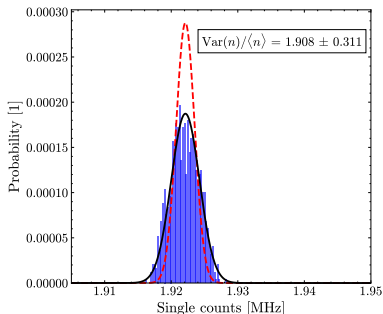
- Poisson: $\langle n \rangle = 1922127$
- mmBE: $m = 1412365$, $\langle n \rangle = 1922127$
- exp. data: $m_{theo} = 1412365$, $\langle n \rangle = 1922127$



Single counts

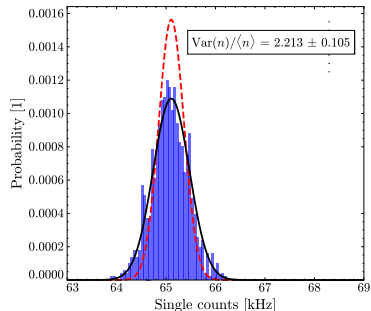
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Idler arm

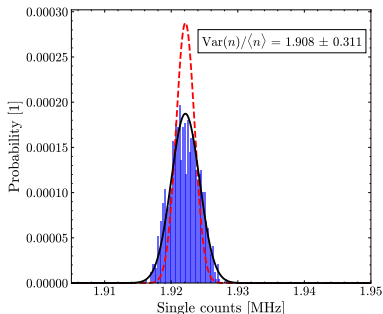
- Poisson: $\langle n \rangle = 65107$
- mmBE: $m = 61384$, $\langle n \rangle = 65107$
- exp. data: $m_{theo} = 61384$, $\langle n \rangle = 65107$



Single counts

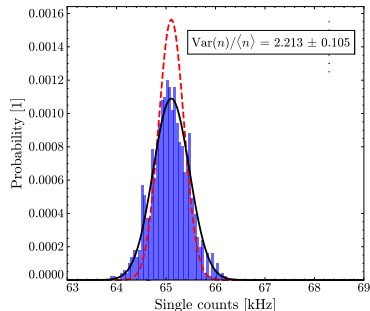
Signal arm

- Poisson: $\langle n \rangle = 1922127$
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Idler arm

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- mmBE: $m = 61384$, $\langle n \rangle = 65107$
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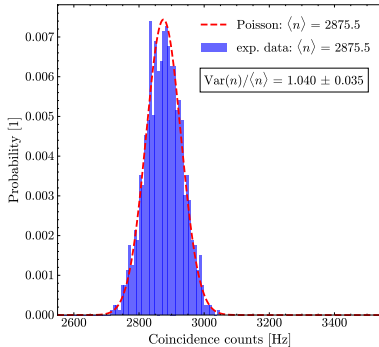


$$\text{Var}(N_{\text{noise}}) = 2.2 \cdot \langle N_{\text{noise}} \rangle$$

Coincidence counts

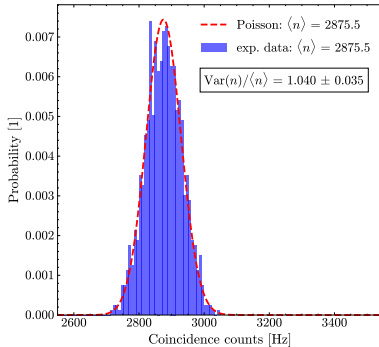
Coincidence counts

Coincidences



Coincidence counts

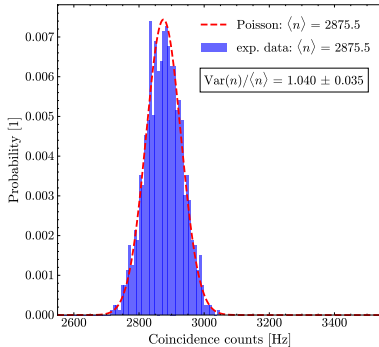
Coincidences



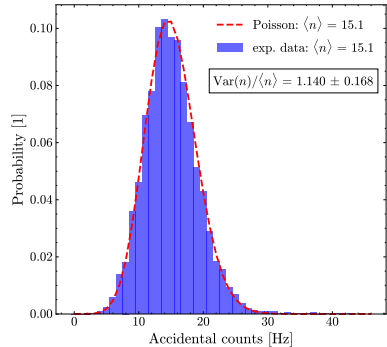
$$\text{Var}(N_{\text{cc}}) = \langle N_{\text{cc}} \rangle$$

Coincidence counts

Coincidences



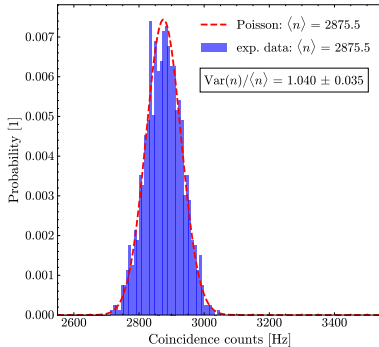
Accidentals



$$\text{Var}(N_{\text{cc}}) = \langle N_{\text{cc}} \rangle$$

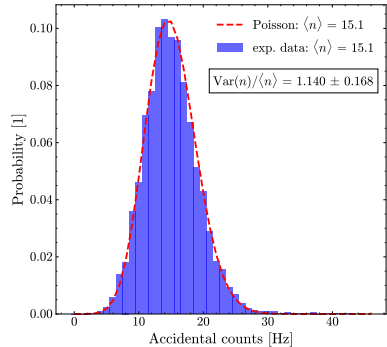
Coincidence counts

Coincidences



$$\text{Var}(N_{\text{cc}}) = \langle N_{\text{cc}} \rangle$$

Accidentals



$$\text{Var}(N_{\text{ac}}) = \langle N_{\text{ac}} \rangle$$

Simulation

Conventional approach:

$$\text{Var}(\mathcal{T}) = (\eta_{\text{idl}} N_{\text{g}})^{-2} \left[\text{Var}(N_{\text{tot}}^{\text{sam}}) + \text{Var}(N_{\text{noise}}^{\text{sam}}) + T^2 \left[\text{Var}(N_{\text{tot}}^{\text{ref}}) + \text{Var}(N_{\text{noise}}^{\text{ref}}) \right] \right]$$

Simulation

Conventional approach:

$$\text{Var}(T) = (\eta_{\text{idl}} N_{\text{g}})^{-2} \left[2.2 \cdot \langle N_{\text{tot}}^{\text{sam}} \rangle + 1.8 \cdot \langle N_{\text{noise}}^{\text{sam}} \rangle + T^2 \left[2.2 \cdot \langle N_{\text{tot}}^{\text{ref}} \rangle + 1.8 \cdot \langle N_{\text{noise}}^{\text{ref}} \rangle \right] \right]$$

Simulation

Conventional approach:

$$\text{Var}(T) = (\eta_{\text{idl}} N_g)^{-2} \left[2.2 \cdot \langle N_{\text{tot}}^{\text{sam}} \rangle + 1.8 \cdot \langle N_{\text{noise}}^{\text{sam}} \rangle + T^2 \left[2.2 \cdot \langle N_{\text{tot}}^{\text{ref}} \rangle + 1.8 \cdot \langle N_{\text{noise}}^{\text{ref}} \rangle \right] \right]$$

Coincidence approach:

$$\text{Var}(T) = (\eta_{\text{sig}} \eta_{\text{idl}} N_g)^{-2} \left[\text{Var}(N_{\text{tot,cc}}^{\text{sam}}) + \text{Var}(N_{\text{ac}}^{\text{sam}}) + T^2 \left[\text{Var}(N_{\text{tot,cc}}^{\text{ref}}) + \text{Var}(N_{\text{ac}}^{\text{ref}}) \right] \right]$$

Simulation

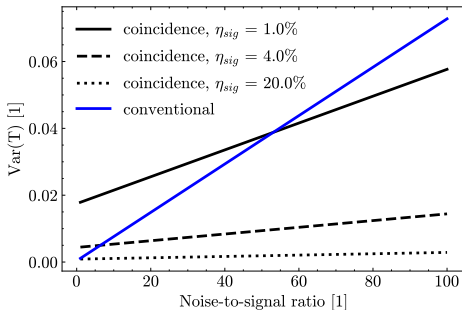
Conventional approach:

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Coincidence approach:

$$\text{Var}(T) = (\eta_{\text{sig}} \eta_{\text{idl}} N_g)^{-2} \left[\langle N_{\text{tot,cc}}^{\text{sam}} \rangle + \langle N_{\text{ac}}^{\text{sam}} \rangle + T^2 \left[\langle N_{\text{tot,cc}}^{\text{ref}} \rangle + \langle N_{\text{ac}}^{\text{ref}} \rangle \right] \right]$$

Noise-to-signal ratio



Parameter

Value

 η_{idl} (%)

0.09

 η_{sig} (%)

1, 4, 20

 R_{idl} (kHz)

10

 $R_{\text{noise,idl}}$ (kHz)

10 - 1000

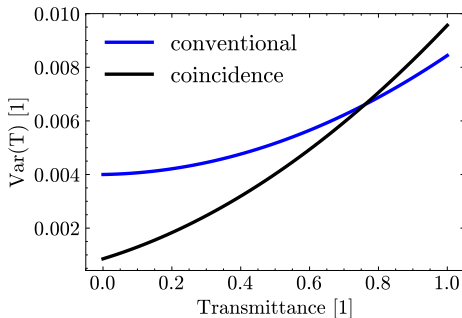
 $R_{\text{noise,sig}}$ (Hz)

7

 T (1)

0.9

Transmittance



Parameter

Value

η_{idl} (%) 0.09

η_{sig} (%) 2.6

R_{idl} (kHz) 10

$R_{\text{noise,idl}}$ (kHz) 100

$R_{\text{noise,sig}}$ (Hz) 7

T (1) 0 - 1

Summary and Outlook

Summary and Outlook

Achievements:

- Model for variance of transmittance
- Experimental verification of photon statistics
- Found regions where coincidence approach offers advantages

Summary and Outlook

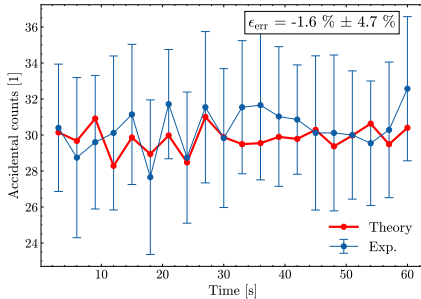
Achievements:

- Model for variance of transmittance
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- Found regions where coincidence approach offers advantages

Outlook:

- Experimental verification of found parameter regions
- Determine the experimental limitations of the variance resolution

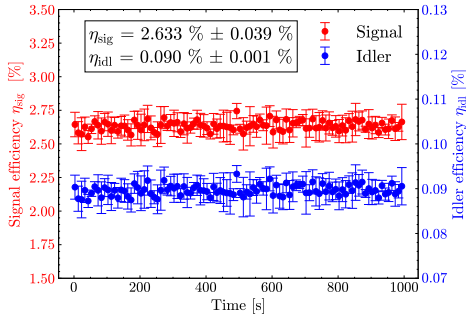
Accidental counts



$$R_{\text{ac}}^{\text{sam}} = \left(T \eta_{\text{idl}} R_{\text{g}} + R_{\text{dc,idl}} - R_{\text{cc,pure}}^{\text{sam}} \right) \cdot \left(\eta_{\text{sig}} R_{\text{g}} + R_{\text{dc,sig}} - R_{\text{cc,pure}}^{\text{sam}} \right) \cdot \tau_{\text{cw}}$$

$$R_{\text{ac}}^{\text{ref}} = \left(\eta_{\text{idl}} R_{\text{g}} + R_{\text{dc,idl}} - R_{\text{cc,pure}}^{\text{ref}} \right) \cdot \left(\eta_{\text{sig}} R_{\text{g}} + R_{\text{dc,sig}} - R_{\text{cc,pure}}^{\text{ref}} \right) \cdot \tau_{\text{cw}}$$

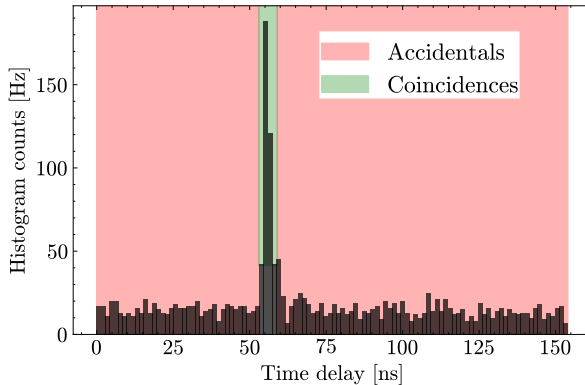
Heralding efficiencies



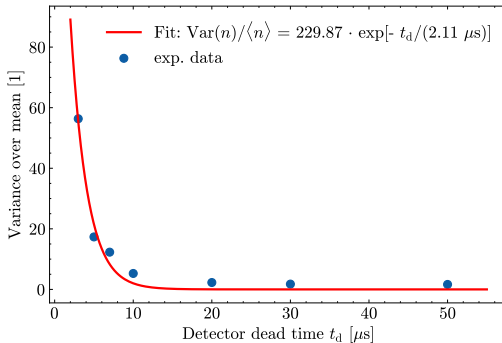
$$\eta_{\text{sig}} = \frac{N_{\text{tot,cc}} - N_{\text{ac}}}{N_{\text{tot,idl}} - N_{\text{noise}}} = \frac{\eta_{\text{sig}} \eta_{\text{idl}} N_g}{\eta_{\text{idl}} N_g}$$

$$\eta_{\text{idl}} = \frac{N_{\text{tot,cc}} - N_{\text{ac}}}{N_{\text{tot,sig}} - N_{\text{noise}}} = \frac{\eta_{\text{sig}} \eta_{\text{idl}} N_g}{\eta_{\text{sig}} N_g}$$

Histogram



Afterpulsing



$$R_{\text{aft}} \propto e^{-t_d}$$