

Parameter estimation with correlated photon pairs

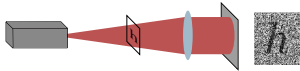
Jan Gößwein

Institute of Applied Physics

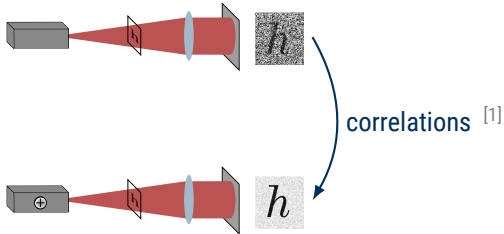
Jena, November 18, 2025

Motivation

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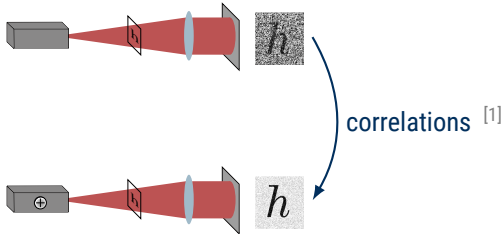


Motivation



[1] Brida, Genovese, and Ruo Berchera, "Experimental Realization of Sub-Shot-Noise Quantum Imaging"

Motivation

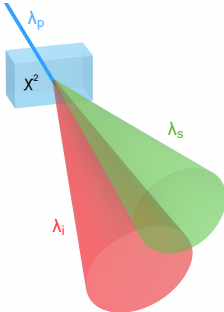


Objective: Can correlated photons provide advantages in terms of precision in noisy regimes for parameter estimation?

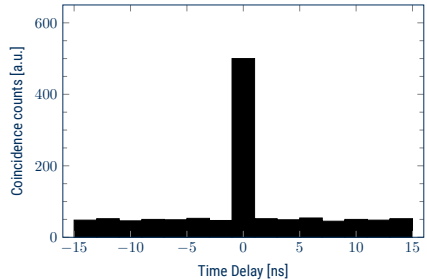
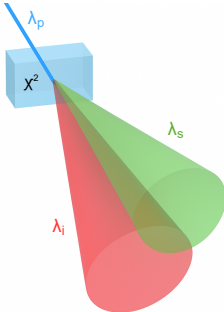
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Spontaneous parametric down-conversion

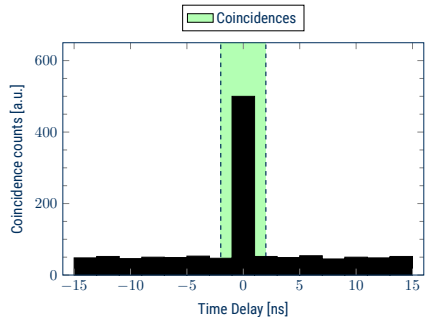
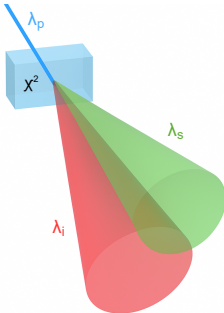
Spontaneous parametric down-conversion



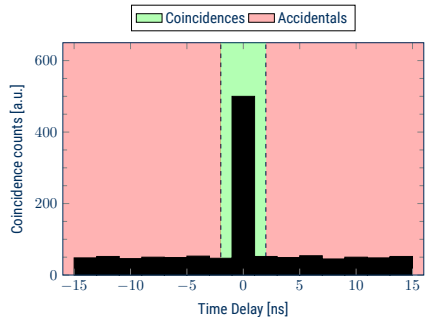
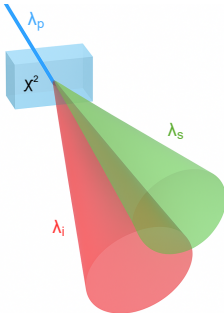
Spontaneous parametric down-conversion



Spontaneous parametric down-conversion



Spontaneous parametric down-conversion



Parameter estimation

Parameter estimation

Setup: Transmission setup

Parameter estimation

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Parameter: Transmittance T

Parameter estimation

Setup: Transmission setup

Parameter: Transmittance T

Precision: $\text{Var}(T)$

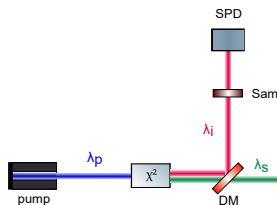
Parameter estimation

Setup: Transmission setup

Parameter: Transmittance T

Precision: $\text{Var}(T)$

Conventional approach:



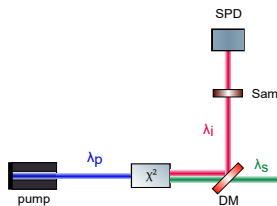
Parameter estimation

Setup: Transmission setup

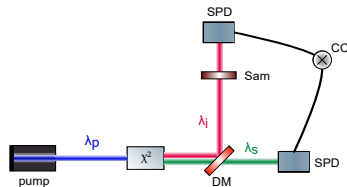
Parameter: Transmittance T

Precision: $\text{Var}(T)$

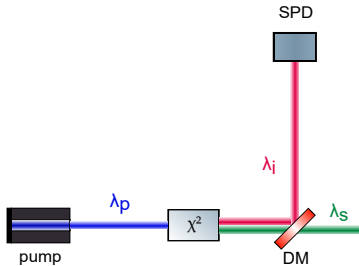
Conventional approach:



Coincidence approach:

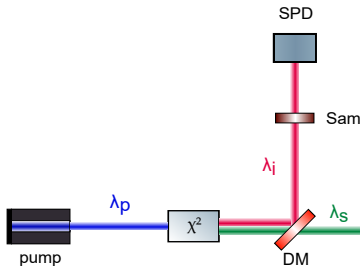


Conventional approach



$$N_{\text{sing}}^{\text{ref}} = \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{ref}}$$

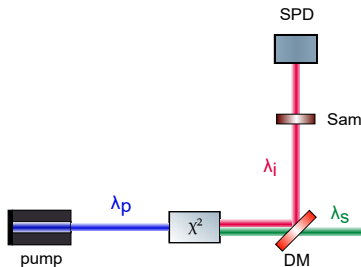
Conventional approach



$$N_{\text{sing}}^{\text{ref}} = \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{ref}}$$

$$N_{\text{sing}}^{\text{sam}} = T \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{sam}}$$

Conventional approach

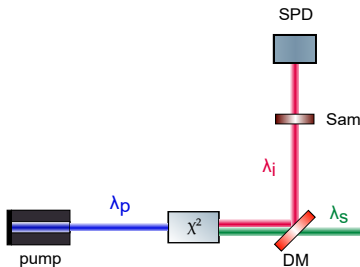


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$$\Rightarrow T = \frac{N_{\text{sing}}^{\text{sam}} - N_{\text{noise}}^{\text{sam}}}{N_{\text{sing}}^{\text{ref}} - N_{\text{noise}}^{\text{ref}}}$$

Conventional approach



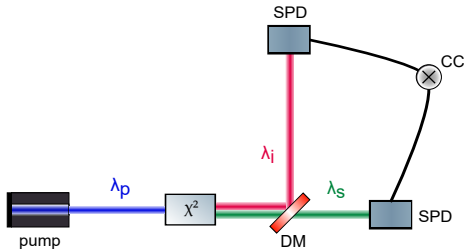
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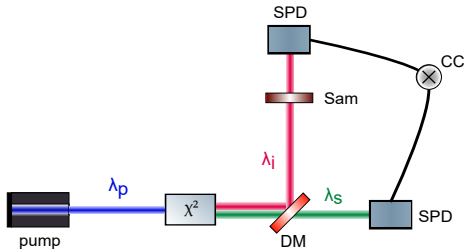
$$\text{Var}(T) = (\eta_{\text{idl}} N_g)^{-2} \left[\text{Var}(N_{\text{sing}}^{\text{sam}}) + \text{Var}(N_{\text{noise}}^{\text{sam}}) + T^2 \left[\text{Var}(N_{\text{sing}}^{\text{ref}}) + \text{Var}(N_{\text{noise}}^{\text{ref}}) \right] \right]$$

Coincidence approach



$$N_{\text{coin}}^{\text{ref}} = \eta_{\text{idl}} \eta_{\text{sig}} N_g + N_{\text{ac}}^{\text{ref}}$$

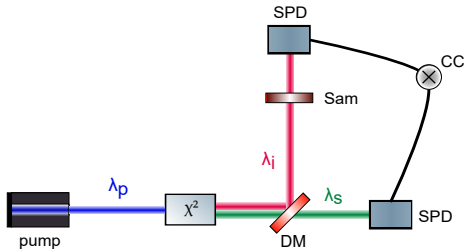
Coincidence approach



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$$N_{\text{coin}}^{\text{sam}} = T \eta_{\text{idl}} \eta_{\text{sig}} N_{\text{g}} + N_{\text{ac}}^{\text{sam}}$$

Coincidence approach

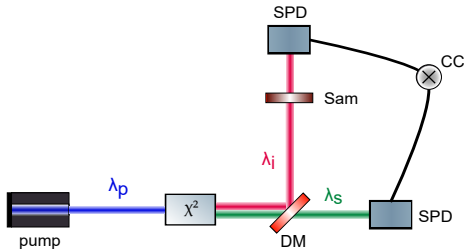


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$$\text{Var}(T) = (\eta_{\text{sig}} \eta_{\text{idl}} N_{\text{g}})^{-2} \left[\text{Var}(N_{\text{coin}}^{\text{sam}}) + \text{Var}(N_{\text{ac}}^{\text{sam}}) + T^2 \left[\text{Var}(N_{\text{coin}}^{\text{ref}}) + \text{Var}(N_{\text{ac}}^{\text{ref}}) \right] \right]$$

Transmittance model

Conventional approach:

$$\text{Var}(T) = (\eta_{\text{idl}} N_g)^{-2} \left[\text{Var}(N_{\text{sing}}^{\text{sam}}) + \text{Var}(N_{\text{noise}}^{\text{sam}}) + T^2 \left[\text{Var}(N_{\text{sing}}^{\text{ref}}) + \text{Var}(N_{\text{noise}}^{\text{ref}}) \right] \right]$$

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Photon statistics

Photon statistics

Poisson distribution (coherent light):

$$\mathcal{P}(n) = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}$$

$$\text{Var}(n) = \langle n \rangle$$

Photon statistics

Poisson distribution (coherent light): ^[2]^[3]

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^[2]Kim et al., "Photon-Counting Statistics-Based Support Vector Machine with Multi-Mode Photon Illumination for Quantum Imaging"

^[3]Fouche, "Detection and False-Alarm Probabilities for Laser Radars That Use Geiger-mode Detectors"

Photon statistics

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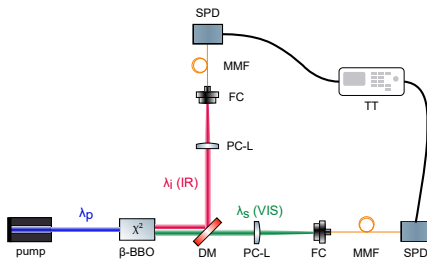
Multi-mode Bose-Einstein distribution (thermal light): ^[2]

$$\mathcal{P}_m(n) = \frac{(n+m-1)!}{(m-1)! n!} \frac{m^m \langle n \rangle^n}{(m + \langle n \rangle)^{n+m}}$$
$$\text{Var}(n) = \langle n \rangle \left(1 + \frac{\langle n \rangle}{m} \right)$$

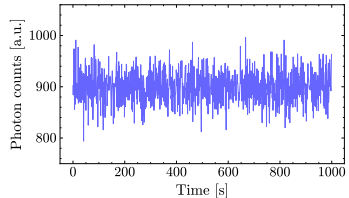
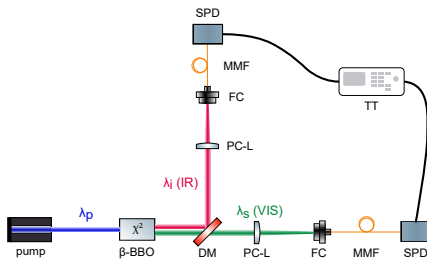
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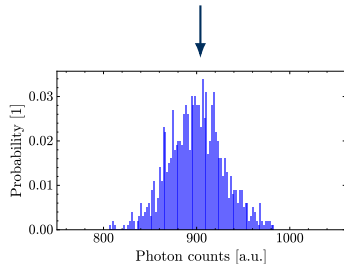
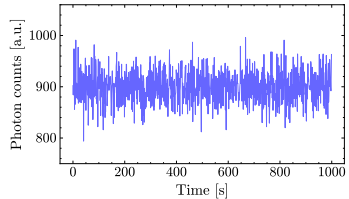
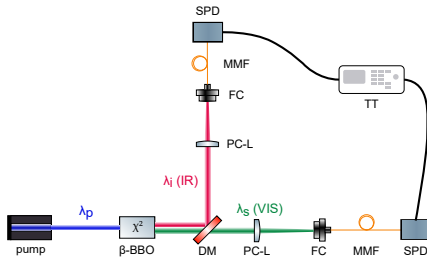
Experimental setup



Experimental setup



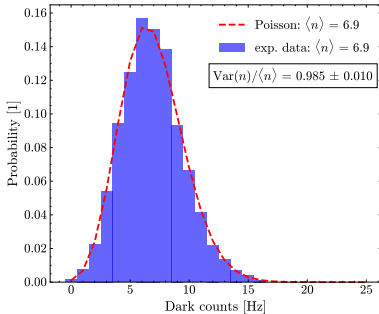
Experimental setup



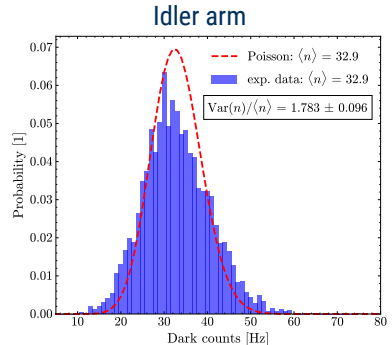
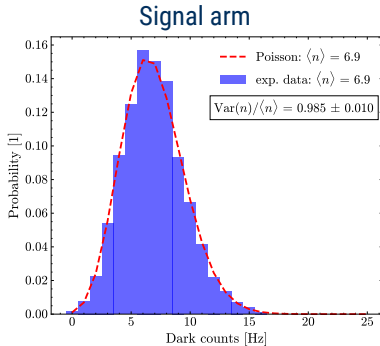
Dark counts, $\text{Var}(N_{\text{noise}})$

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Signal arm

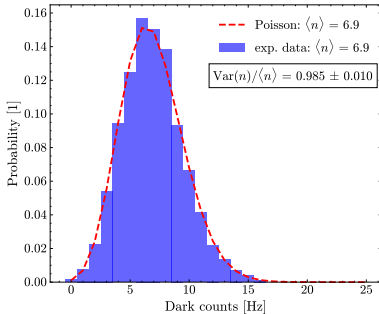


Dark counts, $\text{Var}(N_{\text{noise}})$

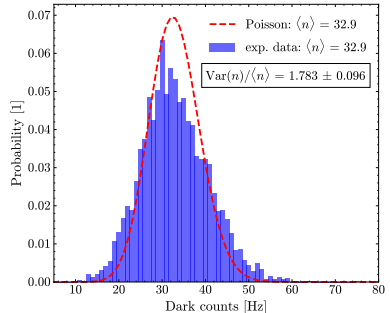


Dark counts, $\text{Var}(N_{\text{noise}})$

Signal arm



Idler arm



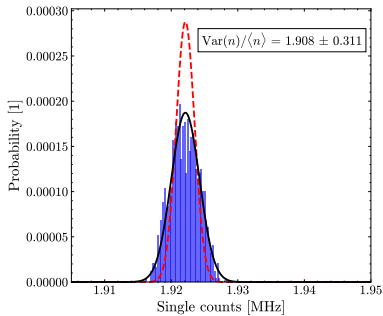
$$\text{Var}(N_{\text{noise}}) = 1.8 \cdot \langle N_{\text{noise}} \rangle$$

Single counts, $\text{Var}(N_{\text{sing}})$

Single counts, $\text{Var}(N_{\text{sing}})$

Signal arm

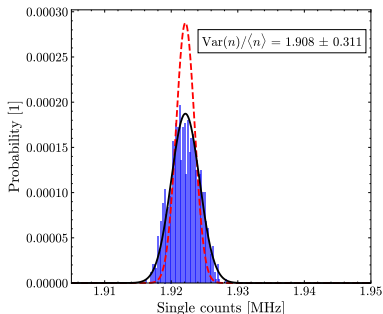
- Poisson: $\langle n \rangle = 1922127$
- mmBE: $m = 1412365$, $\langle n \rangle = 1922127$
- exp. data: $m_{\text{theo}} = 1412365$, $\langle n \rangle = 1922127$



Single counts, $\text{Var}(N_{\text{sing}})$

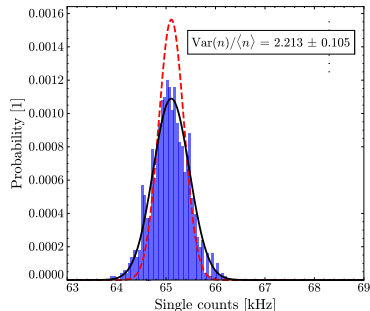
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Idler arm

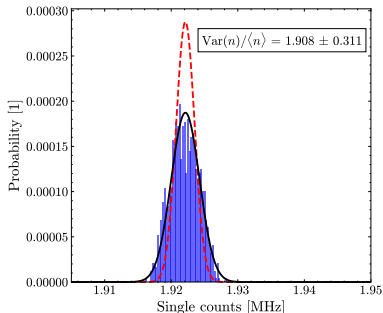
- Poisson: $\langle n \rangle = 65107$
- mmBE: $m = 61384$, $\langle n \rangle = 65107$
- exp. data: $m_{\text{theo}} = 61384$, $\langle n \rangle = 65107$



Single counts, $\text{Var}(N_{\text{sing}})$

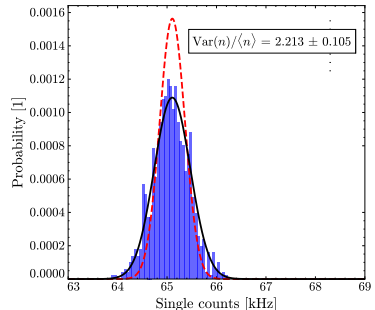
Signal arm

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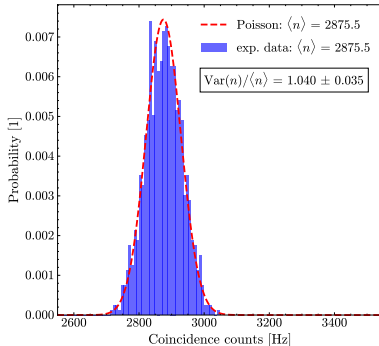


$$\text{Var}(N_{\text{sing}}) = 2.2 \cdot \langle N_{\text{sing}} \rangle$$

Coincidence counts, $\text{Var}(N_{\text{coin}})$

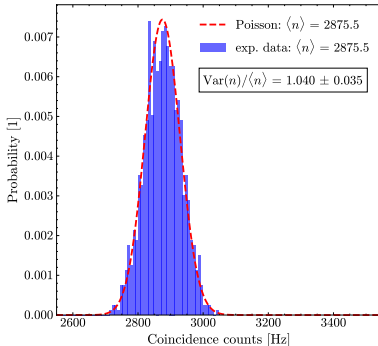
Coincidence counts, $\text{Var}(N_{\text{coin}})$

Coincidences



Coincidence counts, $\text{Var}(N_{\text{coin}})$

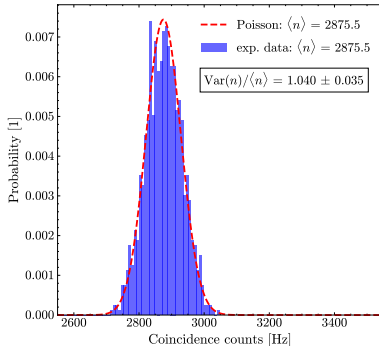
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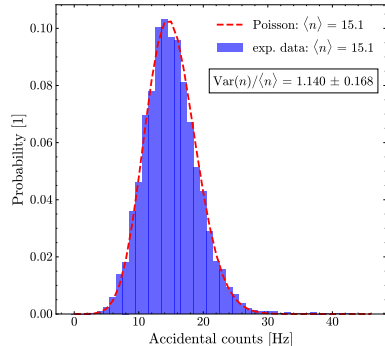
$$\text{Var}(N_{\text{coin}}) = \langle N_{\text{coin}} \rangle$$

Coincidence counts, $\text{Var}(N_{\text{coin}})$

Coincidences



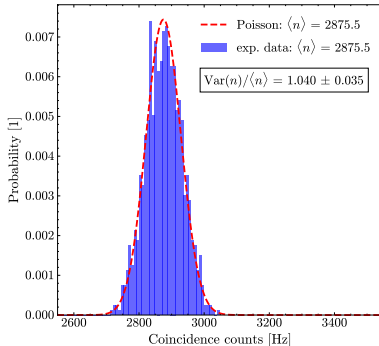
Accidentals



$$\text{Var}(N_{\text{coin}}) = \langle N_{\text{coin}} \rangle$$

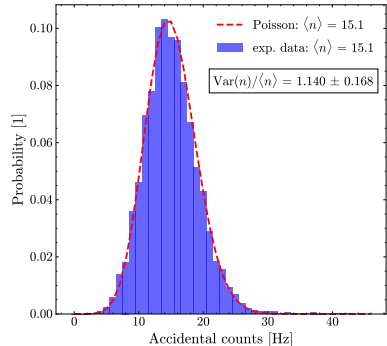
Coincidence counts, $\text{Var}(N_{\text{coin}})$

Coincidences



$$\text{Var}(N_{\text{coin}}) = \langle N_{\text{coin}} \rangle$$

Accidentals



$$\text{Var}(N_{\text{ac}}) = \langle N_{\text{ac}} \rangle$$

Simulation

Conventional approach:

$$\text{Var}(\mathcal{T}) = (\eta_{\text{idl}} N_{\text{g}})^{-2} \left[\text{Var}(N_{\text{sing}}^{\text{sam}}) + \text{Var}(N_{\text{noise}}^{\text{sam}}) + \mathcal{T}^2 \left[\text{Var}(N_{\text{sing}}^{\text{ref}}) + \text{Var}(N_{\text{noise}}^{\text{ref}}) \right] \right]$$

Simulation

Conventional approach:

$$\text{Var}(T) = (\eta_{\text{idl}} N_g)^{-2} \left[2.2 \cdot \langle N_{\text{sing}}^{\text{sam}} \rangle + 1.8 \cdot \langle N_{\text{noise}}^{\text{sam}} \rangle + T^2 \left[2.2 \cdot \langle N_{\text{sing}}^{\text{ref}} \rangle + 1.8 \cdot \langle N_{\text{noise}}^{\text{ref}} \rangle \right] \right]$$

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Coincidence approach:

$$\text{Var}(T) = (\eta_{\text{sig}} \eta_{\text{idl}} N_g)^{-2} \left[\text{Var}(N_{\text{coin}}^{\text{sam}}) + \text{Var}(N_{\text{ac}}^{\text{sam}}) + T^2 \left[\text{Var}(N_{\text{coin}}^{\text{ref}}) + \text{Var}(N_{\text{ac}}^{\text{ref}}) \right] \right]$$

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Conventional approach:

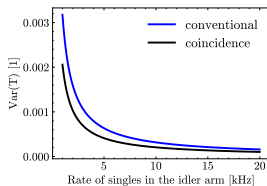
$$\text{Var}(T) = (\eta_{\text{idl}} N_g)^{-2} \left[2.2 \cdot \langle N_{\text{sing}}^{\text{sam}} \rangle + 1.8 \cdot \langle N_{\text{noise}}^{\text{sam}} \rangle + T^2 \left[2.2 \cdot \langle N_{\text{sing}}^{\text{ref}} \rangle + 1.8 \cdot \langle N_{\text{noise}}^{\text{ref}} \rangle \right] \right]$$

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$$\text{Var}(T) = (\eta_{\text{sig}} \eta_{\text{idl}} N_g)^{-2} \left[\langle N_{\text{coin}}^{\text{sam}} \rangle + \langle N_{\text{ac}}^{\text{sam}} \rangle + T^2 \left[\langle N_{\text{coin}}^{\text{ref}} \rangle + \langle N_{\text{ac}}^{\text{ref}} \rangle \right] \right]$$

Simulation

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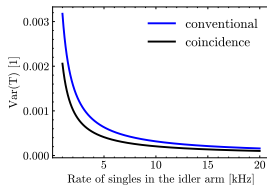


$$\eta_{\text{idl}} = 70\%$$

$$\eta_{\text{sig}} = 70\%$$

$$R_{\text{noise,idl}} = 0 \text{ Hz}$$

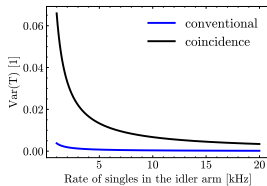
Simulation



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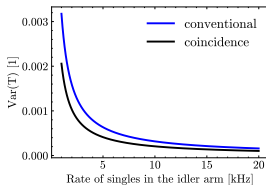


$$\eta_{\text{idl}} = 0.09\%$$

$$\eta_{\text{sig}} = 2.6\%$$

$$R_{\text{noise,idl}} = 0 \text{ Hz}$$

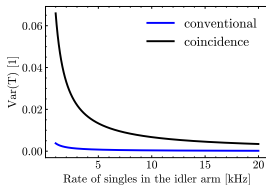
Simulation



$$\eta_{\text{idl}} = 70\%$$

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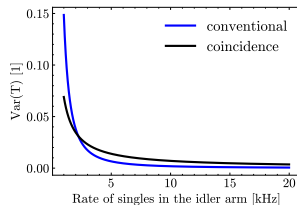
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$$\eta_{\text{idl}} = 0.09\%$$

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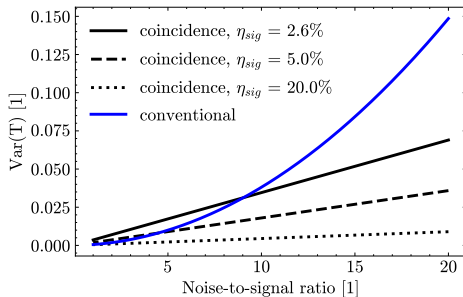


$$\eta_{\text{idl}} = 0.09\%$$

$$\eta_{\text{sig}} = 2.6\%$$

$$R_{\text{noise,idl}} = 20 \text{ kHz}$$

Noise-to-signal ratio



| Parameter | Value |
|-----------------------|------------|
| η_{idl} (%) | 0.09 |
| η_{sig} (%) | 2.6, 5, 20 |
| R_{idl} (kHz) | 1 - 20 |
| $R_{noise,idl}$ (kHz) | 20 |
| $R_{noise,sig}$ (Hz) | 7 |
| T (1) | 0.9 |

Summary and Outlook

Summary and Outlook

Summary:

- Established a model for the variance of transmittance
- Experimental verification of photon statistics
- Found regions where coincidence approach offers higher precision

Summary and Outlook

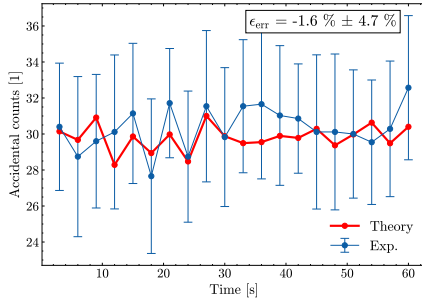
Summary:

- Established a model for the variance of transmittance
- Experimental verification of photon statistics
- Found regions where coincidence approach offers higher precision

Outlook:

- Experimental verification of the found parameter regions
- Determine the experimental limitations for the variance measurement

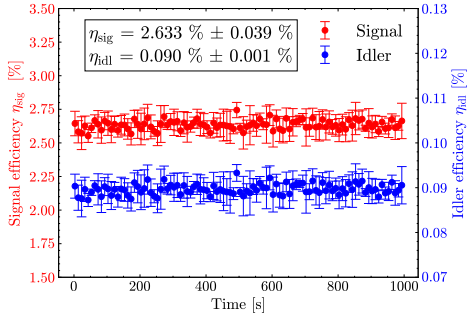
Accidental counts



$$R_{\text{ac}}^{\text{sam}} = \left(T \eta_{\text{idl}} R_{\text{g}} + R_{\text{dc,idl}} - R_{\text{cc,pure}}^{\text{sam}} \right) \cdot \left(\eta_{\text{sig}} R_{\text{g}} + R_{\text{dc,sig}} - R_{\text{cc,pure}}^{\text{sam}} \right) \cdot \tau_{\text{cw}}$$

$$R_{\text{ac}}^{\text{ref}} = \left(\eta_{\text{idl}} R_{\text{g}} + R_{\text{dc,idl}} - R_{\text{cc,pure}}^{\text{ref}} \right) \cdot \left(\eta_{\text{sig}} R_{\text{g}} + R_{\text{dc,sig}} - R_{\text{cc,pure}}^{\text{ref}} \right) \cdot \tau_{\text{cw}}$$

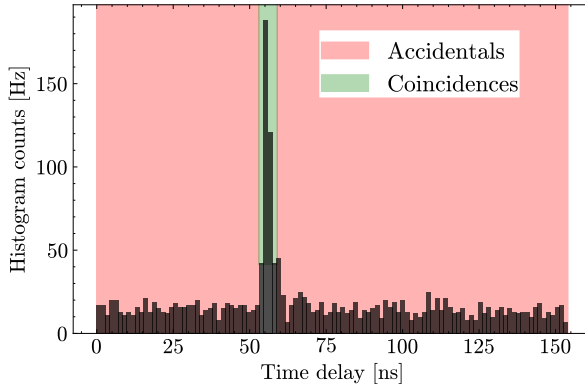
Heralding efficiencies



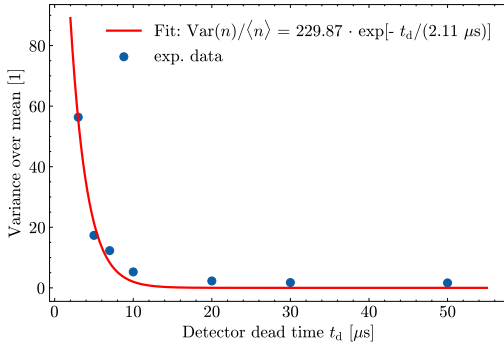
$$\eta_{\text{sig}} = \frac{N_{\text{coin}} - N_{\text{ac}}}{N_{\text{sing, idl}} - N_{\text{noise}}} = \frac{\eta_{\text{sig}} \eta_{\text{idl}} N_{\text{g}}}{\eta_{\text{idl}} N_{\text{g}}}$$

$$\eta_{\text{idl}} = \frac{N_{\text{coin}} - N_{\text{ac}}}{N_{\text{sing, sig}} - N_{\text{noise}}} = \frac{\eta_{\text{sig}} \eta_{\text{idl}} N_{\text{g}}}{\eta_{\text{sig}} N_{\text{g}}}$$

Histogram



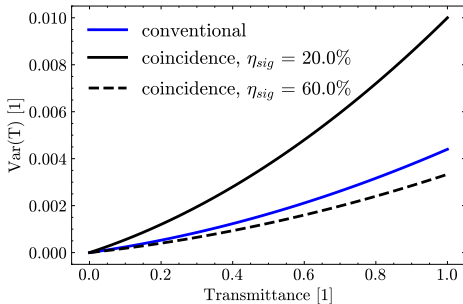
Afterpulsing



$$R_{aft} \propto e^{-\frac{t_d}{\tau_{de}}} \quad [4]$$

[4] Humer et al., "A Simple and Robust Method for Estimating Afterpulsing in Single Photon Detectors"

Noiseless case

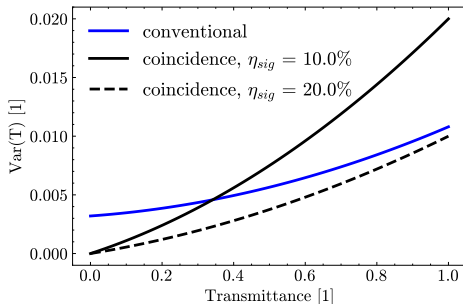


Parameter

Value

| | |
|------------------------------|--------|
| η_{idl} (%) | 60 |
| η_{sig} (%) | 20, 60 |
| R_{idl} (kHz) | 1 |
| $R_{\text{noise,idl}}$ (kHz) | 0 |
| $R_{\text{noise,sig}}$ (Hz) | 7 |
| T (1) | 0 - 1 |

In presence of noise

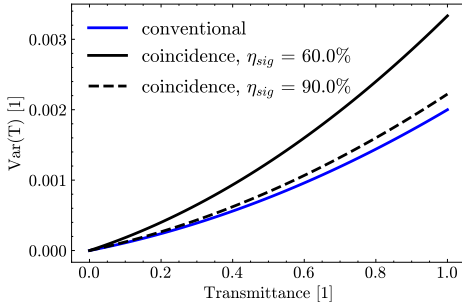


Parameter

Value

| | |
|------------------------|--------|
| η_{idl} (%) | 60 |
| η_{sig} (%) | 10, 20 |
| R_{idl} (kHz) | 1 |
| $R_{noise, idl}$ (kHz) | 1 |
| $R_{noise, sig}$ (Hz) | 7 |
| T (1) | 0 - 1 |

Coherent Illumination, noiseless

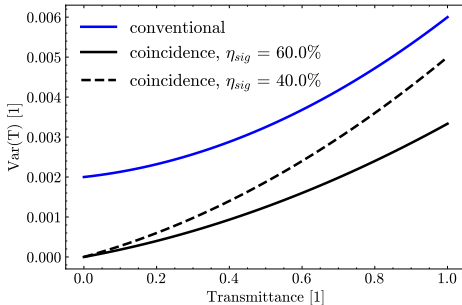


Parameter

Value

| | |
|------------------------------|--------|
| η_{idl} (%) | 60 |
| η_{sig} (%) | 60, 90 |
| R_{idl} (kHz) | 1 |
| $R_{\text{noise,idl}}$ (kHz) | 0 |
| $R_{\text{noise,sig}}$ (Hz) | 7 |
| T (1) | 0 - 1 |

Coherent Illumination, with noise



Parameter

Value

| | |
|------------------------------|--------|
| η_{idl} (%) | 60 |
| η_{sig} (%) | 40, 60 |
| R_{idl} (kHz) | 1 |
| $R_{\text{noise,idl}}$ (kHz) | 1 |
| $R_{\text{noise,sig}}$ (Hz) | 7 |
| T (1) | 0 - 1 |