

Parameter estimation with correlated photon pairs

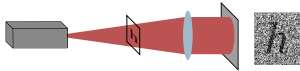
Jan Gößwein

Institute of Applied Physics

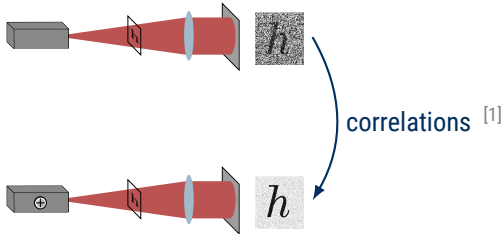
Jena, November 13, 2025

Motivation

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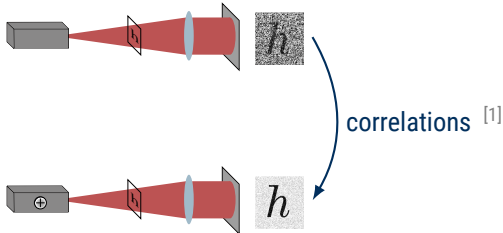


Motivation



[1] Brida, Genovese, and Ruo Berchera, "Experimental Realization of Sub-Shot-Noise Quantum Imaging"

Motivation

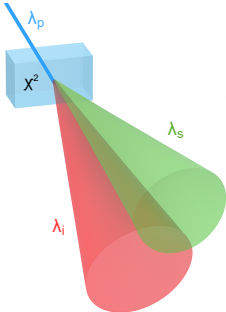


Objective: Can correlated photons provide advantages in terms of precision in noisy regimes for parameter estimation?

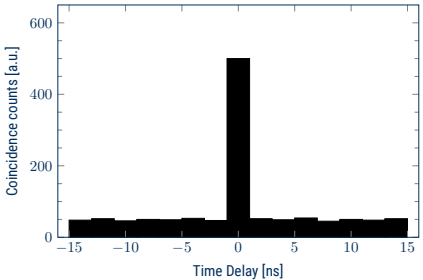
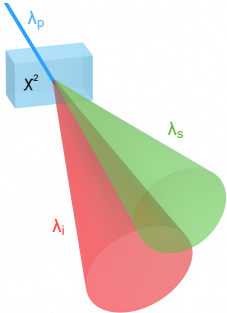
[1] Brida, Genovese, and Ruo Berchera, "Experimental Realization of Sub-Shot-Noise Quantum Imaging"

Spontaneous parametric down-conversion

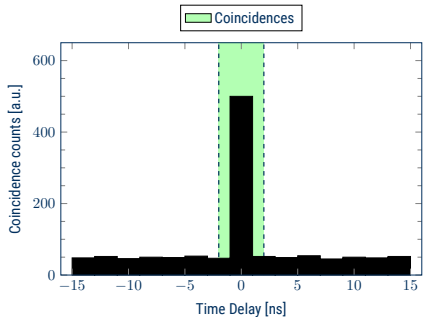
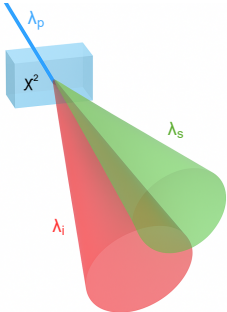
Spontaneous parametric down-conversion



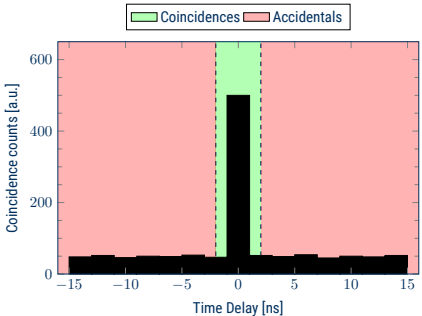
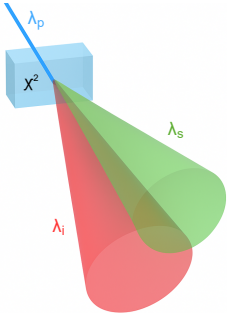
Spontaneous parametric down-conversion



Spontaneous parametric down-conversion



Spontaneous parametric down-conversion



Parameter estimation

Parameter estimation

Setup: Transmission setup

Parameter estimation

Setup: Transmission setup

Parameter: Transmittance T

Parameter estimation

Setup: Transmission setup

Parameter: Transmittance T

Precision: $\text{Var}(T)$

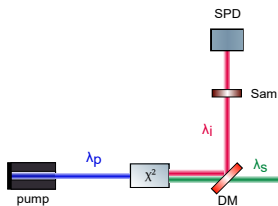
Parameter estimation

Setup: Transmission setup

Parameter: Transmittance T

Precision: $\text{Var}(T)$

Conventional approach:



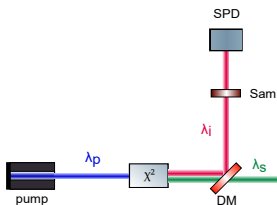
Parameter estimation

Setup: Transmission setup

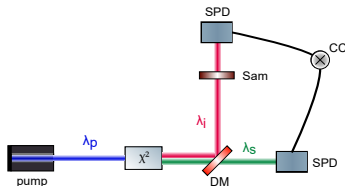
Parameter: Transmittance T

Precision: $\text{Var}(T)$

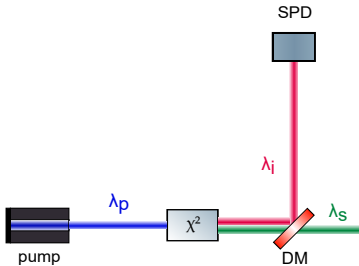
Conventional approach:



Coincidence approach:

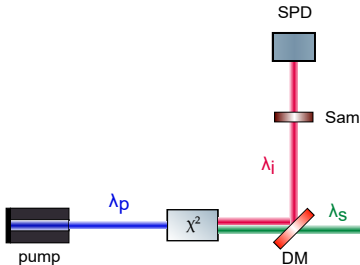


Conventional approach



$$N_{\text{sing}}^{\text{ref}} = \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{ref}}$$

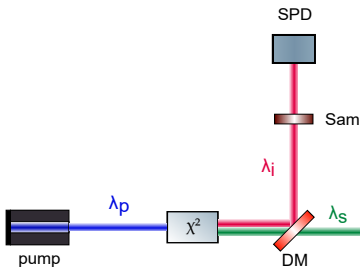
Conventional approach



$$N_{\text{sing}}^{\text{ref}} = \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{ref}}$$

$$N_{\text{sing}}^{\text{sam}} = T \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{sam}}$$

Conventional approach

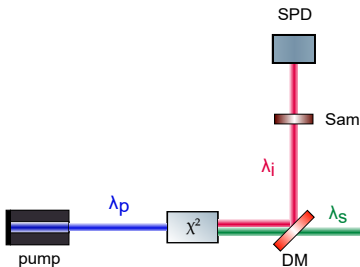


$$N_{\text{sing}}^{\text{ref}} = \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{ref}}$$

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$$\Rightarrow T = \frac{N_{\text{sing}}^{\text{sam}} - N_{\text{noise}}^{\text{sam}}}{N_{\text{sing}}^{\text{ref}} - N_{\text{noise}}^{\text{ref}}}$$

Conventional approach



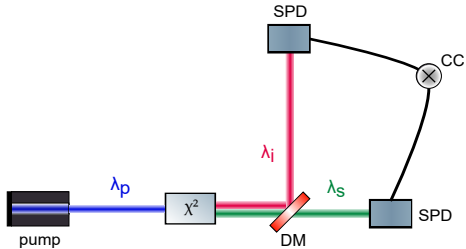
$$N_{\text{sing}}^{\text{ref}} = \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{ref}}$$

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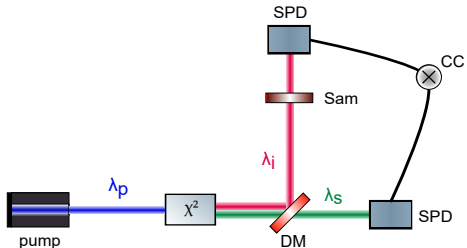
$$\text{Var}(T) = (\eta_{\text{idl}} N_g)^{-2} \left[\text{Var}(N_{\text{sing}}^{\text{sam}}) + \text{Var}(N_{\text{noise}}^{\text{sam}}) + T^2 \left[\text{Var}(N_{\text{sing}}^{\text{ref}}) + \text{Var}(N_{\text{noise}}^{\text{ref}}) \right] \right]$$

Coincidence approach



$$N_{\text{coin}}^{\text{ref}} = \eta_{\text{idl}} \eta_{\text{sig}} N_g + N_{\text{ac}}^{\text{ref}}$$

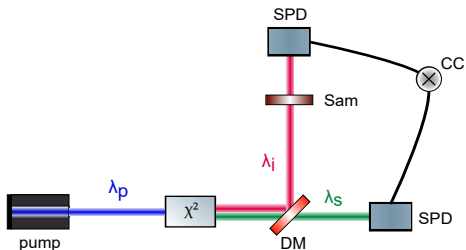
Coincidence approach



$$N_{\text{coin}}^{\text{ref}} = \eta_{\text{idl}} \eta_{\text{sig}} N_g + N_{\text{ac}}^{\text{ref}}$$

$$N_{\text{coin}}^{\text{sam}} = T \eta_{\text{idl}} \eta_{\text{sig}} N_g + N_{\text{ac}}^{\text{sam}}$$

Coincidence approach

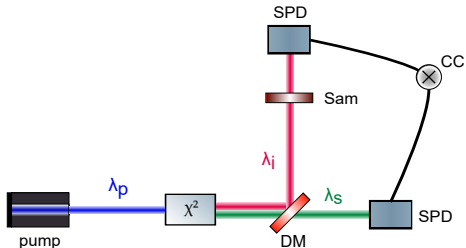


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Coincidence approach



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$$\text{Var}(T) = (\eta_{\text{sig}} \eta_{\text{idl}} N_g)^{-2} \left[\text{Var}(N_{\text{coin}}^{\text{sam}}) + \text{Var}(N_{\text{ac}}^{\text{sam}}) + T^2 \left[\text{Var}(N_{\text{coin}}^{\text{ref}}) + \text{Var}(N_{\text{ac}}^{\text{ref}}) \right] \right]$$

Transmittance model

Conventional approach:

$$\text{Var}(T) = (\eta_{\text{idl}} N_g)^{-2} \left[\text{Var}(N_{\text{sing}}^{\text{sam}}) + \text{Var}(N_{\text{noise}}^{\text{sam}}) + T^2 \left[\text{Var}(N_{\text{sing}}^{\text{ref}}) + \text{Var}(N_{\text{noise}}^{\text{ref}}) \right] \right]$$

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Photon statistics

Photon statistics

Poisson distribution (coherent light):

$$\mathcal{P}(n) = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}$$

$$\text{Var}(n) = \langle n \rangle$$

Photon statistics

Poisson distribution (coherent light): ^[2]^[3]

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^[2]Kim et al., "Photon-Counting Statistics-Based Support Vector Machine with Multi-Mode Photon Illumination for Quantum Imaging"

^[3]Fouche, "Detection and False-Alarm Probabilities for Laser Radars That Use Geiger-mode Detectors"

Photon statistics

Poisson distribution (coherent light): ^{[2][3]}

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$$\text{Var}(n) = \langle n \rangle$$

Multi-mode Bose-Einstein distribution (thermal light): ^[2]

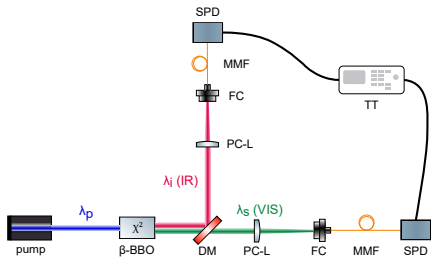
$$\mathcal{P}_m(n) = \frac{(n+m-1)!}{(m-1)!n!} \frac{m^m \langle n \rangle^n}{(m + \langle n \rangle)^{n+m}}$$

$$\text{Var}(n) = \langle n \rangle \left(1 + \frac{\langle n \rangle}{m} \right)$$

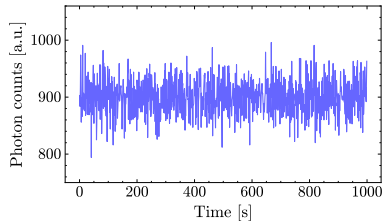
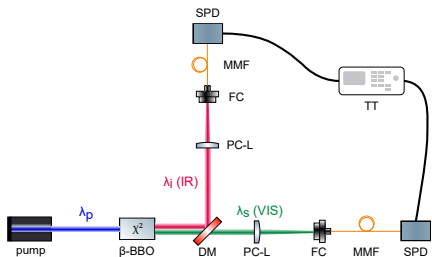
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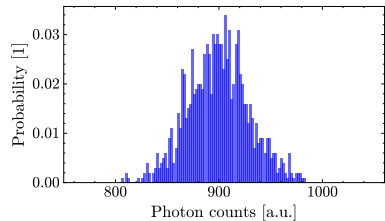
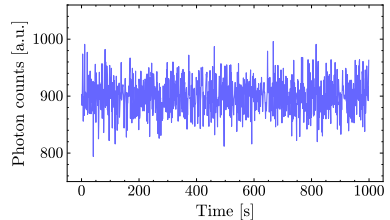
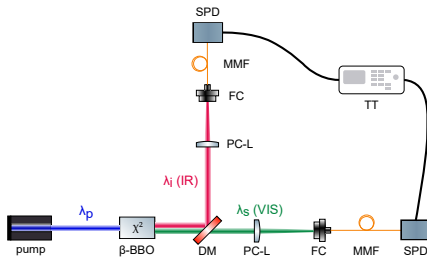
Experimental setup



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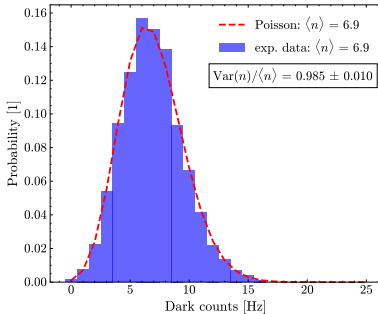
Experimental setup



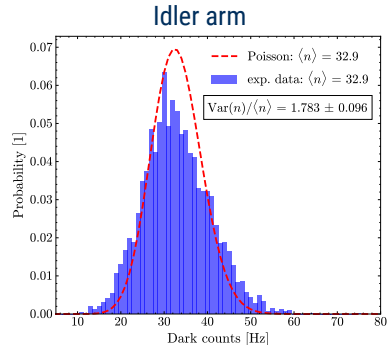
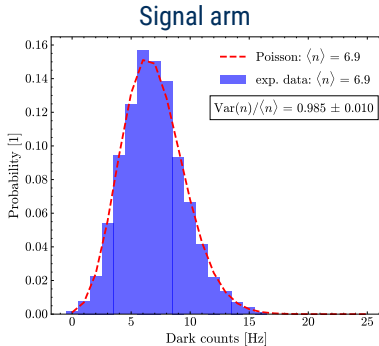
Dark counts, $\text{Var}(N_{\text{noise}})$

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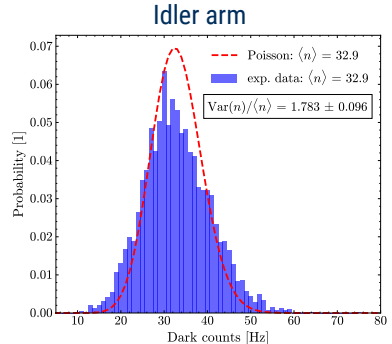
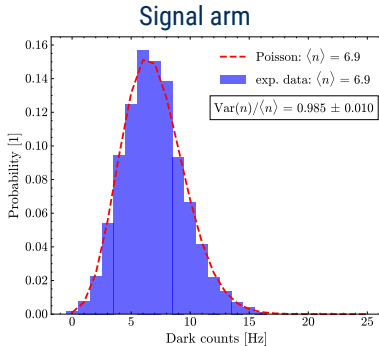
Signal arm



Dark counts, $\text{Var}(N_{\text{noise}})$



Dark counts, $\text{Var}(N_{\text{noise}})$



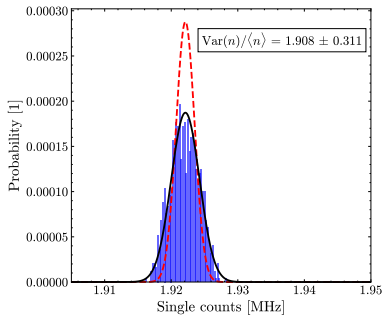
$$\text{Var}(N_{\text{noise}}) = 1.8 \cdot \langle N_{\text{noise}} \rangle$$

Single counts, $\text{Var}(N_{\text{sing}})$

Single counts, $\text{Var}(N_{\text{sing}})$

Signal arm

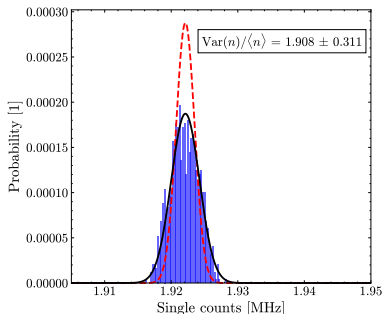
- Poisson: $\langle n \rangle = 1922127$
- mmBE: $m = 1412365$, $\langle n \rangle = 1922127$
- exp. data: $m_{\text{theo}} = 1412365$, $\langle n \rangle = 1922127$



Single counts, $\text{Var}(N_{\text{sing}})$

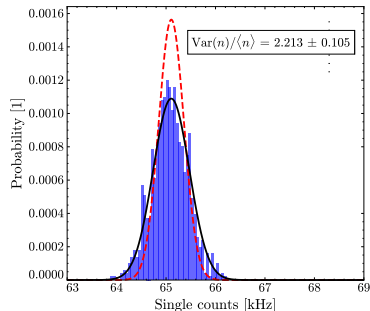
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Idler arm

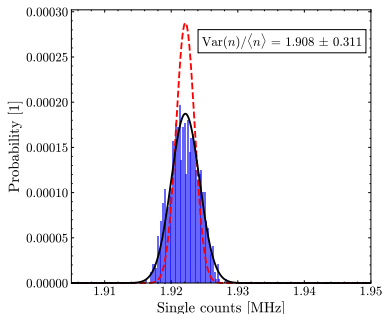
- Poisson: $\langle n \rangle = 65107$
- mmBE: $m = 61384$, $\langle n \rangle = 65107$
- exp. data: $m_{\text{theo}} = 61384$, $\langle n \rangle = 65107$



Single counts, $\text{Var}(N_{\text{sing}})$

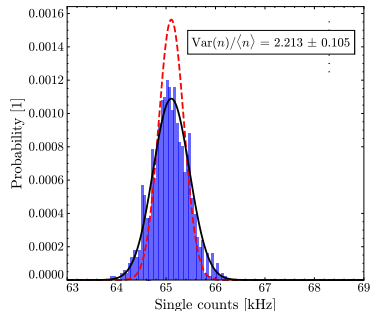
Signal arm

- Poisson: $\langle n \rangle = 1922127$
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Idler arm

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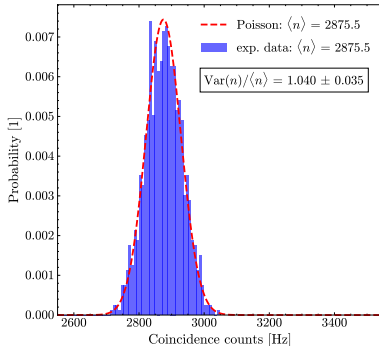


$$\text{Var}(N_{\text{sing}}) = 2.2 \cdot \langle N_{\text{sing}} \rangle$$

Coincidence counts, $\text{Var}(N_{\text{coin}})$

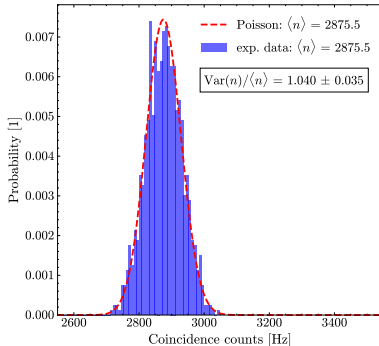
Coincidence counts, $\text{Var}(N_{\text{coin}})$

Coincidences



Coincidence counts, $\text{Var}(N_{\text{coin}})$

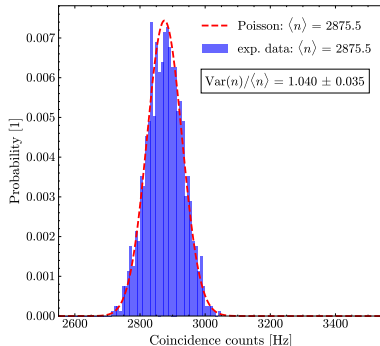
Coincidences



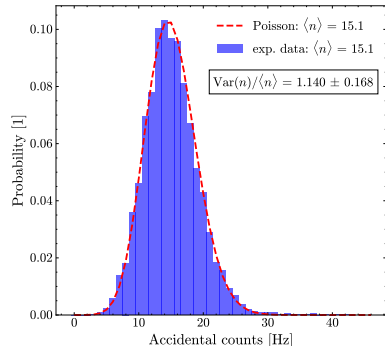
$$\text{Var}(N_{\text{coin}}) = \langle N_{\text{coin}} \rangle$$

Coincidence counts, $\text{Var}(N_{\text{coin}})$

Coincidences



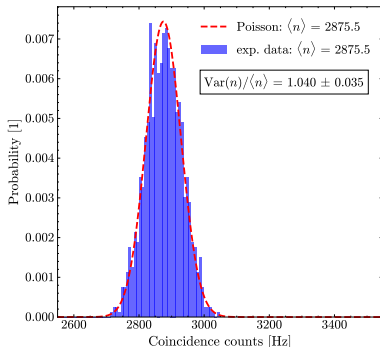
Accidentals



$$\text{Var}(N_{\text{coin}}) = \langle N_{\text{coin}} \rangle$$

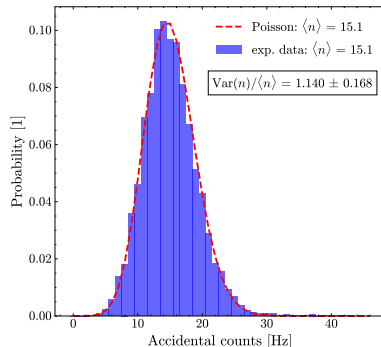
Coincidence counts, $\text{Var}(N_{\text{coin}})$

Coincidences



$$\text{Var}(N_{\text{coin}}) = \langle N_{\text{coin}} \rangle$$

Accidentals



$$\text{Var}(N_{\text{ac}}) = \langle N_{\text{ac}} \rangle$$

Simulation

Conventional approach:

$$\text{Var}(\mathcal{T}) = (\eta_{\text{idl}} N_g)^{-2} \left[\text{Var}(N_{\text{sing}}^{\text{sam}}) + \text{Var}(N_{\text{noise}}^{\text{sam}}) + \mathcal{T}^2 \left[\text{Var}(N_{\text{sing}}^{\text{ref}}) + \text{Var}(N_{\text{noise}}^{\text{ref}}) \right] \right]$$

Simulation

Conventional approach:

$$\text{Var}(T) = (\eta_{\text{idl}} N_g)^{-2} \left[2.2 \cdot \langle N_{\text{sing}}^{\text{sam}} \rangle + 1.8 \cdot \langle N_{\text{noise}}^{\text{sam}} \rangle + T^2 \left[2.2 \cdot \langle N_{\text{sing}}^{\text{ref}} \rangle + 1.8 \cdot \langle N_{\text{noise}}^{\text{ref}} \rangle \right] \right]$$

Simulation

Conventional approach:

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Coincidence approach:

$$\text{Var}(T) = (\eta_{\text{sig}} \eta_{\text{idl}} N_g)^{-2} \left[\text{Var}(N_{\text{coin}}^{\text{sam}}) + \text{Var}(N_{\text{ac}}^{\text{sam}}) + T^2 \left[\text{Var}(N_{\text{coin}}^{\text{ref}}) + \text{Var}(N_{\text{ac}}^{\text{ref}}) \right] \right]$$

Simulation

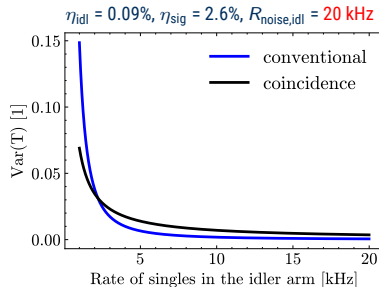
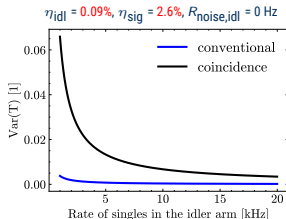
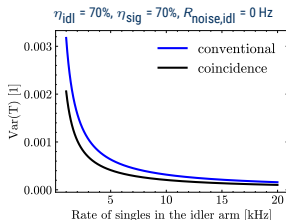
Conventional approach:

$$\text{Var}(T) = (\eta_{\text{idl}} N_g)^{-2} \left[2.2 \cdot \langle N_{\text{sing}}^{\text{sam}} \rangle + 1.8 \cdot \langle N_{\text{noise}}^{\text{sam}} \rangle + T^2 \left[2.2 \cdot \langle N_{\text{sing}}^{\text{ref}} \rangle + 1.8 \cdot \langle N_{\text{noise}}^{\text{ref}} \rangle \right] \right]$$

Coincidence approach:

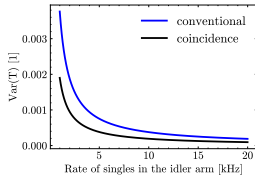
$$\text{Var}(T) = (\eta_{\text{sig}} \eta_{\text{idl}} N_g)^{-2} \left[\langle N_{\text{coin}}^{\text{sam}} \rangle + \langle N_{\text{ac}}^{\text{sam}} \rangle + T^2 \left[\langle N_{\text{coin}}^{\text{ref}} \rangle + \langle N_{\text{ac}}^{\text{ref}} \rangle \right] \right]$$

Simulation



Simulation

Simulation

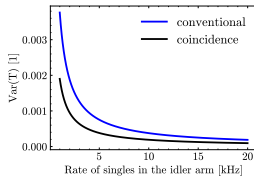


$$\eta_{\text{idl}} = 90\%$$

$$\eta_{\text{sig}} = 90\%$$

$$R_{\text{noise,idl}} = 0 \text{ Hz}$$

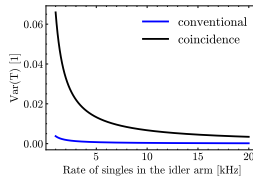
Simulation



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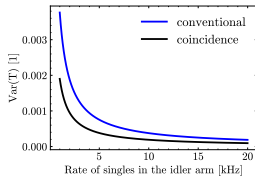


$$\eta_{\text{idl}} = 0.09\%$$

$$\eta_{\text{sig}} = 2.6\%$$

$$R_{\text{noise,idl}} = 0 \text{ Hz}$$

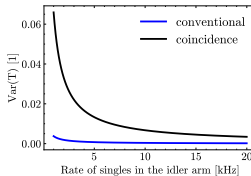
Simulation



$$\eta_{\text{idl}} = 90\%$$

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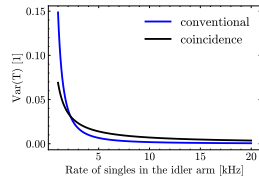
$$R_{\text{noise,idl}} = 0 \text{ Hz}$$



$$\eta_{\text{idl}} = 0.09\%$$

$$\eta_{\text{sig}} = 2.6\%$$

$$R_{\text{noise,idl}} = 0 \text{ Hz}$$

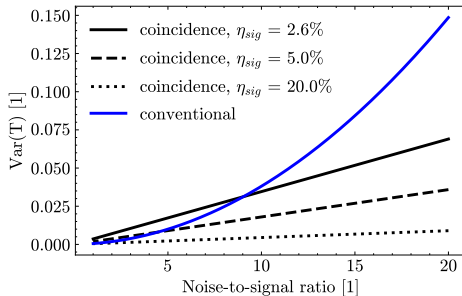


$$\eta_{\text{idl}} = 0.09\%$$

$$\eta_{\text{sig}} = 2.6\%$$

$$R_{\text{noise,idl}} = 20 \text{ kHz}$$

Noise-to-signal ratio



Parameter

Value

η_{idl} (%)	0.09
η_{sig} (%)	2.6, 5, 20
R_{idl} (kHz)	1 - 20
$R_{\text{noise,idl}}$ (kHz)	20
$R_{\text{noise,sig}}$ (Hz)	7
T (1)	0.9

Summary and Outlook

Summary and Outlook

Summary:

- Established a model for the variance of transmittance
- Experimental verification of photon statistics
- Found regions where coincidence approach offers higher precision

Summary and Outlook

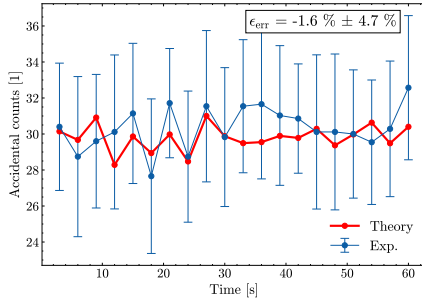
Summary:

- Established a model for the variance of transmittance
- Experimental verification of photon statistics
- Found regions where coincidence approach offers higher precision

Outlook:

- Experimental verification of the found parameter regions
- Determine the experimental limitations for the variance measurement

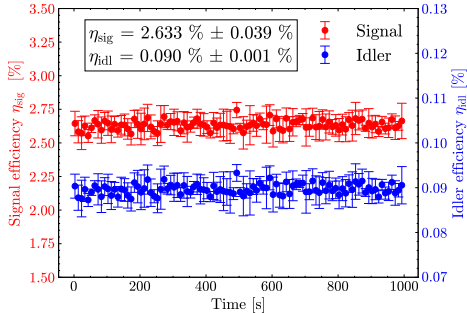
Accidental counts



$$R_{\text{ac}}^{\text{sam}} = \left(T \eta_{\text{idl}} R_{\text{g}} + R_{\text{dc,idl}} - R_{\text{cc,pure}}^{\text{sam}} \right) \cdot \left(\eta_{\text{sig}} R_{\text{g}} + R_{\text{dc,sig}} - R_{\text{cc,pure}}^{\text{sam}} \right) \cdot \tau_{\text{cw}}$$

$$R_{\text{ac}}^{\text{ref}} = \left(\eta_{\text{idl}} R_{\text{g}} + R_{\text{dc,idl}} - R_{\text{cc,pure}}^{\text{ref}} \right) \cdot \left(\eta_{\text{sig}} R_{\text{g}} + R_{\text{dc,sig}} - R_{\text{cc,pure}}^{\text{ref}} \right) \cdot \tau_{\text{cw}}$$

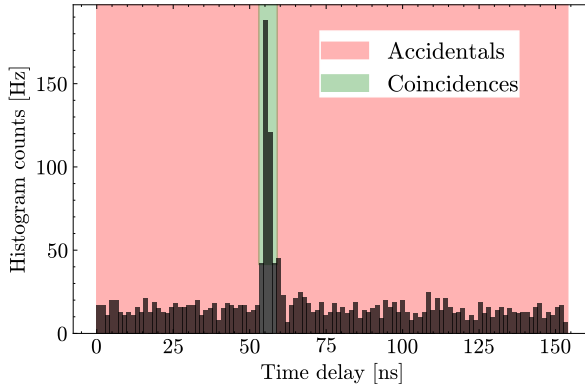
Heralding efficiencies



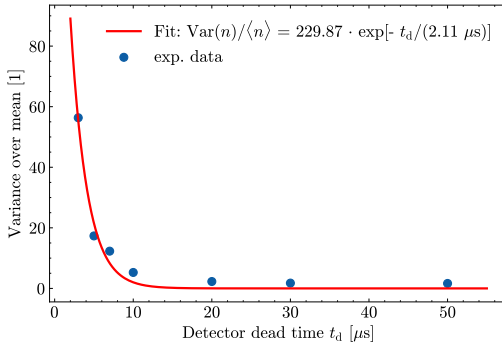
$$\eta_{\text{sig}} = \frac{N_{\text{coin}} - N_{\text{ac}}}{N_{\text{sing, idl}} - N_{\text{noise}}} = \frac{\eta_{\text{sig}} \eta_{\text{idl}} N_{\text{g}}}{\eta_{\text{idl}} N_{\text{g}}}$$

$$\eta_{\text{idl}} = \frac{N_{\text{coin}} - N_{\text{ac}}}{N_{\text{sing, sig}} - N_{\text{noise}}} = \frac{\eta_{\text{sig}} \eta_{\text{idl}} N_{\text{g}}}{\eta_{\text{sig}} N_{\text{g}}}$$

Histogram

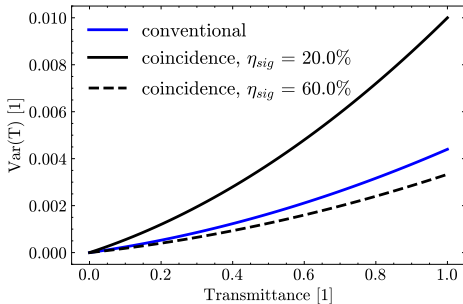


Afterpulsing



$$R_{aft} \propto e^{-\frac{t_d}{\tau_{de}}}$$

Noiseless case

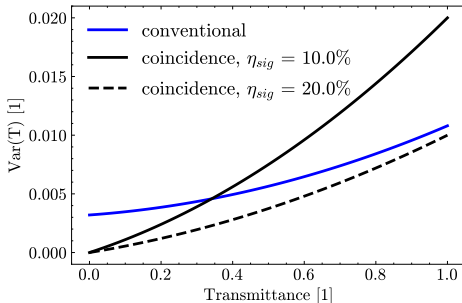


Parameter

Value

η_{idl} (%)	60
η_{sig} (%)	20, 60
R_{idl} (kHz)	1
$R_{\text{noise,idl}}$ (kHz)	0
$R_{\text{noise,sig}}$ (Hz)	7
T (1)	0 - 1

In presence of noise

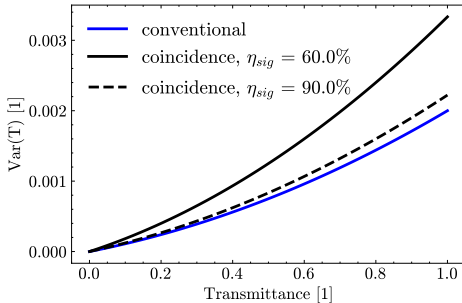


Parameter

Value

η_{idl} (%)	60
η_{sig} (%)	10, 20
R_{idl} (kHz)	1
$R_{\text{noise,idl}}$ (kHz)	1
$R_{\text{noise,sig}}$ (Hz)	7
T (1)	0 - 1

Coherent Illumination, noiseless

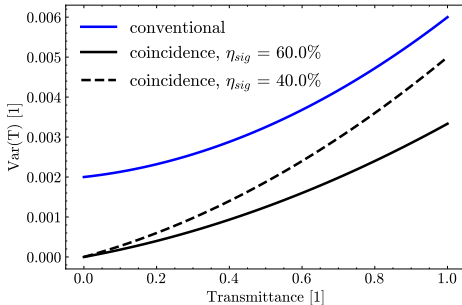


Parameter

Value

η_{idl} (%)	60
η_{sig} (%)	60, 90
R_{idl} (kHz)	1
$R_{\text{noise,idl}}$ (kHz)	0
$R_{\text{noise,sig}}$ (Hz)	7
T (1)	0 - 1

Coherent Illumination, with noise



Parameter

Value

η_{idl} (%)	60
η_{sig} (%)	40, 60
R_{idl} (kHz)	1
$R_{\text{noise,idl}}$ (kHz)	1
$R_{\text{noise,sig}}$ (Hz)	7
T (1)	0 - 1