Parameter estimation with correlated photon pairs

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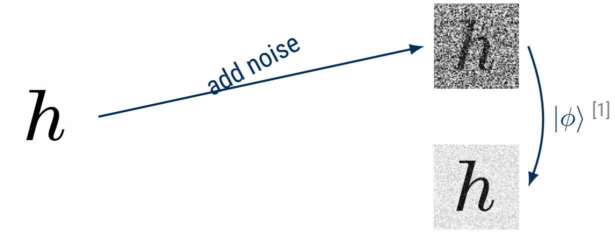
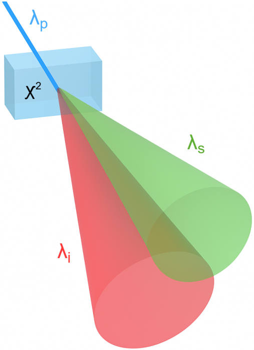
Motivation



Objective: Can coincidence measurements provide advantages in terms of precision in noisy regimes?

[1] Brida, Genovese, and Ruo Berchera, "Experimental Realization of Sub-Shot-Noise Quantum Imaging"

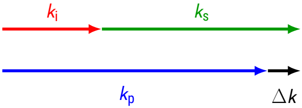
SPDC



Energy conservation

$$\omega_p = \omega_s + \omega_i$$

Momentum conservation



$$\vec{k_p} = \vec{k_s} + \vec{k_i} - \Delta\vec{k}$$

Parameter estimation

Setup: Transmission setup

Parameter estimation

Setup: Transmission setup
Parameter: Transmittance T

Parameter estimation

Setup: Transmission setup
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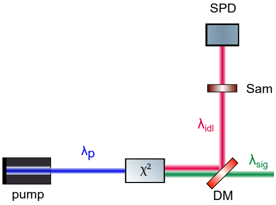
Estimator: $\mathrm{Var}(T)$

Parameter estimation

Setup: Transmission setup
Parameter: Transmittance T
Estimator: Var(T)

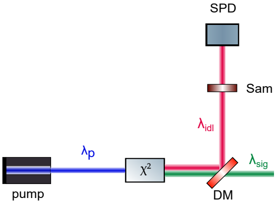
Conventional approach:



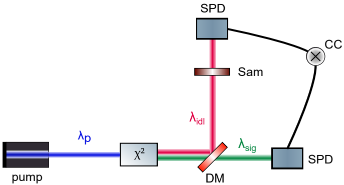
Parameter estimation

Setup: Transmission setup
Parameter: Transmittance T
Estimator: Var(T)

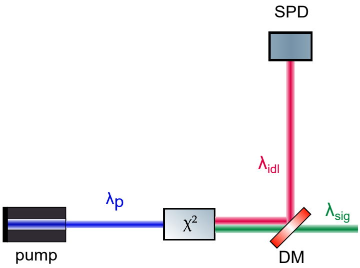
Conventional approach:



Coincidence approach:

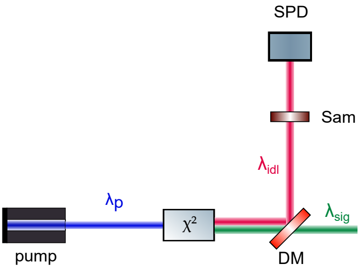


Conventional approach



$$N_{\mathrm{tot}}^{\mathrm{ref}} = \eta_{\mathrm{idl}} N_{\mathrm{g}} + N_{\mathrm{noise}}^{\mathrm{ref}}$$

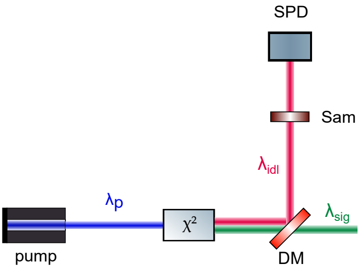
Conventional approach



$$N_{\mathrm{tot}}^{\mathrm{ref}} = \eta_{\mathrm{idl}} N_{\mathrm{g}} + N_{\mathrm{noise}}^{\mathrm{ref}}$$

$$N_{\mathrm{tot}}^{\mathrm{sam}} = T\,\eta_{\mathrm{idl}} N_{\mathrm{g}} + N_{\mathrm{noise}}^{\mathrm{sam}}$$

Conventional approach

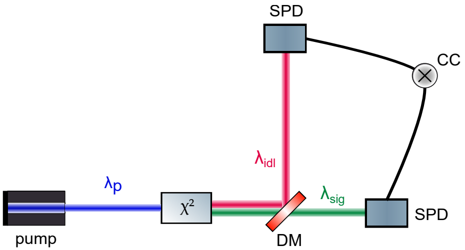


$$N_{\text{tot}}^{\text{ref}} = \eta_{\text{idl}} N_{\text{g}} + N_{\text{noise}}^{\text{ref}}$$

$$N_{\text{tot}}^{\text{sam}} = T\, \eta_{\text{idl}} N_{\text{g}} + N_{\text{noise}}^{\text{sam}}$$

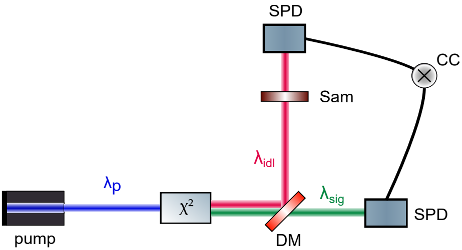
$$\Rightarrow T = \frac{N_{\text{tot}}^{\text{sam}} - N_{\text{noise}}^{\text{sam}}}{N_{\text{tot}}^{\text{ref}} - N_{\text{noise}}^{\text{ref}}}$$

Coincidence approach



$$N_{cc,tot}^{ref} = \eta_{idl}\,\eta_{sig}\,N_g + N_{ac}^{ref}$$

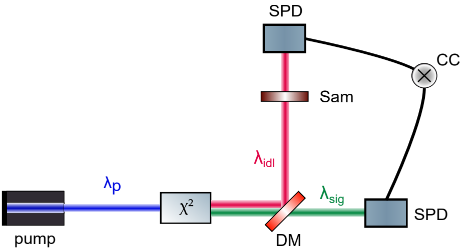
Coincidence approach



$$N_{\text{cc,tot}}^{\text{ref}} = \eta_{\text{idl}}\,\eta_{\text{sig}}\,N_{\text{g}} + N_{\text{ac}}^{\text{ref}}$$

$$N_{\text{cc,tot}}^{\text{sam}} = T\,\eta_{\text{idl}}\,\eta_{\text{sig}}\,N_{\text{g}} + N_{\text{ac}}^{\text{sam}}$$

Coincidence approach



$$N^{ref}_{cc,tot} = \eta_{idl}\,\eta_{sig}\,N_g + N^{ref}_{ac}$$

$$N^{sam}_{cc,tot} = T\,\eta_{idl}\,\eta_{sig}\,N_g + N^{sam}_{ac}$$

$$\Rightarrow T = \frac{N^{sam}_{tot,cc} - N^{sam}_{ac}}{N^{ref}_{tot,cc} - N^{ref}_{ac}}$$

Transmittance model

Conventional approach:

$$\mathrm{Var}(T) = (\eta_{\mathrm{idl}}\, N_{\mathrm{g}})^{-2} \left[\mathrm{Var}(N_{\mathrm{tot}}^{\mathrm{sam}}) + \mathrm{Var}\left(N_{\mathrm{noise}}^{\mathrm{sam}}\right) + T^2 \left[\mathrm{Var}\left(N_{\mathrm{tot}}^{\mathrm{ref}}\right) + \mathrm{Var}\left(N_{\mathrm{noise}}^{\mathrm{ref}}\right) \right] \right]$$

Coincidence approach:

$$\mathrm{Var}(T) = \left(\eta_{\mathrm{sig}}\, \eta_{\mathrm{idl}}\, N_{\mathrm{g}}\right)^{-2} \left[\mathrm{Var}\left(N_{\mathrm{tot,cc}}^{\mathrm{sam}}\right) + \mathrm{Var}(N_{\mathrm{ac}}^{\mathrm{sam}}) + T^2 \left[\mathrm{Var}\left(N_{\mathrm{tot,cc}}^{\mathrm{ref}}\right) + \mathrm{Var}\left(N_{\mathrm{ac}}^{\mathrm{ref}}\right) \right] \right]$$

Transmittance model

Conventional approach:

$$\mathrm{Var}(T) = (\eta_{\mathrm{idl}}\, N_{\mathrm{g}})^{-2} \left[\mathrm{Var}(N_{\mathrm{tot}}^{\mathrm{sam}}) + \mathrm{Var}(N_{\mathrm{noise}}^{\mathrm{sam}}) + T^2 \left[\mathrm{Var}(N_{\mathrm{tot}}^{\mathrm{ref}}) + \mathrm{Var}(N_{\mathrm{noise}}^{\mathrm{ref}}) \right] \right]$$

Coincidence approach:

$$\mathrm{Var}(T) = (\eta_{\mathrm{sig}}\, \eta_{\mathrm{idl}}\, N_{\mathrm{g}})^{-2} \left[\mathrm{Var}(N_{\mathrm{tot,cc}}^{\mathrm{sam}}) + \mathrm{Var}(N_{\mathrm{ac}}^{\mathrm{sam}}) + T^2 \left[\mathrm{Var}(N_{\mathrm{tot,cc}}^{\mathrm{ref}}) + \mathrm{Var}(N_{\mathrm{ac}}^{\mathrm{ref}}) \right] \right]$$

Photon statistics

Photon statistics

Poisson distribution (coherent light): [2][3]

$$\mathcal{P}(n) = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}$$
$$\mathrm{Var}(n) = \langle n \rangle$$

Photon statistics

Poisson distribution (coherent light): [2][3]

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[2] Kim et al., "Photon-Counting Statistics-Based Support Vector Machine with Multi-Mode Photon Illumination for Quantum Imaging"

[3] Fouche, "Detection and False-Alarm Probabilities for Laser Radars That Use Geiger-mode Detectors"

Photon statistics

Poisson distribution (coherent light): [2][3]

$$\mathcal{P}(n) = \frac{\langle n \rangle^n}{n!} \, e^{-\langle n \rangle}$$

$$\mathrm{Var}(n) = \langle n \rangle$$

multi-mode Bose-Einstein distribution (thermal light): [4]<5->[2]

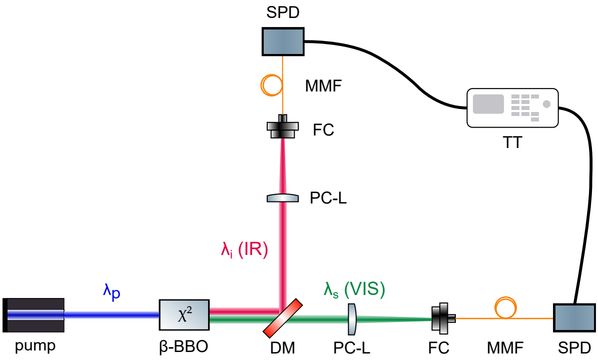
$$\mathcal{P}_m(n) = \frac{(n+m-1)!}{(m-1)!\, n!} \, \frac{m^m \langle n \rangle^n}{(m + \langle n \rangle)^{n+m}}$$

$$\mathrm{Var}(n) = \langle n \rangle \left(1 + \frac{\langle n \rangle}{m}\right)$$

[2] Kim et al., "Photon-Counting Statistics-Based Support Vector Machine with Multi-Mode Photon Illumination for Quantum Imaging"

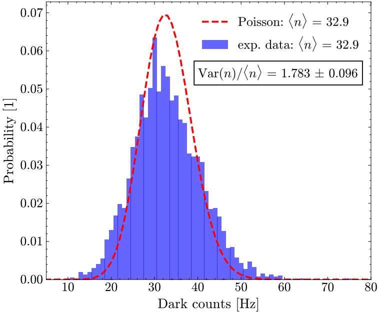
[3] Fouche, "Detection and False-Alarm Probabilities for Laser Radars That Use Geiger-mode Detectors"

Experimental setup

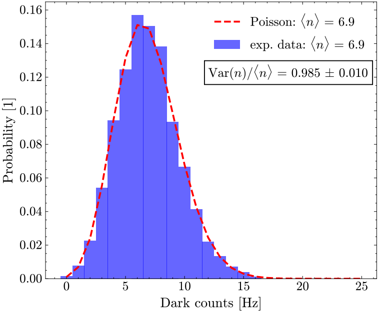


Dark counts

Idler arm



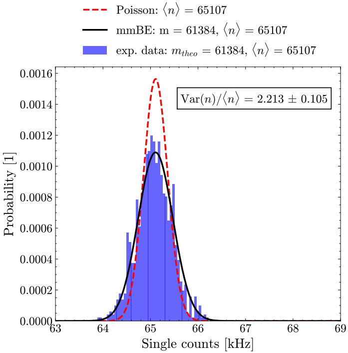
Signal arm



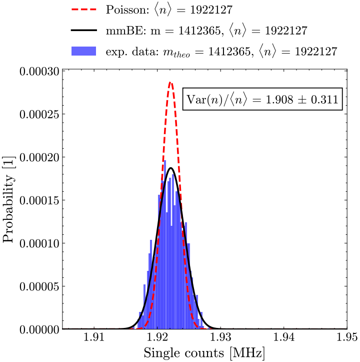
$$\mathrm{Var}(N_{\mathrm{noise}}) = 1.8 \cdot \langle N_{\mathrm{noise}} \rangle$$

Single counts

Idler arm



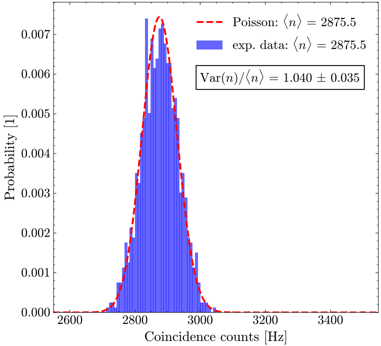
Signal arm



$$\mathrm{Var}(N_{\mathrm{noise}}) = 2.2 \cdot \langle N_{\mathrm{noise}} \rangle$$

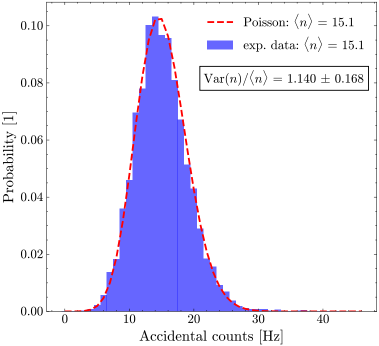
Coincidence counts

Coincidences



$$\mathrm{Var}(N_{\mathrm{cc}}) = \langle N_{\mathrm{cc}} \rangle$$

Accidentals



$$\mathrm{Var}(N_{\mathrm{ac}}) = \langle N_{\mathrm{ac}} \rangle$$

Simulation

Conventional approach:

$$\mathrm{Var}(T) = (\eta_{\mathrm{idl}}\, N_{\mathrm{g}})^{-2} \left[\mathrm{Var}(N_{\mathrm{tot}}^{\mathrm{sam}}) + \mathrm{Var}\left(N_{\mathrm{noise}}^{\mathrm{sam}}\right) + T^2 \left[\mathrm{Var}\left(N_{\mathrm{tot}}^{\mathrm{ref}}\right) + \mathrm{Var}\left(N_{\mathrm{noise}}^{\mathrm{ref}}\right) \right] \right]$$

Simulation

Conventional approach:

$$\mathrm{Var}(T) = (\eta_{\mathrm{idl}}\, N_{\mathrm{g}})^{-2} \left[2.2 \cdot \langle N_{\mathrm{tot}}^{\mathrm{sam}} \rangle + 1.8 \cdot \langle N_{\mathrm{noise}}^{\mathrm{sam}} \rangle + T^2 \left[2.2 \cdot \langle N_{\mathrm{tot}}^{\mathrm{ref}} \rangle + 1.8 \cdot \langle N_{\mathrm{noise}}^{\mathrm{ref}} \rangle \right] \right]$$

Simulation

Conventional approach:

$$\mathrm{Var}(T) = (\eta_{\mathrm{idl}}\, N_{\mathrm{g}})^{-2} \left[2.2 \cdot \langle N_{\mathrm{tot}}^{\mathrm{sam}} \rangle + 1.8 \cdot \langle N_{\mathrm{noise}}^{\mathrm{sam}} \rangle + T^2 \left[2.2 \cdot \langle N_{\mathrm{tot}}^{\mathrm{ref}} \rangle + 1.8 \cdot \langle N_{\mathrm{noise}}^{\mathrm{ref}} \rangle \right] \right]$$

Coincidence approach:

$$\mathrm{Var}(T) = (\eta_{\mathrm{sig}}\, \eta_{\mathrm{idl}}\, N_{\mathrm{g}})^{-2} \left[\mathrm{Var}(N_{\mathrm{tot,cc}}^{\mathrm{sam}}) + \mathrm{Var}(N_{\mathrm{ac}}^{\mathrm{sam}}) + T^2 \left[\mathrm{Var}(N_{\mathrm{tot,cc}}^{\mathrm{ref}}) + \mathrm{Var}(N_{\mathrm{ac}}^{\mathrm{ref}}) \right] \right]$$

Simulation

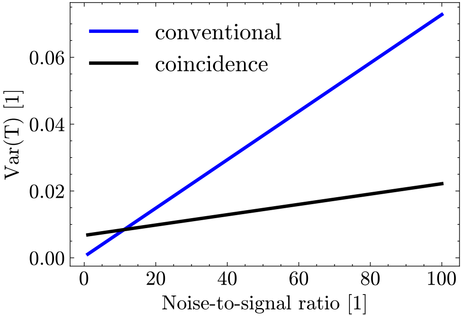
Conventional approach:

$$\mathrm{Var}(T) = (\eta_{\mathrm{idl}}\, N_{\mathrm{g}})^{-2} \left[2.2 \cdot \langle N_{\mathrm{tot}}^{\mathrm{sam}} \rangle + 1.8 \cdot \langle N_{\mathrm{noise}}^{\mathrm{sam}} \rangle + T^2 \left[2.2 \cdot \langle N_{\mathrm{tot}}^{\mathrm{ref}} \rangle + 1.8 \cdot \langle N_{\mathrm{noise}}^{\mathrm{ref}} \rangle \right] \right]$$

Coincidence approach:

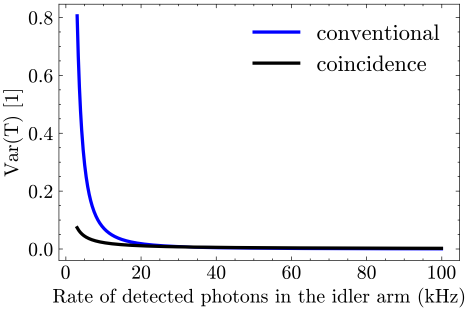
$$\mathrm{Var}(T) = \left(\eta_{\mathrm{sig}}\, \eta_{\mathrm{idl}}\, N_{\mathrm{g}}\right)^{-2} \left[\langle N_{\mathrm{tot,cc}}^{\mathrm{sam}} \rangle + \langle N_{\mathrm{ac}}^{\mathrm{sam}} \rangle + T^2 \left[\langle N_{\mathrm{tot,cc}}^{\mathrm{ref}} \rangle + \langle N_{\mathrm{ac}}^{\mathrm{ref}} \rangle \right] \right]$$

Simulation



Parameter	**Value**
η_{idl} (%)	0.09
η_{sig} (%)	2.6
R_{idl} (kHz)	10
$R_{\mathrm{noise,idl}}$ (kHz)	10 - 1000
$R_{\mathrm{noise,sig}}$ (Hz)	7
T (1)	0.9

Simulation



Parameter	Value
η_{idl} (%)	0.09
η_{sig} (%)	2.6
R_{idl} (kHz)	3 - 100
$R_{noise,idl}$ (kHz)	1000
$R_{noise,sig}$ (Hz)	7
T (1)	0.9

Summary and Outlook

Can coincidence measurements provide more precise results?

Achievements:

- Model for variance of transmittance
- Experimental verification of photon statistics
- Found regions where coincidence approach offers advantages

Summary and Outlook

Git repository

public accessible:

`https://git.tpi.uni-jena.de/mstnhsr/latexbeamer_corporatedesign`

Feedback

marc.steinhauser@uni-jena.de

Slide title in Palatino Linotype Font

block environment (lower-case b)

itemize:

- First Level
  - Second Level
    - Third Level has no item mark

Block environment (upper-case B)

enumerate:

1. First Level
   1.1 Second Level
      1.1.1 Third Level