

Parameter estimation with correlated photon pairs

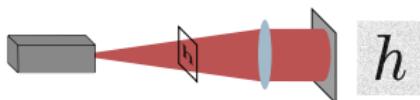
Jan Gößwein

Institute of Applied Physics

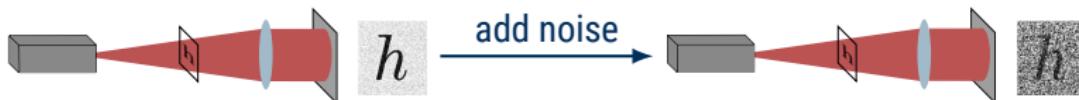
Jena, November 10, 2025

Motivation

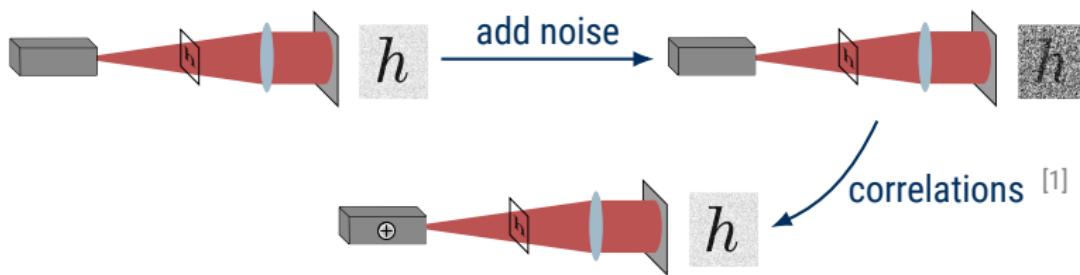
Motivation



Motivation

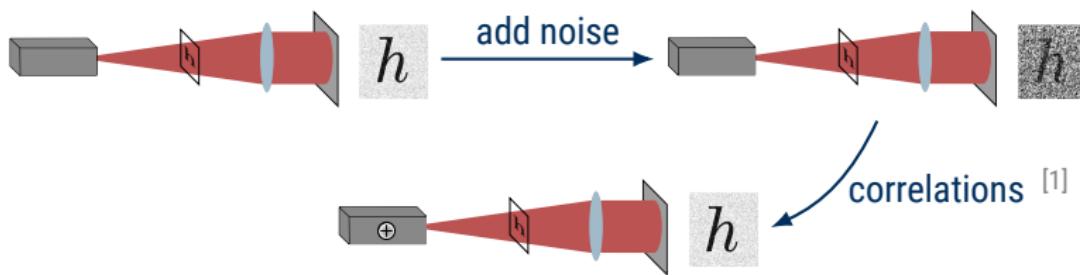


Motivation



[1] Brida, Genovese, and Ruo Berchera, "Experimental Realization of Sub-Shot-Noise Quantum Imaging"

Motivation

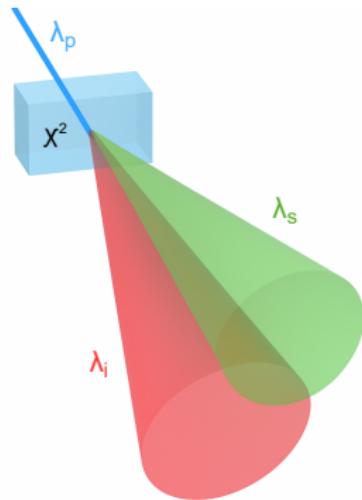


Objective: Can correlated photons provide advantages in terms of precision in noisy regimes for parameter estimation?

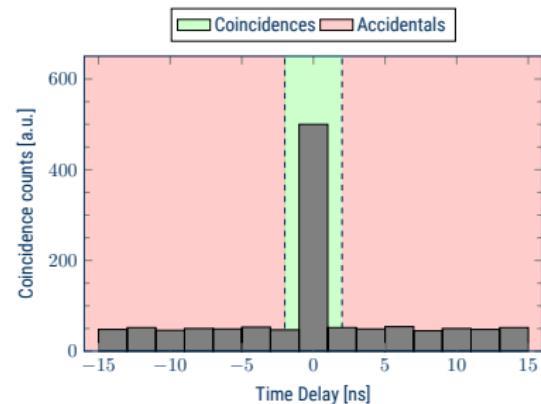
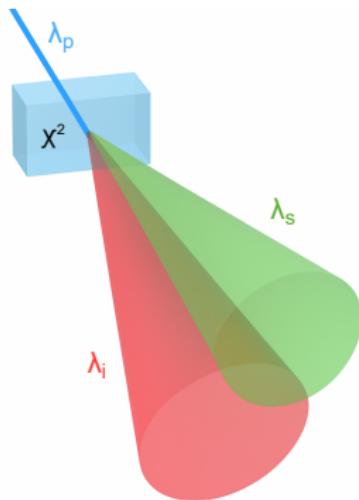
[1] Brida, Genovese, and Ruo Berchera, "Experimental Realization of Sub-Shot-Noise Quantum Imaging"

Spontaneous parametric down-conversion

Spontaneous parametric down-conversion



Spontaneous parametric down-conversion



Parameter estimation

Parameter estimation

Setup: Transmission setup

Parameter estimation

Setup: Transmission setup

Parameter: Transmittance T

Parameter estimation

Setup: Transmission setup

Parameter: Transmittance T

Precision: $\text{Var}(T)$

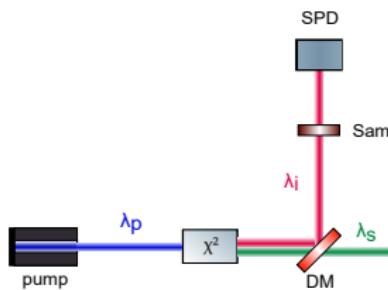
Parameter estimation

Setup: Transmission setup

Parameter: Transmittance T

Precision: $\text{Var}(T)$

Conventional approach:



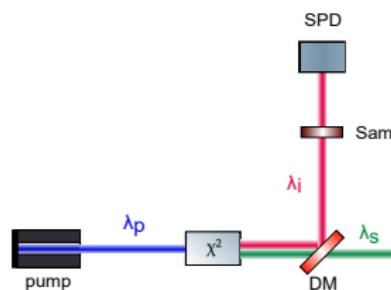
Parameter estimation

Setup: Transmission setup

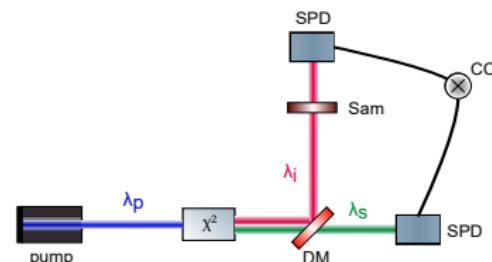
Parameter: Transmittance T

Precision: $\text{Var}(T)$

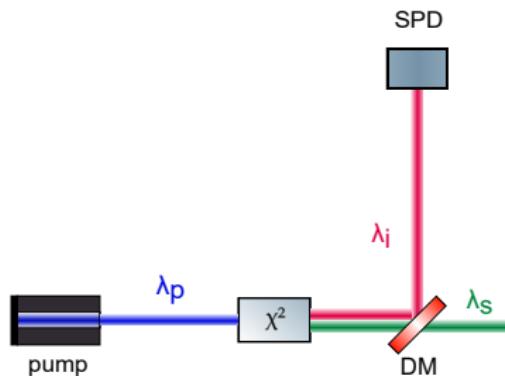
Conventional approach:



Coincidence approach:

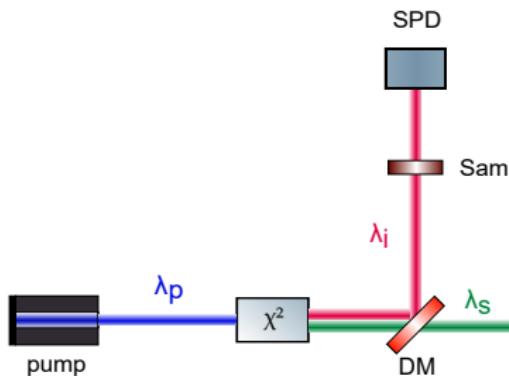


Conventional approach



$$N_{\text{sing}}^{\text{ref}} = \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{ref}}$$

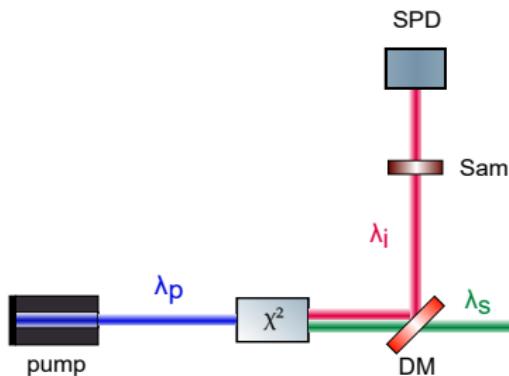
Conventional approach



$$N_{\text{sing}}^{\text{ref}} = \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{ref}}$$

$$N_{\text{sing}}^{\text{sam}} = T \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{sam}}$$

Conventional approach

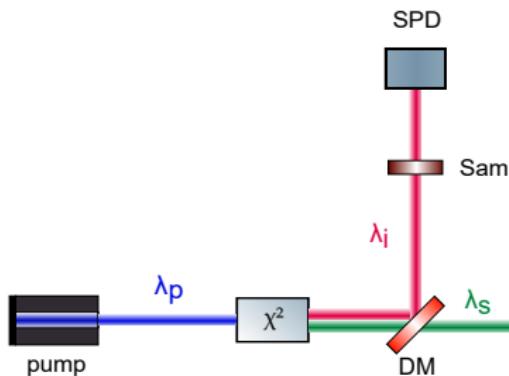


$$N_{\text{sing}}^{\text{ref}} = \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{ref}}$$

$$N_{\text{sing}}^{\text{sam}} = T \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{sam}}$$

$$\Rightarrow T = \frac{N_{\text{sing}}^{\text{sam}} - N_{\text{noise}}^{\text{sam}}}{N_{\text{sing}}^{\text{ref}} - N_{\text{noise}}^{\text{ref}}}$$

Conventional approach



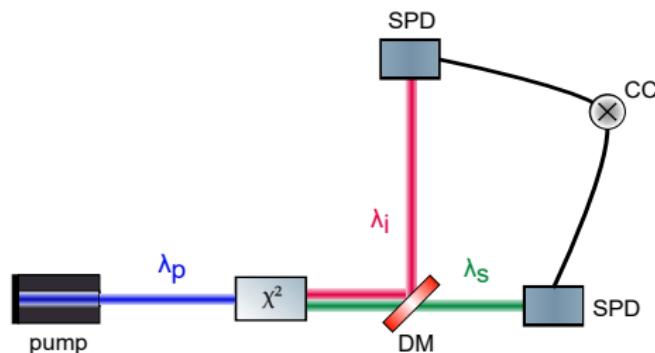
$$N_{\text{sing}}^{\text{ref}} = \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{ref}}$$

$$N_{\text{sing}}^{\text{sam}} = T \eta_{\text{idl}} N_g + N_{\text{noise}}^{\text{sam}}$$

$$\Rightarrow T = \frac{N_{\text{sing}}^{\text{sam}} - N_{\text{noise}}^{\text{sam}}}{N_{\text{sing}}^{\text{ref}} - N_{\text{noise}}^{\text{ref}}}$$

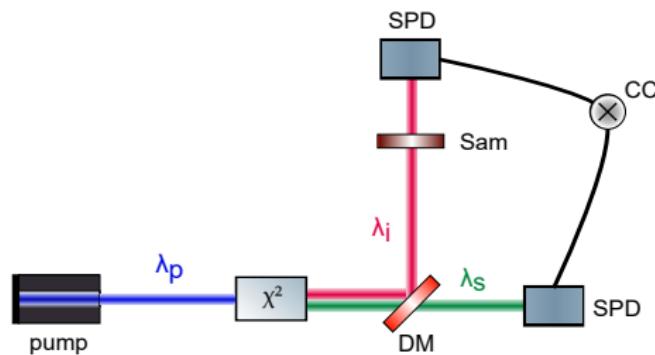
$$\text{Var}(T) = (\eta_{\text{idl}} N_g)^{-2} \left[\text{Var}(N_{\text{sing}}^{\text{sam}}) + \text{Var}(N_{\text{noise}}^{\text{sam}}) + T^2 \left[\text{Var}(N_{\text{sing}}^{\text{ref}}) + \text{Var}(N_{\text{noise}}^{\text{ref}}) \right] \right]$$

Coincidence approach



$$N_{\text{coin}}^{\text{ref}} = \eta_{\text{id}} \eta_{\text{sig}} N_g + N_{\text{ac}}^{\text{ref}}$$

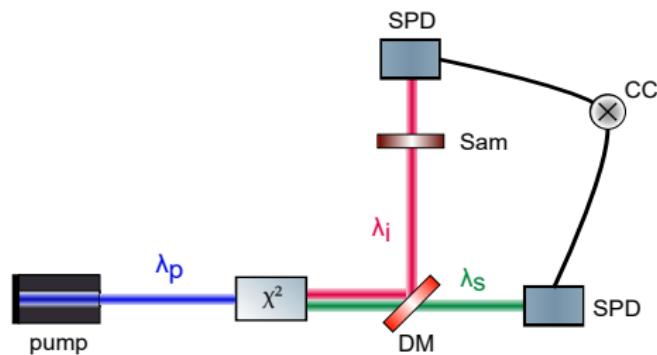
Coincidence approach



$$N_{\text{coin}}^{\text{ref}} = \eta_{\text{idl}} \eta_{\text{sig}} N_g + N_{\text{ac}}^{\text{ref}}$$

$$N_{\text{coin}}^{\text{sam}} = T \eta_{\text{idl}} \eta_{\text{sig}} N_g + N_{\text{ac}}^{\text{sam}}$$

Coincidence approach

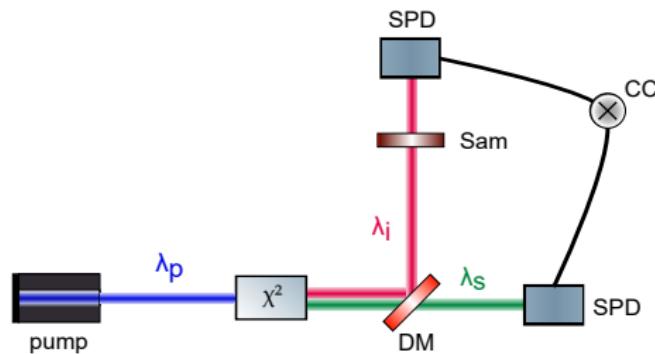


$$N_{\text{coin}}^{\text{ref}} = \eta_{\text{idl}} \eta_{\text{sig}} N_g + N_{\text{ac}}^{\text{ref}}$$

$$N_{\text{coin}}^{\text{sam}} = T \eta_{\text{idl}} \eta_{\text{sig}} N_g + N_{\text{ac}}^{\text{sam}}$$

$$\Rightarrow T = \frac{N_{\text{coin}}^{\text{sam}} - N_{\text{ac}}^{\text{sam}}}{N_{\text{coin}}^{\text{ref}} - N_{\text{ac}}^{\text{ref}}}$$

Coincidence approach



$$N_{\text{coin}}^{\text{ref}} = \eta_{\text{idl}} \eta_{\text{sig}} N_g + N_{\text{ac}}^{\text{ref}}$$

$$N_{\text{coin}}^{\text{sam}} = T \eta_{\text{idl}} \eta_{\text{sig}} N_g + N_{\text{ac}}^{\text{sam}}$$

$$\Rightarrow T = \frac{N_{\text{coin}}^{\text{sam}} - N_{\text{ac}}^{\text{sam}}}{N_{\text{coin}}^{\text{ref}} - N_{\text{ac}}^{\text{ref}}}$$

$$\text{Var}(T) = (\eta_{\text{sig}} \eta_{\text{idl}} N_g)^{-2} \left[\text{Var}(N_{\text{coin}}^{\text{sam}}) + \text{Var}(N_{\text{ac}}^{\text{sam}}) + T^2 \left[\text{Var}(N_{\text{coin}}^{\text{ref}}) + \text{Var}(N_{\text{ac}}^{\text{ref}}) \right] \right]$$

Transmittance model

Conventional approach:

$$\text{Var}(T) = (\eta_{\text{idl}} N_g)^{-2} \left[\text{Var}(N_{\text{sing}}^{\text{sam}}) + \text{Var}(N_{\text{noise}}^{\text{sam}}) + T^2 [\text{Var}(N_{\text{sing}}^{\text{ref}}) + \text{Var}(N_{\text{noise}}^{\text{ref}})] \right]$$

Coincidence approach:

$$\text{Var}(T) = (\eta_{\text{sig}} \eta_{\text{idl}} N_g)^{-2} \left[\text{Var}(N_{\text{coin}}^{\text{sam}}) + \text{Var}(N_{\text{ac}}^{\text{sam}}) + T^2 [\text{Var}(N_{\text{coin}}^{\text{ref}}) + \text{Var}(N_{\text{ac}}^{\text{ref}})] \right]$$

Transmittance model

Conventional approach:

$$\text{Var}(T) = (\eta_{\text{idl}} N_g)^{-2} \left[\text{Var}(N_{\text{sing}}^{\text{sam}}) + \text{Var}(N_{\text{noise}}^{\text{sam}}) + T^2 \left[\text{Var}(N_{\text{sing}}^{\text{ref}}) + \text{Var}(N_{\text{noise}}^{\text{ref}}) \right] \right]$$

Coincidence approach:

$$\text{Var}(T) = (\eta_{\text{sig}} \eta_{\text{idl}} N_g)^{-2} \left[\text{Var}(N_{\text{coin}}^{\text{sam}}) + \text{Var}(N_{\text{ac}}^{\text{sam}}) + T^2 \left[\text{Var}(N_{\text{coin}}^{\text{ref}}) + \text{Var}(N_{\text{ac}}^{\text{ref}}) \right] \right]$$

Photon statistics

Photon statistics

Poisson distribution (coherent light):

$$\mathcal{P}(n) = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}$$

$$\text{Var}(n) = \langle n \rangle$$

Photon statistics

Poisson distribution (coherent light): [2][3]

$$\mathcal{P}(n) = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}$$

$$\text{Var}(n) = \langle n \rangle$$

[2] Kim et al., "Photon-Counting Statistics-Based Support Vector Machine with Multi-Mode Photon Illumination for Quantum Imaging"

[3] Fouche, "Detection and False-Alarm Probabilities for Laser Radars That Use Geiger-mode Detectors"

Photon statistics

Poisson distribution (coherent light): [2][3]

$$\mathcal{P}(n) = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}$$
$$\text{Var}(n) = \langle n \rangle$$

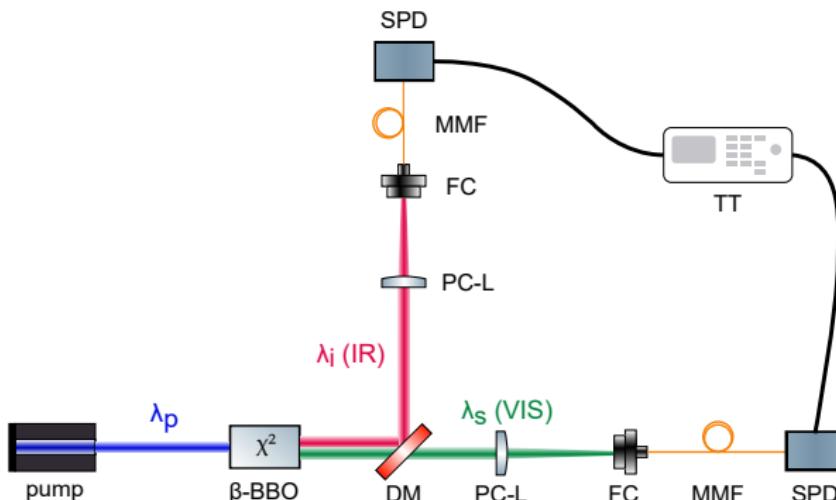
Multi-mode Bose-Einstein distribution (thermal light): [2]

$$\mathcal{P}_m(n) = \frac{(n+m-1)!}{(m-1)! n!} \frac{m^m \langle n \rangle^n}{(m + \langle n \rangle)^{n+m}}$$
$$\text{Var}(n) = \langle n \rangle \left(1 + \frac{\langle n \rangle}{m} \right)$$

[2] Kim et al., "Photon-Counting Statistics-Based Support Vector Machine with Multi-Mode Photon Illumination for Quantum Imaging"

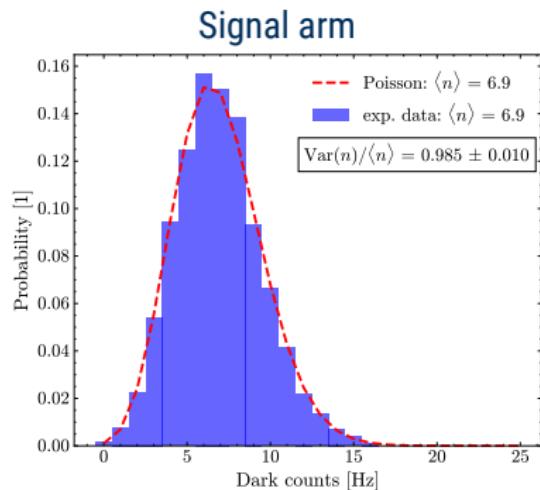
[3] Fouche, "Detection and False-Alarm Probabilities for Laser Radars That Use Geiger-mode Detectors"

Experimental setup

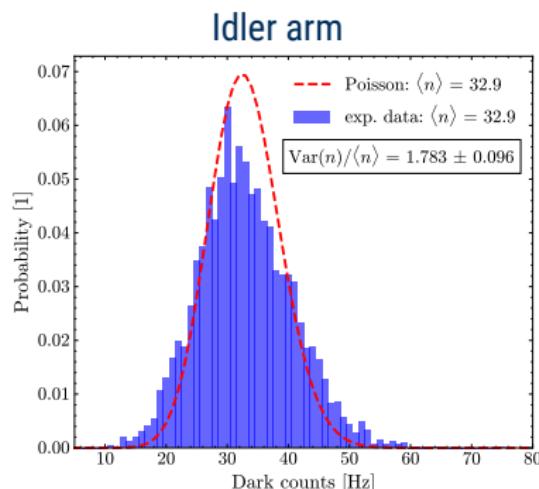
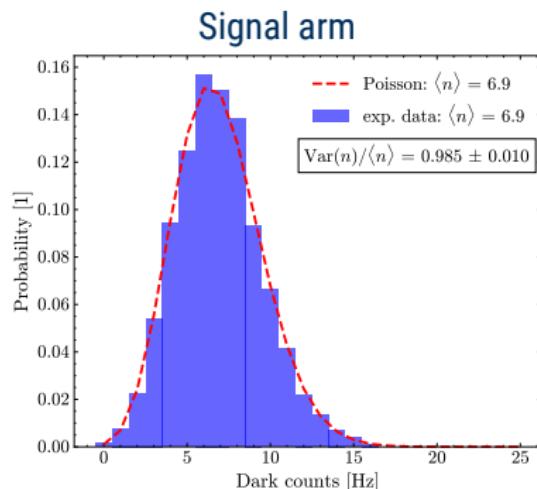


Dark counts, $\text{Var}(N_{\text{noise}})$

Dark counts, $\text{Var}(N_{\text{noise}})$

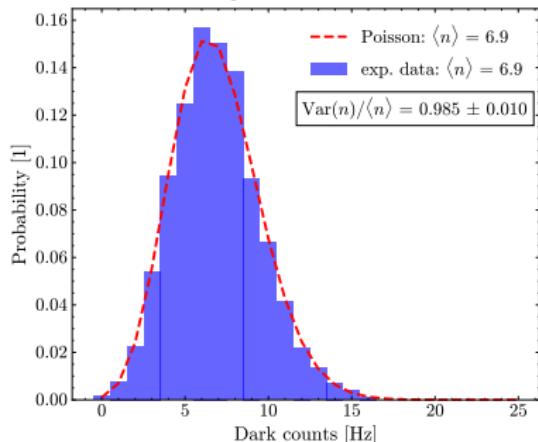


Dark counts, $\text{Var}(N_{\text{noise}})$

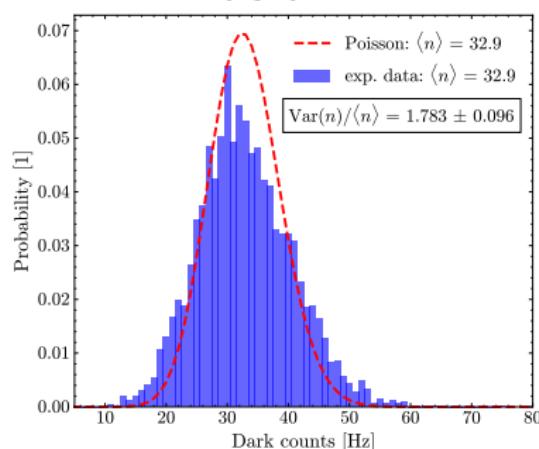


Dark counts, $\text{Var}(N_{\text{noise}})$

Signal arm



Idler arm



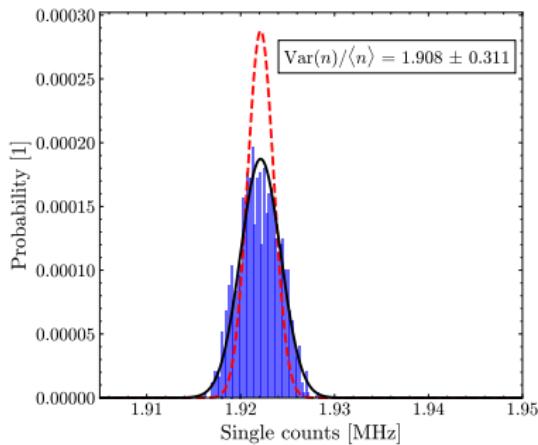
$$\text{Var}(N_{\text{noise}}) = 1.8 \cdot \langle N_{\text{noise}} \rangle$$

Single counts, $\text{Var}(N_{\text{sing}})$

Single counts, $\text{Var}(N_{\text{sing}})$

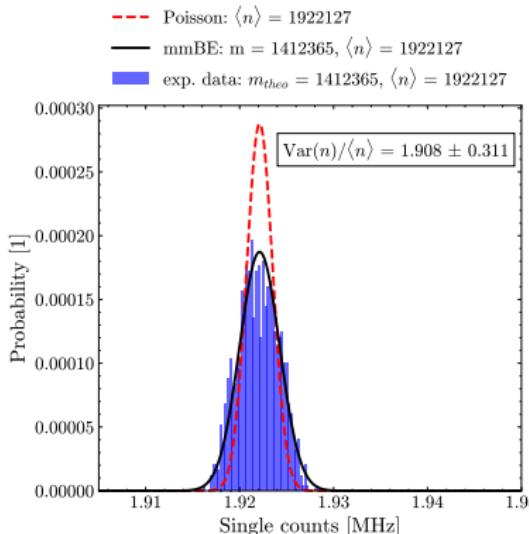
Signal arm

- Poisson: $\langle n \rangle = 1922127$
- mmBE: $m = 1412365$, $\langle n \rangle = 1922127$
- exp. data: $m_{\text{theo}} = 1412365$, $\langle n \rangle = 1922127$

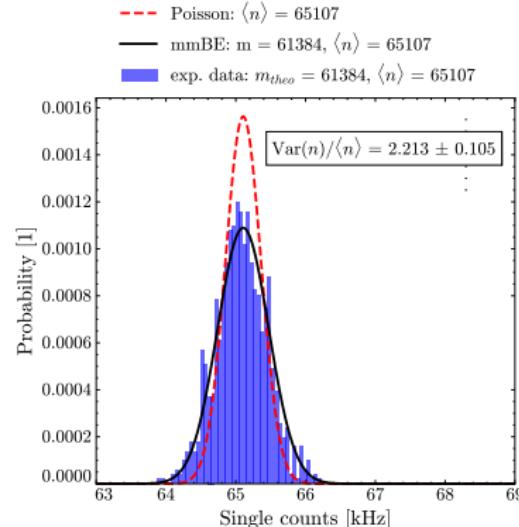


Single counts, $\text{Var}(N_{\text{sing}})$

Signal arm

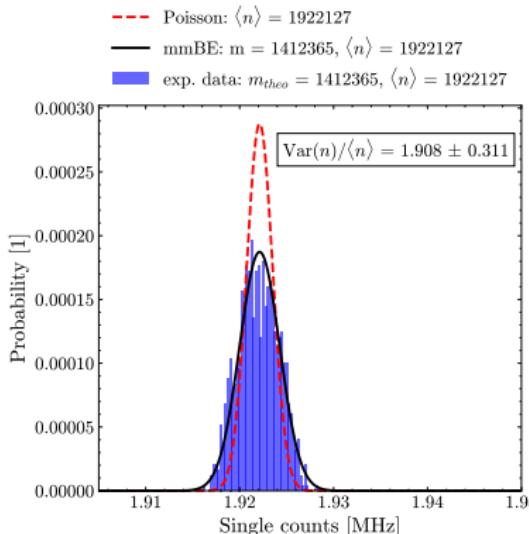


Idler arm

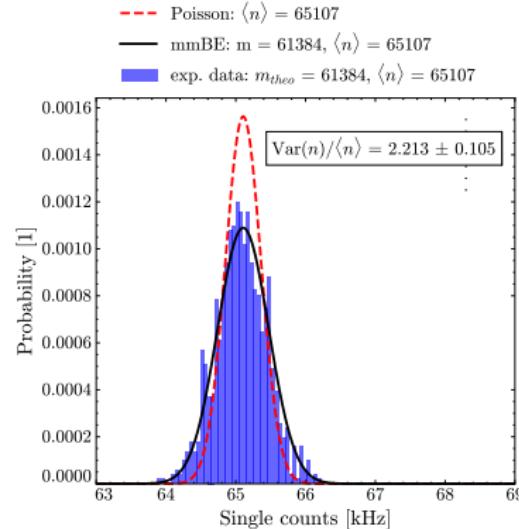


Single counts, $\text{Var}(N_{\text{sing}})$

Signal arm



Idler arm

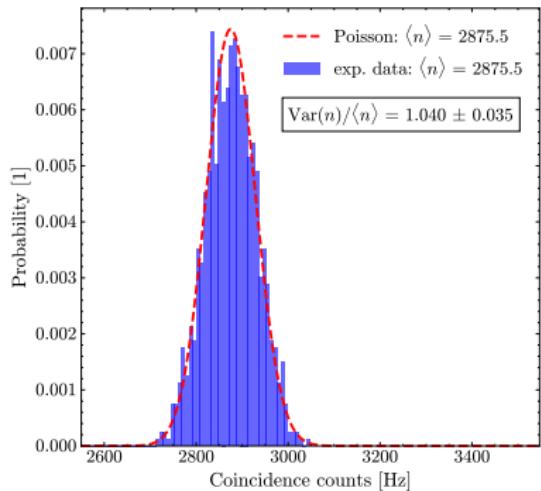


$$\text{Var}(N_{\text{sing}}) = 2.2 \cdot \langle N_{\text{sing}} \rangle$$

Coincidence counts, $\text{Var}(N_{\text{coin}})$

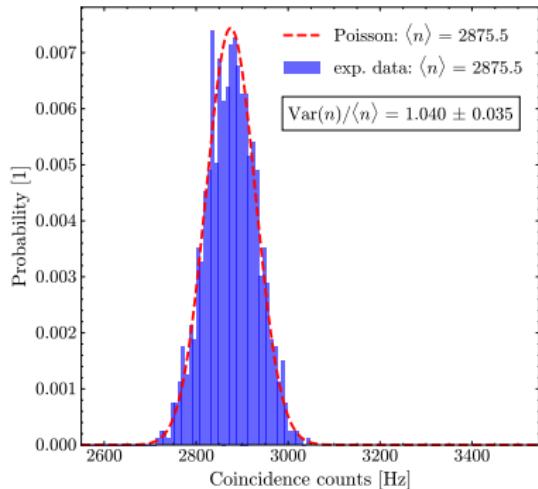
Coincidence counts, $\text{Var}(N_{\text{coin}})$

Coincidences



Coincidence counts, $\text{Var}(N_{\text{coin}})$

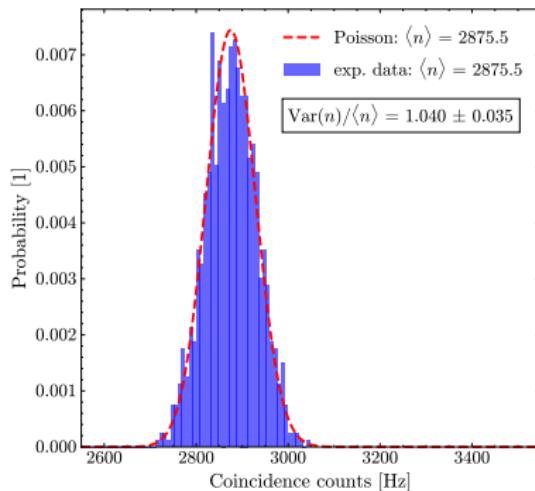
Coincidences



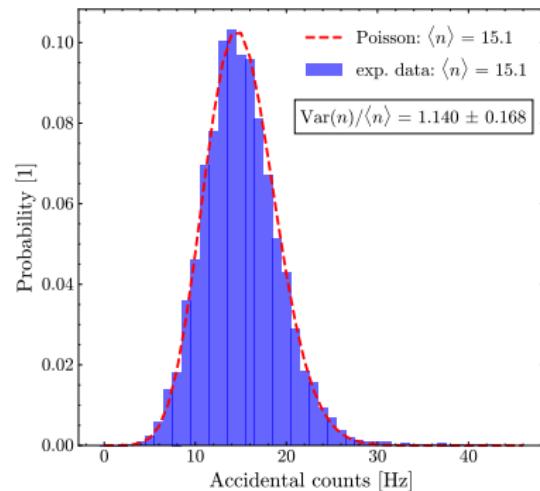
$$\text{Var}(N_{\text{coin}}) = \langle N_{\text{coin}} \rangle$$

Coincidence counts, $\text{Var}(N_{\text{coin}})$

Coincidences



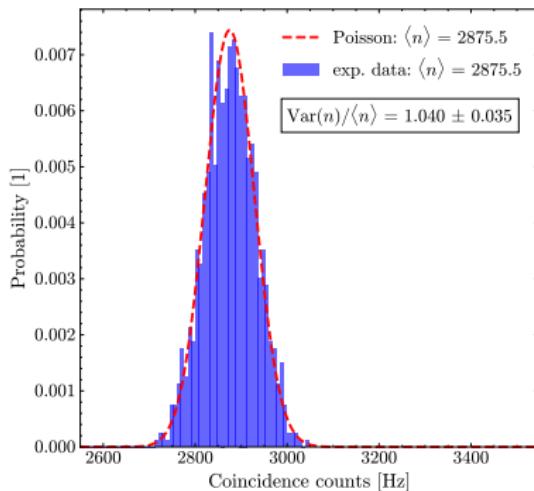
Accidentals



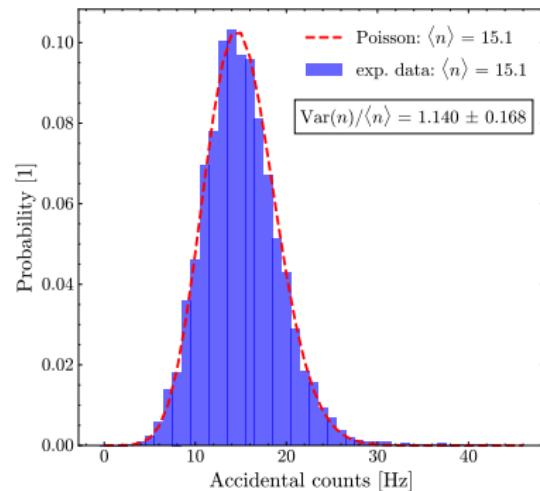
$$\text{Var}(N_{\text{coin}}) = \langle N_{\text{coin}} \rangle$$

Coincidence counts, $\text{Var}(N_{\text{coin}})$

Coincidences



Accidentals



$$\text{Var}(N_{\text{coin}}) = \langle N_{\text{coin}} \rangle$$

$$\text{Var}(N_{\text{ac}}) = \langle N_{\text{ac}} \rangle$$

Simulation

Conventional approach:

$$\text{Var}(T) = (\eta_{\text{idl}} N_g)^{-2} \left[\text{Var}(N_{\text{sing}}^{\text{sam}}) + \text{Var}(N_{\text{noise}}^{\text{sam}}) + T^2 \left[\text{Var}(N_{\text{sing}}^{\text{ref}}) + \text{Var}(N_{\text{noise}}^{\text{ref}}) \right] \right]$$

Simulation

Conventional approach:

$$\text{Var}(T) = (\eta_{\text{idl}} N_g)^{-2} \left[2.2 \cdot \langle N_{\text{sing}}^{\text{sam}} \rangle + 1.8 \cdot \langle N_{\text{noise}}^{\text{sam}} \rangle + T^2 [2.2 \cdot \langle N_{\text{sing}}^{\text{ref}} \rangle + 1.8 \cdot \langle N_{\text{noise}}^{\text{ref}} \rangle] \right]$$

Simulation

Conventional approach:

$$\text{Var}(T) = (\eta_{\text{idl}} N_g)^{-2} \left[2.2 \cdot \langle N_{\text{sing}}^{\text{sam}} \rangle + 1.8 \cdot \langle N_{\text{noise}}^{\text{sam}} \rangle + T^2 [2.2 \cdot \langle N_{\text{sing}}^{\text{ref}} \rangle + 1.8 \cdot \langle N_{\text{noise}}^{\text{ref}} \rangle] \right]$$

Coincidence approach:

$$\text{Var}(T) = (\eta_{\text{sig}} \eta_{\text{idl}} N_g)^{-2} \left[\text{Var}(N_{\text{coin}}^{\text{sam}}) + \text{Var}(N_{\text{ac}}^{\text{sam}}) + T^2 [\text{Var}(N_{\text{coin}}^{\text{ref}}) + \text{Var}(N_{\text{ac}}^{\text{ref}})] \right]$$

Simulation

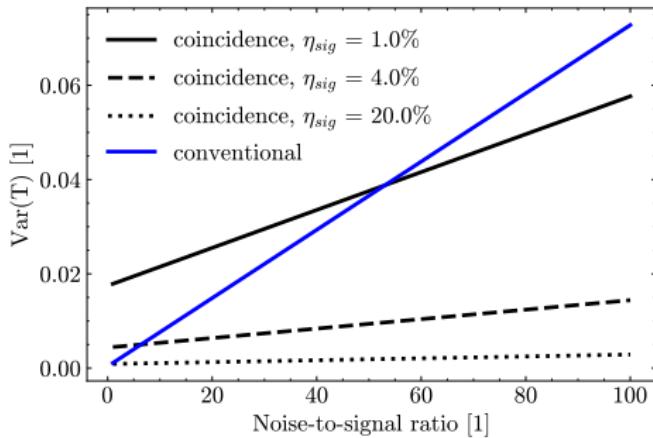
Conventional approach:

$$\text{Var}(T) = (\eta_{\text{idl}} N_g)^{-2} \left[2.2 \cdot \langle N_{\text{sing}}^{\text{sam}} \rangle + 1.8 \cdot \langle N_{\text{noise}}^{\text{sam}} \rangle + T^2 [2.2 \cdot \langle N_{\text{sing}}^{\text{ref}} \rangle + 1.8 \cdot \langle N_{\text{noise}}^{\text{ref}} \rangle] \right]$$

Coincidence approach:

$$\text{Var}(T) = (\eta_{\text{sig}} \eta_{\text{idl}} N_g)^{-2} \left[\langle N_{\text{coin}}^{\text{sam}} \rangle + \langle N_{\text{ac}}^{\text{sam}} \rangle + T^2 [\langle N_{\text{coin}}^{\text{ref}} \rangle + \langle N_{\text{ac}}^{\text{ref}} \rangle] \right]$$

Noise-to-signal ratio



Parameter	Value
$\eta_{\text{idl}} (\%)$	0.09
$\eta_{\text{sig}} (\%)$	1, 4, 20
$R_{\text{idl}} (\text{kHz})$	10
$R_{\text{noise,idl}} (\text{kHz})$	10 - 1000
$R_{\text{noise,sig}} (\text{Hz})$	7
$T (1)$	0.9

Summary and Outlook

Summary and Outlook

Summary:

- Established a model for the variance of transmittance
- Experimental verification of photon statistics
- Found regions where coincidence approach offers higher precision

Summary and Outlook

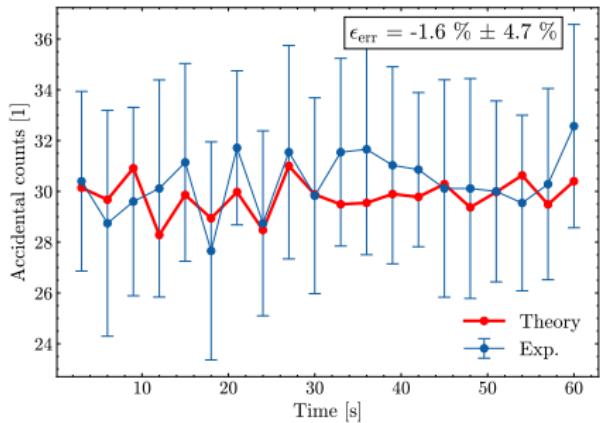
Summary:

- Established a model for the variance of transmittance
- Experimental verification of photon statistics
- Found regions where coincidence approach offers higher precision

Outlook:

- Experimental verification of the found parameter regions
- Determine the experimental limitations for the variance measurement

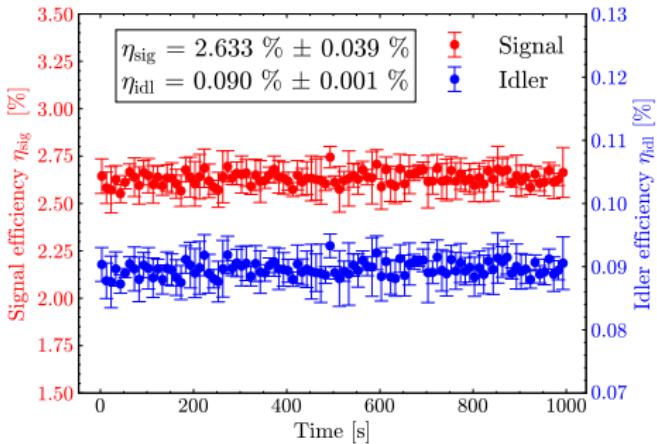
Accidental counts



$$R_{\text{ac}}^{\text{sam}} = \left(T \eta_{\text{idl}} R_g + R_{\text{dc,idl}} - R_{\text{cc,pure}}^{\text{sam}} \right) \cdot \\ \left(\eta_{\text{sig}} R_g + R_{\text{dc,sig}} - R_{\text{cc,pure}}^{\text{sam}} \right) \cdot \tau_{\text{cw}}$$

$$R_{\text{ac}}^{\text{ref}} = \left(\eta_{\text{idl}} R_g + R_{\text{dc,idl}} - R_{\text{cc,pure}}^{\text{ref}} \right) \cdot \\ \left(\eta_{\text{sig}} R_g + R_{\text{dc,sig}} - R_{\text{cc,pure}}^{\text{ref}} \right) \cdot \tau_{\text{cw}}$$

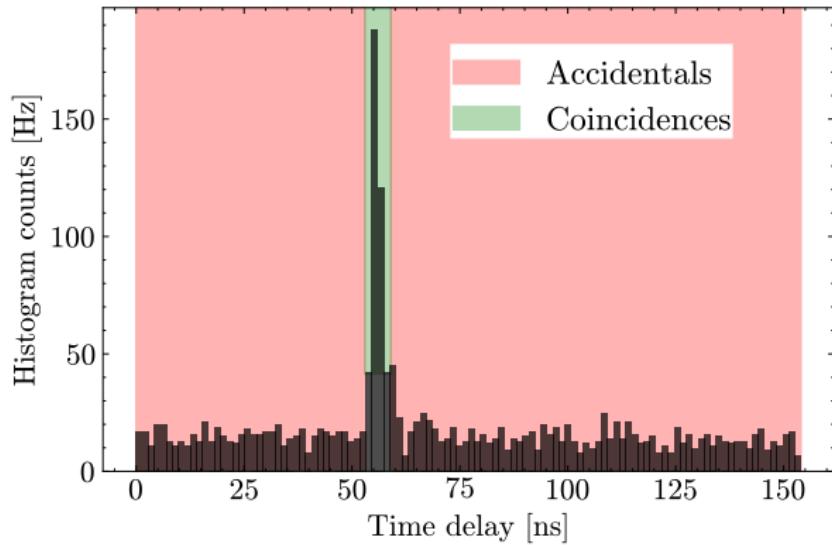
Heralding efficiencies



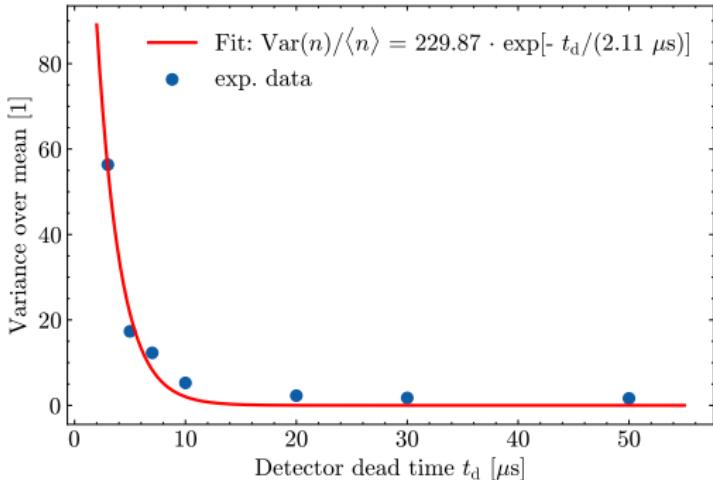
$$\eta_{\text{sig}} = \frac{N_{\text{coin}} - N_{\text{ac}}}{N_{\text{sing,idl}} - N_{\text{noise}}} = \frac{\eta_{\text{sig}} \eta_{\text{idl}} N_g}{\eta_{\text{idl}} N_g}$$

$$\eta_{\text{idl}} = \frac{N_{\text{coin}} - N_{\text{ac}}}{N_{\text{sing,sig}} - N_{\text{noise}}} = \frac{\eta_{\text{sig}} \eta_{\text{idl}} N_g}{\eta_{\text{sig}} N_g}$$

Histogram

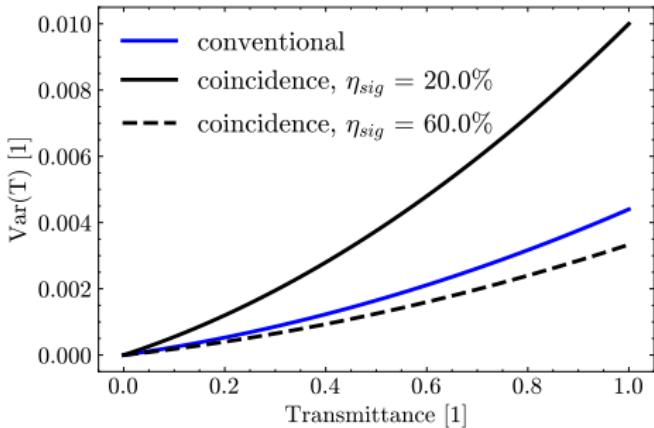


Afterpulsing



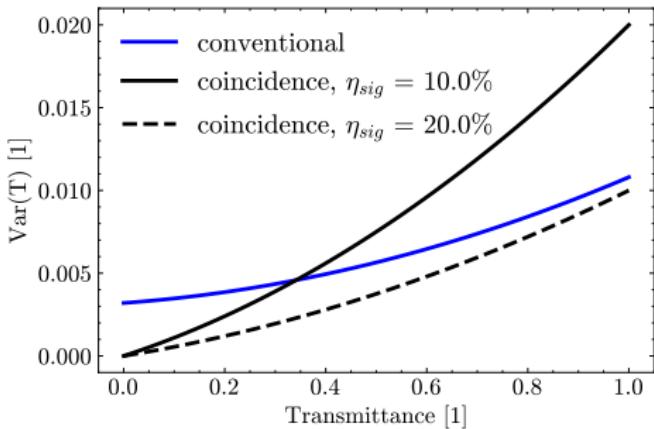
$$R_{aft} \propto e^{-\frac{t_d}{\tau_{de}}}$$

Noiseless case



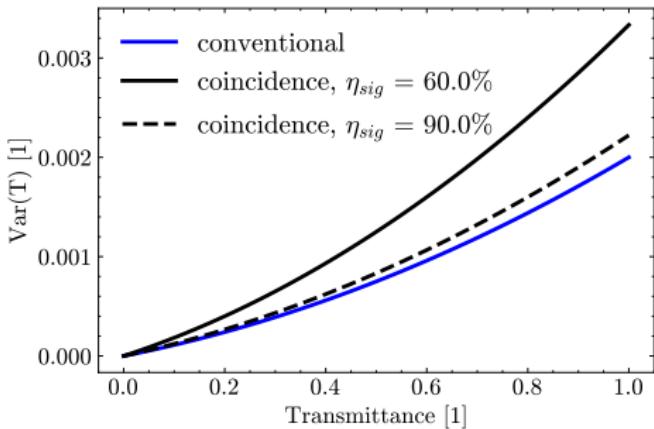
Parameter	Value
η_{idl} (%)	60
η_{sig} (%)	20, 60
R_{idl} (kHz)	1
$R_{noise,idl}$ (kHz)	0
$R_{noise,sig}$ (Hz)	7
$T(1)$	0 - 1

In presence of noise



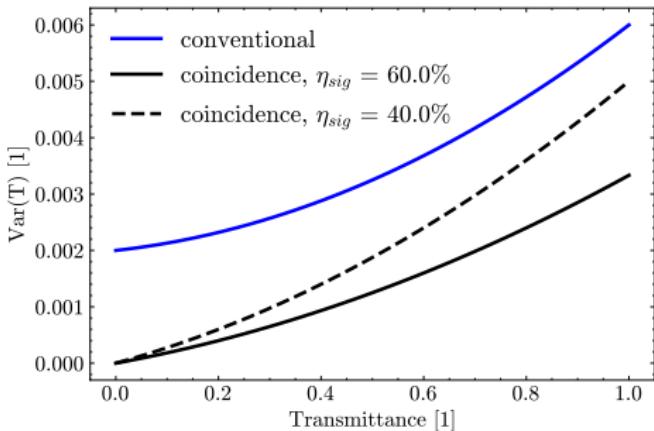
Parameter	Value
η_{idl} (%)	60
η_{sig} (%)	10, 20
R_{idl} (kHz)	1
$R_{noise,idl}$ (kHz)	1
$R_{noise,sig}$ (Hz)	7
$T(1)$	0 - 1

Coherent Illumination, noiseless



Parameter	Value
η_{idl} (%)	60
η_{sig} (%)	60, 90
R_{idl} (kHz)	1
$R_{noise,idl}$ (kHz)	0
$R_{noise,sig}$ (Hz)	7
$T(1)$	0 - 1

Coherent Illumination, with noise



Parameter	Value
η_{idl} (%)	60
η_{sig} (%)	40, 60
R_{idl} (kHz)	1
$R_{noise,idl}$ (kHz)	1
$R_{noise,sig}$ (Hz)	7
$T(1)$	0 - 1