Parameter estimation with correlated photon pairs

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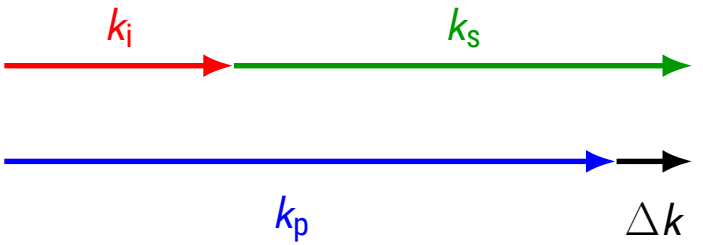
Motivation

SPDC

Energy conservation

$$\omega_p = \omega_s + \omega_i$$

Momentum conservation



$$\vec{k}_p = \vec{k}_s + \vec{k}_i - \Delta\vec{k}$$

Transmittance model

Conventional approach:

$$N_{\text{tot}}^{\text{ref}} = \eta_{\text{idl}}\, N_{\text{g}} + N_{\text{noise}}^{\text{ref}}$$

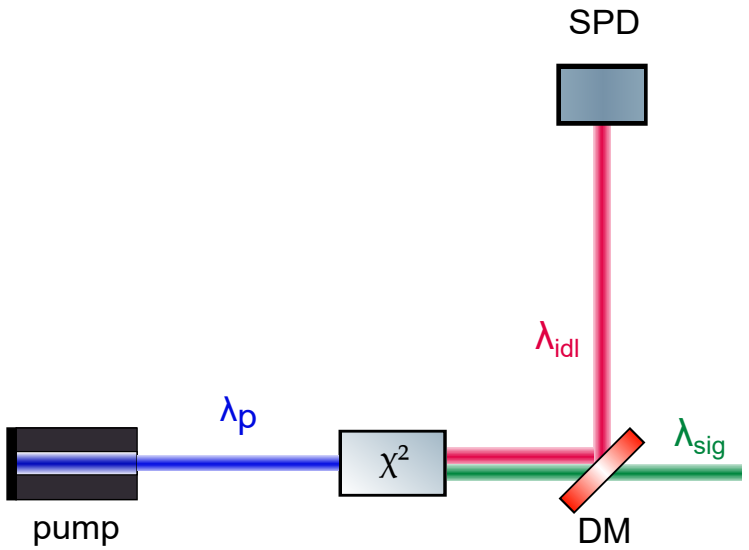
$$N_{\text{tot}}^{\text{sam}} = T\, \eta_{\text{idl}}\, N_{\text{g}} + N_{\text{noise}}^{\text{sam}}$$

Coincidence approach:

$$N_{\text{cc}}^{\text{pure,sam}} = T\, \eta_{\text{idl}}\, \eta_{\text{sig}}\, N_{\text{g}},$$

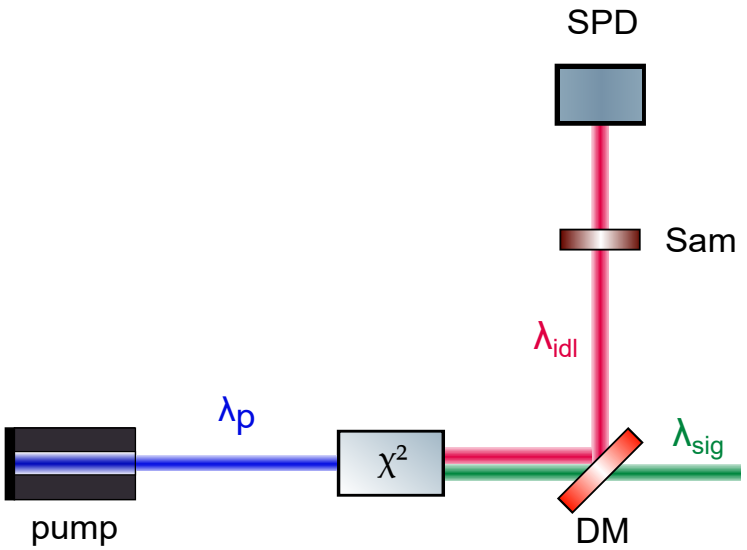
$$N_{\text{cc}}^{\text{pure,ref}} = \eta_{\text{idl}}\, \eta_{\text{sig}}\, N_{\text{g}}$$

Conventional approach



$$N_{\text{tot}}^{\text{ref}} = \eta_{\text{idl}} N_{\text{g}} + N_{\text{noise}}^{\text{ref}}$$

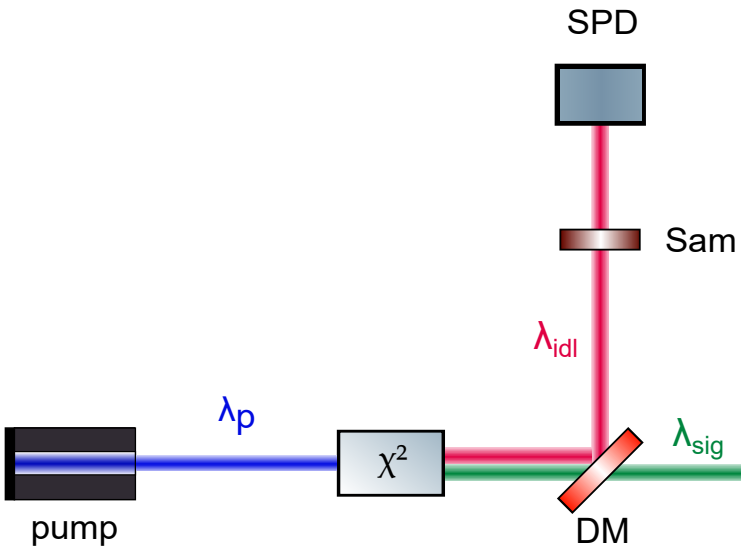
Conventional approach



$$N_{\text{tot}}^{\text{ref}} = \eta_{\text{idl}} N_{\text{g}} + N_{\text{noise}}^{\text{ref}}$$

$$N_{\text{tot}}^{\text{sam}} = T\,\eta_{\text{idl}} N_{\text{g}} + N_{\text{noise}}^{\text{sam}}$$

Conventional approach

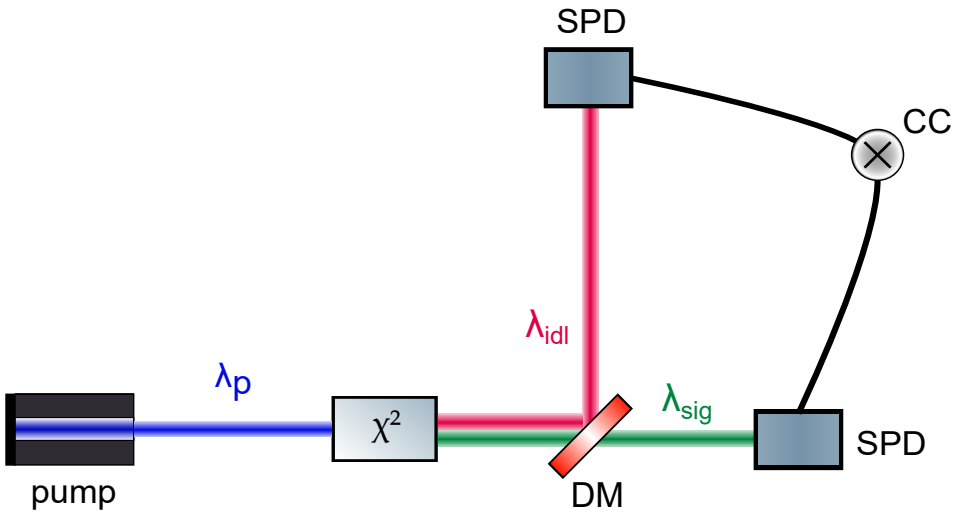


$$N_{\text{tot}}^{\text{ref}} = \eta_{\text{idl}} N_{\text{g}} + N_{\text{noise}}^{\text{ref}}$$

$$N_{\text{tot}}^{\text{sam}} = T\,\eta_{\text{idl}} N_{\text{g}} + N_{\text{noise}}^{\text{sam}}$$

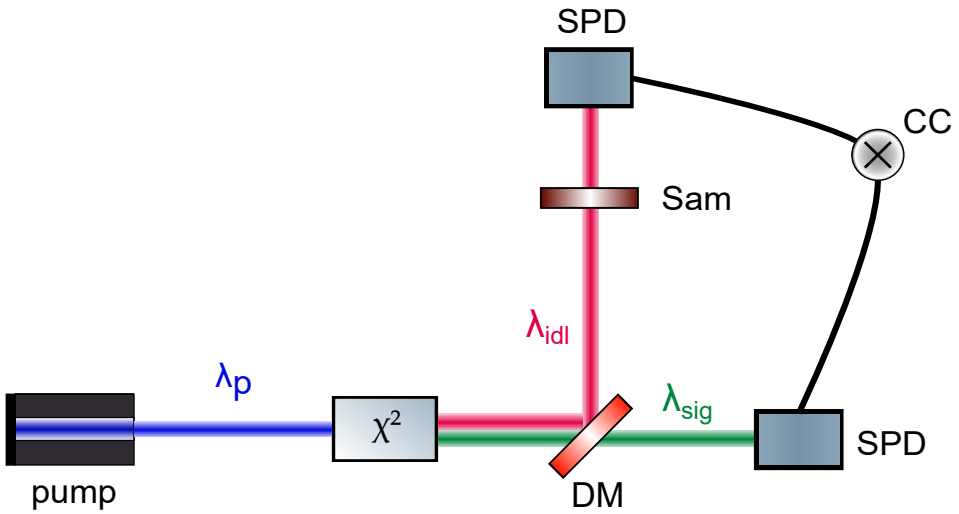
$$\Rightarrow T = \frac{N_{\text{tot}}^{\text{sam}} - N_{\text{noise}}^{\text{sam}}}{N_{\text{tot}}^{\text{ref}} - N_{\text{noise}}^{\text{ref}}}$$

Coincidence approach



$$N^{\text{ref}}_{\text{cc,tot}} = \eta_{\text{idl}}\,\eta_{\text{sig}}\,N_{\text{g}} + N^{\text{ref}}_{\text{ac}}$$

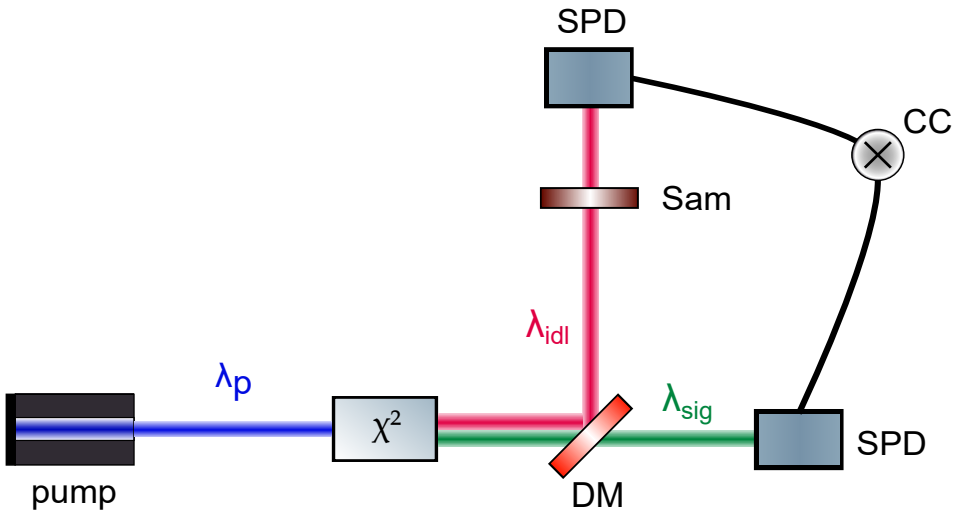
Coincidence approach



$$N^{\text{ref}}_{\text{cc,tot}} = \eta_{\text{idl}}\,\eta_{\text{sig}}\,N_{\text{g}} + N^{\text{ref}}_{\text{ac}}$$

$$N^{\text{sam}}_{\text{cc,tot}} = T\,\eta_{\text{idl}}\,\eta_{\text{sig}}\,N_{\text{g}} + N^{\text{sam}}_{\text{ac}}$$

Coincidence approach



$$N^{\text{ref}}_{\text{cc,tot}} = \eta_{\text{idl}}\,\eta_{\text{sig}}\,N_{\text{g}} + N^{\text{ref}}_{\text{ac}}$$

$$N^{\text{sam}}_{\text{cc,tot}} = T\,\eta_{\text{idl}}\,\eta_{\text{sig}}\,N_{\text{g}} + N^{\text{sam}}_{\text{ac}}$$

$$\Rightarrow T = \frac{N^{\text{sam}}_{\text{tot,cc}} - N^{\text{sam}}_{\text{ac}}}{N^{\text{ref}}_{\text{tot,cc}} - N^{\text{ref}}_{\text{ac}}}$$

Transmittance model

Conventional approach:

$$\mathrm{Var}(T) = (\eta_{\mathrm{idl}}\, N_{\mathrm{g}})^{-2} \left[\mathrm{Var}(N_{\mathrm{tot}}^{\mathrm{sam}}) + \mathrm{Var}(N_{\mathrm{noise}}^{\mathrm{sam}}) + T^2 \left[\mathrm{Var}(N_{\mathrm{tot}}^{\mathrm{ref}}) + \mathrm{Var}(N_{\mathrm{noise}}^{\mathrm{ref}}) \right] \right]$$

Coincidence approach:

$$\mathrm{Var}(T) = (\eta_{\mathrm{sig}}\, \eta_{\mathrm{idl}}\, N_{\mathrm{g}})^{-2} \left[\mathrm{Var}(N_{\mathrm{tot,cc}}^{\mathrm{sam}}) + \mathrm{Var}(N_{\mathrm{ac}}^{\mathrm{sam}}) + T^2 \left[\mathrm{Var}(N_{\mathrm{tot,cc}}^{\mathrm{ref}}) + \mathrm{Var}(N_{\mathrm{ac}}^{\mathrm{ref}}) \right] \right]$$

Transmittance model

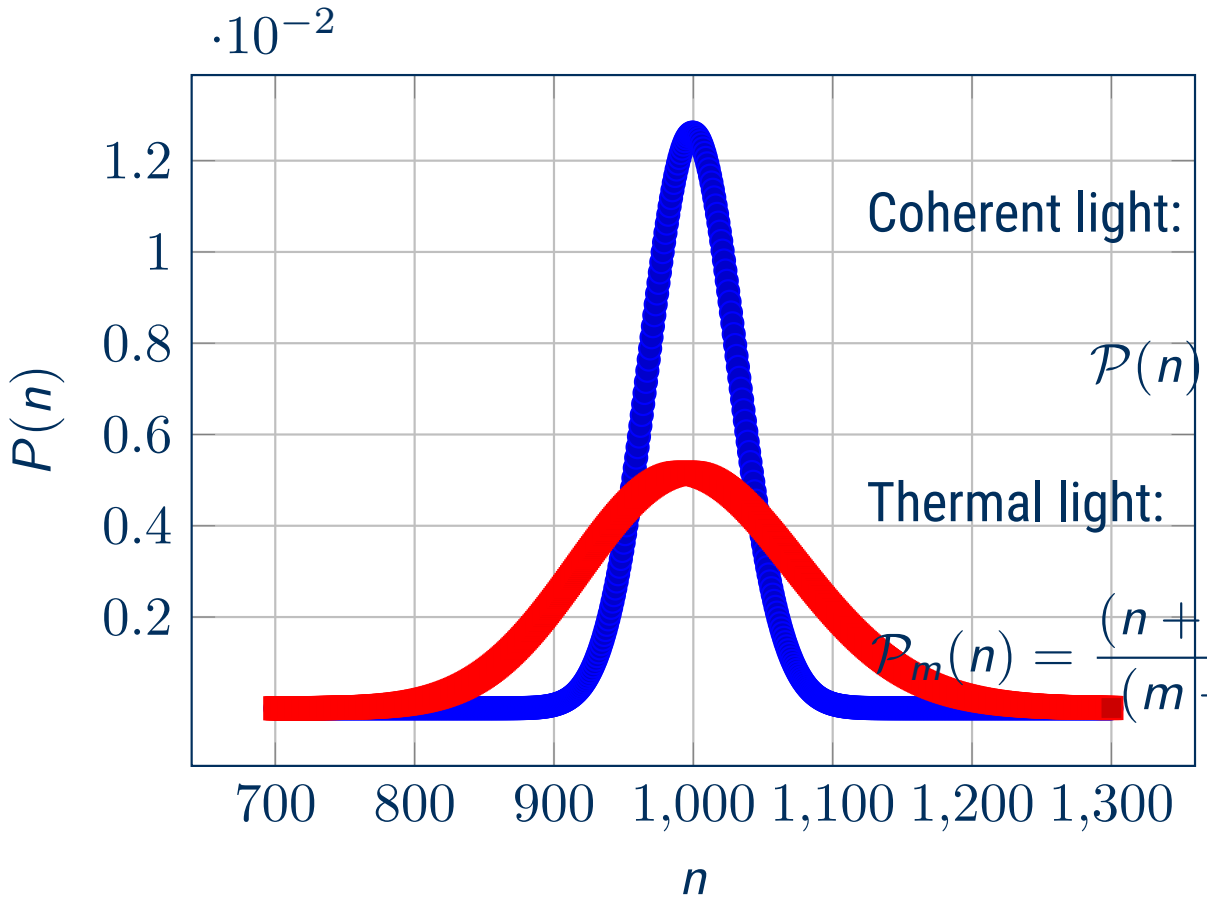
Conventional approach:

$$\mathrm{Var}(T) = (\eta_{\mathrm{idl}}\, N_{\mathrm{g}})^{-2} \left[\mathrm{Var}(N_{\mathrm{tot}}^{\mathrm{sam}}) + \mathrm{Var}(N_{\mathrm{noise}}^{\mathrm{sam}}) + T^2 \left[\mathrm{Var}(N_{\mathrm{tot}}^{\mathrm{ref}}) + \mathrm{Var}(N_{\mathrm{noise}}^{\mathrm{ref}}) \right] \right]$$

Coincidence approach:

$$\mathrm{Var}(T) = (\eta_{\mathrm{sig}}\, \eta_{\mathrm{idl}}\, N_{\mathrm{g}})^{-2} \left[\mathrm{Var}(N_{\mathrm{tot,cc}}^{\mathrm{sam}}) + \mathrm{Var}(N_{\mathrm{ac}}^{\mathrm{sam}}) + T^2 \left[\mathrm{Var}(N_{\mathrm{tot,cc}}^{\mathrm{ref}}) + \mathrm{Var}(N_{\mathrm{ac}}^{\mathrm{ref}}) \right] \right]$$

Photon statistics



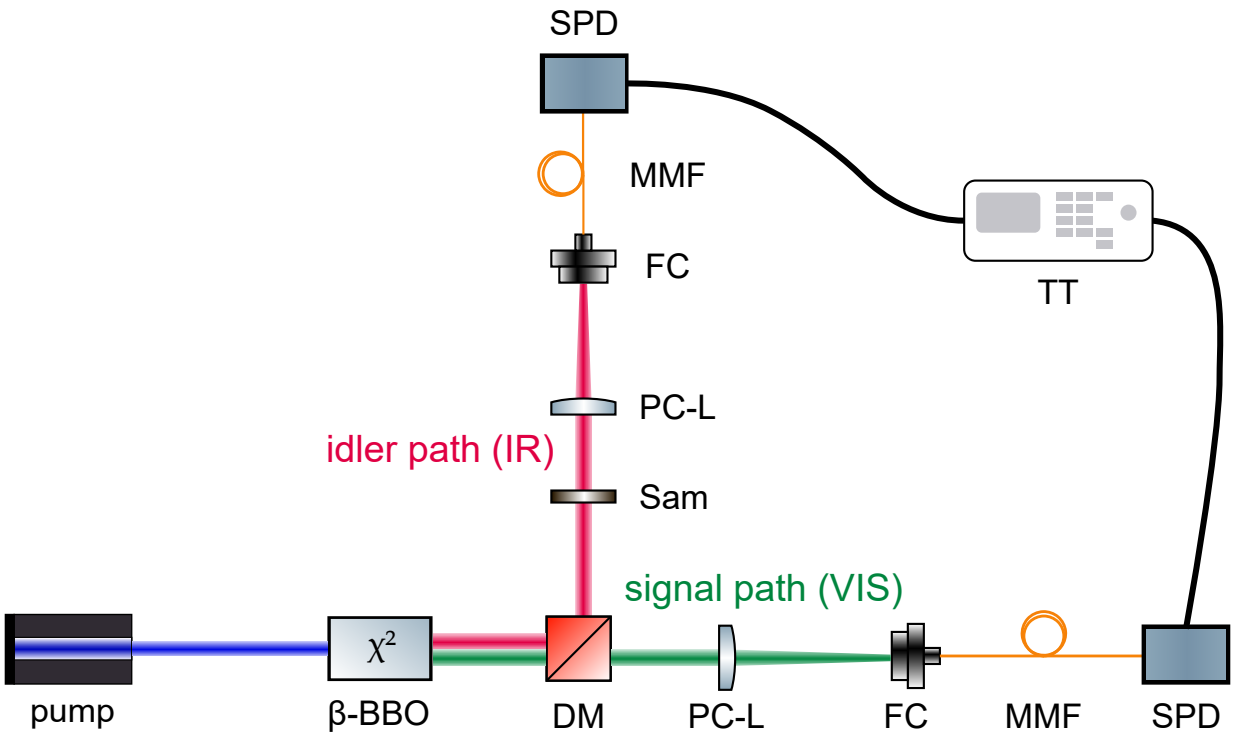
Coherent light:

$$\mathcal{P}(n) = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}$$

Thermal light:

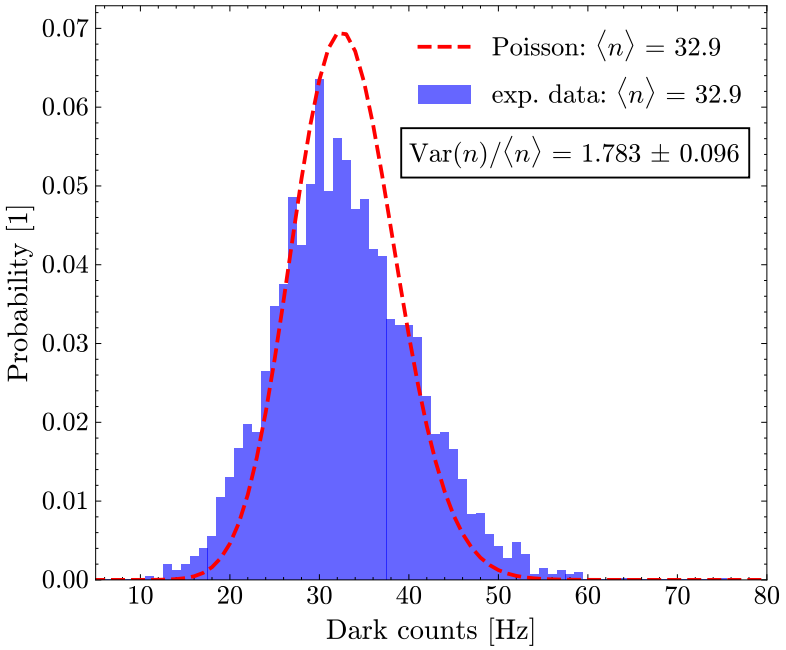
$$\mathcal{P}_m(n) = \frac{(n+m-1)!}{(m-1)!\,n!} \frac{m^m \langle n \rangle^n}{(m+\langle n \rangle)^{n+m}}$$

Experimental setup

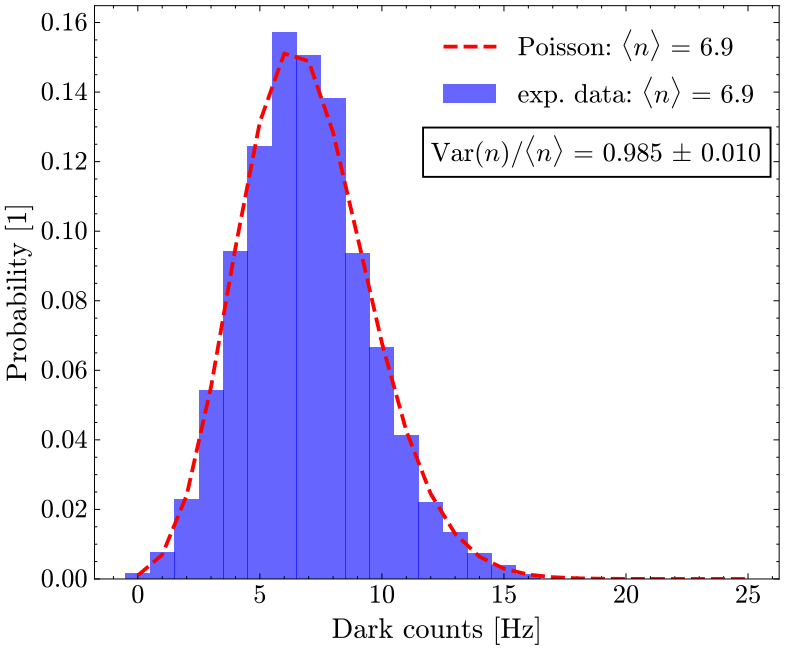


Dark counts

Idler arm

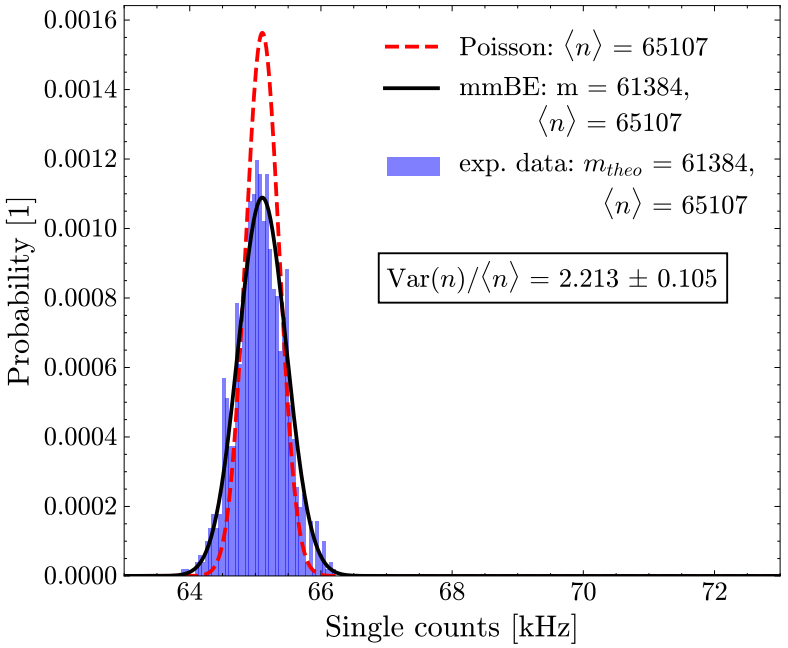


Signal arm

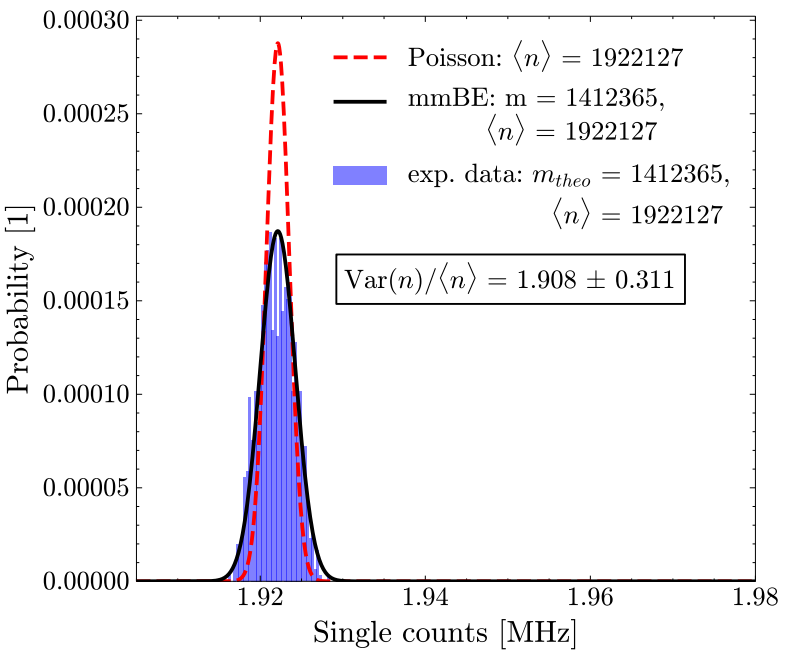


Single counts

Idler arm



Signal arm



Slide title in Palatino Linotype Font

block environment (lower-case b)

itemize:

- First Level
  - Second Level
    - Third Level has no item mark

Block environment (upper-case B)

enumerate:

1. First Level
   1.1 Second Level
   1.1.1 Third Level

Font types

Normal	Lorem ipsum dolor sit amet, consectetur adipiscing elit.
Bold	**Lorem ipsum dolor sit amet, consectetur adipiscing elit.**
Italic	*Lorem ipsum dolor sit amet, consectetur adipiscing elit.*
BoldItalic	***Lorem ipsum dolor sit amet, consectetur adipiscing elit.***

$$e^{i\pi} + 1 = 0 \tag{1}$$

Equations like eq. (1) use the beamer default font computer modern.

Summary and Outlook

Git repository

public accessible:

`https://git.tpi.uni-jena.de/mstnhsr/latexbeamer_corporatedesign`

Feedback

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