Parameter estimation with correlated photon pairs

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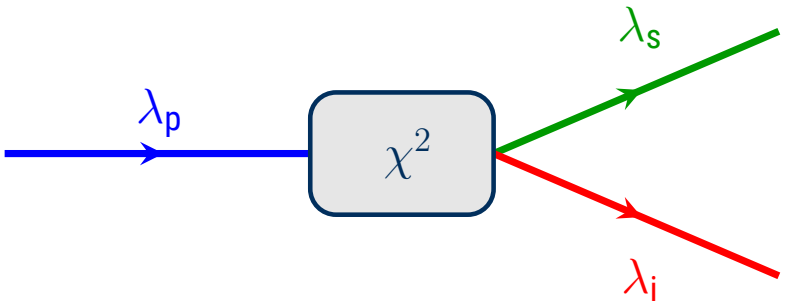
Motivation

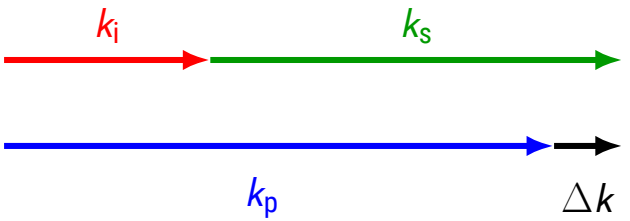
SPDC



Energy conservation

$$\omega_p = \omega_s + \omega_i$$

Momentum conservation



$$\vec{k_p} = \vec{k_s} + \vec{k_i} - \Delta\vec{k}$$

Transmittance model

Conventional approach:

$$N_{\text{tot}}^{\text{ref}} = \eta_{\text{idl}}\, N_{\text{g}} + N_{\text{noise}}^{\text{ref}}$$

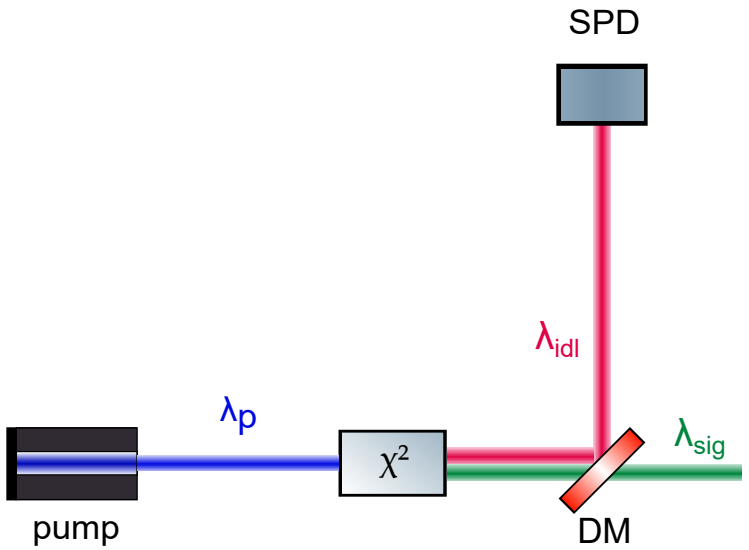
$$N_{\text{tot}}^{\text{sam}} = T\, \eta_{\text{idl}}\, N_{\text{g}} + N_{\text{noise}}^{\text{sam}}$$

Coincidence approach:

$$N_{\text{cc}}^{\text{pure,sam}} = T\, \eta_{\text{idl}}\, \eta_{\text{sig}}\, N_{\text{g}},$$

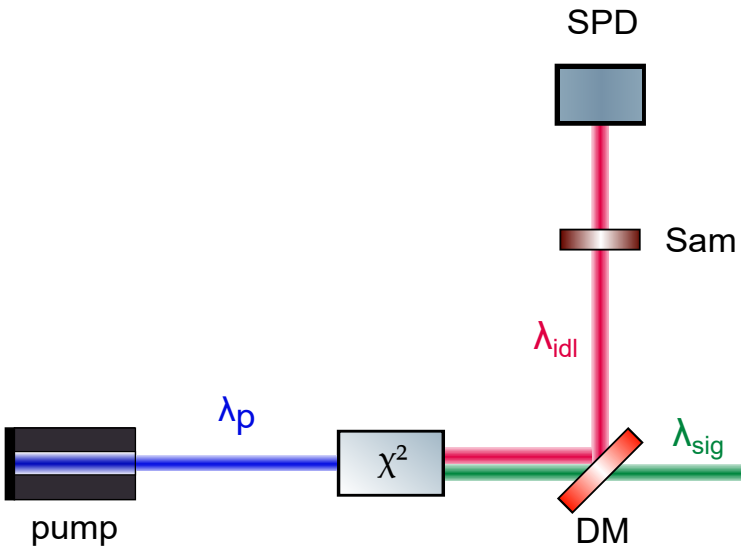
$$N_{\text{cc}}^{\text{pure,ref}} = \eta_{\text{idl}}\, \eta_{\text{sig}}\, N_{\text{g}}$$

Conventional approach



$$N_{\text{tot}}^{\text{ref}} = \eta_{\text{idl}} N_{\text{g}} + N_{\text{noise}}^{\text{ref}}$$

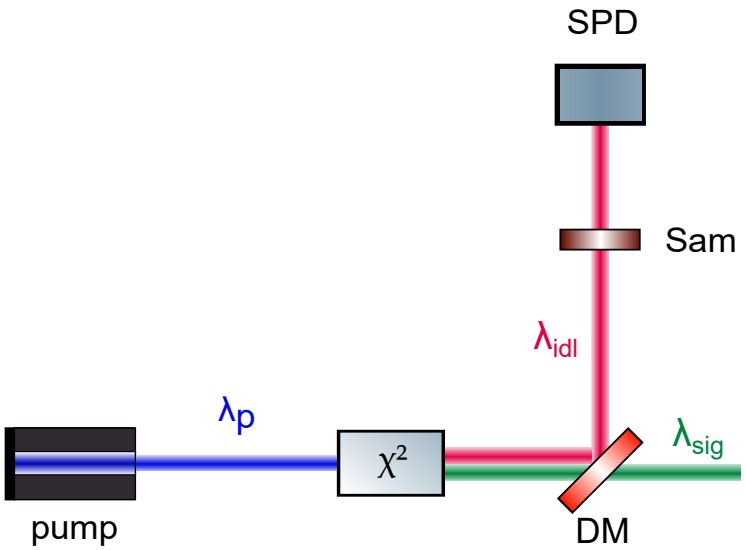
Conventional approach



$$N_{\text{tot}}^{\text{ref}} = \eta_{\text{idl}} N_{\text{g}} + N_{\text{noise}}^{\text{ref}}$$

$$N_{\text{tot}}^{\text{sam}} = T\,\eta_{\text{idl}} N_{\text{g}} + N_{\text{noise}}^{\text{sam}}$$

Conventional approach

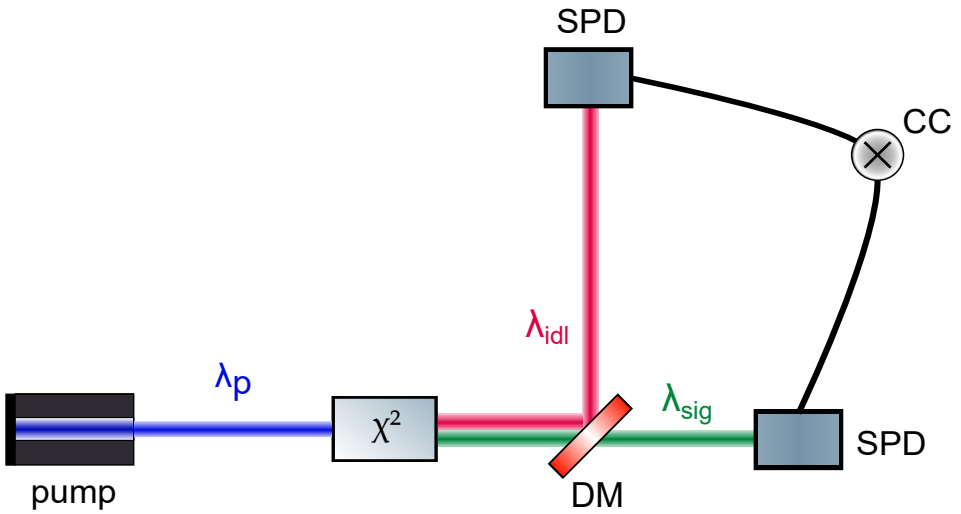


$$N_{\text{tot}}^{\text{ref}} = \eta_{\text{idl}} N_{\text{g}} + N_{\text{noise}}^{\text{ref}}$$

$$N_{\text{tot}}^{\text{sam}} = T\,\eta_{\text{idl}} N_{\text{g}} + N_{\text{noise}}^{\text{sam}}$$

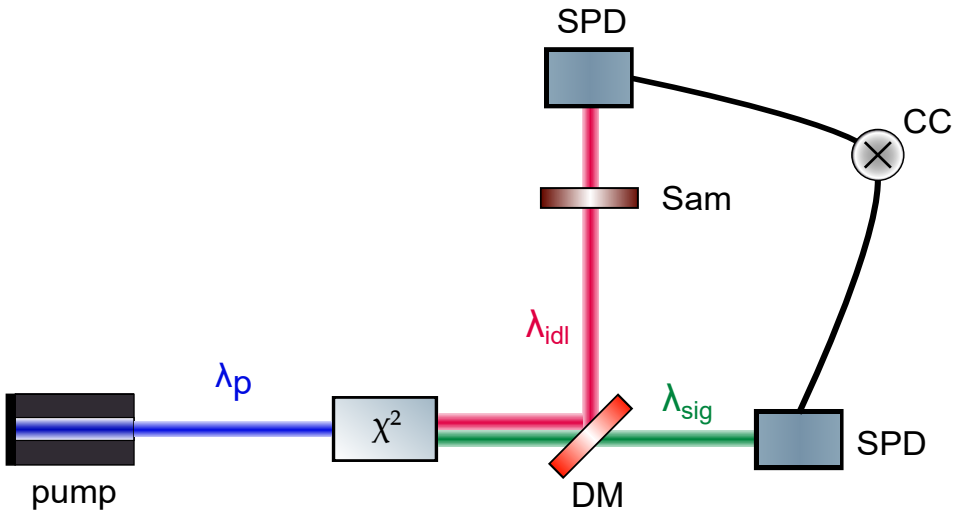
$$\Rightarrow T = \frac{N_{\text{tot}}^{\text{sam}} - N_{\text{noise}}^{\text{sam}}}{N_{\text{tot}}^{\text{ref}} - N_{\text{noise}}^{\text{ref}}}$$

Coincidence approach



$$N^{\mathrm{ref}}_{\mathrm{cc,tot}} = \eta_{\mathrm{idl}}\,\eta_{\mathrm{sig}}\,N_{\mathrm{g}} + N^{\mathrm{ref}}_{\mathrm{ac}}$$

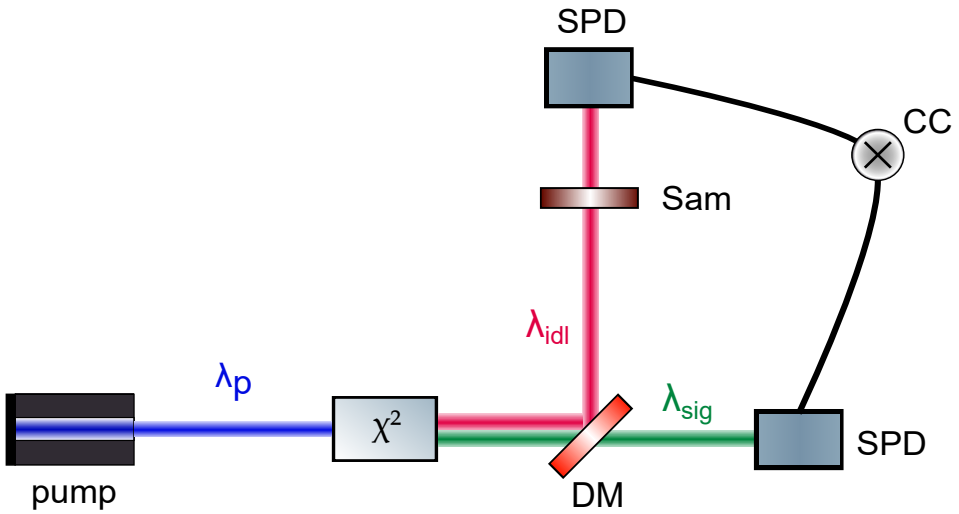
Coincidence approach



$$N^{\text{ref}}_{\text{cc,tot}} = \eta_{\text{idl}}\,\eta_{\text{sig}}\,N_{\text{g}} + N^{\text{ref}}_{\text{ac}}$$

$$N^{\text{sam}}_{\text{cc,tot}} = T\,\eta_{\text{idl}}\,\eta_{\text{sig}}\,N_{\text{g}} + N^{\text{sam}}_{\text{ac}}$$

Coincidence approach



$$N^{\text{ref}}_{\text{cc,tot}} = \eta_{\text{idl}}\,\eta_{\text{sig}}\,N_{\text{g}} + N^{\text{ref}}_{\text{ac}}$$

$$N^{\text{sam}}_{\text{cc,tot}} = T\,\eta_{\text{idl}}\,\eta_{\text{sig}}\,N_{\text{g}} + N^{\text{sam}}_{\text{ac}}$$

$$\Rightarrow T = \frac{N^{\text{sam}}_{\text{tot,cc}} - N^{\text{sam}}_{\text{ac}}}{N^{\text{ref}}_{\text{tot,cc}} - N^{\text{ref}}_{\text{ac}}}$$

Transmittance model

Conventional approach:

$$\mathrm{Var}(T) = (\eta_{\mathrm{idl}}\, N_{\mathrm{g}})^{-2} \left[\mathrm{Var}(N_{\mathrm{tot}}^{\mathrm{sam}}) + \mathrm{Var}\left(N_{\mathrm{noise}}^{\mathrm{sam}}\right) + T^2 \left[\mathrm{Var}\left(N_{\mathrm{tot}}^{\mathrm{ref}}\right) + \mathrm{Var}\left(N_{\mathrm{noise}}^{\mathrm{ref}}\right) \right] \right]$$

Coincidence approach:

$$\mathrm{Var}(T) = (\eta_{\mathrm{sig}}\, \eta_{\mathrm{idl}}\, N_{\mathrm{g}})^{-2} \left[\mathrm{Var}\left(N_{\mathrm{tot,cc}}^{\mathrm{sam}}\right) + \mathrm{Var}(N_{\mathrm{ac}}^{\mathrm{sam}}) + T^2 \left[\mathrm{Var}\left(N_{\mathrm{tot,cc}}^{\mathrm{ref}}\right) + \mathrm{Var}\left(N_{\mathrm{ac}}^{\mathrm{ref}}\right) \right] \right]$$

Transmittance model

Conventional approach:

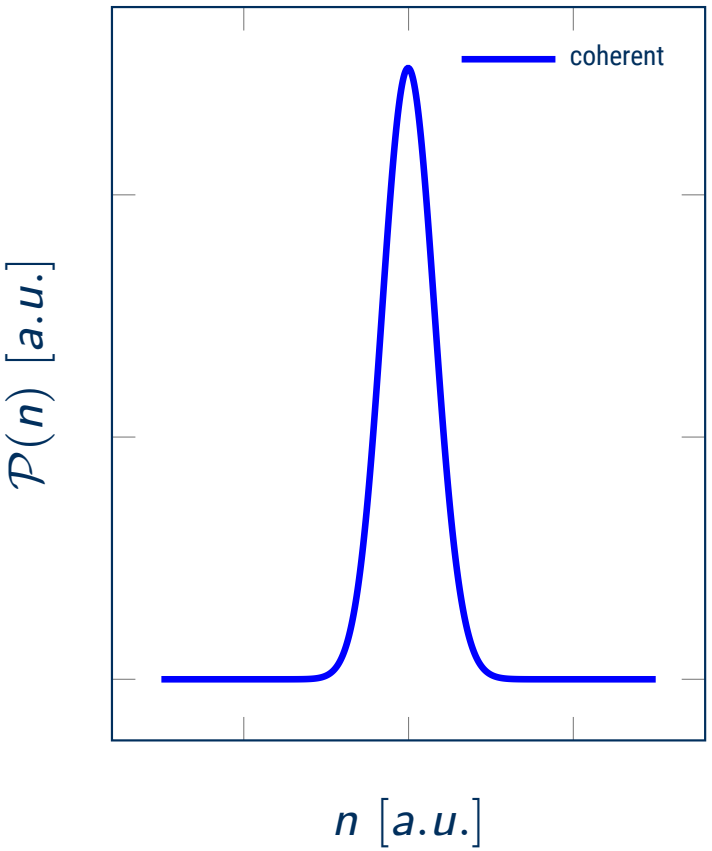
$$\mathrm{Var}(T) = (\eta_{\mathrm{idl}}\, N_{\mathrm{g}})^{-2} \left[\mathrm{Var}(N_{\mathrm{tot}}^{\mathrm{sam}}) + \mathrm{Var}(N_{\mathrm{noise}}^{\mathrm{sam}}) + T^2 \left[\mathrm{Var}(N_{\mathrm{tot}}^{\mathrm{ref}}) + \mathrm{Var}(N_{\mathrm{noise}}^{\mathrm{ref}}) \right] \right]$$

Coincidence approach:

$$\mathrm{Var}(T) = (\eta_{\mathrm{sig}}\, \eta_{\mathrm{idl}}\, N_{\mathrm{g}})^{-2} \left[\mathrm{Var}(N_{\mathrm{tot,cc}}^{\mathrm{sam}}) + \mathrm{Var}(N_{\mathrm{ac}}^{\mathrm{sam}}) + T^2 \left[\mathrm{Var}(N_{\mathrm{tot,cc}}^{\mathrm{ref}}) + \mathrm{Var}(N_{\mathrm{ac}}^{\mathrm{ref}}) \right] \right]$$

Photon statistics

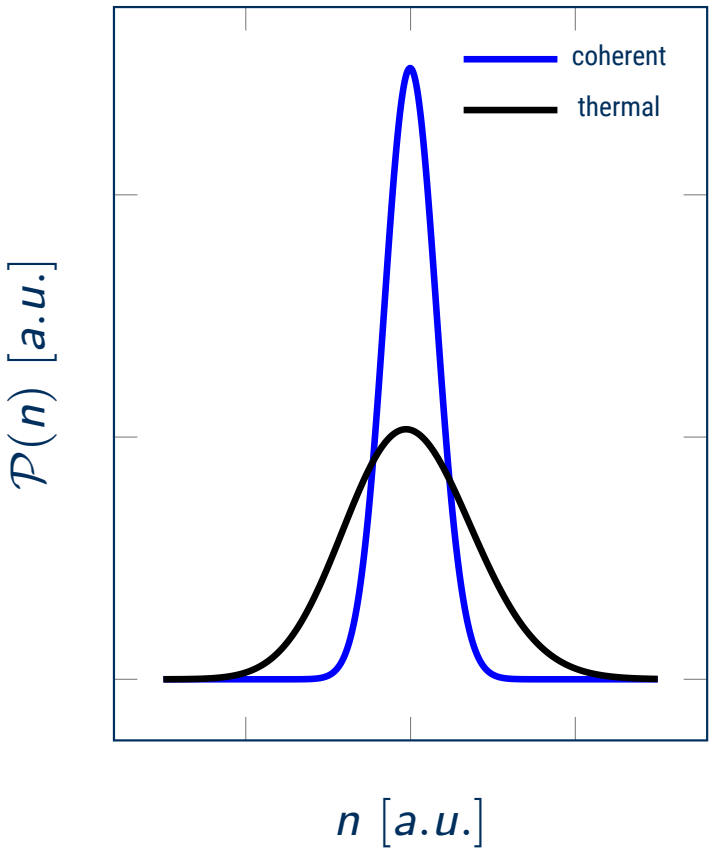
Photon statistics



Coherent light:

$$\mathcal{P}(n) = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}$$
$$\mathrm{Var}(n) = \langle n \rangle$$

Photon statistics



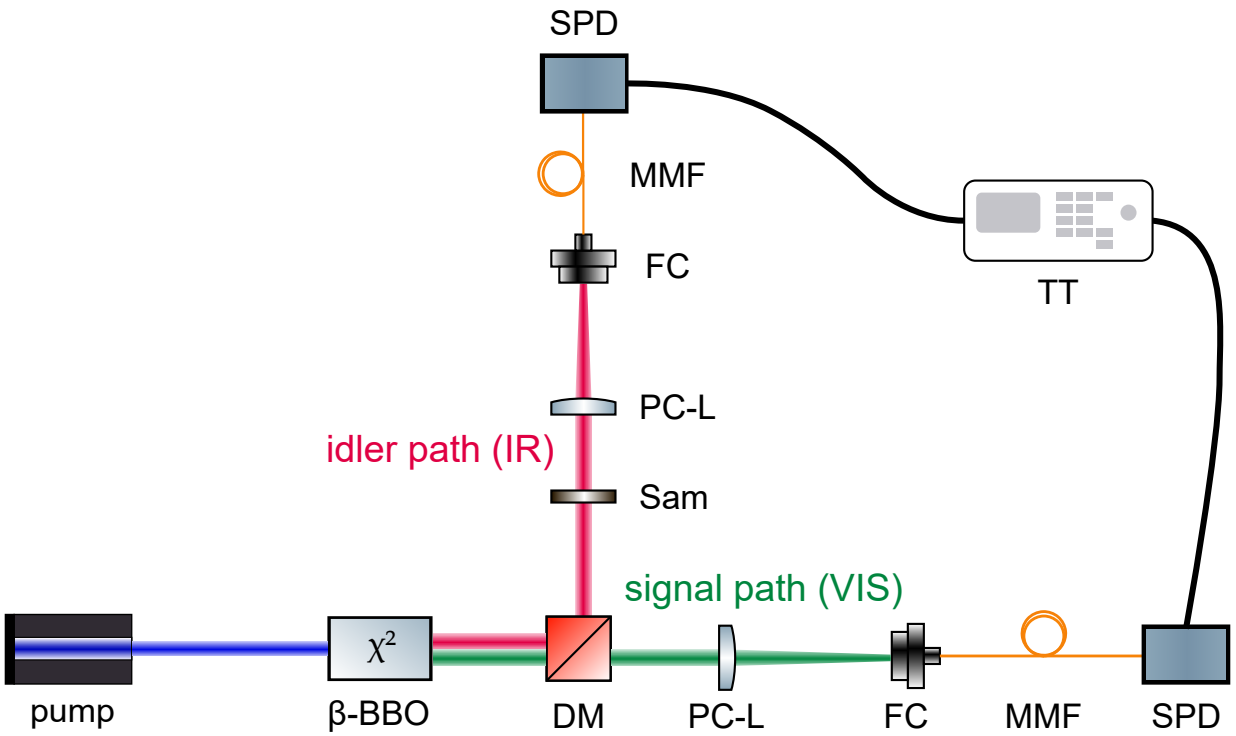
Coherent light:

$$\mathcal{P}(n) = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}$$
$$\mathrm{Var}(n) = \langle n \rangle$$

Thermal light:

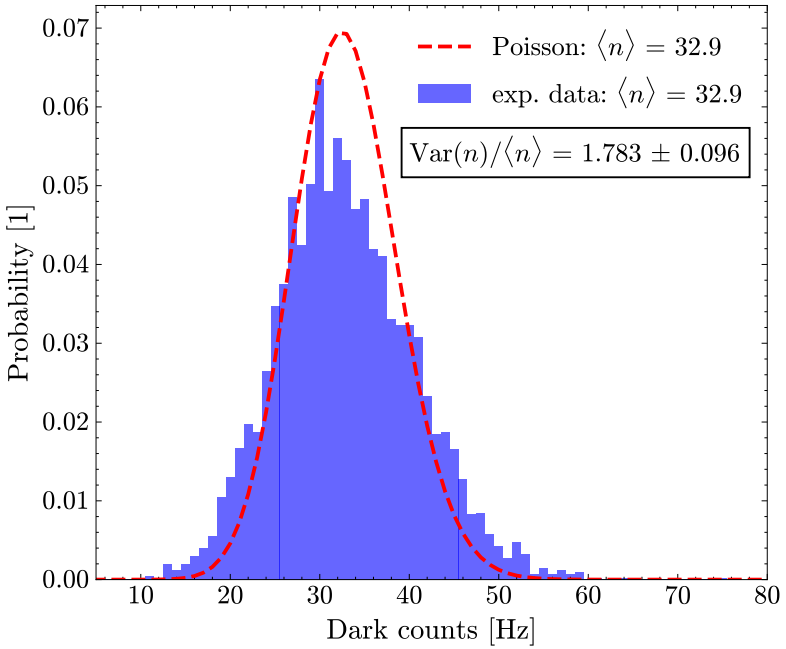
$$\mathcal{P}_m(n) = \frac{(n+m-1)!}{(m-1)!\,n!} \frac{m^m \langle n \rangle^n}{(m+\langle n \rangle)^{n+m}}$$
$$\mathrm{Var}(n) = \langle n \rangle \left(1 + \frac{\langle n \rangle}{m}\right)$$

Experimental setup

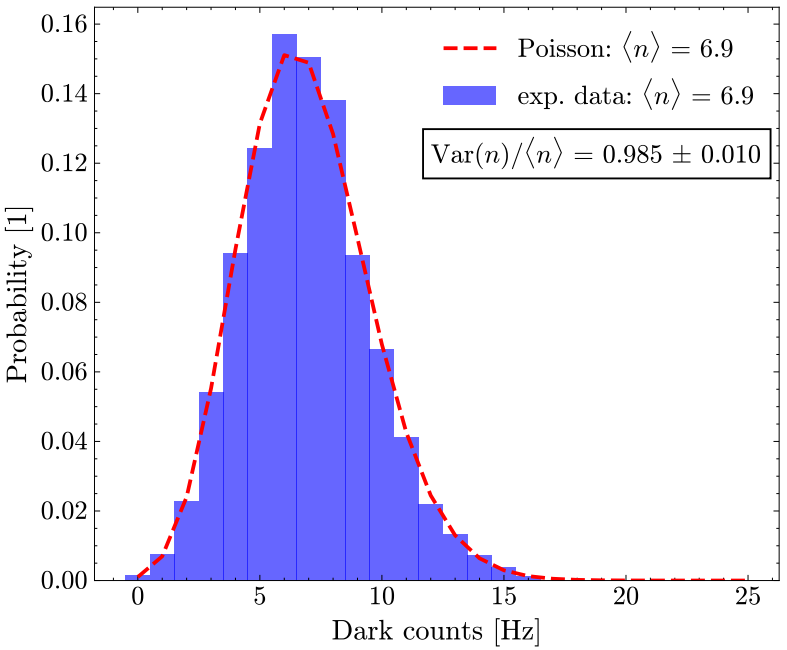


Dark counts

Idler arm



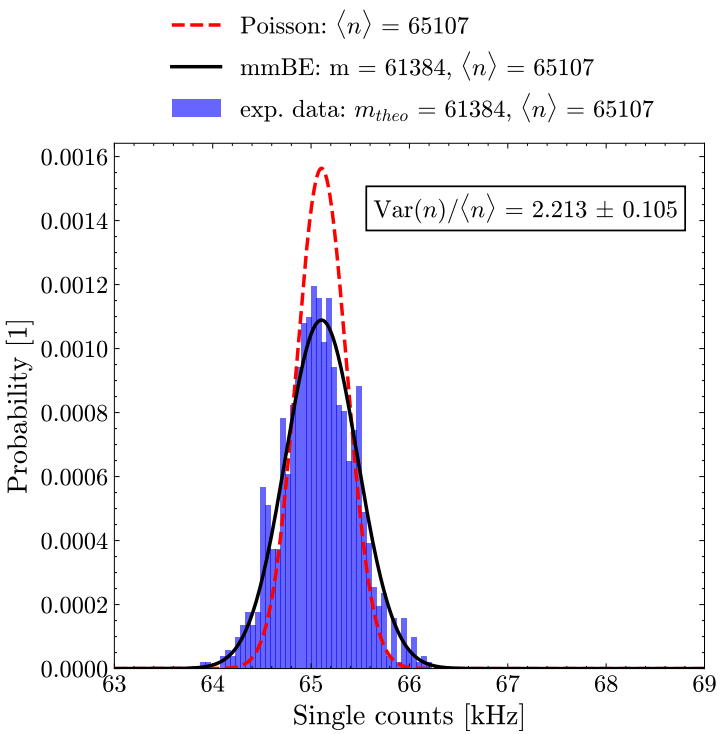
Signal arm



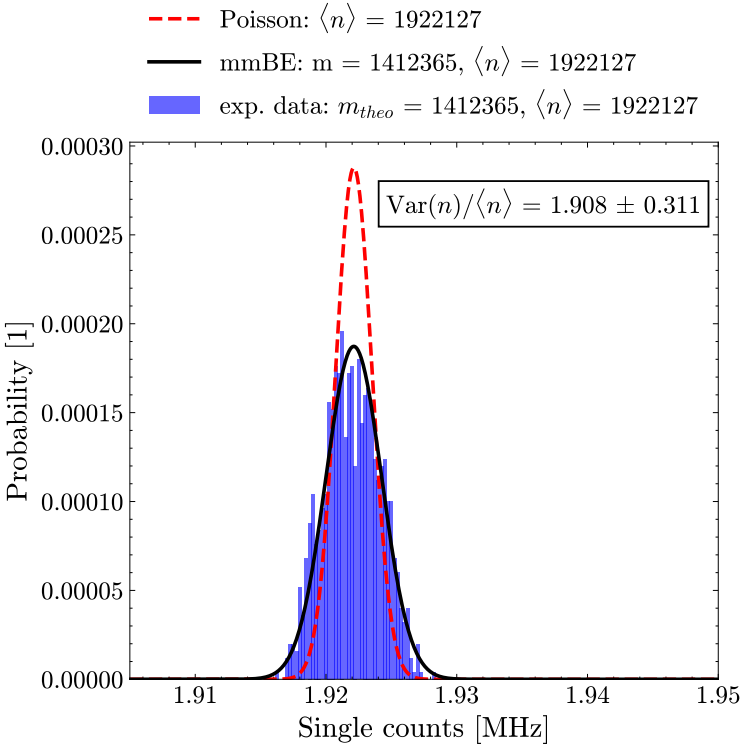
$$\mathrm{Var}(N_{\mathrm{noise}}) = 1.8 \cdot \langle N_{\mathrm{noise}} \rangle$$

Single counts

Idler arm

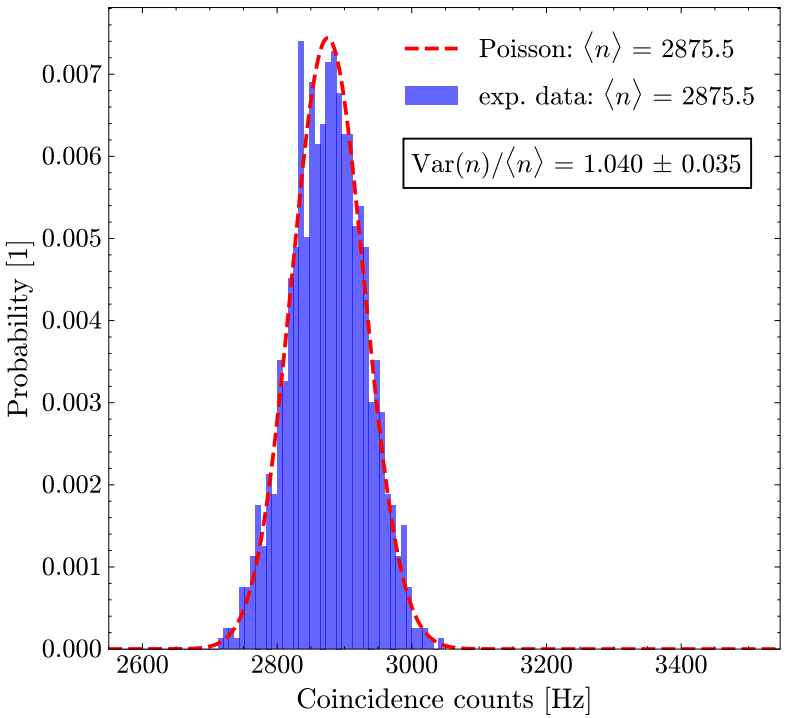


Signal arm

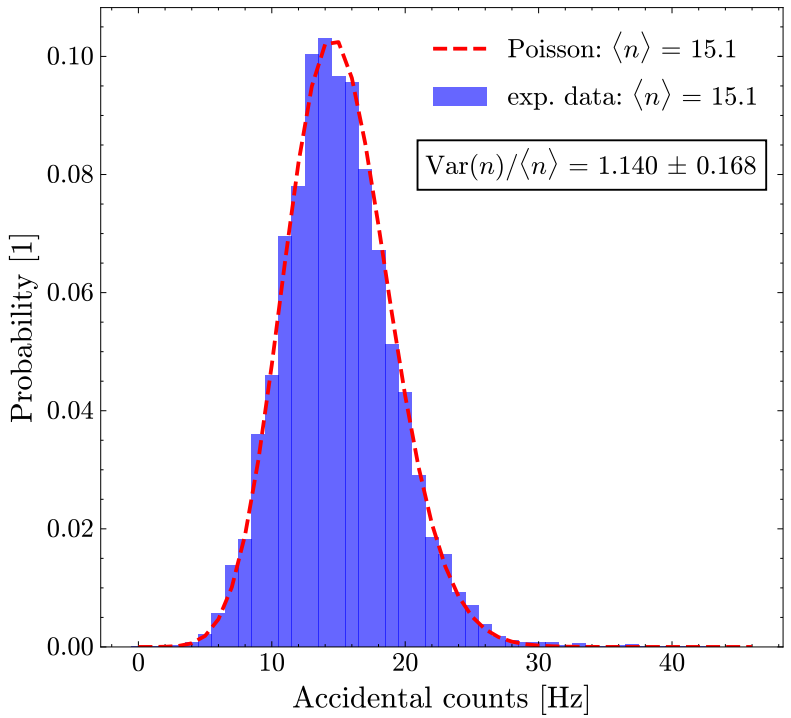


Coincidence counts

Coincidences



Accidentals



$$\mathrm{Var}(N_{\mathrm{cc}}) = \langle N_{\mathrm{cc}} \rangle$$

$$\mathrm{Var}(N_{\mathrm{ac}}) = \langle N_{\mathrm{ac}} \rangle$$

Simulation

Conventional approach:

$$\mathrm{Var}(T) = (\eta_{\mathrm{idl}}\, N_{\mathrm{g}})^{-2} \left[\mathrm{Var}(N_{\mathrm{tot}}^{\mathrm{sam}}) + \mathrm{Var}\left(N_{\mathrm{noise}}^{\mathrm{sam}}\right) + T^2 \left[\mathrm{Var}\left(N_{\mathrm{tot}}^{\mathrm{ref}}\right) + \mathrm{Var}\left(N_{\mathrm{noise}}^{\mathrm{ref}}\right) \right] \right]$$

Simulation

Conventional approach:

$$\mathrm{Var}(T) = (\eta_{\mathrm{idl}}\, N_{\mathrm{g}})^{-2} \left[2.2 \cdot \langle N_{\mathrm{tot}}^{\mathrm{sam}} \rangle + 1.8 \cdot \langle N_{\mathrm{noise}}^{\mathrm{sam}} \rangle + T^2 \left[2.2 \cdot \langle N_{\mathrm{tot}}^{\mathrm{ref}} \rangle + 1.8 \cdot \langle N_{\mathrm{noise}}^{\mathrm{ref}} \rangle \right] \right]$$

Simulation

Conventional approach:

$$\mathrm{Var}(T) = (\eta_{\mathrm{idl}}\, N_{\mathrm{g}})^{-2} \left[2.2 \cdot \langle N_{\mathrm{tot}}^{\mathrm{sam}} \rangle + 1.8 \cdot \langle N_{\mathrm{noise}}^{\mathrm{sam}} \rangle + T^2 \left[2.2 \cdot \langle N_{\mathrm{tot}}^{\mathrm{ref}} \rangle + 1.8 \cdot \langle N_{\mathrm{noise}}^{\mathrm{ref}} \rangle \right] \right]$$

Coincidence approach:

$$\mathrm{Var}(T) = (\eta_{\mathrm{sig}}\, \eta_{\mathrm{idl}}\, N_{\mathrm{g}})^{-2} \left[\mathrm{Var}(N_{\mathrm{tot,cc}}^{\mathrm{sam}}) + \mathrm{Var}(N_{\mathrm{ac}}^{\mathrm{sam}}) + T^2 \left[\mathrm{Var}(N_{\mathrm{tot,cc}}^{\mathrm{ref}}) + \mathrm{Var}(N_{\mathrm{ac}}^{\mathrm{ref}}) \right] \right]$$

Simulation

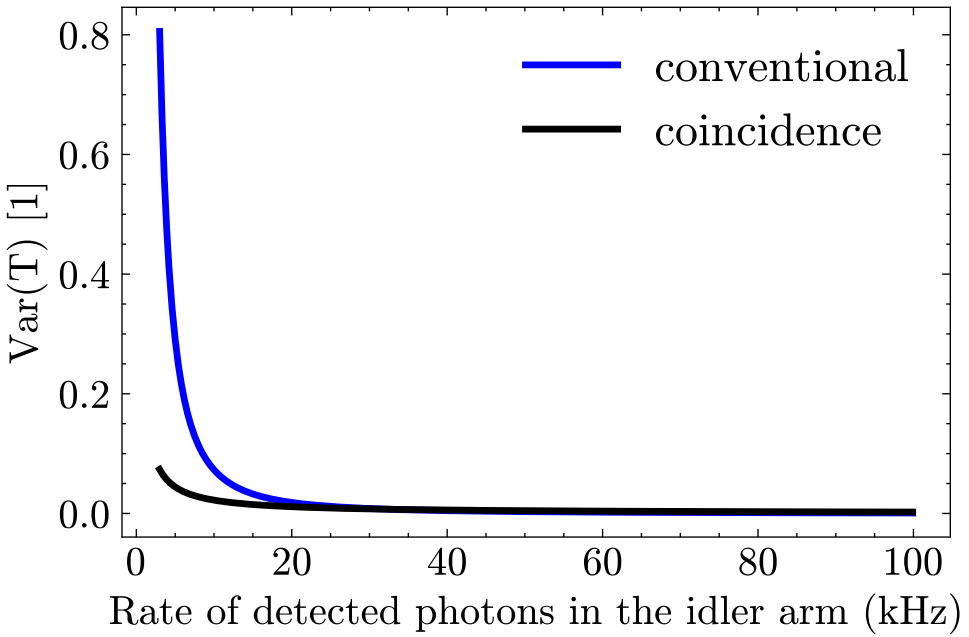
Conventional approach:

$$\mathrm{Var}(T) = (\eta_{\mathrm{idl}}\, N_{\mathrm{g}})^{-2} \left[2.2 \cdot \langle N_{\mathrm{tot}}^{\mathrm{sam}} \rangle + 1.8 \cdot \langle N_{\mathrm{noise}}^{\mathrm{sam}} \rangle + T^2 \left[2.2 \cdot \langle N_{\mathrm{tot}}^{\mathrm{ref}} \rangle + 1.8 \cdot \langle N_{\mathrm{noise}}^{\mathrm{ref}} \rangle \right] \right]$$

Coincidence approach:

$$\mathrm{Var}(T) = \left(\eta_{\mathrm{sig}}\, \eta_{\mathrm{idl}}\, N_{\mathrm{g}}\right)^{-2} \left[\langle N_{\mathrm{tot,cc}}^{\mathrm{sam}} \rangle + \langle N_{\mathrm{ac}}^{\mathrm{sam}} \rangle + T^2 \left[\langle N_{\mathrm{tot,cc}}^{\mathrm{ref}} \rangle + \langle N_{\mathrm{ac}}^{\mathrm{ref}} \rangle \right] \right]$$

Simulation



Parameter	Value
η_{idl} (%)	0.09
η_{sig} (%)	2.6
R_{idl} (kHz)	3 - 100
$R_{noise,idl}$ (kHz)	1000
$R_{noise,sig}$ (Hz)	7
T (1)	0.9

Summary and Outlook

Git repository

public accessible:

`https://git.tpi.uni-jena.de/mstnhsr/latexbeamer_corporatedesign`

Feedback

marc.steinhauser@uni-jena.de

Slide title in Palatino Linotype Font

block environment (lower-case b)

itemize:

- First Level
  - Second Level
    - Third Level has no item mark

Block environment (upper-case B)

enumerate:

1. First Level
   1.1 Second Level
   1.1.1 Third Level