Parameter estimation of correlated photon pairs

Internship report

submitted by

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Glossary

Abbreviation	Description	Page
BBO	beta-barium borate	2
KTP	potassium titanyl phosphate	2
LN	lithium niobate	2
SNSPD	Superconducting nanowire single-photon detector	1
SPDC	Spontaneous parametric down-conversion	1, 2

1 Introduction

- why quantum light
- goal of parameter estimation
- Approach of adding external noise

In recent years, several fields got influenced by quantum technologies, such as communcation, computing, sensing or imaging. All of them have in common that non-classical properties of particles are utilized to improve state of the art technology beyond existing limits.
In imaging and sensing specifically, the quantum nature of light is exploited and therefore not only a light beam but single photon events are used.
Imaging is a central tool for exploring and understanding the physical world, from revealing microscopic biological structures to mapping distant astronomical objects. Despite the remarkable capabilities of modern classical imaging systems, they remain fundamentally constrained by limits such as diffraction and shot noise, both of which arise from the wave–particle duality of light. These constraints impose trade-offs between spatial resolution, sensitivity, and illumination intensity, often limiting the ability to study low-signal or delicate samples. Quantum imaging seeks to surpass these boundaries by exploiting uniquely quantum properties of light—including entanglement, squeezing, and photon-number correlations, to achieve capabilities unattainable with classical techniques [6, 11].

Among the various sources of quantum light, Spontaneous parametric down-conversion (SPDC) has emerged as a very powerful one for generating photon pairs exhibiting strong correlations in multiple degrees of freedom, such as position, momentum, polarization, and time [11]. These correlations form a versatile resource for imaging, enabling protocols that reduce noise below the shot-noise limit, improve spatial resolution beyond the Rayleigh criterion detector Superconducting nanowire single-photon detector (SNSPD), enhance phase sensitivity, and even form images from photons that never directly interact with the object [6, 11].

Temporal correlations between SPDC photons offer particular advantages in imaging. When one photon of a pair interacts with a sample, its detection time can be used to herald the arrival of its twin, enabling time-gated acquisition that suppresses background noise and improves signal fidelity [11]. Heralded imaging schemes can reduce the effective detection window to a few nanoseconds, substantially lowering dark counts and ambient light interference. Furthermore, temporal coincidence measurements isolate true photon-pair events from uncorrelated noise, which is particularly valuable in low-light-level imaging and remote sensing. By exploiting such correlations, quantum imaging can operate with fewer photons, thereby minimizing sample exposure and reducing photo-damage, which is a critical benefit for fragile biological specimens or light-sensitive materials.

2 Theory

2.1 Spontaneous parametric down-conversion

To exploit the advantages of quantum imaging and sensing, one needs to create correlated biphoton states of light. The most efficient technology to create such quantum states are Spontaneous parametric down-conversion (SPDC) sources.
The underlying process is as follows: an incident pump photon with frequency ω_{p} causes a nonlinear material response resulting in the spontaneous emission of a photon pair with lower frequencies ω_{s} and ω_{i}. The subscripts i and s represent the signal and idler photons, as they are usually referred to.
Most experiments use crystals, such as potassium titanyl phosphate (KTP) and beta-barium borate (BBO), or lithium niobate (LN) based optical waveguides, because they exhibit second-order nonlinearity [7, 10, 13]. To achieve efficient SPDC processes, the energy and momentum must be conserved. This means that the photon pair must interfere constructively and fulfill the phase-matching conditions of the wave vector k [8] :

$$\begin{aligned} \omega_{\mathrm{p}} &= \omega_{\mathrm{s}} + \omega_{\mathrm{i}} \\ \vec{k_{\mathrm{p}}} &= k_{\mathrm{s}} + k_{\mathrm{i}} - \Delta k \end{aligned} \tag{2.1}$$

where the indices p, s and i refer to the pump, signal and idler photon. Δk represents the phase mismatch caused by dispersion, which results in zero produced photon pairs. There are two approaches to compensate for the mismatch.
One is called quasi phase matching (QPM) and exploits that in periodically poled crystals, e.g. KTP or LN, the nonlinear response also changes periodically, resulting in a phase mismatch of $\Delta k = 0$. It allows for a process called type-0 SPDC to happen, which means that all three photons (pump, signal, idler) have the same polarization direction.
Another way to compensate the mismatch is called birefringent phase matching (BPM) and uses anisotropic materials, such as BBO, as the refractive index changes with the polarization of the incident photon. The effect of using BPM is that the extraordinary photon is always polarized perpendicular to the pump. Therefore, no type-0 SPDC can be achieved with this approach. The two other possible cases are called type-I SPDC, which means that the signal and idler photons share the same polarization and are polarized perpendicular to the pump. Type-II SPDC means that the signal and idler photons are polarized perpendicular to each other [3].

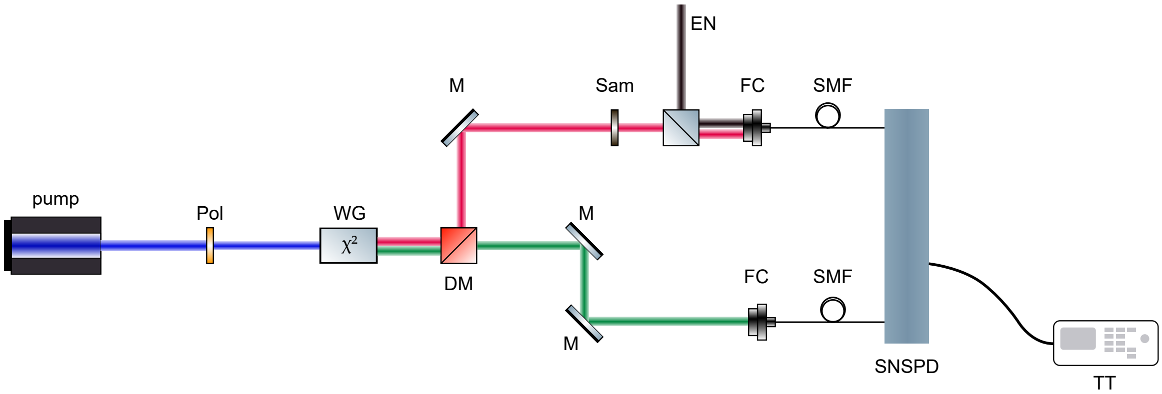

Figure 1: Caption

2.2 Coherent light

2.3 Thermal light

- quantum description of light
- squeezed light
- correlated photons
- SPDC
- SNSPD

3 Experimental Setup

- experimental setup
- measurement procedure

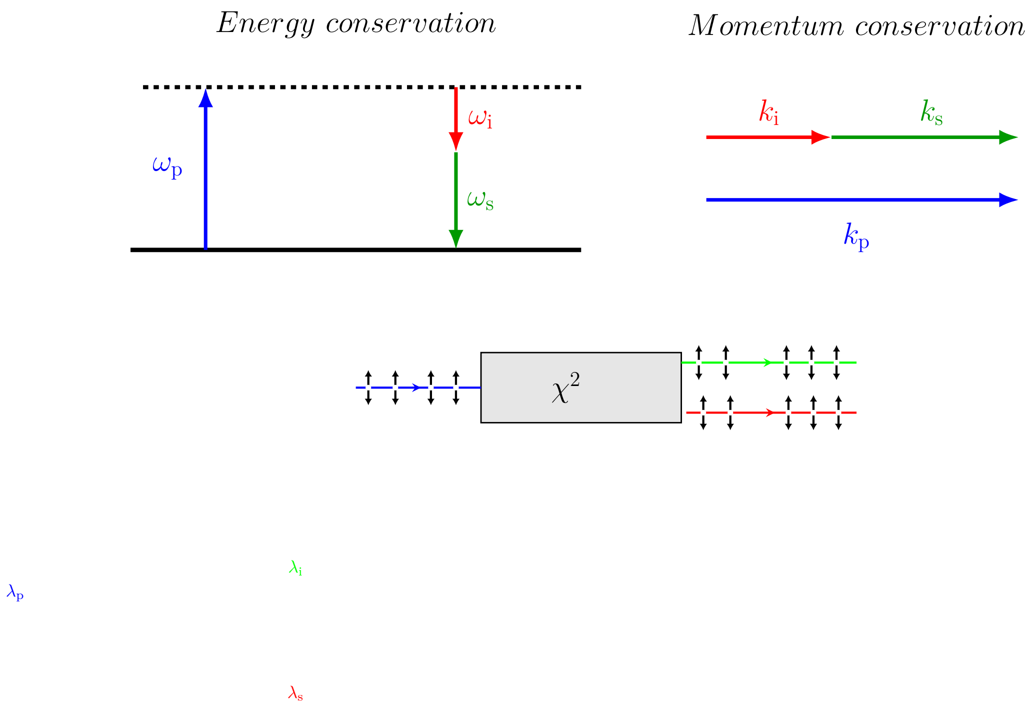

Figure 2: Conservation processes of collinear Type-0 SPDC

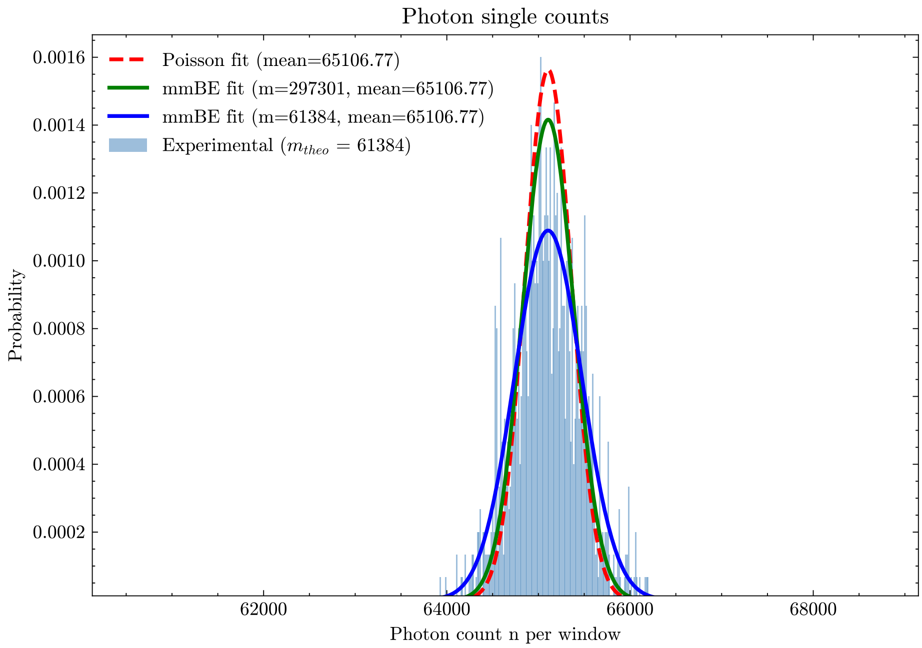

Figure 3: Multi-mode Bose-Einstein fit to experimental data

4 Results

5 Conclusion

6 Summary

Single-arm (unheralded) counts

In SPDC, each output arm alone behaves like a thermal (Bose–Einstein) source. Theory predicts that a single-mode two-mode squeezed vacuum has marginal photon-number variance $\mathrm{Var} = \langle n \rangle^2 + \langle n \rangle$, i.e. "bunched" super-Poissonian statistics [5]. In contrast, joint (signal–idler) coincidences follow Poisson statistics at low pair rates, since downconversion events occur randomly in time [1, 5].

Experimentally, unheralded single-arm counts exhibit thermal statistics. For example, [2] measured a CW-pumped bulk SPDC source and found $g^{(2)}(0) \approx 2$, consistent with a thermal field. Integrated SPDC sources show similar results: in an AlN microring with CW pump, the measured self-correlation was $g^{(2)}(0) = 2.07 \pm 0.12$, matching the ideal value of 2 for a single-mode thermal state [9]. Likewise, a periodically-poled $LiNbO_3$ micro-ring with pulsed pump and photon-number-resolving detection yielded $g^{(2)}(0) \approx 1.99$ in each arm, verifying the thermal statistics of each channel. In that work, all higher-order photon counts (up to three photons) fit a Bose–Einstein (negative-binomial) distribution [5], with variance $\gg \langle n \rangle$. Recent experiments using multiplexed detectors have reconstructed full photon-number histograms: for both CW and pulsed SPDC, the observed single-arm distributions are best described by a thermal model, whereas a Poisson model underestimates the probability of multi-photon events [5].

These observations confirm that under low-gain CW pumping, the statistics of unheralded SPDC photons in each arm are super-Poissonian (thermal), as predicted by theory [1, 5].

Coincidence counts

By contrast, coincidence (paired) counts are nearly Poissonian at low brightness. When the pair-generation probability is $\ll 1$ per mode, individual pair events are essentially independent. Theory shows that in this regime the joint (signal+idler) photon count follows a Poisson distribution [1, 5]. Experimentally, high-visibility signal–idler coincidences (large coincidence-to-accidental ratios) are routinely observed, and the fluctuations of the coincidence counts match Poisson (shot-noise) behavior. SPDC source characterizations often assume Poissonian error bars on singles and coincidences [12]. One modeling study found that measured singles and total coincidence counts in a pulsed SPDC source could be equally well described by Poisson or thermal statistics, but in practice coincidences scale linearly with pump power (as expected for independent events) [1, 4].

In summary, experiments agree with theory: CW-pumped SPDC in single spatial/spectral modes yields thermal, super-Poissonian photon statistics in each arm (variance $> \langle n \rangle$) [5, 9], whereas coincidence counts follow a near-Poissonian distribution at low pump. This conclusion is confirmed by measurements of the mean and variance (or $g^{(2)}$) of the singles and correlated counts [5, 9] and by fitting full photon-count histograms to Bose–Einstein versus Poisson models. These results hold under the low pair-generation probability ($\ll 1$ per coherence time) required for heralded single-photon sources; in that limit, multi-pair contributions are negligible and the above distributions dominate [1, 5].

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