① $-k(x)u''(x)=0$ ② $u(2)=3$ ③ $u'(0)+u(0)=20$

④ $k(x)\begin{cases}1, & x\in\langle 0;1)\\ 2, & x\in\langle 1;2\rangle\end{cases}$

1° Dokonuję podstawienia $w=u-3$, wtedy

$$\frac{dw}{dx}=\frac{du}{dx} \wedge \frac{d^2w}{dx^2}=\frac{d^2u}{dx^2} \wedge w(2)=0 \wedge w'(0)+w(0)=20$$

$$\wedge\ -k(x)w''(x)=0$$

2° Niech $v(x)$ będzie testową taką, że $v(2)=0$, wtedy

$$-k(x)v(x)w''(x)=0$$

$$-\int_0^2 k(x)v(x)w''(x)=0$$

$$-k(x)v(x)w'(x)\Big|_0^2-\int_0^2 k(x)v'(x)w'(x)\,dx=0$$

$$\int_0^2 k(x)v'(x)w'(x)\,dx+k(0)\cdot v(0)\cdot w'(0)-k(2)v(2)w'(2)=0$$

$$\int_0^2 k(x)v'(x)w'(x)\,dx+v(0)(17-w(0))=0$$

$$\underbrace{\int_0^2 k(x)v'(x)w'(x)\,dx-v(0)w(0)}_{B(w,v)}=\underbrace{-17w(0)}_{L(v)}$$

$w\approx\sum_{j=1}^{N}w_je_j$, $v\approx\sum_{j=1}^{N}v_je_j$, wstawmy to do równania liniowego

$$B\left(\sum_{j=1}^{N}w_je_j,\sum_{j=1}^{N}v_je_j\right)=L\left(\sum_{j=1}^{N}v_je_j\right)$$

$$\sum_{j=1}^{N}B(e_j,e_i)w_j=L(e_i),\ i=1,\dots,N$$

$$\begin{bmatrix}B(e_1,e_1), & B(e_2,e_1) & \dots & B(e_N,e_1)\\ B(e_1,e_2) & \dots & & \vdots\\ \vdots & & \ddots & \vdots\\ B(e_1,e_N) & \dots & \dots & B(e_N,e_N)\end{bmatrix}\begin{bmatrix}w_1\\ w_2\\ \vdots\\ w_N\end{bmatrix}=\begin{bmatrix}L(e_1)\\ L(e_2)\\ \vdots\\ L(e_N)\end{bmatrix}$$