$AUSM^{+}$-up

Tato verze algoritmu nevyžaduje transformaci do systmu n,t. Jenom je potřeba ve vzorcích brát v úvahu normálovou složku rychlosti.

3.3. Algorithm: $AUSM^{+}$-up for all speeds

The final algorithm is given as follows. First, one defines,

$$M_{L/R} = \frac{u_{L/R}}{a_{1/2}}, \tag{69}$$

where $a_{1/2}$ is defined either by Eq. (30) or a simple average of a_L and a_R. For multi-dimensional flows, $u = V \cdot \mathbf{n}$, with $\mathbf{n}$ being the unit normal vector of the cell face under consideration.

$$\bar{M}^2 = \frac{(u_L^2 + u_R^2)}{2a_{1/2}^2}, \tag{70}$$

$$M_o^2 = \min(1, \max(\bar{M}^2, M_\infty^2)) \in [0,1], \tag{71}$$

$$f_a(M_o) = M_o(2 - M_o) \in [0,1], \tag{72}$$

$$M_{1/2} = \mathcal{M}^{+}_{(4)}(M_L) + \mathcal{M}^{-}_{(4)}(M_R) - \frac{K_p}{f_a}\max(1-\sigma\bar{M}^2, 0)\frac{p_R - p_L}{\rho_{1/2}a_{1/2}^2}, \quad \rho_{1/2} = (\rho_L + \rho_R)/2, \tag{73}$$

Then, the mass and pressure fluxes are readily defined

$$\dot{m}_{1/2} = a_{1/2}M_{1/2}\begin{cases} \rho_L & \text{if } M_{1/2} > 0, \\ \rho_R & \text{otherwise} \end{cases} \tag{74}$$

and

$$p_{1/2} = \mathcal{P}^{+}_{(5)}(M_L)p_L + \mathcal{P}^{-}_{(5)}(M_R)p_R - K_u\mathcal{P}^{+}_{(5)}\mathcal{P}^{-}_{(5)}(\rho_L + \rho_R)(f_a a_{1/2})(u_R - u_L) \tag{75}$$

using the parameters

$$\alpha = \frac{3}{16}(-4 + 5f_a^2) \in \left[-\frac{3}{4}, \frac{3}{16}\right], \quad \beta = \frac{1}{8}, \tag{76}$$

with $0 \leqslant K_u \leqslant 1$.

Finally, the whole flux is

$$\mathbf{f}_{1/2} = \dot{m}_{1/2}\begin{cases} \vec{\psi}_L & \text{if } \dot{m}_{1/2} > 0, \\ \vec{\psi}_R & \text{otherwise,} \end{cases} + \mathbf{p}_{1/2}. \tag{77}$$

Tlak se samozřejmě přičítá pouze ke složce hybnosti.

příslušné polynomy:

The split Mach numbers $\mathcal{M}^{\pm}_{(m)}$ are polynomial functions of degree m (= 1,2,4), as given in [16]:

$$\mathcal{M}^{\pm}_{(1)}(M) = \tfrac{1}{2}(M \pm |M|), \tag{18}$$

$$\mathcal{M}^{\pm}_{(2)}(M) = \pm\tfrac{1}{4}(M \pm 1)^2 \tag{19}$$

and

$$\mathcal{M}^{\pm}_{(4)}(M) = \begin{cases} \mathcal{M}^{\pm}_{(1)} & \text{if } |M| \geqslant 1, \\ \mathcal{M}^{\pm}_{(2)}(1 \mp 16\beta\mathcal{M}^{\mp}_{(2)}) & \text{otherwise.} \end{cases} \tag{20}$$

$$\mathscr{P}^{\pm}_{(5)}(M) = \begin{cases} \frac{1}{M}\mathscr{M}^{\pm}_{(1)} & \text{if } |M| \geqslant 1, \\ \mathscr{M}^{\pm}_{(2)}[(\pm 2 - M) \mp 16\alpha M \mathscr{M}^{\mp}_{(2)}] & \text{otherwise.} \end{cases} \tag{24}$$

$$\vec{\psi} = (1, u, H)^{\mathrm{T}}.$$

$$a_{1/2} = \frac{1}{2}(a_L + a_R)$$

Úprava pro 2D: $\vec{\psi} = (1, u, v, H)^T$, v rovnici (77) bude zmena. Ke druhé složce toku se přičte $p_{1/2} \cdot n_x$, ke třetí složce se přičte $p_{1/2} \cdot n_y$

$$H = \frac{e + p}{\rho}$$

Použité konstanty:

$K_p = 0.25,\ K_u = 0.75$ and $\sigma = 1.0$, $M_\infty = 0.1$

HLLC

The two-dimensional (2D) Euler equations may be written in integral form as

$$\frac{\partial}{\partial t}\int_{\Omega} \mathbf{U} d\Omega + \oint_{\partial\Omega} \mathbf{F} dS = 0, \tag{1}$$

where $\partial\Omega$ denote boundaries of the control volume Ω. The state vector and flux vector are defined as

$$\mathbf{U} = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho e \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} \rho q \\ \rho u q + p n_x \\ \rho v q + p n_y \\ (\rho e + p) q \end{bmatrix}, \tag{2}$$

My používáme místo vektoru U vektor W (vektor neznámých), tok F zde představuje náš součet $F \cdot n_x + G \cdot n_y$, kde q je normálová složka rychlosti. Opět se zde nemusí dělat rotace do systému n,t. Dále je potřeba si zde dát pozor na značení: součin ρe v našem programu značíme pouze e .

$$
\mathbf{U}_{\mathrm{HLLC}} = \begin{cases} \mathbf{U}_L, & 0 \le S_L \\ \mathbf{U}_L^*, & S_L \le 0 \le S^* \\ \mathbf{U}_R^*, & S^* \le 0 \le S_R \\ \mathbf{U}_R, & 0 \ge S_R, \end{cases} \tag{5}
$$

where $\mathbf{U}_L^*$ and $\mathbf{U}_R^*$ represent intermediate states at the left and right sides of the contact discontinuity respectively,

$$
\mathbf{U}_K^* = \left[\rho_K^*,\ \rho_K^* u_K^*,\ \rho_K^* v_K^*,\ \rho_K^* e_K^*\right], \quad K = L, R, \tag{6}
$$

and S_L, S_R denote the left and right wave speeds. The corresponding interface flux, denoted by $\mathbf{\Phi}_{\mathrm{HLLC}}$, is defined as

$$
\mathbf{\Phi}_{\mathrm{HLLC}} = \begin{cases} \mathbf{F}_L, & \text{if} \quad 0 \le S_L, \\ \mathbf{F}_L^*, & \text{if} \quad S_L \le 0 \le S^*, \\ \mathbf{F}_R^*, & \text{if} \quad S^* \le 0 \le S_R, \\ \mathbf{F}_R, & \text{if} \quad 0 \ge S_R. \end{cases} \tag{7}
$$

To determine the intermediate fluxes $\mathbf{F}_L^*$ and $\mathbf{F}_R^*$, one need to consider the following Rankine-Hugoniot conditions across each of the waves of speeds S_L, S^* and S_R:

$$
\begin{aligned} \mathbf{F}_L^* &= \mathbf{F}_L + S_L\left(\mathbf{U}_L^* - \mathbf{U}_L\right) \\ \mathbf{F}_R^* &= \mathbf{F}_L^* + S^*\left(\mathbf{U}_R^* - \mathbf{U}_L^*\right) \\ \mathbf{F}_R^* &= \mathbf{F}_R + S_R\left(\mathbf{U}_R^* - \mathbf{U}_R\right). \end{aligned} \tag{8}
$$

By jump conditions (8), the intermediate states in the star region can be derived as

$$
\begin{aligned} \rho_K^* &= \frac{\alpha_K}{S_K - S^*} \\ u_K^* &= u_K + n_x\left(S^* - q_K\right) \\ v_K^* &= v_K + n_y\left(S^* - q_K\right) \\ e_K^* &= e_K + \left(S^* - q_K\right) / \left(S^* + p_K/\alpha_K\right), \end{aligned} \tag{9}
$$

where the contact velocity and pressure in the star region can be obtained by

$$
\begin{aligned} S^* &= \frac{\alpha_R q_R - \alpha_L q_L + p_L - p_R}{\alpha_R - \alpha_L} \\ p^* &= \frac{\alpha_R p_L - \alpha_L p_R - \alpha_L \alpha_R (q_L - q_R)}{\alpha_R - \alpha_L}. \end{aligned} \tag{10}
$$

In Equation (9) and Equation (10), we use the following simple notations that are defined by Shen et al[19]:

$$
\alpha_L = \rho_L(S_L - q_L), \quad \alpha_R = \rho_R(S_R - q_R). \tag{11}
$$

$$
S_L = \min\left(q_L - c_L, q_R - c_R\right), \quad S_R = \max\left(q_L + c_L, q_R + c_R\right),
$$

c_L, c_R jsou rychlosti zvuku.