Poglavje 15

Potence in koreni

15.1 Koreni poljubnih stopenj

Za sodo naravno število n je **n-ti koren** $\sqrt[n]{a}$ realnega števila $a \geqslant 0$ tisto nenegativno realno število x, za katerega velja $a = x^n$.

$$\sqrt[n]{a} = x \Leftrightarrow a = x^n; \quad a, x \in \mathbb{R}^+$$

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Število a imenujemo **korenjenec**, simbol $\sqrt{\ }$ **korenski znak**, število n pa **korenski eksponent**.

Pravila za računanje s koreni poljubnih stopenj

- $\left(\sqrt[n]{a}\right)^n = a$
- $\sqrt[n]{a^n} = \begin{cases} |a|, & n = 2k, k \in \mathbb{N} \\ a, & n = 2k-1, k \in \mathbb{N} \end{cases}$
- $\sqrt[n]{a^w} = \left(\sqrt[n]{a}\right)^w$
- $\sqrt[n]{a^w} = \sqrt[nz]{a^{wz}}$
- $\sqrt[n]{\sqrt[m]{a}} = \sqrt[nm]{a}$
- $\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
- $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}};\ b \neq 0$
- $\sqrt[n]{a^w} \cdot \sqrt[n]{a^z} = \sqrt[n]{a^{w+z}}$
- $\frac{\sqrt[n]{a^w}}{\sqrt[n]{a^z}} = \sqrt[n]{a^{w-z}};\ a \neq 0$

Pri tem za sode korenske stopnje n privzamemo $a, b \in [0, \infty)$; za lihe stopnje n pa $a, b \in \mathbb{R}$.

Naloga 15.1. *Poenostavite izraz in ga delno korenite.*

- $\sqrt[3]{xy^2\sqrt{x^5y}}$
- $\sqrt{a\sqrt{a^2\sqrt{a^3}}}$
- $\sqrt[4]{a^3b^2\sqrt{ab^5}}$
- $\sqrt[4]{ab^2\sqrt[3]{ab}}$
- $\sqrt[3]{a\sqrt[4]{a\sqrt[5]{a}}}$
- $\sqrt[5]{x^4y\sqrt[4]{x^5y^3}}$
- $\sqrt[6]{a^2b^3\sqrt{a^8\sqrt[3]{b}}}$
- $\sqrt[3]{x\sqrt{y^3\sqrt[4]{x^3\sqrt[5]{y^6y^{-1}}}}}$

Naloga 15.2. *Izračunajte.*

- $\sqrt[5]{\frac{1}{32}}$
- $\sqrt[4]{\frac{16}{81}}$
- $\sqrt[3]{-8}$
- $\sqrt[4]{-625}$
- $\sqrt[3]{0.125}$
- $\sqrt[4]{0.0016}$

Naloga 15.3. *Poenostavite.*

- $\sqrt[18]{x^{15}}$
- $\sqrt[9]{a^6}$
- $\sqrt[30]{y^{18}}$
- $\sqrt[20]{b^{30}}$

Naloga 15.4. *Racionalizirajte ulomke.*

- $\frac{1}{3-\sqrt{x}}$
- $\frac{1}{2-4\sqrt[3]{a}}$
- $\frac{2}{a-\sqrt[3]{b}}$
- $\frac{x-1}{\sqrt[3]{x}-1}$
- $\frac{8x}{2\sqrt[3]{x}+1}$
- $\frac{1}{2-\sqrt[4]{3}}$
- $\frac{1}{\sqrt[4]{2}-1}$
- $\frac{\sqrt[4]{y}}{2-\sqrt[4]{y}}$
- $\frac{3}{1+\sqrt[5]{2}}$

Naloga 15.5. *Poenostavite in delno korenite izraz.*

- $\frac{\sqrt[4]{2}}{\sqrt{2\sqrt{8}}}$
- $\frac{\sqrt[3]{9}}{\sqrt[5]{3\sqrt{27}}}$
- $\frac{\sqrt{\sqrt{\sqrt{1}}}}{\sqrt[17]{1}}$
- $\frac{\sqrt{\sqrt{a}}}{\sqrt[3]{a^2}}$
- $\frac{\sqrt{a\sqrt[3]{a^{-1}}\cdot\sqrt[3]{a^2\sqrt[5]{a}}}}{\sqrt[5]{a\sqrt{a^{-5}}}}$
- $\frac{\sqrt{x^3\sqrt[4]{x^3\sqrt{x}}}}{\sqrt[4]{x^{-3}\sqrt[4]{x}}}$
- $\frac{\sqrt[7]{b^{13}\sqrt{b^{-2}}}}{\sqrt{\sqrt{b^{-1}}}}$
- $\frac{\sqrt[3]{x^2\sqrt[4]{x^{-1}}}\cdot\sqrt[4]{x^3\sqrt{x}}}{\sqrt[4]{x\sqrt{x\sqrt[3]{x^{-1}}}}}$
- $\frac{\sqrt{8ab^{-1}}}{\sqrt{0.5}\sqrt[3]{8ab^2}}$

Naloga 15.6. *Izračunajte natančno vrednost korena.*

- $\sqrt{31-12\sqrt{3}}$
- $\sqrt{18+8\sqrt{2}}$
- $\sqrt{9-4\sqrt{5}}$
- $\sqrt{17+2\sqrt{2}}$

Naloga 15.7. *Poenostavite izraz in ga delno korenite.*

- $\frac{\sqrt[5]{xy^3\sqrt[4]{x^2y^3}}}{\sqrt[10]{\sqrt{x}}}$
- $\frac{\sqrt[4]{ab^3\sqrt[3]{a^2b^3}}}{\sqrt{\sqrt[6]{a}}}$
- $\left(\frac{1-z}{1-\sqrt[3]{z}}-\sqrt[3]{z}\right)\left(1-\sqrt[6]{z^4}\right)$
- $\sqrt[3]{\sqrt{\sqrt{4096}}}+\sqrt{\sqrt{\sqrt{16}}}-\sqrt[5]{32}$
- $\frac{\sqrt[6]{ab^3\sqrt{a^3b}}}{\sqrt[4]{b^{-3}\sqrt[3]{a}}}$