

Problem -1. [Crash course - complexity]

Is there anything unclear regarding considerations of time complexity of algorithms?

Problem 0. [Any questions?]

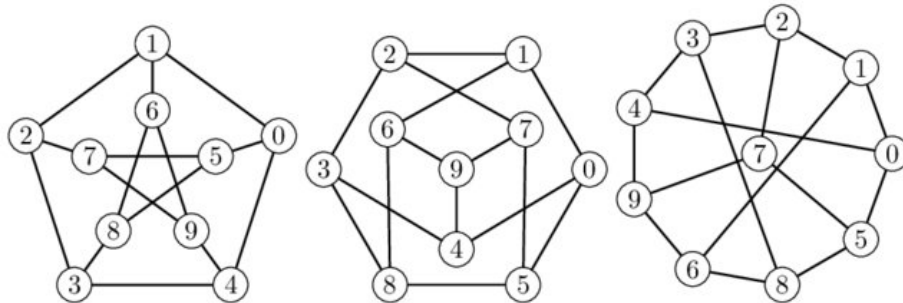
Is there anything unclear from the lectures?

Problem 1. [Enumerations]

- (a) How many k -vertex induced subgraphs does any graph Γ with n vertices have?
- (b) How many induced subgraphs does Graph Γ with n vertices have?
- (c) Is it possible to enumerate all (not only induced) subgraphs? If yes, provide an algorithm (think also about its time complexity). Is there a closed formula for the enumeration? If not, is it possible to bound the number?

Problem 2. [Graph properties]

- (a) Determine diameter, eccentricities, radius, central vertex of these graphs. (Hint: use a matrix to mark the distances.)



- (b) Is $C = \{1, 5\}$ a vertex cut? If yes, give an argument. If not, adjust C so that it will be a vertex cut.
- (c) Is $C = \{\{1, 6\}, \{6, 8\}\}$ an edge cut? If yes, give an argument. If not, adjust C so that it will be an edge cut.

Problem 3. [Graphical sequences]

(a) Are these sequences **graphical**? Prove. (Hint: Havel-Hakimi alg. & Handshake theorem & logic)

1. $\{2, 2, 2, 1, 1\}$
2. $\{2, 2\}$
3. $\{2, 2, 2, 2\}$
4. $\{3, 3, 5, 1, 7, 9, 1\}$
5. $\{3, 3, 3, 3, 3, 3, 3, 3\}$
6. $\{6, 5, 5, 4, 3, 3, 3, 2, 2\}$
7. $\{3, 3, 2, 2, 1, 1\}$
8. $\{7, 6, 4, 3, 3, 2\}$
9. $\{3, 3, 1, 1\}$
10. $\{5, 4, 3, 3, 2, 2, 2, 1, 1, 1\}$

(b) How would you construct graphs out of these graphical sequences?

Problem 4. [Graph properties of families of graphs]

Can something be proved about diameter, eccentricities, radius of simple graph families such as hypercubes, cycles, paths, complete graphs?