Graphs, Graph Algorithms, and Optimization
Problem set 1

W 24/25

GitHub https://github.com/JanPastorek/1-AIN-413-22-Graphs

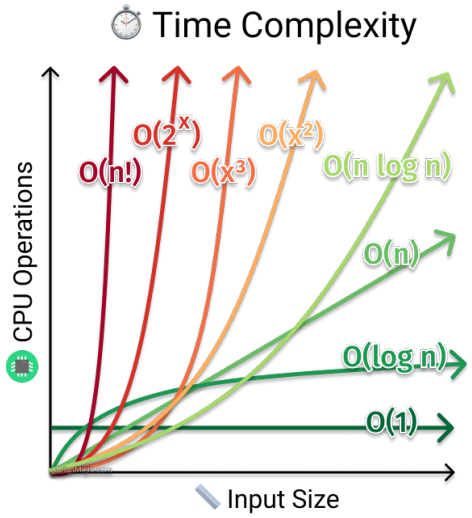
LIST

50 points for the semester from the exercises.

- activity and participation
- solving problem sets
- homework assignments (≈ 3)

Note

When asked for an algorithm you must give (1) a brief informal description of the algorithm, (2) a precise description using pseudo-code, (3) an informal argument for termination and correctness of the algorithm, and (4) an analysis of the running time of the algorithm. Be clear about what the input to the algorithm is, how you measure the size of the input, and what constitutes a "step" in your running-time analysis.



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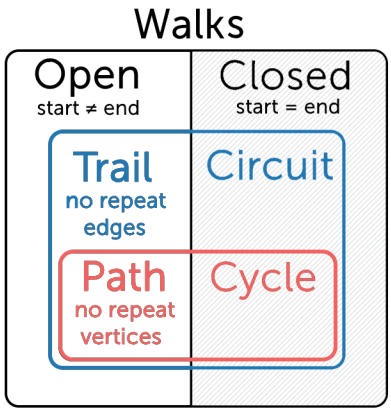
A **walk** is an alternating list $v_0, e_1, v_1, e_2, ..., v_k$, of vertices and edges such that for $1 \leq i \leq k$, the edge e_i is a $\{v_{i-1}, v_i\}$.



Problem 0. [Any questions?]

Is there anything unclear from the lectures?

Problem 1. [Graph Property]

Is it possible to construct an undirected graph that has an odd number of vertices and where each vertex has an odd degree? If so, show an example. If not, show the reason. (Graphs are not allowed to have self-loops or multiple edges between the same pair of vertices.)

Problem 2. [Cycle Detection]

Design linear time algorithms for the following problems:

(a) Determine whether a directed graph $G = (V, E)$ has a cycle. Your algorithm should run in $O(|V| + |E|)$ time.

(b) Determine whether an undirected graph $G = (V, E)$ has a cycle. Your algorithm should run in $O(|V|)$ time, independent of $|E|$.

Problem 3. [Graph Traversal]

Let G be a strongly connected graph with the property that (u, v) is an edge iff (v, u) is also an edge. Does there always exist a path that traverses each edge of G exactly once? If so, design an algorithm for finding such a path; otherwise, show why not.