Graphs, Graph Algorithms, and Optimization
Problem set 11

W 24/25

- **GitHub** https://github.com/JanPastorek/1-AIN-413-22-Graphs

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Today's exercises are based on:

[1] Stanoyevitch, A. (2011). *Discrete structures with contemporary applications.* CRC Press, Taylor & Francis Group.

[2] West, D. B. (2001). *Introduction to graph theory* (2nd ed). Prentice Hall.

Recall

Dominating set of graph $G = (V, E)$ is the set of vertices $S \subseteq V$ such that every vertex in $V \setminus S$ is adjacent to a vertex in S.

Domination number of graph G, $\gamma(G)$ is the minimum order of a dominating set.

Problem 0. [Any questions?]

Is there anything unclear from the lectures?

Problem 0,5. [Prep. for Rehearsal]

1. Formulate three specific non-trivial questions related to concepts and algorithms from this course that you do not understand. By non-trivial I mean that the answer is not just reading the definition. If some definitions are unclear, the unclarity must be specified. (10-15 minutes)
2. For each answered question, you get one (or more for harder questions) point. (at most 5 for each of you). For each asked non-trivial question you get one point. (45 minutes)

Problem 1. [Domination number]

1. Determine domination number of Cycle on n vertices for any n.
2. Determine domination number of Path on n vertices for any n.
3. For any $1 \leq k \leq \frac{n}{2}$ construct a connected n-vertex graph G with $\gamma(G) = k$
4. From [2], 3.1.52, Prove that if the diameter of G is at least 3, then $\gamma(G^C) \leq 2$

Tutorial 2. [Algorithm for determining domination number]

1. Invent a greedy algorithm for determining upper bound on the domination number
2. Dynamic programming algorithm