Graphs, Graph Algorithms, and Optimization
Problem set 10

W 24/25

- **GitHub** https://github.com/JanPastorek/1-AIN-413-22-Graphs

---

Today's exercises are based on:

- Stanoyevitch, A. (2011). *Discrete structures with contemporary applications*. CRC Press, Taylor & Francis Group.
- West, D. B. (2001). *Introduction to graph theory* (2nd ed). Prentice Hall.

Problem 0. [Any questions?]

Is there anything unclear from the lectures?

Problem 1. [Random Graphs]

A random graph $G(n, p)$ is a probability space of all labeled graphs on n vertices $1, 2, \ldots, n$, where for each pair $1 \leq i < j \leq n$, (i, j) is an edge of $G(n, p)$ with probability p, independently of any other edge (you can think of a sequence of independent coin tosses for each edge).

1. Compute the following:

(a) The expected number of edges in $G(n, p)$;
(b) The expected degree of a vertex in $G(n, p)$;
(c) The expected number of triangles (cycles of length 3) in $G(n, p)$;
(d) The expected number of paths of length 2 in $G(n, p)$;
(e) The probability that the degree of a given vertex v is exactly k.

2. Let G be a graph with m edges, and let $X \subseteq V(G)$ be a random set that contains each vertex of G independently with probability 1/2. What is the expected number of edges in the induced subgraph $G[X]$? (Here $G[X]$ is the subgraph of G with vertex set X, and contains all edges in G with both ends in X.)

3. Homework - tutorial 7 on the github