Graphs, Graph Algorithms, and Optimization
Exercises - Home Project 2

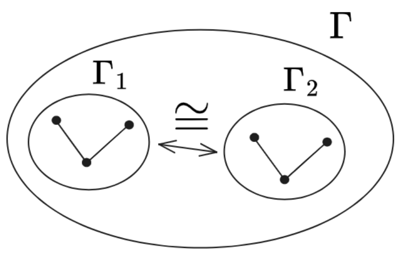
W 24/25

Include all your reasoning steps, but only the neccesary ones. The following home projects will most probably force you to use computers. You can use any language, libraries, and functions available. Your output for this home project must include code, adjacency list representation of the best record graph you have found, d , and your reasoning about how have you approached this problem (heuristics, constructions, etc.). Almost all graphs are asymmetric, so I apriori assume the probability that you will output the same graphs as almost 0.

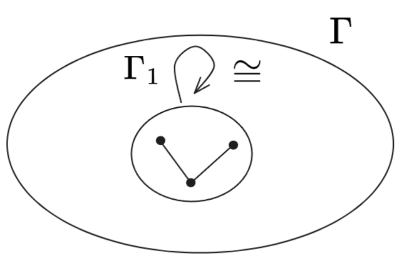
Total points ($2 \times d$ + bonus)

Graph Γ is **asymmetric** if it has only a trivial automorphism, i.e. identity map (mapping each vertex to itself). Partial automorphism is an isomorphism between any two induced subgraphs Γ_1, Γ_2 (of the same order) of Γ. A rank of a partial automorphism is given by the size of its domain.

For instance, the following graph Γ has a nontrivial partial automorphism φ mapping swapping Γ_1 and Γ_2. Since the size of its domain is 6, the rank of this partial automorphism that swaps Γ_1 and Γ_2 is 6.



For instance, the following graph Γ has a one trivial partial automorphism that maps each vertex of Γ_1 to itself; and one nontrivial partial automorphism mapping Γ_1 to Γ_1 such that the leaves are switched. Both have rank 3.



We define the following measure of asymmetry, **asymmetry depth** d of a graph Γ as the $d = n - |\text{largest rank of a nontrivial partial automorphism}|$ where n is the number of vertices of a graph Γ. Currently the best record graph has asymmetric depth $d = 8$. The smallest asymmetric graph has 6 vertices and $d = 1$. You can get as much points as $2 \times d$ for a graph you find!

One of the possible useful heuristics: $d \geq n - min(\Delta_{v_i v_j})$, i.e. asymmetric depth is definitely bigger or equal to the minimum of the symmetric difference of neighbourhoods over all pairs of vertices.

Bonus (hard): Find asymmetric graph where $d = n - min(\Delta_{v_i v_j}) = n - avg(\Delta_{v_i v_j})$. This might even earn you being mentioned in one of my future research papers!