GitHub https://github.com/JanPastorek/1-AIN-413-22-Graphs

Problem 0. [Any questions?]

Is there anything unclear from the lectures?

Problem 1. [Bipartite graphs]

Are these claims true? Argue. (Stanoyevitch, 8.1 exercises 13 & 14)

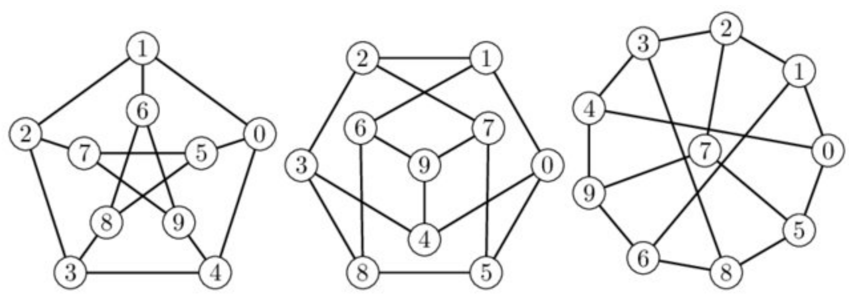
1. A subgraph of a bipartite graph is bipartite.
2. A bipartite graph cannot have a self-loop.

Problem 2. [& Isomorphism]

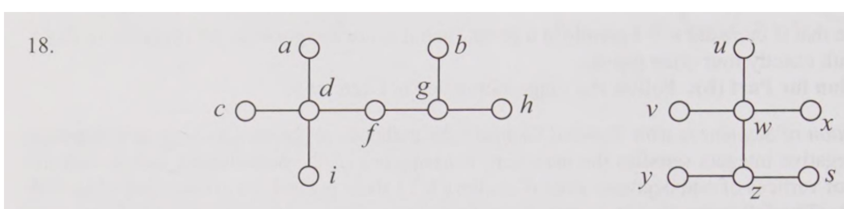
(x) Prove that isomorphism is an equivalence relation.

Determine which of these graphs are isomorphic, bipartite. Hint: use the fact that isomorphism is an **equivalence relation** to shorten your proofs (reflexive, **symmetric** and **transitive**)

(a)



(b)



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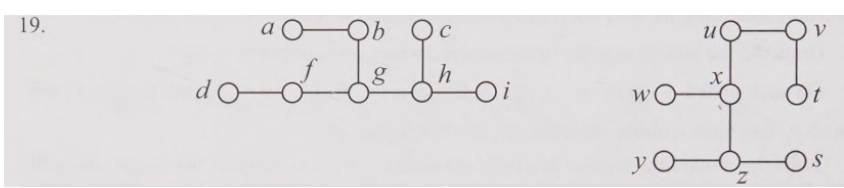
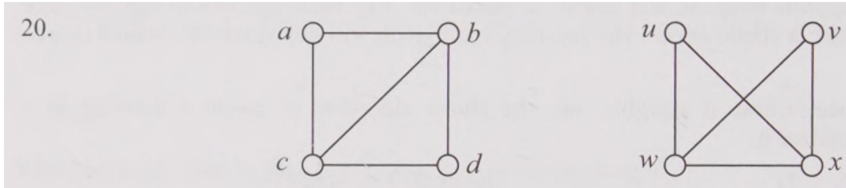
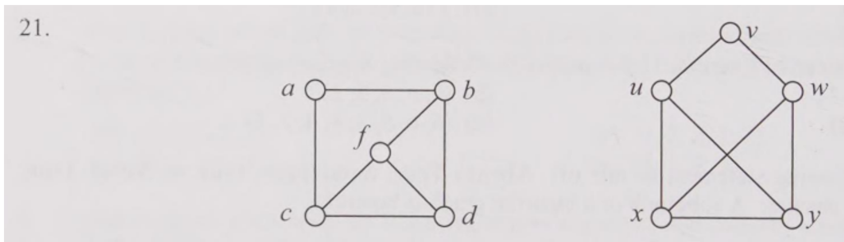
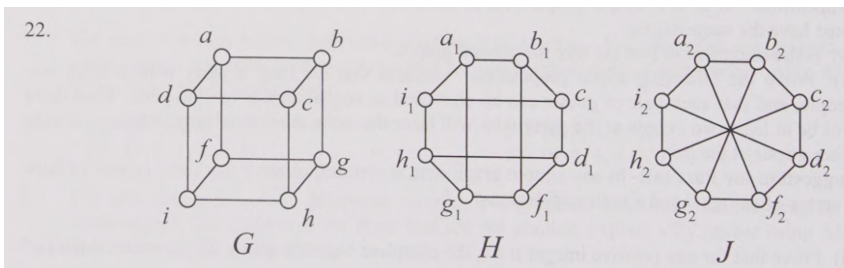
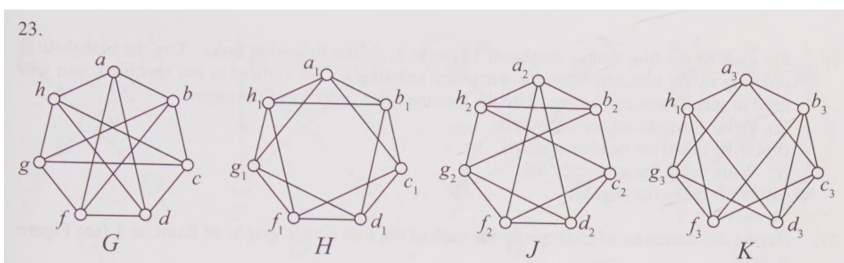
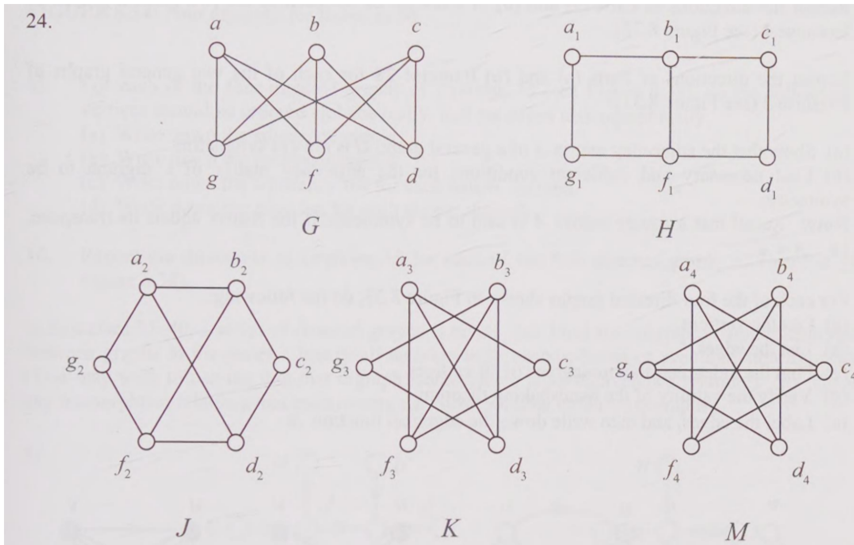


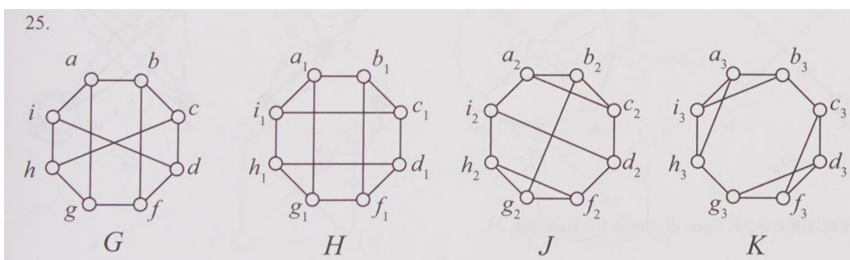
(d) 

(e) 

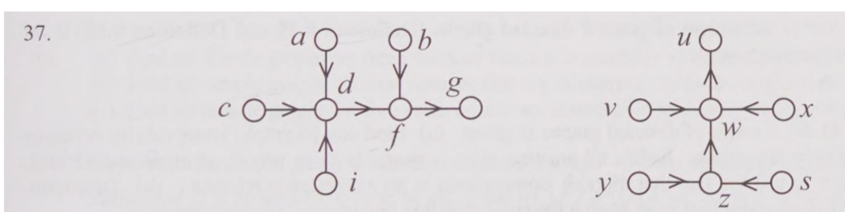
(f) 

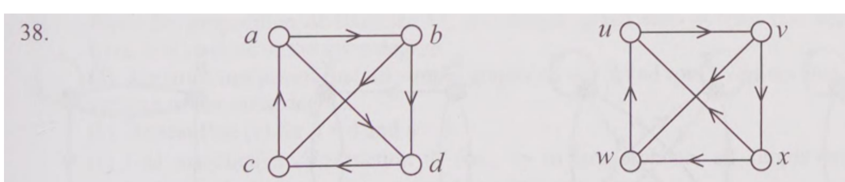
(g) 

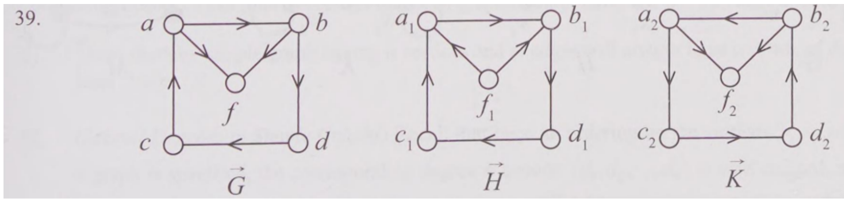
(h) 

(i) 

If it is a directed graph, for it to be isomorphic to another graph, it needs to also preserve the directionality of edges.

(j) 

(k) 

(l) 

Problem 3. [Trees]

(a) Prove that in any tree every vertex of degree greater than one is a cut vertex. (Stanoyevitch 8.3 Exercise 38)

(b) Prove that every tree is bipartite.

(c) Prove this theorem. (Stanoyevitch 8.3 Exercise 41) Suggestion: Use induction (or another theorem).

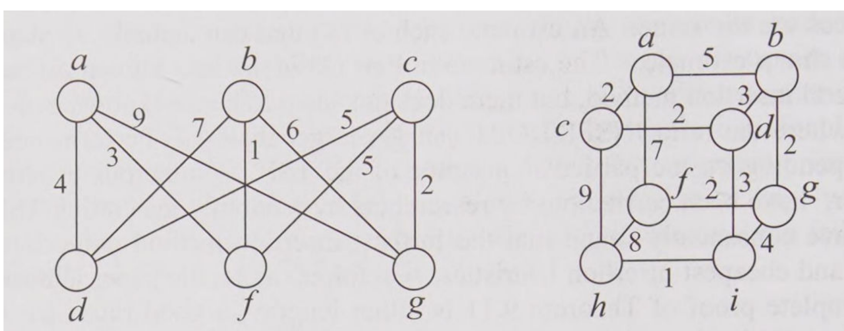
Theorem 1 (Degree Sequences of Trees) *There is no simple criterion to check whether a given sequence of nonnegative integers is a degree sequence for a simple graph. (Recall that Algorithm (Havel-Hakimi) was used for this determination.) For trees, however, there is the following very simple characterization... Sequence of positive integers with $n > 1$, is the degree sequence of a tree* iff $\sum_{i=1}^{n} d_i = 2n - 2$.

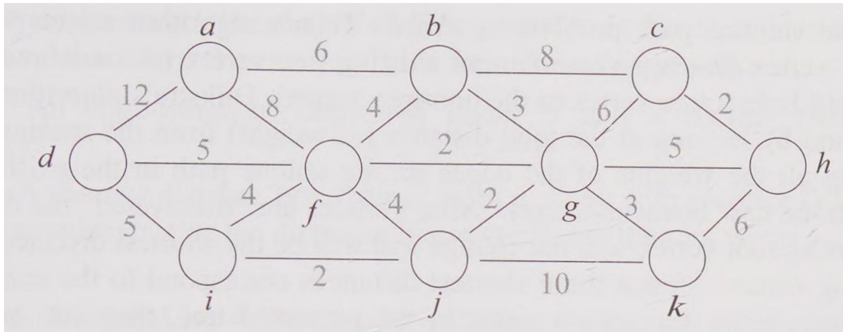
Problem 4. [Spanning subgraphs]

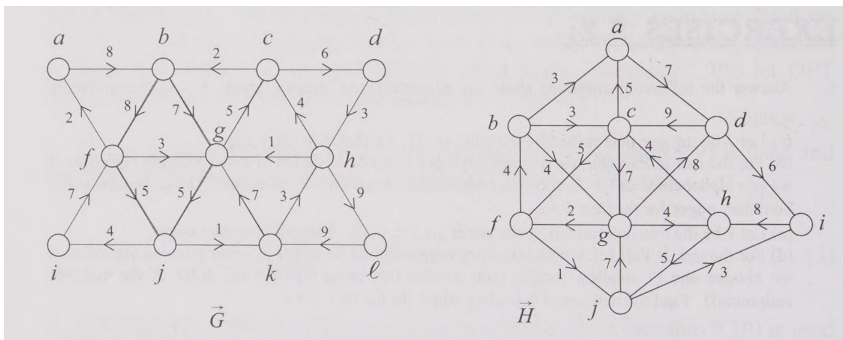
(a) Prove this theorem:

Theorem 2 *Every connected graph has at least one spanning tree.*

(b) Apply algorithms of 1. Kruskal and 2. Prim to find the minimum spanning tree of the following graphs (try different starting vertices)

- 

- 

- 

(c) Prove the correctness of Prim's and Kruskal's algorithm. (Stanoyevitch 9.2 Exercise 26-27)