DIVIDE AND CONQUER

PROBLEM A: Exponentiation

This problem is very simple. Given two positive integers a and x, compute a^x.

$$2 \leq a \leq 13$$

$$0 \leq x \leq 10^{18}$$

However, using ** operator is not allowed here. (we are coding in Python 3)

To keep size of answer convenient to read, print the output as being modulo by 2147483647.

1) Write a straightforward program that computes a^x by using for loop that repeats x times.

2) The running time of program in step 1 is O(_____).

3) Utilize the following fact, rewrite a fast version of exponentiating program.

$$a^x = \begin{cases} \sqrt{a^x} \times \sqrt{a^x} & ; x \text{ is even} \\ a \times \sqrt{a^x} \times \sqrt{a^x} & ; x \text{ is odd} \end{cases}$$

In essence,

- Divide the large problem (with respect to x) into smaller subproblems
- Conquer the subproblems
- Combine the results

NOTE Dividing can automatically result in a floating-point number. Be aware to keep x integer.

4) The running time of program in step 3 is O(_____).

Subproblems in Depth-First-Search are explored "options".

In Divide-and-Conquer, the subproblems are explicitly specified.
Therefore, each subproblem is unique, always getting smaller, and will not be repeated.

PROBLEM B: Fibonacci for a huge n

The Fibonacci number is defined as $F_0 = F_1 = 1, F_n = F_{n-1} + F_{n-2}; n \geq 2$

Given an integer *n*, write a Python 3 program to compute the value of Fibonacci number for a huge value of *n*, $10^8 \leq n \leq 10^{50}$.

INPUT: An integer n

OUTPUT: The residue of the corresponding Fibonacci number divided by 2147483647

EXAMPLE

INPUT	OUTPUT
123456789	2053005829
12345678901234567890	268002575

1) Study the following mathematical fact. F_n can be obtained by computing M^{n-1}.

$$\begin{bmatrix} F_n \\ F_{n-1} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} F_{n-1} \\ F_{n-2} \end{bmatrix}$$

$$\begin{bmatrix} F_n \\ F_{n-1} \end{bmatrix} = M \begin{bmatrix} F_{n-1} \\ F_{n-2} \end{bmatrix}$$

$$\begin{bmatrix} F_n \\ F_{n-1} \end{bmatrix} = M^2 \begin{bmatrix} F_{n-2} \\ F_{n-3} \end{bmatrix}$$

$$\begin{bmatrix} F_n \\ F_{n-1} \end{bmatrix} = M^{n-1} \begin{bmatrix} F_1 \\ F_0 \end{bmatrix}$$

2) Write a function mutiply2D(A, B) that takes 2x2 matrices A and B, and then return a 2x2 matrix that is the multiplication of A and B, denoted as AB.

3) Apply the function created in step 2 to calculate M^{n-1}, given a huge input n. Keep elements of the matrix under 2147483647.
Some useful mathematical facts : $(a \times b)\%m = a\%m \times b\%m$ and $(a \pm b)\%m = a\%m \pm b\%m$

4) Write a program for this problem B.