# Logic of Compound Statements

CSX2008 Mathematics Foundation for Computer Science

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# Big Picture of the next 3 weeks

- Propositional Logic
- Predicate Logic
- Each individual statement: True or False?
- The argument as a whole: Valid or Invalid?

# Session Outline

- Fundamental of Propositional Logic
- Mathematical Sentences and Statements
  - Open Sentence vs. Closed Sentence
  - Mathematical Statements (Propositions)
  - Compound Statements
- Fundamental Logical Connectives
  - and, or, not, xor
- Forms of Statements (Propositional Forms)
  - Statements vs. Statement Forms
- Truth Table of Statement Forms
- Rules of Logical Equivalences
- Conditional & Bi-Conditional Statements

# Logical Argument

- An argument is a sequence of statements aimed at demonstrating the truth of assertion.
  - conclusion: the assertion at the end of sequence
  - premises: preceding statements

- Argument Forms
  - All cats have big bellies.
  - Tom is a cat.
  - Therefore, Tom has a big belly.

# Example:
# Argument Form vs. Argument Content

- Argument #1
  - If the program syntax is faulty or if program execution results in division by zero, then the computer will generate an error message.
  - Therefore, if the computer does not generate an error message, then the program syntax is correct and program execution does not result in division by zero.

- Argument #2
  - If $x$ is a real number such that $x < -2$ or $x > 2$, then $x^2 > 4$.
  - Therefore, if $x^2 \not> 4$, then $x \not< -2$ and $x \not> 2$.

- Both have the same argument form
  - If $p$ or $q$, then $r$
  - Therefore, if not $r$, then not $p$ and not $q$.

p , q and r are propositional variables representing any statements.

# Example:
# Argument Form vs. Argument Content

- Argument #1
  - If the program syntax is faulty or if program execution results in division by zero, then the computer will generate an error message.
  - Therefore, if the computer does not generate an error message, then the program syntax is correct and program execution does not result in division by zero.
- Argument #2
  - If x is a real number such that $x < -2$ or $x > 2$, then $x^2 > 4$.
  - Therefore, if $x^2 \not> 4$, then $x \not< -2$ and $x \not> 2$.
- Is Argument #1 valid?
- Is Argument #2 valid?
- All arguments of the same valid form are valid.

# Exercise

Fill in the blanks below so that argument (b) has the same form as argument (a). Then represent the common form of the arguments using variables to stand for component sentences.

a) If Jane is a math major or Jane is a computer science major, then Jane will take Math 150. Jane is a computer science major. Therefore, Jane will take Math 150.

b) If logic is easy or ________________, then ___________. I will study hard. Therefore, I will get an A in this course.

- What is the argument form of (a)?
- Argument Form:

  If p or q, then r.

  q.

  Therefore, r.

# Mathematical Sentence vs. Statement

- Sentence
  - Specifies information or idea (says something).
  - For instance,
    - The sun is shining
    - He majors in Computer Science.
    - $3 + 4 = 9$
    - $x + y > 17$
- Closed Sentence (*aka* Statement or Proposition)
  - A sentence that can be determined either true or false.
  - We treat statement and proposition same in this class.
- Open Sentence
  - A sentence whose truth cannot be determined because the truth or falsity depends on some other unknown facts.

# Compound Statements

- Algebraic Expressions
  - x * (3 + y) / (z – 27)
  - Build more complex expression by associating operands with operators.
  - Output of one operation is used as an input to the next operation.
- Similarly, we can build more complex logical expressions by combining simpler statements (propositions) with the logical connectives
- Compound Statements
  - Somchai is smart and strong but not happy
  - Somchai is smart and Somchai is strong and it’s not the case that Somchai is happy
  - Argument Form: $p \wedge q \wedge \sim r$

# Logical Connectives

- Negation (not) Symbol: ~ (unary operator) or ¬
  - ~p
  - ¬p
- Conjunction (and) Symbol: $\wedge$ (binary operator)
  - $p \wedge q$
- Disjunction (or) Symbol: $\vee$ (binary operator)
  - $p \vee q$
- Exclusive Disjunction (xor) Symbol: $\oplus$
  - $p \oplus q$
- Operands of the logical connectives are statements (or propositions), *i.e.*, a sentence whose truth value is known.

Truth Tables of different Logical Operators

# Example: English to Logical Statements

- If we let h = “it is hot” and s = “it is sunny”.
- It is not hot but it is sunny
  - It is not hot and it is sunny
  - $\sim h \wedge s$
- It is neither hot nor sunny.
  - It is not hot and it is not sunny
  - $\sim h \wedge \sim s$

# Example:
# English to Logical Statements

- You go to hell if you kill or steal
  - $k \vee s \rightarrow h$
- "A free drink comes with your meal, Sir. Would you like tea or coffee?"
  - $t \oplus c$

Be careful with possible ambiguities in English.

Any human language is ambiguous because they are not formal language like mathematics.

# Example:
# Inequalities to Logical Statements

- $x \leq a$
- $x < a$ or $x = a$
- $(x < a) \vee (x = a)$

- $a \leq x \leq b$
- $a \leq x$ and $x \leq b$
- $(a \leq x) \wedge (x \leq b)$
- $((a < x) \vee (a = x)) \wedge ((x < b) \vee (x = b))$

# Exercise:

- Let x be any real number.
- Let p = “0 < x”, q = “x < 3”, and r = “x = 3”.
- Write the following inequalities symbolically using the variables.
  - $x \leq 3$
  - $0 < x < 3$
  - $0 < x \leq 3$

# Operator Precedence and Associativity

- Define the order of evaluation for compound statements
- Operator Precedence
  - Operator of higher precedence will be evaluated first.
  - not has higher precedence than and, or, xor
  - and, or, xor have same precedence
  - $\sim p \wedge q \equiv (\sim p) \wedge q$
- Associativity
  - Defines the order of evaluation for operators of the same precedence.
  - Binary operators have Left-to-Right associativity
    - $p \wedge q \vee r \equiv (p \wedge q) \vee r$
  - Negation operator has Right-to-Left associativity
    - $\sim\sim p \equiv \sim(\sim p)$

$X \equiv Y$ denotes *logical equivalence* of statement forms $X$ and $Y$

# Notes on Operator Precedence and Associativity

- Do make use of parenthesis to clearly indicate the intended order of evaluation.
  - Especially in your homework and exercises.
- For instance, $p \wedge q \vee r$ can be ambiguous!
- Write either
  - $(p \wedge q) \vee r$
  - $p \wedge (q \vee r)$

# Statement Forms

- Statement Form (or Propositional Form)
  - An expression made of statement variables and logical connectives that becomes a statement when actual statements are substituted for the component statement variables.

e.g. p, q, and r

e.g. ~, $\wedge$, and v

- For instance,
  - He must be either rich or handsome and smart.
  - $r \vee (h \wedge s)$

# Truth Table

- The truth table for a given statement form displays the truth values that correspond to all possible combinations of truth values for its component statement variables.

- For instance, $r \vee (h \wedge s)$

| $r$ | $h$ | $s$ | $h \wedge s$ | $r \vee (h \wedge s)$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | T | F | F | T |
| T | F | T | F | T |
| T | F | F | F | T |
| F | T | T | T | T |
| F | T | F | F | F |
| F | F | T | F | F |
| F | F | F | F | F |

# Truth Tables

| $p$ | $\sim p$ |
|---|---|
| T | F |
| F | T |

Truth Table for $\sim p$

| $p$ | $q$ | $p \wedge q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

Truth Table for $p \wedge q$

| $p$ | $q$ | $p \vee q$ |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

Truth Table for $p \vee q$

| $p$ | $q$ | $p \oplus q$ |
|---|---|---|
| T | T | F |
| T | F | T |
| F | T | T |
| F | F | F |

Truth Table for $p \oplus q$

# Exercise

- Construct a truth table for the statement form $(p \wedge q) \vee \sim r$

# Logical Equivalence

- $x$ is greater than $y$. VS. $y$ is smaller than $x$.
- It is not case that Jim is handsome or Jim is smart.
- Jim is not handsome and Jim is not smart.

# Logical Equivalence

- $P \equiv Q$ denotes the logical equivalence of statement forms $P$ and $Q$.
- Logical equivalence of the statements have nothing to do with the meanings of the words.
- Logical equivalence is purely based on the forms of the statements
- $p \wedge q \equiv q \wedge p$
  - regardless of the meanings of $p$ and $q$

| $p$ | $q$ | $p \wedge q$ | $q \wedge p$ |
|---|---|---|---|
| T | T | T | T |
| T | F | F | F |
| F | T | F | F |
| F | F | F | F |

$p \wedge q$ and $q \wedge p$ always have the same truth values, so they are logically equivalent

# Logical Equivalence

- Two statement forms are logically equivalent *iff* they have identical truth values for each possible substitution of statements for their statement variables.

- Two statements are logically equivalent *iff* they have logically equivalent forms when identical component statement variables are used to replace identical component statements

# What have been covered so far?

- Mathematical or Logical Sentence?
  - Open Sentence?
  - Closed Sentence?
- Statement?
- Proposition?
  - Propositional Variables?
- Compound Statement (Compound Proposition)
  - Logical Connectives (Logical Operators)
- Form of Statement (Propositional Form)
  - Statement vs. Statement Form
  - Truth Table for a given statement form
- Logical Equivalence?

# Logical Equivalence

# Proof of Double Negation Law

- Construct a truth table to show that the negation of the negation of a statement is logically equivalent to the statement, annotating the table with a sentence of explanation.

- $\sim(\sim p) \equiv p$

| $p$ | $\sim p$ | $\sim(\sim p)$ |
|---|---|---|
| T | F | T |
| F | T | F |

$p$ and $\sim(\sim p)$ always have the same truth values, so they are logically equivalent

# Exercise

- Show that $p \oplus q \equiv (p \vee q) \wedge \sim(p \wedge q)$

# Exercise

- Show that the statement forms $\sim(p \wedge q)$ and $\sim p \wedge \sim q$ are not logically equivalent.

# Alternative Solution

- Show that the statement forms ~(p ∧ q) and ~p ∧ ~q are not logically equivalent.

An example which disproves a proposition

- Showing a counterexample:
  - Let *p* be the statement "0 < 1" , and let *q* be the statement "1 < 0."
  - Then ~(*p* ∧ *q*) is "it is not the case that both 0 < 1 and 1 < 0, which is actually true.
  - However, $\sim p \wedge \sim q$ is "$0 \not< 1$ and $1 \not< 0$,"
    - which is actually false. ( false ∧ true)

  - Therefore, ~(*p* ∧ *q*) and ~*p* ∧ ~*q* are not logically equivalent

# De Morgan's Laws

$$\sim(p \wedge q) \equiv \sim p \vee \sim q$$

$$\sim(p \vee q) \equiv \sim p \wedge \sim q.$$

- Implications
  - Negation of a Conjunction is logically equivalent to the Disjunction of the negated components
  - Negation of a Disjunction is logically equivalent to the Conjunction of the negated components.

# Proof of De Morgan's Laws

- Show that $\boxed{\sim(p \wedge q) \equiv \sim p \vee \sim q}$ and $\boxed{\sim(p \vee q) \equiv \sim p \wedge \sim q.}$

# Exercise: De Morgan’s Laws

- What’s the negation of “John is 6 feet tall and he weighs at least 200 pounds”?

# Exercise: De Morgan’s Laws

- What’s the negation of “The bus was late or Tom’s watch was slow”?

# Caution #1

- p: Jim is tall and Jim is thin
  - $s \wedge t$
- ~p: It's not the case that Jim is tall and Jim is thin
  - $\sim(s \wedge t) \equiv \sim s \vee \sim t$
  - Jim is not tall or Jim is not thin
- q: Jim is tall and thin
- ~q: Jim is not tall and thin
  - It's not a violation of De Morgan's law
- In formal logic, the connectives (and, or) are allowed only between complete statements, not between sentence fragments.

# Caution #2

- $x \nless 2$
  - $x$ is not less than 2
  - It's not the case $x < 2$
  - $x = 2$ or $x > 2$ ($x$ is equal to 2 or greater than 2)
- $x \nless 2 \equiv x \geq 2$
- $x \ngtr 2 \equiv x \leq 2$
- $x \nleq 2 \equiv x > 2$
- $x \ngeq 2 \equiv x < 2$

# Example

Let $p$: $-1 < x \leq 4$, define $\sim p$

$$
\begin{aligned}
\sim(-1 < x \leq 4) \quad &\equiv \sim(-1 < x \text{ and } x \leq 4) \\
&\equiv \sim(-1 < x \land x \leq 4) \\
&\equiv \sim(-1 < x) \lor \sim(x \leq 4) \\
&\equiv (-1 \not< x) \lor (x \not\leq 4) \\
&\equiv (-1 \geq x) \lor (x > 4) \\
&\equiv -1 \geq x \text{ or } x > 4
\end{aligned}
$$

# Tautologies

- A tautology is a statement form that is always true regardless of the truth values of individual statements substituted for its variables.
  - *e.g.* $p \vee {\sim}p$
  - *e.g.* $(p \wedge q) \vee {\sim}p \vee {\sim}q$ (verify using truth table)
- A tautological statement: a statement whose form is a tautology

# Contradictions

- A contradiction is a statement form that is always false regardless of the truth values of the individual statements substituted for its variables. (A contradictory statement)
  - e.g.) $p \wedge \sim p$
  - *e.g.)* $(p \vee q) \wedge (\sim p \wedge \sim q)$ (verify using truth table)
- A contradictory statement: a statement whose form is a contradiction

# Logical Equivalence involving Tautologies and Contradiction

- Let **t** stands for a tautology and **c** stands for a contradiction
  - $p \vee {\sim}p \equiv \mathbf{t}$
  - $p \wedge {\sim}p \equiv \mathbf{c}$
  - $p \vee \mathbf{c} \equiv p$
  - $p \wedge \mathbf{t} \equiv p$
  - $p \vee \mathbf{t} \equiv \mathbf{t}$
  - $p \wedge \mathbf{c} \equiv \mathbf{c}$
  - ${\sim}\mathbf{t} \equiv \mathbf{c}$
  - ${\sim}\mathbf{c} \equiv \mathbf{t}$

A tautology is a form of statement that is always true.

True is a value

A contradiction is a form of statement that is always false.

False is a value

Verify each of the equivalence shown here using truth table.

# Distributive Laws

- Show the following logical equivalences using truth tables
  - $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
  - $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

| $p$ | $q$ | $r$ | $q \wedge r$ | $p \vee (q \wedge r)$ | $p \vee q$ | $p \vee r$ | $(p \vee q) \wedge (p \vee r)$ |
|---|---|---|---|---|---|---|---|
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# Absorption Laws

- Show the following logical equivalences using truth tables.
  - $p \lor (p \land q) \equiv p$
  - $p \land (p \lor q) \equiv p$

| $p$ | $q$ | $p \land q$ | $p \lor (p \land q)$ | $p \lor q$ | $p \land (p \lor q)$ |
|---|---|---|---|---|---|
| | | | | | |
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# Logical Equivalences

- De Morgan's Laws
  - $\sim(p \vee q) \equiv \sim p \wedge \sim q$
  - $\sim(p \wedge q) \equiv \sim p \vee \sim q$
- Distributive Laws
  - $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
  - $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
- Absorption Laws
  - $p \vee (p \wedge q) \equiv p$
  - $p \wedge (p \vee q) \equiv p$

# Exercise

- Use the laws of logical equivalence to show that

$$\sim(\sim p \wedge q) \wedge (p \vee q) \equiv p$$

# Exercise

- Use the laws of logical equivalence to show that

$$\sim(\sim p \wedge q) \wedge (p \vee q) \equiv p$$

- What if we make use of a Truth Table to show?

- Assume that a compound statement P consists of 10 different statement variables and a compound statement Q consists of 6 different statement variables.
  - How long does it take to show $P \equiv Q$ if using a Truth Table?
  - How many rows are there in the two truth tables to be built?

# Conditional Statements

# Conditional Statement

- If you get a full mark in the final exam, you will get a grade of 'C' or higher.
- Let p and q be statements; a statement of form "if p then q" is called a conditional statement.
  - $p \rightarrow q$ ( $\rightarrow$ is a logical connective like ~, $\wedge$ , $\vee$)
  - p is called the hypothesis and q the conclusion

# Conditional Statement

- Conditional statement: $p \longrightarrow q$
  - Conditional of $q$ by $p$
  - If $p$ then $q$ ($p$ implies $q$)
  - $p$: hypothesis (antecedent)
  - $q$: conclusion (consequent)

- Logic helps to determine the validity of the logical forms of arguments, not the meaning of individual statements.

| $p$ | $q$ | $p \to q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

Truth Table for $p \longrightarrow q$

# Conditional Statement

- Vacuously true (true by default)
  - A conditional statement that is true by virtue of the fact that its hypothesis is false.
- If 0 = 1 then 1 = 2.
  - Is the (whole conditional) statement true or false?
- If 2 + 2 = 7, then Santa Clause is the PM of Thailand

# Exercise

- Use truth tables to show that

$$p \lor q \rightarrow r \equiv (p \rightarrow r) \land (q \rightarrow r)$$

- Reminders:
  - Order of Evaluation (Operator Precedence)
  - How many rows in the truth table?, Why?
  - What columns to show in the truth table?
- What does the above logical equivalence tell?
  - When truth of r follows from truth of p and truth of r follows from truth of q, we know that r must be true if at least one of p or q is true.

# Exercise

- Use truth tables to show that

$$p \to q \equiv \sim p \lor q$$

- Another way of saying "If p then q" is "not p or q"
- Do not be late to the classes or you will not be given attendance.
  - If you are late to the classes, then you will not be given attendance.
- If you do not come to class on time, then you will not be given attendance.
  - Either you come to class on time or you will not be given attendance.

# Negation of Conditional Statements

- By definition, $p \rightarrow q$ is false *iff* p is true and q is false.
- Therefore, $\sim(p \rightarrow q)$ is true *iff* p is true and q is false
- In other words, the negation of "if p then q" is logically equivalent to "p and not q"
  - $\sim(p \rightarrow q) \equiv p \wedge \sim q$
    - Use truth tables to show the equivalence;
    - Use the laws of logical equivalences to show
    - $\sim(p \rightarrow q) \equiv \sim(\sim p \vee q) \equiv \sim(\sim p) \wedge (\sim q) \equiv p \wedge \sim q$
- Negation of an "if-then" statement does not start with the word if!
  - P: If there is a heavy traffic in Bang-Na road, then I cannot get to class.
  - ~P: There is a heavy traffic in Bang-Na road and (but) I can get to class.

# Converse of Conditional Statements

- If you get a full mark in the final exam, you will get a grade of ‘C’ or higher. $(p \rightarrow q)$

- Converse of $p \rightarrow q$ : $q \rightarrow p$
  - If you get a grade of ‘C’ or higher, then you would have got a full mark in the final exam.

- Note that a conditional statement and its converse is not logically equivalent
  - Why?
  - Because it’s possible to get a grade of ‘C’ or higher by other means even though you don’t get a full mark in the final exam, getting a grade of ‘C’ or higher does not imply that you got a full mark in the final exam.

# Inverse of Conditional Statements

- If you get a full mark in the final exam, you will get a grade of ‘C’ or higher. $(p \rightarrow q)$

- Inverse of $p \rightarrow q$ : $\sim p \rightarrow \sim q$
  - If you don’t get a full mark in the final exam, you will not get a grade of ‘C’ or higher

- Note that a conditional statement and its inverse is not logically equivalent
  - Why?
  - It’s possible that you get a grade of ‘C’ or higher even though you don’t get a full mark in the final exam.

# Exercise

- Show using truth tables that
  - $p \rightarrow q \not\equiv q \rightarrow p$
  - $p \rightarrow q \not\equiv {\sim}p \rightarrow {\sim}q$

# Exercise

- Write the converse and inverse of each of the following statements:
  - If Howard can swim across the lake, then Howard can swim to the island.
  - If today is Easter, then tomorrow is Monday.
- Are the inverse and converse of a conditional statement logically equivalent to each other?

# Contrapositive of a Conditional Statement

- Contrapositive of $p \rightarrow q$ is $\sim q \rightarrow \sim p$
- For instance,
  - If you get a full mark in the final exam, you will get a grade of ‘C’ or higher. ($p \rightarrow q$)
  - If you don’t get a grade of ‘C’ or higher, then you would not have a full mark in the final exam. ( $\sim q \rightarrow \sim p$ )
- Are they equivalent?
- Reason using the given example
- Confirm by use of Truth tables (Exercise)

# Exercise

- Write each of the following statements in its equivalent contrapositive form:
  - If Howard can swim across the lake, then Howard can swim to the island.
  - If today is Easter, then tomorrow is Monday.

# Notes on Inverse and Converse of Conditional Statement

- The inverse and the converse of a conditional statement are logically equivalent to each other because they are contrapositive of each other.
  - $\sim p \rightarrow \sim q \equiv q \rightarrow p$
  - $p \rightarrow q \equiv \sim q \rightarrow \sim p$
  - $p \rightarrow q \not\equiv \sim p \rightarrow \sim q$
  - $p \rightarrow q \not\equiv q \rightarrow p$

# Only If

- p only if q
  - I will give you 100 Baht only if you eat all the veggies.
  - p can take place only if q takes place also
  - If q does not take place, then p cannot take place
    - $\sim q \rightarrow \sim p$
  - If p occurs then q must also occur since p occurs only if q occurs
    - $\sim q \rightarrow \sim p \equiv p \rightarrow q$
- p only if q means
  - “if p then q”, which is equivalent to
  - “if not q then not p”

# Exercise

Rewrite the following statement in if-then form in two ways, one of which is the contrapositive of the other.

- John will break the world's record for the mile run only if he runs the mile in under four minutes.

- If John does not run the mile in four minutes, then he will not break the world's record.

- If John breaks the world's record, then he will have run the mile in under four minutes.

# Biconditional (if and only if)

- Biconditional of p and q is a statement of form

  “p if, and only if q”

- $p \leftrightarrow q$
- true if both p and q have the same truth values
- false if p and q have opposite truth values

# Biconditional (if and only if)

- Show that $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

| $p$ | $q$ | $p \leftrightarrow q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | T |

| $p$ | $q$ | $p \rightarrow q$ | $q \rightarrow p$ | $p \leftrightarrow q$ | $(p \rightarrow q) \wedge (q \rightarrow p)$ |
|---|---|---|---|---|---|
| T | T | T | T | T | T |
| T | F | F | T | F | F |
| F | T | T | F | F | F |
| F | F | T | T | T | T |

$p \leftrightarrow q$ and $(p \rightarrow q) \wedge (q \rightarrow p)$ always have the same truth values, so they are logically equivalent

# Exercise

- Rewrite the following statement as a conjunction of two “if-then” statements:
  - This computer program is correct if, and only if, it produces correct answers for all possible sets of input data.

# Necessary and Sufficient Conditions

- if p then q
  - if p happens then q also happens
  - p is a sufficient condition for q
- if not q then not p (equivalent to if p then q)
  - If q does not occur then p cannot occur either
  - q is a necessary condition for p
- p only if q (equivalent to if p then q)
  - q is a necessary condition for p
- p if and only if q
  - p is a necessary and sufficient condition for q

# Exercise

- Manchester United will win the UEFA Champions League title only if they beat Barcelona in the semi-final.
- Transform into equivalent if p then q form
- Transform into the contrapositive form

# Exercise

- Manchester United will win the UEFA Champions League title only if they beat Barcelona in the semi-final.
  - Re-write an equivalent statement using 'necessary condition'
  - Re-write an equivalent statement using 'sufficient condition'

# Exercise

- Manchester United will win the UEFA Champions League title only if they beat Barcelona in the semi-final.
  - Is Beating Barcelona in the semi-final a sufficient condition for Manchester United to win the UEFA Champions League title?
    Explain.
  - Is Manchester United winning the UEFA Champions League title a necessary condition for Manchester United to beat Barcelona in the semi-final?
    Explain.

# Caution!

- Be careful with ambiguities in everyday English!
- Jenny, if you eat all the veggies, then you will get an ice-cream. (to emphasize reward)
  - Jenny, you will get an ice-cream only if you eat all the veggies. (to emphasize punishment)
  - Jenny, if you don't eat all the veggies, you will not get an ice-cream
- Either way, what does Jenny's mum really mean?
  - If you eat all the veggies then you will get an ice-cream AND if you do not eat all the veggies, then you will not get dessert (you will get an ice-cream only if you eat all the veggies)
- She actually means "Jenny, you will get an ice-cream if d only if you eat all the veggies"
  - she means "bi-conditional" not "conditional"
  - But not many ordinary people say such a way. (if and onl

In formal logic, such ambiguities are not allowed.