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ANALYSING INFLATION BY THE FRACTIONALLY INTEGRATED ARFIMA–GARCH MODEL

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SUMMARY

This paper considers the application of long-memory processes to describing inflation for ten countries. We implement a new procedure to obtain approximate maximum likelihood estimates of an ARFIMA–GARCH process; which is fractionally integrated $I(d)$ with a superimposed stationary ARMA component in its conditional mean. Additionally, this long memory process is allowed to have GARCH type conditional heteroscedasticity. On analysing monthly post-World War II CPI inflation for ten different countries, we find strong evidence of long memory with mean reverting behaviour for all countries except Japan, which appears stationary. For three high inflation economies there is evidence that the mean and volatility of inflation interact in a way that is consistent with the Friedman hypothesis.

1. INTRODUCTION

This paper is concerned with the long-memory, fractionally integrated ARFIMA process with conditionally heteroscedastic innovations. Following the work of Granger (1980), Granger and Joyeux (1980), and Hosking (1981) several recent studies have dealt with the estimation of the ARFIMA process. For data with time-dependent conditional heteroscedasticity, one convenient extension is to consider the ARFIMA model with GARCH type innovations. This model can provide a useful way of analysing the relationships between the conditional mean and variance of a process exhibiting long memory and slow decay in its level, yet with time-varying volatility.

The aggregate price level is clearly one of the key variables in understanding the macroeconomy. One important issue concerns how aggregate prices respond to shocks. Many studies such as Nelson and Schwert (1977), Barsky (1987), Ball and Cecchetti (1990), and Kim (1993) have found evidence of two unit roots in prices, so that any shock has a permanent effect on inflation. Brunner and Hess (1993) argue that there is evidence of a unit root in inflation subsequent to 1960, but evidence that inflation is $I(0)$ prior to this time. The apparent non-stationarity of inflation is far from innocuous in terms of its economic implications. For example, there is widespread evidence that nominal interest rates contain a unit root (e.g. Fama and Gibbons, 1982; Mankiw and Miron, 1986). Accepting these findings implies that for real rate of interest to be stationary (e.g. Fama, 1975); then inflation should necessarily have a unit root and also be cointegrated with the nominal interest rate. As noted by Rose (1988), the alternative possibility of *ex-post* real rates of interest being non-stationary poses problems for

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the interpretation of conventional capital asset pricing models. An inflation process which is non-stationary also presents difficulties in the construction of optimal policy rules of the type suggested by McCallum (1988). Some of these issues are further described by Baillie (1989) in the context of attempting to construct a commodity price rule to use for monetary policy.

This paper considers the possibility that inflation is a long-memory, fractionally integrated process with time-dependent heteroscedasticity. Considerable motivation exists for this assumption. On taking monthly CPI inflation from 1948 to 1990 for the G7 countries and also for three high-inflation economies, the application of standard unit root tests with a null hypothesis of a unit root and the new KPSS test with a null hypotheses of stationarity indicate rejection of both I(1) and I(0) behaviour for inflation. On the application of an approximate MLE technique, described in more detail in Chung and Baillie (1993), the subsequent econometric estimation results indicate the existence of long-memory, fractional integration for all countries' inflation, except Japan.

One contribution of this paper is to extend the ARFIMA process, which has a fractionally integrated conditional mean with the Generalized Autoregressive Conditional Heteroskedastic (GARCH) process to describe time-dependent heteroscedasticity. The conditional density of the innovations is Student t, or Normal, depending on the degree of kurtosis in the data. For the high-inflation countries there is support for the Friedman hypothesis with strong interactions between the conditional mean and the variance of inflation.

2. ESTIMATION OF THE ARFIMA–GARCH PROCESS

The most general model used in this paper is denoted as the ARFIMA(p, d, q)–GARCH(P, Q) process,

$$\phi(L)(1-L)^d(y_t - \mu - b'x_{1t} - \delta\sigma_t) = \theta(L)\varepsilon_t \tag{1}$$

$$\varepsilon_t \,|\, \Omega_{t-1} \sim D(0, \sigma_t^2) \tag{2}$$

$$\beta(L)\sigma_t^2 = \omega + \alpha(L)\varepsilon_t^2 + \gamma' x_{2t} \tag{3}$$

where $y_t = 100\Delta \log \mathrm{CPI}_t$ and is CPI inflation, x_{1t} and x_{2t} are vectors of predetermined variables, μ is the mean of the process, $\phi(L) = 1 - \phi_1 L - \cdots - \phi_p L^p$, $\theta(L) = 1 + \theta_1 L + \cdots + \theta_q L^q$, $\beta(L) = 1 - \beta_1 L - \cdots - \beta_p L^P$, $\alpha(L) = \alpha_1 L + \alpha_2 L^2 + \cdots + \alpha_p L^Q$ and all the roots of $\phi(L)$, $\theta(L)$, $\beta(L)$, and $\alpha(L)$ lie outside the unit circle. With $\delta = 0$ and $b = 0$, equations (1) and (2) describe the ARFIMA process introduced by Granger (1980, 1981) and Granger and Joyeux (1980). With $\delta \neq 0$, the model is extended to allow volatility to influence mean inflation. The innovations ε_t are assumed to follow a conditional density D, which is either Normal or Student t and the time-dependent heteroscedasticity σ_t^2 follows the Generalized Autoregressive Conditionally Heteroskedastic, or GARCH(P, Q) model of Engle (1982) and Bollerslev (1986). Lagged inflation, which is predetermined, is allowed to possibly enter the conditional variance equation (3) through being included in x_{2t}.

The population characteristics of ARFIMA processes have been extensively studied by Granger (1980), Granger and Joyeux (1980), and Hosking (1981). For $-0{\cdot}5 < d < 0{\cdot}5$ the process y_t in equation (1) is covariance stationary and the moving average coefficients decay at a relatively slow hyperbolic rate compared with the stationary and invertible ARMA process where the moving average coefficients decline exponentially with increasing lag. A detailed discussion of these processes is given in Baillie (1995).

Geweke and Porter-Hudak (1983) initially suggested a semi-parametric estimate of d, which was subsequently used in applied work by Diebold and Rudebusch (1989) and others. However, as shown by Agiakoglou *et al.* (1992), there are many situations where this estimator has poor small sample properties and interest has shifted towards maximum likelihood estimation. Sowell (1992) has recently derived the full maximum likelihood estimator (MLE) for the ARFIMA(p, d, q) process with normally distributed innovations. Under normality, the logarithm of the likelihood can be expressed in the time domain as

$$L(\mu, d, \phi, \theta, \sigma^2) = -(T/2)\log(2\pi) - (1/2)\log|\Sigma| - (1/2)(y-\mu)'\Sigma^{-1}(y-\mu) \tag{4}$$

where y is the T-dimensional vector of y_t and Σ is the corresponding $T \times T$ autocovariance matrix, where each element is a nonlinear function of hypergeometric functions. Sowell (1992) has provided an approach for computing full MLEs of the basic ARFIMA model with unconditional normality and no ARCH effects. The approach is computationally demanding, with every iteration of the likelihood requiring inversion of a T-dimensional covariance matrix, each element of which is a nonlinear function of hypergeometric functions. See Chung (1994) for further details.

For estimating the complicated ARFIMA–GARCH process (1) to (3), with a conditional Normal density, we use an alternative conditional sum of squares (CSS) estimator which minimizes the quantity,

$$S(\mu, d, \phi, b, \delta, \theta, \alpha, \beta, \omega) = (1/2)\log(\sigma_t^2) + (1/2)\Sigma_{t=1,T}\varepsilon_t^2/\sigma_t^2 \tag{5}$$

where

$$\varepsilon_t = \theta(L)^{-1}\phi(L)(1-L)^d(y_t - b'x_{1t} - \delta\sigma_t - \mu)$$

and

$$\sigma_t^2 = \beta(L)^{-1}[\omega + \gamma' x_{2t} + \alpha(L)\varepsilon_t^2]$$

In our application, $b = \delta = \gamma = 0$. When the conditional density D in equation (2) is Student t with ν degrees of freedom, then following Bollerslev (1987), the CSS estimator maximizes the logarithmic likelihood,

$$\begin{aligned} L(\mu, d, \phi, \theta, \alpha, \beta, \sigma^2, \nu) = {} & T[\log\Gamma\{(\nu+1)/2\} - \log\Gamma(\nu/2) - (1/2)\log(\nu-2)] \\ & - (1/2)\Sigma_{t=1,T}\{\log(\sigma_t^2) + (\nu+1)[\log(1+\varepsilon_t^2\sigma_t^{-2}(\nu-2)^{-1}]\} \end{aligned}$$

If the initial observations $y_0, y_{-1}, y_{-2}, \ldots$ are assumed fixed, then minimizing the conditional sum of squares function will be asymptotically equivalent to MLE.

The CSS estimation procedure in the context of ARFIMA processes was originally suggested by Hosking (1984). It is worth noting that similar estimation methods have been implemented in the stationary and invertible class of ARMA models. Box and Jenkins (1976) used the minimum CSS estimator, while Newbold (1974) considered the full MLE with the initial observations treated as stochastic. Some of the properties of the CSS estimator in the context of ARFIMA(p, d, q) models are discussed by Chung and Baillie (1993). In particular, they show that the effect of initial observations are asymptotically negligible so that the CSS estimator is asymptotically equivalent to MLE. Chung and Baillie also report simulation results which shows the CSS estimator performs well when estimating ARFIMA(p, d, q) models with p and q being 0, 1 or 2; $-1/2 < d < 1/2$ and for sample sizes greater than 100. The CSS estimator can be regarded as being a time domain alternative to the Fox and Taqqu (1986) frequency-domain estimator of the ARFIMA model, which is known to also be approximate MLE under Normality. Results on th asymptotic properties of MLEs of the parameters in ARFIMA models are given by Dahlhaus (1988, 1989), Moehring (1990), and Yajima (1988). The Fox Taqqu estimator has been used in empirical studies by Diebold *et al.* (1991).

Table I. Simulation results of estimating the ARFIMA(0, d, 0) model $(1-L)^d(y_t-\mu)=\varepsilon_t$

True d	Mean	MSE	Mean SE	True SE
Estimation of the d parameter				
Sample size of $T=500$				
0·45	0·444	0·038	0·036	0·035
0·25	0·242	0·037	0·036	0·035
0·05	0·042	0·037	0·036	0·035
0·00	−0·011	0·038	0·036	0·035
−0·05	−0·059	0·036	0·036	0·035
−0·25	−0·259	0·037	0·036	0·035
−0·45	−0·454	0·037	0·036	0·035
Sample size of $T=300$				
0·45	0·440	0·050	0·048	0·045
0·25	0·234	0·048	0·048	0·045
0·05	0·035	0·050	0·048	0·045
0·00	−0·014	0·049	0·048	0·045
−0·05	−0·065	0·048	0·048	0·045
−0·25	−0·264	0·050	0·048	0·045
−0·45	−0·457	0·052	0·047	0·045
Sample size of $T=100$				
0·45	0·418	0·093	0·088	0·078
0·25	0·205	0·095	0·088	0·078
0·05	0·005	0·096	0·088	0·078
0·00	−0·047	0·096	0·088	0·078
−0·05	−0·096	0·093	0·088	0·078
−0·25	−0·300	0·091	0·088	0·078
−0·45	−0·482	0·093	0·087	0·078
Estimation of the μ parameter				
Sample size of $T=500$				
0·45	1·006	0·858	0·473	0·478
0·25	0·999	0·215	0·180	0·184
0·05	1·004	0·061	0·058	0·060
0·00	1·001	0·047	0·043	0·045
−0·05	1·001	0·033	0·032	0·033
−0·25	0·999	0·011	0·010	0·011
−0·45	1·000	0·004	0·003	0·003
Sample size $T=300$				
0·45	1·020	0·893	0·491	0·499
0·25	0·998	0·244	0·198	0·209
0·05	0·998	0·076	0·071	0·075
0·00	1·000	0·059	0·055	0·058
−0·05	1·000	0·045	0·042	0·044
−0·25	1·001	0·016	0·015	0·015
−0·45	0·999	0·032	0·005	0·005
Sample size of $T=100$				
0·45	1·029	0·881	0·520	0·550
0·25	0·994	0·338	0·247	0·277
0·05	0·999	0·120	0·109	0·123
0·00	1·007	0·100	0·089	0·100
−0·05	1·000	0·081	0·071	0·081
−0·25	0·999	0·037	0·030	0·035
−0·45	0·999	0·020	0·014	0·015

Note:
Without loss of generality, μ is set as unity in all experiments.

Chung and Baillie (1993) have provided evidence on the biases of parameters estimated by the CSS method in ARFIMA(p, d, q) processes, with p and $q \leq 2$ and with a sample size of 100. The study particularly focused on the effects generated by unknown μ. Table I reports complementary evidence on applying the CSS estimator to sample sizes of 100, 300, and 500 in the fractional white noise model,

$$(1-L)^d(y_t-\mu)=\varepsilon_t \qquad \varepsilon_t \sim \mathrm{NID}(o, \sigma^2)$$

and indicates the relative biases of the estimates of d and μ and the performance of the Hessian when used to compute the asymptotic standard errors. For each replication in the simulation study, T values from a standard normal random variable were placed into a column vector, e. The analytic autocovariance matrix is Σ and C is the Cholesky decomposition matrix, where $CC' = \Sigma$. The vector of T observations for $T = 100$, 300, and 500 was then constructed as $y = \mu + Ce$. Without loss of generality, the values of μ and σ^2 are set to unity in all the simulation experiments.

It is known that the MLE of μ converges at the relatively slow rate of $T^{d-1/2}$, while the other parameter estimates converge at the usual $T^{-1/2}$ rate. As can be seen from Table I, the average bias of the parameter estimates are very reasonable and the Hessian appears to provide a good estimate of the true asymptotic standard errors of the parameter estimates. Similar results are also available for the estimates of σ^2 but are omitted for reasons of space. These results provide further evidence to those in Chung and Baillie (1993) on the satisfactory nature of the CSS estimator.

A number of experiments to compare the CSS estimator with that of Sowell's (1992) full MLE were also conducted. For sample sizes of 100 or more, there was virtually no difference between the estimators. These comparative simulation results are consistent with the findings of Cheung and Diebold (1994) who compared Sowell's full MLE with the Fox–Taqqu approximate MLE. Further details of the comparison between the CSS estimator and Sowell's full MLE are available from the authors on request.

While the above results and discussion provide reassuring evidence on the finite sample performance of the CSS estimator for the ARFIMA model, the truly great advantage with the CSS estimator is that it allows approximate MLE to be calculated for quite complicated models such as the ARFIMA–GARCH process in equations (1) to (3). It is well known that even for relatively simple nonlinear dynamic models (e.g. Engel's (1982) ARCH process) full MLE is either computationally extremely demanding or completely intractable, so that a type of CSS estimator has to be used.

3. THE PERSISTENCE AND VARIABILITY OF INFLATION

Many previous studies have examined the characteristics of aggregate US CPI (Consumer Price Index) inflation. Klein (1976) and Nelson and Schwert (1977) impose a unit root on the inflation process; while Ball and Cecchetti (1990) and Kim (1993) model inflation as a transitory and a permanent component, which is represented as a random walk. Barsky (1987) and Brunner and Hess (1993) argue that inflation was covariance stationary until 1960, but is subsequently well represented as an I(1) process. In order to develop some sense of the international similarities of inflation, Table II gives the autocorrelations for CPI inflation for Argentina, Brazil, Canada, France, Germany, Israel, Italy, Japan, the UK and the USA. The data are monthly, not seasonally adjusted, and generally exhibit the clear pattern of slow decay and persistence. The autocorrelations of the first differenced inflation series in Table III generally appear to be overdifferenced with large negative autocorrelations at lag one.

Table II. Autocorrelations of inflation series

Lag	Argentina	Brazil	Canada	France	Germany	Israel	Italy	Japan	UK	USA
1	0·758	0·886	0·434	0·428	0·362	0·736	0·253	0·121	0·267	0·467
2	0·561	0·789	0·369	0·169	0·275	0·653	0·280	0·092	0·232	0·423
3	0·368	0·698	0·409	0·120	0·210	0·671	0·316	0·180	0·197	0·399
4	0·272	0·645	0·380	0·088	0·011	0·635	0·240	0·036	0·203	0·360
5	0·373	0·611	0·362	−0·102	0·063	0·658	0·368	0·098	0·223	0·316
6	0·410	0·588	0·288	−0·077	−0·026	0·658	0·204	0·082	0·313	0·305
7	0·464	0·557	0·311	−0·078	−0·024	0·602	0·310	0·025	0·160	0·312
8	0·510	0·531	0·349	−0·022	0·047	0·614	0·326	0·166	0·146	0·359
9	0·420	0·510	0·317	−0·025	0·000	0·615	0·194	0·112	0·210	0·386
10	0·355	0·506	0·272	0·057	0·044	0·600	0·220	−0·021	0·173	0·349
11	0·293	0·531	0·311	0·155	0·069	0·575	0·194	−0·031	0·201	0·320
12	0·281	0·549	0·419	0·222	0·055	0·624	0·370	0·151	0·403	0·278
13	0·228	0·552	0·286	0·193	0·070	0·507	0·229	−0·076	0·156	0·234
14	0·219	0·550	0·216	0·113	−0·016	0·470	0·193	−0·135	0·144	0·180
15	0·192	0·512	0·235	0·140	0·095	0·456	0·246	−0·083	0·168	0·211
16	0·173	0·498	0·225	0·102	0·021	0·445	0·188	−0·019	0·106	0·229
17	0·164	0·773	0·195	−0·035	0·184	0·461	0·296	−0·088	0·138	0·160
18	0·163	0·449	0·144	−0·043	−0·135	0·487	0·199	0·013	0·192	0·129
First	Jan. 57	Jan. 57	Jan. 48	Jan. 48	Jan. 48	Jan. 57	Jan. 48	Jan. 48	Jan. 48	Jan. 48
Last	Dec. 90	Jan. 91	Aug. 90	July 90	March 90	Feb. 91	June 90	July 90	Aug. 90	July 90

Note:
The inflation series are defined as 100Δ log CPI_t.

Table III. Autocorrelations of first differenced inflation series

Lag	Argentina	Brazil	Canada	France	Germany	Israel	Italy	Japan	UK	USA
1	−0·092	−0·074	−0·437	−0·191	−0·419	−0·343	−0·505	−0·467	−0·471	−0·499
2	−0·009	−0·024	−0·094	−0·214	−0·017	−0·191	−0·018	−0·075	−0·003	0·112
3	−0·201	−0·169	0·057	−0·030	0·063	0·102	0·072	0·130	−0·024	−0·086
4	−0·304	−0·087	−0·004	0·150	−0·082	−0·114	−0·138	−0·108	−0·018	0·032
5	−0·077	−0·015	0·038	−0·189	0·069	0·047	0·185	0·028	−0·033	−0·050
6	0·072	−0·019	−0·075	−0·100	−0·025	0·104	−0·153	0·038	0·160	−0·013
7	0·013	0·002	−0·018	−0·018	−0·054	−0·130	0·057	−0·119	−0·095	−0·024
8	0·281	−0·024	0·068	0·108	0·093	0·021	0·087	0·095	−0·053	0·028
9	−0·050	−0·066	0·017	−0·033	−0·074	0·030	−0·098	0·058	0·065	0·042
10	−0·008	−0·151	−0·070	−0·050	−0·012	0·020	0·030	−0·069	−0·041	−0·029
11	−0·104	−0·094	−0·072	0·041	0·035	−0·140	−0·132	−0·105	−0·120	0·024
12	0·086	0·092	0·222	0·163	0·003	0·314	0·209	0·226	0·309	0·037
13	−0·091	0·013	−0·052	0·051	0·089	−0·151	−0·070	−0·094	−0·161	−0·034
14	0·039	0·204	−0·084	−0·151	−0·035	−0·044	−0·052	−0·065	−0·025	−0·089
15	−0·016	−0·102	0·025	0·062	−0·134	−0·005	0·067	−0·005	0·050	0·073
16	−0·021	0·069	0·018	0·082	0·178	−0·051	−0·116	0·069	−0·056	0·008
17	−0·018	−0·004	0·008	−0·110	−0·127	−0·019	0·130	−0·100	−0·016	0·084
18	−0·033	−0·001	−0·034	−0·051	0·050	0·159	−0·064	0·117	0·078	−0·036

Note:
The series are defined as $100(\Delta^2 \log CPI_t)$.

Wichern (1973) has noted that such an autocorrelation pattern is consistent with the properties of an ARIMA(0, 1, 1) process where the moving-average parameter is negative and is relatively close to minus one. Schwert (1987) has discussed the performance of the Dickey and Fuller (1979, 1981) and Phillips Perron tests of Phillips (1987), Phillips and Perron (1988), and Perron (1988), when the true data-generating process is I(1) with a large negative moving-average coefficient. Schwert notes that size distortions can give rise to rejecting a unit root too often in favour of an I(0) stationary process. On finding the true distribution of the Dickey and Fuller $\hat{\tau}$ test analogous to a regression *t*-statistic, Schwert (1987, p. 98) concludes that for the US CPI series, the 'unit root hypothesis should not be rejected'. In a study involving seven countries' inflation rates, Baillie (1989, p. 211), notes that 'CPI series are borderline I(2), or ... "near integrated" I(2)', and concludes that 'aggregate prices and inflation are not well described by the ARIMA class of models'.

Most of the standard tests for stationarity involve a null hypothesis containing a unit root. Classical statistical hypothesis testing generally ensures that a null hypothesis is only rejected in the face of very strong evidence against it. If an investigator wishes to test stationarity as a null and has strong priors in its favour, then it is not clear that the conventional Dickey–Fuller parameterization is very useful. Kwiatkowski, Phillips, Schmidt, and Shin (1992) (henceforth KPSS) have developed an alternative approach of testing for unit roots and they impose stationarity under the null. On assuming that a time series can be decomposed into the sum of a deterministic trend, random walk, and stationary I(0) disturbance, KPSS show that a score test of the null of stationarity can be based on the statistic

$$\eta = T^{-2}\Sigma_{t=1,T} S_t^2 / s^2(k)$$

where

$$S_t = \Sigma_{i=1,t} e_i$$

is the partial sum of the residuals e_i, when the series has been regressed on an intercept and possibly also a time trend, and T is the sample size. $s^2(k)$ is a consistent non-parametric estimate of the disturbance variance; it is computed in an identical manner to its equivalent in the Phillips and Perron test by using a Bartlett window adjustment based on the first k sample autocovariances as suggested by Newey and West (1987). When the residuals are computed from an equation with only an intercept, the test statistic is denoted by $\hat{\eta}_\mu$, and when a time trend is included in the initial regression, the test statistic is denoted by $\hat{\eta}_\tau$. Under the null hypothesis of the series being I(0), KPSS show that both $\hat{\eta}_\mu$ and $\hat{\eta}_\tau$ are asymptotically functionals of a Brownian bridge and they produce tables of critical values. The critical values for $\hat{\eta}_\mu$ and $\hat{\eta}_\tau$ are 0·739 and 0·216 at the 0·01 level and 0·463 and 0·146 at the 0·05 level, respectively. The test is an upper tail test.

The combined use of the Phillips Perron (PP) and KPSS test statistics gives rise to four possible outcomes:

(1) Rejection by the PP statistic and failure to reject by the KPSS statistic is viewed as strong evidence of covariance stationarity, i.e. an I(0) process.
(2) Failure to reject by the PP and rejection by the KPSS statistic appears to be strongly indicative of a unit root, i.e. an I(1) process.
(3) Failure to reject by both the PP and KPSS statistics is probably due to the data being insufficiently informative on the long-run characteristics of the process.
(4) Rejection by both the PP and KPSS statistics presumably indicates evidence of some process that is well described by neither an I(1) or an I(0) process.

While KPSS describe their null hypothesis as one of stationarity, it is more readily described as one of mixing, since it satisfies the regularity conditions of Phillips (1987) and Phillips and Perron (1988). Since fractionally integrated $I(d)$ processes do not satisfy these conditions, rejection by the KPSS statistic is consistent with fractionally integrated $I(d)$ behaviour. In the results reported in Table IV, the number of lags in the Bartlett kernel for computing $s(k)$ in the KPSS tests was chosen at 8. This number is consistent with the recommendations in Kwiatkowski *et al.* (1992) and also in Schwert (1989), who recommended basing the number of lags in residual autocorrelations for unit root tests as $\mathrm{int}\{12(T100)^{1/4}\}$. We also computed the KPSS test statistics using lags, but this did not change the qualitative conclusions of the KPSS tests and are not reported to conserve space. Considerable additional motivation for fractional integration has been provided by Diebold and Rudebusch (1991), who show that Dickey–Fuller unit root tests have low power in distinguishing unit roots from fractional alternatives.

Table IV presents the results of applying the PP and KPSS tests to the ten inflation series. For Argentina, Brazil, Canada, France, Italy, Israel, the UK, and the USA situation (4) above arises where it is possible to reject both a unit root and stationarity. Hence for eight countries there is evidence that inflation may not be generated by an I(0) or I(1) process and is at least indicative of fractional integration. However, for both Germany and Japan, the PP and KPSS test statistics are indicative of inflation being I(0) in these countries. Indeed, subsequent estimation of an ARFIMA–GARCH models for Japan realized an estimated value of d close to and not significantly different from zero. For Germany, the estimated value of d was relatively small (0·18), but significantly different from zero.

Table IV. Tests for order of integration of different countries' inflation series

	H_0: I(1)		H_0: I(0)	
Country	$Z(t_{\alpha^*})$	$Z(t_{\tilde{\alpha}})$	$\hat{\eta}_\mu$	$\hat{\eta}_\tau$
Argentina	$-4{\cdot}78^b$	$-3{\cdot}67^a$	$2{\cdot}56^b$	$0{\cdot}21^a$
Brazil	$-3{\cdot}48^b$	$-2{\cdot}50$	$1{\cdot}44^b$	$0{\cdot}44^b$
Canada	$-10{\cdot}61^b$	$-6{\cdot}23^b$	$1{\cdot}13^b$	$0{\cdot}26^b$
France	$-12{\cdot}78^b$	$-10{\cdot}48^b$	$0{\cdot}20$	$0{\cdot}22^b$
Germany	$-15{\cdot}11^b$	$-13{\cdot}12^b$	$0{\cdot}24$	$0{\cdot}14$
Italy	$-15{\cdot}19^b$	$-9{\cdot}06^b$	$2{\cdot}70^b$	$0{\cdot}46^b$
Israel	$-4{\cdot}51^b$	$-3{\cdot}48^a$	$2{\cdot}16^b$	$0{\cdot}36^b$
Japan	$-18{\cdot}38^b$	$-15{\cdot}76^b$	$0{\cdot}33$	$0{\cdot}17$
UK	$-14{\cdot}35^b$	$-8{\cdot}76^b$	$0{\cdot}88^b$	$0{\cdot}26^b$
USA	$-9{\cdot}64^b$	$-5{\cdot}66^b$	$1{\cdot}80^b$	$0{\cdot}36^b$

Notes:
$Z(t_{\alpha^*})$ and $Z(t_{\tilde{\alpha}})$ are the Phillips–Perron adjusted t-statistics of the lagged dependent variable in a regression with intercept only, and intercept and time trend included, respectively. The 0·05 critical values for $Z(t_{\alpha^*})$ and $Z(t_{\tilde{\alpha}})$ are $-2{\cdot}86$ and $-3{\cdot}41$, respectively; while the 0·01 critical values are $-3{\cdot}43$ and $-3{\cdot}96$, respectively.
$\hat{\eta}_\mu$ and $\hat{\eta}_\tau$ are the KPSS test statistics described in the text and are based on residuals from regressions with an intercept and intercept and time trend, respectively. The 0·05 critical values for $\hat{\eta}_\mu$ and $\hat{\eta}_\tau$ are 0·463 and 0·146, respectively; while the 0·01 critical values are 0·739 and 0·216, respectively.
All the test statistics reported in this table were based on Newey and West (1987) adjustments using 8 lags.
[a] Significant at the 0·05 level.
[b] Significant at the 0·01 level.

Table V. Autocorrelations of filtered US CPI inflation series

lag/d	0·0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0
1	0·467	0·313	0·121	−0·028	−0·142	−0·231	−0·302	−0·360	−0·409	−0·451	−0·499
2	0·423	0·353	0·220	0·137	0·090	0·067	0·058	0·058	0·064	0·073	0·112
3	0·399	0·225	0·084	0·000	−0·045	−0·067	−0·076	−0·078	−0·076	−0·073	−0·096
4	0·360	0·221	0·095	0·028	−0·003	−0·015	−0·015	−0·011	−0·006	0·001	0·032
5	0·316	0·172	0·046	−0·020	−0·050	−0·060	−0·061	−0·057	−0·051	−0·046	−0·050
6	0·305	0·185	0·069	0·009	−0·016	−0·024	−0·024	−0·020	−0·014	−0·009	−0·013
7	0·312	0·207	0·094	0·034	0·005	−0·009	−0·014	−0·016	−0·016	−0·016	−0·024
8	0·359	0·257	0·154	0·097	0·006	0·049	0·039	0·032	0·027	0·023	0·028
9	0·386	0·275	0·179	0·125	0·096	0·079	0·069	0·061	0·056	0·052	0·042
10	0·349	0·235	0·132	0·073	0·038	0·017	0·003	−0·008	−0·016	−0·022	−0·029
11	0·320	0·242	0·156	0·101	0·076	0·062	0·054	0·049	0·045	0·042	0·024
12	0·278	0·213	0·122	0·076	0·054	0·043	0·037	0·034	0·031	0·030	0·037
13	0·234	0·135	0·035	−0·016	−0·038	−0·048	−0·050	−0·050	−0·048	−0·045	−0·034
14	0·180	0·103	0·001	−0·050	−0·074	−0·083	−0·086	−0·086	−0·084	−0·082	−0·089
15	0·211	0·187	0·110	0·076	0·063	0·060	0·060	0·061	0·062	0·063	0·073

Notes:
Each series is $100(1-L)^d \, \Delta \log \mathrm{CPI}_t = (1-L)^d y_t$. The CPI series were obtained from January 1947 to September 1990 and the first 30 observations were omitted before the autocorrelations were computed for each filtered series.

Table V reports the autocorrelations of the US inflation series filtered by $(1-L)^d$, for different values of d. Although far from providing any formal evidence for the presence of fractional integration, the autocorrelations in Table V are at least supportive of the notion that US inflation is $\mathrm{I}(d)$. The extremes are of substantial persistence in the original inflation series ($d=0$) to the case of apparent overdifferencing ($d=1$). Between these extreme values of d there is a range of values, particularly in the 0·3–0·5 range, where the US inflation series appears covariance stationary. Similar patterns are also apparent for the other countries' inflation series, but are not reported for reasons of space.

4. RESULTS OF ESTIMATING THE ARFIMA–GARCH MODELS

Table VI reports estimated ARFIMA$(0, d, 1)$–GARCH$(1, 1)$ models for the US inflation series. The models were estimated by minimizing the CSS conditioned on different values of d. The log likelihood appeared relatively flat in the range $0{\cdot}3 < d < 0{\cdot}5$, with an apparent trade-off between the fractional differencing parameter d and the moving-average parameter θ. As expected from the previous discussion the point estimate of μ and its standard error also change markedly with d. The speed of convergence of $\hat{\mu}$ to its limiting distribution is dependent on the value of d.

The CSS estimates of ARFIMA–GARCH models for all ten countries are reported in Table VII. The most general estimated models are seasonal and can be described as ARFIMA$(0, d, 1) \times (0, 0, 2)_{12}$–GARCH$(1, 1) \sim$ Student t. The seasonal moving-average parameters were necessary to account for the significant seasonality, which is evident for all the countries except the USA. The estimated value for d for the USA is 0·472, which is quite close to the semi-parametric estimate obtained by Geweke and Porter-Hudak (1983). Figure 1 plots the monthly rate of inflation and its corresponding conditional standard deviation from the model in Table VII.

Table VI. Estimated ARFIMA(0, d, 1)–GARCH(1, 1) ~ Student t models for US CPI inflation

$$100(1-L)^d(y_t-\mu)=(1+\theta L)\varepsilon_t$$
$$\varepsilon_t \mid \Omega_{t-1} \sim t(0, \sigma_t^2, \nu^{-1})$$
$$\sigma_t^2=\omega+\alpha\varepsilon_{t-1}^2+\beta\sigma_{t-1}^2$$

$\hat{d}$	0·00	0·10	0·20	0·30	0·40	0·50	0·60	0·70	0·80	0·90	1·00
$\hat{\mu}$	0·317	0·313	0·317	0·329	0·352	0·392	0·470	0·634	1·091	3·855	−7·460
	(0·019)	(0·029)	(0·046)	(0·076)	(0·129)	(0·225)	(0·404)	(0·753)	(1·530)	(2·641)	(5·755)
$\hat{\theta}$	0·301	0·180	0·067	−0·040	−0·143	−0·247	−0·354	−0·470	−0·609	−0·721	−0·810
	(0·037)	(0·040)	(0·042)	(0·043)	(0·045)	(0·047)	(0·049)	(0·052)	(0·052)	(0·043)	(0·031)
$\hat{\omega}$	0·0039	0·0041	0·0040	0·0037	0·0034	0·0031	0·0030	0·0030	0·0030	0·0029	0·0038
	(0·0019)	(0·0020)	(0·0020)	(0·0019)	(0·0018)	(0·0017)	(0·0016)	(0·0016)	(0·0016)	(0·0013)	(0·0016)
$\hat{\alpha}$	0·127	0·107	0·089	0·072	0·061	0·055	0·052	0·052	0·054	0·047	0·046
	(0·038)	(0·038)	(0·037)	(0·034)	(0·031)	(0·029)	(0·028)	(0·028)	(0·029)	(0·029)	(0·028)
$\hat{\beta}$	0·839	0·847	0·861	0·879	0·893	0·901	0·906	0·907	0·905	0·912	0·898
	(0·042)	(0·048)	(0·050)	(0·049)	(0·046)	(0·043)	(0·041)	(0·041)	(0·042)	(0·037)	(0·038)
$\hat{\nu}$	12·580	11·764	10·963	10·045	9·311	8·886	8·712	8·722	8·858	8·930	8·575
	(5·268)	(4·746)	(4·256)	(3·703)	(3·270)	(3·032)	(2·946)	(2·965)	(3·047)	(3·047)	(2·857)
$Q(10)$	281·09	135·67	61·97	30·40	20·50	20·52	24·54	29·93	34·56	33·94	29·02
$Q^2(10)$	6·37	7·83	9·82	11·35	11·90	11·54	10·79	10·17	9·99	10·01	9·55
m_3	0·25	0·27	0·28	0·27	0·24	0·20	0·17	0·14	0·13	0·13	0·11
m_4	4·61	4·55	4·53	4·56	4·59	4·61	4·59	4·55	4·50	4·60	4·68
$\mathcal{L}$‘	−125·30	−91·824	−72·11	−61·53	−56·91	−56·03	−57·24	−59·17	−60·50	−59·89	−57·84

Notes:
The variable y_t is defined as 100Δ log CPI. The statistics $Q(10)$ and $Q^2(10)$ are the Ljung–Box tests based on the residuals and squared residuals, m_3 and m_4 are the sample skewness and kurtosis statistics based on the standardized residuals and $\mathcal{L}$ is the maximized value of the log likelihood. All the models were estimated with their first 30 observations omitted, so that estimation uses these values for initialization.

Table VII. Estimated ARFIMA–GARCH ~ Student t models for countries' CPI inflation

$$100(1-L)^d(y_t-\mu)=(1+\theta_1 L)(1+\Phi_1 L^{12}+\Phi_2 L^{24})\varepsilon_t$$
$$\varepsilon_t \mid \Omega_{t-1} \sim t(0,\sigma_t^2,\nu)$$
$$\sigma_t^2=\omega+\alpha\varepsilon_{t-1}^2+\beta\sigma_{t-1}^2$$

Parameter	Argentina	Brazil	Canada	France	Germany	Israel	Italy	Japan	UK	USA
$\hat{d}$	0·598	0·595	0·386	0·452	0·181	0·591	0·449	0·084	0·202	0·472
	(0·086)	(0·061)	(0·083)	(0·058)	(0·051)	(0·080)	(0·056)	(0·056)	(0·048)	(0·065)
$\hat{\mu}$	−0·422	1·264	0·377	0·204	0·189	1·305	−0·549	0·230	0·306	0·306
	(5·064)	(2·631)	(0·159)	(0·304)	(0·061)	(0·794)	(0·553)	(0·086)	(0·112)	(0·176)
$\hat{\theta}_1$	−0·048	0·003	−0·237	−0·135	0·108	−0·385	−0·334	−0·026	−0·093	−0·223
	(0·109)	(0·092)	(0·108)	(0·076)	(0·063)	(0·267)	(0·103)	(0·077)	(0·065)	(0·083)
$\hat{\Phi}_1$	0·091	0·105	0·206	0·186	0·148	0·295	0·092	0·251	0·213	–
	(0·042)	(0·043)	(0·044)	(0·034)	(0·043)	(0·041)	(0·043)	(0·042)	(0·046)	
$\hat{\Phi}_2$	–	–	0·174	–	0·192	0·141	0·157	0·224	−0·193	–
			(0·040)		(0·036)	(0·028)	(0·031)	(0·033)	(0·035)	
$\hat{\omega}$	3·7150	0·0164	0·0025	0·0007	0·0008	0·1681	0·0045	0·0168	0·0099	0·0034
	(1·0088)	(0·0340)	(0·0024)	(0·0002)	(0·0007)	(0·1029)	(0·0037)	(0·0103)	(0·0100)	(0·0017)
$\hat{\alpha}$	0·961	0·224	0·055	0·012	0·038	0·248	0·121	0·099	0·046	0·094
	(0·255)	(0·150)	(0·026)	(0·005)	(0·015)	(0·097)	(0·040)	(0·033)	(0·029)	(0·032)
$\hat{\beta}$	0·174	0·820	0·923	0·972	0·951	0·778	0·881	0·882	0·932	0·870
	(0·070)	(0·123)	(0·040)	(0·006)	(0·016)	(0·056)	(0·031)	(0·034)	(0·044)	(0·040)
$\hat{\nu}$	3·927	5·367	11·663	5·640	5·074	3·190	3·944	5·492	3·983	7·817
	(1·023)	(1·533)	(5·183)	(1·324)	(1·118)	(0·576)	(1·877)	(1·231)	(0·719)	(2·304)
$Q(10)$	1·31	9·41	7·59	38·35	14·94	19·78	11·34	11·34	15·58	17·55
$Q^2(10)$	12·99	4·15	9·03	7·32	5·50	8·08	9·54	6·56	6·76	15·91
m_3	0·63	0·71	0·36	0·59	1·51	1·30	1·00	0·94	1·13	0·37
m_4	4·34	7·12	3·80	5·45	12·96	8·92	6·30	7·69	7·25	4·94
$3(\hat{\nu}-2)/(\hat{\nu}-4)$	n.a.	7·39	3·78	6·66	8·59	n.a.	n.a.	7·02	n.a.	4·57
$\log \mathcal{L}$	−1002·53	−722·12	−165·31	−186·38	−184·67	−788·44	−412·44	−670·16	−459·50	−99·64

Notes:
As for Table V. Additionally, $3(\nu-2)/(\nu-4)$ is the implied kurtosis of the standardized residuals and should be compared with m_4, the sample equivalent. The statistic is only applicable when $\nu>4$; otherwise the implied kurtosis is infinite.

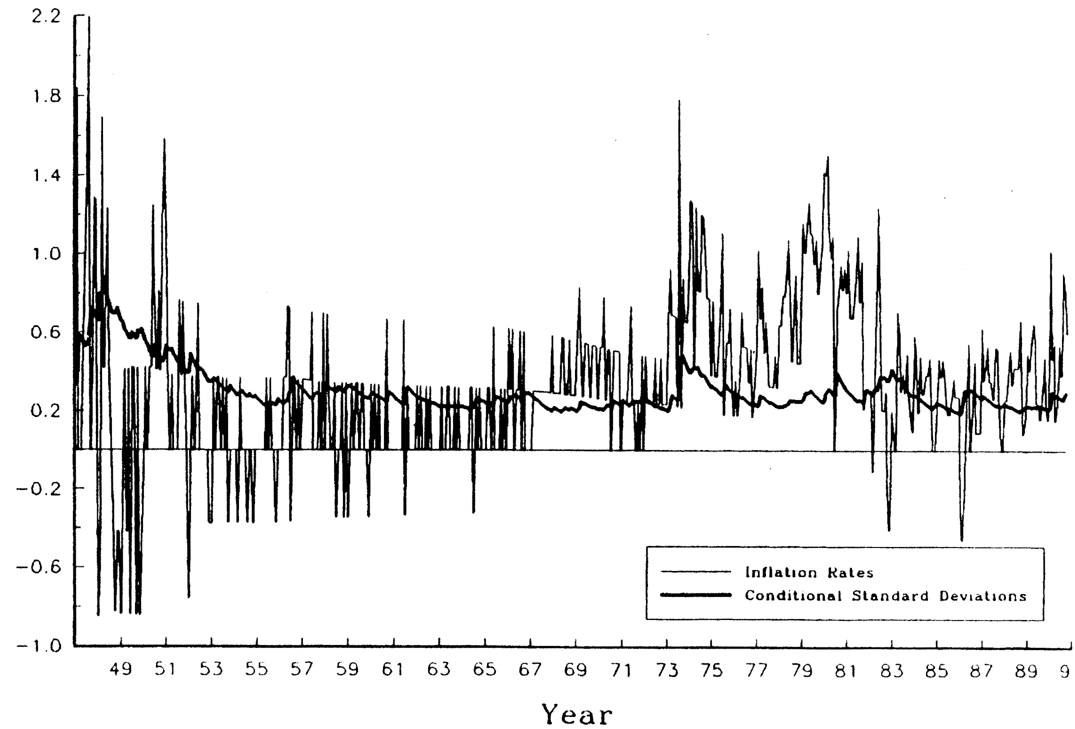

Figure 1. The conditional standard deviations based on the ARFIMA–GARCH model and the inflation rates for the USA

Certain stylized features are apparent for all the estimated models in Table VII. For all three high-inflation economies of Argentina, Brazil, and Israel, the estimated value of d is approximately 0·59. This implies that the inflation series for these three countries have an infinite variance but are still mean reverting. Confirmation of these results was obtained by estimating similar models for the first differenced inflation series, i.e. imposing two unit roots in prices. The resulting estimated parameters were very close to those in Table VII; only d was now estimated at approximately −0·41. Hence there is consistent evidence that prices are integrated of order 1·59, i.e. I(1·59), for the three high-inflation economies. Further details of the estimated models are available in Tieslau (1992) and from the authors on request.

The estimated fractional differencing parameter was in the range of $0 < d < 0{\cdot}50$ for the seven low-inflation economies implying covariance stationarity of the inflation process. Interestingly enough, the estimated standard errors of $\hat{d}$ are relatively small and two-sided confidence intervals for d are correspondingly tight. Only for Japan can the hypothesis that $d = 0$ not be rejected. From Table IV, Germany as well as Japan appeared to possess a stationary inflation process. The estimated value of d for Germany in Table VII is 0·18, which is significantly different from zero and implies some mild long-memory features. For all other low-inflation economies, the estimated values of d are relatively large and are clearly significantly different from zero.

The estimated models for all ten countries also have strong persistence in their conditional variances, as evidenced by the sum of α and β, the GARCH parameters, which is at least 0·95. The information matrix of this model is block diagonal between the parameters in the conditional mean and parameters in the conditional variance, so that the estimates of the GARCH parameters do not significantly change in relation to the specification of the conditional mean part of the model. The inflation data also exhibited considerable excess kurtosis which necessitated the estimation of a Student t conditional density. The estimated degrees of freedom parameter, ν, was generally small for all ten countries, implying considerable departures from conditional normality.

5. FURTHER INTERPRETATION OF THE ESTIMATED MODELS

An important question concerns the overall merit of the ARFIMA modelling strategy when applied to inflation. To shed light on this issue, we also estimated some high-order ARMA models to compare with the ARFIMA models reported earlier in the paper. One of the main advantages with the ARFIMA approach lies in the flexible nature of the impulse response weights, which provide useful information on the importance of past shocks. The usual definition of cumulative impulse response weights is to consider the differenced inflation series, y_t, or $\Delta \log(\mathrm{CPI}_t)$, in equation (1), to obtain

$$\begin{aligned} y_t = \Delta \log(\mathrm{CPI}_t) &= \mu + (1-L)^{-d}\phi(L)^{-1}\theta(L)\varepsilon_t \\ &= \mu + \Sigma_{j=0,\infty}\lambda_j L^j \varepsilon_t \end{aligned} \tag{6}$$

Then the cumulative impulse response weights are defined as

$$\lambda_j^* = \Sigma_{k=0,j}\lambda_k = \delta \log(\mathrm{CPI}_t)/\delta\varepsilon_t \tag{7}$$

the computation of which is discussed by Chung (1994). Figure 2 plots the cumulative impulse response weights derived from the estimated ARFIMA model for the USA in Table VII, and

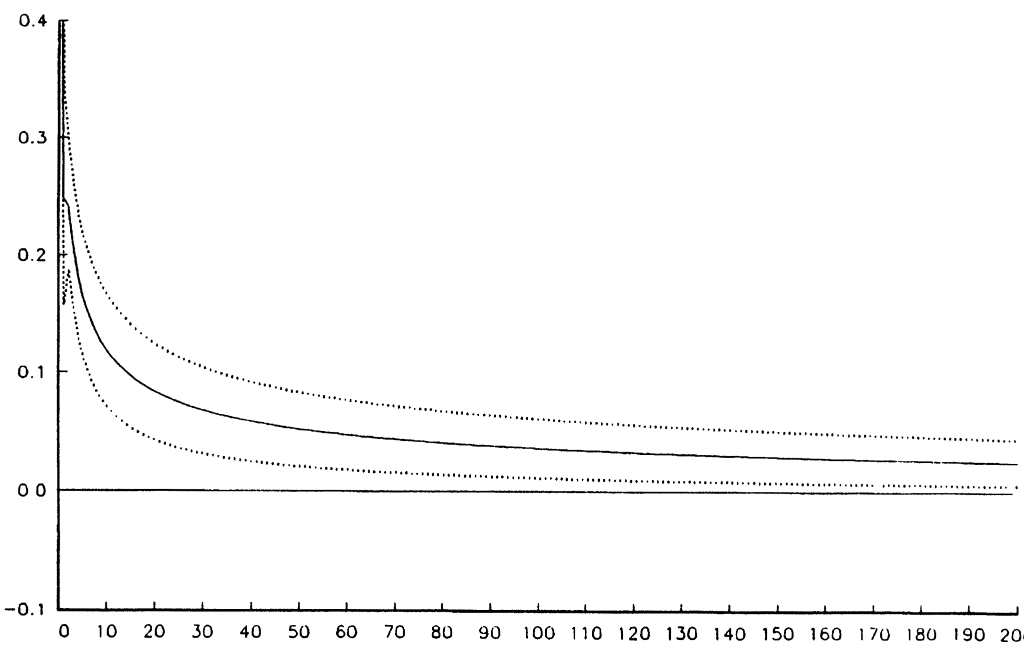

Figure 2. The cumulative impulse response function based on the ARFIMA(0, 0·471, 1) model: the US inflation rate

their associated 95% confidence intervals. The eventual slow hyperbolic rate of decay consistent with an I(d) process is very evident in the plot of the cumulative impulse response weights.

In order to compare the ARFIMA model with more conventional approaches, we also used information criteria to select the most appropriate low-order ARMA model. The preferred model was an ARMA(6, 1) and the estimated cumulative impulse response weights from this model, together with their 95% confidence intervals, are plotted in Figure 3. It is very noticeable that the impulse response weights from the ARMA(6, 1) model exhibit a relatively rapid exponential decay, which is in marked contrast to those of the ARFIMA model. Also the 95% confidence intervals are relatively wide and even include the possibility of negative cumulative weights at lag of 40. The width of the confidence intervals is partly due to the model being poorly identified compared with the parsimoniously parameterized ARFIMA process. An alternative strategy is to impose a unit root and to estimate the ARIMA(0, 1, 1) model presented in Table VI. The cumulative impulse response weights from this model are, of course, completely persistent and impose quite different long-run properties on inflation. Since many of the competing models are non-nested, a comparison of their maximized log likelihoods is not valid. However, it is clear that high-order AR(p) models do begin to possess similar degrees of fit, compared to the ARFIMA model, for values of $p \geq 15$.

Although prolifigately parameterized models provide similar degrees of goodness of fit, they do not possess the attractive interpretation of the ARFIMA model in terms of slow hyperbolic decay of shocks. The distinction between inflation being I(0·47) as opposed to I(1) is potentially important in situations such as the analysis of *ex-post* real rates of interest, and for the analysis of the international Fisher equation, which implies parity conditions between interest rate

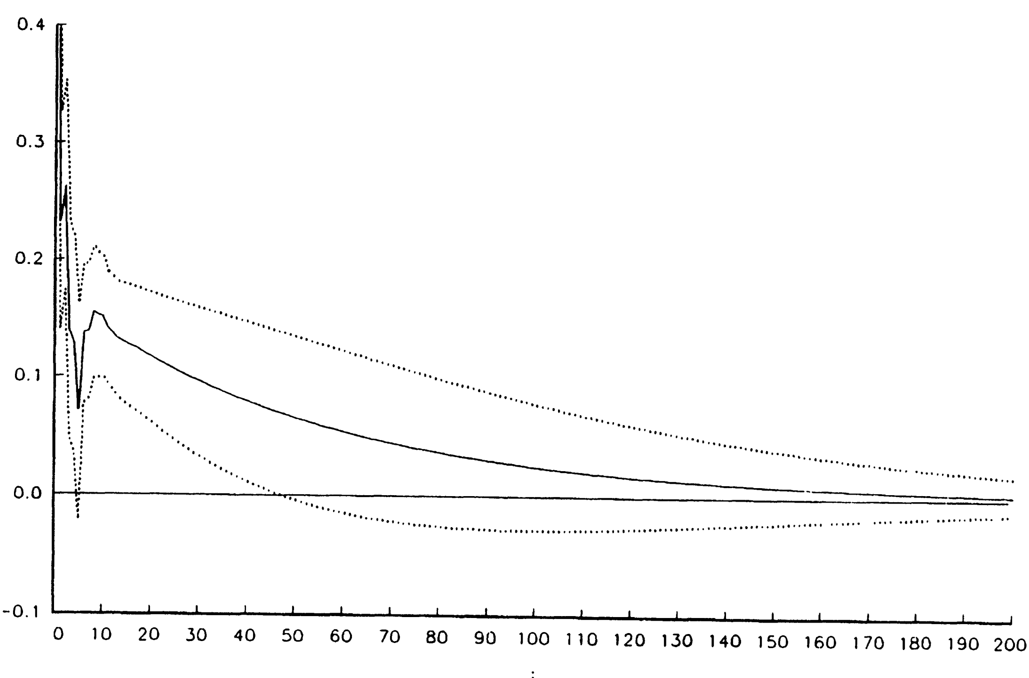

Figure 3. The cumulative impulse response function based on the ARMA(6, 1) model: the US inflation rate

Table VIII. Likelihood ratio tests of relationship between mean and variability of inflation, y_t

$$(1-L)^d(y_t-\mu)=(1+\theta_1 L)(1+\Phi_1 L^{12}+\Phi_2 L^{24})\varepsilon_t+\delta\sigma_t$$
$$\varepsilon_t \mid \Omega_{t-1} \sim D(0,\sigma_t^2,\nu)$$
$$\sigma_t^2=\omega+\alpha\varepsilon_{t-1}^2+\beta\sigma_{t-1}^2+\gamma y_{t-1}$$

LR tests	Argentina	Brazil	France	Germany	Israel	Italy	Japan	UK	USA
$\delta=0$	8·90[a]	4·26[a]	0·00	2·40	5·46[a]	0·72	1·44	3·88[a]	0·40
$\gamma=0$	12·92[b]	7·14[b]	1·54	0·80	10·66[b]	1·12	0·44	9·12[b]	1·28

Notes:
y_t is 100Δ log CPI$_t$, the conditional density D is Student t for France and Israel and is Normal otherwise. Under null hypothesis all the test statistics are distributed as asymptotic χ_1^2 random variables.
[a] Significant at the 0·05 level.
[b] Significant at the 0·01 level.

differentials and inflation rate differentials. We intend to report further analysis based on fractional integration of this parity relationship, which has recently been analysed along more conventional lines by Vlaar and Palm (1993).

A further interesting issue concerns the possibility of regime switches, and we also estimated ARFIMA–GARCH models for various subperiods. Following the results in Brunner and Hess (1993), particular attention was placed on the possibility of 1960 being a break point. Although based on only twelve years of data, the pre-1960 period appears less persistent. However, analysis of the 1960–92 data realized an estimated model with all the parameter values in both the conditional mean and variance to be very close to those over the full-sample period. This is in contrast to model of Brunner and Hess (1993), who impose a unit root in this period. However, further research might well benefit from considering ARFIMA models with switching regimes. This is a possibly interesting avenue for future research, but is beyond the scope of the present paper.

Table 8 reports likelihood ratio test statistics of whether lagged inflation Granger causes volatility and of whether lagged volatility Granger causes the mean of inflation. The first test, where lagged inflation is included in the conditional variance equation, can be interpreted as a direct test of Friedman's hypothesis, which states that volatility or uncertainty increases in high-inflation regimes. The results in Table 8 are quite striking. For the low-inflation countries of Canada, France, Germany, Italy, Japan, and the USA there is no apparent relationship between the mean and variance of inflation. The results for the USA are consistent with the findings of previous studies by Engle (1983) and Cosimano and Jansen (1988). However, for the high-inflation economies of Argentina, Brazil, and Israel, and also surprisingly for the UK, there is strong evidence of joint feedback between the conditional mean and variance of inflation, and hence strong support for the Friedman (1977) hypothesis.

6. CONCLUSION

Approximate MLE of the ARFIMA–GARCH-Student t process has revealed interesting similarities for ten different countries' monthly CPI inflation. The approximate MLE provides strong evidence against the widespread assumption of inflation having a unit root. Apart from Japan, which appears stationary, the other six G7, low-inflation countries have an estimated order of integration between 0·18 and 0·47. Our approximate MLE indicates relatively small standard errors on the estimates of the fractional differencing parameter and suggests the model is significantly different from assuming I(0) or I(1) behaviour. One interesting interpretation of these models is that an inflationary shock will have long memory and persistence; but that ultimately will be mean reverting. For three high-inflation economies the estimated order of integration is 0·59, which implies mean reversion but non-stationarity of inflation. Hence one potentially important aspect of fractionally integrated processes is that they provide more insight for macroeconomists on the persistence of shocks. The results in this study indicate that it is possible to distinguish between a process with a unit root and one which is fractionally integrated. There is also clear evidence for the G7 countries that although inflationary shocks have long memory they are nevertheless mean reverting.

Furthermore, all ten countries possess substantial persistence in their conditional variances, which are well described by GARCH(1, 1) processes. For the three high-inflation economies there is substantial evidence in favour of the Friedman hypothesis, with current inflation tending to lead to additional variability of inflation.

The empirical regularities of the persistence of inflation across countries raises interesting questions as to the type of monetary policy rules and price-transmission mechanism that would be consistent with this form of behaviour.

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