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Long memory and structural change in the G7 inflation dynamics

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ABSTRACT

In order to contribute to the inflation persistence debate, we extend the ARFIMA–GARCH model by allowing for time varying baseline mean and volatility using logistic functions. The proposed time-varying ARFIMA–GARCH model is applied to the monthly CPI inflation rates of the seven advanced economies (G7) from 1955 to 2014. The main finding of this study is that neglecting structural changes in the inflation level and volatility appears to overestimate the long run and GARCH persistence. Moreover, the identified shifts in the inflation dynamics are in line with economic and political events that marked the examined period.

1. Introduction

Inflation is an important indicator of the economy growth, the cost of living and thus the well-being of the population. Consequently, it has been one of the most appealing topics in macroeconomics, on the theoretical and empirical front. The principal aim of monetary policy is to keep inflation at low and stable levels as much as possible. Consequently, to design an optimal monetary policy, the monetary authorities in industrial countries monitor the persistence of inflation which reflects their ability to recover equilibrium after a shock to price levels.

A rich body of research has aimed to assess empirically the extreme persistence of inflation in order to address the issue of lasting effect of shocks and, hence, to improve forecasting quality, but studies appear to reach quite diverging conclusions. At first, the literature investigating whether inflation is a unit root $I(1)$ or stationary $I(0)$ process does not offer any consensus. Nelson and Schwert (1977), Ball and Cecchetti (1990), Kim (1993), Banerjee et al. (2001) and Banerjee and Russell (2001) find the evidence of a unit root in the inflation rate. Rose (1988) and Grier and Perry (1998) claim that the inflation rate is an $I(0)$ process. However, most others like Kirchgässner and Wolters (1993) and Bos et al. (1999) found mixed results. More recently, Narayan (2014) and Narayan and Liu (2015) argued that unit root tests based on standard linear models are inappropriate if the data has level shifts along with significant ARCH effects. They propose a unit root test with two endogenous structural breaks that specifically includes a GARCH(1,1) specification for the volatility. The results brought clearer and valuable answer about the inflation non-stationarity issue.

Another stream of studies argued that inflation is better characterized by a long memory process (ARFIMA) to capture the persistence e.g., Hassler and Wolters (1995), Baillie et al. (1996), Baum et al. (1999), Gadea and Mayoral (2006), Bagliano and Morana (2007) and Bos et al. (2014). An alternative modeling approach has been to use structural changes to explain the switching persistence of the inflation rate as in Osborn and Sensier (2009), Gonzalez et al. (2009), Boero et al. (2010)and Narayan (2014). The aforementioned studies provided consistent evidence across time periods and countries that inflation rates exhibit long memory properties or regime switching until another strand of studies combines these two features following Diebold and Inoue (2001), Granger and Hyung (2004) and Hyung et al. (2006) who gave evidence that neglecting structural breaks in the time series can bias the long memory estimates. Cecchetti and Debelle (2006) investigated the inflation persistence for major industrial economies and found that, conditional on a break in the intercept, inflation is much less persistent than neglecting shifts in the mean. Cecchetti et al. (2007) provided empirical evidences on the need of using models allowing for changes in mean, volatility and persistence of inflation. Belkhouja and Boutahar (2009) used an ARFIMA model for the U.S. inflation rate and showed that accounting for regime shifts in level and persistence leads to a lower long memory persistence estimate. Kang et al. (2009) investigated the structural change in US inflation persistence using a model with Markov switching parameters and found two sudden regime shifts. Baillie and Morana (2012) introduced the Adaptive ARFIMA Adaptive FIGARCH allowing the baseline mean and baseline volatility to be time dependent according to Gallant's (1984) specification of a flexible Fourier form (FFF). Applied to the G7 inflation rates, Baillie and Morana (2012) found that the estimates of the fractional differencing parameters are in general smaller in magnitude than implied by conventional ARFIMA models with no adjustment for

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the unconditional mean changing. A separate strand of the literature studied the changing long memory persistence of the inflation process. Kumar and Okimoto (2007) find a structural break in the order of fractional integration of the monthly US CPI inflation coinciding with 1982. Hassler and Meller (2011) extended this approach by testing for multiple structural changes in the degree of long memory. Thus, all these alternative characterizations of the inflation persistence yield interesting economic implications because if the occurrence and/or timing of structural changes are not accounted for properly, then inflation forecasts and policy decisions might be misguided.

Many studies have presented structural changes in the time series properties of inflation dynamics as consequences of changes in monetary policy or exogenous shocks. While the statistical significance and the economic importance of changes in mean and persistence are more or less debatable during the Great Inflation period of the 1970s and early 1980s, there is a consensus on the decline in the inflation level and persistence over the 1980s of the most industrialized economies in the world. This decline has been attributed to the central bank's increased focus on price stability.

The cornerstone of this paper is the ARFIMA–GARCH model introduced by Baillie et al. (1996) to describe the inflation dynamics for ten countries. The authors provided strong evidence of long memory with mean reverting behavior; furthermore, they found that the sum of the GARCH coefficients is usually close to 1, implying infinite variances and a strong persistence of the shocks effect on volatility over time. In the light of these facts, including a non-linear specification in the ARFIMA–GARCH model would capture a part of the persistence in the conditional mean, as pointed by Baillie and Morana (2012) among many others. The same holds for the inflation volatility following the arguments of Lastrapes (1989) and Lamoureux and Lastrapes (1990) which state that neglecting structural breaks in volatility could lead to spuriously high estimates of the sum of the GARCH coefficients.

In light of this discussion, this article presents both methodological and economic contributions. From a methodological viewpoint, the present paper contributes to the applied econometrics literature by addressing the relationship between persistence and regime shifts issue in two ways. First, we developed a new model called time-varying ARFIMA time-varying GARCH or TV-ARFIMA–TV-GARCH. In addition to the long memory property, our model accounts for changing dynamics in the mean and variance utilizing a more flexible specification than the abrupt regime shifts often employed in the literature. More specifically, our model allows to the baseline mean and volatility to evolve according to logistic functions where the transition between regimes might be smooth over time, which better captures any changing dynamics pattern. Second, we designed a modeling strategy based on a sequence of LM-type test in order to advance our knowledge of the source of persistence in the mean and volatility. From an economic perspective, the article contributes to the inflation modeling literature by connecting the identified changes in the G7 post-World War II inflation dynamics with historical macroeconomic events. We provided also a more fine-grained interpretation of the persistence in the context of monetary policy shifts, oil shocks and economic recession. In order to investigate whether the source of persistence in the mean and variance is genuine or partially spurious due to neglected structural breaks, we applied our modeling strategy to the monthly CPI inflation rates of the seven advanced economies (G7) from 1955 to 2014. The results based on monthly inflation rates provide strong evidence that the TV-ARFIMA–TV-GARCH model performs better than the historical ARFIMA–GARCH model and support the time-variation hypothesis of the unconditional mean and variance. More specifically, taking into account the estimated number of breaks, our model results show that all the inflation rates still exhibit a long memory property in their conditional means, however, with a less magnitude than obtained with the ARFIMA models. In other words, our findings suggest that the G7 inflation series are less backward looking, which is consistent with the results provided by Baillie and Morana (2012) among others. Likewise, the null hypothesis of the unconditional variance stability is rejected for all the inflation time series, which reduces the estimated persistence when the TV-GARCH specification is employed to capture the volatility dynamics. More, interestingly the patterns and the smoothness degrees of the transitions between regimes, assessed by logistic functions, vary from one country to another. Taken together, we believe that our contribution lead to a better understanding of the G7 inflation dynamics.

The rest of the paper is organized as follows. In Section 2, we introduce the class of TV-ARFIMA–TV-GARCH models and we discuss their properties. Section 3 deals with the parameter constancy using the Lagrange Multiplier (LM) test. In Section 4, we present our modeling strategy. Section 5 is dedicated to empirical results of the monthly G7 inflation ratemodeling and their economic interpretations. The last section concludes with the main findings and perspectives.

2. A time varying ARFIMA–GARCH model

In this section we present the time varying ARFIMA–GARCH model or the TV-ARFIMA–TV-GARCH allowing for long memory and structural changes in the conditional mean and regime shifts in the conditional variance. First, the conditional mean is expressed as an ARFIMA model:

$$\Phi(L)(1-L)^d y_t = \mu_0 + \Psi(L)\varepsilon_t \tag{1}$$

where μ_0 is the unconditional mean of the process, d is the long memory parameter, $\Phi(L) = 1-\phi_1 L - \ldots - \phi_p L^p$ and $\Psi(L) = 1+\psi_1 L + \ldots + \psi_q L^q$ have all their roots outside the unit circle and L is the lag operator. ε_t is an innovation sequence with a potentially time-varying conditional variance $E(\varepsilon_t^2|\Omega_{t-1}) = h_t$, where Ω_{t-1} is the information setup to time $t-1$. That is $\varepsilon_t|\Omega_{t-1} \sim N(0,h_t)$ and:

$$\varepsilon_t = z_t\sqrt{h_t}$$

where z_t is normal distributed with mean 0 and variance 1. Thus, the process will be covariance stationary and invertible for $-0.5<d<0.5$. When $0<d<0.5$, we have a stationary long memory process characterized by an hyperbolic decay of the autocorrelation function toward zero. If $0.5 \le d<1$, the process does not have a finite variance and is non-stationary, but it is still mean reverting. Finally, when $d=1$, the process has a unit root and shocks have a permanent effect on the data.

To allow the baseline mean to vary over time, we extended the specification of the ARFIMA(p,d,q) to the TV-ARFIMA(p,d,q,R_1). The TV-ARFIMA model has the feature to capture systematic movements of the baseline mean and, consequently, the model in Eq. (1) becomes:

$$\Phi(L)(1-L)^d y_t = \mu_0 + F_{1t} + \Psi(L)\varepsilon_t \tag{2}$$

$$F_{1t} = \sum_{r=1}^{R_1} \mu_r f_{1r}(s_t, \gamma_{1r}, c_{1r}) \tag{3}$$

where $f_{1r}(s_t,\gamma_{1r},c_{1r})$, $r=1,\ldots,R_1$, are the transition functions governing the regime shifts. These functions are continuous, non-negative and bounded between zero and one allowing the intercept of the ARFIMA model to fluctuate over time between μ_0 and $\mu_0+\sum_{r=1}^{R_1}\mu_r$. The order $R_1 \in \mathbb{N}$ determines the dynamics of the baseline mean. A suitable choice for $f_{1r}(s_t,\gamma_{1r},c_{1r})$, $r=1,\ldots,R_1$, is the logistic function defined as follows:

$$f_{1r}(s_t,\gamma_{1r},c_{1r}) = (1+\exp\{-\gamma_{1r}(s_t-c_{1r})\})^{-1}. \tag{4}$$

The slope parameter $\gamma_{1r}(\gamma_{1r}>0)$ reflects the smoothness of transition from one regime to another. The larger γ_{1r} is, the faster the transition between regimes will be. When $\gamma_{1r}\to\infty$, the switch from one regime to another is abrupt. c_{1r} is the threshold parameter or the location of transition between different regimes, such as $c_{11} \leq c_{12} \leq \cdots \leq c_{1R_1}$. $s_t = t/T$ is the transition variable and T is the total number of observations.

To capture the volatility in the conditional variance process, Bollerslev (1986) introduced the GARCH(m,n) model, which defines the conditional variance equation as follows:

$$h_t = \omega_0 + \sum_{i=1}^{n} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{m} \beta_j h_{t-j}. \tag{5}$$

The parameters in Eq. (5) satisfy the restrictions $\omega_0 > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$, $i = 1, \ldots, n$ and $j = 1, \ldots, m$ in order to ensure the positivity of h_t. Similarly, as for the conditional mean, we extended the GARCH(m,n) model to a TV-GARCH(m,n,R_2) model allowing the baseline volatility to be time dependent and, hence, the model (5) becomes:

$$h_t = \omega_0 + F_{2t} + \sum_{i=1}^{n} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{m} \beta_j h_{t-j} \tag{6}$$

$$f_{2t} = \sum_{r=1}^{R_2} \omega_r F_{2r}(s_t, \gamma_{2r}, c_{2r}). \tag{7}$$

As for F_{1t}, F_{2t} governs the movement of GARCH model intercept (baseline volatility) over time between ω_0 and $\omega_0 + \sum_{r=1}^{R_2} \omega_r$ according to a combination of logistic functions. c_{2r} and γ_{2r} determine the location and the speed of transitions between different regimes according to the calendar time s_t. The following constraints are set to guarantee the stationarity and the non-negativity of the conditional variance in Eq. (6):

$$\begin{cases} \omega_0 + \sum_{r=1}^{R_2} \omega_r > 0 \\ \alpha_i \geq 0, \quad i = 1, .., n \\ \beta_j \geq 0, \quad j = 1, .., m \\ \sum_{i=1}^{n} \alpha_i + \sum_{j=1}^{m} \beta_{0j} < 1 \end{cases}$$

Thus, the resulting TV-ARFIMA–TV-GARCH model is simply an association between models (2) and (6):

$$\begin{cases} \Phi(L)(1-L)^d y_t = \mu_0 + F_{1t} + \Psi(L)\varepsilon_t \\ \varepsilon_t = z_t \sqrt{h_t}, \qquad \varepsilon_t | \Omega_{t-1} \sim N(0, h_t) \\ h_t = \omega_0 + F_{2t} + \sum_{i=1}^{n} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{m} \beta_j h_{t-j} \end{cases} \tag{8}$$

3. Testing parameter constancy

3.1. Testing the baseline mean constancy

This test has previously been used by Lundbergh and Teräsvirta (2002) and Teräsvirta and Amado (2008) for GARCH models. For our purpose we will apply it to ARFIMA model to check whether the unconditional mean varies over time. In order to derive the test statistic, let us rewrite the model (2) with only one transition logistic function:

$$\Phi(L)(1-L)^d y_t = \mu_0 + \mu_1 f_{11}(s_t, \gamma_{11}, c_{11}) + \Psi(L)\varepsilon_t.$$

The first step of the test consists in estimating the ARFIMA(p,d,q) assuming that the conditional variance is constant over time. Once the residuals $\hat{\varepsilon}_t$ are obtained, we test the null hypothesis of the intercept constancy corresponding to $H_0: \gamma_{11} = 0$ against $H_1: \gamma_{11} > 0$. Under null hypothesis, μ_1 and c_{11} are not identified but this identification problem has been resolved by Luukkonen et al. (1988) replacing the transition function[1] by its first order Taylor approximation around $\gamma_{11} = 0$. Thus, the first order Taylor expansion of the logistic transition function around $\gamma_{11} = 0$ is given by:

$$T_{11}(s_t, \gamma_{11}, c_{11}) = \frac{1}{4}\gamma_{11}(s_t - c_{11}) + R_1(s_t, \gamma_{11}, c_{11})$$

where $R_{11}(s_t, \gamma_{11}, c_{11})$ is a remainder term. Substituting $F_{11}(s_t, \gamma_{11}, c_{11})$ by $T_{11}(s_t, \gamma_{11}, c_{11})$ in Eq. (8), and after rearranging terms we have:

$$\Phi(L)(1-L)^d y_t = \mu_0^* + \mu_1^* s_t + \Psi(L)\varepsilon_t + R_1$$

where $\mu_0^* = \mu_0 - \frac{1}{4}\mu_1\gamma_{11}c_{11}$ and $\mu_1^* = \frac{1}{4}\mu_1\gamma_{11}$. Therefore, the null hypothesis of unconditional mean constancy becomes: $H_0: \mu_1^* = 0$. Under H_0 the remainder ($R_1 = 0$) does not affect the asymptotic null distribution of the Lagrange Multiplier test. Let $\theta_1 = (d, \mu_0^*, \mu_1^*, \Phi', \Psi', \sigma^2)'$ the vector of the TV-function as follow:

$$l_t = -\frac{1}{2}\ln 2\pi - \frac{1}{2}\ln\sigma^2 - \frac{1}{2}\frac{\varepsilon_t^2}{\sigma^2}.$$

The partial derivatives evaluated under H_0 are given by:

$$\left.\frac{\partial l_t}{\partial \theta_1}\right|_{H_o} = -\frac{\hat{\varepsilon}_t}{\sigma^2}\left.\frac{\partial \hat{\varepsilon}_t}{\partial \theta_1}\right|_{H_o}$$

- $\frac{\partial \hat{\varepsilon}_t}{\partial d}|_{H_0} = (1-L)^d \hat{\Phi}(L) \sum_{j}^{t-1} \frac{y_{t-j}}{j} - \sum_{j=1}^{q} \hat{\Psi}_j \frac{\partial \hat{\varepsilon}_{t-j}}{\partial d}$
- $\frac{\partial \hat{\varepsilon}_t}{\partial \mu_0^*}|_{H_0} = -1 - \sum_{j=1}^{q} \hat{\Psi}_j \frac{\partial \hat{\varepsilon}_{t-j}}{\partial \mu_0^*}$
- $\frac{\partial \hat{\varepsilon}_t}{\partial \mu_1^*}|_{H_0} = -s_t - \sum_{j=1}^{q} \hat{\Psi}_j \frac{\partial \hat{\varepsilon}_{t-j}}{\partial \mu_0^*}$
- $\frac{\partial \hat{\varepsilon}_t}{\partial \Phi}|_{H_0} = -(1-L)^d (y_{t-1}, \ldots, y_{t-p}) - \sum_{j=1}^{q} \hat{\Psi}_j \frac{\partial \hat{\varepsilon}_{t-j}}{\partial \Phi}$
- $\frac{\partial \hat{\varepsilon}_t}{\partial \Psi}|_{H_0} = -(\varepsilon_{t-1}, \ldots, \varepsilon_{t-q}) - \sum_{j=1}^{q} \hat{\Psi}_j \frac{\partial \hat{\varepsilon}_{t-j}}{\partial \Psi}$
- $\frac{\partial \hat{\varepsilon}_t}{\partial \sigma^2}|_{H_0} = -\frac{1}{2\sigma^2} + \frac{\varepsilon_t^2}{2\sigma^4}.$

Under H_0, the LM-type statistic is asymptotically distributed as χ^2 with one degree of freedom:

$$LM_1 = \frac{1}{2}\sum_{t=1}^{T}\left.\frac{\partial l_t}{\partial \theta_1}\right||_{H_o}\left(\sum_{t=1}^{T}\left[\frac{\partial \hat{\varepsilon}_t}{\partial \theta_1}|_{H_o}\right]\left[\left.\frac{\partial \hat{\varepsilon}_t}{\partial \theta_1}\right||_{H_o}\right]'\right)^{-1}\sum_{t=1}^{T}\left.\frac{\partial l_t}{\partial \theta_1}\right|_{H_0}.$$

3.2. Testing the baseline volatility constancy

Once the TV-ARFIMA model is estimated and the residuals ε_t are obtained, we test, similarly, the alternative hypothesis of TV-GARCH specification with one transition function against the null hypothesis of GARCH model. First, let us rewrite the model (6) with one transition function:

$$\begin{cases} \varepsilon_t = z_t \sqrt{h_t}, \quad \varepsilon_t | \Omega_{t-1} \sim N(0, h_t) \\ h_t = \omega_0 + \omega_1 f_{21}(s_t, \gamma_{21}, c_{21}) + \alpha(L)\varepsilon_t^2 + \beta(L)h_t \end{cases}$$

[1] For the purpose of deriving the test, we replace $F(s_t,\gamma,c)$ by $F(s_t,\gamma,c)$-1/2, (See Teräsvirta. and Amado (2008)).

The null hypothesis of the test corresponds to $H_0:\gamma_{21}=0$ against $H_1:\gamma_{21}>0$, but under the null hypothesis, ω_1 and c_{21} are not identified. As for the baseline mean constancy test, we use the first order Taylor approximation around $\gamma_{21}=0$. Thus, the resulting model is:

$$\begin{cases} \varepsilon_t = z_t\sqrt{h_t}, \quad \varepsilon_t|\Omega_{t-1} \sim N(0,h_t) \\ h_t = \omega_0^* + \omega_1^* s_t + \alpha(L)\varepsilon_t^2 + \beta(L)h_t + R_2(s_t,\gamma_{21},c_{21}) \end{cases}$$

where $\omega_0^* = \omega_0 - \frac{1}{4}\omega_1\gamma_{21}c_{21}$, $\omega_1^* = \frac{1}{4}\omega_1\gamma_{21}$. Therefore, the null hypothesis of the unconditional variance constancy test becomes: $H_0:\omega_1^*=0$. Under H_0, the remainder ($R_2=0$) does not affect the asymptotic null distribution of the Lagrange Multiplier test.

$\theta_2=(\omega_0^*,\omega_1^*,\alpha',\beta')'$ is the vector of the TV-GARCH model parameters and the quasi log-likelihood function is given by:

$$l_t(\theta_2) = -\frac{1}{2}\ln 2\pi - \frac{1}{2}\ln h_t - \frac{1}{2}\frac{\varepsilon_t^2}{h_t}.$$

The partial derivatives evaluated under H_0 are given by:

$$\left.\frac{\partial l_t}{\partial\theta_2}\right|_{H_0} = \frac{1}{2}\left(\frac{\hat{\varepsilon}_t^2}{\hat{h}_{0t}}-1\right)\left.\frac{\partial \ln h_t}{\partial\theta_2}\right|_{H_0}$$

- $\left.\frac{\partial \ln h_t}{\partial\omega_0^*}\right|_{H_0} = \left(\hat{h}_{0t}\right)^{-1}\left(1+\sum_{j=1}^{p}\hat{\beta}_j\frac{\partial\hat{h}_{t-j}}{\partial\omega_0^*}\right)$
- $\left.\frac{\partial \ln h_t}{\partial\omega_1^*}\right|_{H_0} = \left(\hat{h}_{0t}\right)^{-1}\left(s_t+\sum_{j=1}^{p}\hat{\beta}_j\frac{\partial\hat{h}_{t-j}}{\partial\omega_1^*}\right)$
- $\left.\frac{\partial \ln h_t}{\partial\alpha}\right|_{H_0} = \left(\hat{h}_{0t}\right)^{-1}\left(\left(\varepsilon_{t-1}^2,\ldots,\varepsilon_{t-n}^2\right)'+\sum_{j=1}^{m}\hat{\beta}_j\frac{\partial\hat{h}_{t-j}}{\partial\alpha}\right)$
- $\left.\frac{\partial \ln h_t}{\partial\beta}\right|_{H_0} = \left(\hat{h}_{0t}\right)^{-1}\left(\left(h_{t-1},\ldots,h_{t-p}\right)'+\sum_{j=1}^{m}\hat{\beta}_j\frac{\partial\hat{h}_{t-j}}{\partial\beta}\right).$

Under the null hypothesis, the "hats" indicate the maximum likelihood estimators and $\hat{h}_{0t}$ denotes the conditional variance estimated at time t. Under H_0, the LM-type statistic is asymptotically distributed as χ^2 with one degree of freedom:

$$LM_2 = \frac{1}{2}\sum_{t=1}^{T}\left.\frac{\partial l_t}{\partial\theta_2}\right|_{H_0}\left(\sum_{t=1}^{T}\left[\left.\frac{\partial\ln h_t}{\partial\theta_2}\right|_{H_0}\right]\left[\left.\frac{\partial\ln h_t}{\partial\theta_2}\right|_{H_0}\right]'\right)^{-1}\sum_{t=1}^{T}\left.\frac{\partial l_t}{\partial\theta_2}\right|_{H_0}.$$

4. Specification and estimation of the model

In order to build the TV-ARFIMA–TV-GARCH model in Eq. (8), we start with a simple and restricted specification without time-varying parameters. Our strategy modeling is based on the following steps:

- First of all, we remove any seasonal effect and then estimate a parsimonious ARFIMA model assuming that the conditional variance is constant. We obtain the residuals $\hat{\varepsilon}_t$ and make sure they are free of serial correlation because neglected autocorrelation may bias the mean baseline constancy test.
- Second, we select the number of transitions if they do exist. We use a selection rule based on a sequence of LM-type tests assuming that $R_1\max=5$ for the transition function F_{1t} in Eq. (8). Consequently, we have five hypotheses to test:

$$\begin{aligned} &H_{05}: \mu_5^* = 0, \\ &H_{04}: \mu_4^* = 0 \,|\, \mu_5^* = 0, \\ &H_{03}: \mu_3^* = 0 \,|\, \mu_4^* = \mu_5^* = 0, \\ &H_{02}: \mu_2^* = 0 \,|\, \mu_3^* = \mu_4^* = \mu_5^* = 0, \\ &H_{01}: \mu_1^* = 0 \,|\, \mu_2^* = \mu_3^* = \mu_4^* = \mu_5^* = 0 \end{aligned}$$

- Of course the selected order R_1 corresponds to the lowest p-value among those of rejected null hypotheses. Thus, no reject of null hypothesis simply implies no structural changes in the conditional mean.
- We estimate the selected TV-ARFIMA model and we specify a parsimonious GARCH model. Once again, we have to check if any serial correlation is left in the residuals in order to avoid any bias of the test.
- As for the conditional mean, a similar methodology was adopted to identify the number of breaks in the conditional variance assuming $R_2\max=5$ in the transition function F_{2t} in Eq. (8). The five hypotheses to test are as follow:

$$\begin{aligned} &H_{05}: \omega_5^* = 0, \\ &H_{04}: \omega_4^* = 0 \,|\, \omega_5^* = 0, \\ &H_{03}: \omega_3^* = 0 \,|\, \omega_4^* = \omega_5^* = 0, \\ &H_{02}: \omega_2^* = 0 \,|\, \omega_3^* = \omega_4^* = \omega_5^* = 0, \\ &H_{01}: \omega_1^* = 0 \,|\, \omega_2^* = \omega_3^* = \omega_4^* = \omega_5^* = 0 \end{aligned}$$

  Here again, the selected order R_2 corresponds to the lowest p-value among those of rejected null hypotheses. Hence, no reject of null hypotheses means no regime shifts in the volatility.
- Despite the non-stationarity of the TV-ARFIMA–TV-GARCH process due to the time varying parameters, we use the quasi maximum likelihood method for estimation, which is valid in non-standard frameworks.[2] The final model is evaluated based on diagnostic tests.

Let $\theta=(\theta_1',\theta_2')'$, where $\theta_1=(d,\mu_0^*,\mu_1^*,\Phi',\Psi',\sigma^2)'$ is the vector of the TV-ARFIMA model parameters and $\theta_2=(\omega_0^*,\omega_1^*,\alpha',\beta')'$ is the vector of the TV-GARCH model parameters. The quasi maximum likelihood estimates of the parameters in θ are obtained by:

$$\hat{\theta} = \arg\max_{\theta\in\Theta}\frac{1}{T}\sum_{t=1}^{T}l_t(\theta)$$

where $l_t(\theta)$ is the quasi log-likelihood of model for observation t:

$$l_t(\theta) = -\frac{1}{2}\ln 2\pi - \frac{1}{2}\ln h_t - \frac{1}{2}\frac{\varepsilon_t^2}{h_t}.$$

Under fairly general conditions, the asymptotic distribution of the *QMLE* is

$$T^{1/2}\left(\hat{\theta}-\theta_0\right)\rightarrow N\left\{0,A(\theta_0)^{-1}B(\theta_0)A(\theta_0)^{-1}\right\}$$

where θ_0 denotes the true values of the vector parameters, $A(\theta_0)$ is the hessian and $B(\theta_0)$ the outer product gradient. The proposed method-allows for jointly estimating all parameters at once. Otherwise, we note that large estimates of the smoothness parameter γ may lead to numerical problems when testing the parameter constancy. To remedy this issue, which does not affect the value of the test statistic, Eitrheim and Teräsvirta (1996)suggested to omit score elements that are partial derivatives of the transition function parameters $f(s_t,\gamma,c)$.

5. Empirical analysis

5.1. Data

We used monthly data of the consumer price index (CPI) inflation rate. The data range from 1955:01 to 2014:12 for a total of 720 monthly observations and cover the G7 countries: USA, Japan, Canada, United Kingdom, Germany, France and Italy. The inflation rate is measured by the monthly difference of the log CPI, i.e., $y_t=100.\log(CPI_t/CPI_{t-1})$, where CPI_t denotes the consumer price index at month t. The inflation time series are collected from the Federal Reserve Economic Data (FRED) database and are seasonally adjusted. The top left panels of Figs. 1–7 present the time series plots of the G7 monthly inflation rates. A first interesting feature is that there are several countries' inflations following, to some extent, a similar hump-shaped pattern. Furthermore, in most countries, we observe high inflation rates in the 1970s, followed by a marked reduction in volatility in the mid-1980s.

[2] See Dahlhaus and Subba Rao (2006) for more details.

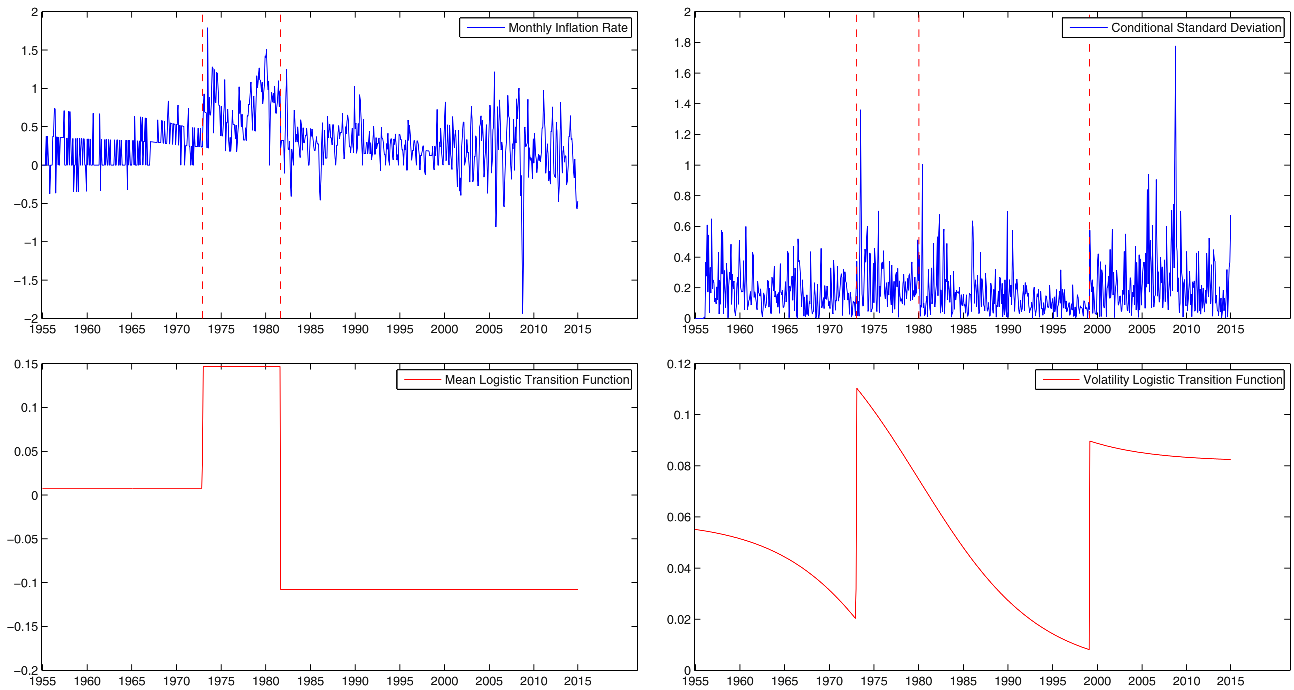

Fig. 1. Plots of the US inflation rate, the conditional standard deviations, the mean baseline logistic transition function F_{1t} and the volatility baseline logistic transition function F_{2t}.

For these reasons, the eventual time-varying character of the inflation level and the baseline volatility should be investigated further.

Table 1 reports summary statistics for the inflation series. The results reveal that sample means are positive and smaller than the standard deviations. All the inflation distributions seem to be positively skewed and most of them are heavy-tailed according to the kurtosis coefficients estimates. Testing for heteroskedasticity with 12 lags, we can comfortably reject the null of "no ARCH" effect in all the time series. To check the presence of a time trend, we run OLS regressions of inflation rate variables on a constant and a time trend. The estimated coefficients of the time trend are almost equal to zero and their p-values suggest that the null hypothesis of "no time-trend effect" cannot be rejected. This statistical evidence is in line with what we observed from the simple time-series plots of the inflation rates in the top left panels of Figs. 1–7. As none of these inflation series is characterized by a time trend, we utilized the standard augmented Dickey–Fuller (ADF) including a drift to test for unit root, where serial correlation is controlled through lags of the dependent variable. The optimal lags in the ADF regression are chosen based on the first significant lagged effects. As presented in Table 1, none of the inflation rates is a unit root process,

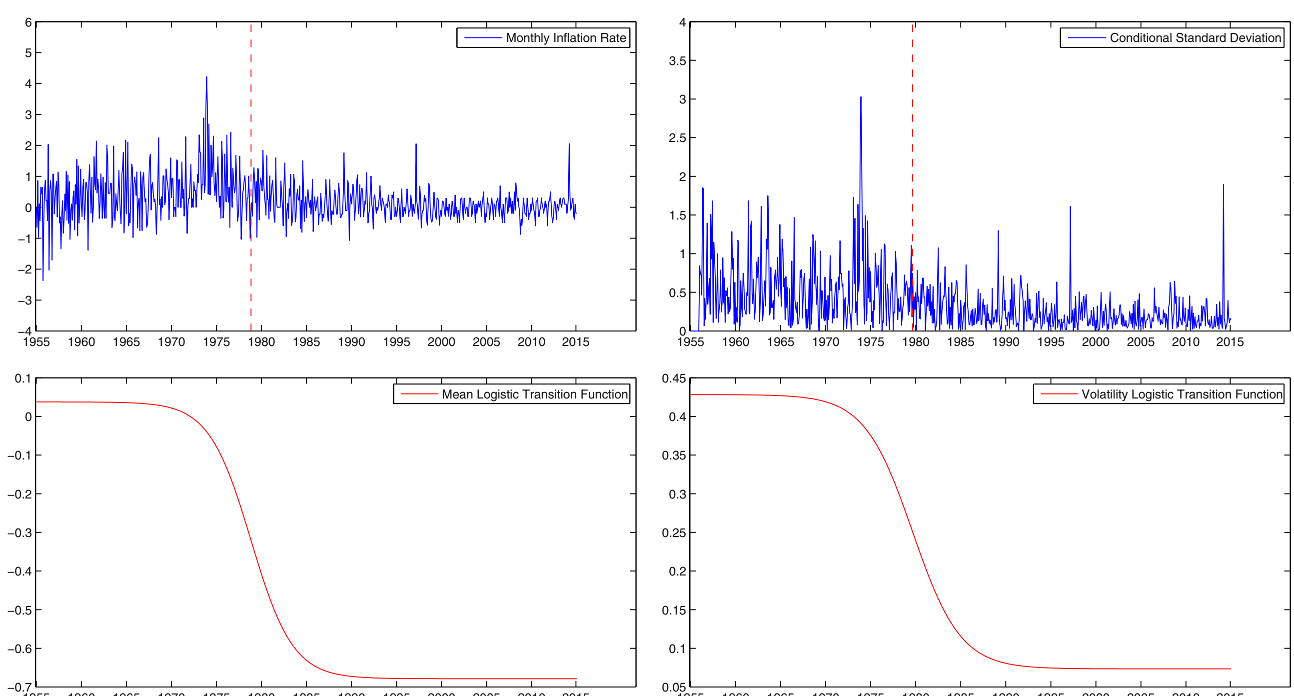

Fig. 2. Plots of the Japan inflation rate, the conditional standard deviations, the mean baseline logistic transition function F_{1t} and the volatility baseline logistic transition function F_{2t}.

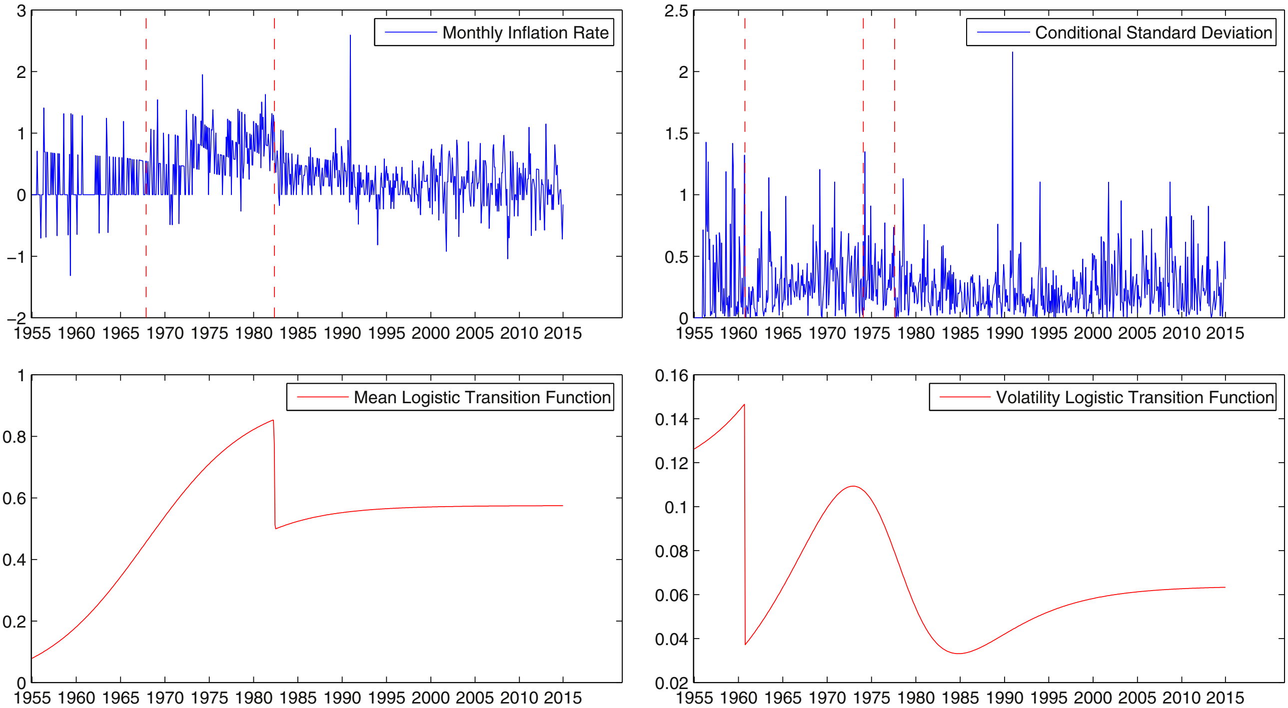

Fig. 3. Plots of the Canada inflation rate, the conditional standard deviations, the mean baseline logistic transition function F_{1t} and the volatility baseline logistic transition function F_{2t}.

which means that shocks to inflation rates have a finite life for those countries. As a final point, we attempt to confirm the ADF results and gain power in rejecting the unit root null hypothesis accounting for both structural breaks that are easily noticeable in the inflation rate plots, and heteroskedasticity. To do this, we run the Narayan et al. (2015b) unit root test with structural breaks in the intercept including a GARCH(1,1) specification for innovations. The number of structural breaks used in the Narayan et al. (2015b) unit root test are given in Table 2 based on the sequence of LM-type tests. As expected, the results of the unit root test accounting for both structural breaks and heteroskedasticity show that the unit root null hypothesis is rejected at the 5% level of signification for all the inflation series.

5.2. Results

First of all, we select the number of regime shifts; if they do exist, in level and unconditional variance of the G7 monthly inflation rates according to a sequence of LM-type tests, as explained in the modeling

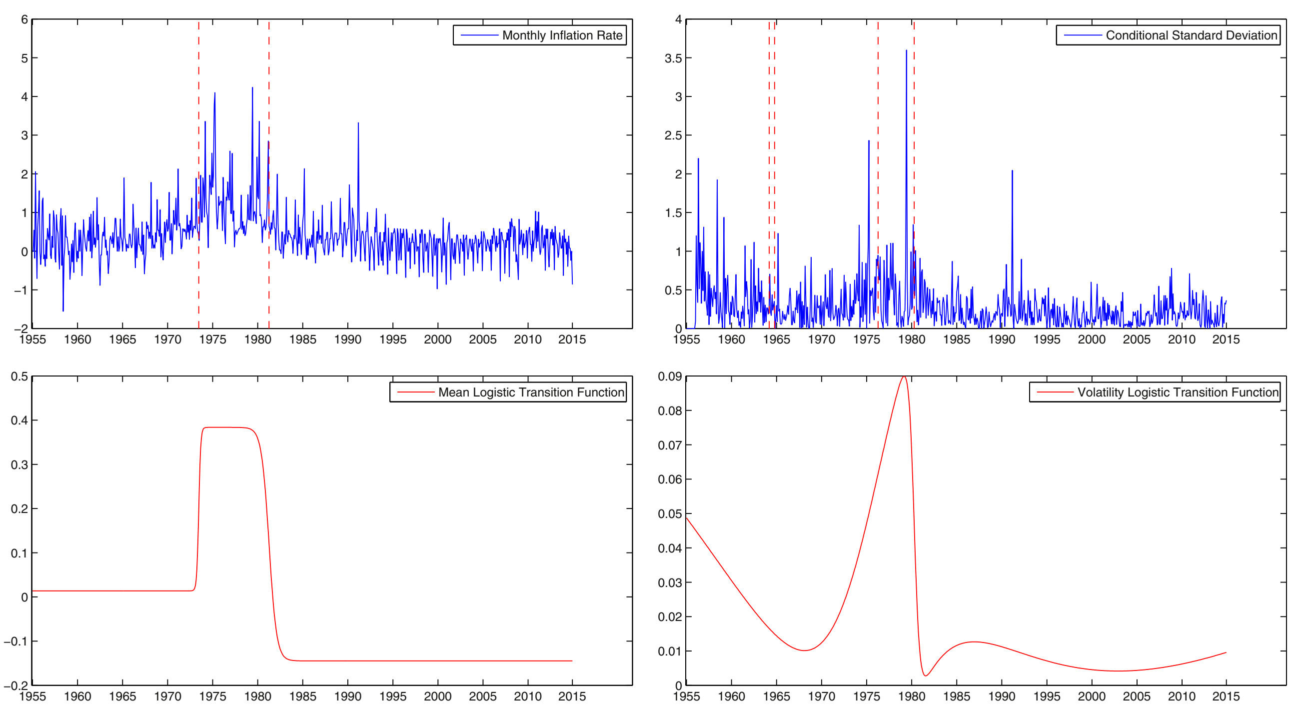

Fig. 4. Plots of the UK inflation rate, the conditional standard deviations, the mean baseline logistic transition function F_{1t} and the volatility baseline logistic transition function F_{2t}.

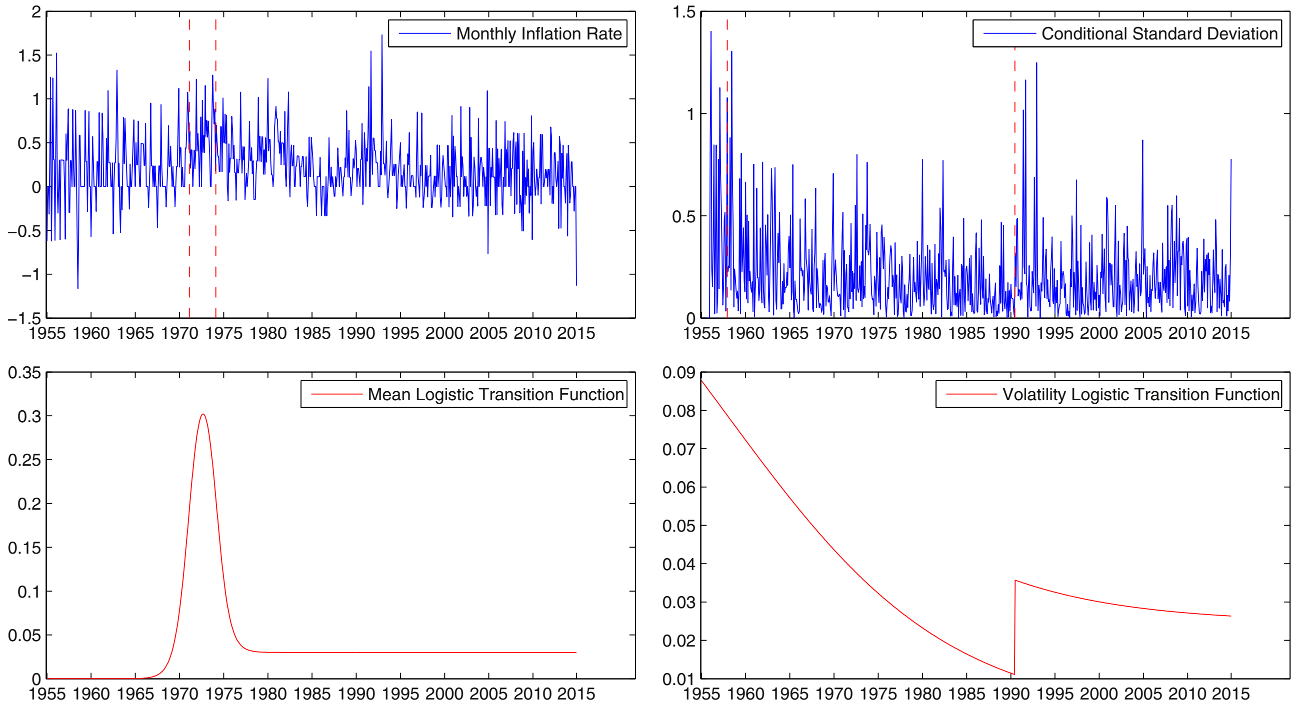

Fig. 5. Plots of the Germany inflation rate, the conditional standard deviations, the mean baseline logistic transition function F_{1t} and the volatility baseline logistic transition function F_{2t}.

strategy (see Section 4). The test results reported in Tables 2–3 reveal that, whatever the country, the inflation level and its baseline volatility are instable over time with at least one structural break detected. However, as stressed by Narayan and Sharma (2015), Narayan et al. (2015a), Phan et al. (2015a) and Phan et al. (2015b) among others, data frequency matters in financial economics research and this is not specific to a particular strand of the literature. Thus, as a robustness check, we assess the sensitivity of the hypotheses test results to alternative data frequency using quarterly and yearly data, even it is a well-known fact that a higher frequency data provides additional information. We follow the same modeling strategy as for the monthly inflation rates with one exception for the yearly data. As the annual inflation rates are not long memory featured, we used the TV-ARMA model to account for regime shifts in the mean instead of the TV-ARFIMA process. The results of the LM-type test based on the quarterly and yearly data (see Tables 6–9) are mostly consistent with those obtained in Tables 2 and 3 for monthly

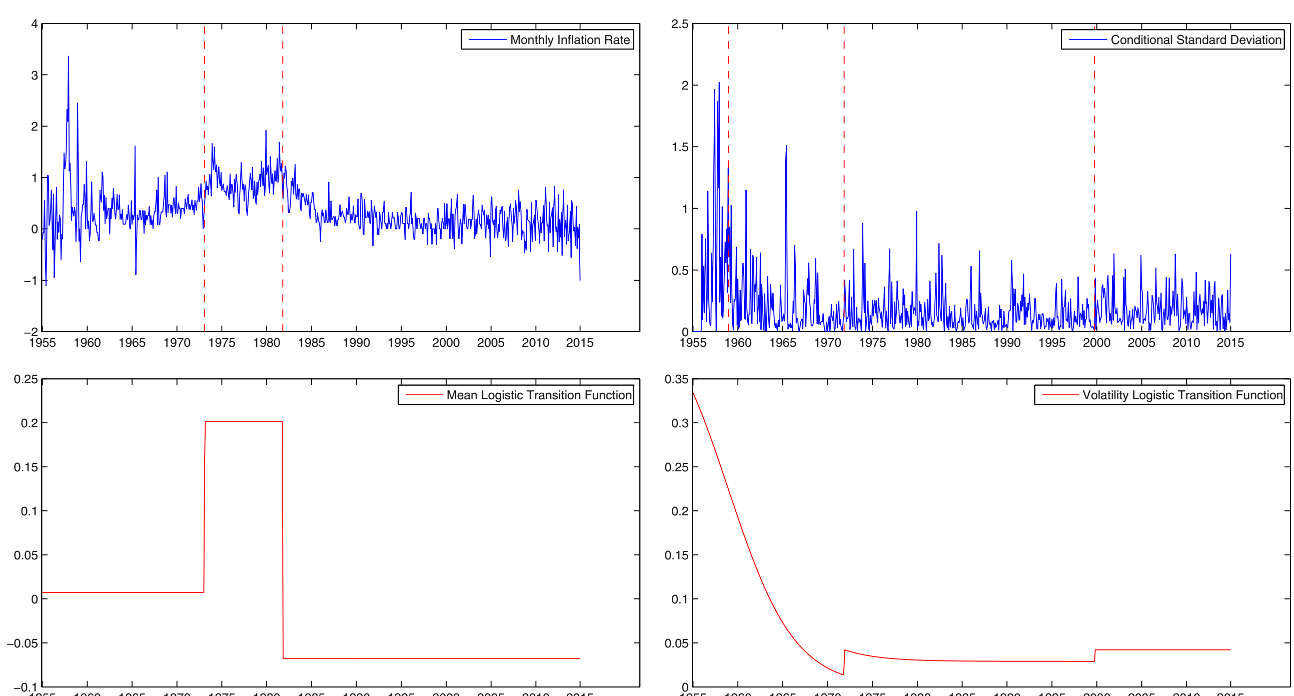

Fig. 6. Plots of the France inflation rate, the conditional standard deviations, the mean baseline logistic transition function F_{1t} and the volatility baseline logistic transition function F_{2t}.

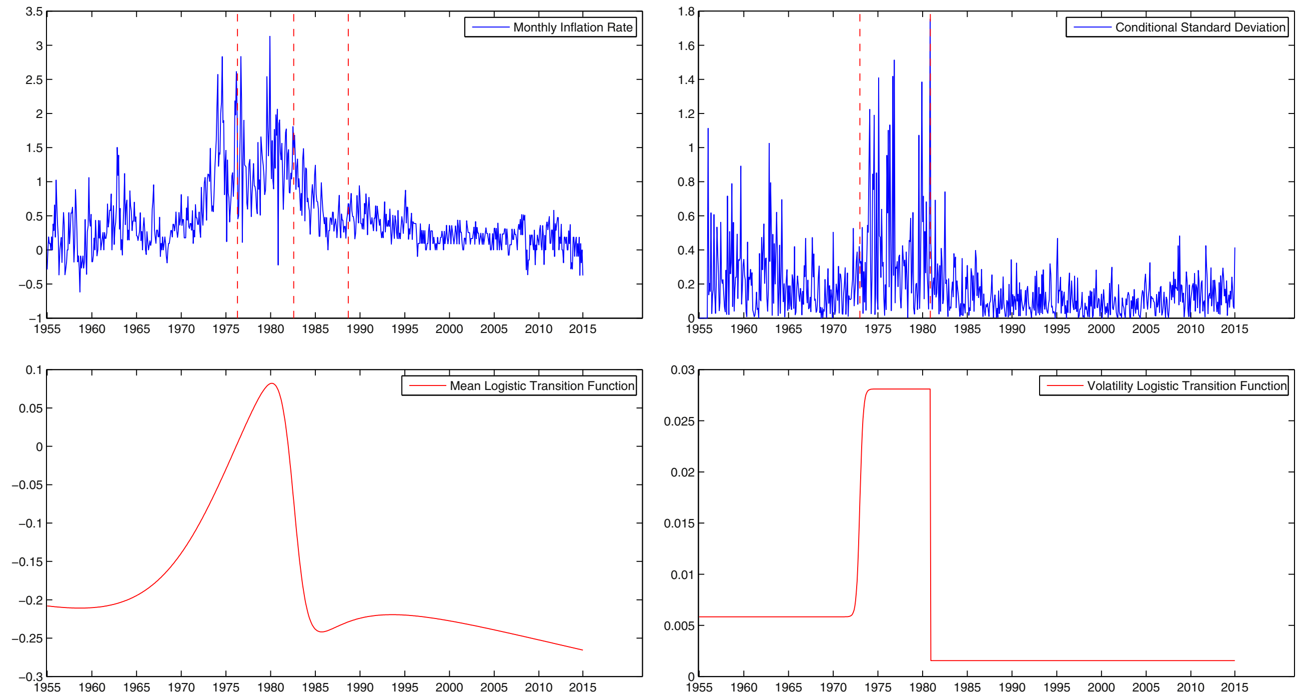

Fig. 7. Plots of the Italy inflation rate, the conditional standard deviations, the mean baseline logistic transition function F_{1t} and the volatility baseline logistic transition function F_{2t}.

inflation rates. In particular, it is clearly depicted that the null hypothesis of the baseline mean and volatility stability can be rejected for all the inflation rates. Let us take a closer look at these results. Testing the intercept stability, we detected one additional structural break for Canada using both quarterly and yearly inflation rates. One reason behind these few differences in test results could well be attributed to the aggregation of monthly data to quarterly and annually data which makes regime switches more distinct. By contrast, one less break was detected testing the baseline volatility stability of the quarterly inflation rates of Canada, UK and France, while we identified only two breaks instead of four in the case of UK based on the annual inflation rates. This discrepancy in results is not surprising, as it is most likely due to the fact that high frequency data are more volatile, that is to say quarterly and yearly data may fail to capture information contained in monthly data such as regime shifts. Thus, we conclude that our baseline results are robust enough with respect to the number of shifts detected in mean and volatility baselines of the inflation dynamics and, thus, changing the data frequency do not alter seriously the findings.

Turning back our attention to the monthly data, two classes of models were considered for modeling the G7 inflation rates; the ARFIMA–GARCH model proposed by Baillie et al. (1996) and its extension the TV-ARFIMA–TV-GARCH model which accounts for regime shifts. Both models were estimated using the quasi-maximum likelihood estimation method (QMLE) and nested in the following most general specification:

$$\begin{cases} \Phi(L)(1-L)^d y_t = \mu_0 + F_{1t} + \Psi(L)\varepsilon_t \\ \varepsilon_t = z_t\sqrt{h_t}, \quad \varepsilon_t|\Omega_{t-1} \sim N(0, h_t) \\ h_t = \omega_0 + F_{2t} + \sum_{i=1}^{n}\alpha_i\varepsilon_{t-i}^2 + \sum_{j=1}^{m}\beta_j h_{t-j} \end{cases}$$

where F_{1t} and F_{2t} are the logistic transition functions allowing to unconditional mean and unconditional variance to vary over time. At first sight, it is worth noting that the TV-ARFIMA–TV-GARCH model outperforms the ARFIMA–GARCH model of Baillie et al. (1996) according to AIC and RMSE criteria (see Tables 4 and 5). Such results suggest that accounting for structural changes can provide better fit of the data set. Focusing on the parameter estimates, the results reveal that in all series the long memory orders have less magnitude using the TV-ARFIMA–TV-GARCH model than those estimated from ARFIMA–GARCH model. This demonstrates that a part of the long run persistence in the inflation mean is due to regime shifts, which is in line with Belkhouja and Boutahar (2009) and Baillie and Morana (2012) findings among others. Moreover, while the GARCH parameter sums in Table 4 are very close to one, we notice a considerable reduction in volatility persistence when shifts in the variance are taken into account for all of the countries investigated here (see Table 5). These findings also are consistent with many others in the literature including Malik et al. (2005), Mansur et al. (2007) and Marcelo et al. (2008), as GARCH specification might overestimates the volatility persistence if changes in the unconditional variance are neglected. So, as expected, accounting for structural changes in our model specification reduces significantly the mean and the variance persistence. Regarding the logistic transition functions, we detect, in most cases, two structural breaks in the inflation level and at least one regime switching in the unconditional variance. The estimated smoothness parameters $\hat{\gamma}$ are relatively high for certain inflation rates indicating an abrupt transition from one regime to another, while the estimated threshold parameters $\hat{c}$ vary around very common dates (see bottom left and the bottom right panels of Figs. 1–7).

To visualize our results, let us start with the case of the US inflation rate. As shown in the top left panel of Fig. 1, two break points or three regimes are detected in the inflation level dynamics over the sample investigated. The inflation mean increases sharply in 1973 ($\hat{c} = 0.30$) and is maintained relatively high until the beginning of the 1980s as clearly displayed by the logistic transition function F_{1t} in the bottom left panel of Fig. 1. From this empirical finding, a natural interpretation of the US inflation rate instability is related to specific economic and political events occurring after the 1970s. Actually, this high regime of inflation is aligned with the most important oil shocks ever known where in September 1973 the Arab–Israeli War led to a drop of 7.8% in the world production of oil, whereas in July 1980 the Iran–Iraq War led to a 7.2% decline. This period of oil price turbulence caused a dramatic increase in production costs which led to an immediate acceleration of the inflation rate. While the first structural break had an exogenous trigger, the second structural break ($\hat{c} = 0.44$) was country specific. To remedy the unstable economic situation of the 1970s, the federal reserve of the United States, during the chairmanship of Volker and Greenspan, began moving toward an inflation targeting strategy based on a new price stabilization program. This monetary policy has been very successful and its trend was confirmed during the 1990s. On the other hand, the unconditional variance also demonstrates three regimes but with different speeds of transition. After the sharp increase of the US inflation volatility owing to the first oil shock of 1973, there was a gradual and smooth drop during the Volcker–Greenspan. Subsequently, the US volatility stabilization episode was followed by a sudden rise in the beginning of the millennium ($\hat{c} = 0.74$), as exposed by the logistic transition function F_{2t} in the bottom right panel of Fig. 1. The relatively high volatility of the last fifteen years is probably explained by the recession and the uncertain economic climate characterizing that period.

Turning our attention to the case of Canada and UK, it is noticeable that their inflation dynamics are very close to that of US. The first inflation break in the UK corresponds to the first oil shock along with the breakdown of income policies over the years 1975–1977. Subsequently, the UK inflation rate dropped briefly but rapidly increased again due to the increase in VAT from 8% to 15% in 1979–1980, in parallel to the second petroleum shock. The second identified structural break coincides with the marked decline in inflation during the 1980s as a result of the monetary policy adopted by the Thatcher administration. A similar hump-shaped pattern is observed in the Canadian inflation level but with a smoother dynamics (see the logistic transition function F_{1t} in the bottom left panel of Figs. 3–4). Regarding the UK inflation volatility, we detected five regimes with different smoothing degrees of transition. The logistic transition function F_{2t} in the bottom right panel of Fig. 4 shows how the baseline volatility evolves over time. The first break is explained by the same reasons as the upward movement of the inflation level. The second break took place around 1990 and was followed by a sizable decline in inflation volatility as a result of the adoption of the inflation targeting framework in October 1992. Finally, following the recent financial crisis in the period 2007 to 2009 and consequently the huge increase in the budget deficit, UK inflation volatility has been moving upward once again. Canada also has experienced an official inflation target in the early 1990s, which reduced its inflation level and volatility, to some extent. However, uncertainty measures have risen again during the Global Financial Crisis, similarly to the majority of the G7 economies.

Interestingly, the picture of Japan contrasts with the pattern of the other countries and there are at least two features characterizing it. First, Japan exhibits the lowest long memory persistence among the other G7 countries. Second the Japanese inflation level and volatility seem to be the most stable over the period under examination with only one regime switching in the mean and variance as depicted in the bottom left and right panels of Fig. 2. This outcome can be attributed to rigorous and more control-based economic policies held by the Japanese Bank after the first oil shock of 1973.

Concerning Germany, we detected two structural breaks around the first oil shock (see the logistic transition function F_{1t} in the bottom left panel of Fig. 5), demonstrating a very short rise in inflation before it moves back to a lower level. This observed behavior is in line with the anti-inflationary reputation of the German monetary authorities. The gradual decrease of the inflation volatility, from the beginning of the sample, is followed in the early 1990s by a sudden rise. Such change may be attributed to the reunification of Germany which led to significant budget deficits. Subsequently, the inflation-targeting monetary policy has moderated this higher inflation uncertainty (see the logistic transition function F_{2t} in the bottom right panel of Fig. 5).

In France and Italy also, the inflation rate rises noticeably during the 70s in view of the two petroleum shocks (see the logistic transition function F_{1t} in the bottom left panel of Figs. 3–4). In order to reverse this rise, the French government abandoned the 'encadrement du credit' policy and moved to the 'Franc fort' policy, while the Banca d'Italia announced its independence from the Italian Treasury in the early 1980s, leading to a more aggressive approach and hence lower and more stable inflation rate. Despite the Italian government efforts, the estimated results reveal that the Italian inflation presents the largest long run persistence in the group G7, revealing a certain economic fragility due to the stronger and longer persistence of shocks' effect on the national inflation rate.

To check the goodness of fit of our models, we used diagnostic tests on the standardized residuals and the squared standardized residuals. Judging the results for all the countries, the hypothesis of uncorrelated standardized and squared standardized residuals is well supported, indicating that allowing for regime shifts in the conditional mean and variance successfully accounts for any remaining nonlinearity in the inflation series. Otherwise, the Jarque–Bera test results indicate that the standardized residuals are not normally distributed, which might be set using heavy-tailed conditional densities such as Student or generalized error distributions.

5.3. Discussion

To sum up, within the economic situation over the last five decades, the oil price spikes were a significant trigger for a wave of high inflation across the world. In order to dampen the transmission of the petroleum shocks, the monetary authorities have undertaken major legislative reforms and this disinflation episode has been effective relatively until the end of 1990s. Toward the end of the sample, the variance has risen again in almost all countries reflecting the recession of the 2000s along with the global financial crisis, whereas, the inflation rate has been maintained under control. All the changes in the inflation level and volatility dynamics of the G7 inflation rates over the period under examination were accurately depicted with different levels of smoothness, using the logistic function. The logistic function has proved to be very useful in capturing not only sudden structural changes, but also gradual and smooth regime shifts, which would not be possible using other models accounting only for the first type of breaks.

Central bankers and financial institutions require accurate measurements for inflation persistence and uncertainty to design an optimal monetary policy. To this end, we sought to capture any potential spurious persistence generated by structural changes in the mean and volatility of inflation rates and provided evidence that ignoring such nonlinearities overestimates the long run and volatility persistence. Such a result might help monetary authorities in the decision process toward adjusting the policy instrument to achieve the desired target more accurately.

On the other hand, our findings concur with a research consensus that, in the developed countries, there is now a better understanding of how to implement monetary policy. In this vein, countries exhibiting a stronger commitment to price stability, following an inflation targeting strategy and showing strengthened domestic monetary policy independence (such as Japan and Germany) are less sensitive to international shocks than those with weaker inflation discipline (such as Italy). Such differences, reflected by a low long run persistence, contribute to these countries' macroeconomic performance improvements.

A final conclusion from our results is that higher inflation uncertainty is typically associated with higher inflation level, as has been stressed in the literature e.g., Friedman (1977) and Ball (1992) among others. Similarly, when policy regulations reduce the inflation level, this also contributes to a stabilization of inflation uncertainty and, hence, more stability.

6. Conclusion

It is widely agreed that many inflation rates, in the past few decades, exhibit persistence which could be partially explained by changes in the time series characteristics due to specific economic and political events. In this regard, we propose a new model named TV-ARFIMA–TV-GARCH which accounts jointly for long memory and nonlinearity properties of data. This extension of the ARFIMA–GARCH model was applied to the G7 (USA, Japan, Canada, United Kingdom, Germany, France and Italy) monthly inflation rates in order to better understand the persistence origins of these time series.

Based on our modeling strategy, the analysis of the structural stability reveals that all the G7 inflation rates demonstrate regime shifts in their baseline means and volatilities. On one hand, the estimated TV-ARFIMA–TV-GARCH model provides lower fractional differencing orders and GARCH parameter estimates than those obtained with the ARFIMA–GARCH model. This finding suggests that, as a matter of fact, the inflation expectation is less backward looking and the volatility is less persistent than it appears using an ARFIMA–GARCH specification. On the other hand, the estimated logistic functions reflect the changing dynamics of the G7 inflation rates driven by three major events that marked the examined period: Oil price shocks, changes in monetary policy behavior and the recession of 2000s followed by the global financial crisis. As a conclusion, we believe that our modeling strategy, based on the TV-ARFIMA–TV-GARCH specification, could be very useful when structural changes are likely to occur in persistent time series.

In a future research, we intend to pursue the estimation of structural breaks in both intercepts and fractional integration degrees, and investigate whether inflation expectation sensitivity to transitory shocks is also time varying. To this end, we will adjust and test the effectiveness of our modeling strategy using the logistic function in order to capture the long run persistence dynamics.

Appendix A. Empirical results

Table 1
Summary statistics.

	US	JA	CA	UK	GE	FR	IT
μ	0.30	0.24	0.30	0.41	0.21	0.36	0.46
σ	0.36	0.68	0.44	0.62	0.35	0.44	0.51
Min.	−1.93	−2.37	−1.32	−1.55	−1.16	−1.11	−0.62
Max.	1.79	4.22	2.59	4.24	1.73	3.36	3.13
k	3.09	3.52	1.48	7.36	1.55	4.39	3.70
sk	0.02	1.10	0.53	1.78	0.44	1.04	1.67
Arch	266.95 (0.00)	154.44 (0.00)	119.50 (0.00)	197.41 (0.00)	73.55 (0.00)	302.58 (0.00)	389.28 (0.00)
Trend	0.00 (0.16)	0.00 (0.50)	0.00 (0.22)	0.00 (0.46)	0.00 (0.14)	0.00 (0.26)	0.00 (0.36)
ADF	−10.93*** [2]	−22.14*** [1]	−6.87*** [4]	−4.75*** [5]	−22.71*** [1]	−6.53*** [3]	−4.89*** [3]
NLW	−43.73**	−23.33**	−19.10**	−20.30**	−29.77**	−16.44**	−21.98**

$[\alpha, \beta]$		N = 150			N = 250			N = 500		
	$t1/t2$	0.4	0.6	0.8	0.4	0.6	0.8	0.4	0.6	0.8
[0.05, 0.90]	0.2	−3.83	−3.83	−3.82	−3.75	−3.77	−3.74	−3.65	−3.69	−3.66
	0.4		−3.82	−3.82		−3.75	−3.74		−3.66	−3.65
	0.6			−3.82			−3.75			−3.67
		N = 150			N = 250			N = 500		
	t1/t2	0.4	0.6	0.8	0.4	0.6	0.8	0.4	0.6	0.8
[0.45, 0.50]	0.2	−3.87	−3.78	−3.76	−3.69	−3.69	−3.66	−3.60	−3.60	−3.58
	0.4		−3.77	−3.75		−3.66	−3.65		−3.59	−3.58
	0.6			−3.76			−3.65			−3.60
		N = 150			N = 250			N = 500		
	t1/t2	0.4	0.6	0.8	0.4	0.6	0.8	0.4	0.6	0.8
[0.90, 0.05]	0.2	−3.74	−3.75	−3.70	−3.65	−3.64	−3.64	−3.56	−3.54	−3.55
	0.4		−3.71	−3.74		−3.61	−3.65		−3.51	−3.58
	0.6			−3.71			−3.62			−3.52

Notes: μ denotes the average inflation rate, σ its standard deviation, its minimum (min.) and its maximum (max.). k is the Kurtosis, sk is the Skewness coefficient. The statistics of the heteroskedasticity test with 12 lags and the p-values (in parenthesis) are reported. Trend is an OLS regression model with time trend. The Augmented Dickey and Fuller (ADF) unit root test statistics including a drift are also reported along with the optimal lag length in the ADF regression (in square brackets). NLW refers to the GARCH two-structural break unit root test statistics proposed by Narayan et al. (2015b) and the 5% critical values are provided in the table below at different sample sizes (N) ranging from 150 to 500, GARCH parameters ($[\alpha, \beta]$) combinations and different structural break locations ($t1/t2$) ranging from 0.2 to 0.8.
** Denotes significance at 5% level.
*** Denotes significance at 1% level.

Table 2
Testing the baseline mean constancy for monthly inflation rates.

	US		JA		CA		UK		GE		FR		IT	
H_{01}	16.52	(0.00)	31.61	(0.00)	9.08	(0.00)	11.17	(0.00)	7.81	(0.01)	4.65	(0.03)	7.64	(0.01)
H_{02}	22.00	(0.00)	29.85	(0.00)	156.13	(0.00)	58.56	(0.00)	10.69	(0.00)	14.33	(0.00)	8.21	(0.00)
H_{03}	14.48	(0.00)	23.45	(0.00)	150.73	(0.00)	29.29	(0.00)	7.43	(0.01)	5.68	(0.02)	15.23	0.00)
H_{04}	11.28	(0.00)	24.92	(0.00)	153.06	(0.00)	51.35	(0.00)	6.35	(0.01)	5.47	(0.02)	5.64	(0.02)
H_{05}	13.81	(0.00)	23.18	(0.00)	144.33	(0.00)	45.34	(0.00)	9.16	(0.00)	4.04	(0.04)	7.54	(0.01)
R_1	2		1		2		2		2		2		3	

Notes: The numbers in parenthesis are the p-values. R_1 is the number of structural changes in the conditional mean for the monthly inflation rates.

Table 3
Testing the baseline volatility constancy for monthly inflation rates.

	US		JA		CA		UK		GE		FR		IT	
H_{01}	2.46	(0.11)	9.17	(0.00)	2.38	(0.12)	8.18	(0.00)	3.03	(0.08)	2.66	(0.10)	0.69	(0.40)
H_{02}	0.04	(0.84)	0.57	(0.44)	4.05	(0.04)	0.69	(0.41)	7.08	(0.00)	6.66	(0.01)	12.43	(0.00)
H_{03}	4.99	(0.02)	0.25	(0.61)	6.93	(0.01)	5.05	(0.02)	1.06	(0.30)	7.60	(0.01)	1.87	(0.17)
H_{04}	0.35	(0.55)	1.74	(0.18)	3.27	(0.07)	8.59	(0.00)	1.18	(0.28)	2.15	(0.14)	11.85	(0.00)
H_{05}	0.39	(0.52)	1.12	(0.28)	2.80	(0.09)	3.79	(0.05)	0.59	(0.44)	1.48	(0.22)	3.14	(0.08)
R_2	3		1		3		4		2		3		2	

Notes: The numbers in brackets are the p-values. R_2 is the number of structural changes in the conditional variance for the monthly inflation rates.

Table 4
Estimation of ARFIMA–GARCH models for inflation.

	US	JA	CA	UK	GE	FR	IT
$\hat{d}$	0.32 (0.04)	0.10 (0.04)	0.17 (0.05)	0.22 (0.04)	0.13 (0.04)	0.29 (0.04)	0.37 (0.04)
$\hat{\mu}_0$	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.10 (0.00)	0.00 (0.00)	−0.00 (0.00)
$\hat{\phi}_1$	–	–	0.11 (0.05)	–	–	–	–
$\hat{\phi}_6$	–	–	–	–	–	–	−0.04 (0.01)
$\hat{\psi}_1$	–	–	−0.17 (0.06)	–	–	–	–
$\hat{\psi}_3$	−0.05 (0.02)	–	–	–	–	–	–
$\hat{\psi}_6$	–	–	–	0.05 (0.02)	–	0.11 (0.04)	0.07 (0.04)
$\hat{\psi}_9$	–	0.01 (0.00)	–	–	0.07 (0.03)	–	–
$\hat{\psi}_{10}$	–	0.04 (0.01)	–	–	0.07 (0.03)	–	–
$\hat{\psi}_{11}$	0.06 (0.02)	–	–	–	–	–	–
$\hat{\omega}_0$	0.00 (0.00)	0.00 (0.00)	0.03 (0.00)	0.00 (0.00)	0.00 (0.00)	0.01 (0.00)	0.00 (0.00)
$\hat{\alpha}_1$	0.19 (0.06)	0.05 (0.02)	0.11 (0.05)	0.07 (0.03)	0.12 (0.05)	0.16 (0.07)	0.10 (0.05)
$\hat{\beta}_1$	0.75 (0.06)	0.94 (0.08)	0.67 (0.26)	0.92 (0.03)	0.26 (0.12)	0.32 (0.11)	0.52 (0.13)
$\hat{\beta}_2$	–	–	–	–	0.54 (0.15)	0.41 (0.20)	0.37 (0.19)
log L	−3.50	−443.16	−299.06	−299.52	−95.51	−37.64	47.89
AIC	25.00	904.32	616.12	615.04	211.03	93.28	−75.79
$RMSE$	0.2672	0.5204	0.3717	0.4143	0.2876	0.2994	0.2879
$Q(12)$	15.26 [0.12]	14.96 [0.13]	10.96 [0.36]	4.87 [0.89]	5.63 [0.84]	2.77 [0.98]	15.30 [0.12]
$Q^2(12)$	3.81 [0.95]	2.01 [0.99]	7.65 [0.66]	5.29 [0.87]	10.19 [0.42]	4.35 [0.93]	4.29 [0.93]
$ARCH(4)$	1.09 [0.89]	1.23 [0.87]	1.72 [0.87]	3.06 [0.54]	4.25 [0.37]	1.77 [0.77]	1.62 [0.80]
JB	151.88 [0.00]	1036.92 [0.00]	294.64 [0.00]	1969.21 [0.00]	116.25 [0.00]	702.01 [0.00]	155.90 [0.00]

Notes: Log L denotes the maximum value of the log likelihood. $Q(12)$ and $Q^2(12)$ are respectively the 12th order Ljung–Box tests for serial correlation in the standardized and squared standardized residuals. JB is the Jarque–Bera normality test. The asymptotic standard errors are reported in parenthesis and the numbers in brackets are the p-values.

Table 5
Estimation of TV-ARFIMA–TV-GARCH models for inflation.

	US	JA	CA	UK	GE	FR	IT
$\hat{d}$	0.23 (0.04)	0.08 (0.02)	0.13 (0.02)	0.17 (0.06)	0.10 (0.03)	0.23 (0.05)	0.32 (0.04)
$\hat{\mu}_0$	0.00 (0.00)	0.04 (0.00)	−0.02 (0.00)	0.01 (0.00)	0.00 (0.00)	0.01 (0.00)	0.04 (0.00)
$\hat{\phi}_1$	–	–	0.19 (0.03)	–	–	–	–
$\hat{\phi}_6$	–	–	–	–	–	–	−0.03 (0.01)
$\hat{\psi}_1$	–	–	−0.36 (0.05)	–	–	–	–
$\hat{\psi}_3$	−0.07 (0.02)	–	–	–	–	–	–
$\hat{\psi}_6$	–	–	–	–	–	0.09 (0.03)	0.04 (0.01)
$\hat{\psi}_9$	–	–	–	–	0.05 (0.03)	–	–
$\hat{\psi}_{10}$	–	–	–	–	0.05 (0.03)	–	–
$\hat{\psi}_{11}$	0.03 (0.01)	–	–	–	–	–	–
$\hat{\mu}_1$	0.14 (0.05)	−0.72 (0.07)	0.95 (0.05)	0.37 (0.18)	0.40 (0.07)	0.19 (0.07)	0.55 (0.26)
$\hat{\mu}_2$	−0.25 (0.05)	–	−0.36 (0.03)	−0.52 (0.23)	−0.37 (0.04)	−0.26 (0.09)	−0.44 (0.15)
$\hat{\mu}_3$	–	–	–	–	–	–	−0.69 (0.22)
$\hat{\gamma}_{11}$	1311.30 (11.65)	25.77 (9.24)	10.06 (2.18)	544.64 (18.98)	74.84 (13.67)	1760.33 (76.29)	12.69 (4.36)
$\hat{\gamma}_{12}$	5875.30 (15.09)	–	3232.57 (76.83)	142.57 (36.83)	79.43 (27.37)	4784.63 (37.04)	77.71 (24.83)
$\hat{\gamma}_{13}$	–	–	–	–	–	–	1.06 (0.39)
$\hat{c}_{11}$	0.30 (0.01)	0.40 (0.02)	0.22 (0.03)	0.31 (0.01)	0.27 (0.01)	0.30 (0.01)	0.36 (0.02)
$\hat{c}_{12}$	0.44 (0.01)	–	0.46 (0.01)	0.44 (0.02)	0.32 (0.02)	0.45 (0.01)	0.46 (0.04)
$\hat{c}_{13}$	–	–	–	–	–	–	0.56 (0.08)
$\hat{\omega}_0$	0.06 (0.01)	0.42 (0.03)	0.11 (0.03)	0.01 (0.00)	0.16 (0.05)	0.44 (0.17)	0.00 (0.00)
$\hat{\alpha}_1$	0.13 (0.06)	0.13 (0.05)	0.14 (0.04)	0.06 (0.01)	0.16 (0.04)	0.16 (0.07)	0.05 (0.02)
$\hat{\beta}_1$	–	–	0.34 (0.07)	0.65 (0.05)	0.06 (0.02)	0.06 (0.02)	0.66 (0.26)
$\hat{\beta}_2$	–	–	–	–	0.34 (0.11)	0.05 (0.02)	0.05 (0.02)
$\hat{\omega}_1$	0.09 (0.03)	−0.35 (0.02)	−0.11 (0.03)	0.65 (0.03)	−0.16 (0.03)	−0.45 (0.17)	0.02 (0.01)
$\hat{\omega}_2$	−0.14 (0.03)	–	0.35 (0.05)	−0.66 (0.13)	0.03 (0.01)	0.03 (0.01)	−0.03 (0.01)
$\hat{\omega}_3$	0.08 (0.02)	–	−0.28 (0.03)	0.19 (0.03)	–	0.02 (0.01)	–
$\hat{\omega}_4$	–	–	–	−0.11 (0.02)	–	–	–
$\hat{\gamma}_{21}$	9311.29 (5.34)	22.42 (4.59)	2126.32 (112.19)	1.81 (0.30)	4.75 (1.26)	16.23 (13.42)	321.29 (54.57)

Table 5 (*continued*)

	US	JA	CA	UK	GE	FR	IT
$\hat{\gamma}_{22}$	9.00 (2.76)	–	9.42 (4.87)	3.28 (0.37)	1376.55 (5.32)	1179.41 (22.46)	5183.56 (45.23)
$\hat{\gamma}_{23}$	3747.31 (64.75)	–	20.03 (8.87)	18.64 (9.31)		3263.41 (22.46)	–
$\hat{\gamma}_{24}$	–	–	–	194.19 (40.23)	–	–	–
$\hat{c}_{21}$	0.30 (0.00)	0.41 (0.02)	0.10 (0.02)	0.15 (0.02)	0.05 (0.02)	0.07 (0.02)	0.30 (0.00)
$\hat{c}_{22}$	0.42 (0.03)	–	0.32 (0.11)	0.17 (0.03)	0.95 (0.00)	0.28 (0.00)	0.43 (0.00)
$\hat{c}_{23}$	0.74 (0.00)	–	0.38 (0.13)	0.35 (0.07)	–	0.75 (0.00)	–
$\hat{c}_{24}$	–	–	–	0.42 (0.02)	–	–	–
log *L*	28.50	−408.92	−257.27	−255.73	−76.10	1.59	76.74
AIC	−11.01	843.85	562.54	561.47	209.17	44.81	−102.88
RMSE	0.2580	0.5147	0.3601	0.4002	0.2789	0.2890	0.2837
$Q(12)$	14.04 [0.18]	12.76 [0.24]	4.46 [0.92]	7.43 [0.68]	4.87 [0.89]	4.34 [0.93]	8.01 [0.62]
$Q^2(12)$	10.14 [0.41]	2.1 [0.99]	5.64 [0.84]	9.95 [0.44]	8.87 [0.54]	6.37 [0.78]	1.03 [0.99]
ARCH(4)	5.82 [0.22]	0.54 [0.96]	2.14 [0.70]	1.82 [0.77]	4.39 [0.35]	2.99 [0.55]	0.47 [0.97]
JB	41.08 [0.00]	811.14 [0.00]	473.25 [0.00]	406.36 [0.00]	115.04 [0.00]	249.46 [0.00]	49.74 [0.00]

Notes: Log *L* denotes the maximum value of the log likelihood. $Q(12)$ and $Q^2(12)$ are respectively the 12th order Ljung–Box tests for serial correlation in the standardized and squared standardized residuals. *JB* is the Jarque–Bera normality test. The asymptotic standard errors are reported in parenthesis and the numbers in brackets are the p-values.

Table 6
Testing the baseline mean constancy for quarterly inflation rates.

	US		JA		CA		UK		GE		FR		IT	
H_{01}	0.80	(0.36)	5.47	(0.02)	72.21	(0.00)	0.05	(0.81)	1.67	(0.19)	72.74	(0.00)	8.89	(0.01)
H_{02}	4.32	(0.03)	3.62	(0.06)	166.19	(0.00)	6.01	(0.01)	3.87	(0.04)	179.53	(0.00)	9.98	(0.00)
H_{03}	2.12	(0.14)	1.15	(0.28)	174.33	(0.00)	0.13	(0.71)	0.76	(0.38)	179.02	(0.00)	11.67	0.00)
H_{04}	2.33	(0.12)	2.21	(0.13)	139.81	(0.00)	1.51	(0.21)	0.97	(0.32)	159.59	(0.00)	7.93	(0.00)
H_{05}	3.05	(0.08)	3.01	(0.08)	150.44	(0.00)	0.12	(0.72)	1.11	(0.29)	153.69	(0.00)	6.99	(0.00)
R_1	2		1		3		2		2		2		3	

Notes: The numbers in parenthesis are the p-values. R_1 is the number of structural changes in the conditional mean for the quarterly inflation rates.

Table 7
Testing the baseline volatility constancy for quarterly inflation rates.

	US		JA		CA		UK		GE		FR		IT	
H_{01}	0.41	(0.51)	8.41	(0.00)	0.57	(0.44)	1.71	(0.19)	3.25	(0.07)	1.49	(0.22)	0.81	(0.36)
H_{02}	3.81	(0.05)	0.94	(0.33)	3.65	(0.05)	2.31	(0.12)	8.63	(0.00)	9.36	(0.00)	15.95	(0.00)
H_{03}	9.89	(0.00)	2.10	(0.14)	0.55	(0.45)	9.23	(0.00)	7.06	(0.00)	8.32	(0.00)	10.33	(0.00)
H_{04}	5.02	(0.02)	0.43	(0.51)	1.16	(0.27)	0.89	(0.34)	6.48	(0.01)	6.95	(0.00)	9.29	(0.00)
H_{05}	5.25	(0.02)	2.29	(0.12)	0.55	(0.45)	5.21	(0.02)	1.09	(0.30)	2.25	(0.12)	1.55	(0.22)
R_2	3		1		2		3		2		2		2	

Notes: The numbers in brackets are the p-values. R_2 is the number of structural changes in the conditional variance for the quarterly inflation rates.

Table 8
Testing the baseline mean constancy for annual inflation rates.

	US		JA		CA		UK		GE		FR		IT	
H_{01}	4.64	(0.03)	11.18	(0.00)	2.51	(0.11)	4.34	(0.03)	0.17	(0.67)	8.27	(0.00)	2.15	(0.14)
H_{02}	13.26	(0.00)	8.63	(0.00)	0.32	(0.56)	12.96	(0.00)	9.94	(0.00)	15.52	(0.00)	2.83	(0.09)
H_{03}	3.91	(0.04)	3.70	(0.05)	8.33	(0.00)	2.05	(0.15)	2.65	(0.10)	2.88	(0.08)	12.83	0.00)
H_{04}	8.58	(0.00)	1.39	(0.23)	3.21	(0.07)	4.07	(0.04)	6.04	(0.01)	13.23	(0.00)	2.42	(0.13)
H_{05}	1.90	(0.16)	3.65	(0.05)	4.39	(0.03)	5.45	(0.02)	8.41	(0.0)	8.44	(0.00)	9.75	(0.00)
R_1	2		1		3		2		2		2		3	

Notes: The numbers in parenthesis are the p-values. R_1 is the number of structural changes in the conditional mean for the annually inflation rates.

Table 9
Testing the baseline volatility constancy for annual inflation rates.

	US		JA		CA		UK		GE		FR		IT	
H_{01}	2.69	(0.10)	18.30	(0.00)	9.19	(0.00)	3.68	(0.05)	11.84	(0.07)	3.37	(0.06)	7.74	(0.00)
H_{02}	0.42	(0.51)	1.74	(0.18)	8.93	(0.00)	10.54	(0.00)	22.01	(0.00)	10.58	(0.00)	8.84	(0.00)
H_{03}	7.18	(0.00)	0.68	(0.40)	10.80	(0.00)	7.08	(0.00)	1.56	(0.21)	11.69	(0.00)	5.16	(0.02)
H_{04}	4.81	(0.02)	2.98	(0.08)	10.08	(0.00)	9.31	(0.00)	7.32	(0.00)	1.59	(0.20)	3.45	(0.06)
H_{05}	0.85	(0.34)	4.23	(0.03)	6.09	(0.01)	1.20	(0.25)	9.86	(0.00)	7.57	(0.12)	3.55	(0.06)
R_2	3		1		3		2		2		3		2	

Notes: The numbers in brackets are the p-values. R_2 is the number of structural changes in the conditional variance for the annually inflation rates.

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