# 1 Research Topics to read

1. What is optimal control?
2. Read papers provided by Max.
3. Summarize papers to best knowledge.

# 2 Research

## 2.1 Artificial Viscosity

Necessary additional term introduced to achieve numerical stability that comes from the discretization of the convective term in the Navier-Stokes equation. It is only active in the streamline direction of the calculation and is dependent on $\Delta t$ therefore yielding only higher order effects.

## 2.2 Optimization Techniques

**Least-squares** minimizes the sum of the squares of the residuals between a given fitting model and data. Linear least-squares, where the residuals are linear in the unknowns, has a closed form solution which can be computed by taking the derivative of the residual with respect to each unknown and setting it to zero. It is commonly used in the engineering and applied sciences for fitting polynomial functions. Nonlinear least-squares typically requires iterative refinement based upon approximating the nonlinear least-squares with a linear least-squares at each iteration.

**Gradient descent** is the industry leading, convex optimization method for high-dimensional systems. It minimizes residuals by computing the gradient of a given fitting function. The iterative procedure updates the solution by moving downhill in the residual space. The Newton- Raphson method is a one-dimensional version of gradient descent. Since it is often applied in high-dimensional settings, it is prone to find only local minima. Critical innovations for big data applications include stochastic gradient descent and the backpropagation algorithm which makes the optimization amenable to computing the gradient itself.

**Alternating descent method (ADM)** avoids computations of the gradient by optimizing in one unknown at a time. Thus all unknowns are held constant while a line search (non-convex optimization) can be performed in a single variable. This variable is then updated and held constant while another of the unknowns is updated. The iterative procedure continues through all unknowns and the iteration procedure is repeated until a desired level of accuracy is achieved.

**Augmented Lagrange method (ALM)** is a class of algorithms for solving constrained optimization problems. They are similar to penalty methods in that they replace a constrained optimization problem by a series of unconstrained problems and add a penalty term to the objective which helps enforce the desired constraint. ALM adds another term designed to mimic a Lagrange multiplier. The augmented Lagrangian is not the same as the method of Lagrange multipliers.

**Linear program and simplex method** are the workhorse algorithms for convex optimization. A linear program has an objective function which is linear in the unknown and the constraints consist of linear inequalities and equalities. By computing its feasible region, which is a convex polytope, the linear programming algorithm finds a point in the polyhedron where this function has the smallest (or largest) value if such a point exists. The simplex method is a specific iterative technique for linear programs which aims to take a given basic feasible solution to another basic feasible solution for which the objective function is smaller, thus producing an iterative procedure for optimizing. [1]

## 2.3 Derivation of Navier-Stokes Equations

# Bibliography


[1] Steven L. Brunton and J. Nathan Kutz. *Data-Driven Science and Engineering: Machine Learning, Dynamical Systems, and Control*. Cambridge University Press, 2019.

[2] tec science. Derivation of the navier-stokes equations. 2020.