On control and stabilization of a rotating beam by applying moments at the base only*

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Abstract

A decomposition-method introduced in [Le1] is used to decompose the rigid-body motion from the elastic vibration of a slowly rotating beam. On the base of the transformed system, a controller is constructed that steers all oscillations of the beam to rest in finite time. In addition, the beam is thereby driven to zero angular velocity. A second result is concerned with strong feed-back stabilizability.

1 The control problem

In this note I want to give another example of how a simple idea of decomposing complex systems, introduced in [Le1], can be effectively used for the design of controls. The example which I will discuss here is that of a slowly rotating beam. This mechanical substructure has been the subject of many research articles in control theory. It is of some importance in the area of control of flexible space-structures. The typical model is given by a long thin flexible beam which is attached to a cylinder -called the base - at one end. The cylinder has its axis perpendicular to the plane of bending of the beam (plane motion is assumed), and this is also the axis of rotation. The control objective is to counteract disturbations of a slewing maneuver at a given angular velocity, which may be introduced to the system from various sources. In particular, it is of great importance to extinguish all vibrations of the structure due to bending. In addition, the controlled - non oscillatory - structure should approach a desired angular velocity (here set to be equal to zero) as fast as possible. For the sake of brevity, I dispense with any mechanical derivation of such systems, and refer to [DK], [BL]. Let w represent the vertical displacement of the centerline of the beam in the rotated frame. Let φ denote the angular velocity at which the frame is rotated. The equlibrium towards which the system should ultimately be

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controlled is $w = 0$, $\varphi = 0$. ($\varphi \neq 0$ could be handled equally well.) The only control that can be applied is a torque, i.e. the moment applied to the base-cylinder. The system is then given by

$$(CP) \quad \begin{cases} \ddot{w} + \gamma^2 w'''' + \alpha x \ddot{\varphi} = & 0, \qquad (x,t) \in (0,\ell) \times (0,\infty) \\ \ddot{\varphi} + \alpha \int_0^\ell x \ddot{w} ds + \nu \dot{\varphi} = & M, \qquad t \in (0,\infty) \end{cases}$$

together with the boundary conditions

$$(bc) \quad w(0) = w'(0) = w''(\ell) = w'''(\ell) = 0, \quad t \in (0,\infty),$$

and initial conditions

$$(ic) \quad w(\cdot,0) = w_0, \quad \dot{w}(\cdot,0) = w_1, \quad x \in (0,\ell), \quad \varphi(0) = \varphi_0.$$

For simplicity of notation, the variables x, t are supressed wherever it is felt unmistakable. A dot indicates a time-derivative, a prime denotes a spatial derivation. It should be emphazised that the term $\nu\dot{\varphi}$ is viewed as a previously implemented feed-back control which, in the absence of the active control M, guarantees $w = 0$, $\varphi = 0$ as the eqilibrium. In Fact, it already gives asymptotic stability. One might, therefore, think of just cancelling this term by M and insert the second equation into the first, to end up with a Sobolev-type equation in terms of w. However, this procedure would obscure the perturbation-viewpoint, i.e. that active - open-loop - controllers should be used to perform control actions related to disturbations, rather than to change the whole built-in control set- up. *In addition, asymptotic stability of $\varphi = 0$ could not be achieved that way!* The control process (CP) is also assumed to be already properly rescaled to a non-dimensional form. A system of this sort can be classified under hybrid control systems, as it consists of a partial differential equation together with an ordinary differential equation. As for the mathematical analysis of (CP) I refer to [DM],[Le2] and the bibliographies therein. The purpose of this note is to decompose the rigid body motion - the second equation in (CP) - from the purely oscillatory motion - the first equation in (CP) - by means of a similarity transform. The procedure is as in [Le1], eventhough the theorem there does not apply to this situation directly.

2 Decomposition

It is obvious that, upon integration and use of (bc), the system (CP) is equivalent to

$$(CP') \quad \begin{cases} \ddot{w} + \gamma^2 w'''' + \alpha x \ddot{\varphi} & = & 0 \\ \dot{\varphi} - \alpha\beta w''(0) + \mu\varphi & = & m \end{cases}$$

Here, with $(1 - \frac{\alpha}{3}\ell^3) =: \zeta$ and α small enough, one has $\beta = \gamma^2/\zeta > 1$, $\mu = \nu/\zeta$, $m = M/\zeta$. The paramter α is supposed to model the strenght of the coupling between rotation and bending. This parameter is small in real systems. The second equation in (CP') can be integrated to

$$\varphi(t) = \alpha\beta \int_0^\infty e^{-\mu s} w''(0, t-s) ds + \int_{-\infty}^t e^{-\mu(t-s)} m(s) ds$$

and differentiated again to give

$$\begin{aligned}
\dot{\varphi} &= \alpha\beta \int_0^{\infty} e^{-\mu s} \dot{w}''(0, t-s) ds \\
&\quad + m(t) - \mu \int_{-\infty}^{t} e^{-\mu(t-s)} m(s) ds \\
&= \alpha\beta \int_{-\infty}^{t} e^{-\mu(t-s)} (\dot{w}''(0,s) - \frac{\mu}{\alpha\beta} m(s)) ds + m(t) \\
&=: \beta y(t) + m(t)
\end{aligned}$$

This results in the equivalent system

$$(CP'') \quad \begin{cases} \ddot{w} + \gamma^2 w'''' + \alpha\beta x \cdot y + \alpha x \cdot m = 0 \\ \dot{y} = -\mu y + \alpha(\dot{w}''(0) - \frac{\mu}{\alpha\beta} m) \end{cases}.$$

Upon defining $w =: x_1$, $\dot{w} = x_2$, as usual, on obtains the matrix formulation

$$(M) \quad \dot{\begin{pmatrix} x_1 \\ x_2 \\ y \end{pmatrix}} = \begin{pmatrix} O & I & O \\ -A & O & -\alpha\beta P \\ O & \alpha T & -\mu I \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ y \end{pmatrix} + \begin{pmatrix} 0 \\ \alpha p \\ -\frac{\mu}{\beta} \end{pmatrix} \cdot m$$

Here A is the self-adjoint positive definite operator in $L^2(0,\ell)$

$$Aw = w'''', \quad D(A) = \{w \in H^4(0,\ell) | \; w(0) = w'(0) = w''(\ell) = w'''(\ell) = 0\},$$

with

$$\begin{aligned}
&\sigma(A) = \sigma_d(A) = \{0 < \lambda_1 < \lambda_2 < \ldots < \lambda_n < \ldots \lim_{n\to\infty} \lambda_n = \infty\}, \\
&\{\phi_n\}_1^{\infty}: \quad A\phi_n = \lambda_n \phi_n, \quad <\phi_n, \phi_m> = \delta_{n,m}, \quad w = \sum_n <w, \phi_n> \phi_n \\
&A^s w := \sum \lambda_n^s <w, \phi_n> \phi_n, \\
&D(A^s) = \{w \in H := L^2(0,\ell) | \sum_n \lambda^{2s} | <w, \phi_n> |^2 < \infty\},
\end{aligned}$$

and T, P and p are defined by

$$\begin{aligned}
Tw &= w''(0), \quad D(T) = D(A^{\frac{3}{4}}), \\
Pw &= p \cdot w, \quad p = x.
\end{aligned}$$

T is relatively A–compact and has, therefore, a small A–bound. In addition to that it satisifies the inequality

$$|Tw| \le \ell^2 \|Aw\|, \quad \forall w \in D(A),$$

and even more,

$$|Tw| \le \ell^2 \|A^{\frac{3}{4}} w\|, \quad \forall w \in D(A^{\frac{3}{4}}) \Rightarrow T \cong \delta_0'' \in D(A^{\frac{3}{4}})^*$$

In order for the third component in (M) to make any sense, it seems appropriate to introduce a somewhat stronger energy space than the usual one, i.e.

$$E = D(A^{\frac{3}{4}}) \times D(A^{\frac{1}{4}}) \times \mathbf{R}. \tag{1}$$

This can be called a shifted energy space. It has the property that the wave-type operator $\mathcal{A}$ associated with the beam equation, namely

$$\mathcal{A} = \begin{pmatrix} O & I \\ -A & O \end{pmatrix}, \tag{2}$$

has the dense domain $D(\mathcal{A}) := D(A^{\frac{5}{4}}) \times D(A^{\frac{3}{4}})$ in E, and is maximally dissipative there. In this space set-up one certainly has well-posedness in the strong sense for the system (M).

As in [Le1] one may introduce a (similarity) transformation Γ acting within the "energy-space" as follows

$$\mathbf{x} := (x_1, x_2)^T, \quad \mathcal{Q} := (Q_1, Q_2) : D(A^{\frac{3}{4}}) \times D(A^{\frac{1}{4}}) \longrightarrow \mathbf{R},$$

$$\begin{pmatrix} \mathbf{x} \\ y \end{pmatrix} = \begin{pmatrix} I & O \\ \mathcal{Q} & I \end{pmatrix} \begin{pmatrix} \xi \\ \eta \end{pmatrix} =: \Gamma \begin{pmatrix} \xi \\ \eta \end{pmatrix}.$$

This defines a boundedly invertible operator, with the inverse obtained from Γ by just changing the sign in front of $\mathcal{Q}$. In addition, $\Gamma D(\mathcal{A}) \subseteq D(\mathcal{A})$. Multiplying from the left by Γ^{-1} one obtains

$$\begin{pmatrix} \dot{\xi}_1 \\ \xi_2 \\ \eta \end{pmatrix}$$

$$= \begin{pmatrix} O & I & O \\ -A - \alpha\beta P Q_1 & -\alpha\beta P Q_2 & -\alpha P \\ \left\{ \begin{matrix} -\mu Q_1 + \\ Q_2(A + \alpha P Q_1) \end{matrix} \right\} & \left\{ \begin{matrix} -\mu Q_2 - Q_1 + \\ \alpha\beta Q_2 P Q_2 + \alpha T \end{matrix} \right\} & \left\{ \begin{matrix} -\mu I + \\ \alpha\beta Q_2 P \end{matrix} \right\} \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \\ \eta \end{pmatrix} + \begin{pmatrix} 0 \\ \alpha p \\ -Q_2 p - \frac{\mu}{\beta} \end{pmatrix} \cdot m$$

As in [Le1], one tries to eliminate the first two entries in the third row of the matrix above. This ammounts to solving the operator equations

$$\begin{aligned} -\mu Q_1 + Q_2(A + \alpha P Q_1) &= O \\ -\mu Q_2 - Q_1 + \alpha\beta Q_2 P Q_2 + \alpha T &= O \end{aligned} \tag{3}$$

The meaning of the system (3) is

$$\begin{aligned} -\mu Q_1(f) + A^* Q_2(f) + \alpha\beta Q_2(x) Q_1(f) &= 0 \\ -\mu Q_2(g) - Q_1(g) + \alpha\beta Q_2(x) Q_2(g) + \alpha\delta_0''(g) &= 0 \end{aligned}$$

for all $(f,g) \in D(\mathcal{A})$. The functionals Q_1, Q_2 are in fact elements $q_1 \in D(A^{\frac{3}{4}})^*$, $q_2 \in D(A^{\frac{1}{4}})^*$. Upon eliminating q_1 with the aid of the first equation, one obtains the following inhomogenous nonlinear elliptic boundary value problem.

$$\mu^2 q_2 + (1 - \frac{\alpha\beta}{\mu} q_2(x))^{-1} A q_2 + \alpha\beta\mu q_2(x) q_2 = -\alpha\mu\delta_0'' \tag{4}$$

This semi-linear problem admits a unique small solution in $D(A^{\frac{1}{4}})$, if α is small. This is because, after the application of the resolvent $R(-\mu^2, A) := (\mu^2 + A)^{-1}$ of A to (4) from the left, one obtains

$$q_2 + C(q_2) R(-\mu^2, A) A q_2 + \alpha\beta\mu q_2(x) R(-\mu^2, A) q_2 = -\mu\alpha R(-\mu^2, A)\delta_0'',$$

where $C(q_2)$ is the expression in front of Aq_2 in (4). This, in turn, may be solved by a fixed-point argument, for a small enough α. The fixed-point-map is

$$\begin{aligned} &\Lambda(r) = q, \quad \|r\|_{D(A^{\frac{1}{4}})} \leq \varepsilon, \quad \text{where } q \text{ is the solution of} \\ &q + C(r) A R(-\mu^2, A) q + \alpha\beta\mu r(x) R(-\mu^2, A) q = -R(-\mu^2, A)\delta_0''. \end{aligned} \tag{5}$$

This equation, obtained by freezing the occurence of q_2 in the nonlinear parts, is a linear inhomogenous problem, which admits a unique small solution q for small α. In addition, it is easy to verify that Λ is in fact a contraction, mapping the ε-ball of r's into itself. The resulting fixed-point is q_2 and, a forteriori, q_1 is determined by q_2, and has the right regularity, namely $q_1 \in D(A^{\frac{3}{4}})^*$. Having solved the equations (3), the matrix-system takes on the following block-triangular form

$$\begin{aligned} \begin{pmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \\ \dot{y} \end{pmatrix} &= \begin{pmatrix} O & I & O \\ -A - \alpha\beta P Q_1 & -\alpha\beta P Q_2 & -\alpha P \\ O & O & -\mu I + \alpha\beta Q_2 P \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \\ \eta \end{pmatrix} \\ &\quad + \begin{pmatrix} 0 \\ \alpha p \\ -Q_2 p - \frac{\mu}{\beta} \end{pmatrix} \cdot m \end{aligned} \tag{6}$$

Using the fact that $w = \xi_1, \dot{w} = \xi_2$, this can now be rewritten as follows

$$(\Gamma CP) \begin{cases} \ddot{w} + Aw + \alpha x \cdot q_1(w) + \alpha x \cdot q_2(\dot{w}) + \alpha x \cdot \eta = \alpha x \cdot m \\ \dot{\eta} = -\mu\eta + \alpha\beta q_2(x)\eta - (q_2(x) + \frac{\mu}{\beta}) \cdot m \end{cases}$$

3 Exact controllability

Upon integration, the second equation of (ΓCP) gives

$$\eta = -\frac{1}{\beta}(\rho + \mu) \int_0^t e^{-\sigma(t-s)} m(s) ds \tag{7}$$

where $\sigma := \mu - \alpha\beta q_2(x)$, $\rho := \beta q_2(x)$. In fact, we have assumed zero initial-histories for the variable y and the control m, which is in accordance with the applications, since the process is considered at rest before the loading impacts occur. Once again inserted into the first equation, this ammounts to

$$\begin{aligned} \ddot{w} + Aw + \alpha x \cdot q_1(w) + \alpha x \cdot q_2(\dot{w}) = \\ \alpha x \cdot (m + \frac{1}{\beta}(\rho + \mu) \int_0^t e^{-\sigma(t-s)} m(s) ds \end{aligned} \tag{8}$$

One considers the control problem for (8), neglecting the perturbations in terms of q_1, q_2 first. Set $\frac{1}{\beta}(\rho + \mu) := c$ and multiply the simplified equation (8) with ϕ_j. Note that the original equation is not diagonal. This leads to the infinite sequence of initial value problems

$$\begin{aligned} \ddot{w}_j + \lambda_j w_j &= \alpha < x, \phi_j > (m(t) + c \int_0^t e^{-\sigma(t-s)} m(s) ds), \\ w_j(0) = w_{0,j}, & \qquad \dot{w}_j(0) = w_{1,j} \ \forall j. \end{aligned}$$

With

$$\begin{aligned} h_j &:= \alpha < x, \phi_j > \\ w_j^0 &:= \cos \lambda_j^{\frac{1}{2}}(T) w_{0,j} + \frac{1}{\lambda_j^{\frac{1}{2}}} \sin \lambda_j^{\frac{1}{2}}(T) w_{1,j} \\ w_j^1 &:= -\lambda_j^{\frac{1}{2}} \sin \lambda_j^{\frac{1}{2}}(T) w_{0,j} + \cos \lambda_j^{\frac{1}{2}}(T) w_{1,j} \end{aligned}$$

and the requirement that

$$w_j(T) = 0, \ \ \dot{w}_j(T) = 0, \ \ \forall j,$$

one obtains

$$\begin{aligned} -w_j^0 \frac{\lambda_j^{\frac{1}{2}}}{h_j} &= \Im\{(1 + \frac{c}{i\lambda_j^{\frac{1}{2}} + \sigma}) \int_0^T e^{i\lambda_j^{\frac{1}{2}}(T-s)} m(s) ds \\ &\quad - \frac{c}{i\lambda_j^{\frac{1}{2}} + \sigma} \int_0^T e^{-\sigma(T-s)} m(s) ds\} \\ -w_j^1 \frac{1}{h_j} &= \Re(1 + \frac{c}{i\lambda_j^{\frac{1}{2}} + \sigma}) \int_0^T e^{i\lambda_j^{\frac{1}{2}}(T-s)} m(s) ds \\ &\quad - \frac{c\sigma}{\lambda_j^{\frac{1}{2}}} \Im \frac{c}{i\lambda_j^{\frac{1}{2}} + \sigma} \int_0^T e^{-\sigma(T-s)} m(s) ds. \end{aligned} \tag{9}$$

Here it is first assumed that $m \in H_0^1(0,T)$, and then, by density, the equations are extended to $L^2(0,T)$-controls . Now, by (7)

$$0 = \eta(T) = -c \int_0^T e^{-\sigma(T-s)} m(s) ds. \tag{10}$$

(Nonzero initial history for η would not cause any difficulty as well.) Hence, (9) constitutes what is known as the moment problem associated with a beam under the distributed control $x \cdot m$. It is plain that $x \in D(A^{\frac{1}{4}})$. The condition (10) is just an additional moment condition. But the system of exponentials $e^{\lambda_j^{\frac{1}{2}} t} \cup e^{-\sigma t}$ still admits a unique, in fact, minimal norm– solution of (9),(10) in an arbitrarily small time interval $(0,T)$, for all initial data satisfying $w_0 \in D(A^{\frac{3}{4}})$, $w_1 \in D(A^{\frac{1}{4}})$. See Krabs[Kra], for a very nice treatment of moment problems related to beam (and other) equations. By standard perturbation arguments one may now include the neglected terms $\alpha x \cdot (q_1(w) + q_2(\dot{w}))$. On the state space E, these are bounded and small perturbations. The resulting norm–minimal control $m \in L^2(0,T)$ is then extended by $m(T+t) \equiv 0$, $\forall t > 0$, so that the whole state remains at equilibrium. The variable η being controlled to zero, implies $y(T+t) = 0$ and in turn $\dot{\varphi}(T+t) = 0$. But from $m(T+t) \equiv 0$, one infers that, because of $w''(0,T+t) \equiv 0$, $\varphi(T+t) \equiv 0$. Hence, exact null–controllability of the original system (CP) obtains.

Theorem: (Exact controllability)
Let $w_0 \in D(A^{\frac{3}{4}})$, $w_1 \in D(A^{\frac{1}{4}})$, $\varphi_0 \in \mathbf{R}$ and $T > 0$ be given, and let α be sufficiently small. Then there exists a norm–minimal control $M \in L^2(0,\infty)$ such that the solution w, φ of (CP) has the following properties

$$w_M(T+t) = \dot{w}_M(T+t) = 0, \varphi_M(T+t) = 0 \quad \forall t > 0$$

Concerning the well-posedness of the control–process, as stated in the theorem, some remarks are in order.

Remark:
Formally, the transform Γ leads to the desired decomposition (6), only if applied to strong data. Therfore, in order to obtain from (6) well–posedness in the "mild" sense of the theorem, one first of all starts with strong initial data, obtains strong solutions of (6) and, hence, strong solutions of (CP). The mild initial–data of (6) are then – by density – approximated by strong data. One, naturally, defines the resulting limits of solutions to (CP) as mild solutions thereof.

4 Stabilizability

It is interesting to observe from (ΓCP) – and this is another nice feature of the decomposition method – that the frequency spectrum of (CP), in the absence of controls $m(t)$, consists of two parts. An "oscillatory" part, and a "creep" part. The oscillatory spectrum consists of infinetely many eigenfrequencies in the stable half–plane *approaching the imaginary axis at infinity.* The creep part consists of a negative real eigenvalue, namely $\sigma = \mu - \alpha\beta q_2(x)$. *As a result, there is no uniform decay without further control !* This turns out to be very important for stabilization devices based on M. Again, one starts with (8), where α is – for the time being – set equal to zero. The spatial part x of the control satifies the following property

$$< x, \phi_j >= \frac{1}{\lambda_j} < x, A\phi_j >= \frac{1}{\lambda_j}\phi_j''(0).$$

Hence, the inner product $< A^{\frac{1}{4}}\dot{w}, A^{\frac{1}{4}}x >$, which naturally appears in any energy estimate based on the topology of $E_0 := D(A^{\frac{3}{4}}) \times D(A^{\frac{1}{4}})$, satisfies

$$< A^{\frac{1}{4}}\dot{w}, A^{\frac{1}{4}}x >= \sum_j \frac{1}{\lambda_j^{\frac{1}{2}}} < \dot{w}, \phi_j > \phi_j''(0) = (A^{-\frac{1}{2}}\dot{w})''(0),$$

which is well defined for $\dot{w} \in D(A^{\frac{1}{4}})$. It, therefore, makes perfect (mathematical) sense to define a feed-back control as follows

$$h(t) := -(A^{-\frac{1}{2}}\dot{w})''(0).$$

As a matter of fact, $(A^{-\frac{1}{2}}\cdot)''$ is a bounded operator which is given explicitly. This definition gives dissipation of energy.

$$\frac{d}{dt}E_0(t) = -\alpha((A^{-\frac{1}{2}}\dot{w}(t))''(0))^2. \tag{11}$$

One wonders, whether this dissipation is enough to ensure uniform exponential decay of $w, \dot{w}$. The answer is in the negative. As the operator

$$\mathcal{A} := \begin{pmatrix} O & I \\ -A & O \end{pmatrix}$$

generates a strongly conitinous (unitary) group $S(t)$ of bounded operators in the "energy-space" $D(A^{\frac{3}{4}}) \times D(A^{\frac{1}{4}})$, and the input-map

$$\mathcal{B}\dot{w}(t) := -\begin{pmatrix} 0 \\ \alpha x \end{pmatrix} \cdot (A^{-\frac{1}{2}}\dot{w})''(0)$$

is bounded and has a finite dimensional range, we conclude by Proposition 2.1 of Lasiecka and Triggiani [LT] that the semigroup associated with the feedback law above, i.e. the semigroup $S_{\mathcal{B}}(t)$ generated by the operator $\mathcal{A} + \mathcal{B}$ in $D(A^{\frac{3}{4}}) \times D(A^{\frac{1}{4}}) := Z$ satisfies $\|S_{\mathcal{B}}(t)\| \geq 1 \ \forall t \geq 0$. However, we do have strong stability. This is true, since the resolvent of $\mathcal{A} + \mathcal{B}$ is compact in Z and there is no purely imaginary eigenvalue of $\mathcal{A} + \mathcal{B}$. With hindsight to the problem of stabilizability of a cantilever, where the control enters the first derivative at the clamped end of the beam, we might suggest that another control-law may indeed lead to uniform exponential decay of solutions, if we switch to a stronger topology, i.e. to smaller spaces, such as $D(A) \times D(A^{\frac{1}{2}})$ rather than Z. See Lasiecka[Las], for the corresponding plate problem. In particular her problem (1.4) with boundary condition (1.5) – in the linear case – can be converted by transposition to the problem addressed here. For the sake of selfconsistency, I provide the argument leading to the "right" Ansatz for the feedback control which is much easier to derive in the given one-dimensional case. The procedure is as follows. One defines a "the energy" as

$$E_1(t) := \frac{1}{2}\{\|A^{\frac{1}{2}}\dot{w}(t)\|^2 + \|Aw(t)\|^2\}$$

Upon taking the time derivative of $E_1(t)$ and using the equation (8) with $q_1 = q_2 = 0$ one derives

$$\frac{d}{dt}E_1(t) = -\alpha \int_0^\ell A\dot{w}x dx \cdot h(t)$$

But, upon integration by parts

$$A^{\frac{1}{4}}x = \sum_j \lambda_j^{-\frac{3}{4}} \phi_j''(0)\phi_j.$$

This shows that

$$< A\dot{w}, x > = < A^{\frac{3}{4}}\dot{w}, A^{\frac{1}{4}}x > = \sum_j < \dot{w}_j, \phi_j > \phi_j''(0) = \dot{w}''(0).$$

Therefore, the "right" feed-back, i.e. the one giving dissipation of the energy, is $h(t) = \dot{w}''(0)$. This leads to

$$\frac{d}{dt}E_1(t) = -(\dot{w}''(0))^2.$$

This feedback is not of the type given above, in the sense that it is not bounded. It does not seem to be known though whether uniform exponential decay obtains. What is known is that the solution decays strongly to zero. We do not dwell on this further here, and leave the question of uniform energy decay to the future.

Remark:

The analoguous control-problem for thin plates is subject of current research. In addition, the control problem for a nonlinear version of (CP) is currently under investigation as well. However, the decomposition method is essentially limited to linear processes, so that it contributes information on the linearization, i.e. (CP), only. The corresponding results are then obtained on the local level by the use of some version of the implicit function theorem.

The motivation for this additional work is the fact that light robot arms in recent robot structures are often more adequately modelled by thin plates rather than beams. In addition, the usually high angular velocity during a rotation maneuver constitutively introduces some nonlinearity to the system – the centrifugal– and Coriolis forces.

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