An introduction to Anderson localization

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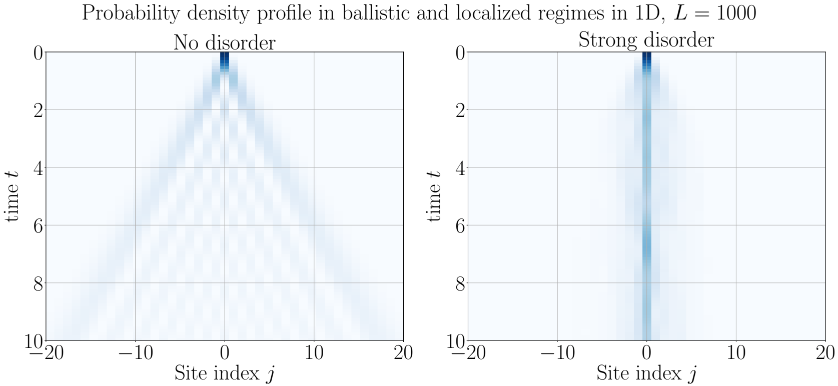
What is it about?

- Conduction in **NON-INTERACTING** systems with **DISORDER**
- Describes the role of **IMPURITIES**
- Completely different than the **Drude** model:

$$\sigma \propto l, \qquad l : \text{the mean-free path}$$

What does it predict?

- for some disorder: $\sigma = 0$
- seminal paper by **P. W. Anderson (1958)** [1]
- Nobel prize in **1977**



Why does it (still) matter?

What began in 1958 ...

PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Absence of Diffusion in Certain Random Lattices

P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey

(Received October 10, 1957)

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... still remains relevant today

PHYSICAL REVIEW B **96**, 214201 (2017)

Anderson localization transitions with and without random potentials

Trithep Devakul and David A. Huse
Department of Physics, Princeton University, New Jersey 08544, USA
(Received 20 October 2017; published 6 December 2017)

The current “hot topic”

- **Many-body localization (MBL)** - includes **INTERACTIONS**

Published in 2015:

Many-Body Localization and Thermalization in Quantum Statistical Mechanics

Rahul Nandkishore[1] and David A. Huse[1,2]

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672 citations as of April 2018 acc. to Google Scholar.

- not our today’s topic

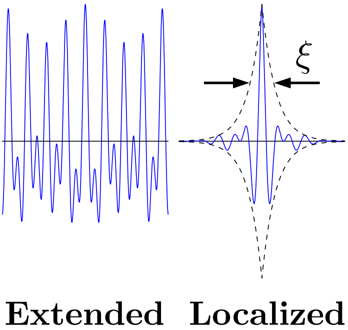
Outline

1. The basic concepts of the Anderson localization
2. Models of disorder
3. Numerical simulations
4. Conclusion

The basics

- **DISORDER** $\rightarrow$ **(eigen)**states can localize

Localization:



- A localized state:

$$|\psi(\mathbf{r})| \sim \exp\left(|\mathbf{r}-\mathbf{r}_0|/\xi\right)$$

- explains **vanishing** transport

The important keynotes

- An **interference** phenomenon

- Strong **dimensionality** dependence

- Energy dependence $\rightarrow$ the **mobility edge**

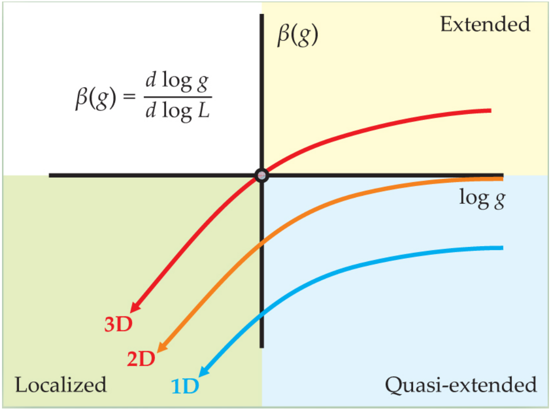
- scaling of the **conductance** g of a **hypercube** L^d [2]

- **Ohmic** conductor:

$$g = \sigma L^{d-2}$$

- **Localized** regime:

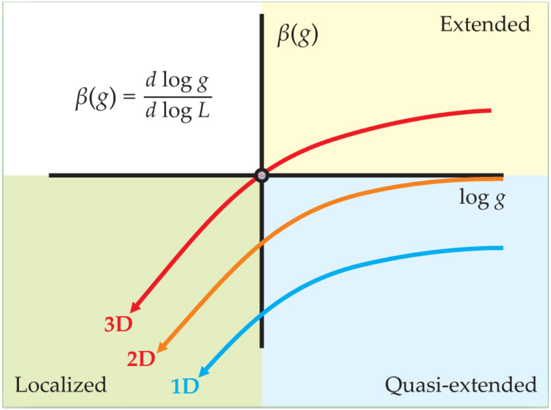
$$g \propto \exp(-L)$$



Transition between ext. and loc. states is only possible in 3D. Taken from [4].

1D, 2D
localization for any finite disorder

3D
localization for some critical disorder



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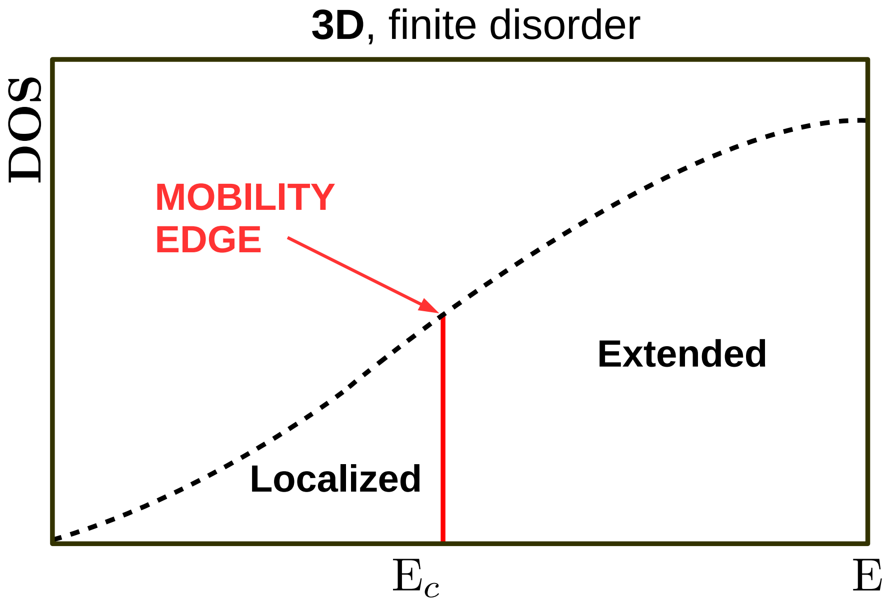
The mobility edge



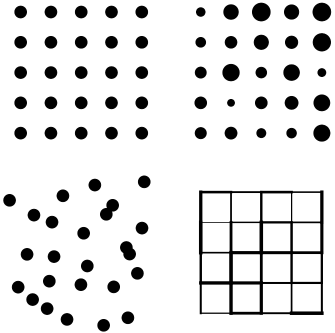
The models of disorder

- somehow distorting the **ideal crystal**

A generic Hamiltonian

$$H = \frac{\hat{p}^2}{2m} + \sum_{j=1}^{N} V_j(\mathbf{r} - \mathbf{R}_j)$$

- we consider the **Anderson model**



Ideal crystal, compositional, structural and kinetic disorder. Adapted acc. to [3].

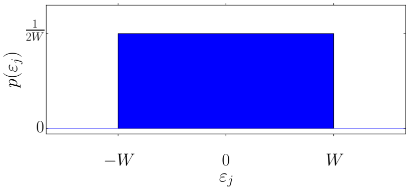
Our model - the **Anderson model**

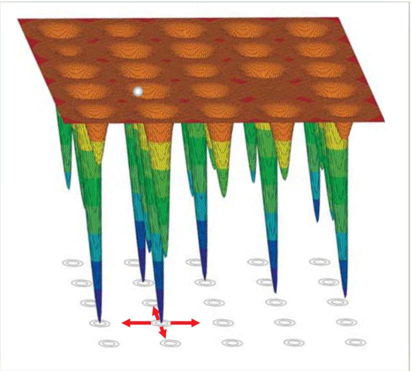
The Anderson Hamiltonian [1]

$$H = \sum_j \varepsilon_j c_j^\dagger c_j - V \sum_{\text{n.n.}} c_i^\dagger c_j + \text{h.c.}$$

Probability distribution of ε_j

$$p(\varepsilon_j) = \frac{1}{2W}\Theta(W - |\varepsilon_j|)$$

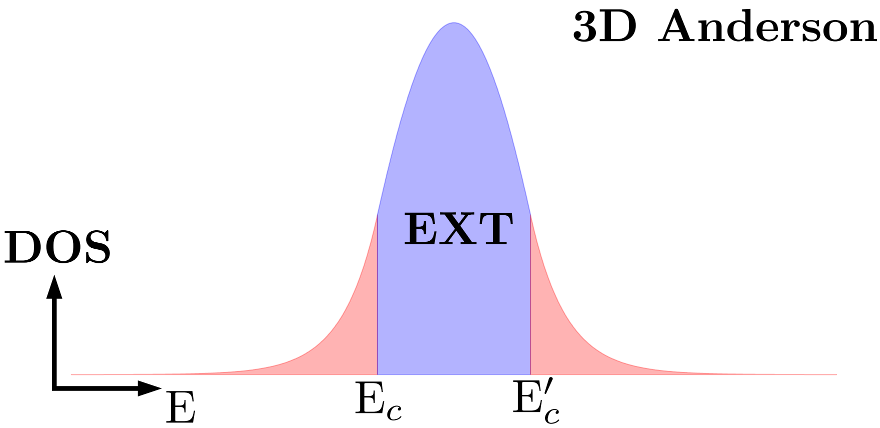




The model in 2D. Taken from [4].

The Anderson model

- Two mobility edges in 3D



My work - the numerical simulations

- How to extract features of the Anderson localization numerically?
  - implementation in 1D, 2D and 3D
  - calculations were run at the F-1 cluster at IJS
  - Full diagonalization and time evolution calculations
- Two localization criteria:
  - the **inverse participation ratio** (IPR)
  - the **absence of diffusion**

Localization criteria - the IPR

The definition

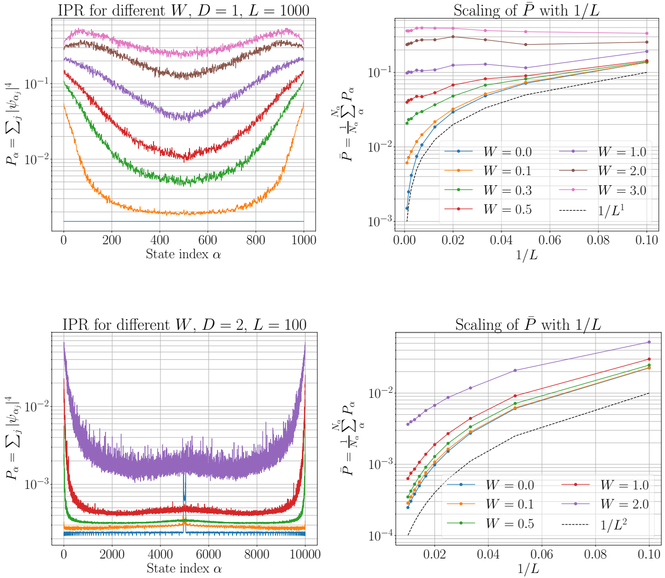
$$P^{-1} = \sum_{\mathbf{r}} |\psi(\mathbf{r})|^4, \qquad \|\psi\| = 1.$$

- sum over the lattice sites

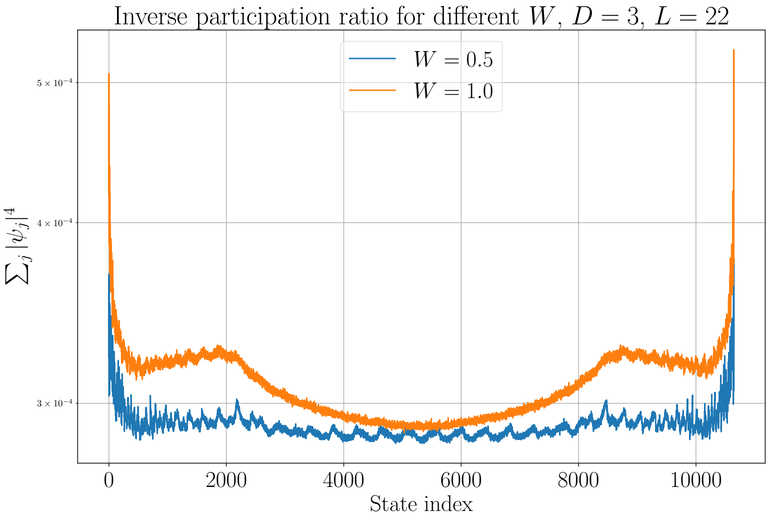
If L large enough

$$P^{-1} \propto \begin{cases} 1/L^d, & \text{extended} \\ \text{const}, & \text{localized} \end{cases}$$

IPR - the results



IPR 3D


Inverse participation ratio for different W, $D = 3$, $L = 22$
$W = 0.5$
$W = 1.0$
5×10^{-4}
4×10^{-4}
3×10^{-4}
$\sum_j |\psi_j|^4$
0
2000
4000
6000
8000
10000
State index

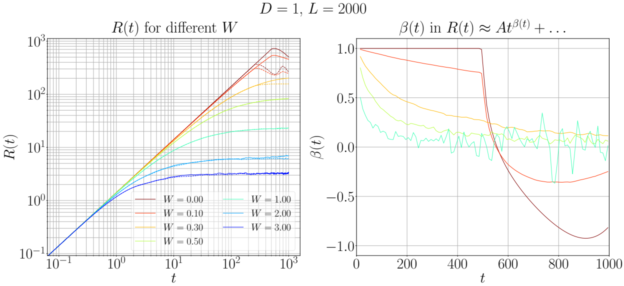
Absence of diffusion

Time evolution

$$|\psi, t + \mathrm{d}t\rangle = \exp\left(-\mathrm{i}\hat{H}\,\mathrm{d}t\right)|\psi, t\rangle,$$

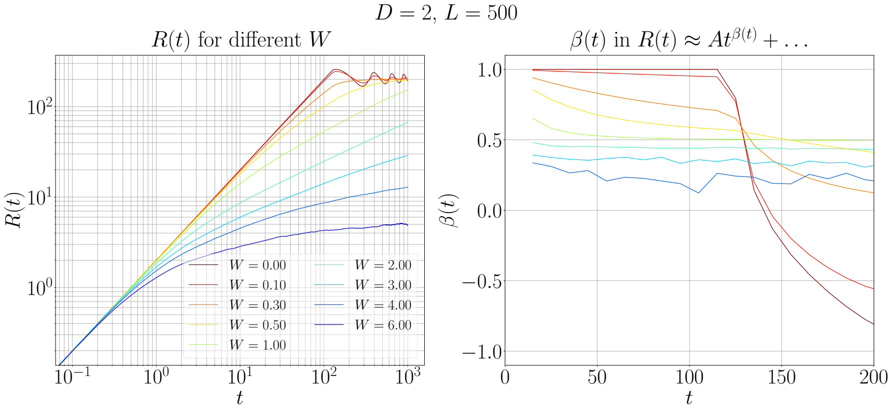
$$\hat{R^2} = \sum_{\mathbf{r}_j} \mathbf{r}_j^2 \hat{n}_{\mathbf{r}_j},$$

$$\beta(t) = \frac{\mathrm{d}\log R}{\mathrm{d}\log t}$$



$$R(t) = \sqrt{\langle\psi, t|\hat{R^2}|\psi, t\rangle - \langle\psi, 0|\hat{R^2}|\psi, 0\rangle}$$

Absence of diffusion, 2D



Conclusion

- A first step towards the description of conduction in real systems.
- A nontrivial numerical implementation.
- Needed in understanding the MBL phenomena, the current “hot topic.”

References and sources of images

- Anderson, P. (1958). *Absence of Diffusion in Certain Random Lattices.* Physical Review, **109**(5), pp.1492-1505.
- Abrahams E., Anderson P. W., Licciardello, D. and Ramakrishnan, T.V. (1979).*Scaling Theory of Localization: Absence of Quantum Diffusion in Two Dimensions.* Phys. Rev. Lett. **42**(10), 673
- Kramer, B. and MacKinnon, A. (1993). *Localization: theory and experiment.* Reports on Progress in Physics, **56**(12), pp.1469-1564.
- Lagendijk, A., Tiggelen, B. and Wiersma, D. (2009). *Fifty years of Anderson localization.* Physics Today, **62**(8), pp.24-29.