

Spatial Economics – Assignment 1

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Exercise A

For our independent variables, we use per capita crime rate by town, average number of rooms per dwelling, Charles River dummy variable (= 1 if tract bounds river; 0 otherwise), nitrogen oxides concentration (parts per 10 million) and a constant.

Below we wrote a function which computes: OLS point estimates for the intercept, slope parameters, and the error variance. Suitable test statistics with corresponding p-values for the relevant coefficients. Intervals of the coefficients for a confidence level of 95%.

```
OLS <- function(X,Y){  
  
  # OLS estimates for coefficients  
  beta_hat <- solve(t(X) %*% X) %*% t(X) %*% Y  
  Y_hat <- X %*% beta_hat    # Fitted values  
  
  e <- Y - Y_hat            # residuals  
  n <- nrow(X)              # Number of observations  
  k <- ncol(X) - 1          # Number of covariates excluding intercept  
  
  s <- as.numeric(t(e)%*%e / (n-k))      # SSR adjusted for degrees of freedom  
  sigma <- s*solve(t(X) %*% X)          # VCV of Beta hat  
  
  se <- sqrt(diag(sigma))                # standard error  
  t_stat <- (beta_hat-0) / se             # Compute t-statistic  
  p <- pt(abs(t_stat), n-k, lower.tail=FALSE) # Compute p-value
```

```

# 95% Confidence interval
th <- qt(0.975, n-k)
conf <- cbind(beta_hat-th*se, beta_hat+th*se)

colnames(beta_hat) <- "estimate"
colnames(conf) <- c("2.5%", "97.5%")
colnames(t_stat) <- "t-statistic"
colnames(p) <- "p-value"

error_variance <- s

list(rbind(beta_hat, error_variance), cbind(t_stat, p), conf)
}

```

The output of the function is presented below:

OLS(X,Y)

```

## [[1]]
##              estimate
## Constant      -17.259625
## Crime          -0.184610
## Rooms          7.706840
## Charles_River  4.673807
## NO_pp10m       -14.960358
## error_variance 35.771378
##
## [[2]]
##              t-statistic      p-value
## Constant      -5.373025 5.935625e-08
## Crime         -5.358215 6.414580e-08
## Rooms         19.155659 4.233091e-62
## Charles_River  4.388052 6.974274e-06
## NO_pp10m      -5.674181 1.178869e-08
##
## [[3]]
##              2.5%      97.5%
## Constant     -23.5707812 -10.9484684
## Crime        -0.2523012 -0.1169189
## Rooms         6.9163879  8.4972927
## Charles_River 2.5811627  6.7664511
## NO_pp10m     -20.1404236 -9.7802928

```

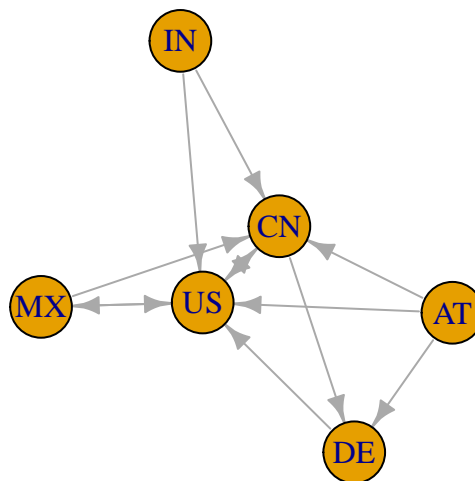
Exercise B

1. Draw a graph of the network; create the adjacency matrix in R.

We could consider 6 different countries and an indicator of a high trade between them. For illustration, we could consider a rule where there is directed edge from A to B, if B is one of the top 5 export destinations of A. We don't actually check for biggest trading partners empirically,

but let's assume that such procedure gives rise to following network (arrows are mostly made up to achieve 12 edges and nice interpretations):

Trade network



The corresponding adjacency matrix is:

	US	MX	DE	AT	CN	IN
US	0	1	0	0	1	0
MX	1	0	0	0	1	0
DE	1	0	0	0	0	0
AT	1	0	1	0	1	0
CN	1	0	1	0	0	0
IN	1	0	0	0	1	0

2. Who are the most and least central agents in the network? Name, explain, and try to quantify different notions of centrality.

A very basic concept of centrality could define an agent as most central, if it has the highest number of directed edges pointing towards itself (i.e. if it is important export country for most other countries). Using this criterion we can see from the graph that or from columns of adjacency matrix that US is the most central agents in this network, because it is among the top 5 trading partner for all 5 other countries. The least central agents would be Austria and India, since they are not a top 5 exporting destination for any country in this network.

Another basic criterion of centrality could be the amount of outward pointing arrows of an agent. Thus a country would be considered central if it exports a lot to high number of countries from this network. In this sense, Germany is least central with only 1 outward arrow, while Austria is most central with 3 outward arrows. This criterion makes perhaps less sense than the first one, but we mention it to explain the concepts. Another possible centrality concept would be function of the two basic concepts above, e.g. sum of in- and outward pointing arrows of an agent.

In the class we briefly introduced the intuition for eigenvector centrality. This tells us how often a node is visited on average if we start at a random node. This method does not work if there are sinks. Many applied algorithms such as page rank introduce jump probabilities for sinks. Once in a sink, i.e. node which has no outward connections, a probability distribution of jumping to other nodes is specified. While defining and computing page rank would be perhaps beyond the scope

of this small exercise, below we display the eigenvector centrality computed using the package igraph.

```
eigvcen1 <- eigen_centrality(  
  graph_from_adjacency_matrix(adj),  
  directed = TRUE,  
  scale = TRUE,  
  weights = NULL,  
  options = arpack_defaults  
)  
  
eigvcen1$vector
```

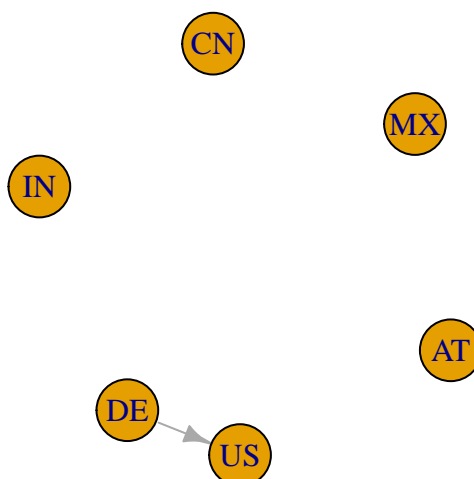
```
##      US      MX      DE      AT      CN      IN  
## 1.0000000 0.5436890 0.4563110 0.0000000 0.8392868 0.0000000
```

Using eigenvector centrality, We observe that US is the most central agent, followed by China. Austria and India are the least central agents with value of 0. This is because they are source only - there are no directed edges pointing to them, meaning that they are not a top 5 trading partner to any country.

- How would centralities change if you considered a row-normalized network instead?

Germany which had only 1 connection, is the only row with sum of 1. Thus its the only edge which survives row normalization. Here, depending on criterion US or Germany would be most central agent. Other countries would be all least central. We can see that there is a strong distortion resulting from row normalization. Possible solutions to this would be to use some weights for the connections between agents. E.g. connection could be there if trade is not zero, weighted by the amount of the trade. Then row normalization would not delete the edge completely. Another possibility would be to normalizing by scaling by a constant instead of row normalization.

Trade network row normalized



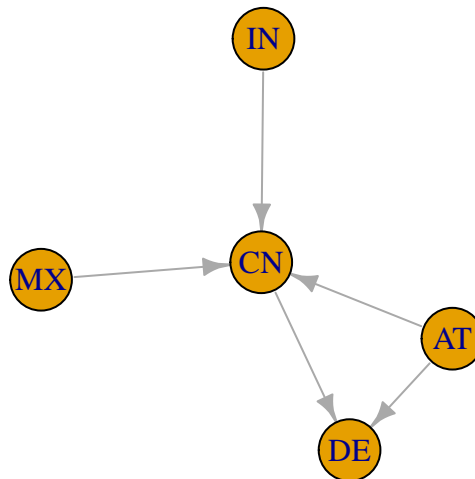
```
eigen_centrality(  
  graph_from_adjacency_matrix(adj.rn),  
  directed = TRUE,  
  scale = TRUE,  
  weights = NULL,
```

```
options = arpack_defaults
)
```

Notice that We get a directed acyclic graph (DAG), for which eigenvector centrality is not defined, since there are no cycles at all and US is a sink.

- How would the network change if you removed or added a specific agent? Lets consider the case of removing US:

Trade network excl. US



Based on Inflowing arrows, China becomes the most central agent and India, Austria and Mexico the least central agents. Based on Outflowing arrows, Austria is still the most central agent, while Germany is the least central agent.

===== Similarly to above, eigenvector centrality is undefined since there is a sink - Germany.

```
eigen_centrality(
  graph_from_adjacency_matrix(adj[-1,-1]),
  directed = TRUE,
  scale = TRUE,
  weights = NULL,
  options = arpack_defaults
)
```

3. Simulate some agent characteristic based on a standard Normal distribution; use this characteristic to simulate responses following a liner-in-means model with your network. Repeat this a couple of times, and compare estimates of a standard linear model ($y_i = x_i\beta + \varepsilon_i$) with the true values that you used to simulate the data

We simulate the data based on the row normalized adjacency matrix for trade partner connections, that we created in the preceding exercise.

Let's say we are interested in estimating the impact of a countries fentanyl precursor production (x) on the log of a countries drug deaths (y). Then Wx denotes the average of neighboring countries fentanyl precursor production. We hypothesize that if a countries close trade partners fentanyl precursor production rate is high (as measured by Wx), then the fentanyl related drug deaths are high. Moreover, there are spillover effects with regards to the trading partners fentanyls related deaths from connected countries (Wy).

We specify the following linear-in-means model to estimate the relationship.

$$y = Wy\lambda + Wx\delta + x\beta + \varepsilon$$

Where Wy denotes the average death rate from drugs of countries that are close neighbors.

To simulate the values for y , we need to solve for the reduced form.

$$y = (I - W\lambda)^{-1}(Wx\delta + x\beta + \varepsilon)$$

```
set.seed(123)
# number of agents
N = 6

# parameters definition
sigma2 = 1
lambda = 0.6
delta = 0.3
beta = 2 # true beta is 2
W = adj.rn

simulations <- 10000
times <- c(1:simulations)
result <- numeric(simulations)

for (i in times) {

  # simulation of vecotrs
  x = rnorm(N, 0, 1)
  e = rnorm(N, 0, sigma2)

  # caluclating means
  Wx <- W %*% x

  # calculating S
  S = diag(N) - lambda * W

  # generating y's
  y = solve(S, Wx * delta + x * beta + e)

  model <- lm(y ~ x)

  result[i] <- coef(model)["x"]
}

inconsistent_estimate <- mean(result)

my_data <- data.frame(
  Name = c("Simulated coef", "Real coef"),
```

```
Estimate = c(inconsistent_estimate, beta)
)
```

```
knitr::kable(my_data, format = "markdown")
```

Name	Estimate
Simulated coef	1.718638
Real coef	2.000000

We set the true value for β to 2, λ to 0.6, and δ to 0.3. We then regress y on x , repeat this a 1000 times, and find that the mean of the estimand is 1.718638. Thus using the simple linear regression fails to recover the true value of β .

Exercise C

Download a suitable shapefile for NUTS2 regions (from [here](#) or using R directly) and some dataset of interest at the same level of aggregation (e.g. from [here](#)). Install and load the `sf` and `ggplot2` packages together with their dependencies.

1. Read in the shapefile, and find out what projection and CRS the file uses. Map the data to use another projection and/or CRS of your choosing:

WGS 84 (EPSG:4326)



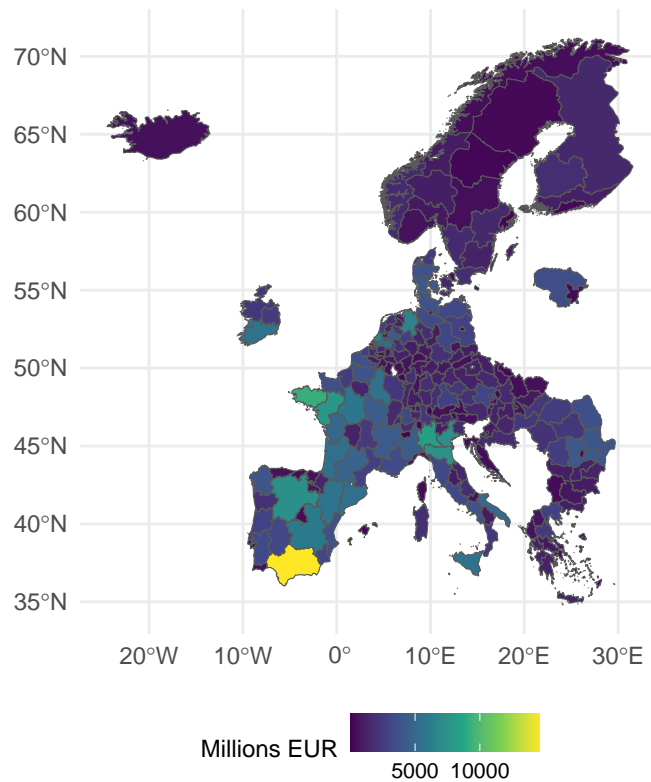
British National Grid (EPSG:27700)



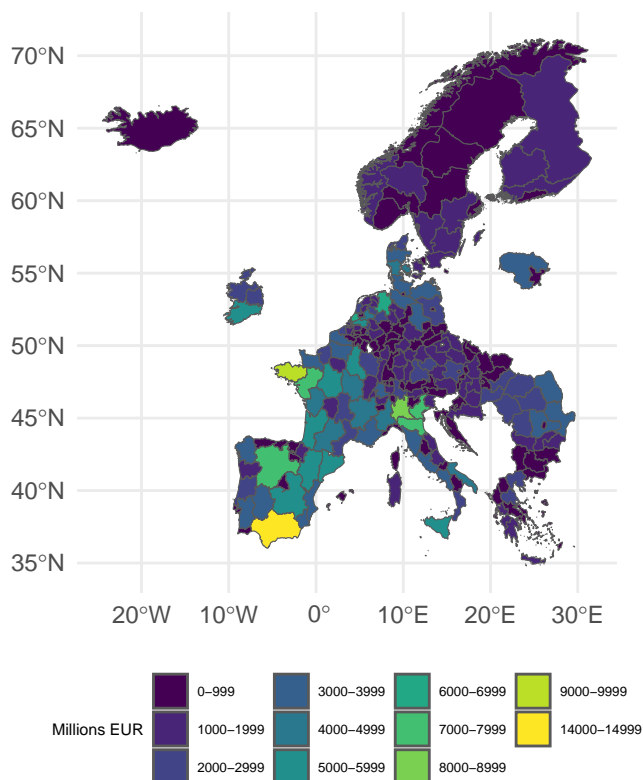
The coordinate reference system for the first map is the “World Geodetic System 1984,” or “WGS 84” (EPSG:4326) for short. The data is then re-projected to the “OSGB36 / British National Grid” coordinate reference system. The first map utilizes the Mercator projection, which is widely used in various global applications, including Google Earth. The Mercator projection is conformal, meaning it preserves angles. The second map is created using the Transverse Mercator projection, which does not have distortions along the central meridian. It’s worth noting that OSGB36 is specifically designed for mapping Great Britain and is optimized for maintaining accurate scale within this region.

2. Merge the shapefile and the chosen dataset. Create two meaningful visualizations of the chosen dataset using different scales (e.g. continuous versus discrete scaling of the data)

Agricultural Value Added by NUTS-2 Region in 2021



Binned Agricultural Value Added per NUTS-2 Region in



The discrete scaling shows the agricultural value added in million Euros at a NUTS-2 Level in 2021. While the first map visualizes the value added in continuous scaling. The second map portrays agricultural value added in a binned scaling. Note that animal production and fishing is also included, which accounts for the high agricultural value added in Western France.

It is interesting to note that the NUTS-2 level visualization might induce a somewhat distorting picture. The regions with the highest output are Andalusia in Spain and some Western regions in

France. However, this is partially driven by the fact that the average NTUSS-2 region in Spain is far larger than in Germany. As the average definition of regions is smaller in Germany the average agricultural value added might appear somewhat smaller in Germany.

3. Briefly explain what two conceptually different ways there are to store visualizations, and name two formats each. Which ones are more appropriate for which types of visualizations and why? Use the one you think is more appropriate to store your visualizations.

Visualizations can be broadly categorized into two distinct types based on the underlying spatial data they represent: raster and vector. Raster graphics are pixel-based, encoding images as arrays of multi-colored pixels. They are ideal for depicting continuous variables, such as elevation, temperature, or (as in the case above) agricultural output, where the variation is smoothly transitioned across the visualization. Common formats for storing raster graphics include PNG, JPEG, and TIFF files. The major caveat with raster graphics is their resolution dependency; zooming in too far reveals the pixel grid, leading to a loss of clarity.

On the other hand, vector graphics are built using geometric shapes such as points, lines, and polygons, defined mathematically by paths rather than pixels. This method enables vector images to retain crispness and clarity regardless of scaling, making them infinitely 'zoomable' without loss of resolution. Vector graphics are commonly stored in SVG (Scalable Vector Graphics) and EPS (Encapsulated PostScript) formats. Due to their scalable nature, vector graphics are typically more compact and require less storage compared to their raster counterparts.

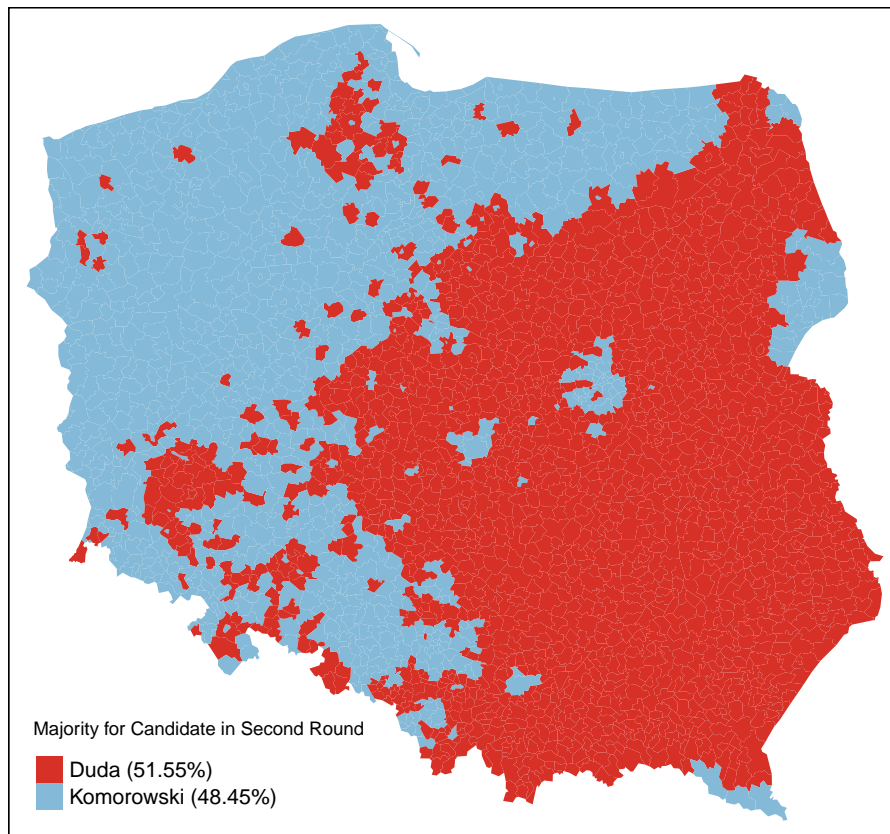
Choosing between raster and vector formats for storing visualizations depends on the nature of the data and the intended use of the visualization. Raster images are best suited for detailed and complex imagery, such as remotely sensed measures and detailed maps with continuous data representation. They offer a more nuanced depiction of variations in data such as agricultural productivity across a landscape, although at the expense of larger file sizes and fixed resolution.

Vector graphics, with their ability to scale without loss of quality, are preferable for diagrams, logos, and maps where clear, clean lines and shapes are needed. Fonts for example are stored as vector graphics.

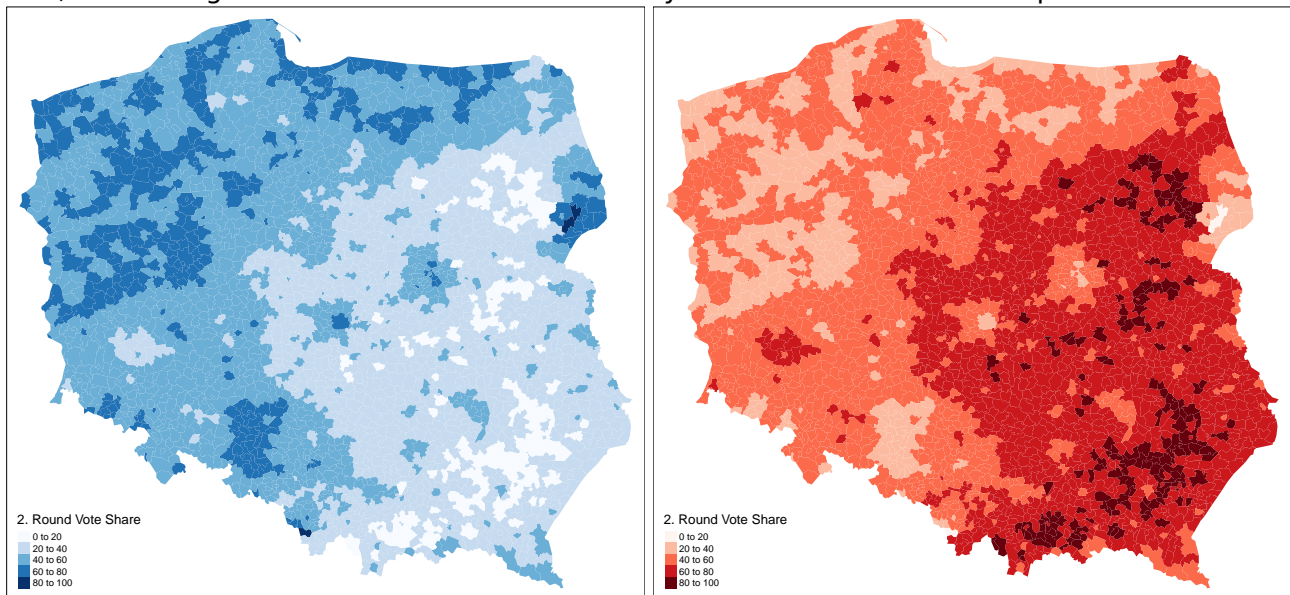
For visualizing agricultural value added in NUTS-2 regions across Europe, a TIFF file would be most appropriate due to its capability to handle complex, continuous data at a high resolution. This choice supports a detailed and accurate representation of agricultural productivity data at the pixel level, accommodating the depth and variation in the dataset, albeit with considerations for the potentially large file size.

Exercise D

We start by visualizing the results of the second round of the Polish election. The first map displays the winner of the respective district. Overall we can see that the election was extremely close, with Duda winning just 51.55% and Komorowski winning 48.45% of the popular vote. One can clearly observe an east west pattern, where the Western part of the country predominantly voted for Duda and likewise the eastern part of the country for Komrowski.



We now zoom in on the shares by using the function `tm_facets`, which allows to display the vote shares of the candidates separately. The maps now display the vote shares of the respective candidates in each district. We can observe a strong polarization, as a large number of districts voted by above 80% for the respective candidate.



We now investigate possible issues with voting envelopes. We check if all envelopes that were sent, were also received. If the number of envelopes received is equal to the number of the envelopes sent, the municipality is coloured as green. If the municipality is visualized in grey, that means that there were no voting envelopes used for the election. If a municipality is red however, that indicates that the total number of envelopes sent is higher than the number of envelopes received (at a threshold off 99 percent). That indicates that there were probably issues with the delivery of the voting envelopes. We can observe red clusters especially in urban regions

(Warsaw, Katowice and Krakow). An interesting side fact however is that one can clearly see that in urban areas, voting envelopes were used more often than in rural areas of Poland:

