

Error Analysis

Distance estimation

The location of each node is estimated using the map of the node locations, Google Maps and afstandmeten.nl¹:



Distance between 42 and 68: 2.73 km = 2730 m.

Distance between 42 and 138: 4.96 km = 4960 m.

Distance between 68 and 138: 6.50 km = 6500 m.

Sources of uncertainty

The main sources of uncertainty in the method above are mostly human errors, starting with reading the node locations from the map PDF. It is hard to accurately pinpoint the exact location, which in turn causes an extra error in finding this location on Google Maps and afstandmeten.nl. This process most likely causes an error in the final results.

Distance uncertainty estimation

Transferring the dots from the map PDF to Google Maps and afstandmeten.nl (again introducing a human reading error) gives a dot size of about 150m. The dot size is the uncertainty of the estimated distances: $\pm 150\text{m}$.

¹ www.afstandmeten.nl

Distance calculation

Using a calculator from www.moveable-type.co.uk² and the GPS locations of the nodes, the distances between the nodes can be determined more accurately:

Node ID	Longitude	Latitude
42	6.531761914930448	52.993261611492905
68	6.555020702187587	52.972656627946741
138	6.571273202912570	53.029347204469147

Distance between 42 and 68: 3.445 km = 3445m.

Distance between 42 and 138: 5.932 km = 5932m.

Distance between 68 and 138: 6.518 km = 6518m.

Calculation uncertainties

Assuming the calculator is free of errors when calculating the distance between two GPS locations, the uncertainty is probably introduced when determining and documenting GPS location of each node. It is unknown how the GPS location of each node is determined so we cannot derive any conclusions from this. Furthermore, errors could be made when copying the GPS data into the assignment document. It seems there is an error in the GPS location of node 42, as the differences between the calculated and estimated distances for this node to other nodes are very large.

6.570319

52.992477

Absolute uncertainty

GPS location of node 160: longitude 6,570319 - latitude 52,992477

Distances to other nodes in meters:

	42	68	138	160
42	-	3445 ± 6	5932 ± 6	2582 ± 5
68	3445 ± 6	-	6518 ± 6	2430 ± 5
138	5932 ± 6	6518 ± 6	-	4100 ± 5
160	2582 ± 5	2430 ± 5	4100 ± 5	-

Distance uncertainty

As the GPS locations were measured within 3 meters of a node, the total uncertainty between nodes is equal to the sum of this deviation: ± 6m.

The sound is produced within 2m of node 160, which leads to an uncertainty between node and sound of ± 5m.

Time uncertainty

The sound has a sharp onset with an order of magnitude of 4 samples in the 96kHz signal:

$$f = 96000$$

$$t = \frac{1}{96000} = 1.04166667 \times 10^{-5}$$

$$\text{Times 4 samples: } 4.16666667 \times 10^{-5}$$

Therefore the uncertainty regarding time is ± 4.17*10⁻⁵ seconds.

² <http://www.moveable-type.co.uk/scripts/latlong.html>

Speed of sound

To be able to calculate the uncertainty of the speed of sound, an assumption of the actual speed of sound must be made. It is assumed the speed of sound is 343.2 m/s, at 20° and sea level³.

Travelling times

In order to calculate the uncertainty of the speed of sound the actual travel time of the sound needs to be calculated, as this is required to fill in the formula for the uncertainty:

Nodes	Distance travelled [m]	Travel time [s]
42 – 160	2582 ± 5	7.52331002
68 – 160	2430 ± 5	7.08041958
138 – 160	4100 ± 5	11.9463869

*Node 160 is used as the sound source

Speed of sound uncertainty

The formula needed to determine the speed of sound:

$$v = \frac{s}{t}$$

In which:

- v = speed
- s = distance
- t = time

In order to determine the uncertainty of v (in this case, the speed of sound) the Product & Quotient rule for uncertainties calculations needs to be used:

$$\frac{\partial v}{|v|} = \frac{\partial s}{|s|} + \frac{\partial t}{|t|}$$

This calculation is done for all three nodes (42, 68 and 138):

$$\text{Node 42: } \frac{\partial v}{|v|} = \frac{5}{2582} + \frac{4.16666667 * 10^{-5}}{7.52331002} = 0.00194202169 \%$$

$$\text{Node 68: } \frac{\partial v}{|v|} = \frac{5}{2430} + \frac{4.16666667 * 10^{-5}}{7.08041958} = 0.00206349794 \%$$

$$\text{Node 138: } \frac{\partial v}{|v|} = \frac{5}{4100} + \frac{4.16666667 * 10^{-5}}{11.9463869} = 0.001223 \%$$

For a more accurate calculation, the average of the values for each node is used in the final calculation to determine the uncertainty of the speed of sound:

$$\frac{0.00194202169 + 0.00206349794 + 0.001223}{3} = 0.00174283988 \%$$

Using the earlier assumption for the speed of sound of 343.2 m/s, the uncertainty is 0.598142647 m/s:

$$\text{Speed of sound} = 343.2 \pm 0.6 \left[\frac{m}{s} \right]$$

(With rules of significance and rounding kept in mind).

³ https://en.wikipedia.org/wiki/Speed_of_sound