Data Structure and Algorithm
TREES

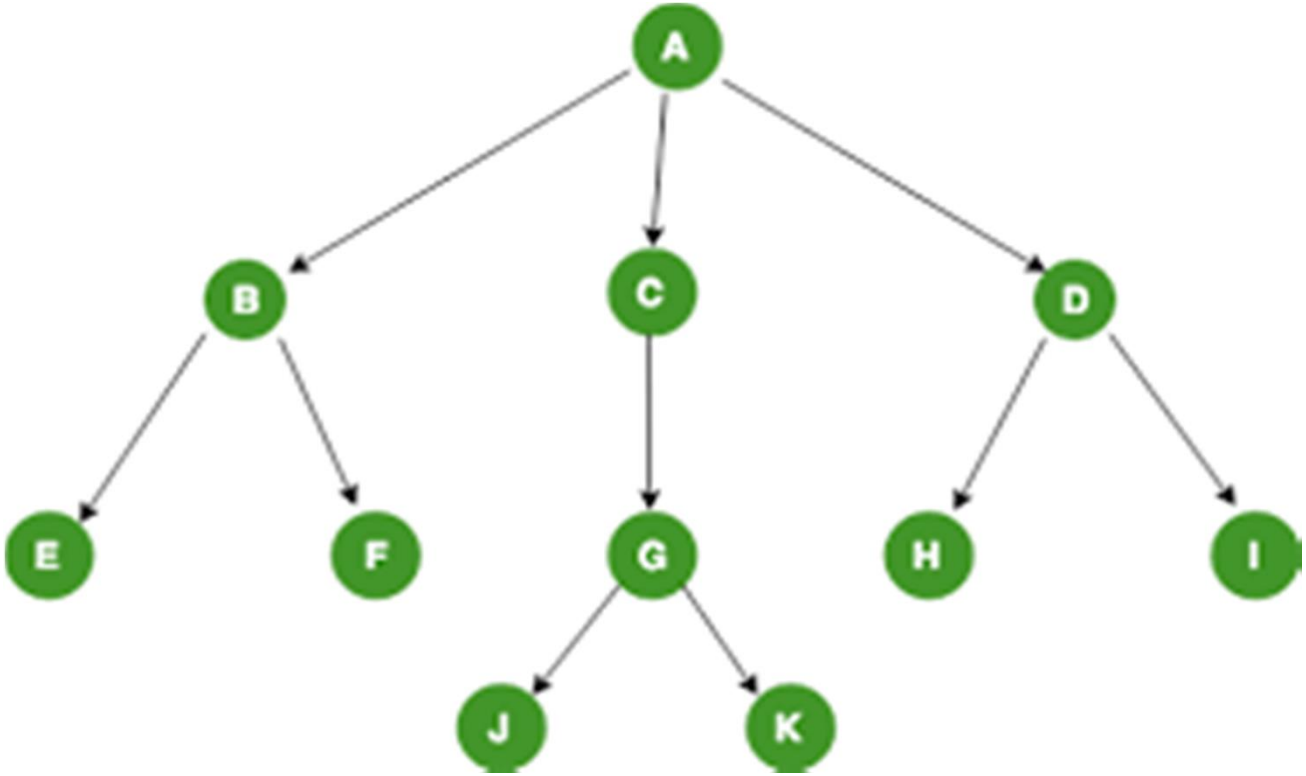
Kanak Kalyani

TREE

A **Tree:**

- is a nonlinear **data structure.**
- **tree** is a **structure** consisting of one node called the root and zero or more subtrees.
- stores the data elements in a hierarchical manner.
- can be empty with no nodes

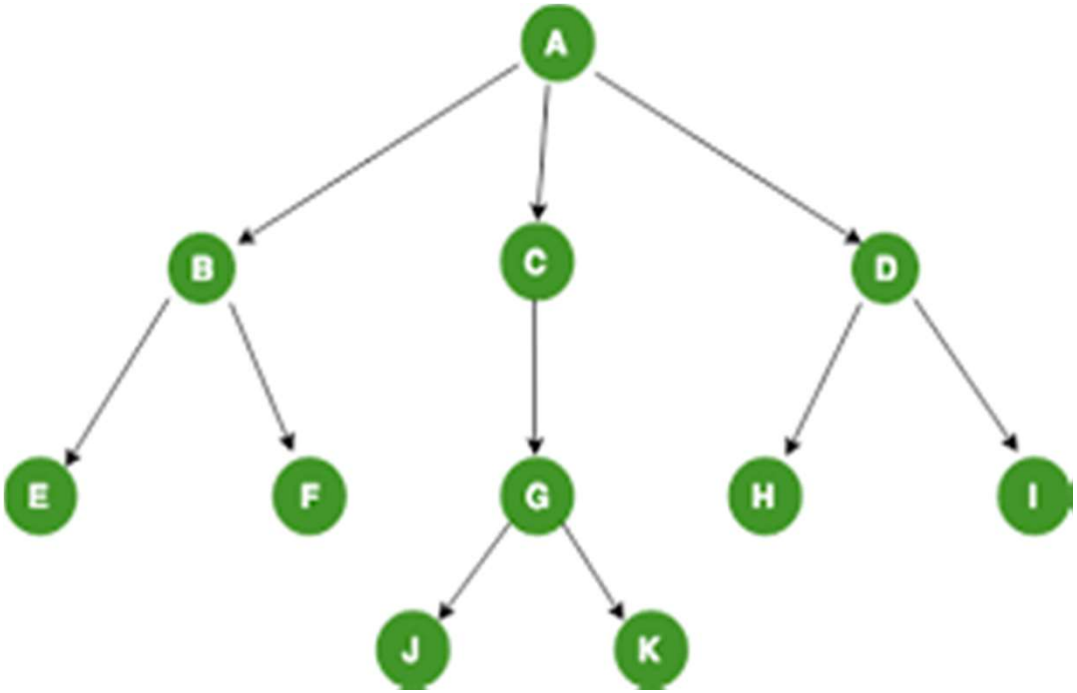
Basic Concept



Tree terminologies

Node→

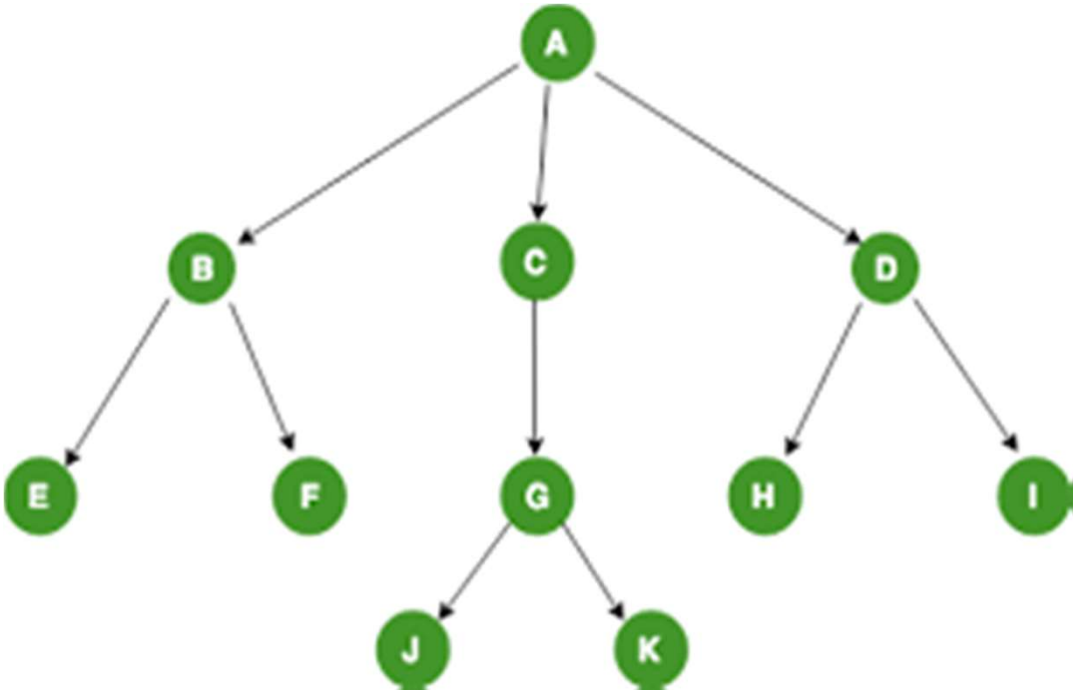
It is the data element of tree. Apart from storing a value it also specifies links to the other nodes



Tree terminologies

Root →

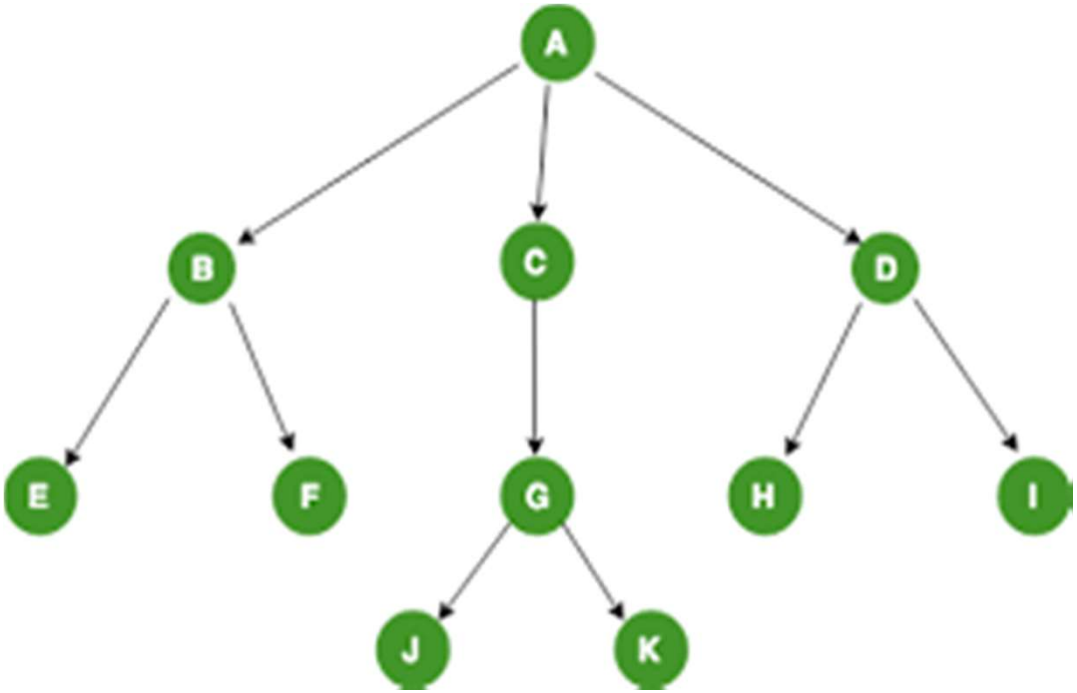
It is the top node in the tree



Tree terminologies

Parent→

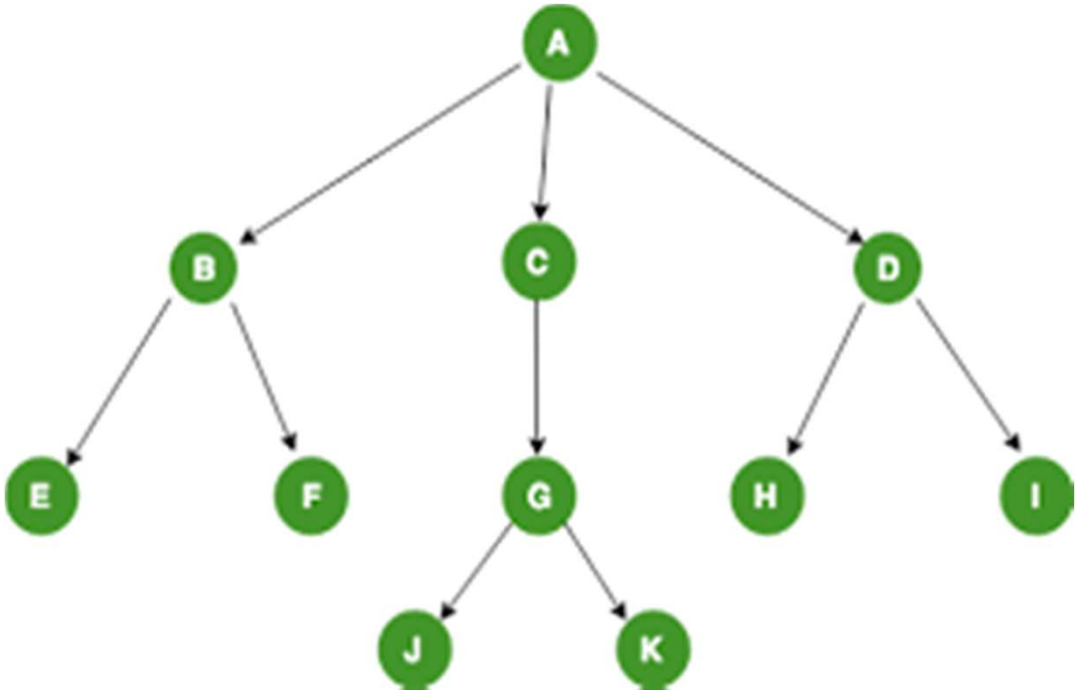
A node that has one or more child nodes



Tree terminologies

Child→

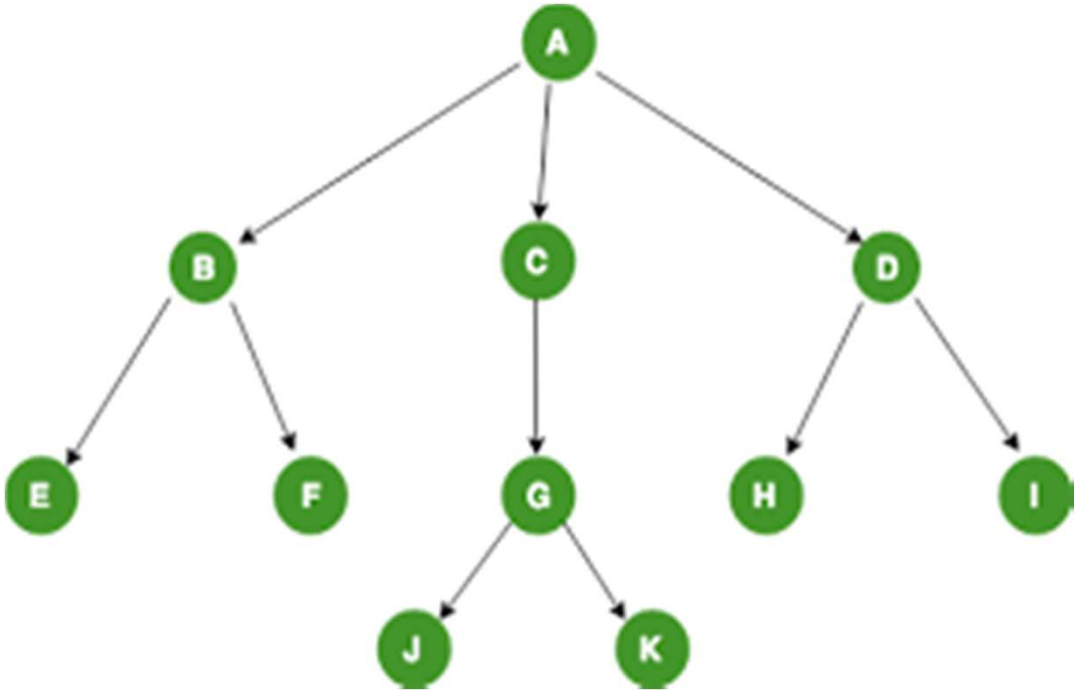
All nodes in a tree except the root node are child nodes of their immediate predecessor nodes



Tree terminologies

Leaf/ Terminal Node→

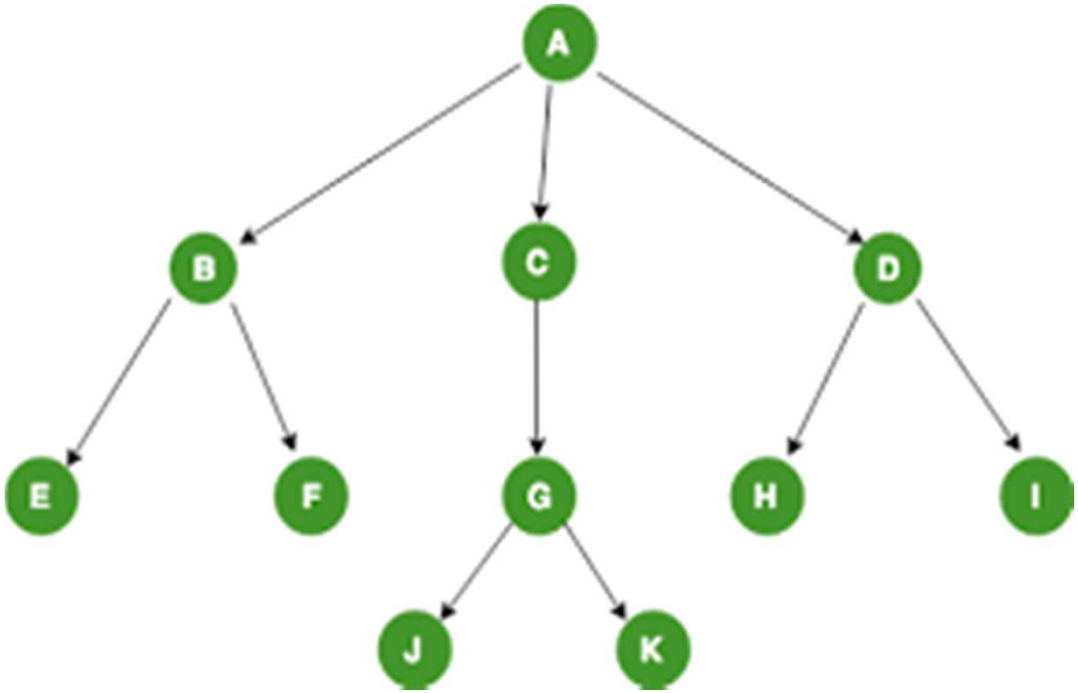
It is the terminal node that does not have any child nodes



Tree terminologies

Internal Node→

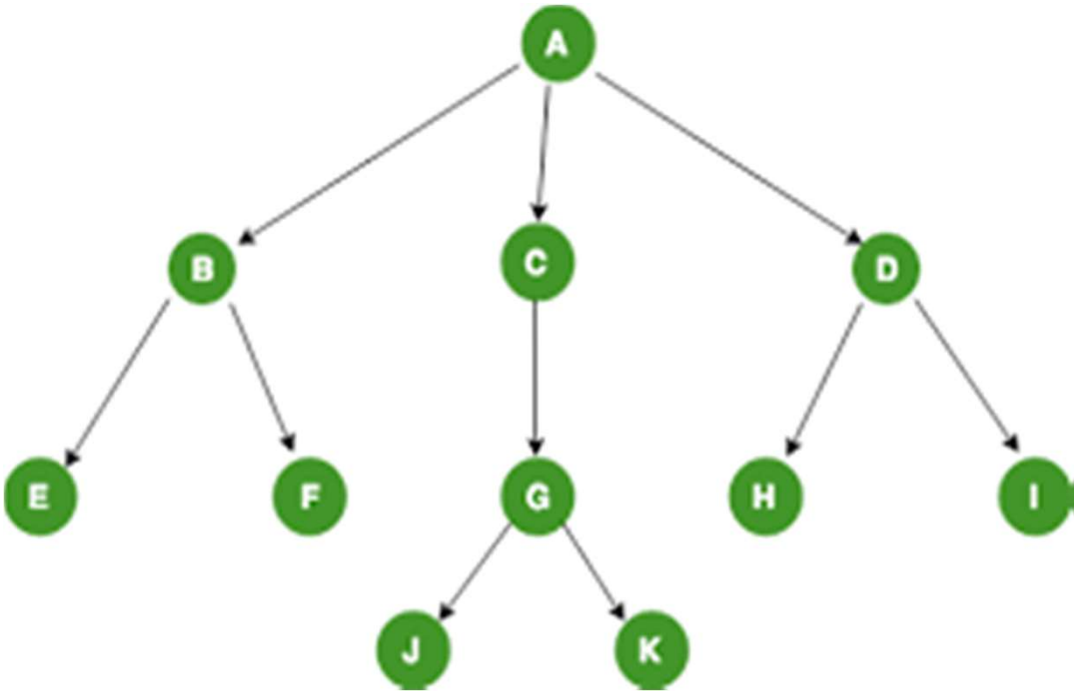
All nodes except root and leaf nodes are referred as internal nodes



Tree terminologies

Non Terminal Node→

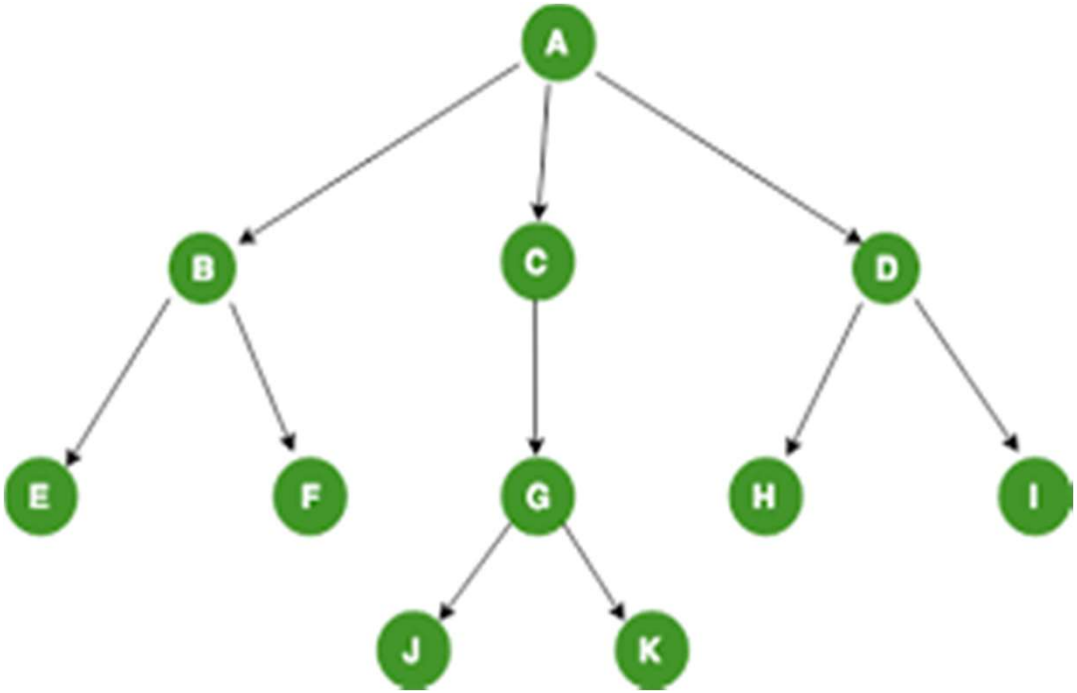
All nodes except leaf nodes are referred as Non Terminal nodes



Tree terminologies

Sibling→

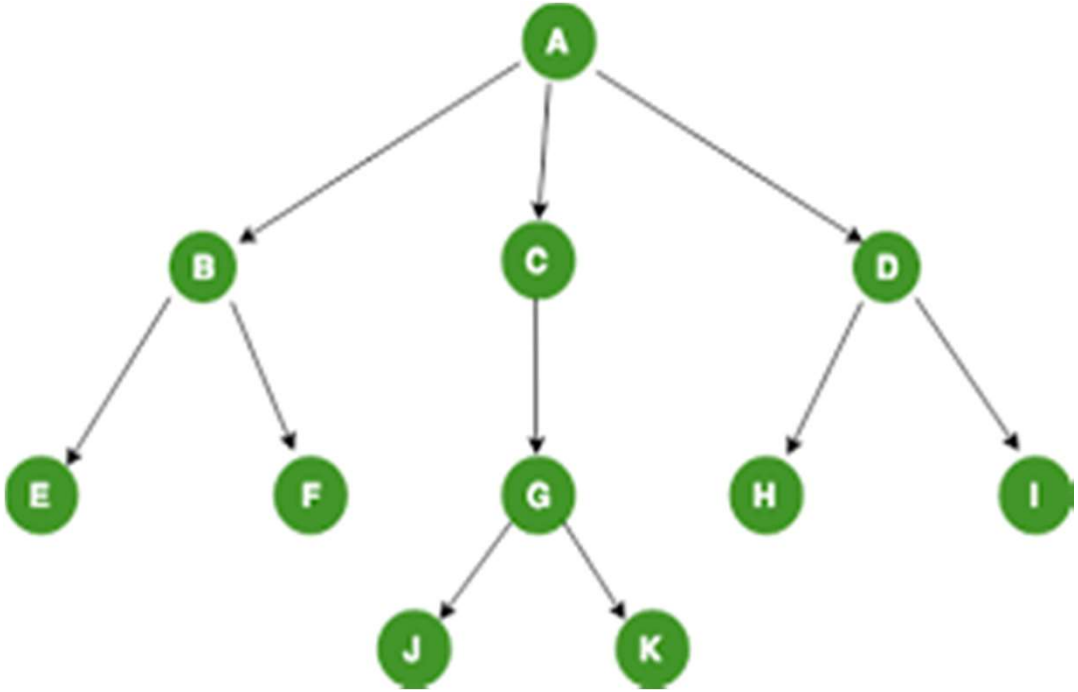
All child nodes of same parent are referred as Siblings



Tree terminologies

Degree→

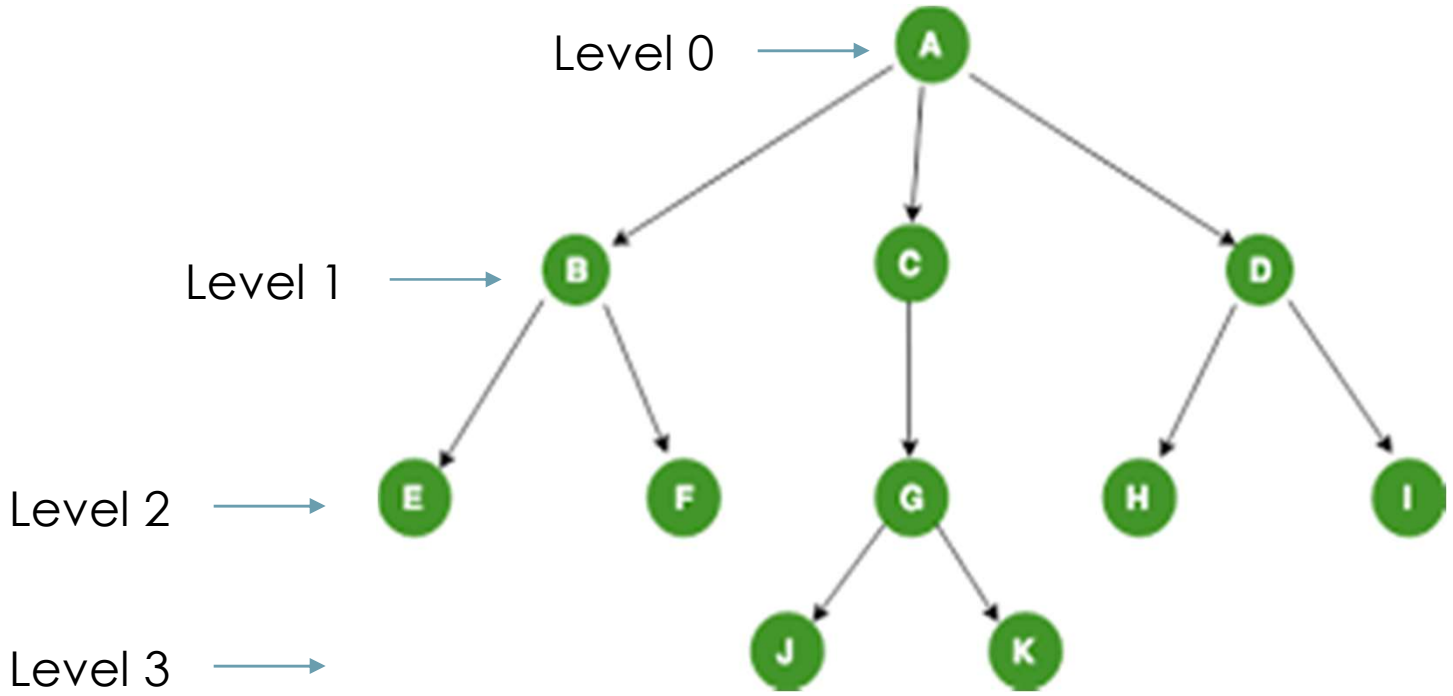
The degree of a node is the number of subtrees coming out of the node



Tree terminologies

Level→

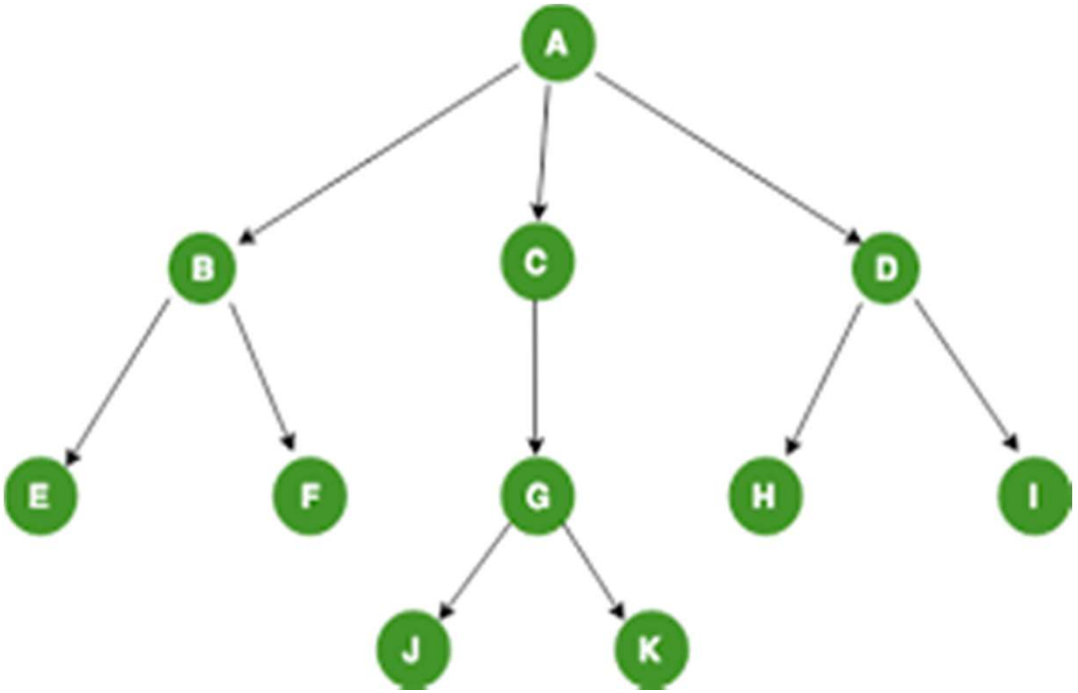
All the tree nodes are present at different levels. Root node is at level 0.



Tree terminologies

Height OR Depth→

It is maximum level of a node in the tree

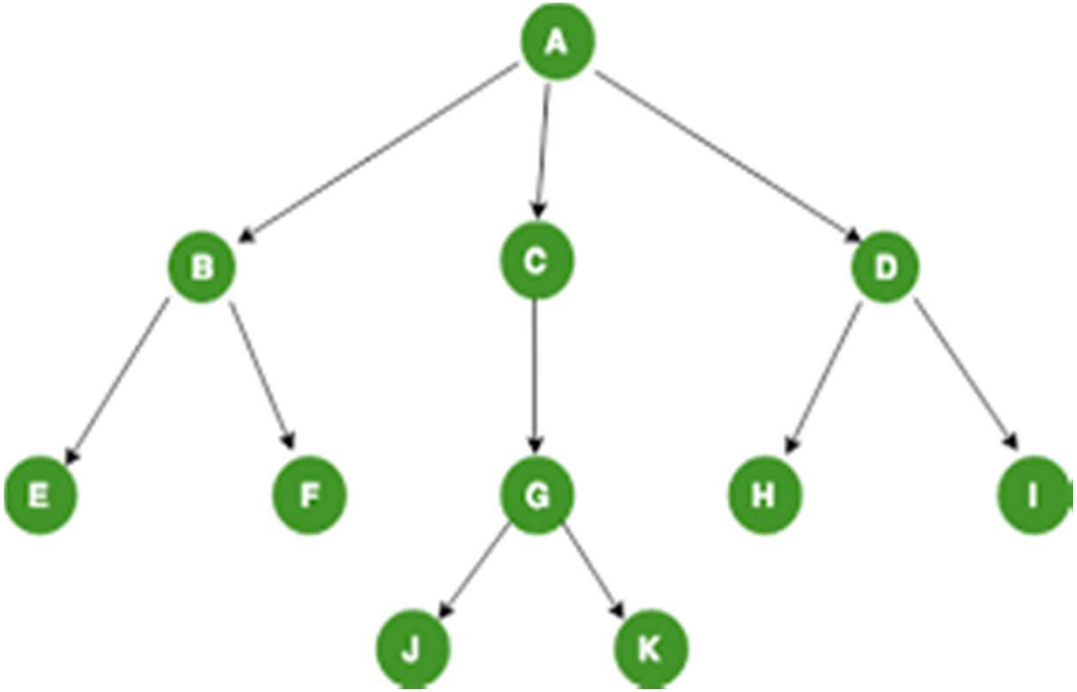


Height= 3

Tree terminologies

Path→

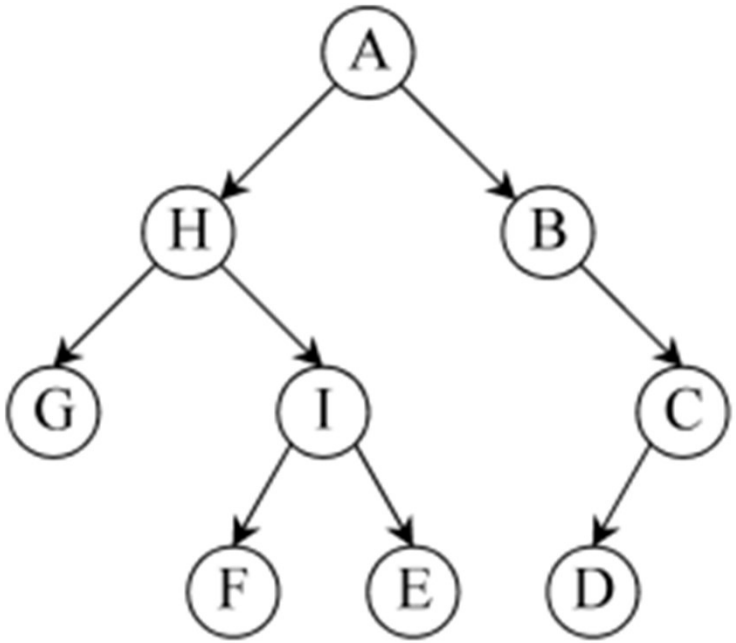
It is the sequence of nodes from source node to destination node



Binary Tree

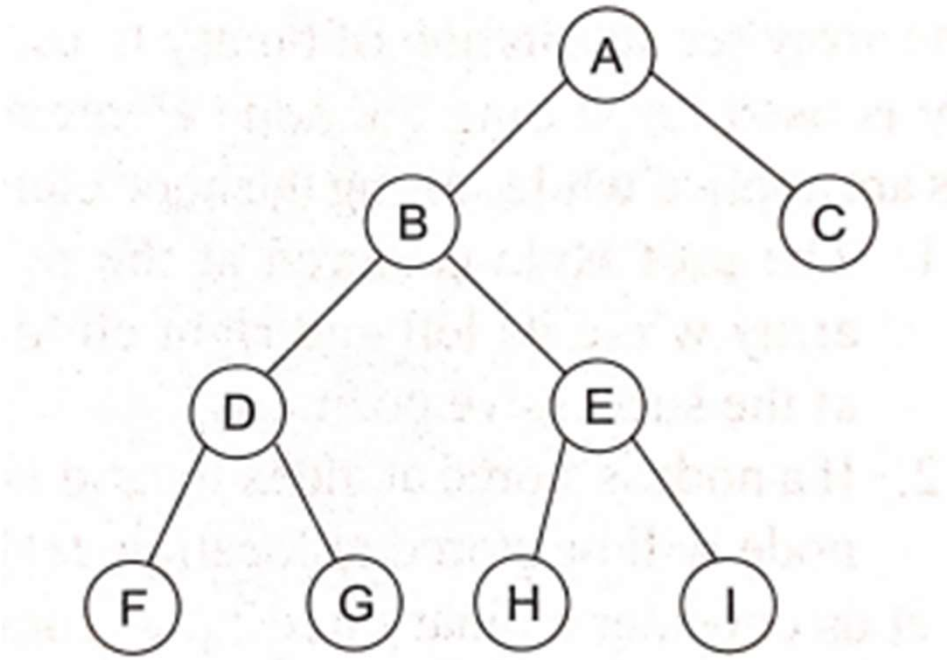
- A binary tree is a hierarchical data structure in which each node has at most two children generally referred as left child and right child.

- Each node contains three components: Pointer to left subtree. Pointer to right subtree



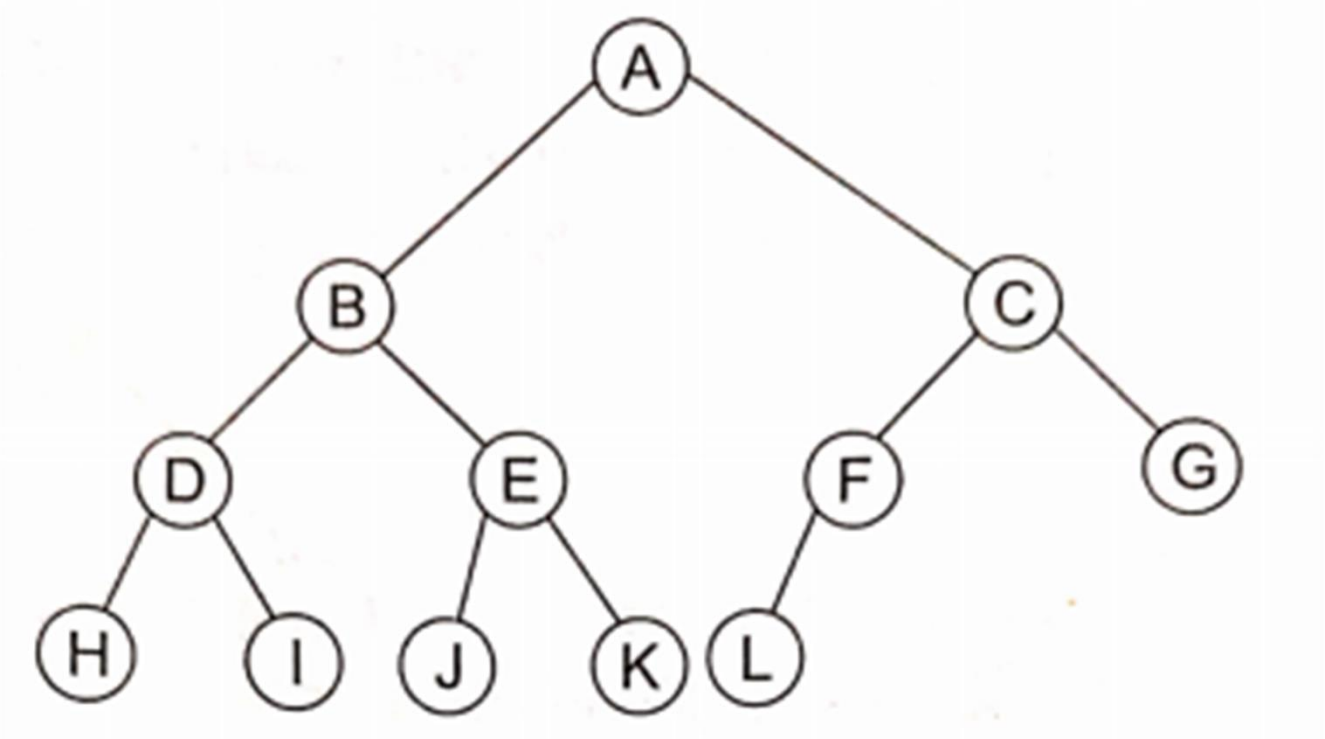
Strictly Binary Tree

A binary tree is called strictly binary if all its nodes except the leaf nodes contain two child nodes



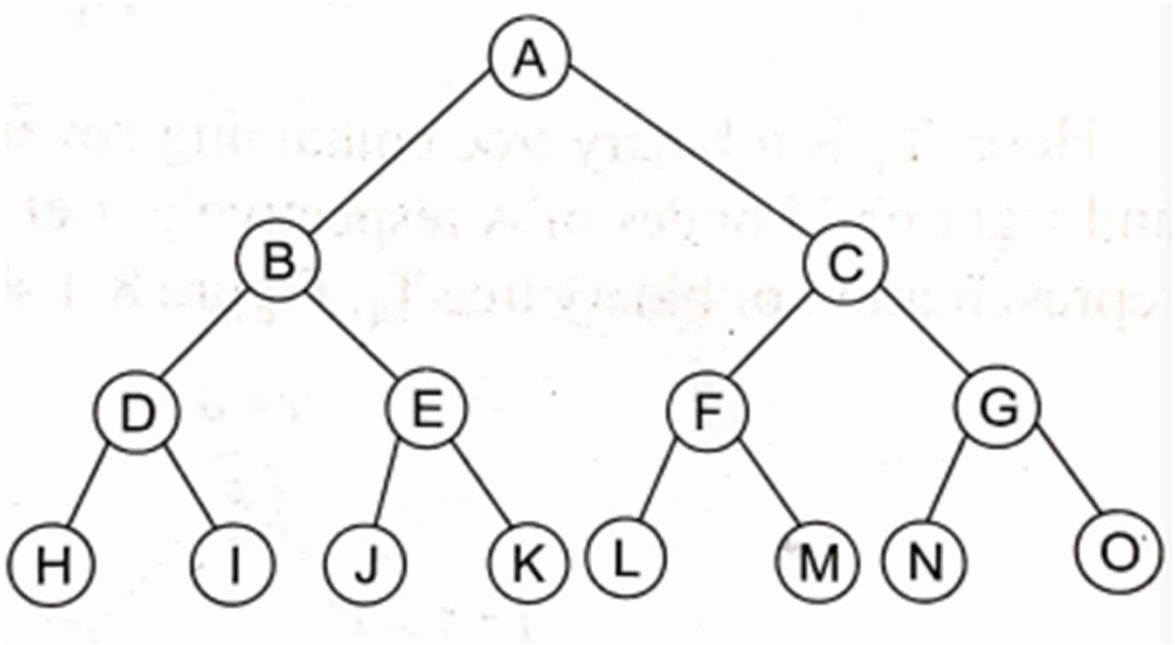
Complete Binary Tree

A binary tree of depth d is called complete binary tree if all its levels from 0 to d-1 contain maximum possible number of nodes and all the leaf nodes present at level d are placed towards the left side



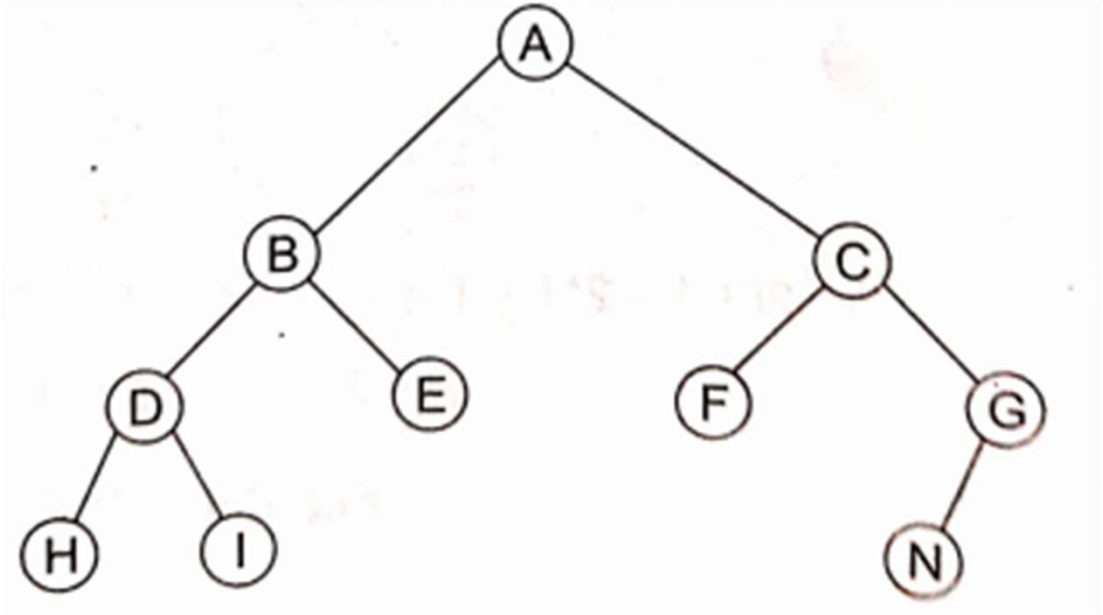
Perfect Binary Tree

A binary tree is called perfect binary tree if all its leaf nodes are at the lowest level (with Maximum Level number)and all the non-leaf nodes contain two child nodes.



Balanced Binary Tree

A binary tree is called balanced binary tree if the depths of the subtrees of all its do not differ by more than 1



Binary Tree Traversal

A binary tree traversal means visiting every node of the tree only once.

Three Traversal methods are:

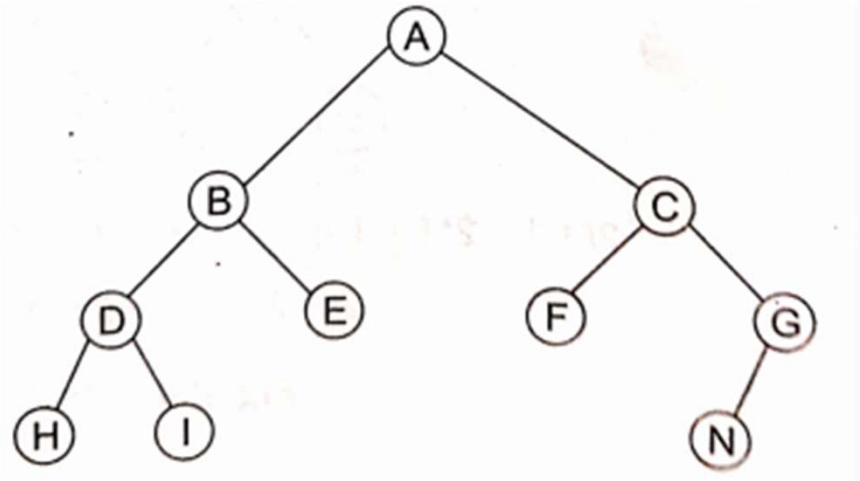
1. Preorder Traversal
2. Inorder Traversal
3. Postorder Traversal

Preorder Traversal

Preorder Traversal:

- Visit and print the root node
- Traverse the left sub-tree
- Traverse the right sub-tree

ROOT - LEFT - RIGHT



A B D H I E C F G N

Inorder Traversal

Inorder Traversal:

- Traverse the left sub-tree
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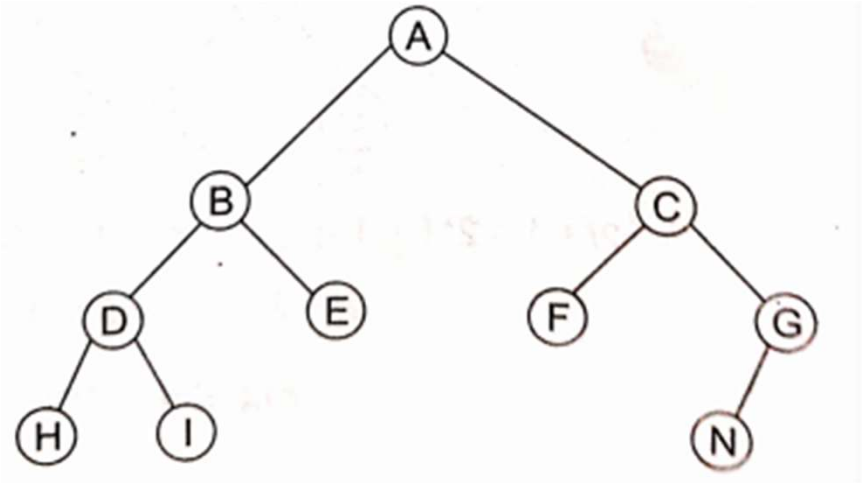
LEFT – ROOT - RIGHT



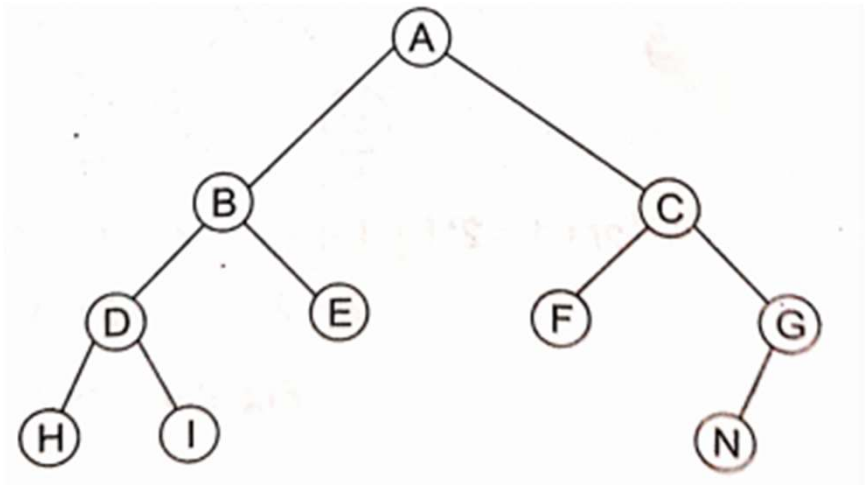
H D I B E A F C N G

Postorder Traversal

Postorder Traversal:

- Traverse the left sub-tree,
- Traverse the right sub-tree, .
- Visit and print the root node.

LEFT - RIGHT - ROOT



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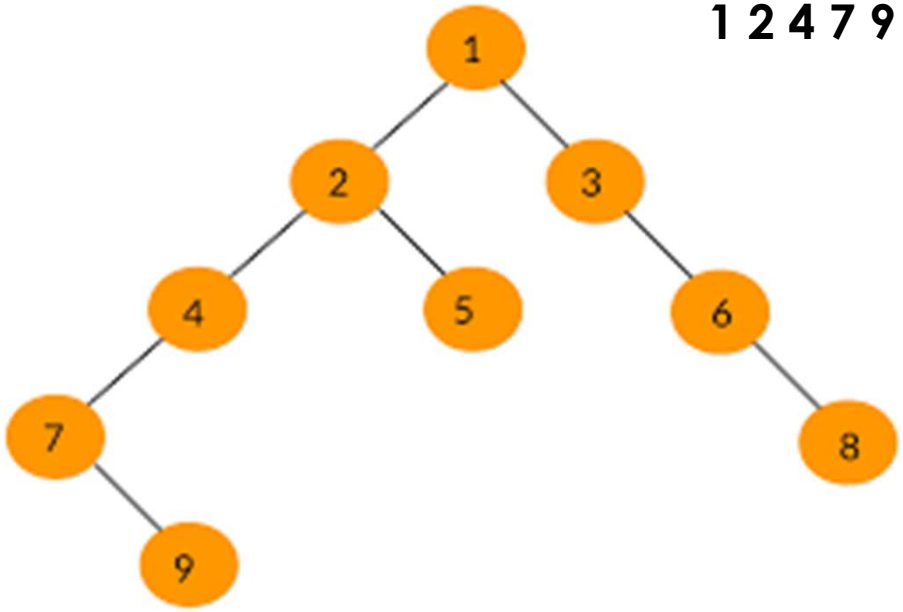

1 2 4 7 9 5 3 6 8

Inorder Traversal

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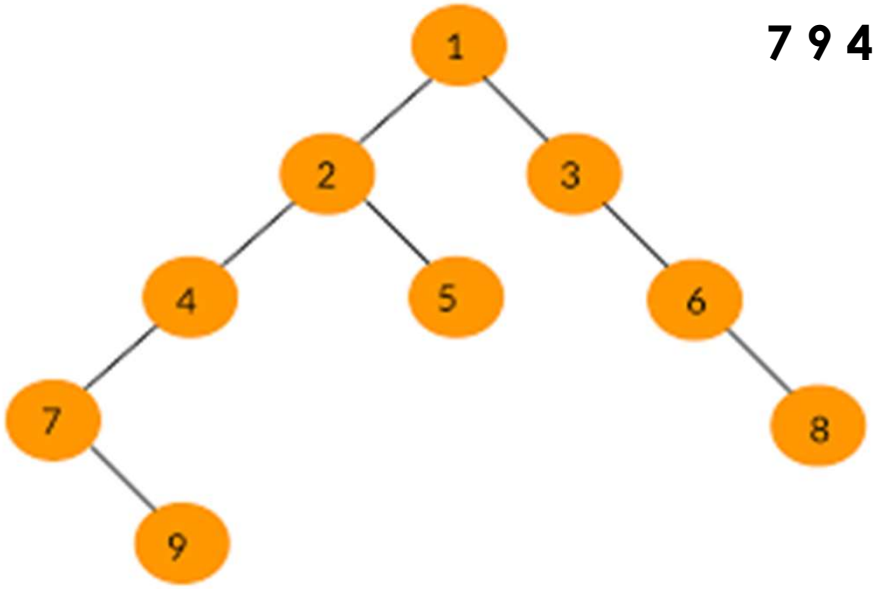

7 9 4 2 5 1 3 6 8

Postorder Traversal

Postorder Traversal:

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- Traverse the right sub-tree
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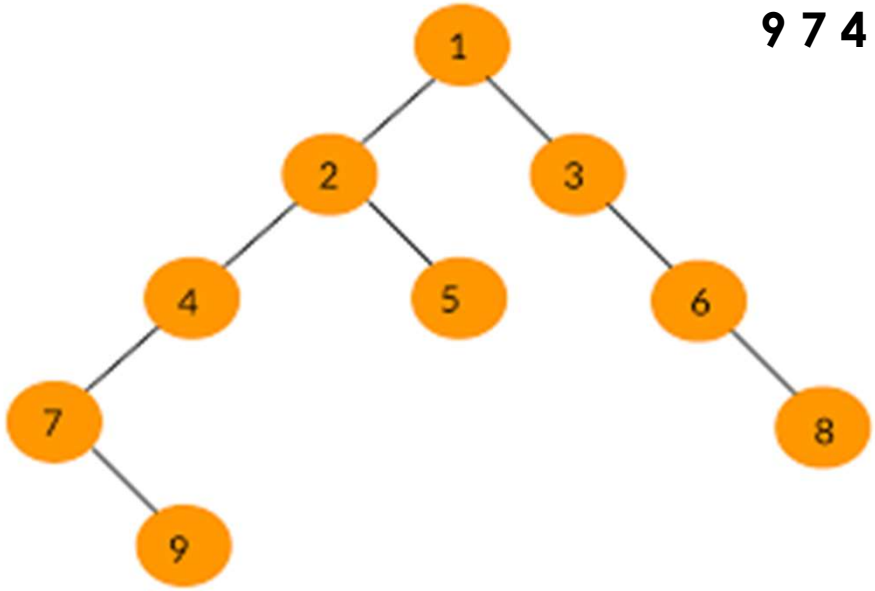
LEFT - RIGHT - ROOT



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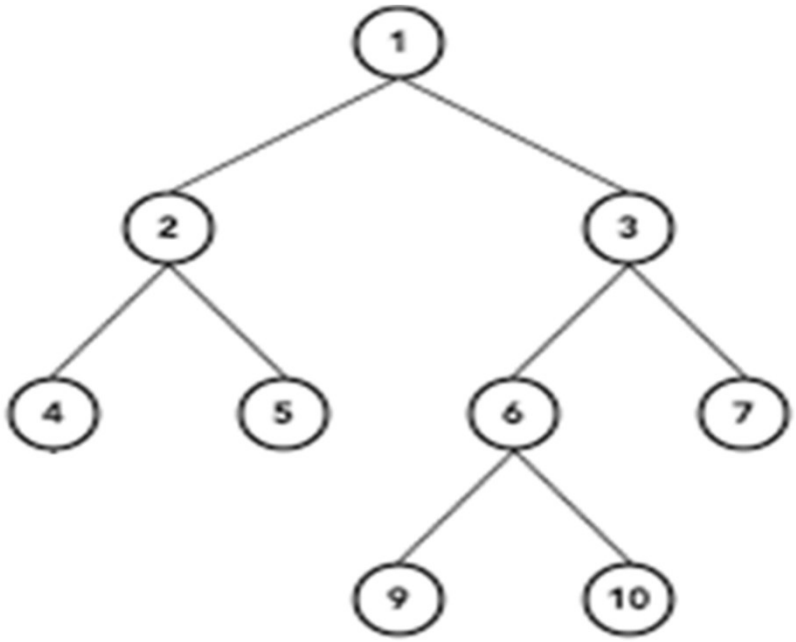

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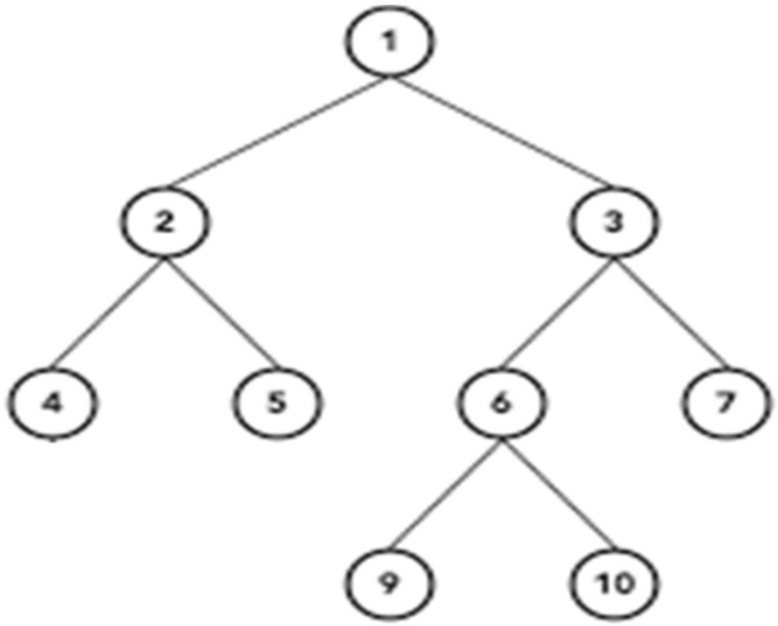

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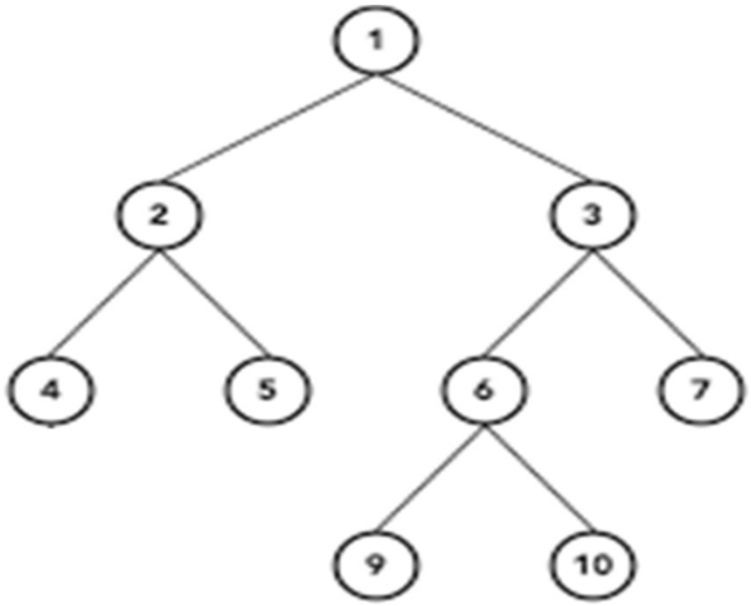

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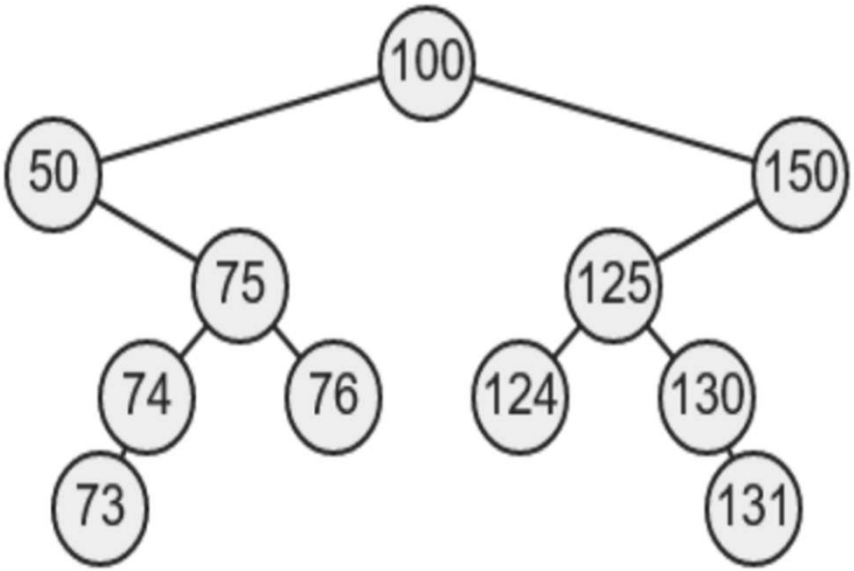

100, 50, 75, 74, 73, 76, 150, 125, 124, 130, 131

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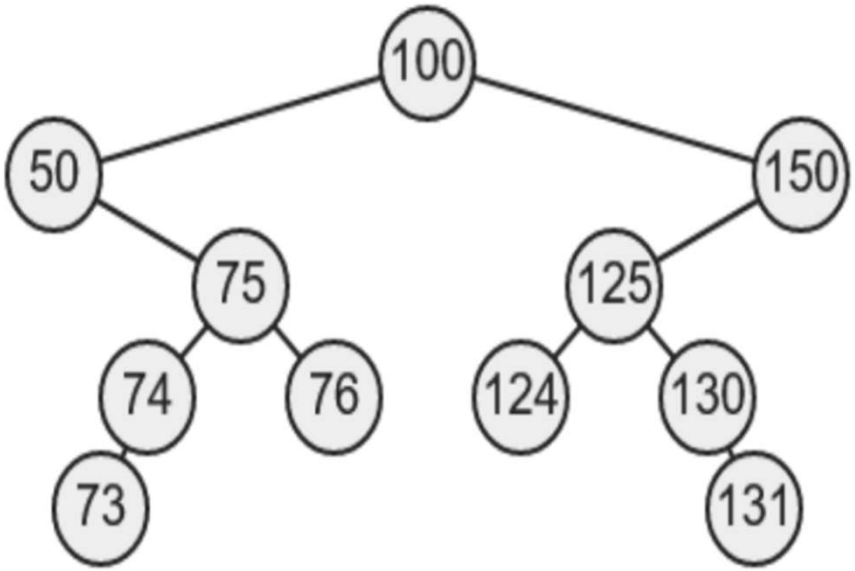

50, 73, 74, 75, 76, 100, 124, 125, 130, 131, 150

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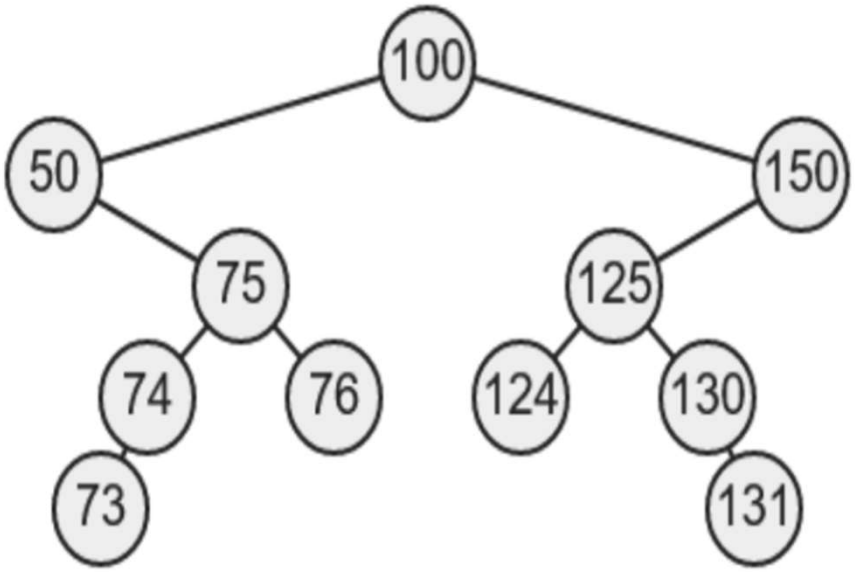

73, 74, 76, 75, 50, 124, 131,130, 125, 150, 100

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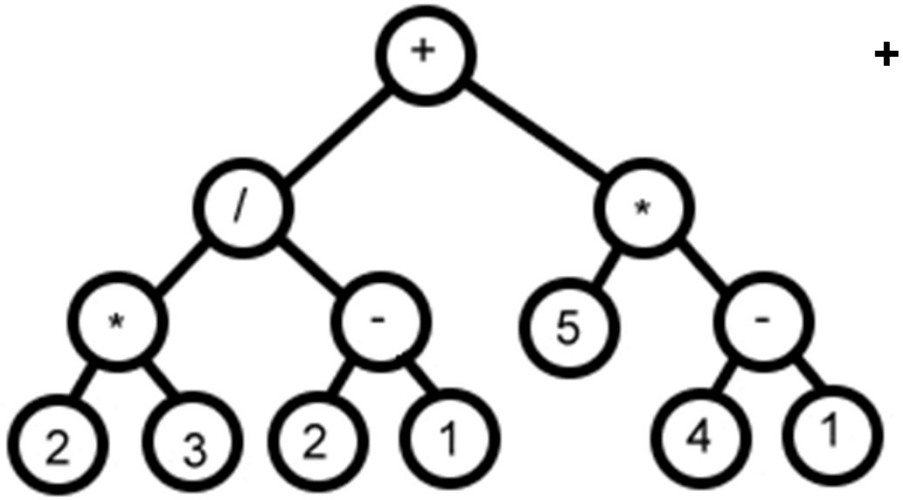

Expression tree for 2*3/(2-1)+5*(4-1)

+ / * 2 3 – 2 1 * 5 – 4 1

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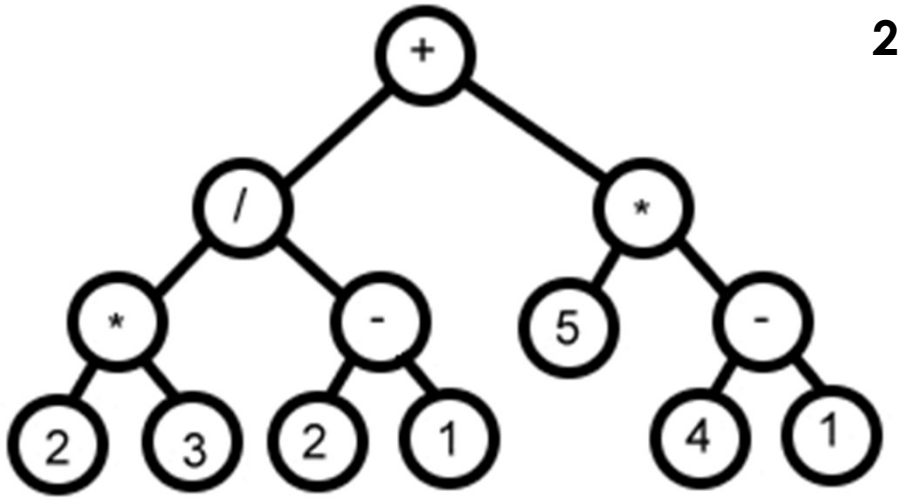

Expression tree for 2*3/(2-1)+5*(4-1)

2 * 3 / 2 – 1 + 5 * 4 - 1

Postorder Traversal

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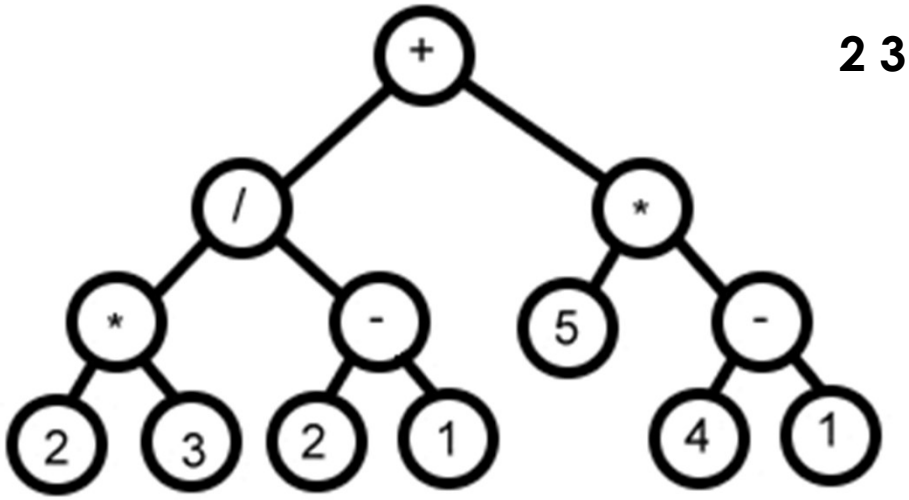

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2 3 * 2 1 - / 5 4 1 - * +

Constructing a Tree

Construct a binary tree with the given traversal techniques

- In–order Traversal: (L-Root-R): D B E A F C G

- Pre–order Traversal: (Root-L-R): A B D E C F G

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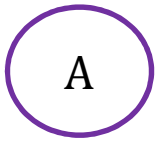

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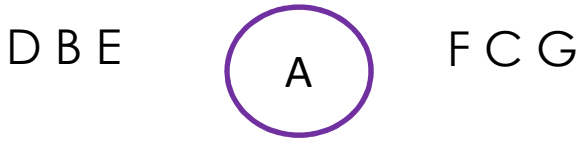

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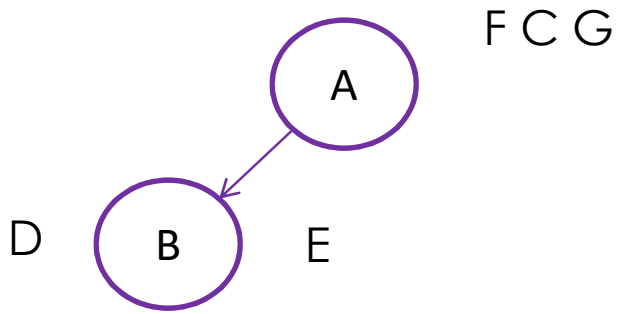

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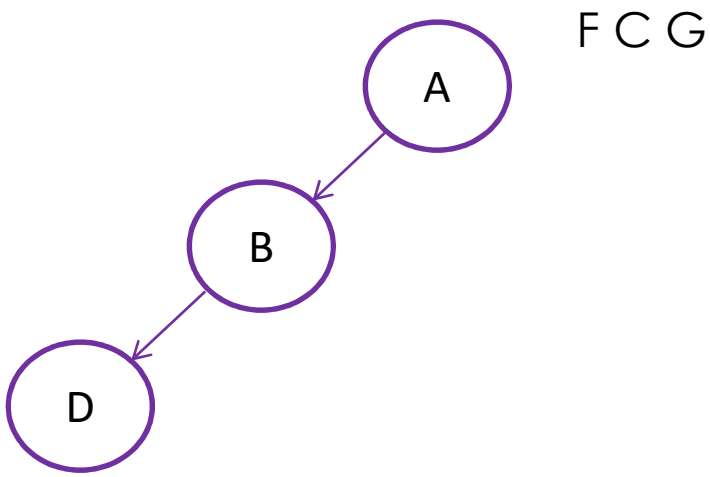

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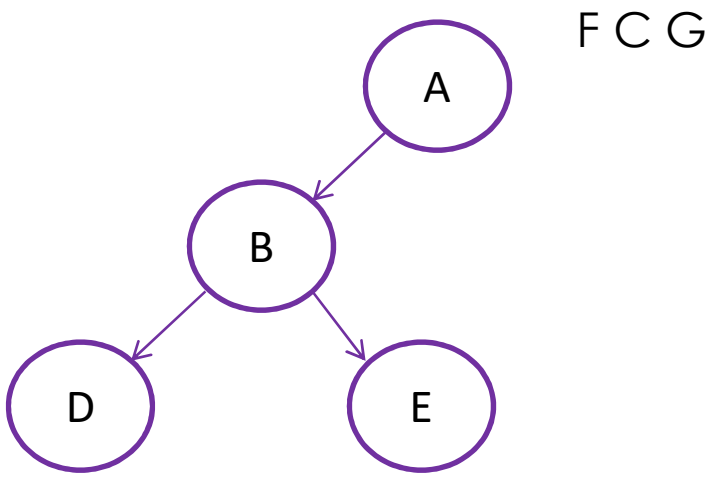

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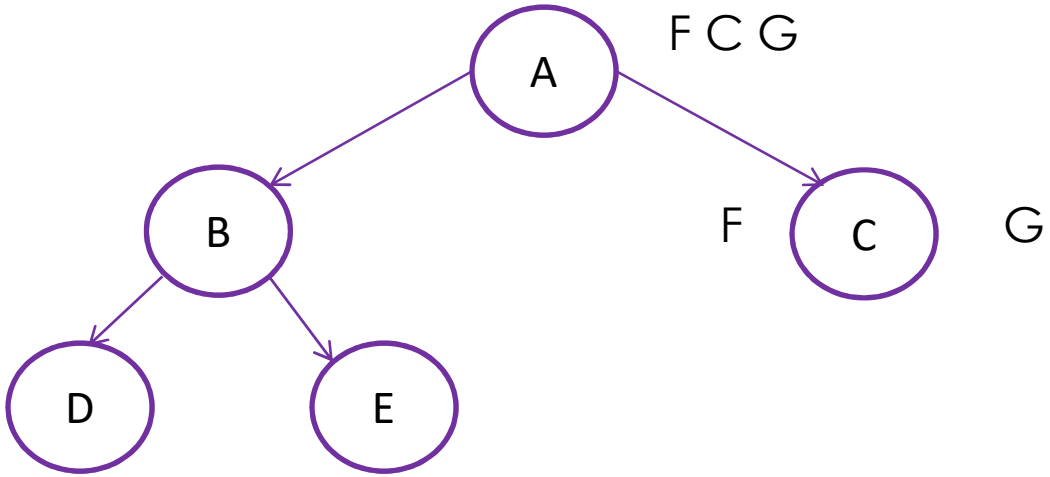

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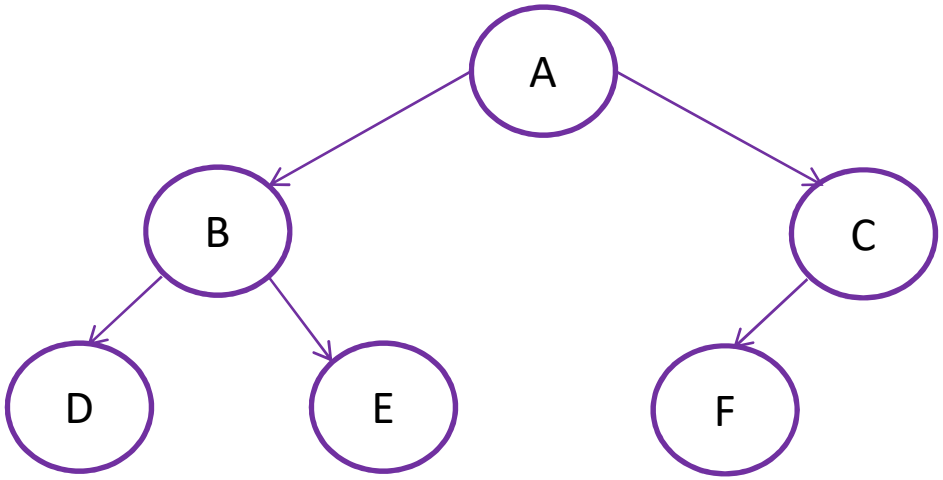

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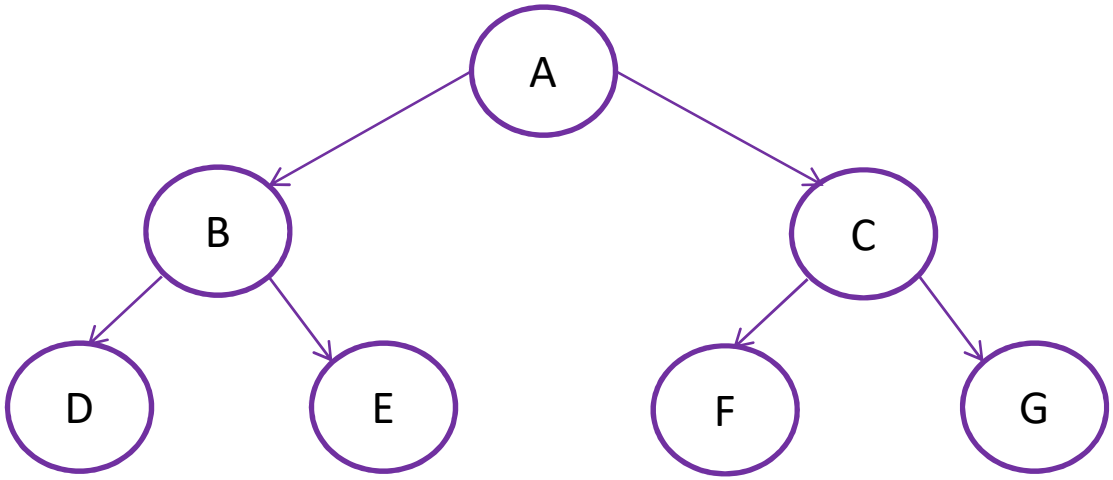

Constructing a Tree

Construct a binary tree with the given traversal techniques

- In–order Traversal (L-Root-R):D H B E I A F C K J L G

- Post–order Traversal(L-R-Root): H D I E B F K L J G C A

Constructing a Tree

Construct a binary tree with the given traversal techniques

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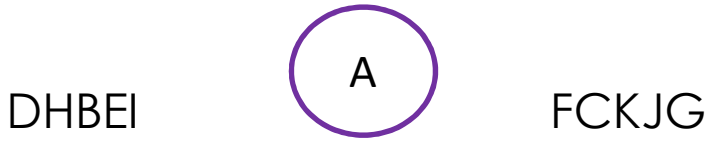

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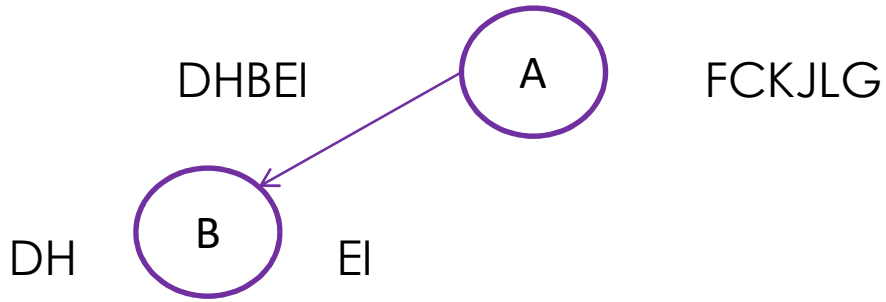
- Post–order Traversal(L-R-Root): H D I E B F K L J G C A



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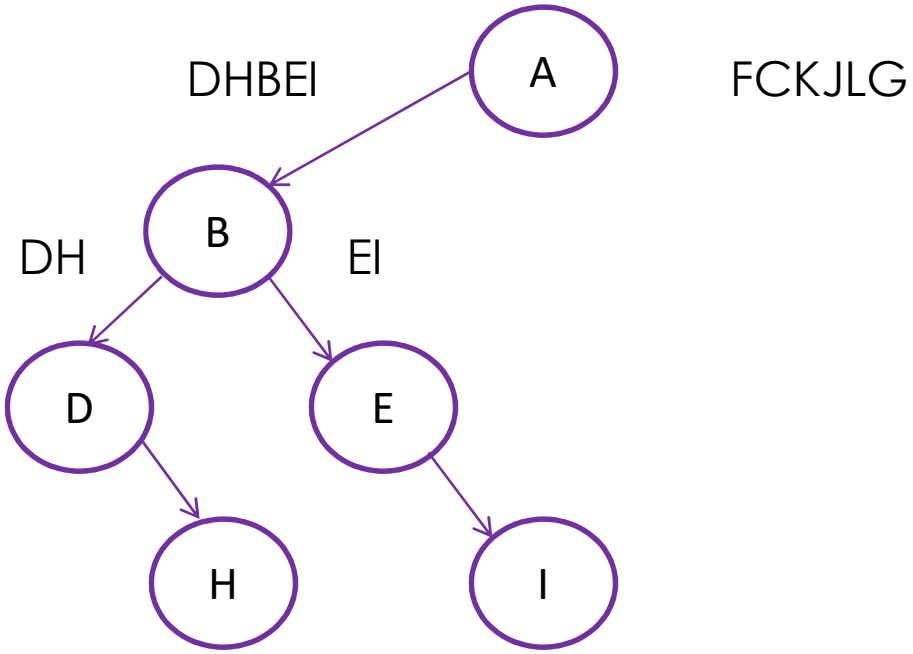
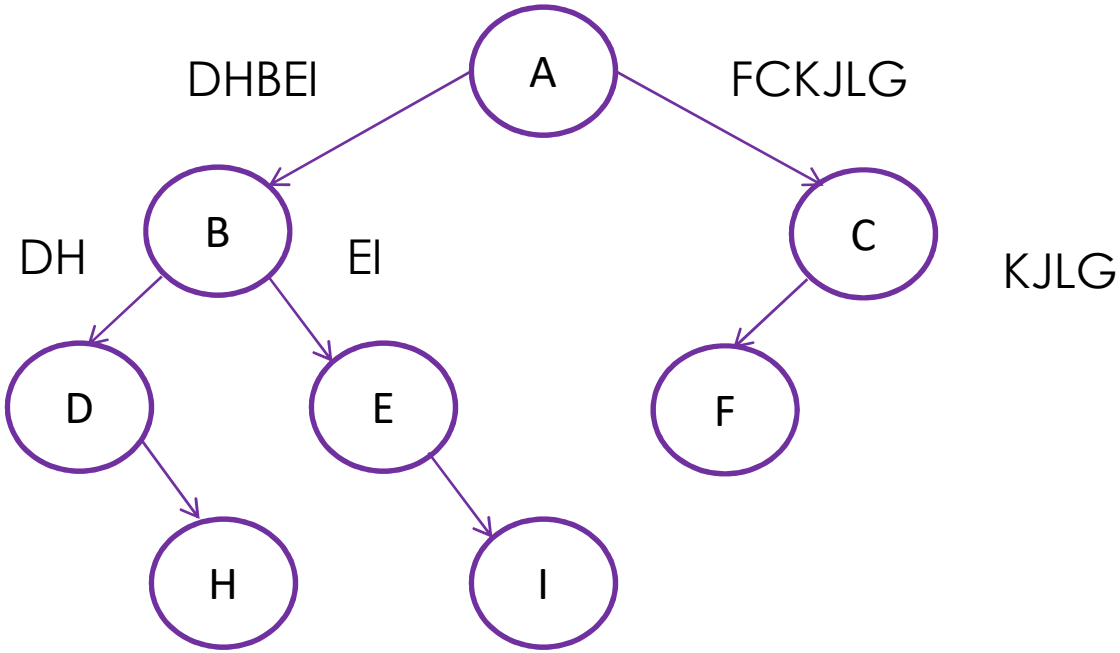
- In–order Traversal (L-Root-R):D H B E I A F C K J L G
- Post–order Traversal(L-R-Root): H D I E B F K L J G C A



Constructing a Tree

Construct a binary tree with the given traversal techniques

- In–order Traversal (L-Root-R):D H B E I A F C K J L G
- Post–order Traversal(L-R-Root): H D I E B F K L J G C A



Constructing a Tree

Construct a binary tree with the given traversal techniques

- In–order Traversal (L-Root-R):D H B E I A F C K J L G
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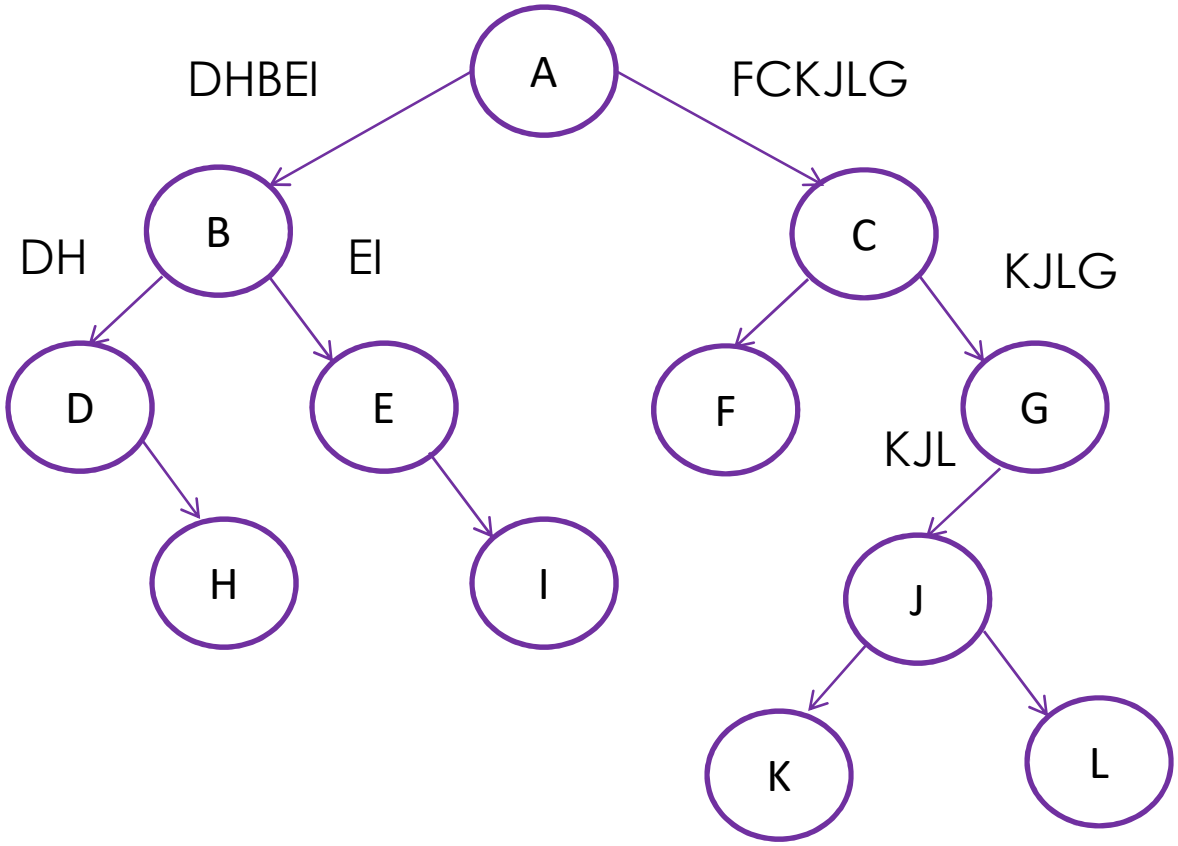
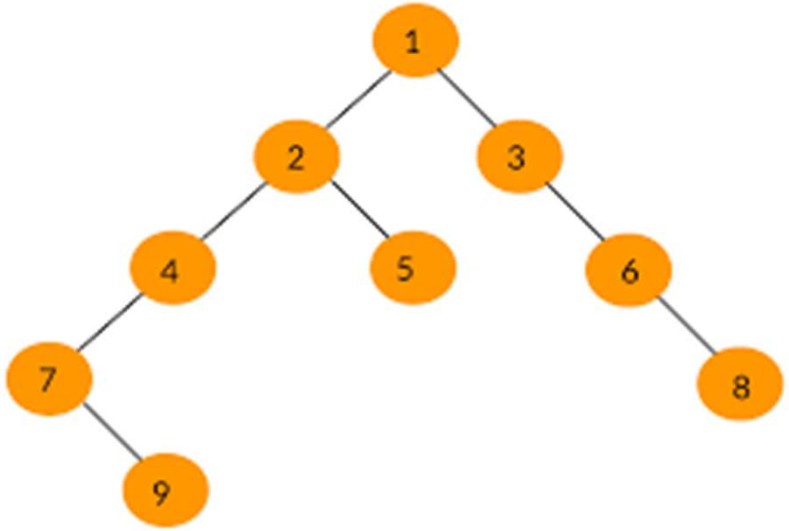


Constructing a Tree

Construct a binary tree with the given traversal techniques

- In–order Traversal: (L-Root-R): **7 9 4 2 5 1 3 6 8**

- Post–order Traversal: (L-R-Root) **9 7 4 5 2 8 6 3 1**



Constructing a Tree

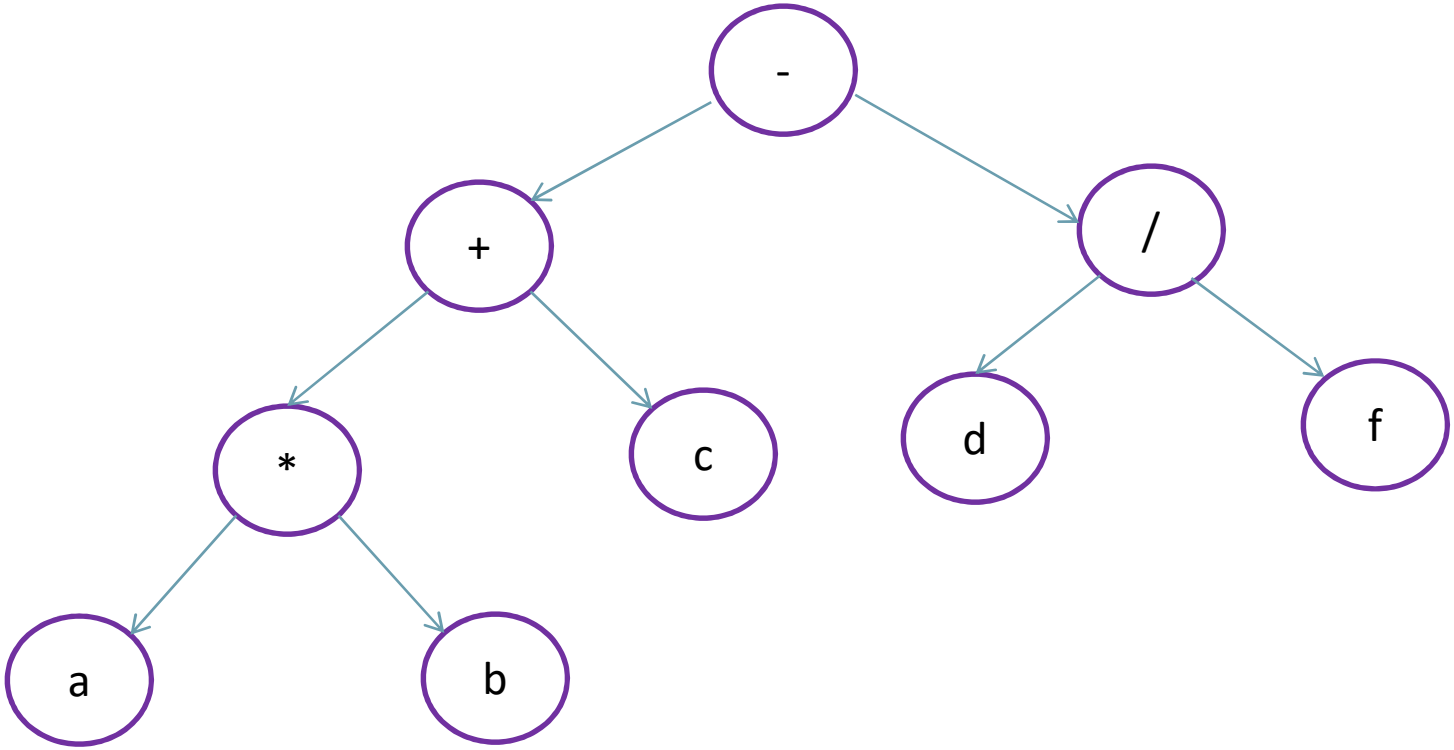
Building a Binary Expression tree from a Prefix expression

- Scan Prefix expression from Left to Right
  - Insert new nodes each time moving to the left until an operand has been inserted
  - Backtrack to the last operator, and put the next node to its right
  - Continue in the same fashion

Constructing a Tree

- Given the Prefix Expression Construct a Expression Tree

  Prefix Expression: - + * a b c / d f



Constructing a Tree

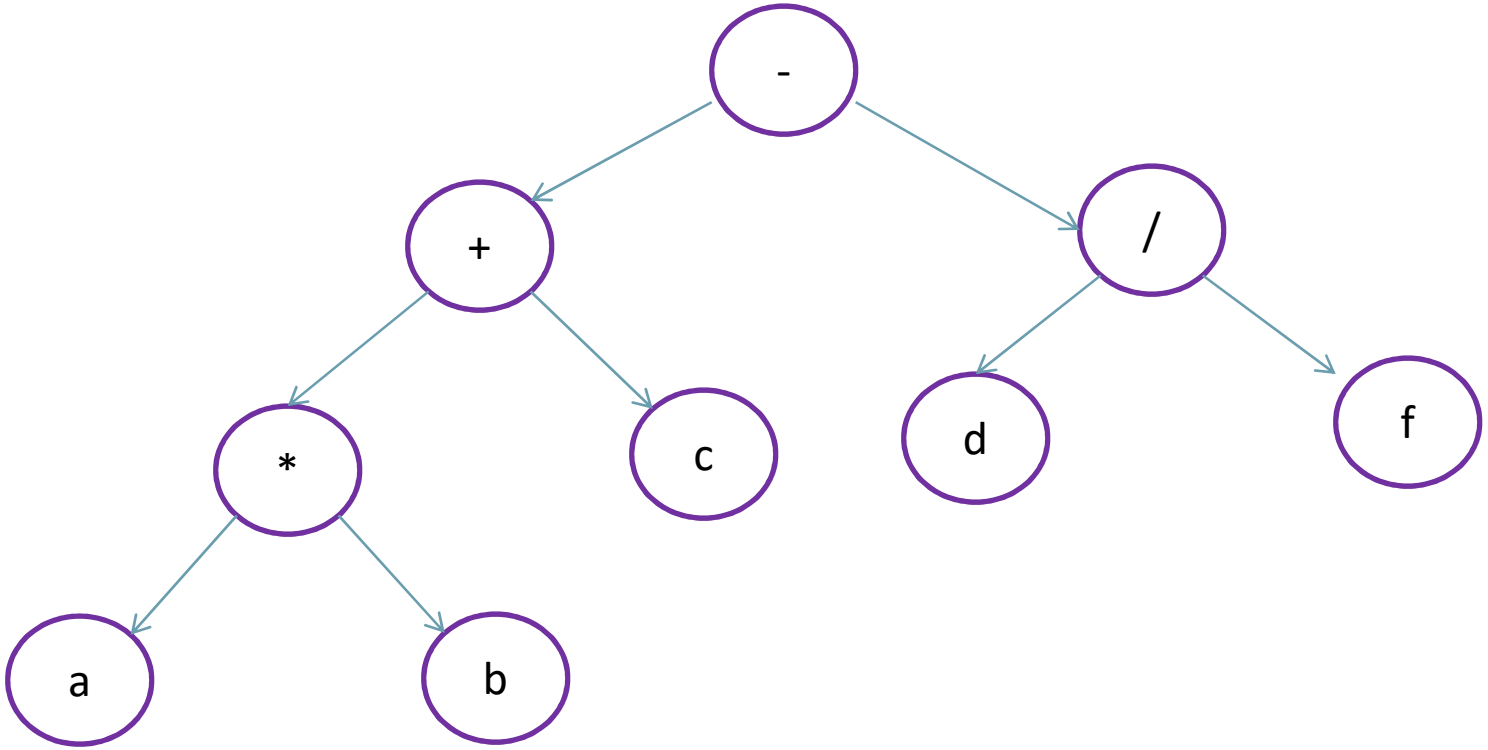
Building a Binary Expression tree from a Postfix expression

- Scan Postfix expression from Right to Left
  - Insert new nodes each time moving to the right until an operand has been inserted
  - Backtrack to the last operator, and put the next node to its left
  - Continue in the same fashion

Constructing a Tree

Given the Postfix Expression Construct a Expression Tree

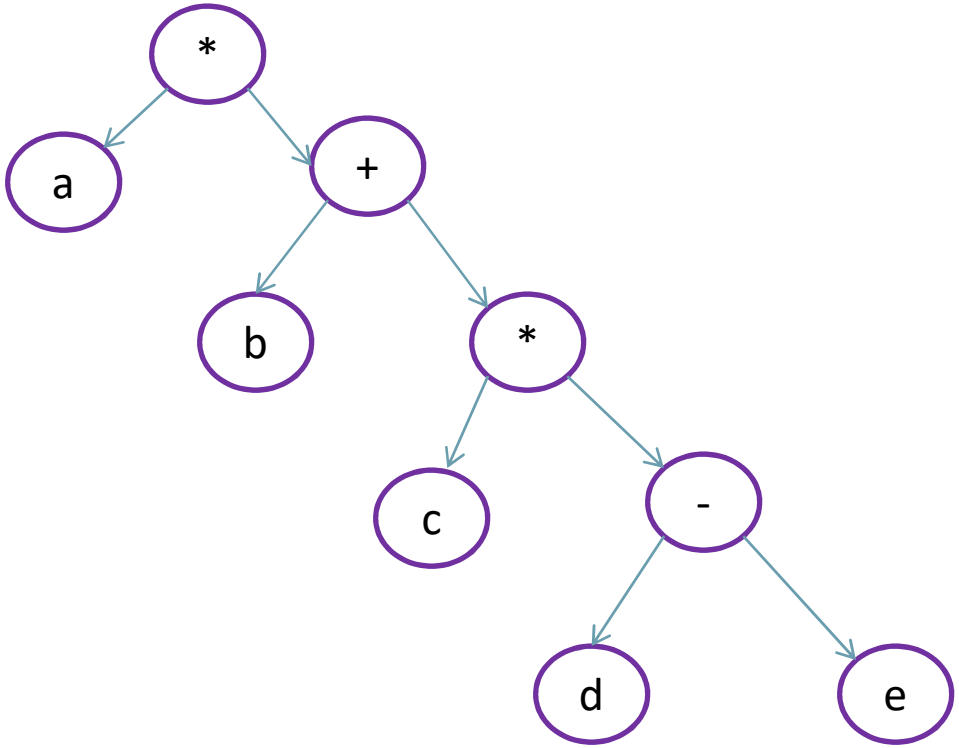
Postfix Expression: (L-R-Root) : a b * c + d f / -



Constructing a Tree

- Given the Prefix Expression Construct a Expression Tree

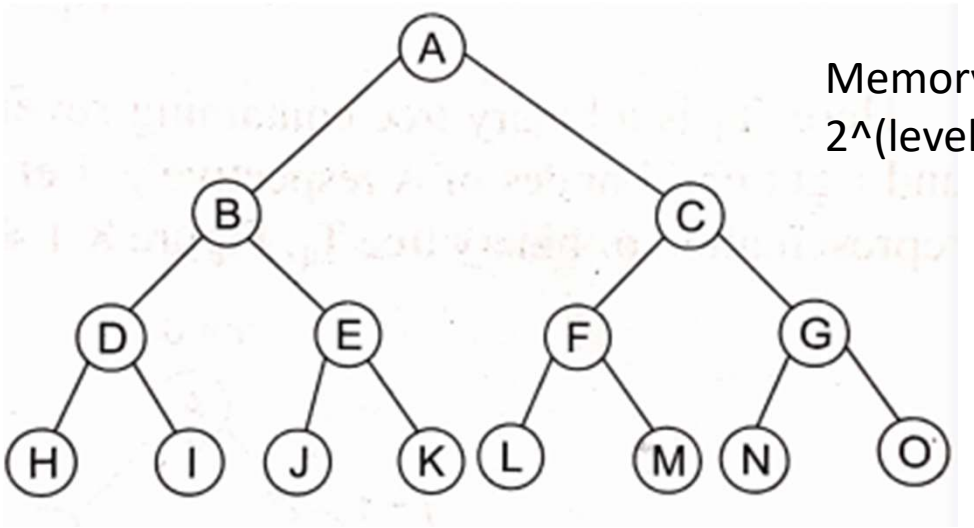
  Prefix Expression:* a + b * c – d e



Binary Tree Representation

Kanak Kalyani

Array Representation of Tree



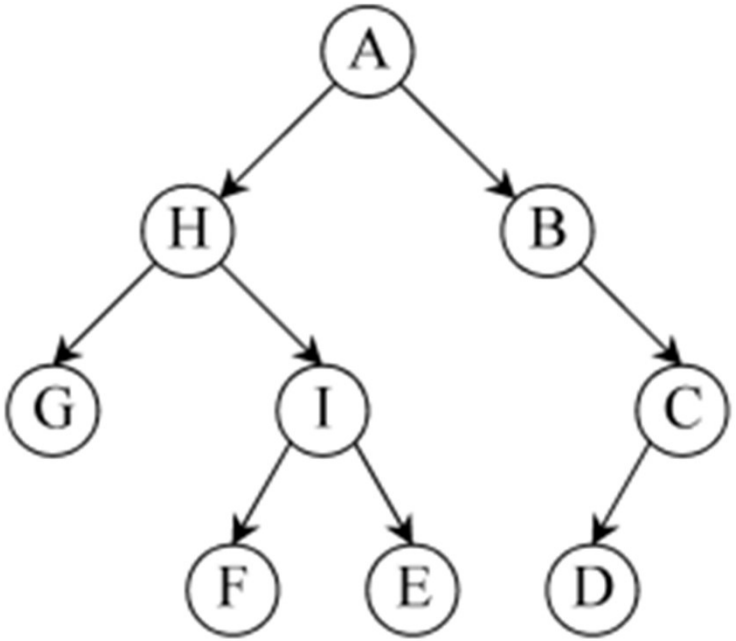
Memory allocated of array:
2^(levels) - 1

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O

- Notice:
  - The left child of index i is at index 2*i+1
  - The right child of index i is at index 2*i+2

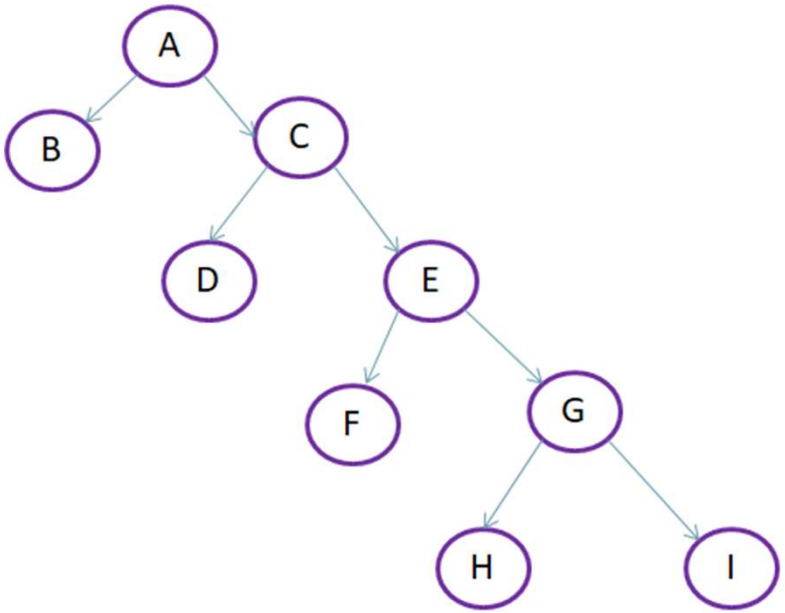
Kanak Kalyani

Array Representation of Tree



0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
A	H	B	G	I		C			F	E			D	

Array Representation of Tree



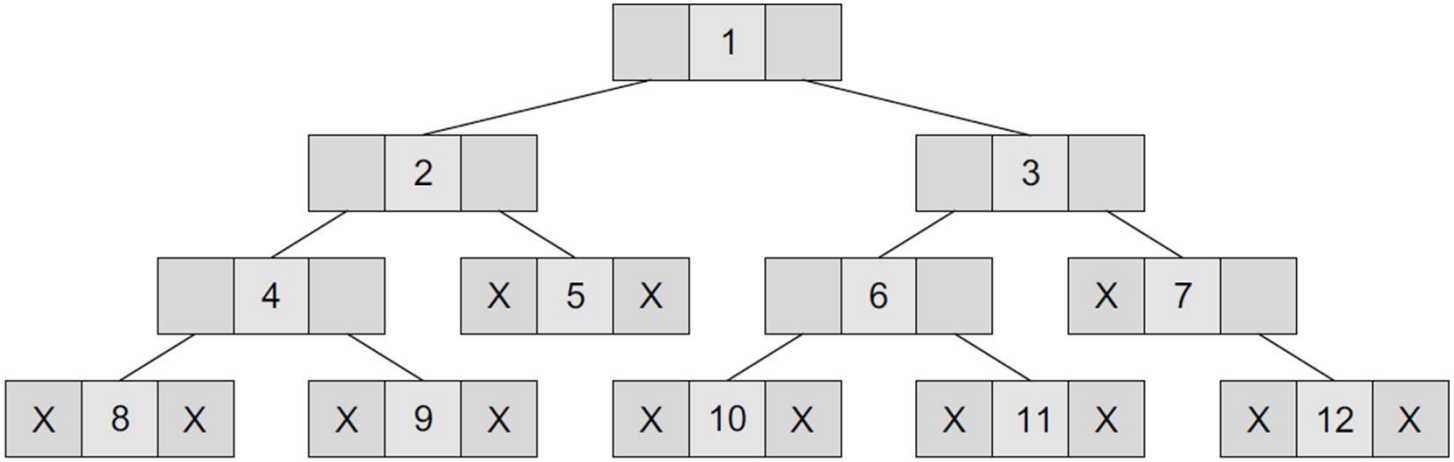
Memory allocated of array:
2^(level) - 1

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
A	B	C			D	E							F	G				

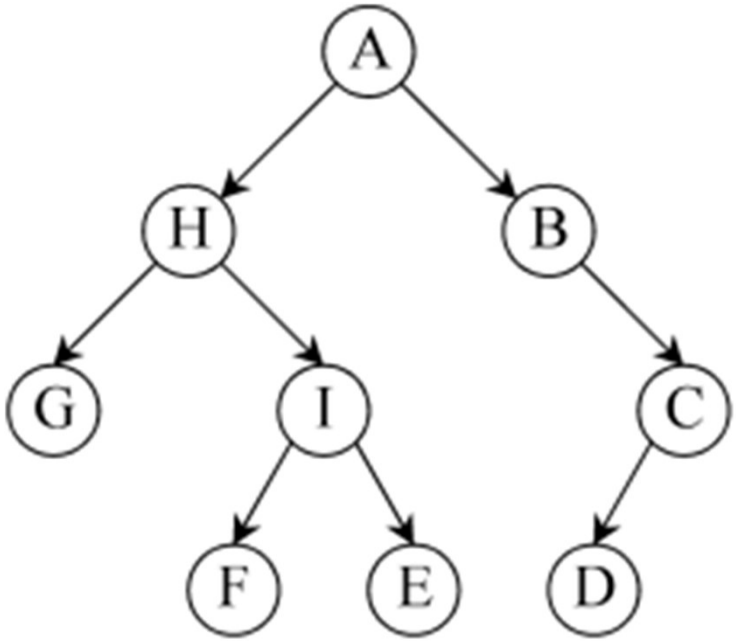
19	20	21	22	23	24	25	26	27	28	29	30							
										H	I							

Linked representation of binary trees

```
struct node {
        struct node *left;
        int data;
        struct node *right;
};
```

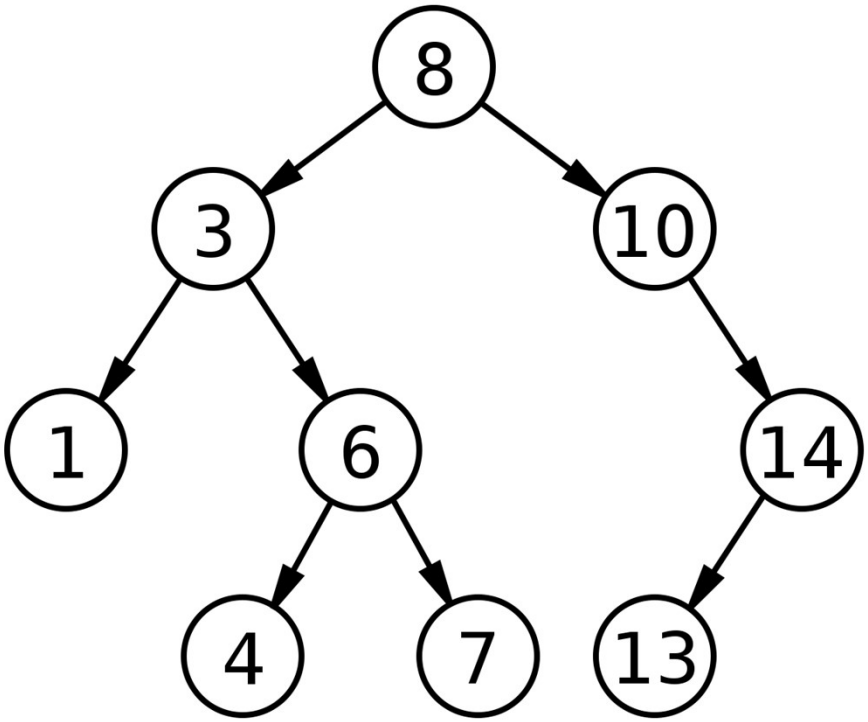


Linked Tree Representation of Binary Tree: Static Allocation



Row#	Left	Data	Right
0	1	A	2
1	3	H	4
2	-1	B	5
3	-1	G	-1
4	6	I	7
5	8	C	-1
6	-1	F	-1
7	-1	E	-1
8	-1	D	-1
9			
10			

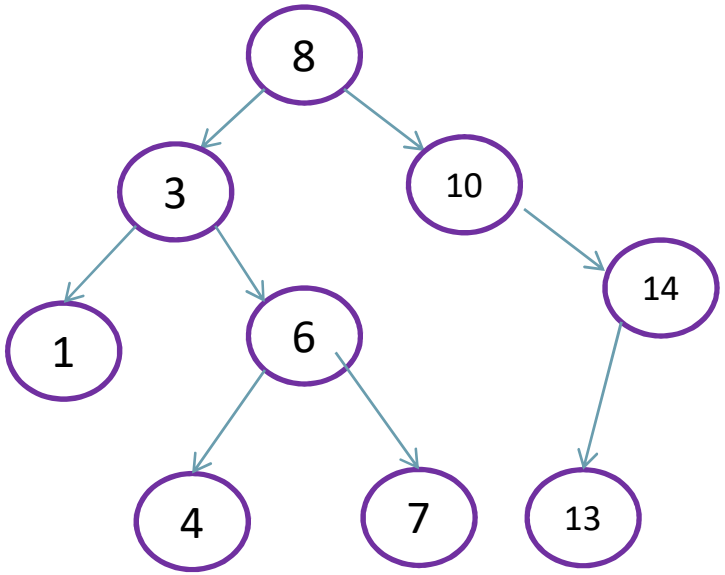
Binary Search Tree (BST)



The left subtree of a node contains nodes with keys lesser or equal than the node's key.
The right subtree of a node contains only nodes with keys greater than the node's key.
The left and right subtree each must also be a binary search tree.

Creation of BST

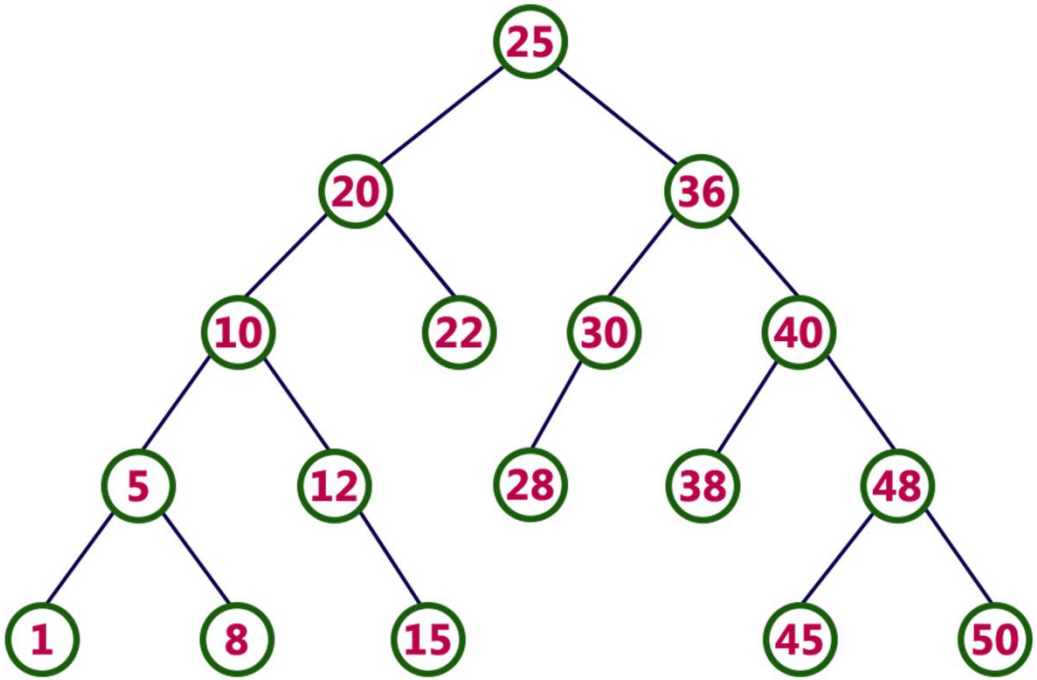
- Create a BST with following nodes:
- 8, 3, 10,14,13,6,7,4,1



Creation of BST

- Create a BST with following nodes:

25,36,20,10,5,22,30, 12,28, 40,38,48,1,8,15,45,50

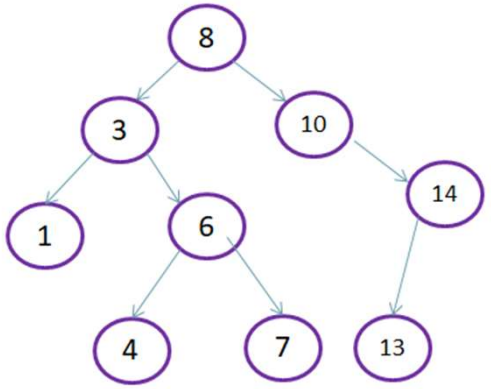


BST: Node Declaration

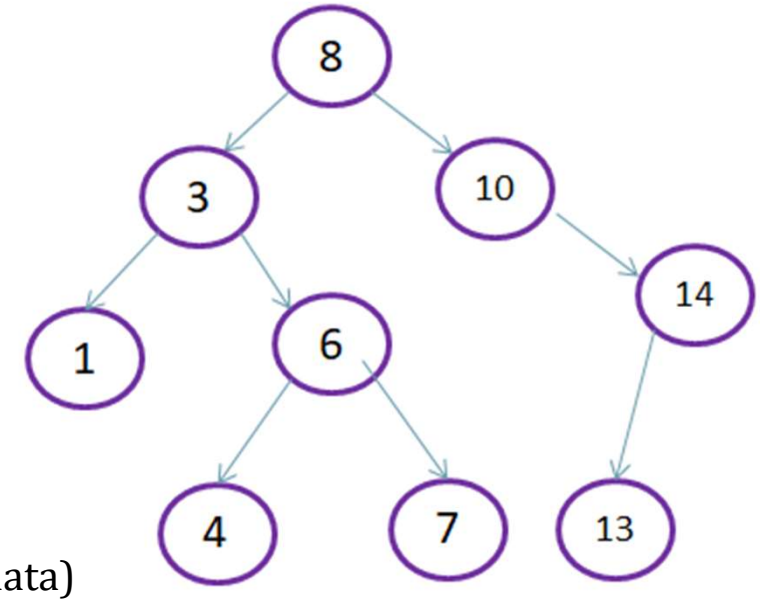
```
struct treenode
{
        struct treenode *left;
        int data;
        struct treenode *right;
};
typedef struct treenode treenode;
```

BST: Create()

```
void create(treenode *root){
        int num,i;
        int a[]={8, 3, 10,14,13,6,7,4,1};
        treenode *ptr,*temp,*prev;
        root=NULL;
        for(i=0;i<8;i++){
        temp=(treenode*)malloc(sizeof(treenode));
        temp->data=a[i];
        temp->left=NULL;
        temp->right=NULL;
```



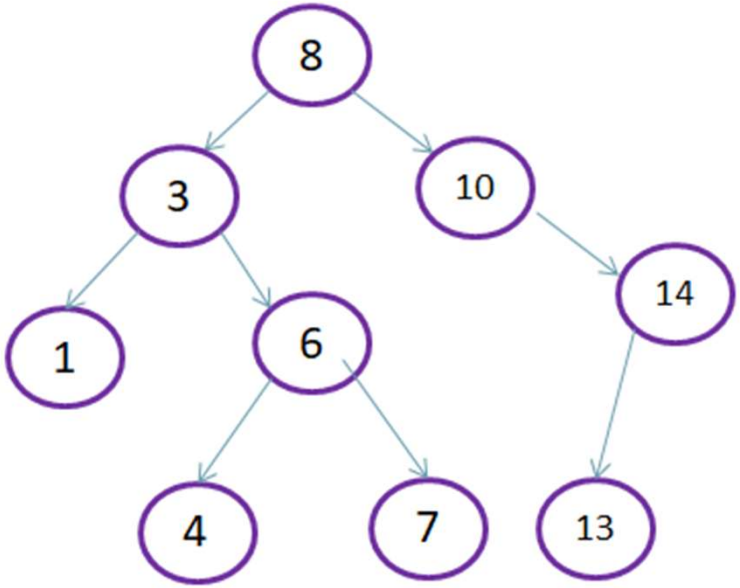
BST: Create()




```
if(root==NULL)
        root=temp;
else{
        ptr=root;
        while(ptr!=NULL){
                prev=ptr;
                if(ptr->data < temp->data)
                        ptr=ptr->right;
                else
                        ptr=ptr->left;
                }
        if(prev->data < temp->data)
                prev->right=temp;
        else
                prev->left=temp;
        }
}
return root;
}
```

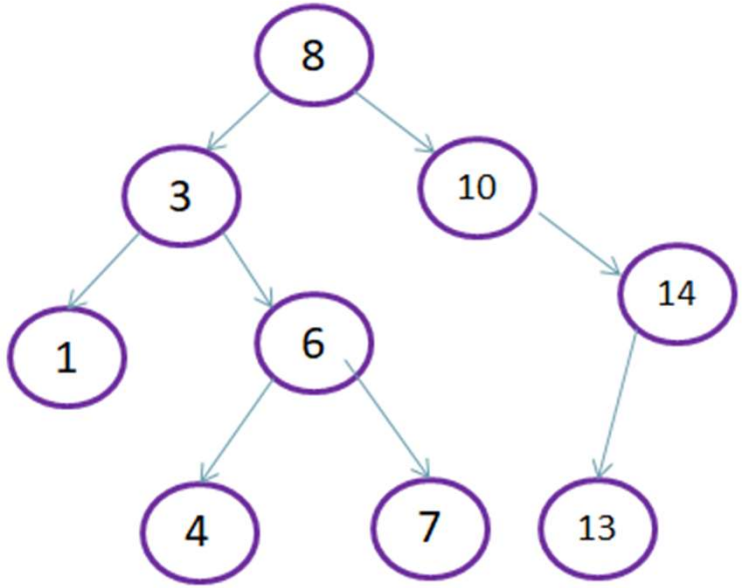
BST: Inorder Traversal

```
void printInorder(treenode* node)
{
   if (node == NULL)
      return;

   printInorder(node->left);
   printf("%d ", node->data);
   printInorder(node->right);
}
```



BST: Preorder Traversal

```
void printPreorder(treenode* node)
{
   if (node == NULL)
      return;

   printf("%d ", node->data);
   printPreorder(node->left);
   printPreorder(node->right);
}
```



BST: Postorder Traversal

```
void printPostorder(treenode* node)
{
   if (node == NULL)
      return;

   printPostorder(node->left);
   printPostorder(node->right);
   printf("%d ", node->data);
}
```