Q.1 a) Let $v = at^3+bt^2+ct+d \in V$

$$at^3+bt^2+ct+d = \alpha_1(t^3) + \alpha_2(t^2+t) + \alpha_3(t^3+t+1)$$
$$= (\alpha_1+\alpha_3)t^3 + \alpha_2 t^2 + (\alpha_2+\alpha_3)t + \alpha_3$$

$$\therefore \left.\begin{aligned} \alpha_1+\alpha_3 &= a \\ \alpha_2 &= b \\ \alpha_2+\alpha_3 &= c \\ \alpha_3 &= d \end{aligned}\right\} \Rightarrow \left[\begin{array}{ccc:c} 1 & 0 & 1 & a \\ 0 & 1 & 0 & b \\ 0 & 1 & 1 & c \\ 0 & 0 & 1 & d \end{array}\right] \sim \left[\begin{array}{ccc:c} 1 & 0 & 1 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c-b \\ 0 & 0 & 1 & d \end{array}\right] \sim \left[\begin{array}{ccc:c} 1 & 0 & 1 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c-b \\ 0 & 0 & 0 & d-c+b \end{array}\right]$$

In consistent. No solution.

Thus every element of V can not be express as linear combination of elements of W.

$\therefore$ W does not span V.

Q. 1 b) $\{x_1, x_2\}$ is linearly independent. $\therefore$ basis of R^2

Let $(x,y) \in R^2$

$\therefore$ $(x,y)^T = \alpha_1 x_1 + \alpha_2 x_2$ — (1)

$\therefore$ $(x,y)^T = \alpha_1(1,3)^T + \alpha_2(4,6)^T$

$$\Rightarrow \left.\begin{aligned} \alpha_1 + 4\alpha_2 &= x \\ 3\alpha_1 + 6\alpha_2 &= y \end{aligned}\right\} \Rightarrow \begin{aligned} \alpha_1 &= \frac{-6x+4y}{6} \\ \alpha_2 &= \frac{3x-y}{6} \end{aligned}$$

(1) $\Rightarrow$ $T(x,y)^T = \alpha_1 T(x_1) + \alpha_2 T(x_2)$

$= \alpha_1(-2,2,-7)^T + \alpha_2(-2,-4,-10)^T$

$= (-2\alpha_1 - 2\alpha_2,\ 2\alpha_1 - 4\alpha_2,\ -7\alpha_1 - 10\alpha_2)$

$T(x,y) = (-x+y,\ -4x+2y,\ 2x-3y)$

or

$$\begin{bmatrix} -1 & 1 \\ -4 & 2 \\ 2 & -3 \end{bmatrix}$$

Q. 2 a) $w_1 = u_1 = (1,3,1,1)$

$w_2 = \left(\frac{5}{6}, \frac{3}{6}, \frac{-7}{6}, \frac{-7}{6}\right)$

$w_3 = \left(\frac{10}{11}, \frac{-5}{11}, \frac{19}{11}, \frac{-14}{11}\right)$

$w_4 = \left(\frac{42}{31}, \frac{-21}{31}, \frac{-7}{31}, \frac{28}{31}\right)$

OR

$w_1 = (1,3,1,1)$

$w_2 = (5,3,-7,-7)$

$w_3 = (10,-5,19,-14)$

$w_4 = (42,-21,-7,28)$

$\{w_1, w_2, w_3, w_4\}$ is orthogonal basis.

and $\left\{\frac{w_1}{\|w_1\|}, \frac{w_2}{\|w_2\|}, \frac{w_3}{\|w_3\|}, \frac{w_4}{\|w_4\|}\right\}$ is orthonormal basis.

Q. 1 c) $T(1,1,0) = (5,3) = \alpha_1(1,2) + \alpha_2(2,3)$

$$\therefore \left.\begin{aligned} \alpha_1 + 2\alpha_2 &= 5 \\ 2\alpha_1 + 3\alpha_2 &= 3 \end{aligned}\right\} \Rightarrow \alpha_1 = -9,\ \alpha_2 = 7$$

$T(1,2,3) = (5,8) = \beta_1(1,2) + \beta_2(2,3)$

$\Rightarrow \beta_1 = 1,\ \beta_2 = 2$

$T(1,3,5) = (6,11) = \gamma_1(1,2) + \gamma_2(2,3)$

$\Rightarrow \gamma_1 = 4,\ \gamma_2 = 1$

$$\therefore [T]_{S,S'} = \begin{bmatrix} -9 & 1 & 4 \\ 7 & 2 & 1 \end{bmatrix} = A$$

Now, $v = (p,q,r) = \alpha_1(1,1,0) + \alpha_2(1,2,3) + \alpha_3(1,3,5)$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & p \\ 1 & 2 & 3 & q \\ 0 & 3 & 5 & r \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & p \\ 0 & 1 & 2 & q-p \\ 0 & 3 & 5 & r \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & p \\ 0 & 1 & 2 & q-p \\ 0 & 0 & 1 & -r+3q-3p \end{bmatrix}$$

$\Rightarrow \alpha_3 = -r+3q-3p$

$\alpha_2 = q-p-2\alpha_3 = 5p-5q+2r$

$\alpha_1 = -p+2q-r$

$$\therefore [v]_S = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} -p+2q-r \\ 5p-5q+2r \\ -r+3q-3p \end{bmatrix}$$

and $Tv = T(p,q,r) = (2p+3q-r,\ 4p-q+2r) = \beta_1(1,2) + \beta_2(2,3)$

$$\Rightarrow \left[\begin{array}{cc|c} 1 & 2 & 2p+3q-r \\ 2 & 3 & 4p-q+2r \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 2 & 2p+3q-r \\ 0 & +1 & 7q-4r \end{array}\right]$$

$\therefore \beta_2 = 7q-4r$

$\beta_1 = 2p-11q+7r.$

$$\therefore [Tv]_{S'} = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}.$$

Now, $$A[v]_S = \begin{bmatrix} -9 & 1 & 4 \\ 7 & 2 & 1 \end{bmatrix} \begin{bmatrix} -p+2q-r \\ 5p-5q+2r \\ -r+3q-3p \end{bmatrix} = \begin{bmatrix} 2p-11q+7r \\ 7q-4r \end{bmatrix} = [Tv]_{S'}$$

Q.2 b) $A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$ $A^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$ $AA^T = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$, $A^TA = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

Eigen values of AA^T $\lambda_1 = \lambda_2 = 2$

$\because$ Eigen values of AA^T & A^TA are same.

Eigen values of A^TA are $\lambda_1 = \lambda_2 = 2$, $\lambda_3 = \lambda_4 = 0$.

$$\therefore \Sigma = \begin{bmatrix} \sqrt{\lambda_1} & 0 & 0 & 0 \\ 0 & \sqrt{\lambda_2} & 0 & 0 \end{bmatrix} = \begin{bmatrix} \sqrt{2} & 0 & 0 & 0 \\ 0 & \sqrt{2} & 0 & 0 \end{bmatrix}$$

To find Eigen vectors of AA^T

$$\begin{bmatrix} 2-\lambda & 0 \\ 0 & 2-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

For $\lambda = 2$, we have only equation

$0x_1 + 0x_2 = 0.$

Nontrivial sol^n we can take

$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$\therefore U = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

To find Eigen vectors of A^TA

For $\lambda = 2$.

we have

$-x_1 + 0x_2 + x_3 + 0x_4 = 0.$

$0x_1 - x_2 + 0x_3 + x_4 = 0.$

Two free variables x_3, x_4

Let $x_3 = 1, x_4 = 0 \Rightarrow x_1 = 1, x_2 = 0$

$x_3 = 0, x_4 = 1 \Rightarrow x_1 = 0, x_2 = 1$

$$\therefore \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \ \& \ \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

For $\lambda = 0$. we have,

$x_1 + 0x_2 + x_3 + 0x_4 = 0$

$0x_1 + x_2 + 0x_3 + x_4 = 0$

Again

$x_3 = 1, x_4 = 0 \Rightarrow x_1 = -1, x_2 = 0$

$x_3 = 0, x_4 = 1 \Rightarrow x_1 = 0, x_2 = -1$

$$\Rightarrow \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \ \& \ \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

$$\therefore V = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Q.3 a

$$\int_{x=0}^{\infty}\int_{y=0}^{\infty} kxy\, e^{-(x^2+y^2)}\,dy\,dx = 1 \Rightarrow k\int_{x=0}^{\infty} x e^{-x^2}dx \int_{y=0}^{\infty} y e^{-y^2}dy = 1$$

$$\Rightarrow k\left(\int_{x=0}^{\infty} x e^{-x^2}dx\right)^2 = 1.$$

put $x^2 = t \Rightarrow xdx = \frac{dt}{2}$

$$\therefore k\left(\int_0^{\infty} e^{-t}\frac{dt}{2}\right)^2 = 1$$

$$\Rightarrow \frac{k}{4}\left(\frac{e^{-t}}{-1}\Big|_0^{\infty}\right)^2 = 1 \Rightarrow \frac{k}{4} = 1 \Rightarrow \boxed{k=4}$$

$$F_1(x) = P(X \le x) = P(0 < X \le x,\ 0 < Y < \infty)$$

$$= \int_{y=0}^{\infty}\int_{u=0}^{x} kuy\, e^{-(u^2+y^2)}\,du\,dy.$$

$$= k\int_{y=0}^{\infty} y e^{-y^2}dy \int_{u=0}^{x} u e^{-u^2}du$$

put $u^2 = t$, $2u\,du = dt$

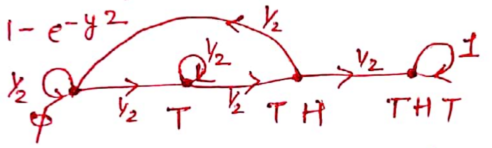
$$= k\cdot\frac{1}{2}\cdot\int_{t=0}^{x^2} e^{-t}\frac{dt}{2}$$

when $u=0$, $t=0$; $u=x$, $t=x^2$.

$$= \frac{k}{4}\,\frac{e^{-t}}{-1}\Big|_0^{x^2} = \frac{-k}{4}\left(e^{-x^2}-1\right) = 1-e^{-x^2}$$

lly $F_2(y) = 1-e^{-y^2}$

Q.3 b)



$$\begin{array}{c|cccc} & \phi & T & TH & THT \\ \hline \phi & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ T & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ TH & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ THT & 0 & 0 & 0 & 1 \end{array}$$

$$P(A) = P(T \to T \to P \to TH \to THT)$$

$$= \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{16}$$

Q.4 a

$$\bar{x} = \frac{\Sigma x}{13} = 2.461538$$

$$s^2 = \frac{\Sigma(x-\bar{x})^2}{12} = \frac{115.2307}{12} = 9.602564 \Rightarrow s = 3.0989$$

At 5% level of significance

Confidence interval

$$\left(\bar{x} \pm \sqrt{\frac{9.602564}{13}} \times 1.782\right) \Rightarrow (0.9299,\ 3.993083)$$

i.e. average change in B.P. of the population is positive

$\therefore$ the stimulus will increase the B.P.

Q.4 b

$\mu = 600$, $\sigma_x = 120 \approx \sigma_{\bar{x}}$

$n = 100$

$\mu_{\bar{x}} = 625$

$H_0:\ \mu = 600$

$H_1:\ \mu > 600$ (Rt tailed test)

$$z = \frac{\mu_{\bar{x}} - \mu}{\sigma_x/\sqrt{n}} = \frac{625-600}{120/\sqrt{100}} = 2.08333$$

At 5% level of significance

$z_\alpha = 1.645 < z_{\text{calculated}}.$

$\therefore$ Ho is rejected.

$\therefore$ New process has increase the mean life of the equipment.

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