Implementing Artificial Neural Networks with TensorFlow - Homework 01

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1 Setup

No Submission for this Assignment.

2 Coding Exercises

Confer cat.py and kittyconcert.py.

3 Math

3.1 Sigmoid Function

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

3.2 Derivative of the Sigmoid Function with Leibniz Notation

$$\frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{dx}$$

$$f_1 = \frac{1}{v}$$

$$v = 1 + e^{-x}$$

$$f_1' = (\frac{1}{v})' = (v^{-1})' = -1 \cdot v^{-1-1} = -v^{-2} = -\frac{1}{v^2} = -\frac{1}{(1 + e^{-x})^2}$$

$$\frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{dw} \cdot \frac{dw}{dx}$$

$$f_2 = 1 + w$$

$$w = e^{-x}$$

$$f_2' = (1 + w)' = (1 + w^1)' = 1 \cdot w^0 = 1 \cdot 1 = 1$$

$$\frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{dw} \cdot \frac{dw}{dt} \cdot \frac{dt}{dx}$$

$$f_3 = e^t$$

$$t = -x$$

$$f_3' = (e^t)' = e^t = e^{-x}$$

$$f_4 = -x$$

$$f_4' = (-x)' = (-x^1)' = 1 \cdot (-x^0) = 1 \cdot (-1) = -1$$

$$\begin{aligned}
\frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{dw} \cdot \frac{dw}{dt} \cdot \frac{dt}{dx} &= -\frac{1}{(1 + e^{-x})^2} \cdot 1 \cdot e^{-x} \cdot (-1) \\
&= \frac{e^{-x}}{(1 + e^{-x})^2} \\
&= \frac{1}{1 + e^{-x}} \cdot \frac{e^{-x}}{1 + e^{-x}} \\
&= \frac{1}{1 + e^{-x}} \cdot \frac{e^{-x} + (1 - 1)}{1 + e^{-x}} \\
&= \frac{1}{1 + e^{-x}} \cdot \frac{(1 + e^{-x}) - 1}{1 + e^{-x}} \\
&= \frac{1}{1 + e^{-x}} \cdot (\frac{1 + e^{-x}}{1 + e^{-x}} - \frac{1}{1 + e^{-x}}) \\
&= \frac{1}{1 + e^{-x}} \cdot (1 - \frac{1}{1 + e^{-x}}) \\
&= \sigma(x) \cdot (1 - \sigma(x))
\end{aligned}$$

3.3 Partial Derivatives

$f(x, z, a, b) := y = (4ax^2 + a) + 3 + \sigma(z) + \sigma(b)^2$

- $\frac{dy}{dx} = 0$
- $\frac{dy}{dz} = \sigma(z) \cdot (1 - \sigma(z))$
- $\frac{dy}{da} = 1$
- $\frac{dy}{db} = \frac{dy}{dv} \cdot \frac{dv}{db}$

$f_1 = v^2$
$v = \sigma(b)$
$f_1' = 2v = 2 \cdot \sigma(b)$

$f_2 = \sigma(b)$
$f_2' = \sigma(b) \cdot (1 - \sigma(b))$

$\frac{dy}{db} = \frac{dy}{dv} \cdot \frac{dv}{db} = 2 \cdot \sigma(b) \cdot \sigma(b) \cdot (1 - \sigma(b)) = 2 \cdot (\sigma(b))^2 \cdot (1 - \sigma(b))$