University of Ottawa

Assignment 5 - NLMS, AP, LS and RLS algorithms

Jana Rusrus (300205310)

ELG5377 - Adaptive Signal Processing April 14, 2021

Question 1

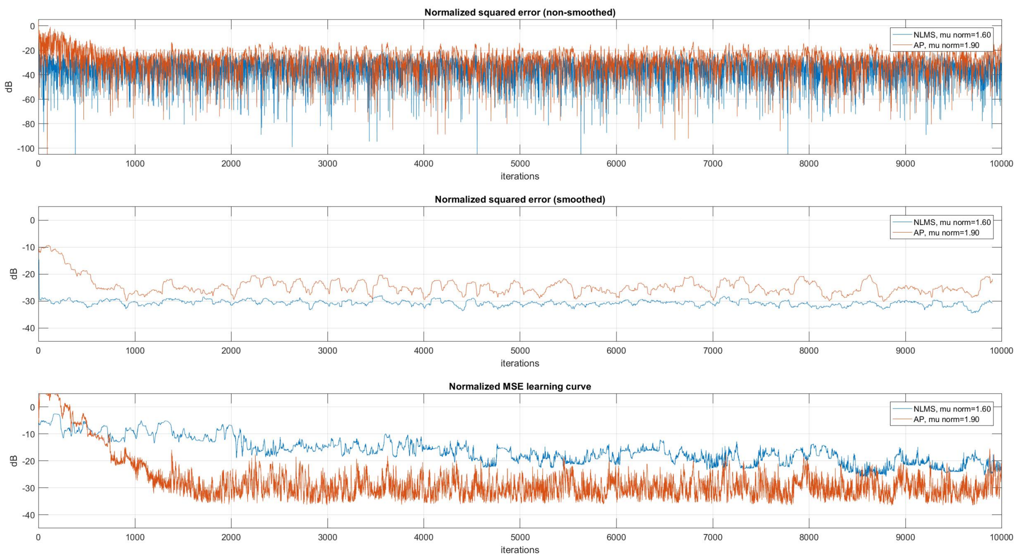
Largest *value of* μ *that leads to* ***Convergence*** *for NLMS and AP*

NLMS Algorithm

- The value that leads to ***largest convergence*** is μ = **1.6**
- The theoretical values of μ of NLMS $\subset$ [0, 2].

AP Algorithm

- The value that leads to ***largest convergence*** is μ = **1.9**
- The theoretical values of μ of AP $\subset$ [0, 2].



Question 2

A. Comparing *Smoothed normalized squared error* with *Normalized MSE learning* for NLMS

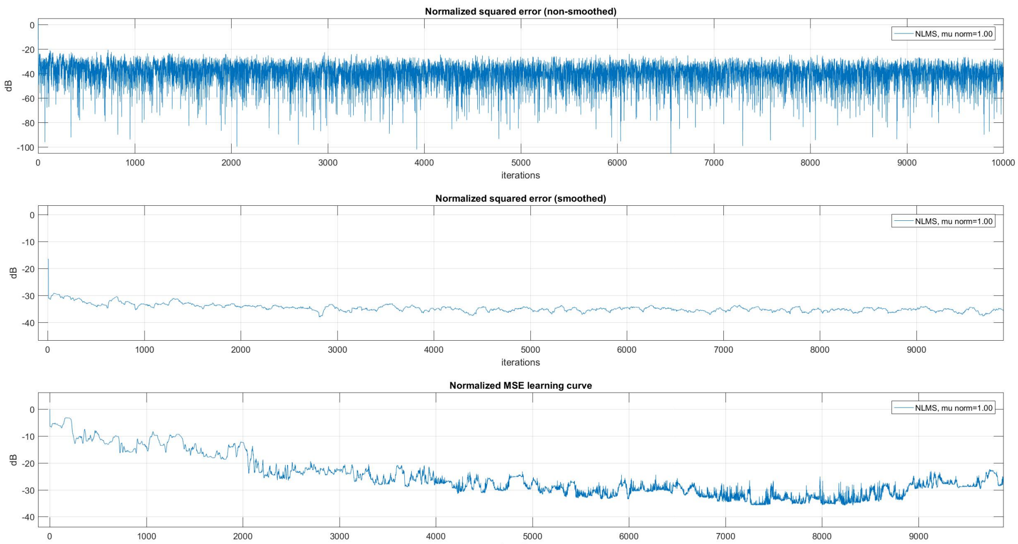
$$\frac{|e(n)|^2}{\sigma^2{}_d} = d(n) - W^H(n) \text{ x } X(n)$$

$$\frac{\zeta(w(n))}{\sigma^2{}_d} = \frac{\sigma^2{}_d - w^H(n)P - w^T(n)P^* + w^H(n)\,\mathrm{R}w(n)}{\sigma^2{}_d}$$

$$w(n+1) = w(n) + \frac{\mu}{x^H(n)\, X(n) + \psi} x(n)\, e^H(n)$$

Using $\mu = 1.0$, we notice a large discrepancy between the *smoothed normalized squared error* and *normalized MSE learning. Smoothed normalized squared error* is quite better because of the following:

- In NLMS, we approximate the MSE by the smoothed square error $\frac{|e(n)|^2}{\sigma^2{}_d}$ and optimize the algorithm with this error.
- Also, NLMS aims at addressing the situations where signal statistics are unknown by an online calculation of the convergence factor μ.
- The MSE on the other hand is dependent on P and R matrix. This is not what NLMS is designed to work with.



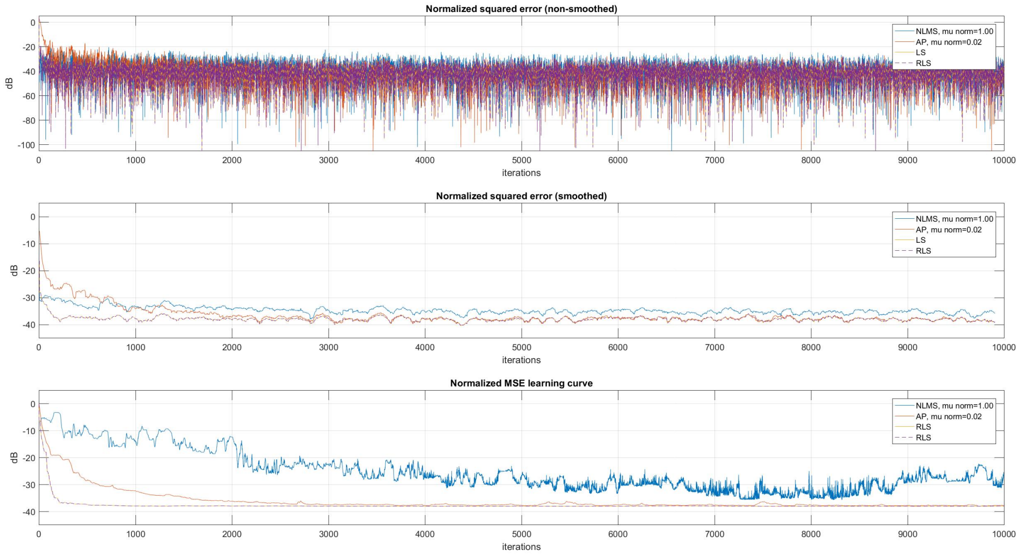
B. $\left| \frac{e(n)^2}{\sigma^2{}_d} \right|$ is more accurate. It gives the optimal solution based on the posteriori error signal. This can still indirectly reduce the MSE, but it would be less accurate measure of this minimization.

Question 3

The *value of μ that leads to* ***fastest Convergence*** *for NLMS and AP*

- **NLMS Algorithm**: the value that leads to ***fastest convergence*** is $\mu = 1.00$
- **AP Algorithm**: the value that leads to ***fastest convergence*** is $\mu = 0.02$

And the value for LS and RLS = $\lambda_{LS} = \lambda_{RLS} = 0.999$

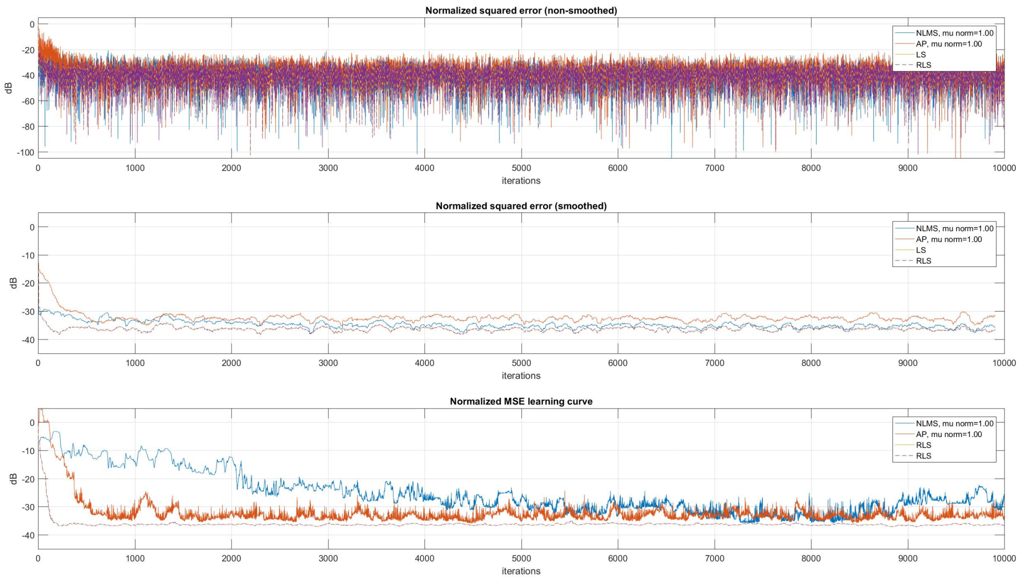


By looking at the result above we can notice the following:

- LS and RLS algorithm are identical. And they have the highest convergence speed since they are independent of the eigenvalue spread of the input correlation matrix.
- For NLMS algorithm ,using $\mu = 1$, the fastest convergence is attained at the price of a higher misadjustment.
- For AP algorithm, it’s the prominent adaptation algorithms that allow trade-off between *fast convergence* and *low computational complexity*. By adjusting the number of projections, or alternatively, the number of data reuses, one obtains adaptation processes ranging from that of the NLMS algorithm to that of the sliding-window RLS algorithm.
- Some significant improvement of convergence speed often comes with M = 5 (low values of M) and there is no need to use high M values (which would require more computations). It results in a lower misadjustemnet than NLMS and a little bit higher than RLS.

Question 4

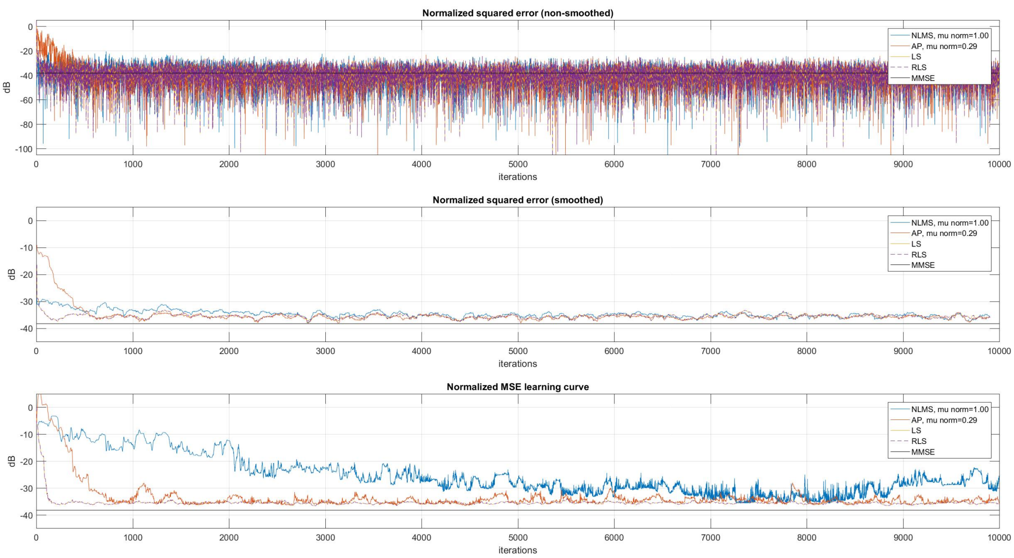
$\mu_{AP} = \mu_{NLMS} = 1.00$



Note

- Using $\mu_{AP} = 1$ (which is quite bigger than the optimal value) leads to higher misadjustment as observed by normalized squared error (middle plot) and even higher than NLMS. That makes sense, since using inappropriate value of μ leads to higher misadjustment or divergence .
- Also, we notice that using $\mu_{AP} = 1$, leads to *a* noisy signal in the third subplot(MSE).

Question 5



Note

- Using the following values of μ, we notice that all the algorithms nearly have the same misadjustment:
  - $\mu_{NLMS} = 1.00$
  - $\mu_{AP} =$ 0.290

  - $\lambda_{LS} = \lambda_{RLS} = 0.9855$
- Theoretically, $\lambda_{LS,RLS}$ can vary within the range $0 < \lambda < 1$
- $M_{LS} = M_{RLS} = 2.5044$; while the theoretical misadjustment for both RLS, LS is $\boldsymbol{M} = \frac{1-\lambda}{1+\lambda} N$ --> $M_{LS} = M_{RLS} = 0.73029$.
- By using common misadjustment, **LS** and **RLS** algorithms still have the fastest convergence. However, AP's convergence was affected and it has a lower convergence than the optimal, and even slower than NLMS.