



ACHARYA NARENDRA DEV COLLEGE

UNIVERSITY OF DELHI

B.Sc. Physics Hons.

Sem IV

Numerical Methods

Practical File

-Ayush Mishra

Submitted To :-

Mr. Brijendra Sir

Dr. Rakesh Sir

Submitted By-

Ayush Mishra

AC-815

B.Sc. Physics Hons

Index

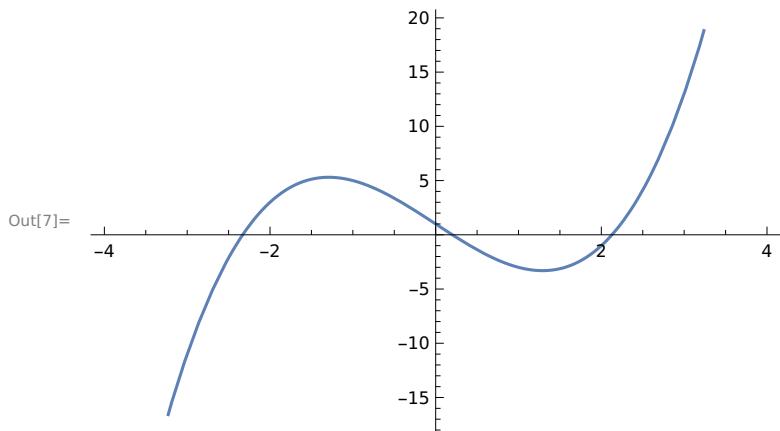
1. Bisection Method
2. Secant Method
3. Regula Falsi Method
4. NewtonRaphson Method
5. Trapezoidal Rule
6. Lagrange's Interpolation
7. Gauss Jacobi Iteration Methd
8. Newton Interpolation
9. Gauss Elimination
10. Euler's Method

Practical No.-1.

Bisection Method

Q-1- Find the roots of $f(x) = \boxed{\quad}$ by using bisection method.

```
In[5]:= bisection[f_, ao_, bo_, n_] := Module[{}, a = N[ao]; b = N[bo];
If[f[a]*f[b] > 0, Print["Bisectionmethod can be applied"];
Return[]];
p = (a + b)/2;
i = 1;
While[i ≤ n,
If[f[a]*f[p] < 0, b = p, a = p];
Print[i, " ", a, " ", b];
i++;
p = (a + b)/2];
Print["Root = ", p]]
f[x_] := x^3 - 5*x + 1
Plot[f[x], {x, -4, 4}]
```



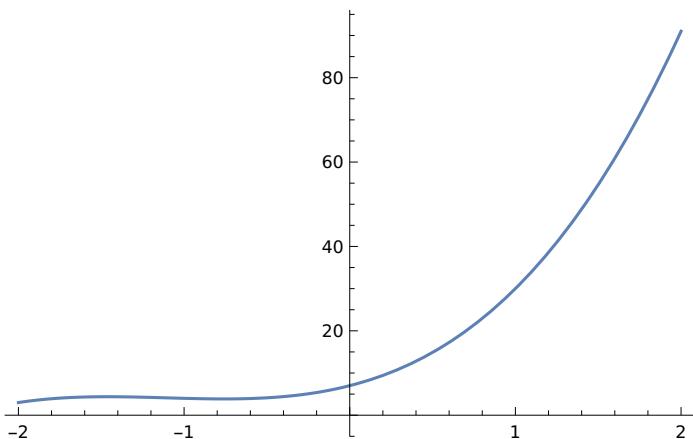
```
In[8]:= bisection[f, 2, 3, 7]
```

1 2. 2.5
2 2. 2.25
3 2.125 2.25
4 2.125 2.1875
5 2.125 2.15625
6 2.125 2.14063
7 2.125 2.13281
Root = 2.12891

Q-2: Find the roots of $f(x) = 3x^3 + 10x^2 + 10x + 7$ by using bisection method.

```
In[9]:= bisection[f_, ao_, bo_, n_] := Module[{}, a = N[ao]; b = N[bo];
If[f[a]*f[b] > 0, Print["Bisectionmethod can be applied"];
Return[]];
p = (a + b)/2;
i = 1;
While[i ≤ n,
If[f[a]*f[p] < 0, b = p, a = p];
Print[i, " ", a, " ", b];
i++;
p = (a + b)/2];
Print["Root = ", p]]
f[x_] := 3*x^3 + 10*x^2 + 10*x + 7
Plot[f[x], {x, -2, 2}]
```

Out[11]=



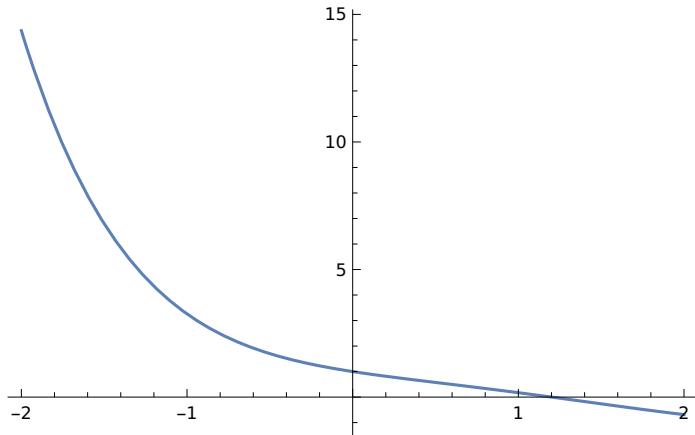
In[12]:= bisection[f, -3, -2, 10]

```
1 -2.5 -2.
2 -2.5 -2.25
3 -2.375 -2.25
4 -2.375 -2.3125
5 -2.34375 -2.3125
6 -2.34375 -2.32813
7 -2.33594 -2.32813
8 -2.33594 -2.33203
9 -2.33398 -2.33203
10 -2.33398 -2.33301
Root = -2.3335
```

Q-3: Find the roots of $f(x) = \boxed{\quad}$ by using bisection method.

```
In[104]:= bisection[f_, ao_, bo_, n_] := Module[{}, a = N[ao]; b = N[bo];
If[f[a]*f[b] > 0, Print["Bisectionmethod can be applied"];
Return[]];
p = (a + b)/2;
i = 1;
While[i ≤ n,
If[f[a]*f[p] < 0, b = p, a = p];
Print[i, " ", a, " ", b];
i++];
p = (a + b)/2];
Print["Root = ", p]]
f[x_] := Cos[x] - x * E^(-x)
Plot[f[x], {x, -2, 2}]
```

Out[106]=



In[42]:= bisection[f, 0, 2, 10]

1 1. 2.
2 1. 1.5
3 1. 1.25
4 1.125 1.25
5 1.1875 1.25
6 1.1875 1.21875
7 1.1875 1.20313
8 1.19531 1.20313
9 1.19922 1.20313
10 1.19922 1.20117
Root = 1.2002

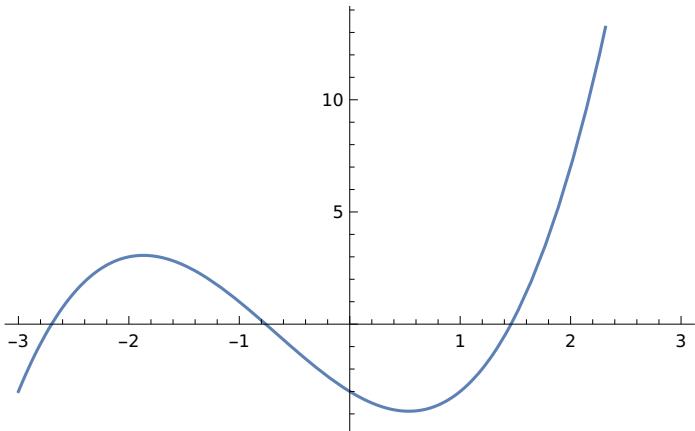
Practical No.-2.

Secant Method

Q-1: Find the root of the function $f(x) = \boxed{}$.

```
In[47]:= secant[f_, ao_, bo_, n_] := Module[], p0 = N[ao];
p1 = N[bo];
If[f[p0] * f[p1] > 0, Print["secant method can be applied "];
Return[]];
i = 1;
While[i ≤ n, p2 = N[p1 - ((p1 - p0) * f[p1] / (f[p1] - f[p0]))];
Print[i, " ", p0, " ", p1];
i++;
p0 = p1;
p1 = p2];
Print["Root =", p2]]
f[x_] := x^3 + 2x^2 - 3x - 3;
Plot[f[x], {x, -3, 3}]
secant[f, 1, 3, 5]
```

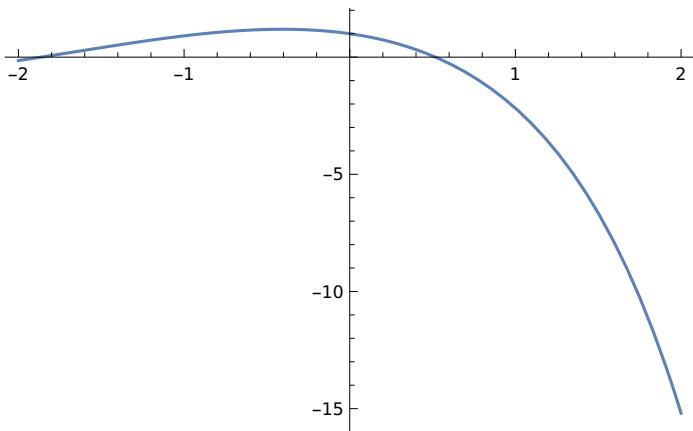
Out[49]=



1 1. 3.
 2 3. 1.16667
 3 1.16667 1.28075
 4 1.28075 1.50933
 5 1.50933 1.454
 Root =1.46029

Q-2- Find the root of the function $f(x) = \boxed{}$.

```
In[51]:= secant[f_, ao_, bo_, n_] := Module[{}, p0 = N[ao];
p1 = N[bo];
If[f[p0] * f[p1] > 0, Print["secant method can be applied "];
Return[]];
i = 1;
While[i ≤ n, p2 = N[p1 - ((p1 - p0) * f[p1] / (f[p1] - f[p0]))];
Print[i, " ", p0, " ", p1];
i++;
p0 = p1;
p1 = p2];
Print["Root =", p2]]
f[x_] := Cos[x] - x * E^x;
Plot[f[x], {x, -2, 2}]
secant[f, 0, 2, 7]
```

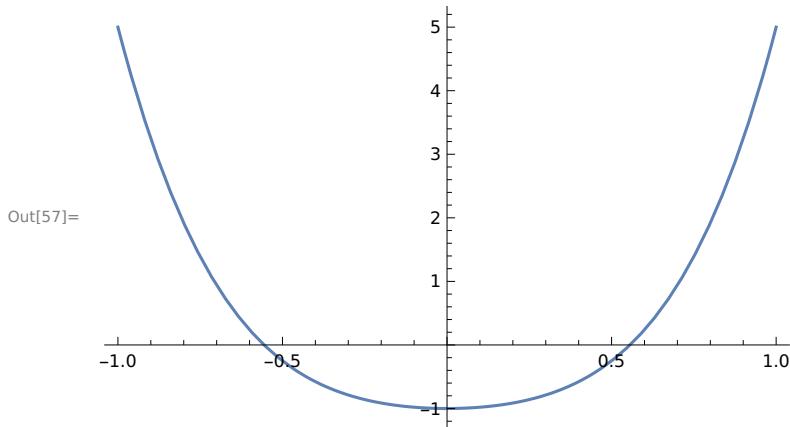


Out[53]=

```
1 0. 2.
2 2. 0.123501
3 0.123501 0.223208
4 0.223208 0.666784
5 0.666784 0.478581
6 0.478581 0.513121
7 0.513121 0.517913
Root = 0.517757
```

Q-3 Find the root of the function $f(x) =$

```
In[55]:= secant[f_, ao_, bo_, n_] := Module[], p0 = N[ao];
p1 = N[bo];
If[f[p0] * f[p1] > 0, Print["secant method can be applied "];
Return[]];
i = 1;
While[i ≤ n, p2 = N[p1 - ((p1 - p0) * f[p1] / (f[p1] - f[p0]))];
Print[i, " ", p0, " ", p1];
i++;
p0 = p1;
p1 = p2];
Print["Root = ", p2]]
f[x_] := 4 * x^4 + 2 * x^2 - 1;
Plot[f[x], {x, -1, 1}]
secant[f, 0, 1, 7]
```



```
1 0. 1.
2 1. 0.166667
3 0.166667 0.298701
4 0.298701 0.986288
5 0.986288 0.397064
6 0.397064 0.461935
7 0.461935 0.592611
Root = 0.549029
```

Practical No.-3.

Regula Falsi Method

Q-1: Solve The Equation $f(x) = \boxed{\quad}$ Using Regula Falsi Method

```
In[62]:= f[x_] = x^3 + 3x^2 - 10;
RegulaFalsi [ao_, bo_, m_] := Module[{}, a = N[ao];
b = N[bo];
c = (a f[b] - b f[a]) / (f[b] - f[a]);
k = 0;
While[k < m, If[Sign[f[b]] == Sign[f[c]], b = c, a = c;];
c = (a f[b] - b f[a]) / (f[b] - f[a]);
k = k + 1;];
Print[" c = ", NumberForm[c, 16]];
Print[" f[c] = ", NumberForm[f[c], 16]];
RegulaFalsi [1, 2, 30]

c = 1.492033301171817
f[c] = -4.440892098500626 × 10-16
```

Q-2 Solve The Equation $f(x) = \boxed{}$ Using Regula Falsi Method.

```
In[65]:= f[x_] = Cos[x] - x * E^x;
RegulaFalsi [ao_, bo_, m_] := Module[{}, a = N[ao];
b = N[bo];
c = (a f[b] - b f[a]) / (f[b] - f[a]);
k = 0;
While[k < m, If[Sign[f[b]] == Sign[f[c]], b = c, a = c;];
c = (a f[b] - b f[a]) / (f[b] - f[a]);
k = k + 1;];
Print[" c = ", NumberForm[c, 16]];
Print[" f[c] = ", NumberForm[f[c], 16]];
RegulaFalsi [0, 2, 30]

c = 0.5177446104539607
f[c] = 0.00003879648444338191
```

Q-3 Solve The Equation $f(x) = \boxed{}$ Using Regula Falsi Method.

```
In[68]:= f[x_] = 4*x^4 + 2*x^2 - 1;
RegulaFalsi [ao_, bo_, m_] := Module[{}, a = N[ao];
b = N[bo];
c = (a f[b] - b f[a]) / (f[b] - f[a]);
k = 0;
While[k < m, If[Sign[f[b]] == Sign[f[c]], b = c, a = c];
c = (a f[b] - b f[a]) / (f[b] - f[a]);
k = k + 1];
Print[" c = ", NumberForm[c, 16]];
Print[" f[c] = ", NumberForm[f[c], 16]];
RegulaFalsi [0, 1, 20]
c = 0.555887532165347
f[c] = -0.00002703820024263504
```

Practical No.-4.

Newton Raphson Method

Q-1: Find the roots of the equation of $f(x) = \boxed{}$ by using Newton Raphson method

```
In[74]:= newtonraphson [f_, p0_, eps_] := Module[{}, pold = N[p0];
i = 1; pnew = 0;
df[x_] = D[f[x], x];
While[i ≤ 50 && Abs[N[f[pold]]] > eps, pnew = N[pold - N[f[pold]] / N[df[pold]]];
Print[i, " ", pnew]; i++; pold = pnew];
Print[" Root = ", pnew]]
f[x_] := x^3 - x - 4;
newtonraphson [f, 0.5, .0000002]
```

```

1 -17.
2 -11.3418
3 -7.57045
4 -5.05309
5 -3.36038
6 -2.18673
7 -1.26733
8 -0.0185779
9 -4.00413
10 -2.64117
11 -1.64841
12 -0.6933
13 7.54196
14 5.08119
15 3.48408
16 2.50125
17 1.98646
18 1.81557
19 1.79655
20 1.79632
21 1.79632
Root = 1.79632

```

Q-2 Find the roots of the equation of $f(x) = \boxed{}$ by using Newton Raphson method.

```

In[86]:= newtonraphson[f_, p0_, eps_] := Module[{}, pold = N[p0];
i = 1; pnew = 0;
df[x_] = D[f[x], x];
While[i ≤ 50 && Abs[N[f[pold]]] > eps, pnew = N[pold - N[f[pold]] / N[df[pold]]];
Print[i, " ", pnew]; i++; pold = pnew];
Print[" Root = ", pnew]]
f[x_] := 3*x^3 + 10*x^2 + 10*x + 7;
newtonraphson[f, -2, .00000002]

```

```

1 -2.5
2 -2.35385
3 -2.3337
4 -2.33333
5 -2.33333
Root = -2.33333

```

Q-3 Find the roots of the equation of $f(x) = \boxed{}$ by using Newton Raphson method.

```

In[89]:= newtonraphson[f_, p0_, eps_] := Module[{}, pold = N[p0];
i = 1; pnew = 0;
df[x_] = D[f[x], x];
While[i <= 50 && Abs[N[f[pold]]] > eps, pnew = N[pold - N[f[pold]] / N[df[pold]]];
Print[i, " ", pnew]; i++; pold = pnew];
Print[" Root = ", pnew]
f[x_] := x^4 - x - 10;
newtonraphson[f, 0.5, .0000002]

1 -20.375
2 -15.2811
3 -11.4607
4 -8.59577
5 -6.44823
6 -4.84098
7 -3.64474
8 -2.77088
9 -2.17017
10 -1.82753
11 -1.71019
12 -1.69761
13 -1.69747
14 -1.69747
Root = -1.69747

```

Practical No.- 5.

Trapezoidal rule

Q-1: To find the approx value of integral $\int x^2 dx$ from $x=2$ to $x=10$, by using trapezoidal rule .

```
In[90]:= ClearAll[n, x, f]
a = 2
b = 10
n = 15
sum = 0
h = (b - a) / n
f[x_] = x^2
For[i = 1, i ≤ n - 1, i++, sum += N[f[x] /. x → (a + i * h)]]
sum = N[(2 * sum + f[x] /. x → b) * h / 2]

Out[91]= 2
Out[92]= 10
Out[93]= 15
Out[94]= 0
Out[95]= 8
Out[96]= x^2
Out[98]= 329.979
```

Q-2: To find the approx value of integral $\int x^3 + 8 dx$ from x= 2 to x= 10, by using trapezoidal rule .

```
In[99]:= ClearAll[n, x, f]
a = 2
b = 10
n = 15
sum = 0
h = (b - a) / n
f[x_] = x^2 + 3 x
For[i = 1, i ≤ n - 1, i++, sum += N[f[x] /. x → (a + i * h)]]
sum = N[(2 * sum + f[x] /. x → b) * h / 2]

Out[100]=
2

Out[101]=
10

Out[102]=
15

Out[103]=
0

Out[104]=
8
—
15

Out[105]=
3 x + x^2

Out[107]=
472.379
```

Q-3: To find the approx value of integral $\int x^3 + 8 dx$ from $x= 1$ to $x = 4$, by using trapezoidal rule.

```
In[108]:= ClearAll[n, x, f]
a = 1
b = 4
n = 15
sum = 0
h = (b - a) / n
f[x_] = x^3 + 8
For[i = 1, i ≤ n - 1, i++, sum += N[f[x] /. x → (a + i * h)]]
sum = N[(2 * sum + f[x] /. x → b) * h / 2]

Out[109]=
1

Out[110]=
4

Out[111]=
15

Out[112]=
0

Out[113]=
1
-
5

Out[114]=
8 + x^3

Out[116]=
87.
```

PRACTICAL-6

LAGRANGE 'S INTERPOLATION METHOD

Q1. The function $y = f(x)$ is given at the points $(-1,-2),(1,0),(4,63),(7,342)$.

Find $f(5.0)$ using Lagrange interpolation .

```
In[1]:= No = 4; sum = 0;
lagrange[No_, n_] :=
  Product[If[Equal[k, n], 1, (x - x[k]) / (x[n] - x[k])], {k, 1, No}];
For[i = 1, i ≤ No, i++, sum += (f[x[i]] * lagrange[No, i])];
Print[sum]
sum = 0;
points = {{-1, -2}, {1, 0}, {4, 63}, {7, 342}};
No = Length[points]
y = points[[All, 1]]
f = points[[All, 2]]
lagrange[No_, n_] :=
  Product[If[Equal[k, n], 1, (x - y[[k]]) / (y[[n]] - y[[k]])], {k, 1, No}]
For[i = 1, i ≤ No, i++, sum += (f[[i]] * lagrange[No, i])]
Expand[sum]
sum /. x → 5.0

```

$$\frac{f[x[1]] (x - x[2]) (x - x[3]) (x - x[4])}{(x[1] - x[2]) (x[1] - x[3]) (x[1] - x[4])} + \frac{f[x[2]] (x - x[1]) (x - x[3]) (x - x[4])}{(-x[1] + x[2]) (x[2] - x[3]) (x[2] - x[4])} +$$
$$\frac{f[x[3]] (x - x[1]) (x - x[2]) (x - x[4])}{(-x[1] + x[3]) (-x[2] + x[3]) (x[3] - x[4])} + \frac{f[x[4]] (x - x[1]) (x - x[2]) (x - x[3])}{(-x[1] + x[4]) (-x[2] + x[4]) (-x[3] + x[4])}$$

```
Out[7]= 4
Out[8]= {-1, 1, 4, 7}
Out[9]= {-2, 0, 63, 342}
Out[12]= -1 + x3
Out[13]= 124.
```

Q2. The function $y = f(x)$ is given at the points $(2,15),(4,5),(5,6),(6,19)$.

Find $f(5.5)$ using Lagrange interpolation .

```

In[40]:= No = 4; sum = 0;
lagrange[No_, n_] :=
  Product[If[Equal[k, n], 1, (x - x[k]) / (x[n] - x[k])], {k, 1, No}];
For[i = 1, i ≤ No, i++, sum += (f[x[i]] * lagrange[No, i])];
Print[sum]
sum = 0;
points = {{2, 15}, {4, 5}, {5, 6}, {6, 19}};
No = Length[points]
y = points[[All, 1]]
f = points[[All, 2]]
lagrange[No_, n_] :=
  Product[If[Equal[k, n], 1, (x - y[[k]]) / (y[[n]] - y[[k]])], {k, 1, No}]
For[i = 1, i ≤ No, i++, sum += (f[[i]] * lagrange[No, i])]
Expand[sum]
sum /. x → 5.5

(x - x[2]) (x - x[3]) (x - x[4]) {210, 5, 6, 19}[x[1]] + (x - x[1]) (x - x[3]) (x - x[4]) {210, 5, 6, 19}[x[2]] +
  (x[1] - x[2]) (x[1] - x[3]) (x[1] - x[4]) (-x[1] + x[2]) (x[2] - x[3]) (x[2] - x[4]) +
  (x - x[1]) (x - x[2]) (x - x[4]) {210, 5, 6, 19}[x[3]] + (x - x[1]) (x - x[2]) (x - x[3]) {210, 5, 6, 19}[x[4]] +
  (-x[1] + x[3]) (-x[2] + x[3]) (x[3] - x[4]) (-x[1] + x[4]) (-x[2] + x[4]) (-x[3] + x[4])

Out[46]= 4
Out[47]= {2, 4, 5, 6}
Out[48]= {15, 5, 6, 19}
Out[51]= 1 + 21 x - 9 x2 + x3
Out[52]= 10.625

```

Q3. The function $y = f(x)$ is given at the points $(0,1), (1,14), (2,15), (4,5)$.

Find $f(8.0)$ using Lagrange interpolation .

```
In[53]:= No = 4; sum = 0;
lagrange[No_, n_] :=
  Product[If[Equal[k, n], 1, (x - x[k]) / (x[n] - x[k])], {k, 1, No}];
For[i = 1, i ≤ No, i++, sum += (f[x[i]] * lagrange[No, i])];
Print[sum]
sum = 0;
points = {{0, 1}, {1, 14}, {2, 15}, {4, 5}};
No = Length[points]
y = points[[All, 1]]
f = points[[All, 2]]
lagrange[No_, n_] :=
  Product[If[Equal[k, n], 1, (x - y[[k]]) / (y[[n]] - y[[k]])], {k, 1, No}]
For[i = 1, i ≤ No, i++, sum += (f[[i]] * lagrange[No, i])]
Expand[sum]
sum /. x → 8.0

$$\frac{(x - x[2])(x - x[3])(x - x[4])\{15, 5, 6, 19\}x[1]}{(x[1] - x[2])(x[1] - x[3])(x[1] - x[4])} + \frac{(x - x[1])(x - x[3])(x - x[4])\{15, 5, 6, 19\}x[2]}{(-x[1] + x[2])(x[2] - x[3])(x[2] - x[4])} +$$


$$\frac{(x - x[1])(x - x[2])(x - x[4])\{15, 5, 6, 19\}x[3]}{(-x[1] + x[3])(-x[2] + x[3])(x[3] - x[4])} + \frac{(x - x[1])(x - x[2])(x - x[3])\{15, 5, 6, 19\}x[4]}{(-x[1] + x[4])(-x[2] + x[4])(-x[3] + x[4])}$$

Out[59]= 4
Out[60]= {0, 1, 2, 4}
Out[61]= {1, 14, 15, 5}
Out[64]= 1 + 21 x - 9 x2 + x3
Out[65]= 105.
```

Practical-07

Gauss Jacobi Iteration Method

Q1 = Solve the following equation using jacobi method

$$10x + y + z = 12$$

$$x + 10y + z = 12$$

$$x + y + 10z = 12$$

Taking initial approx . = [0, 0, 0]

```
In[ ]:= Jacobi[A0_, B0_, P0_, max_] := Module[{A = N[A0], B = N[B0],
  i, j, k = 0, n = Length[P0], P = P0, Pold = P0}, Print["P ", " = ", P];
  While[k < max,
    For[i = 1, i ≤ n, i++,
      P[[i]] =  $\frac{1}{A[[i, i]]} \left( B[[i]] + A[[i, i]] * Pold[[i]] - \sum_{j=1}^n A[[i, j]] * Pold[[j]] \right)$ ;
    Print["P ", k+1, " = ", P];
    Pold = P;
    k = k + 1];
  Return[P];
A =  $\begin{pmatrix} 10 & 1 & 1 \\ 1 & 10 & 1 \\ 1 & 1 & 10 \end{pmatrix}$ ;
B = {12, 12, 12};
vars = {"x1", "x2", "x3", "x4"};
Print["Solve the system"];
Print[MatrixForm[A], MatrixForm[vars], " = ", MatrixForm[B]]
P = {0, 0, 0};
X = Jacobi[A, B, P, 8];
```

Solve the system

$$\left(\begin{array}{ccc|c} 10 & 1 & 1 & x_1 \\ 1 & 10 & 1 & x_2 \\ 1 & 1 & 10 & x_3 \\ \hline & & & x_4 \end{array} \right) = \left(\begin{array}{c} 12 \\ 12 \\ 12 \\ 12 \end{array} \right)$$

$$P_0 = \{0, 0, 0\}$$

$$P_1 = \{1.2, 1.2, 1.2\}$$

$$P_2 = \{0.96, 0.96, 0.96\}$$

$$P_3 = \{1.008, 1.008, 1.008\}$$

$$P_4 = \{0.9984, 0.9984, 0.9984\}$$

$$P_5 = \{1.00032, 1.00032, 1.00032\}$$

$$P_6 = \{0.999936, 0.999936, 0.999936\}$$

$$P_7 = \{1.00001, 1.00001, 1.00001\}$$

$$P_8 = \{0.999997, 0.999997, 0.999997\}$$

Q2. Solve the following equation using jacobi method

$$4x + 3y + 2z = 16$$

$$2x + 3y + 4z = 20$$

$$x + 2y + z = 8$$

Taking initial approx. = [0, 0, 0]

```

In[=]:= Jacobi[A0_, B0_, P0_, max_] := Module[{A = N[A0], B = N[B0],
  i, j, k = 0, n = Length[P0], P = P0, Pold = P0}, Print["P "0, " = ", P];
  While[k < max,
    For[i = 1, i ≤ n, i++,
      P[[i]] =  $\frac{1}{A[[i, i]]} \left( B[[i]] + A[[i, i]] \times Pold[[i]] - \sum_{j=1}^n A[[i, j]] \times Pold[[j]] \right);$ 
      Print["P "k+1, " = ", P];
      Pold = P;
      k = k + 1];
    Return[P];
  ];
  A =  $\begin{pmatrix} 4 & 3 & 2 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{pmatrix};$ 
  B = {16, 20, 8};
  vars = {"x1", "x2", "x3"};
  Print["Solve the system"];
  Print[MatrixForm[A], MatrixForm[vars], " = ", MatrixForm[B]];
  P = {0, 0, 0};
  X = Jacobi[A, B, P, 10];
  Solve the system
   $\begin{pmatrix} 4 & 3 & 2 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} 16 \\ 20 \\ 8 \end{pmatrix}$ 
  P 0 = {0, 0, 0}
  P 1 = {4., 6.66667, 8.}
  P 2 = {-5., -6.66667, -9.33333}
  P 3 = {13.6667, 22.4444, 26.3333}
  P 4 = {-26., -37.5556, -50.5556}
  P 5 = {57.4444, 91.4074, 109.111}
  P 6 = {-119.111, -177.111, -232.259}
  P 7 = {252.963, 395.753, 481.333}
  P 8 = {-533.481, -803.753, -1036.47}
  P 9 = {1125.05, 1744.28, 2148.99}
  P 10 = {-2378.7, -3608.68, -4605.61}

```

Q3.Solve the following equation using jacobi method

$$28x + 4y - z = 32$$

$$2x + 17y + 4z = 35$$

$$x + 3y + 10z = 24$$

Taking initial approx . = [0, 0, 0]

```
In[ ]:= Jacobi[A0_, B0_, P0_, max_] := Module[{A = N[A0], B = N[B0],
  i, j, k = 0, n = Length[P0], P = P0, Pold = P0}, Print[" P ", " = ", P];
  While[k < max,
    For[i = 1, i ≤ n, i++,
      P[[i]] =  $\frac{1}{A[[i, i]]} \left( B[[i]] + A[[i, i]] * Pold[[i]] - \sum_{j=1}^n A[[i, j]] * Pold[[j]] \right)$ ;
      Print["P ", k+1, " = ", P];
      Pold = P;
      k = k + 1];
    Return[P];
  ];
  A =  $\begin{pmatrix} 28 & 4 & 1 \\ 2 & 17 & 4 \\ 1 & 3 & 10 \end{pmatrix}$ ;
  B = {32, 35, 24};
  vars = {"x1", "x2", "x3"};
  Print["Solve the system"];
  Print[MatrixForm[A], MatrixForm[vars], " = ", MatrixForm[B]];
  P = {0, 0, 0};
  X = Jacobi[A, B, P, 10];
  Solve the system
  
$$\begin{pmatrix} 28 & 4 & 1 \\ 2 & 17 & 4 \\ 1 & 3 & 10 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} 32 \\ 35 \\ 24 \end{pmatrix}$$

  P 0 = {0, 0, 0}
  P 1 = {1.14286, 2.05882, 2.4}
  P 2 = {0.763025, 1.35966, 1.66807}
  P 3 = {0.889046, 1.57657, 1.9158}
  P 4 = {0.849212, 1.50345, 1.83812}
  P 5 = {0.862431, 1.52642, 1.86404}
  P 6 = {0.858225, 1.51876, 1.85583}
  P 7 = {0.859611, 1.52119, 1.85855}
  P 8 = {0.859168, 1.52039, 1.85768}
  P 9 = {0.859313, 1.52064, 1.85797}
  P 10 = {0.859266, 1.52056, 1.85788}
```

PRACTICAL NO.:8

NEWTON'S INTERPOLATION METHOD

Q1. The function $y = f(x)$ is given at the points $(-1, -2), (1, 0), (4, 63), (7, 342)$.

Find $f(5.0)$ using Newton interpolation .

In[232]:=

```
In[10] := sum = 0;
points = {{-1, -2}, {1, 0}, {4, 63}, {7, 342}};
n = Length[points]
y = points[[All, 1]]
f = points[[All, 2]]
dd[k_] :=
  Sum[(f[[i]] / Product [If[Equal[j, i], 1, (y[[i]] - y[[j]])], {j, 1, k}]), {i, 1, k}]
p[x_] = Sum[(dd[i] * Product [If[i <= j, 1, x - y[[j]]], {j, 1, i - 1}]),
{i, 1, n}]
Simplify [p[x]]
Evaluate [p[5.0]]
```

Out[234]= 4

Out[235]= {-1, 1, 4, 7}

Out[236]= {-2, 0, 63, 342}

Out[238]= -1 + x + 4 (-1 + x) (1 + x) + (-4 + x) (-1 + x) (1 + x)

Out[239]= -1 + x³

Out[240]= 124.

Q2. The function $y = f(x)$ is given at the points $(2, 15), (4, 5), (5, 6), (6, 19)$.

Find $f(5.5)$ using Newton interpolation .

```
In[241]:= In[10] := sum = 0;
points = {{2, 15}, {4, 5}, {5, 6}, {6, 19}};
n = Length[points]
y = points[[All, 1]]
f = points[[All, 2]]
dd[k_] :=
Sum[(f[[i]] / Product [If[Equal[j, i], 1, (y[[i]] - y[[j]])], {j, 1, k}]), {i, 1, k}]
p[x_] = Sum[(dd[i] * Product [If[i <= j, 1, x - y[[j]]], {j, 1, i - 1}]),
{i, 1, n}]
Simplify [p[x]]
Evaluate [p[5.5]]

Out[243]= 4

Out[244]= {2, 4, 5, 6}

Out[245]= {15, 5, 6, 19}

Out[247]= 15 - 5 (-2 + x) + 2 (-4 + x) (-2 + x) + (-5 + x) (-4 + x) (-2 + x)

Out[248]= 1 + 21 x - 9 x2 + x3

Out[249]= 10.625
```

Q3. The function $y = f(x)$ is given at the points (0,1),(1,14),(2,15),(4,5).

Find $f(8.0)$ using Newton interpolation .

```
In[250]:= In[10] := sum = 0;
points = {{0, 1}, {1, 14}, {2, 15}, {4, 5}};
n = Length[points]
y = points[[All, 1]]
f = points[[All, 2]]
dd[k_] :=
Sum[(f[[i]] / Product [If[Equal[j, i], 1, (y[[i]] - y[[j]])], {j, 1, k}]), {i, 1, k}]
p[x_] = Sum[(dd[i] * Product [If[i <= j, 1, x - y[[j]]], {j, 1, i - 1}]),
{i, 1, n}]
Simplify [p[x]]
Evaluate [p[8.0]]

Out[252]= 4

Out[253]= {0, 1, 2, 4}

Out[254]= {1, 14, 15, 5}

Out[256]= 1 + 13 x - 6 (-1 + x) x + (-2 + x) (-1 + x) x

Out[257]= 1 + 21 x - 9 x2 + x3

Out[258]= 105.
```

Practical N0.-:9

Gauss Elimination Method

Q1.Solve the equation

$$x_1+x_2+x_3=1$$

$$4*x_1+3*x_2-x_3=6$$

$$3*x_1-5*x_2-3*x_3=4$$

```
1 1 1      x1      1  
In[◎]:= m = 4 3 -1; x = x2; b = 6; m.x == b  
3 -5 -3      x3      4
```

```
ArrayFlatten[{{m, b}}] // MatrixForm
```

```
RowReduce[%] // MatrixForm
```

```
LinearSolve[m, b]
```

```
Out[◎]= {{x1 + x2 + x3}, {4 x1 + 3 x2 - x3}, {3 x1 - 5 x2 - 3 x3}} == {{1}, {6}, {4}}
```

```
Out[◎]//MatrixForm=
```

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 4 & 3 & -1 & 6 \\ 3 & -5 & -3 & 4 \end{pmatrix}$$

```
Out[◎]//MatrixForm=
```

$$\begin{pmatrix} 1 & 0 & 0 & \frac{21}{17} \\ 0 & 1 & 0 & \frac{7}{34} \\ 0 & 0 & 1 & -\frac{15}{34} \end{pmatrix}$$

```
Out[◎]= \left\{ \left\{ \frac{21}{17} \right\}, \left\{ \frac{7}{34} \right\}, \left\{ -\frac{15}{34} \right\} \right\}
```

Q2.Solve the equation

$$2x+3y+z=1$$

$$2x+y+z=8$$

$$4x+4y+5z=6$$

```

2 3 1      x      1
In[•]:= m = 2 1 1; x = y; b = 8;
        4 4 5      z      6

ArrayFlatten[{{m, b}}] // MatrixForm
RowReduce[%] // MatrixForm
LinearSolve[m, b]

```

Out[•] //MatrixForm=

$$\begin{pmatrix} 2 & 3 & 1 & 1 \\ 2 & 1 & 1 & 8 \\ 4 & 4 & 5 & 6 \end{pmatrix}$$

Out[•] //MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & \frac{25}{4} \\ 0 & 1 & 0 & -\frac{7}{2} \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

Out[•]= $\left\{ \left\{ \frac{25}{4} \right\}, \left\{ -\frac{7}{2} \right\}, \{-1\} \right\}$

Q3. Solve the equation

$8x+3y+z=5$

$2x+8y-z=13$

$x+7y-2z=12$

```

8 3 1      x      5
In[•]:= m = 2 8 -1; x = y; b = 13;
        1 7 -2      z      12

ArrayFlatten[{{m, b}}] // MatrixForm
RowReduce[%] // MatrixForm
LinearSolve[m, b]

```

Out[•] //MatrixForm=

$$\begin{pmatrix} 8 & 3 & 1 & 5 \\ 2 & 8 & -1 & 13 \\ 1 & 7 & -2 & 12 \end{pmatrix}$$

Out[•] //MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & \frac{8}{57} \\ 0 & 1 & 0 & \frac{86}{57} \\ 0 & 0 & 1 & -\frac{37}{57} \end{pmatrix}$$

Out[•]= $\left\{ \left\{ \frac{8}{57} \right\}, \left\{ \frac{86}{57} \right\}, \left\{ -\frac{37}{57} \right\} \right\}$

Practical N0.-: 9

(Gauss Elimination Method)

Q1.Solve the equation

$$x_1+x_2+x_3=1$$

$$x_1+x_2-x_3=3$$

$$x_1-x_2-x_3=-1$$

$$\begin{array}{cccc} 1 & 1 & 1 & x_1 & 1 \\ \text{In[} & \text{]:= m = } & 1 & 1 & -1; x = x_2; b = 3; m.x == b \\ & & 1 & -1 & -1 & x_3 & -1 \end{array}$$

$$\text{Out[}]=\{\{x_1+x_2+x_3\}, \{x_1+x_2-x_3\}, \{x_1-x_2-x_3\}\} == \{1\}, \{3\}, \{-1\}$$

```
In[ ]:= ArrayFlatten[{{m, b}}] // MatrixForm
RowReduce[%] // MatrixForm
LinearSolve[m, b]
```

Out[]//MatrixForm=

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & 3 \\ 1 & -1 & -1 & -1 \end{pmatrix}$$

Out[]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

$$\text{Out[}]=\{0\}, \{2\}, \{-1\}$$

Q2.Solve the equation

$$3x+2y+4z=7$$

$$2x+y+z=7$$

$$x+3y+5z=2$$

```

3 2 4      x      7
In[◎]:= m = { { 2 1 1 }, { x = y, b = 7 } ;
1 3 5      z      2

ArrayFlatten[{{m, b}}] // MatrixForm
RowReduce[%] // MatrixForm
LinearSolve[m, b]

Out[◎]//MatrixForm=

$$\begin{pmatrix} 3 & 2 & 4 & 7 \\ 2 & 1 & 1 & 7 \\ 1 & 3 & 5 & 2 \end{pmatrix}$$


Out[◎]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -2 \end{pmatrix}$$


Out[◎]= {{3}, {3}, {-2}}

```

Q3.Solve the equation

$$10x+y+z=12$$

$$2x+10y+z=13$$

$$x+y+3z=5$$

```

10 1 1      x      12
In[1]:= m = { { 2 10 1 }, { x = y, b = 13 } ;
1 1 3      z      5

ArrayFlatten[{{m, b}}] // MatrixForm
RowReduce[%] // MatrixForm
LinearSolve[m, b]

```

```

Out[2]//MatrixForm=

$$\begin{pmatrix} 10 & 1 & 1 & 12 \\ 2 & 10 & 1 & 13 \\ 1 & 1 & 3 & 5 \end{pmatrix}$$


```

```

Out[3]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$


```

Out[4]= {{1}, {1}, {1}}