

Central tendency:

* Mean = $\frac{\sum_{i=1}^n x_i}{n}$ * Ungrouped Data

Median \rightarrow $n \rightarrow \text{odd} = x_{(\frac{n+1}{2})}$
 $\rightarrow n \rightarrow \text{even} = \frac{x_{(\frac{n}{2})} + x_{(\frac{n}{2}+1)}}{2}$

Mode \rightarrow Value with most frequency

* Mean * Grouped Data

① MidPoint = $\frac{\text{Start} + \text{end}}{2}$

② Multiply MidPoint by the frequency (P.X)

③ Mean = $\frac{\sum_{i=1}^n (P.X)}{n}$

Note: Mean > Median > Mode \rightarrow Right skewed

Mean < Median < Mode \rightarrow Left skewed

Mean = Median = Mode \rightarrow Symmetric

Range (R) = largest value - smallest value

Interquartile Range (IQR)

① Find Q_2 = mean of data

② Find Q_1 = mean of data before Q_2

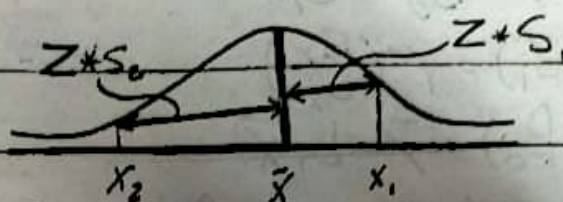
③ Find Q_3 = mean of data after Q_2

$$\text{Variance } (S^2) = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}$$

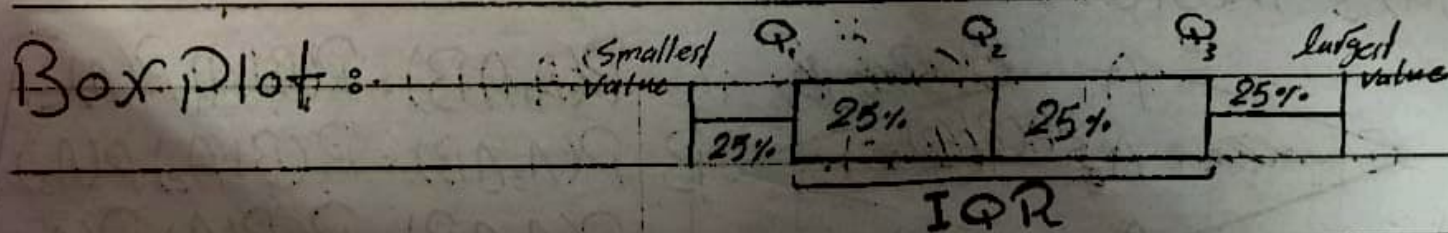
$$\text{Standard deviation } (S) = \sqrt{S^2} = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}}$$

$$\text{Z-Score } (z) = \frac{x - \bar{x}}{S}$$

$$x = \bar{x} + z * S$$



- Positive → score above the mean
- Zero → score is mean score
- Negative → score below the mean



Correlation (r):

$$r = \frac{\sum_{i=1}^n z_{x_i} * z_{y_i}}{n-1}$$

Sign

+ve

-ve

Values increase
or decrease
Together

When value
decrease as
other increase.

Value

- Weak → 0.1, 0.2, 0.3
- Moderate → 0.4, 0.5, 0.6
- Strong → 0.7, 0.8, 0.9
- Perfect → 1

* Note: 0.6898 ≈ 0.7
0.96978 ≈ 1

Simple Linear Regression:

$$y = \beta_0 + \beta_1 x + e$$

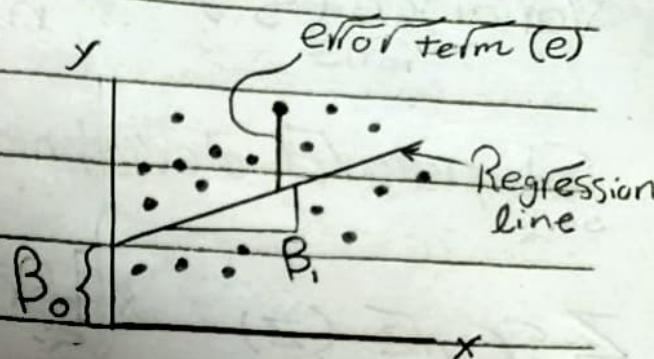
$$E(x) = \hat{y} = b_0 + b_1 x$$

$$b_1 = E(\beta_1) = r \left(\frac{s_y}{s_x} \right)$$

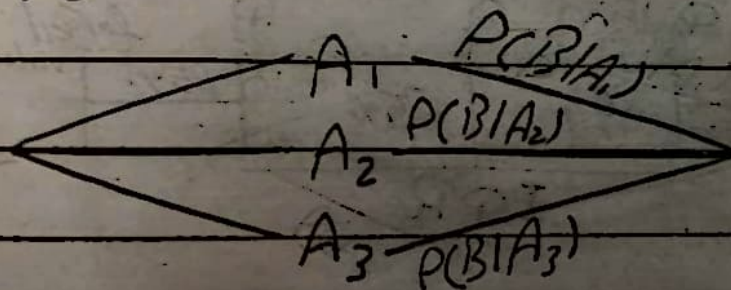
$$b_0 = E(\beta_0) = \bar{y} - b_1 \bar{x}$$

$$\text{Error } (e) = |y - \hat{y}|$$

* Coefficient of determination (r^2) = $\left(\frac{\sum_{i=1}^n Z_{x_i} * Z_{y_i}}{n-1} \right)^2$



Bay's Theorem:-



$$\begin{aligned} P(A_1 \cap B) &= P(B|A_1) P(A_1) \\ P(A_2 \cap B) &= P(B|A_2) P(A_2) \\ P(A_3 \cap B) &= P(B|A_3) P(A_3) \\ P(B) &= \text{Sum} \end{aligned}$$

* Note $P(A|B) = \frac{P(A \cap B)}{P(B)}$

* Random variable:

— A Set of Possible values from Random experiment.

— (a) Discrete: not Interval.

— (b) Continuous: Interval.

* Probability distribution $[P(x)]$:

(P.M.F.)

Probability Mass function

Gives The Probability that a discrete random variable

$P(x)$ is P.M.F. if:

$$(1) 0 \leq P(x) \leq 1$$

$$(2) \sum_{x=1}^n P(x) = 1$$

(P.D.F.)

Probability density function

Gives The Probability that a continuous random variable

$P(x)$ is P.D.F. if:

$$(1) 0 \leq \int_a^b P(x) dx \leq 1$$

$$(2) \int_R P(x) dx = 1$$

Binomial distribution:

— To study number of successes in a sequence of n independent experiments.

n → total number of trials.

P → Probability of success.

q → Probability of failure.

$$P(x) = P(x=r) = \binom{n}{r} P^r q^{n-r}$$

Normal distribution:

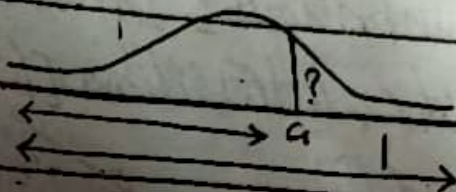
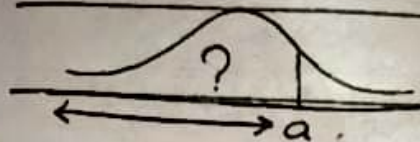
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$$E(X) = \mu, \text{Var}(X) = \sigma^2$$

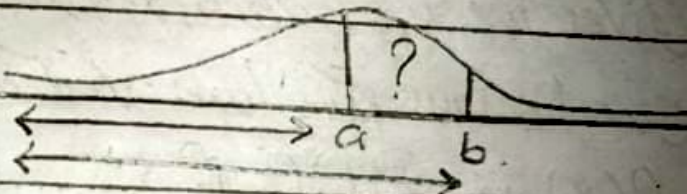
$$Z = \frac{X - \mu}{\sigma}$$

$$* P(Z < a) = \text{Z-Table}$$

$$* P(Z > a) = 1 - P(Z < a)$$

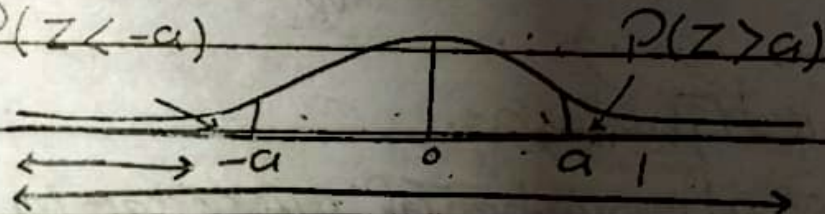


$$* P(a < Z < b) = P(Z < b) - P(Z < a)$$



$$* P(Z < -a) = P(Z > a) = 1 - P(Z < a)$$
$$= P(Z < -a) + P(Z < a) = 1$$

$$P(Z > a) = P(Z < -a)$$



$$* \text{Note: } P(Z < a) = 1 \text{ if } a > 3.49$$

$$P(Z < a) = 0 \text{ if } a < -3.4$$

Central Limit Theorem:

$$\text{Mean } (\mu) = \frac{\sum_{i=1}^n X_i}{n}$$

$$Z = \frac{X - \mu}{\sigma}$$

$$\text{Variance } (\sigma^2) = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1}$$

Subject: _____

Date: _____

Sampling distribution of sample mean:

$$\mu_{\bar{x}} = E(\bar{x})$$

$$= E\left(\frac{\sum_{i=1}^n x_i}{n}\right)$$

$$= \frac{1}{n} E(\sum_{i=1}^n x_i)$$

$$= \frac{1}{n} \sum_{i=1}^n E(x_i)$$

$$= \frac{1}{n} \sum_{i=1}^n \mu$$

$$= \frac{1}{n} \times n \mu \quad \boxed{\mu_{\bar{x}} = \mu}$$

$$\sigma_{\bar{x}}^2 = \text{Var}(\bar{x})$$

$$= \text{Var}\left(\frac{\sum_{i=1}^n x_i}{n}\right)$$

$$= \frac{1}{n^2} \text{Var}(\sum_{i=1}^n x_i)$$

$$= \frac{1}{n^2} \sum_{i=1}^n \text{Var}(x_i)$$

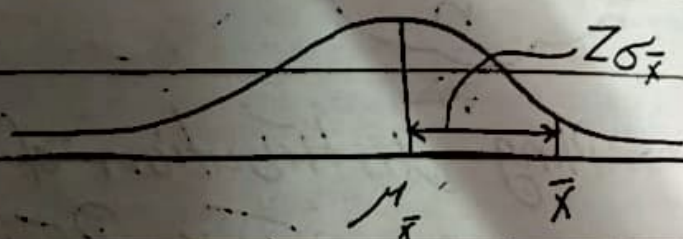
$$= \frac{1}{n^2} \sum_{i=1}^n \sigma^2$$

$$= \frac{1}{n^2} \times n \sigma^2$$

$$\boxed{\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}}$$

$$\boxed{\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}}$$

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$



Sampling distribution of difference between two means:

Two means:

$$\mu_{\bar{x}_1 - \bar{x}_2} = E(\bar{x}_1 - \bar{x}_2)$$

$$= E(\bar{x}_1) - E(\bar{x}_2)$$

$$= \mu_{\bar{x}_1} - \mu_{\bar{x}_2} = \mu_1 - \mu_2$$

$$\sigma_{\bar{x}_1 - \bar{x}_2}^2 = \text{Var}(\bar{x}_1 - \bar{x}_2)$$

$$= \text{Var}(\bar{x}_1) + \text{Var}(\bar{x}_2)$$

$$\sigma_{\bar{x}_1}^2 + \sigma_{\bar{x}_2}^2 = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$$

$$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$Z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

Sampling distribution of sample Proportion:

$$\mu_{\hat{P}} = E(\hat{P})$$

$$= E\left(\frac{X}{n}\right)$$

$$= \frac{1}{n} E(X)$$

$$= \frac{1}{n} (nP) = P$$

$$\therefore \mu_{\hat{P}} = P$$

$$\sigma_{\hat{P}}^2 = \text{Var}(\hat{P})$$

$$= \text{Var}\left(\frac{X}{n}\right)$$

$$= \frac{1}{n^2} \text{Var}(X)$$

$$= \frac{1}{n^2} (nPq)$$

$$\therefore \sigma_{\hat{P}}^2 = \frac{Pq}{n}$$

$$Z = \frac{\hat{P} - P}{\sqrt{\frac{Pq}{n}}}$$

$$\sigma_{\hat{P}} = \sqrt{\frac{Pq}{n}}$$

Sampling distribution of difference between Two Proportions:

$$\mu_{\hat{P}_1 - \hat{P}_2} = E(\hat{P}_1 - \hat{P}_2)$$

$$= E(\hat{P}_1) - E(\hat{P}_2)$$

$$= \mu_{\hat{P}_1} - \mu_{\hat{P}_2} = P_1 - P_2$$

$$\therefore \mu_{\hat{P}_1 - \hat{P}_2} = P_1 - P_2$$

$$\sigma_{\hat{P}_1 - \hat{P}_2}^2 = \text{Var}(\hat{P}_1 - \hat{P}_2)$$

$$= \text{Var}(\hat{P}_1) + \text{Var}(\hat{P}_2)$$

$$= \sigma_{\hat{P}_1}^2 + \sigma_{\hat{P}_2}^2 = \frac{P_1 q_1}{n_1} + \frac{P_2 q_2}{n_2}$$

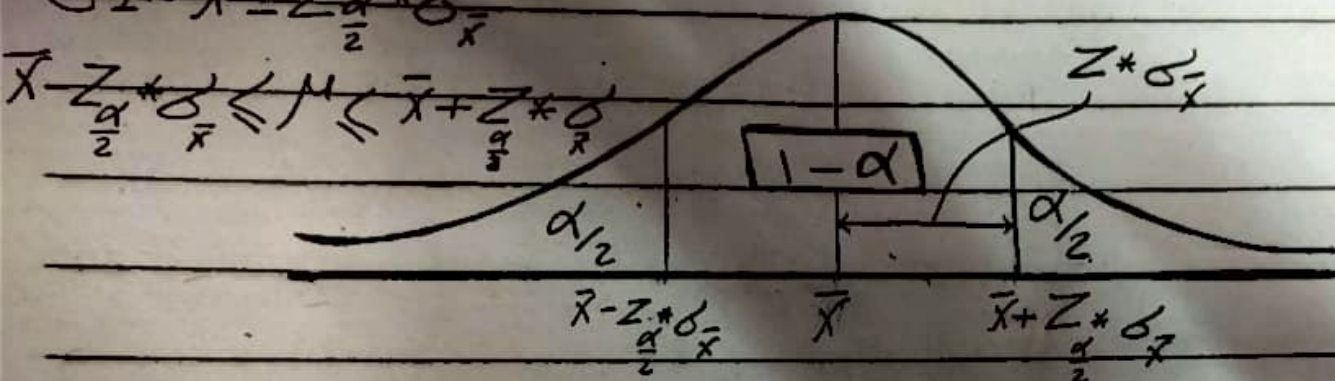
$$\therefore \sigma_{\hat{P}_1 - \hat{P}_2}^2 = \frac{P_1 q_1}{n_1} + \frac{P_2 q_2}{n_2}$$

$$Z = \frac{(\hat{P}_1 - \hat{P}_2) - (P_1 - P_2)}{\sqrt{\frac{P_1 q_1}{n_1} + \frac{P_2 q_2}{n_2}}}$$

$$\sigma_{\hat{P}_1 - \hat{P}_2} = \sqrt{\frac{P_1 q_1}{n_1} + \frac{P_2 q_2}{n_2}}$$

Confidence Intervals:

$$CI = \bar{X} \pm Z_{\frac{\alpha}{2}} * \sigma_{\bar{X}}$$



- ① Find number of observations (n)
and calculate sample mean $\bar{X} = \frac{\sum_{i=1}^n x_i}{n}$
- ② Decide $CI = (1 - \alpha)\%$
- ③ Population standard deviation (σ)

if (σ) Known

use Z-table

$$CI = \bar{X} \pm Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$

if (σ) unknown

(S) Known

if $n \geq 30$

use Z-table

$$CI = \bar{X} \pm Z_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}}$$

if $n < 30$

use t-table

$$CI = \bar{X} \pm t_{\frac{\alpha}{2}, n-1} \frac{S}{\sqrt{n}}$$

$$V = n - 1$$