[1] Decimal numbers:-

⇒ Digits :- 0, 1, 2, --- 9 (Base also called Radix)
⇒ Base = 10

Ex $925 = 900 + 20 + 5$

$= 9 * 100 + 2 * 10 + 5 * 1$

$= (9) * 10^2 + (2) * 10^1 + (5) * 10^0$

Weight — Digits

[2] Binary system:-

⇒ Digits :- 0, 1
⇒ Base = 2

Ex:-

16	8	4	2	1
0	1	0	0	1
2^4	2^3	2^2	2^1	2^0

Magnitude = 8 + 2 = 10

Ex: Convert $(325)_{10}$ in binary

	Quotient	remainder			
325 / 2	162	(1) LSB	2 | 2	1	0
162 / 2	81	0	1/2	0	(1)
81 / 2	40	1			MSB
40 / 2	20	0			
20 / 2	10	0			
10 / 2	5	0			
5 / 2	2	1			

[4] Hexadecimal Systems:

⇒ Digits: 0, 1, 2, ... 9, A, B, C, D, E, F
⇒ Base = 16

⇒ A = 10
B = 11
C = 12
D = 13
E = 14
F = 15
→ in decimal

⇒ 1 Hex Place has 16 different Combinations
⇒ in Binary Stored in 4-bits

Hex	8	4	2	1
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
A	1	0	1	0
B	1	0	1	1
C	1	1	0	0
D	1	1	0	1
E	1	1	1	0
F	1	1	1	1

Binary

Ex: ① $(10110.01)_2 = (?)_{16}$

② $= (?)_{10}$

⇒ 0001 0110.0100

$= (1 \quad 6.4)_{16}$

② $(16.4)_{16} = 1*16^1 + 6*16^0 + 0.4*16^{-1}$

$= (22.025)_{10}$

⇒ Octal in binary representation (in 3 bits)

	4	2	1
0	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	0
5	1	0	1
6	1	1	0
7	1	1	1

Ex: Express $(10110.01)_2$ in octal

⇒ 010 110 . 010

$= (2 \quad 6 . 2)_8$

Ex: Express $(36.5)_8$ in binary

⇒ 3 6 . 5

$= (011 \quad 110 . 101)_2$

3) How many bits needed to represent 205 in binary? (guess number of bits without conversion)

★ Solution:

$\Rightarrow \quad 2^n - 1 \geq 205$

$2^n \geq 206 \Rightarrow \ln 2^n \geq \ln 206$

$n \geq \frac{\ln 206}{\ln 2} \simeq \boxed{8 \text{ bits}}$

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4) What is the largest number (in decimal) that can be obtained with
   a. 7 bits binary
   b. 3 bits hexadecimal

★ Solutions:

[a] $Max = 2^n - 1 = 2^7 - 1 = \boxed{127}$

[b] $Max = 16^n - 1 = 16^3 - 1 = \boxed{4095}$

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5) Convert the following numbers with the indicated bases to decimal:

a. $(10110.0101)_2$	b. $(121)_3$	c. $(345)_6$	d. $(77.7)_8$
e. $(435)_8$	f. $(198)_{12}$	g. $(AC5)_{16}$	h. $(16.5)_{16}$

★ Solution

a) $(10110.0101)_2 = 1*2^4 + 1*2^2 + 1*2^1 + 1*2^{-2} + 1*2^{-4}$

$= \boxed{(22.3125)_{10}}$

b) $(121)_3 = 1*3^2 + 2*3^1 + 1*3^0 = \boxed{(16)_{10}}$

c) $(345)_6 = 3*6^2 + 4*6^1 + 5*6^0 = \boxed{(137)_{10}}$

[3] Octal Systems

⇒ Digits: 0, 1, 2, ... 7
⇒ Base = 8

⇒ Ex $(127.4)_8 = (?)_{10}$

We have

1	2	7	.	4
↓	↓	↓		↓
8^2	8^1	8^0		8^{-1}

Weights:-

∴ Magnitude $= (1*8^2 + 2*8^1 + 7*8^0 + 4*8^{-1})_{10}$

$= (87.5)_{10}$

⇒ Decimal to Octal: $(175)_{10} = (?)_8$

	integer	Remainder
175 | 8	21	$7 \rightarrow a_0$
21 | 8	2	5
2 | 8	0	$2 \rightarrow a_2$

∴ $(175)_{10} = (257)_8$

⇒ Fraction to Octal: $(0.3125)_{10} = (?)_8$

	integer	+ Fraction
0.3125 * 8	$2 \rightarrow a_{-1}$	0.5
0.5 * 8	$4 \rightarrow a_{-2}$	0.00

∴ $(0.3125)_{10} = (0.24)_8$

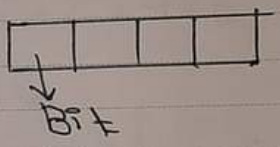
LSB: Least Significat Bit (At right)
MSB: Most Significat Bit (At left)

$\therefore (325)_{10} = (101000101)_2$

Notes:
① For n bits, the number of possible numbers is 2^n (from 0 to $2^n - 1$)

② In Computer Work, 1 Byte = 4 bit

→ Kilo (K) = 2^{10}
Mega (M) = 2^{20}
Giga (G) = 2^{30}
Tera (T) = 2^{40}

Byte ⟹ 

→ Fraction decimal to binary: $(0.625)_{10} = (?)_2$

	integer +	fraction
0.625 * 2	1 → MSB	0.25
0.25 * 2	0	0.5
0.5 * 2	1 → LSB	0.0

$\therefore (0.625)_{10} = (0.101)_2$

d) $(77.7)_8 = 7*8^1 + 7*8^0 + 7*8^{-1} = (63.125)_{10}$

e) $(435)_8 = 4*8^2 + 3*8^1 + 5 = (285)_{10}$

f) $(198)_{12} = 1*12^2 + 9*12^1 + 8 = (260)_{10}$

g) $(AC5)_{16} = 10*16^2 + 12*16 + 5 = (2757)_{10}$

h) $(16.5)_{16} = 1*16 + 6*16^0 + 5*16^{-1} = (22.3125)_{10}$

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6) perform the following conversions

a. $(28.125)_{10}$ to binary

b. $(157.128)_{10}$ to hexadecimal

c. $(67.45)_{10}$ to octal

d. $(2AC5)_{16}$ to octal (without converting to decimal)

* Solution

a) $(28.125)_{10} = (28)_{10} + (0.125)_{10}$

integer + remainder			integer + fraction	
28 | 2	14	LSB ①	0.125*2	0 → MSB 0.25
14 | 2	7	0	0.25*2	0 0.5
7 | 2	3	1	0.5*2	1 → LSB 0.00
3 | 2	1	1		
1 | 2	0	MSB ①		

$\therefore (28)_{10} = (11100)_2$ $\quad \therefore (0.125)_{10} = (0.001)$

d) $(2AC5)_{16} = (\ ?\ )_8$

$\Rightarrow$ $(2 \quad A \quad C \quad 5)_{16}$

0010 1010 1100 0101

= ~~000~~ 010 101 011 000 101

$= (2\ 5\ 3\ 0\ 5)_8$

1) List the numbers from 8 to 28 in base 12.

* Solution:

Decimal	Base 12
8	8
9	9
10	A
11	B
12	10
13	11
14	12
15	13
16	14
17	15

Decimal	Base 12
18	16
19	17
20	18
21	19
22	1A
23	1B
24	20
25	21
26	22
27	23
28	24

2) What is the largest binary number that can be expressed with 16 bits? What are the equivalent decimal and hexadecimal numbers?

* Solution: n=16

⇒ largest number: $(1111\ 1111\ 1111\ 1111)_2$

⇒ in Decimal: Magnitude $= 2^{16} - 1 = (65535)_{10}$

⇒ in Hexadecimal $\equiv (FFFF)_{16}$

$\therefore (28.125)_{10} = (11100.001)_2$

b) $(157.128)_{10} = (\ ?\ )_{16}$

$\Rightarrow (157.128)_{10} = (157)_{10} + (0.128)_{10}$

	integer	Remainder		integer	fraction
$157/16$	9	$13 \equiv D$	0.128×16	2	0.048
$9/16$	0	9	0.048×16	0	0.768
			0.768×16	C	0.288

$\therefore (157)_{10} = (9D)_{16}$

$\therefore (0.128)_{10} = (0.20C)_{16}$

$\therefore (157.128)_{10} = (9D.20C)_{16}$

c) $(67.46)_{10} = (\ ?\ )_8$

	integer	Remainder		integer	fraction
$67/8$	8	$3 \rightarrow$ LSB	0.46×8	(3) MSB	0.68
$8/8$	1	0	0.68×8	5	0.44
$1/8$	0	$1 \rightarrow$ MSB	0.44×8	3	0.52
			0.52×8	4	0.16
			0.16×8	1	0.28

$(67)_{10} = (103)_8$ $\quad (0.46)_{10} = (0.353)_8$

$\therefore (67.46)_{10} = (103.353)_8$