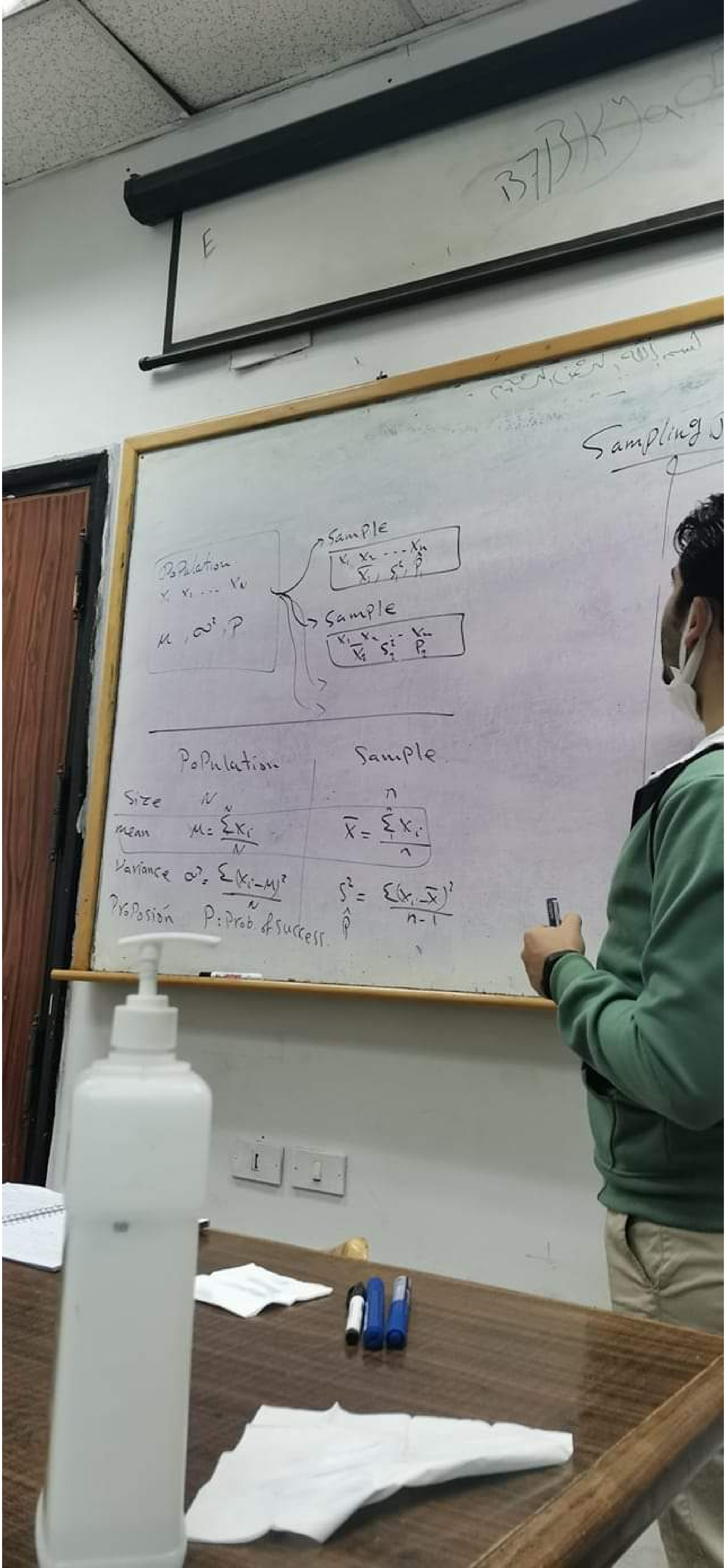
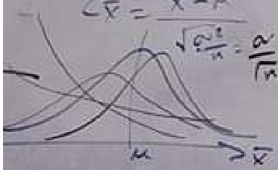

Sampling
Population
Sample
Sample
Population
Sample
Size N n
mean $\mu = \frac{\sum^{N} x_i}{N}$ $\bar{x} = \frac{\sum^{n} x_i}{n}$
Variance $\sigma^2 = \frac{\sum (x_i - \mu)^2}{N}$ $s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$
ProPosion P: Prob. of success $\hat{p}$

$$Z_x = \frac{X-\mu}{\sigma} = \frac{X-E(X)}{\sqrt{V(X)}}$$

$$Z_x = \frac{x_i-\bar{x}}{s}$$

$$Z_{\bar{x}} = \frac{\bar{X}-\mu}{\sqrt{\frac{\sigma^2}{n}}} = \frac{\sigma}{\sqrt{n}}$$



y large.

normal.

Sampling Distn.

① Normal Population with mean = 63 and s.d = 12 Find:

Prob that Random sample of size 16 has mean greater than 61.5.

$$P(\bar{X} \geq 61.5) = P\left(Z \geq \frac{61.5-63}{12/\sqrt{16}}\right)$$

$$= P(Z_{\bar{x}} \geq -0.5)$$

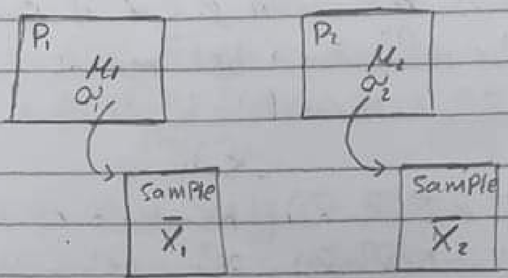
$$= 1 - P(Z_{\bar{x}} < -0.5)$$

$$= P(Z_{\bar{x}} < 0.5)$$

$$= 0.6915$$

- Difference bet. 2-means -

Suppose we have two populations P_1 and P_2 each has it's own mean say μ_1 and μ_2 which can be predicted by a Random samples.



then the difference $(\mu_1 - \mu_2)$ can be predicted by $(\bar{X}_1 - \bar{X}_2)$

$\rightarrow$ mean of difference $E(\bar{X}_1 - \bar{X}_2) = E(\bar{X}_1) - E(\bar{X}_2)$

$= \mu_1 - \mu_2 \Rightarrow \mu_{\bar{x}_1 - \bar{x}_2}$

$\rightarrow$ variance ———— $V(\bar{X}_1 - \bar{X}_2) = V(\bar{X}_1) + V(\bar{X}_2)$

$= \sigma_1^2/n_1 + \sigma_2^2/n_2$

$\rightarrow \sigma^2_{\bar{X}_1 - \bar{X}_2}$

Note: $(\bar{X}_1 - \bar{X}_2) \sim$ Normal if $X_1 \sim$ Normal & $X_2 \sim$ Normal. (according to Central limit or Population distn).

GHARIB

Subject : ....

[4] Population$_1$ has $\mu_1 = 20$ and $\sigma_1^2 = 100$
Population$_2$ — $\mu_2 = 15$ and $\sigma_2^2 = 64$

You sample 20 from Pop$_1$ & 16 from Pop$_2$
→ what is the mean & s.d. of sampling distn of the difference bet means.
if P_1 & P_2 ~ Normal.

-Soln-

$\rightarrow \mu_{\bar{X}_1 - \bar{X}_2} = E(\bar{X}_1 - \bar{X}_2) = \mu_1 - \mu_2 = 20 - 15 = 5$

$\rightarrow \sigma^2_{\bar{X}_1 - \bar{X}_2} = V(\bar{X}_1 - \bar{X}_2) = \sigma_1^2/n_1 + \sigma_2^2/n_2 = \frac{100}{20} + \frac{64}{16}$
$= 5 + 4 = 9$

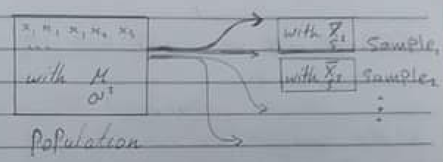
$\therefore \sigma_{\bar{X}_1 - \bar{X}_2} = \sqrt{9} = 3.$

---

[5] the mean of men height is 175 & variance is 64
the mean of girls height is 165 & variance is 64
if 8 boys & 8 girls where sampled.
what is the prob. that mean height of girls would be heigher than the mean height of boys.

Sampling

if we want to study a population with large size N ; we may take a sample of some of it's values say $x_1\, x_2 \dots x_n$ (i.e of size n)



Population		Sample
size	N	n
mean	$\mu = \frac{\sum x_i}{N}$	$\bar{X} = \frac{\sum x_i}{n}$
variance	$\sigma^2 = \frac{\sum^N (x_i-\mu)^2}{N}$	$s^2 = \frac{\sum^n (x_i-\bar{x})^2}{n}$
proportion	P	$\hat{P}$ (Prob. of success)

[1] Samples mean

Now we have.. $\bar{X}_1\ \bar{X}_2\ \bar{X}_3 \dots \approx: \bar{X}$

→ the mean of $\bar{X}$ is $E(\bar{X}) = E\left(\frac{\sum x_i}{n}\right) = \frac{1}{n} E(\sum x_i)$

$= \frac{1}{n} \sum E(x_i) = \frac{1}{n} \sum^n \mu = \mu$

since x_i exist in the population

d. $\sigma = 15$

Population

bove

al limit.

92

Sampling Distn.

$= P(Z_{\bar{x}} < 0.73) - P(Z_{\bar{x}} < -1.83)$

$= P(Z_{\bar{x}} < 0.73) - P(Z_{\bar{x}} > 1.83)$

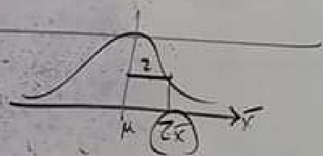
$= \_\_\_\_ - (1 - P(Z_{\bar{x}} < 1.83))$

$= 0.7673 - (1 - 0.9664)$

$= \_\_\_\_ - 0.0336$

$= 0.7337$

② $Z_{\bar{x}} = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$



$\bar{x} = Z_{\bar{x}} \left(\frac{\sigma}{\sqrt{n}}\right) + \mu$

$= 2 \left(\frac{15}{\sqrt{30}}\right) + 90$

$= 95.5$

Sampling Distn.

μ_B

5 & variance $\overset{\sigma_1^2}{64}$

6 ______ $\overset{\sigma_2^2}{64}$

sampled

hight of girls.

t of Boys.

$(Z_{\bar{x}_1-\bar{x}_2} \geq \frac{0-(165-175)}{\sqrt{\frac{64}{8}+\frac{64}{8}}}$

$P(Z_{\bar{x}_1-\bar{x}_2} \geq 2.5) = 1 - P(Z_{\bar{x}_1-\bar{x}_2} < 2.5)$

$= 1 - 0.9938$

$= 0.0062$

---

(4) $P_1 \longrightarrow \mu_1 = 20$ and $\sigma_1^2 = 100$

$P_2 \longrightarrow \mu_2 = 15$ ~ $\sigma_2^2 = 64$

you sampled 20 from P_1 and 16 from P_2

what is the mean & s.d. of sampling dist of Difference of two means.

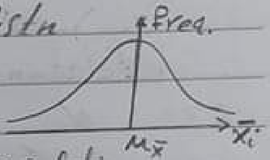
Subject:

→ the variance of $\bar{x}$ ... $V(\bar{x}) = V\left(\frac{\sum x_i}{n}\right)$
$= \frac{1}{n^2} V(\sum x_i)$
since x_i exist in Population $= \frac{1}{n^2} \sum (V(x_i))$.
$= \frac{1}{n^2} \sum \sigma^2 = \frac{\sigma^2}{n}$

$E(\bar{x}) = \mu_{\bar{x}} = \mu$ * $V(\bar{x}) = \sigma^2_{\bar{x}} = \frac{\sigma^2}{n}$ *

_ Centeral limit theorem

as the samples size (n) increasing .. we note that. $\bar{x}_i$ acting like normal distn



when $n \geq 30$ $\bar{x} \sim$ approximately normal ; whatever Population distn.

$\bar{x} \sim$ Normal distn. if Population distn. is a Normal.

GHARIB

Sampling Distn.

→ $(\bar{X}_1 - \bar{X}_2) \sim$ Normal : if each $n_i \geq 30$ according to central limit.

→ 2-Populations ~ Normal.

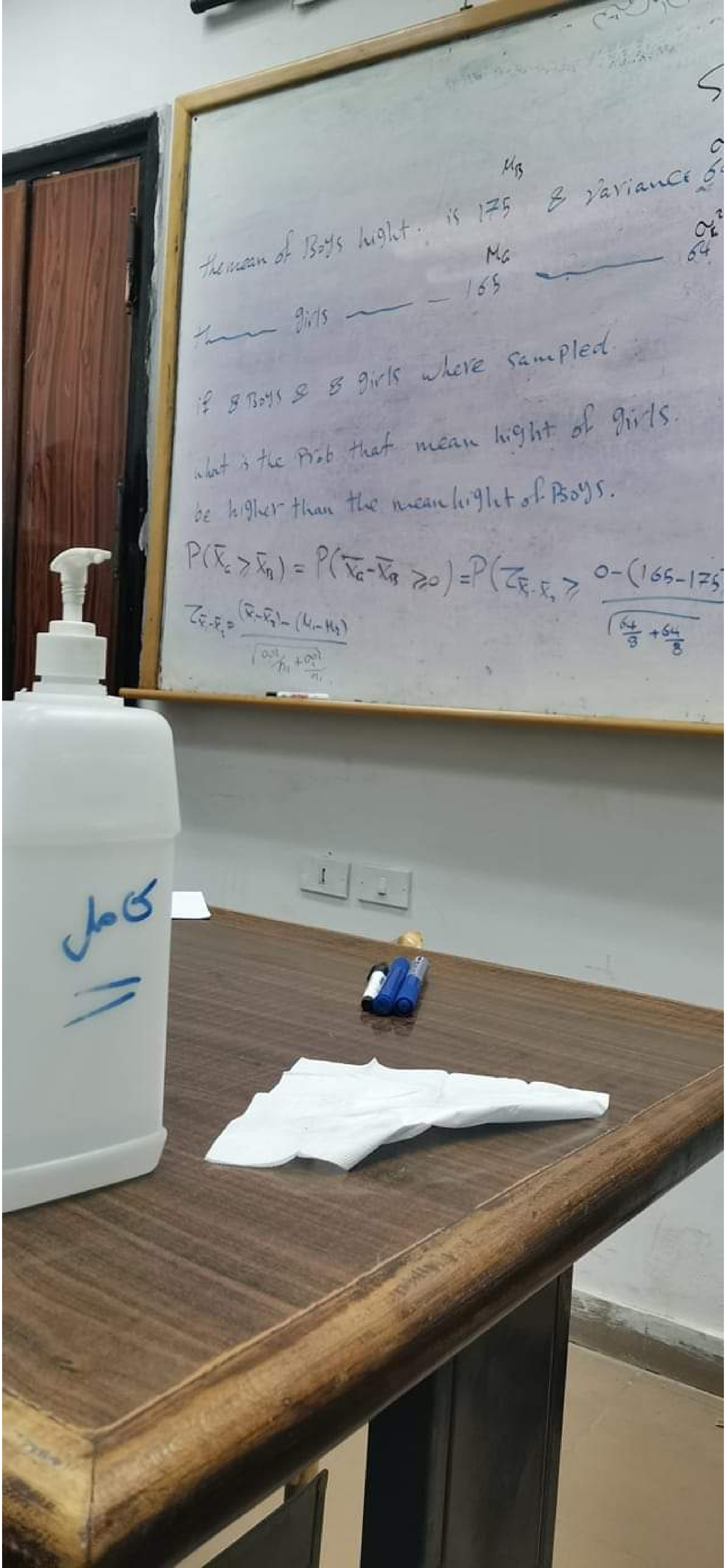
---

(1) $P_1 \longrightarrow \mu_1 = 20$ and $\sigma_1^2 = 100$

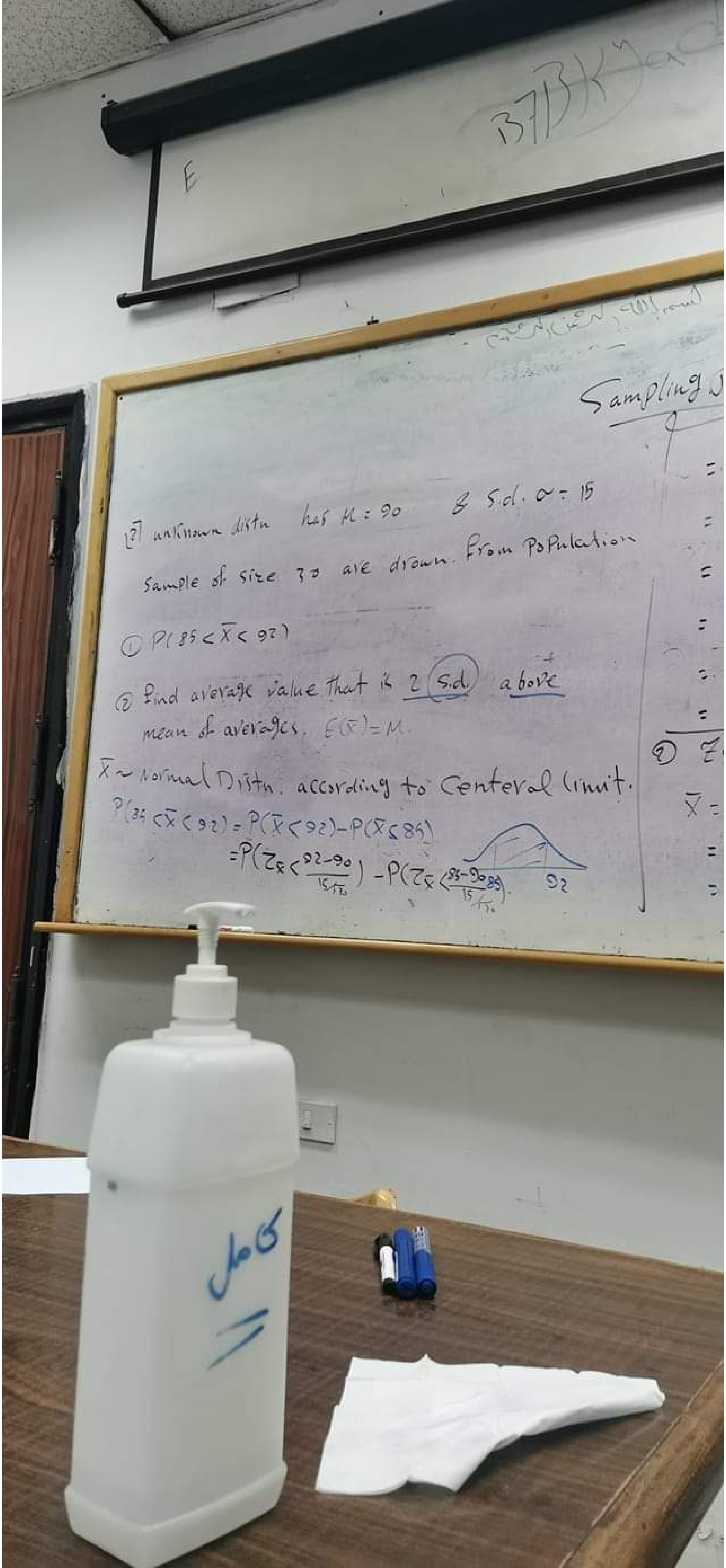
$P_2 \longrightarrow \mu_2 = 15$ ~ $\sigma_2^2 = 64$

you sampled 20 from P_1 and 15 from P_2

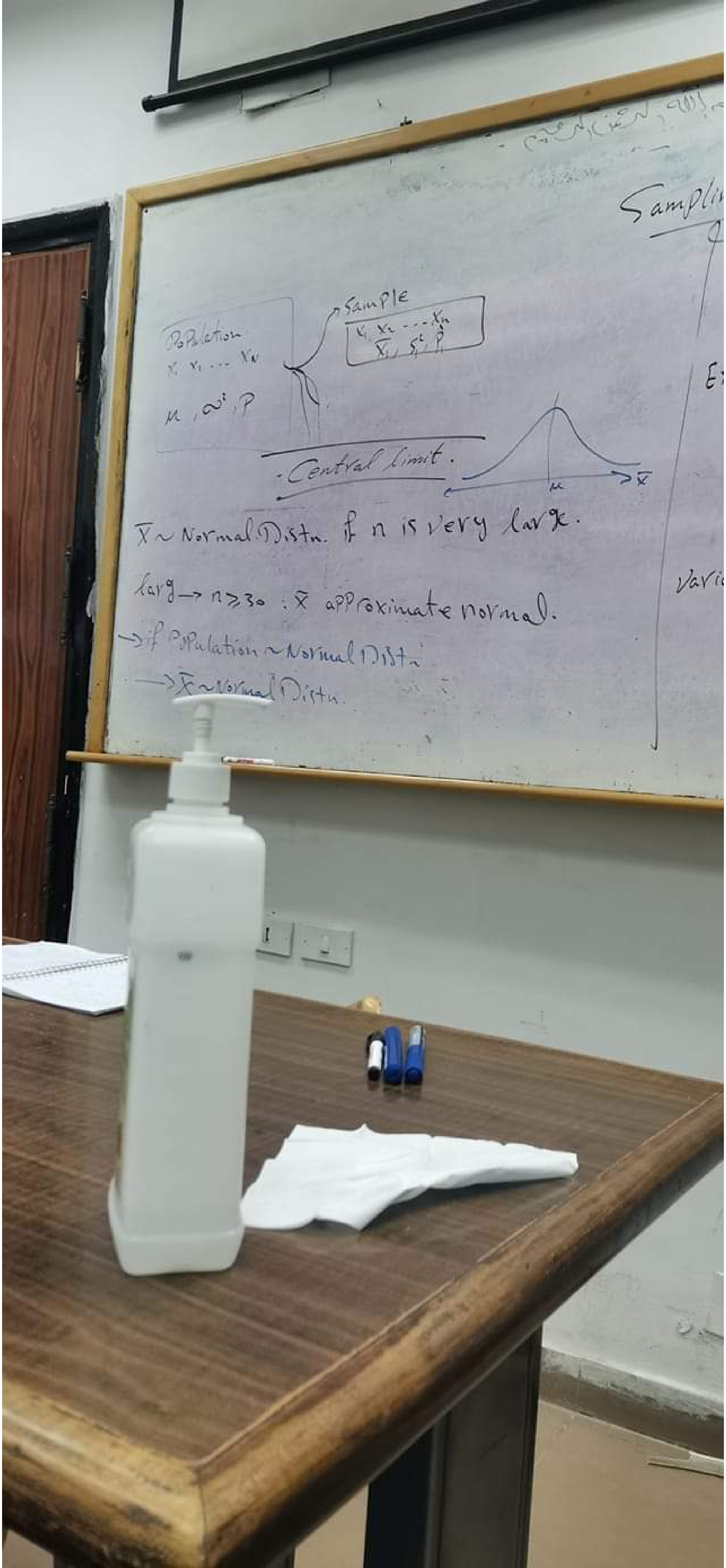
what is the mean & s.d. of sampling distn of Difference of two means.



μ_B
The mean of Boys hight. is 175 & variance σ_B^2
μ_G σ_G^2
The ___ girls ___ 165 ___ 64
if 8 Boys & 8 girls where sampled.
what is the Prob that mean hight of girls.
be higher than the mean hight of Boys.
$P(\bar{X}_G > \bar{X}_B) = P(\bar{X}_G - \bar{X}_B \geq 0) = P\left(Z_{\bar{X}_1-\bar{X}_2} \geq \frac{0-(165-175)}{\sqrt{\frac{64}{8}+\frac{64}{8}}}\right.$
$Z_{\bar{X}_1-\bar{X}_2} = \frac{(\bar{X}_1-\bar{X}_2)-(\mu_1-\mu_2)}{\sqrt{\sigma_1^2/n_1+\frac{\sigma_2^2}{n_2}}}$
كامل



Sampling
[2] unknown distn has $\mu = 90$ & S.d. $\sigma = 15$
sample of size 30 are drawn. from Population
① $P(85 < \bar{X} < 92)$
② find average value that is 2 S.d. above
mean of averages. $E(\bar{X}) = \mu$.
$\bar{X} \sim$ Normal Distn. according to Centeral limit.
$P(85 < \bar{X} < 92) = P(\bar{X} < 92) - P(\bar{X} \le 85)$
$= P(Z_{\bar{x}} < \frac{92-90}{15/\sqrt{30}}) - P(Z_{\bar{x}} < \frac{85-90}{15/\sqrt{30}})$
85
92
كامل


Population
Sample
Central limit.
X̄ ~ Normal Distn. if n is very large.
larg → n ≥ 30 : x̄ approximate normal.
→ if Population ~ Normal Distn
→ X̄ ~ Normal Distn.
Sampli

Subject:

-Soln-

girls

$n_1 = 8$

$\mu_1 = 165$

$\sigma_1^2 = 64$

boys

$n_2 = 8$

$\mu_2 = 175$

$\sigma_2^2 = 64$

$\Rightarrow P(\bar{X}_1 > \bar{X}_2) = P(\bar{X}_1 - \bar{X}_2 > 0)$

$= P\left(\frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}} > \frac{0 - (165 - 175)}{\sqrt{(64/8)2}}\right)$

$= P\left(Z_{\bar{X}_1 - \bar{X}_2} > \frac{10}{4}\right) = P(Z > 2.5)$

$= 1 - P(Z < 2.5)$

$= 1 - 0.9938$

$= 0.0062$

GNARB

Subject: ....

→ the variance of $\bar{x}$ ... $V(\bar{x}) = V\left(\frac{\sum x_i}{n}\right)$

$= \frac{1}{n^2} V(\sum x_i)$

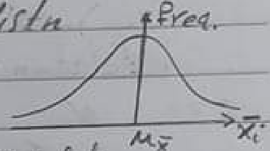
since x_i exist in Population ← $= \frac{1}{n^2} \sum (V(x_i))$

$= \frac{1}{n^2} \sum \sigma^2 = \frac{\sigma^2}{n}$

$E(\bar{X}) = \mu_{\bar{x}} = \mu$ ☆ $V(\bar{x}) = \sigma^2_{\bar{x}} = \frac{\sigma^2}{n}$ ☆

_ Centeral limit theorem

as the samples size (n) increasing .. we note that. $\bar{x}_i$ acting like normal distn



when $n \geq 30$ $\bar{X} \sim$ approximately normal ; whatever Population distn.

$\bar{X} \sim$ Normal distn. if Population distn. is a Normal.

Subject:

-Exercise-

[1] Normal Population with $\mu = 63$ and s.d. $= 12$
find
P(Random sample of size 16 has $\bar{X} \geq 61.5$)

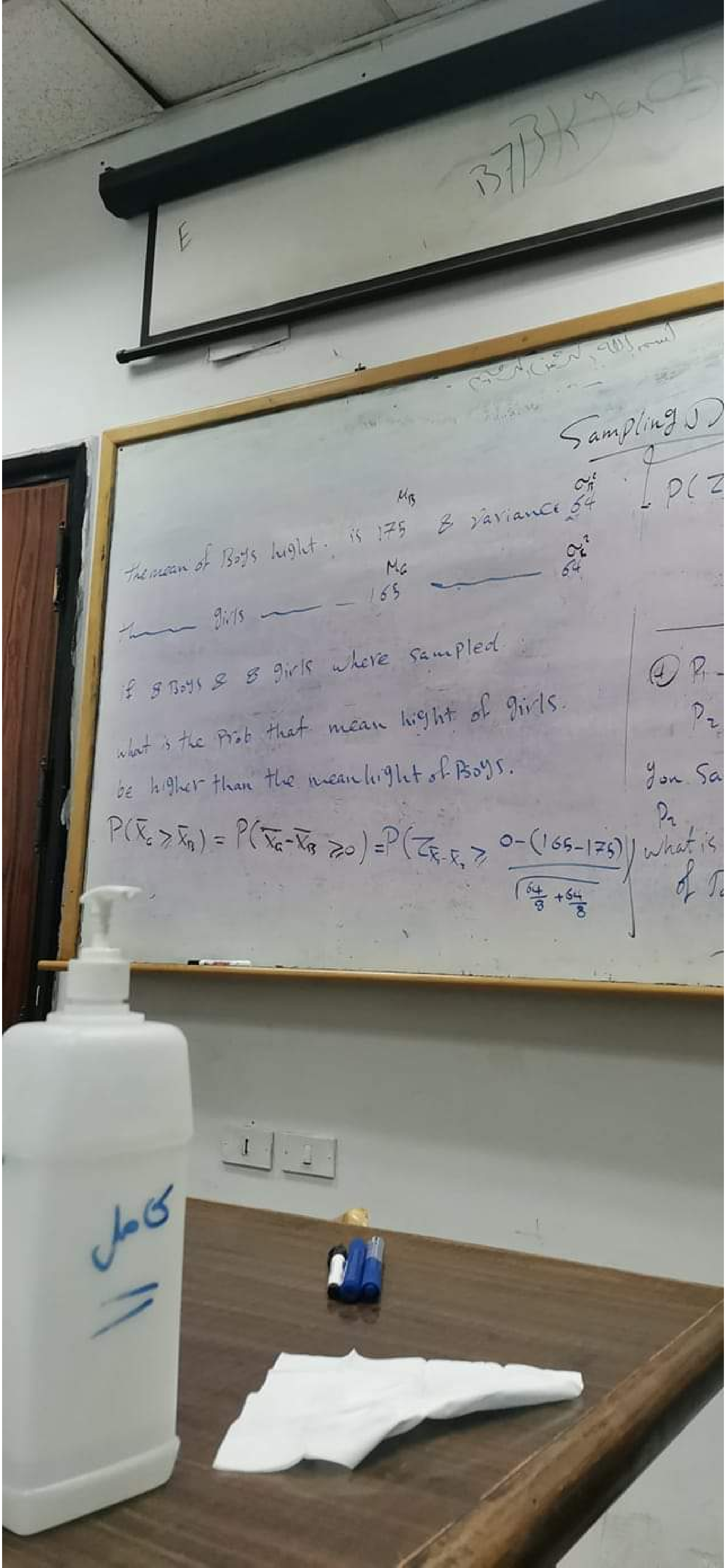
-Soln-

$\mu = 63$ $\sigma = 12$ $n = 16$ $X_i \sim$ Normal
then $\bar{X} \sim$ Normal. $E(\bar{X}) = \mu$ $V(\bar{X}) = \frac{\sigma^2}{n}$
$P(\bar{X} > 61.5) = P(Z > \frac{61.5 - 63}{12/\sqrt{16}})$

$= P(Z > -0.5)$
$= P(Z \leq 0.5)$ oR $1 - P(Z < -0.5)$
$= 0.6915$ oR $(1 - 0.3085) = 0.6915$

---

[2] unknown distn. has $\mu = 90$ & s.d $= \sigma = 15$
samples of size 30 are drawn from the Population
1) $P(85 < \bar{X} < 92)$
2) find the average value that is 2 s.d. above mean of averages.



Sampling D
The mean of Boys hight is 175 (μ_B) & variance 64 (σ_B^2)
—— girls —— 165 (μ_G) —— 64 (σ_G^2)
if 8 Boys & 8 girls where sampled
what is the Prob that mean hight of girls
be higher than the mean hight of Boys.
$P(\bar{X}_G \geq \bar{X}_B) = P(\bar{X}_G - \bar{X}_B \geq 0) = P\left(Z_{\bar{X}_1-\bar{X}_2} \geq \frac{0-(165-175)}{\sqrt{\frac{64}{8}+\frac{64}{8}}}\right)$
P(Z
① P1 -
P2
you Sa
P2
what is
of T
كامل

Sampling Distn.

Sample mean.

$\bar{x}_1 \quad \bar{x}_2 \quad \bar{x}_3 \quad \dots \quad \bar{x}_k \quad \dots$

$E(ax) = aE(x)$.

Expectation: $E(\bar{x}) = E\left(\frac{\sum x_i}{n}\right)$

$= \frac{1}{n} E(\sum x_i) \qquad E(x+y) = E(x) + E(y)$

$= \frac{1}{n} \sum E(x_i)$ ($E(x_i) = \mu$)

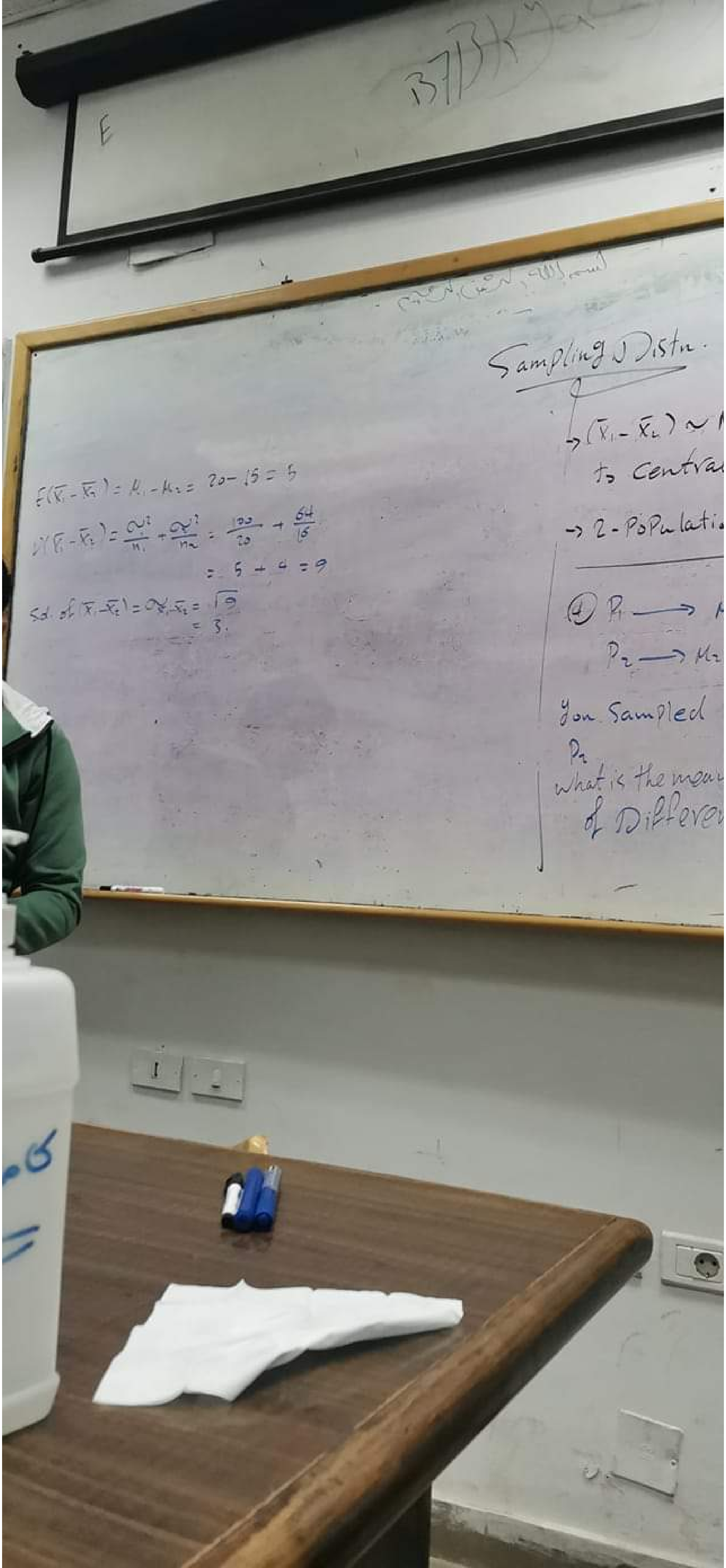
$= \frac{n\mu}{n} = \mu$

Variance $= V(\bar{x}) = V\left(\frac{\sum x_i}{n}\right) \qquad V(ax) = a^2 V(x)$.

$= \frac{1}{n^2} V(\sum x_i)$

$= \frac{1}{n^2} \sum \cancel{V(x_i)}\ \sigma^2$

$= \frac{n\sigma^2}{n^2} = \frac{\sigma^2}{n}$

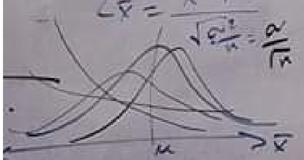

Sampling Distn.
(X̄1 - X̄2) ~
centra
2-Populati
E(X̄1 - X̄2) = μ1 - μ2 = 20 - 15 = 5
V(X̄1 - X̄2) = σ1²/n1 + σ2²/n2 = 100/20 + 64/16
= 5 + 4 = 9
S.d. of (X̄1 - X̄2) = σX̄1-X̄2 = √9
= 3.
P1 →
P2 → μ2
You Sampled
what is the mea
of Differe

Sampling Distn.

$Z_x = \frac{X-\mu}{\sigma} = \frac{X - E(X)}{\sqrt{V(X)}}$

$Z_x = \frac{X_i - \bar{X}}{S}$

$Z_{\bar{X}} = \frac{\bar{X}-\mu}{\sqrt{\frac{\sigma^2}{n}}} = \frac{\sigma}{\sqrt{n}}$



ery large.

e normal.

(1) Normal Population with mean = 63 and s.d = 12 Find:

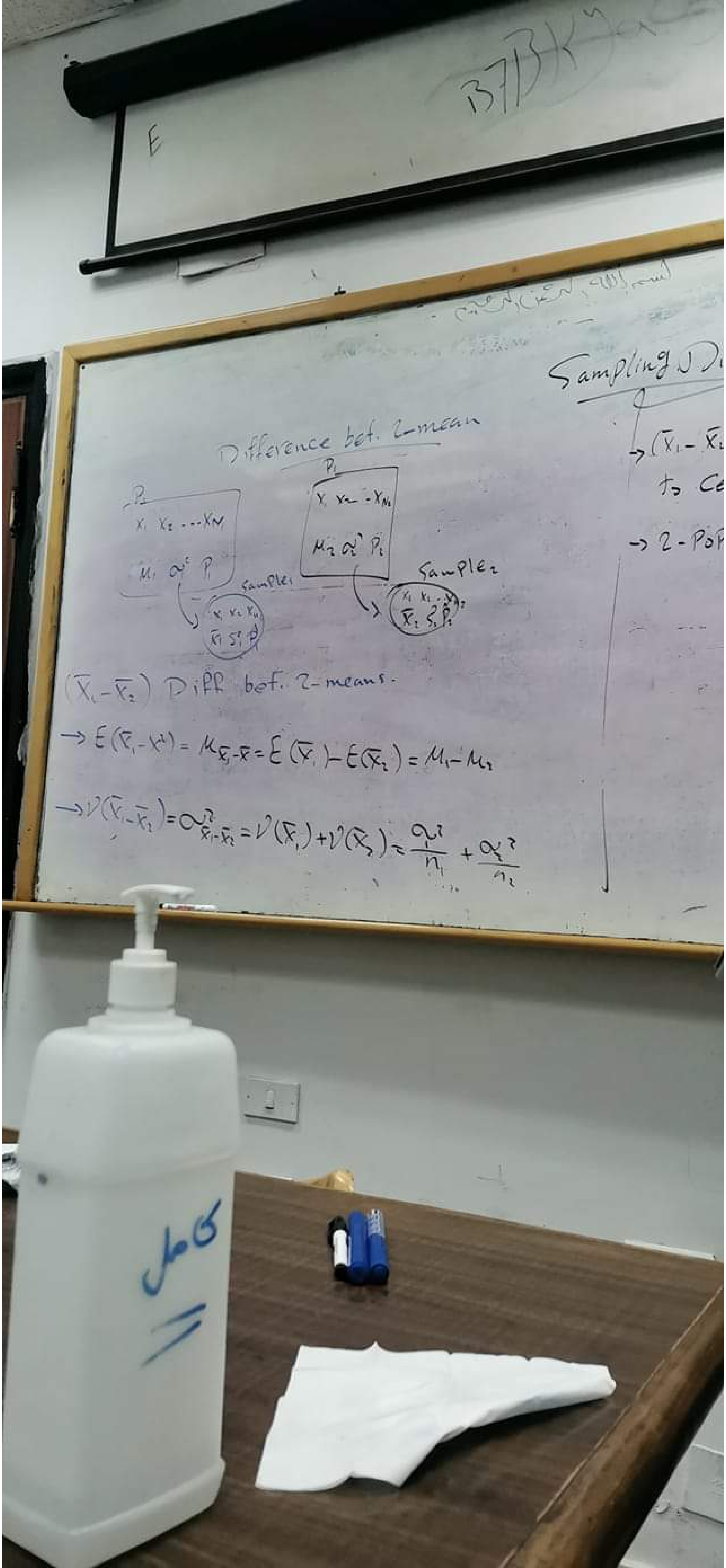
Prob that Random sample of size 16 has mean greater than 61,5.

$P(\bar{X} \geq 61{,}5) = P\left(Z \geq \frac{61{,}5-63}{12/\sqrt{16}}\right)$

$= P(Z_{\bar{X}} \geq -0{,}5)$

$= 1 - P(Z_{\bar{X}} < -0{,}5)$

$= P(Z_{\bar{X}} < 0{,}5)$


E
Sampling Di
Difference bet. 2-mean
P_1
$x_1\ x_2\ \dots x_N$
$\mu_1\ \sigma_1^2\ P_1$
Sample1
$x_1\ x_2\ x_n$
$\bar{x}_1\ s_1^2\ p_1$
P_2
$x_1\ x_2\ \dots x_{N_2}$
$\mu_2\ \sigma_2^2\ P_2$
Sample2
$x_1\ x_2\ \dots x_{n_2}$
$\bar{x}_2\ s_2^2\ p_2$
$(\bar{x}_1-\bar{x}_2)$ Diff bet. 2-means.
$\rightarrow E(\bar{x}_1-\bar{x}_2) = \mu_{\bar{x}_1-\bar{x}_2} = E(\bar{x}_1) - E(\bar{x}_2) = \mu_1 - \mu_2$
$\rightarrow V(\bar{x}_1-\bar{x}_2) = \sigma^2_{\bar{x}_1-\bar{x}_2} = V(\bar{x}_1) + V(\bar{x}_2) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$
$\rightarrow (\bar{x}_1 - \bar{x}_2$
$\rightarrow$ 2-Pop
كامل

Subject :

$\mu = 90 \qquad \sigma = 15 \qquad n = 30$

then $X \sim$ Normal

1) $P(85 < \bar{X} < 92) = P\left(\dfrac{85-90}{15/\sqrt{30}} < Z < \dfrac{92-90}{15/\sqrt{30}}\right)$

$= P(-1{,}83 < Z < 0{,}73)$

$= P(Z < 0{,}73) - P(Z < -1{,}83)$

$= P(Z < 0{,}73) - (P(Z > 1{,}83))$

$= P(Z < 0{,}73) - (1 - P(Z < 1{,}83))$

$= 0{,}7673 - (1 - 0{,}9664)$

$= 0{,}7673 - 0{,}0336$

$= 0{,}7337.$

2) $Z_{\bar{x}} = +2$ s.d. $\qquad$ since $Z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

$\therefore \bar{x} = Z(\sigma/\sqrt{n}) + \mu$

$= 2(15/\sqrt{30}) + 90 = 95{,}5$