

Statistics:

Pr. 1. The Study of The Collection, analysis, it's a mathematical discipline To Collect, Summarize Data

Samples and Population:

It's a Collection of all Possible individuals → Population

A Sample is Portion of The Population of interest → Sample

- Ana basta Random Sample w bsmehm Qse el nas ely basta aw el Falat ely basta. → Qse

- El Faga ely 3rifha 3n el Qse bla3ty ely estima random deh esmha variable → variable

• Qses + Variables = Data Matrix

Data

↓
de variable

Ana 3nid y nogen Mn el Data

* Numerical Data

. Discrete & Continuous

Deh arqam Mofsta

Deh Range

(30, 35, 60, 42, ...)

(30 - 60)

* Categorical or Qualitative

- Nominal → Can't be ordered and Measured → name
- ordinal → Can be ordered and Measured → rank

Graphical Presentation of Data: There are different types of graphical presentation.

① Dot Plot (Dot-Chart) → graphical display of data using dots on x-axis.

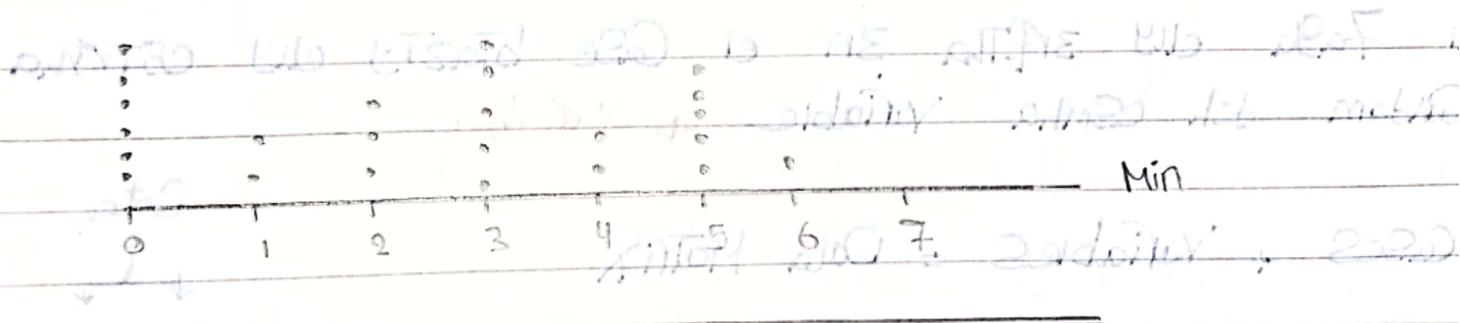
Ex: How long does it take to eat breakfast?

26.9 min	35.1 32.3 42.5 56
People	6 2 3 5 2 1
Time taken by people to eat breakfast	26.9 32.3 35.1 42.5 56

1) What's the sample size?

$$\text{Sample Size} = 6 + 2 + 3 + 5 + 2 + 1 = 24$$

2) Construct a dot plot graph?



② Bar Graph (Bar-Chart) → graphical display of data using bars (rectangles).

How To Create Bar Graph?

X-axis → what is being measured (Case)

Y-axis → No. of numbers (For amount) (Variable)

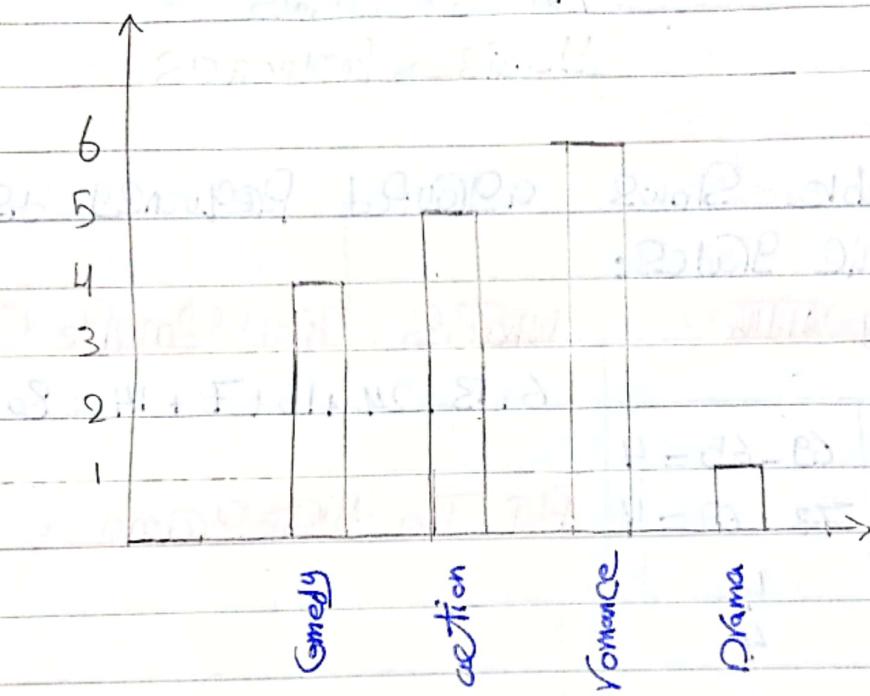
Note → There exist gaps between the bars

E.K.: Movie Genres: Comedy, Action, Romance, Drama

People	4	5	6	1

$$1) \text{ Sample Size} = 4+5+6+1 = 16$$

2) Bar Graph



3) Pie-Chart (circle chart)

movie Comedy Action Romance Drama Total

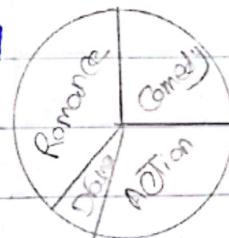
People	4	5	6	1	16
Relative Size	$(4/16) * 100$	$(5/16) * 100$	$(6/16) * 100$	$(1/16) * 100$	

$$\text{Relative Size} = (4/16) * 100 = 25\%$$

$$\text{Relative Size} = (5/16) * 100 = 31.25\%$$

$$\text{Relative Size} = (6/16) * 100 = 37.5\%$$

$$\text{Relative Size} = (1/16) * 100 = 6.25\%$$



→ back to Relative Size ($\text{Sample} / \text{Total} * 100$)

4) Histogram

→ graphical display Data by using bars rectangle and in Histogram There no gaps between the bars.

in Histogram we have Two Cases

1) Histogram for grouped Data:

intervals with intervals width

X-axis → intervals

Y-axis → frequencies

Ex: The following Table Shows a grouped Frequency distribution For The Statistic Grades:

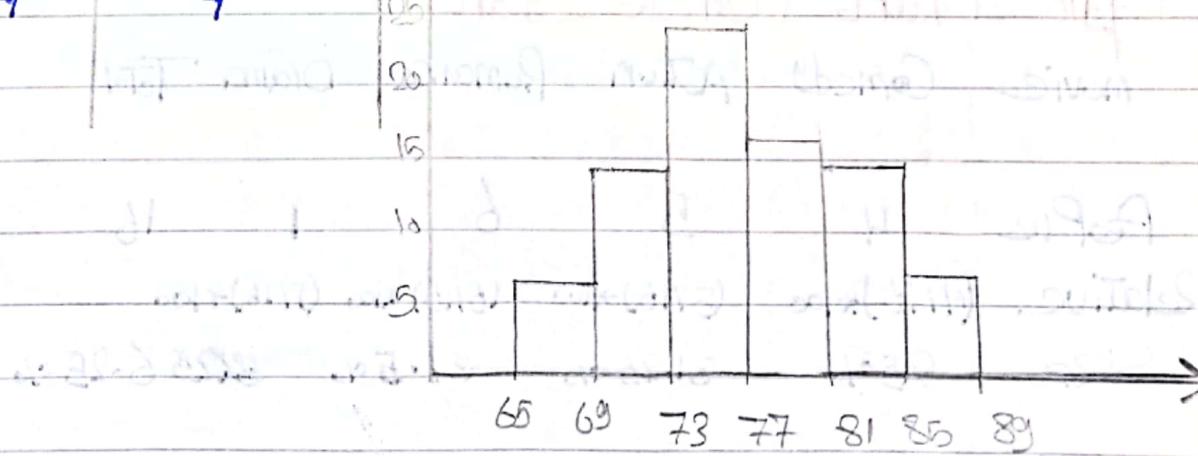
interval Frequency width

65 - 69	6	$69 - 65 = 4$
69 - 73	13	$73 - 69 = 4$
73 - 77	24	4
77 - 81	16	4
85 - 89	17	4
81 - 85	14	4

what is The Sample Size?

$$6 + 13 + 24 + 16 + 7 + 14 = 80$$

Plot The Histogram :



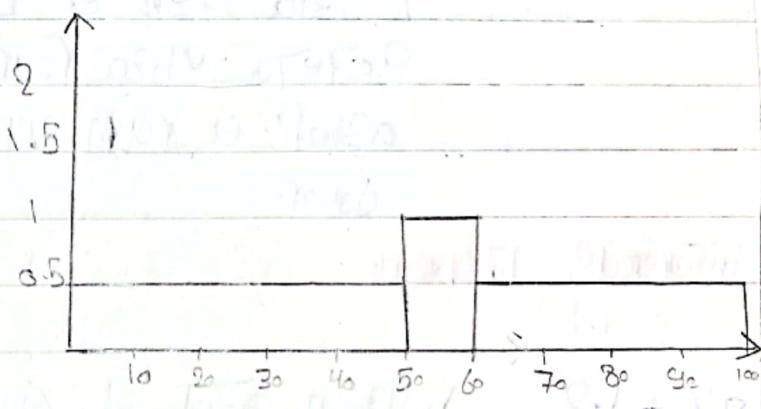
• intervals with unequal width

X-axis → intervals
Y-axis → Height

Area = Height × width → Height = $\frac{\text{Frequency}}{\text{width}}$

Ex: The following Table Shows a Grouped Frequency distribution for the Statistics Grades:

Mark	frequency	width	Height	$\rightarrow \frac{F}{w}$	what is the Sample Size?
3-50	25	50	0.5		$25 + 10 + 20 = 55$
50-60	10	10	1		
60-100	20	40	0.5		Plot The Histogram:



2) Histogram for ungrouped Data:

We Should Following 4 Steps than after el ungrouped Data i grouped Data w o geb el intervals Zdy.

- 1- Find The Smallest and largest Data Point
- 2- Define Sample Size $\rightarrow n$, $K = \sqrt{n}$ (num of intervals)
- 3- Define width of The intervals $L = (\text{largest smallest}) / K$
- 4- Create bin boundaries (smallest el intervals el highest behal)

Start with Smallest number or "less" and adding width L and The last intervals Should contain The largest (S)

Ex: The following Data represent The length of life in years measured to the nearest tenth of 30 similar fuel pumps:

2.6	3.0	2.3	3.3	1.3	0.4
0.2	6.0	5.5	6.5	0.2	2.3
1.5	4.0	5.9	1.8	4.7	0.1
4.5	9.3	1.5	0.5	2.5	5.0
1.0	6.0	5.6	6.6	1.2	0.2

① Largest value = 6.5 Smallest value = 0.2

② n = Sample Size = $5 \times 6 = 30$ $k = \sqrt{n} = \sqrt{30} \approx 5.4 \approx 6$

Note: k is the number of interval that will be used in Table

③ L = width of each interval = $\frac{6.5 - 0.2}{6} = 1.05 \approx 1$

Note: k dayman b3mha round up

L 3la 7sa el Data ehy 3ndy lw kdkta

Sa7etza yb2a round up lw k3or yb2a

ashaf el rom ehy tis h3zmlo round up wala down

Intervals Frequency

Smallest + L

0.2 - 1.2 1. 3shan ar7ot el 6 intervals ehy 3ndy h3dy mn

1.2 - 2.2 el smallest value w ar7ad zleha el L (width)

2.2 - 3.2 0.2 - 1.2 → awl interval

3.2 - 4.2 akmi 17d masis koll 6 interval

4.2 - 5.2 1.2 - 2.2 → tiny interval 17d m awsi li

5.2 - 6.2 → x largest value b2a wasis el intervals ell 6

2. la7m kui el intervals ehy 3ndy ykon el smallest w el largest value fehom TYP lw El 6 interval

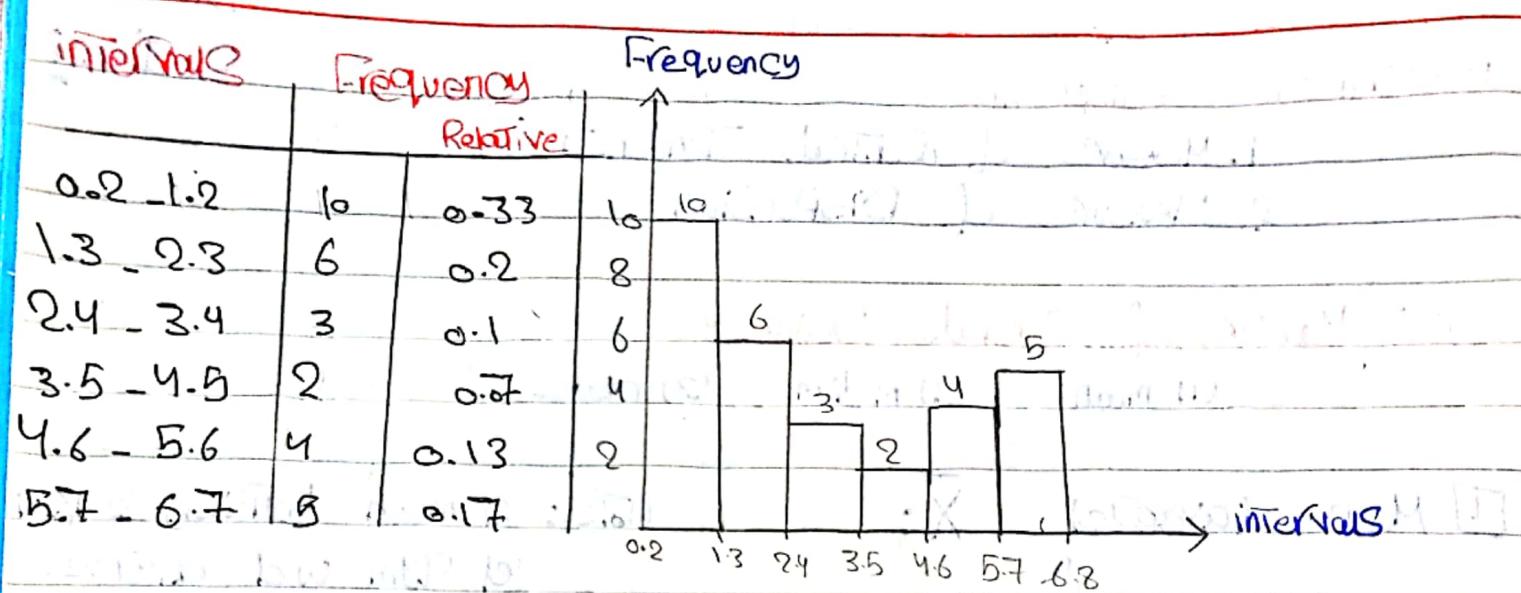
El7to w lsa mustash li large ar7 shayla

el Teekar → 0.2 - 1.2

1.3 - 2.3

2.4 - 3.4

(6)



Relative Frequency = $\frac{f}{n}$ → n = Sample size, f = Frequency

No solo era sayz ausm el relative frequency hñsh nps el steps los bñs M añot el Frequency 3la y axis Hñot el relative Frequency.

En Zally sayz el nearest tenth b2a:

① hegeb el MidPoint = $\frac{\text{lower limit} + \text{upper limit}}{2}$

Mid Point

$$(0.2+1.2) \div 2 = 0.7$$

$$(1.3+2.3) \div 2 = 1.8$$

$$(2.4+3.4) \div 2 = 2.9$$

$$(3.5+4.5) \div 2 = 4$$

$$(4.6+5.6) \div 2 = 5.1$$

$$(5.7+6.7) \div 2 = 6.2$$

② Hñsh 3la el midPoint bñst kult interval w aus ihm b23d

Descriptive Statistics:

1. Measure of Central Tendency
2. Measure of Dispersion

a) Measure of Central Tendency

- (1) mean
- (2) median
- (3) mode

1] Mean (average) \bar{X} :

If we have $\bar{X} = \frac{\sum x_i}{n}$
ungrouped Data

Note: el mean by 2sr b kui

el qym w el extreme

values w msh shirt

y kan f nos dData

$x = (\text{Start} + \text{end}) / 2$ bta3iy

f: frequency

2] Median: it is the value which splits the dataset in half.

1) Order your data from smallest to largest

2) Add el dataset bta3iy even wala odd

a) odd

$$x_{\left(\frac{n+1}{2}\right)}$$

b) even

$$\frac{x_{\left(\frac{n}{2}\right)} + x_{\left(\frac{n}{2}+1\right)}}{2}$$

Ex: 10, 20, 30

$$x_{\left(\frac{3+1}{2}\right)} = x_2$$

Ex: 10, 20, 30, 40

$$\frac{x_2 + x_3}{2} = \frac{2+30}{2} = 16$$

Note: el median ush by 2sr b kui el data elly 3ndy wala
el extreme values hwa maxmd 3ia el data elly f el nos

3] Mode: The mode is the value that occurs the most frequently. There can be one mode, multiple mode or no mode.

Ex: 10, 20, 20, 30, 50

Mode = 20 Ex: 10, 20, 30, 40 No Mode 9

Note : el Mode 3la ek grafhe hwa el A3la frequency
el Mean hwa ely sensitive il extreme value
el Median ely el nos been el values ely 3ndy

Mode < Median < Mean \rightarrow Positively Skewed, Right Skewed

Mode > Median > Mean \rightarrow Negatively Skewed, Left Skewed

Mode = Median = Mean \rightarrow Symmetric

When To Use the Mean?

Symmetric Distribution, because it include all of the Data in Calculations

When You Have Skewed Distribution \rightarrow The Median is a better Measure of Central Tendency Than The Mean

When You Have Categories Data: \rightarrow You must Use The Mode

Ex: The Length of Time, in minutes That 10 Patients waited in a doctor's office before receiving treatment records as following : 5, 11, 9, 5, 10, 15, 6, 10, 5, 10

n 5, 5, 5, 6, 9, 10, 10, 10, 11, 15

1) Mean $\rightarrow \frac{\sum x_i}{n} = 8.6$

2) Median \rightarrow even $= \frac{x_{(\frac{n}{2})} + (x_{(\frac{n}{2})+1})}{2} = \frac{x_5 + x_6}{2} = 9.5$

3) Mode \rightarrow 5, 10 \rightarrow Most Frequency

Q1. feh Examples Tanya f. el See.

(9)

- b) Measure of Dispersion: also called Variability, Scatter, Spread
- (1) Range, (2) Interquartile Range, (3) Variance
 - (4) Standard Deviation

I) Range (R): The Difference between The largest and The Smallest observation in The Data.

Note: It very Sensitive To outliers (extreme) and Doesn't use all The observations in a Data Set.

Ex: Stat Grade (A): 20, 30, 35, 45 $R_A = 45 - 20 = 25$

(B): 1, 30, 35, 45 $R_B = 45 - 1 = 44$

(C): 20, 30, 35, 100 $R_C = 100 - 20 = 80$

Who Had The greater Spread of Marks?

$$R_C > R_B > R_A$$

2) Interquartile Range (IQR): The IQR Describes The Middle 50% of observation

Note: It not effected by extreme value

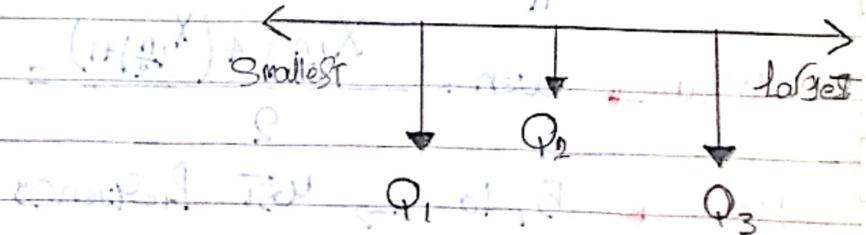
1. Put The Data in order from Smallest To largest.

2. Find Median of all Data : Q_2

3. Find Median of The Data before Q_2 : Q_1

4. " " " " After Q_2 : Q_3

$$IQR = Q_3 - Q_1$$



③ Variance (S^2) and ④ Standard Deviation (S):
 It measure how far a set of number is spread out from their average value.

$$S^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}$$

$$S = \sqrt{S^2}$$

Ex: 9, 2, 5, 4, 12, 7 $n=6$

$$\text{Mean } \bar{x} = \frac{\sum_{i=1}^6 x_i}{n} = \frac{39}{6} = 6.5$$

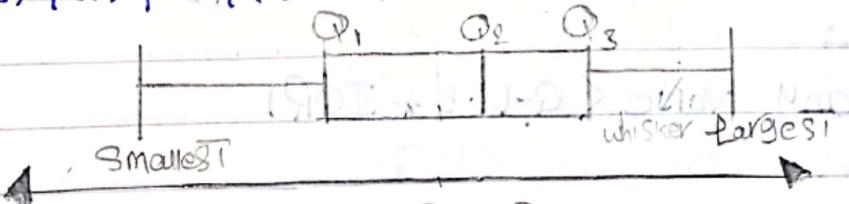
$$\text{Variance } S^2 = \frac{\sum_{i=1}^6 (x_i - \bar{x})^2}{n-1} =$$

$$\sum_{i=1}^6 (x_i - \bar{x})^2 = (9 - 6.5)^2 + (2 - 6.5)^2 + \dots + (7 - 6.5)^2 = 65.5$$

$$S^2 = \frac{65.5}{5} = 13.1 \quad \text{Standard deviation} = \sqrt{13.1} = 3.6$$

Box Plot: a way to show the spread and skewness of Data Set.

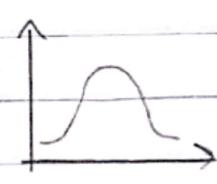
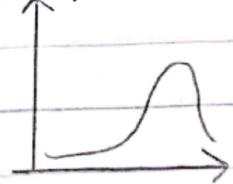
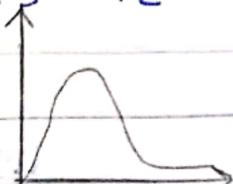
Steps: ①, ②, ③, ④ \rightarrow NP5 Steps IQR



$Q_3 - Q_2 > Q_2 - Q_1$, +ve, right Skew

$Q_3 - Q_2 < Q_2 - Q_1$, -ve, left Skew

$Q_3 - Q_2 = Q_2 - Q_1$, Symmetric



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outliers: any value $> Q_3 + 1.5 \text{ IQR}$

any value $< Q_1 - 1.5 \text{ IQR}$

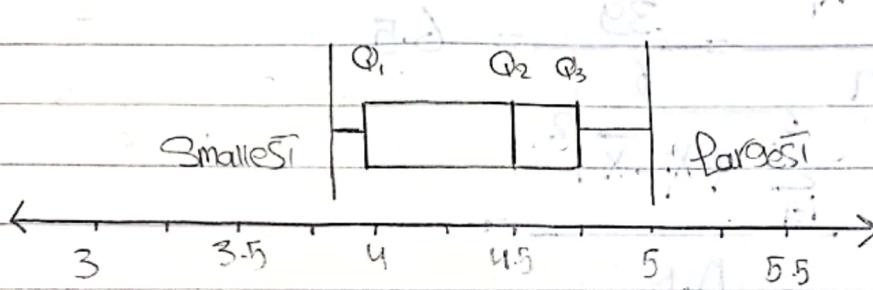
Ex: Draw The boxplot For The following Set of Data:

4.7, 3.8, 3.9, 3.9, 4.6, 4.5, 5

1) 3.8, 3.9, 3.9, 4.5, 4.6, 4.7, 5

2) $Q_2 = 4.5$, $Q_1 = 3.9$, $Q_3 = 4.7$, The largest = 5

The smallest = 3.8



$$Q_3 - Q_2 = 4.7 - 4.5 = 0.2 \quad Q_3 - Q_2 < Q_2 - Q_1 \quad \text{left skew}$$

$$Q_2 - Q_1 = 4.5 - 3.9 = 0.6 \quad -ve$$

outliers: $\text{IQR} = Q_3 - Q_1 = 4.7 - 3.9 = 0.8$

$$1.5 * \text{IQR} = 1.5 * 0.8 = 1.2$$

any value $> (1.5 * \text{IQR}) + Q_3$

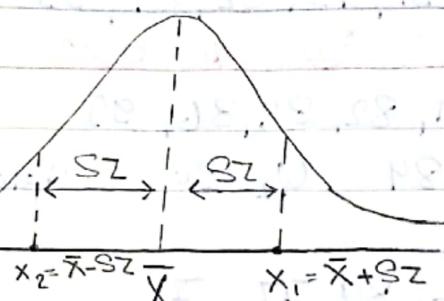
\therefore any value > 5.9

any value $< Q_1 - (1.5 * \text{IQR})$

\therefore any value < 2.7

\therefore No outliers

Z-Score: How Many Standard deviation above or below the Mean it is.



$$\text{Let: } x = \bar{x} + S\sigma$$

$$x - \bar{x} = S\sigma$$

$$\therefore Z = \frac{x - \bar{x}}{\sigma}$$

$\bar{x} \rightarrow \text{Mean}$

$\sigma \rightarrow \text{Standard Deviation}$

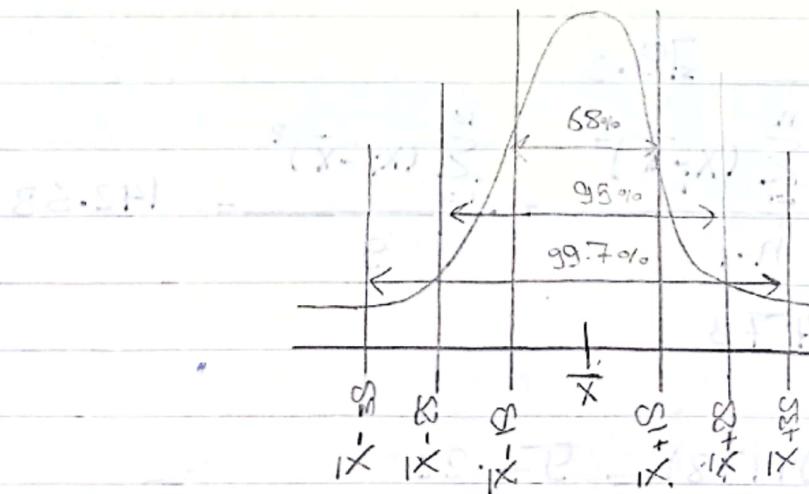
Z-Score if $x = \bar{x}$ $\rightarrow x = \bar{x}$

Z-Score if Positive $\rightarrow x > \bar{x}$

Z-Score if negative $\rightarrow x < \bar{x}$

Empirical Rule: almost all observed Data will fall within Three Standard deviation of The mean

- 68% of The observations Fall within $+/- 1$ Standard Devision from The mean
- 95% \rightarrow Fall within $+/- 2$ Standard Devision from \bar{x}
- 99.7% \rightarrow Fall within $+/- 3$ Standard Devision from \bar{x}



Ex: Student grades on a Chemistry exam were:

77, 78, 76, 81, 86, 51, 79, 82, 84, 99

1) Are there any outliers? If so which score are they?

51, 76, 77, 78, 79, 81, 82, 84, 86, 99

$$Q_3 = \frac{78 + 79}{2} = 78.5 \quad Q_1 = 77 \quad Q_3 = 84 \quad Q_2 = 80 \quad n=10$$

$$\text{IQR} = Q_3 - Q_1 = 84 - 77 = 7$$

$$\text{IQR} * 1.5 = 10.5$$

\therefore Any value $> Q_3 + 1.5 \text{IQR}$

Any value $> 94.5 \rightarrow$ outliers 99

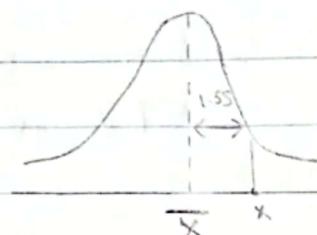
\therefore Any value $< Q_2 - 1.5 \text{IQR}$

Any value $< 68.5 \rightarrow$ outliers 51

2) What would be the score that exceeds the mean by 1.5 standard deviations?
 $X = \bar{X} + 1.5S$

$$X = \bar{X} + 1.5S$$

$$\bar{X} = \text{average} = \frac{\sum_{i=1}^n x_i}{n} = 79.3$$



$$S = \sqrt{S^2} = S^2 = \frac{\frac{n}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2}{n-1} = \frac{\sum_{i=1}^{10} (x_i - \bar{x})^2}{9} = 142.68$$

$$S = \sqrt{142.68} = 11.94478$$

$$X = 79.3 + (1.5)(11.94478) = 97.22$$

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Section 4

CORRELATION: Shows The Strength and The Direction of a relationship between Two Variables and is expressed numerically by The Correlation Coefficient "r"

II Algebraic: Correlation is assumed to be linear (Following line)

$$-1 \leq r \leq 1$$

$r=0$ is no correlation

$r = \frac{\text{Sign}}{\text{Value}}$

↓ direction ↓ strength

+ve → Perfect Strong Moderate Weak

↓ direct & weak

↓ inverse & weak

↓ 0.7, 0.8, 0.9 | 0.1, 0.2, 0.3

↓ 0.4, 0.5, 0.6

Una Tabla el Social la muestra el dirección y el strength
Falta el resto: Direct, Moderate, inverse, weak, +ve perfect

$$r = \frac{\sum Z_x Z_y}{n-1}, \quad Z_x = \frac{X - \bar{X}}{S_x}, \quad Z_y = \frac{Y - \bar{Y}}{S_y}$$

X	Z _x	Y	Z _y	Z _y Z _x

luego se saca el Table de la Fórmula
MÉTRICO CORRELACIÓN

Ex: Calculate The Correlation Coefficient For The Following Data:

$$\bar{X} = \frac{\sum X_i}{n} = 5 \quad S_x = 4.3818 \quad Z_x \quad Z_y \quad Z_y \quad Z_x \quad X \quad Y$$

-2.10 -1.3 1.5 12 1

$$\bar{Y} = \frac{\sum Y_i}{n} = 4 \quad S_y = 2.2804 \quad Z_y \quad 0.3008 1.3 0.68 8 7$$

0 0 0 5 4

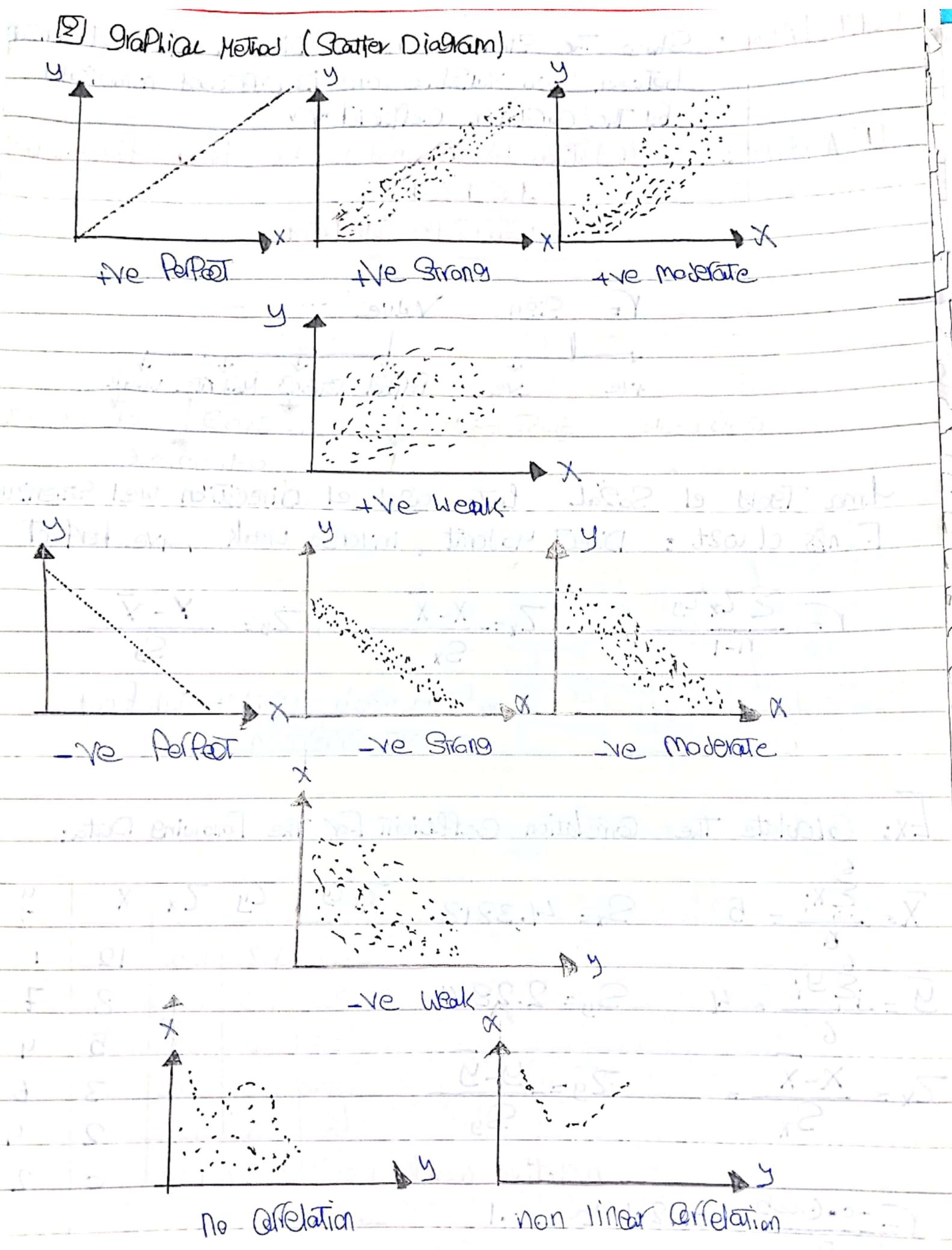
$$Z_x = \frac{X - \bar{X}}{S_x} = \quad Z_y = \frac{Y - \bar{Y}}{S_y} = \quad -0.4003 0.87 0.45 3 6$$

0 0 0.68 2 4

relative weak 1.0007 -0.87 1.14 0 2

$$r = \frac{-0.6005}{5} = 0.1201 \approx -0.1 \quad -0.6005$$

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Simple Linear Regression: It's used to describe or predict the relationship between two variables

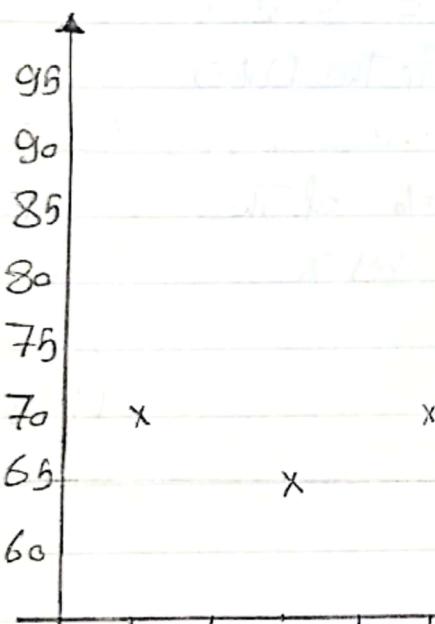
$$y = \beta_0 + \beta_1 x + e \rightarrow \beta_1 = r \left(\frac{s_y}{s_x} \right)$$

y = The Dependent Variable β_0 = Is The y -intercept of The
 x = The independent Variable regression line.

β_1 = IS The Slope

Ex:

a) Draw The Scatter Plot Representing The Data :



x_i	y_i	z_x	z_y	$z_x z_y$
65	70	1.258	0.6378	0.8022
70	75	0.5182	1.4343	0.7433
75	80	0.1480	-0.8878	-0.0826
80	85	-0.592	-0.9362	0.5663
85	90	-1.332	-0.5578	0.7432
				Total = 2.7724

b) What Is The linear regression equation best Predicts Statistics Performance Based on Math aptitude Scores?

$$y = \beta_0 + \beta_1 x + e$$

$$\beta_1 = r \left(\frac{s_y}{s_x} \right), \beta_0 = \bar{y} - \beta_1 \bar{x}$$

$$S_{ys} = \frac{\sum (y - \bar{y})^2}{n-1}$$

$$S_{xs} = \frac{\sum (x - \bar{x})^2}{n-1}$$

$$r = \frac{\sum z_x z_y}{n-1}$$

$$\beta_1 = 0.6931 * \frac{12.5499}{13.5093}$$

$$n=5 \quad \bar{y}=77 \\ S_{ys} = 12.5499$$

$$n=5 \quad \bar{x}=78 \\ S_{xs} = 13.5093$$

$$r = \frac{2.7724}{4} \\ r = 0.6931$$

$$\beta_1 = 0.644 \\ \beta_0 = 77 - 0.644 * 78 \\ \beta_0 = 26.768$$

$$\hat{y} = 26.768 + 0.644x$$

c) If a student made an 80 on the aptitude test, what grade would we expect her to make in statistics?

at $X = 80$

$$\hat{Y} = \beta_0 + \beta_1 X$$

$$\hat{Y} = 26.768 + 0.644 \times 80 =$$

$$\hat{Y}_{80} = 78.288$$

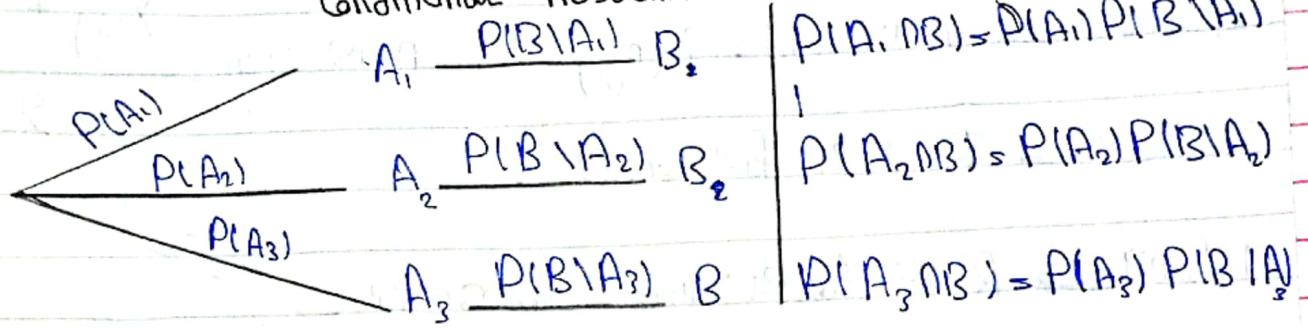
$$\text{Error} = |\hat{Y} - y| = |78.288 - 70| \\ = 8.288$$

d) How well does the regression equation fit the data?

$R^2 = (0.6931)^2 = 0.4804 \times 100 = 48.4\%$ of the variation in y can be described by x

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Bayes' Theorem: IS a mathematical formula for determining Conditional Probability



$$P(B) = P(A_1 \cap B) + P(A_2 \cap B) + P(A_3 \cap B)$$

$$P(B) = P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + P(B|A_3)P(A_3)$$

P(B) = Sum

Ex: All tractors made by a company are produced on one of three assembly lines, named Red, Blue, White. The chances that a tractor will not start when it rolls off a line are 6%, 11%, 8% for lines Red, Blue, White, respectively. 48% of the company's tractors are made on the Red line and 31% are made on the Blue line.

Red line: R, Blue line: B, White line: W, Not start: D (defective)

a) What fraction of the company's tractors don't start when they roll off an assembly line?

$$P(D) = P(R \cap D) + P(B \cap D) + P(W \cap D)$$

$$0.0288 + 0.0848 + 0.0231 = 0.0767$$

$P(R) = 0.48$	R	$P(D R) = 0.06$	$P(D R) = 0.0288$
$P(B) = 0.31$	B	$P(D B) = 0.08$	$P(D B) = 0.0848$
$P(W) = 0.21$	W	$P(D W) = 0.11$	$P(D W) = 0.0231$

b) What is the probability that a tractor came from the Red Company given that it was defective?

$$P(R|D) = \frac{P(R \cap D)}{P(D)}, \quad \frac{0.0288}{0.0767} = 0.3755$$

Best method
for Solution

Binomial Distribution:

To Study the number of Successes in a Sequence of n independent experiments.

$$P(X=r) = P(X=r) = \binom{n}{r} p^r q^{n-r}$$

$$E(X) = np$$

$$\text{Var}(X) = npq$$

Normal Distribution:

Data Tends To be around a Central value with no bias Left or Right \rightarrow get Close To Normal Distribution

$$Z = \frac{X - \mu}{\sigma} \rightarrow \text{Transforms "X" Mn normal Distribution To Standard Normal "Z"}$$

Note: If $P(Z < a) = 1 \therefore a > 3.49$

If $P(Z < 0) = 0 \therefore a < -3.4$

Central Limit Theorem:

X is normal \leftarrow Sample Distribution of Sample Mean Distribution

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$Z = \frac{X - \mu}{\sigma}$$

- Ima tgeley MaSla w mSh
- 3alif emta ASTBdm anhy law
- bta3 el mean w normal DIS
- aShaf el Givens elly f el SoSal
- w el zaga elly talbha kath
- Probability of random
- Probability of Mean of Random

• Sample Distribution of Difference Between Two Means

$$(\bar{X}_1 - \bar{X}_2) - (M_1 - M_2)$$

$$Z = \frac{\text{_____}}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

X is \leftarrow Sample Distribution of The Sample Proportion

Binomial
Distribution

$$Z = \frac{\hat{P} - P}{\sqrt{\frac{P(1-P)}{n}}}$$

$$M_{\hat{P}} = P$$

$$\sigma_{\hat{P}} = \sqrt{\frac{P(1-P)}{n}}$$

Note : The Sample Distribution of Sample Proportion \hat{P}
is approximately normal if $NP \geq 15$ and $N(1-P) \geq 15$

• Sampling Distribution of Difference Between Two Proportions

$$Z = \frac{(\hat{P}_1 - \hat{P}_2) - (P_1 - P_2)}{\sqrt{\frac{P_1 q_1}{n_1} + \frac{P_2 q_2}{n_2}}}$$

$$\sqrt{\frac{P_1 q_1}{n_1} + \frac{P_2 q_2}{n_2}}$$

- Confidence Intervals (CI) of The Population Mean "M"

There is A Confidence Interval for mean

$$CI : \bar{X} \pm Z_{\alpha/2} \sigma_{\bar{X}}$$

Step1 : n, \bar{X} , σ

Step2 : $1-\alpha = ... \%$

Step3 : If σ known \rightarrow Z-table $CI = \bar{X} \pm Z_{\alpha/2} \sigma_{\bar{X}}$
If σ unknown \rightarrow If $n < 30 \rightarrow$ T-Table
If $n > 30 \rightarrow$ Z-Table

Confidence Intervals of The Difference between Two Population Means

$$\bar{X} \pm Z_{\frac{\alpha}{2}} S_x$$

Step 1: n_1, n_2 \bar{X}_1, \bar{X}_2 σ_1, σ_2 OR S_1, S_2

Step 2: $1 - \alpha = -\%$

Step 3: If σ_1, σ_2 known \rightarrow use Z-table

If σ_1, σ_2 unknown

$$\bar{X}_1 - \bar{X}_2 \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$\sigma_1 = \sigma_2$$

$$\sigma_1 \neq \sigma_2$$

T-Table

$$n_1, n_2 > 30$$

Z-Table

$$n_1, n_2 < 30$$

T-Table

$$\bar{X}_1 - \bar{X}_2 \pm T_{\frac{\alpha}{2}, n_1 + n_2 - 2} \sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}$$

$$\text{Note: } V = n_1 + n_2 - 2 = n_1 - 1 + n_2 - 1 = n_1 + n_2 - 2$$

Pooled Variance s_p^2 :

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

$$n_1 + n_2 - 2$$

• Confidence interval of The Population Proportion \hat{P}

$$CI = \hat{P} \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{P}\hat{q}}{n}}$$

Step 1: q, n, P, \hat{P} $\rightarrow P = \frac{x}{n} \rightarrow x$ num of defective

Step 2: $1 - \alpha = - \%$

Step 3: Calculate CI: $\hat{P} \pm Z_{\frac{\alpha}{2}} \sigma_{\hat{P}}$

• Confidence interval of The difference between Two Population Proportions:

$$CI = (\hat{P}_1 - \hat{P}_2) \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{P_1 q_1}{n_1} + \frac{P_2 q_2}{n_2}}$$

Step 1: $P_1, P_2, q_1, q_2, n_1, n_2, X_1, X_2$

Step 2: $1 - \alpha = - \%$

Step 3: Calculate CI: $(\hat{P}_1 - \hat{P}_2) \pm Z_{\frac{\alpha}{2}} (\sigma_{\hat{P}_1} - \sigma_{\hat{P}_2})$