Lista 1

2. Exemplo 7.2.7

- $\theta \in \{0.1, 0.2\}$
- $\xi(0.1) = 0.7$
- $X_1, \dots, X_8 \overset{iid}{\sim} \text{Bernoulli}(\theta)$, pois são experimentos de sucesso e falha de um mesmo processo. Também podemos supor independência, pois selecionamos ao acaso.
- Queremos $\xi(\theta | x_1, \dots, x_8)$
- Se $X = \sum_{i=1}^{8} x_i$, $f_8(x_1, \dots, x_8 | \theta) = \prod_{i=1}^{8} f(x_i|\theta) = \prod_{i=1}^{8} \theta^{x_i}(1-\theta)^{1-x_i} = \theta^x (1-\theta)^{8-x}$
  Nesse caso $X = 2$, logo $f_8(x|\theta) = \theta^2(1-\theta)^6$
- Logo:
  $\xi(0.1 | x_1, \dots, x_8) \propto \theta^2(1-\theta)^6 \cdot 0.7$
- $g_n(x) = \int_\Omega f(x|\theta) \cdot \xi(\theta)\, d\theta = (0.1)^2 \cdot 0.9^6 \cdot 0.7 + (0.2)^2 (0.8)^6 \cdot 0.3 = 6.86 \cdot 10^{-3}$
- Concluo que $\xi(0.1 | x_1, \dots, x_8) = \dfrac{(0.1)^2 (0.9)^6 \cdot 0.7}{6.86 \cdot 10^{-3}} \approx 0.542$
  $\xi(0.2 | x_1, \dots, x_8) \approx 0.458$

3. Seja $X = \#$ defeitos $\sim \text{Poisson}(\lambda)$

- $\lambda \in \{1, 1.5\}$
- $\xi(1.0) = 0.4$
- Queremos saber $\xi(\lambda | X = 3)$
- $f(x|\lambda) = \dfrac{e^{-\lambda} \lambda^x}{x!}$
- $\xi(\lambda | x = 3) \propto \dfrac{e^{-1} \cdot 1^3}{3!} \cdot \xi(\lambda)$
- $g(x) = \int_\Omega f(x|\lambda) \xi(\lambda)\, d\lambda = \dfrac{e^{-1} \cdot 1^x}{x!} \cdot 0.4 + \dfrac{e^{-1.5} \cdot (1.5)^x}{x!} \cdot 0.6$
- Logo:
  $\xi(1.0 | x = 3) = \dfrac{\dfrac{e^{-1} \cdot 1^3}{\cancel{3!}} \cdot 0.4}{\dfrac{e^{-1} \cdot 1^3}{\cancel{3!}} \cdot 0.4 + \dfrac{e^{-1.5} (1.5)^3}{\cancel{3!}} \cdot 0.6} \approx 0.2457$
  $\xi(1.5 | x = 3) \approx 1 - 0.2457 = 0.7543$

10. $X \sim \text{Unif}[\theta - 1/2, \theta + 1/2]$
$\Theta \sim \text{Unif}[10, 20]$
$x = 12$

- $f(x|\theta) = \dfrac{1}{\theta + 1/2 - (\theta - 1/2)} = \mathbb{1}[\theta - 1/2, \theta + 1/2]$
- $\xi(\theta) = 1/(20 - 10) = 1/10\ \mathbb{1}[10, 20]$
- $\xi(\theta | x = 12) \propto \mathbb{1}[\theta - 1/2, \theta + 1/2] \cdot \mathbb{1}[10, 20] \cdot 1/10$
  Sabemos, então, que será uma uniforme. Mas qual o intervalo?
- $\left.\begin{array}{l} \theta - 1/2 < x \Rightarrow \theta < x + 1/2 \\ \theta + 1/2 > x \Rightarrow \theta > x - 1/2 \end{array}\right\} \Rightarrow x - 1/2 < \theta < x + 1/2$ e $10 < \theta < 20$
- Ora, como $x = 12$, $11.5 < \theta < 12.5$, então:
  $\xi(\theta | x = 12) = \mathbb{1}[11.5, 12.5]$

11. $\Theta \sim \text{Unif}[10, 20]$
$X_1, \dots, X_6 \sim [\theta - 1/2, \theta + 1/2]$, $x_1 = 11$, $x_2 = 11.5$, $x_3 = 11.7$, $x_4 = 11.1$, $x_5 = 11.4$ e $x_6 = 10.9$

- $\xi(\theta) = \begin{cases} 1, & \text{se } \theta \in [10, 20] \\ 0, & \text{c.c.} \end{cases}$
- $\mathcal{L}(\theta | \vec{X}) = f(\vec{x}|\theta) = \prod_{i=1}^{6} f(x_i | \theta) = \prod_{i=1}^{6} \mathbb{1}\{\theta \in A_i\}$, onde $\mathbb{1}\{\theta \in A_i\} = \begin{cases} 1, & \theta \in A_i \\ 0, & \text{c.c.} \end{cases}$
  $\theta \mapsto L(\theta)$, função de θ
  e A_i é o suporte da distribuição. Temos que defini-lo.
  suporte de $f(x_i|\theta)$
- Se $\theta - 1/2 \leq x \leq \theta + 1/2 \Rightarrow x + 1/2 \geq \theta$ e $x - 1/2 \leq \theta \Rightarrow x - 1/2 \leq \theta \leq x + 1/2$ e $10 \leq \theta \leq 20$.
  suporte $f(\theta|x)$ — Priori
- Logo, $f(x|\theta) = \prod_{i=1}^{6} \mathbb{1}\{\theta \in [x_i - 1/2, x_i + 1/2]\}$ → Quando isso é 1? Quando TODAS são 1!
  $= \mathbb{1}\{\theta \in \bigcap_{i=1}^{6} [x_i - 1/2, x_i + 1/2]\}$
  $= \mathbb{1}\{\theta \in [10.5, 11.5] \cap [11, 12] \cap [11.2, 12.2] \cap [10.6, 11.6] \cap [10.9, 11.9] \cap [10.4, 11.4]\}$
  $= \mathbb{1}\{\theta \in [11.2, 11.4]\} = \mathbb{1}\{\theta \in [\max\{x_i - 1/2\}, \min\{x_i + 1/2\}]_{i=1}^{6}\}$
- $\xi(\theta | x) \propto f(x|\theta)\xi(\theta) = \mathbb{1}\{\theta \in [11.2, 11.4]\}\, \mathbb{1}\{\theta \in [10, 20]\}$
  $= \mathbb{1}\{\theta \in [11.2, 11.4]\}$
- Logo $\xi(\theta|x) = 5 \cdot \mathbb{1}\{\theta \in [11.2, 11.4]\}$, pois $\int_{\mathbb{R}} \xi(\theta|x) = 1$