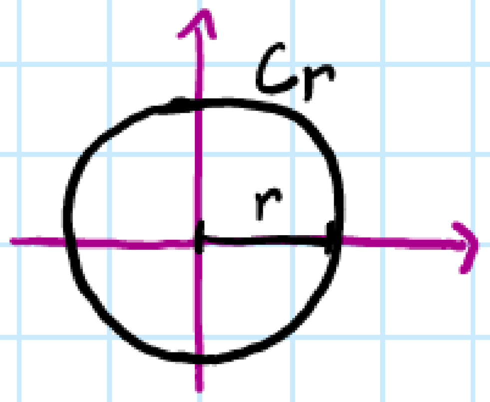
Capitulo_8.2

Isso significa que $(x,y) \in C_r$

4. $X, Y \overset{iid}{\sim} N(0,1)$



$r = \underset{r>0}{\text{argmin}}\ \mathbb{P}(X^2 + Y^2 \leq r^2) = 0.99$

Sabemos que $X^2 \sim \chi^2(1)$ com um grau de liberdade e $Y^2 \sim \chi^2(1)$

Logo $X^2 + Y^2 \sim \chi^2(2)$

Queremos que $\mathbb{P}(X^2 + Y^2 \leq r^2) = 0.99$. Olhando no Stat Treck, vamos que $r^2 = 9.21 \Rightarrow r = \sqrt{9.21} \approx 3{,}03$

7. $X_1, \dots, X_n$ independentes com cdf contínua F_i. Defina $Y := -2 \sum_{i=1}^n \log F_i(x_i)$

Defina $T_i := F_i(X_i)$. Tome $x \in [0,1]$ (imagem de $F_i(X_i)$)

$\mathbb{P}(T_i \leq x) = \mathbb{P}(F_i(X_i) \leq x) = \mathbb{P}(X_i \leq F_i^{-1}(x)) = F_i(F_i^{-1}(x)) = x$ ↗ Universalidade da Uniforme

Logo $T_i \sim Unif[0,1]$.

Defina $Z_i := -2\log T_i$. Vou encontrar a distribuição de Z_i. Tome $x \in [0, \infty)$ ↗ intervalo de $-\log[0,1]$

$\mathbb{P}(Z_i \leq x) = \mathbb{P}(\log T_i \geq -x/2) = \mathbb{P}(T_i \geq e^{-\frac{x}{2}}) = 1 - \mathbb{P}(T_i \leq e^{-x/2}) = 1 - e^{-\frac{x}{2}}$

Ora, logo $Z_i \sim \exp(1/2)$

Assim $Y = \sum_{i=1}^n Z_i \sim Gamma(n, 1/2) \equiv Gamma(2n/2, 1/2) \equiv \chi^2(2n)$

10. $X_1, \dots, X_6 \sim N(0,1)$

$Y = (X_1 + X_2 + X_3)^2 + (X_4 + X_5 + X_6)^2$

Observe que $(X_1 + X_2 + X_3), (X_4 + X_5 + X_6) \sim N(0,3) \Rightarrow \frac{1}{\sqrt{3}}(X_1 + X_2 + X_3), \frac{1}{\sqrt{3}}(X_4 + X_5 + X_6) \sim N(0,1)$

Portanto $\frac{1}{3}(X_1 + X_2 + X_3)^2 + \frac{1}{3}(X_4 + X_5 + X_6)^2 \sim \chi^2(2)$

Portanto $c = \frac{1}{3}$

13. $\hat{\sigma}_0^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \mu)^2$

Veja que $\frac{X_i - \mu}{\sigma} \sim N(0,1) \Rightarrow \sum_{i=1}^n \left(\frac{X_i - \mu}{\sigma}\right)^2 \sim \chi^2(n)$

Logo $\frac{n\hat{\sigma}_0^2}{\sigma^2} \sim \chi^2(n) \equiv Gamma(n/2, 1/2)$

Portanto $\hat{\sigma}_0^2 \sim Gamma(n/2, n/2\sigma^2)$

$X \sim Gamma(\alpha, \beta)$
$cX \sim Gamma(\alpha, \beta/c)$
No caso $c = \frac{\sigma^2}{n}$