Capítulo_9.5

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4. $X_1, \dots, X_8 \overset{iid}{\sim} N(\mu, \sigma^2)$, (μ, σ^2) desconhecidos.

$$H_0: \mu = 0$$
$$H_1: \mu \neq 0$$

$\sum_{i=1}^n x_i = -11{,}2$
$\sum_{i=1}^n X_i^2 = 43.7$
$\alpha_0 = 0.1$

Vamos rejeitar H_0 se $|U| \geq c$, onde
$$U = \frac{n^{1/2}\,\bar{X}_n}{\sigma'}$$

e
$$\alpha_0 = \mathbb{P}(|U| \geq c \mid \mu = 0)$$
$$= \mathbb{P}(U \leq -c \mid \mu = 0) + \mathbb{P}(U \geq c \mid \mu = 0)$$
$$= 2\mathbb{P}(U \geq c \mid \mu = 0)$$
$$= 2(1 - \mathbb{P}(U \leq c)), \quad U \sim t(n-1) \equiv t(7)$$
$$\Rightarrow c = T_7^{-1}(1 - \alpha_0/2) = T_7^{-1}(0.95) \cong 1{,}89$$

$$\sigma'^2 = \frac{1}{n-1}\sum_{i=1}^n (x_i - \bar{x}_n)^2 = \frac{1}{n-1}\sum_{i=1}^n (X_i^2 - 2X_i\bar{X}_n + \bar{X}_n^2)$$
$$= \frac{1}{n-1}\left(\sum_{i=1}^n X_i^2 - 2\bar{X}_n \sum_{i=1}^n X_i + n\bar{X}_n^2\right)$$
$$= \frac{1}{n-1}\left(\sum_{i=1}^n x_i^2 - n\bar{X}_n^2\right)$$
$$= \frac{1}{7}\left(43.7 - 8(-11.2/8)^2\right) \approx 4$$

Logo
$$u = \frac{\sqrt{8}\cdot(-11{,}2)}{8\cdot\sqrt{4}} \approx -1{,}94$$

Como $|u| > c$, rejeitamos H_0.

5. Nesse caso:

$\mathbb{P}(U \leq c_1) = 0.01 \Rightarrow c_1 = T_7^{-1}(0.01) \approx -3$
$\mathbb{P}(U \geq c_2) = 0.09 \Rightarrow c_2 = 1 - T_7^{-1}(0.91) \approx 1{,}48$

Calculamos em 4 que
$$u = -1{,}94$$
Como $c_1 < u < c_2$, não rejeitamos H_0.

8. $X_1, \dots, X_n \sim N(\mu, \sigma^2)$, μ, σ desconhecidos

$$H_0: \sigma^2 \leq \sigma_0^2$$
$$H_1: \sigma^2 > \sigma_0^2$$

Queremos um teste nível α_0. $S_n^2 = \sum_{i=1}^n (X_i - \bar{X}_n)^2$
Rejeitamos H_0 se $S_n^2/\sigma_0^2 \geq c$

$$\pi(\mu, \sigma^2 \mid \delta) = \mathbb{P}(S_n^2 \geq c\cdot\sigma_0^2 \mid (\mu, \sigma^2))$$

Se $\sigma^2 = \sigma_0^2$, provamos que $S_n^2 \sim \chi^2_{n-1}$

Logo queremos que $\mathbb{P}(S_n^2/\sigma_0^2 \geq c \mid \sigma^2 = \sigma_0^2) = \alpha_0$
$\hookrightarrow 1 - \mathbb{P}(S_n^2/\sigma_0^2 \leq c)$

Tome $c = (\chi^2_{n-1})^{-1}(1 - \alpha_0) \Leftrightarrow 1 - \chi^2_{n-1}(c) = \alpha_0$

Agora suponha que $\sigma^2 \neq \sigma_0^2$. Então $S_n^2/\sigma^2 \sim \chi^2_{n-1}$
e, então, $T = \frac{\sigma_0^2}{\sigma^2}\cdot\frac{S_n^2}{\sigma_0^2} \sim \chi^2_{n-1}$.

$$\pi(\mu, \sigma^2 \mid \delta) = \mathbb{P}(S_n^2/\sigma_0^2 \geq c \mid \mu, \sigma^2)$$
$$= \mathbb{P}(T \geq c\sigma_0^2/\sigma^2 \mid \mu, \sigma^2) = 1 - \chi^2_{n-1}(c\,\sigma_0^2/\sigma^2)$$

Se $\sigma^2 < \sigma_0^2 \Rightarrow \sigma_0^2/\sigma^2 > 1$, logo $\chi^2_{n-1}(c) < \chi^2_{n-1}(c\,\sigma_0^2/\sigma^2)$
E assim $\pi(\mu, \sigma^2 \mid \delta) < 1 - \chi^2_{n-1}(c) = \alpha_0$

Se $\sigma^2 > \sigma_0^2$, temos o análogo, logo $\pi(\mu, \sigma^2 \mid \delta) > \alpha_0$.