Experiment 6

Aim

Implementation of Quantum Fourier Transform circuit in IBM Quantum Composer.

Theory

The Quantum Fourier Transform (QFT) is a powerful tool in quantum computing, used for tasks such as quantum phase estimation and quantum algorithms like Shor's algorithm. The QFT is the quantum analogue of the classical Fourier transform, which is used to analyze signals in classical information processing.

The QFT operates on a quantum state $|\psi\rangle = \sum_{x=0}^{2^n-1} c_x|x\rangle$, where n is the number of qubits and c_x are complex amplitudes that describe the state of the qubits. The QFT maps this input state to a new state $|\tilde{\psi}\rangle = \sum_{y=0}^{2^n-1} \tilde{c}_y|\tilde{y}\rangle$, where $\tilde{c}_y$ are new complex amplitudes and $\tilde{y}$ is the Fourier transform of x.

The QFT circuit is composed of several fundamental quantum gates, including the Hadamard gate and the controlled phase gate. The Hadamard gate is a single-qubit gate that transforms the state $|0\rangle$ to $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and the state $|1\rangle$ to $\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$. The controlled phase gate is a two-qubit gate that introduces a phase shift of $e^{i\theta}$ on the target qubit if the control qubit is in the state $|1\rangle$, where θ is a constant.

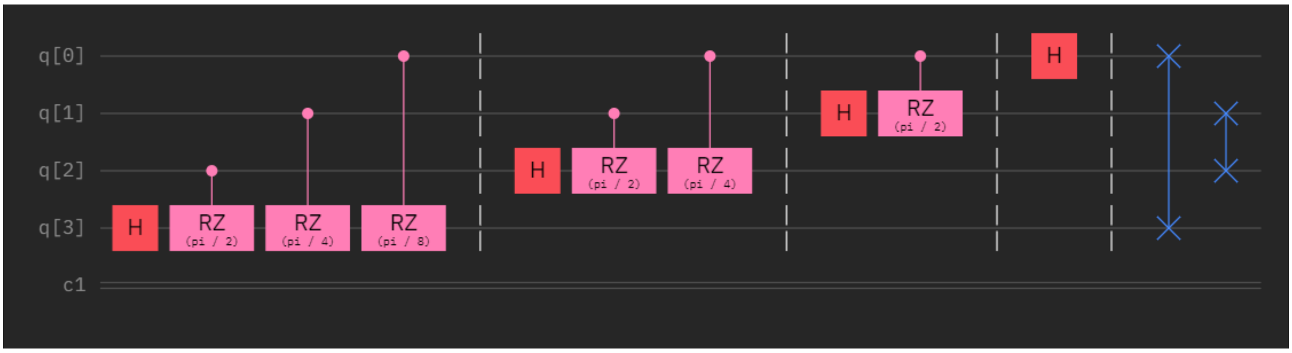
To implement the QFT circuit on n qubits, we can follow the following steps:

- Initialize n qubits in the state $|x\rangle$, where x is an integer in binary representation.
- Apply a Hadamard gate to the first qubit.
- Apply a controlled phase gate between the first qubit and the second qubit with a phase of $e^{i\frac{\pi}{2}}$.
- Apply a controlled phase gate between the first qubit and the third qubit with a phase of $e^{i\frac{\pi}{4}}$.
- Continue applying controlled phase gates between the first qubit and each of the remaining qubits, with phases of $e^{i\frac{\pi}{2^k}}$ for the kth qubit.
- Apply a Hadamard gate to the second qubit, and continue applying Hadamard gates to each of the remaining qubits in succession.
- Swap the i^{th} qu-bit with $(n-i)^{th}$ qu-bit.

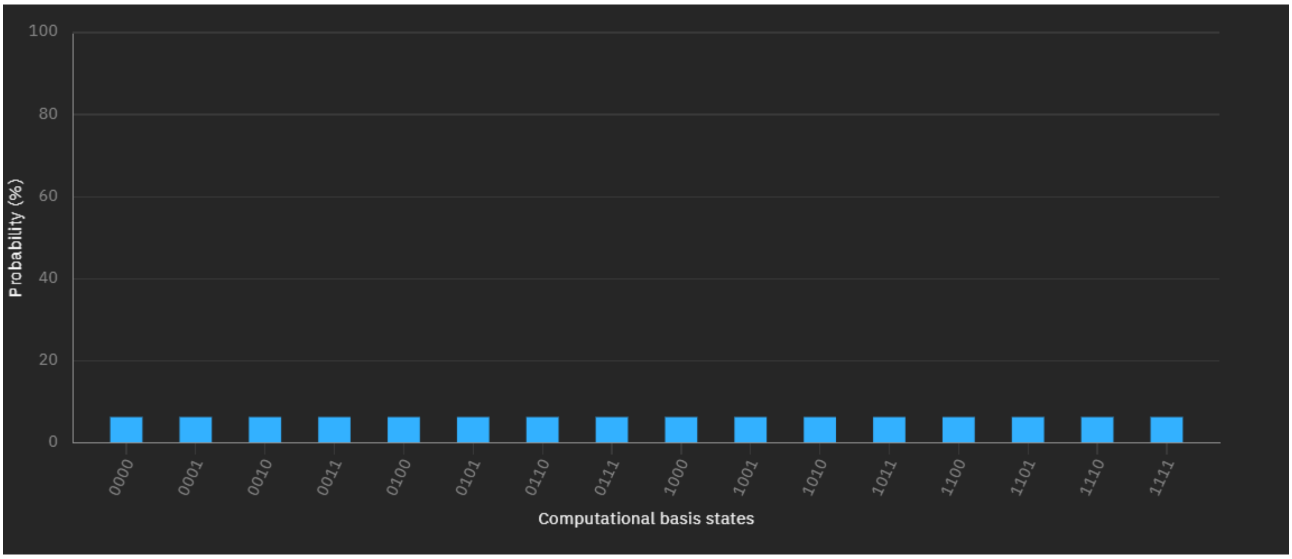
This completes the QFT circuit, which maps the input state $|x\rangle$ to the output state $|\tilde{x}\rangle$, where $\tilde{x}$ is the Fourier transform of x. The QFT circuit can be implemented on real quantum hardware, such as the IBM Quantum systems, using quantum gates and circuits designed for this purpose.

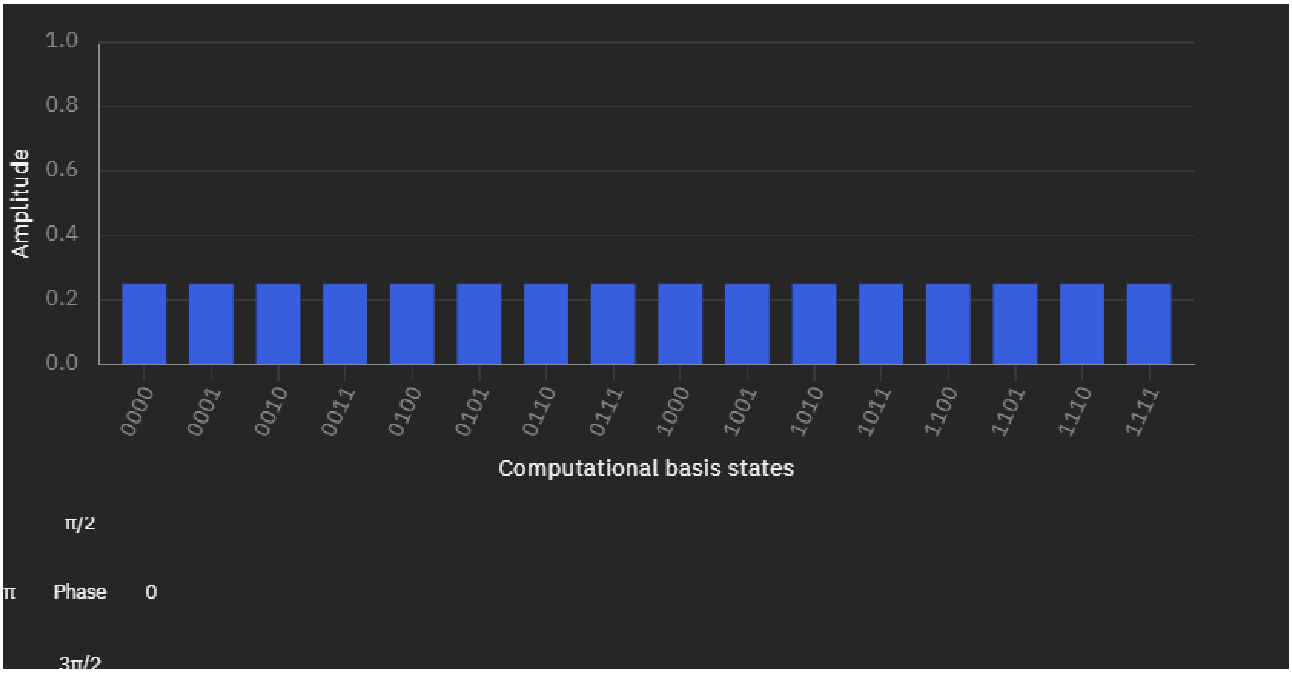
Lab Exercise

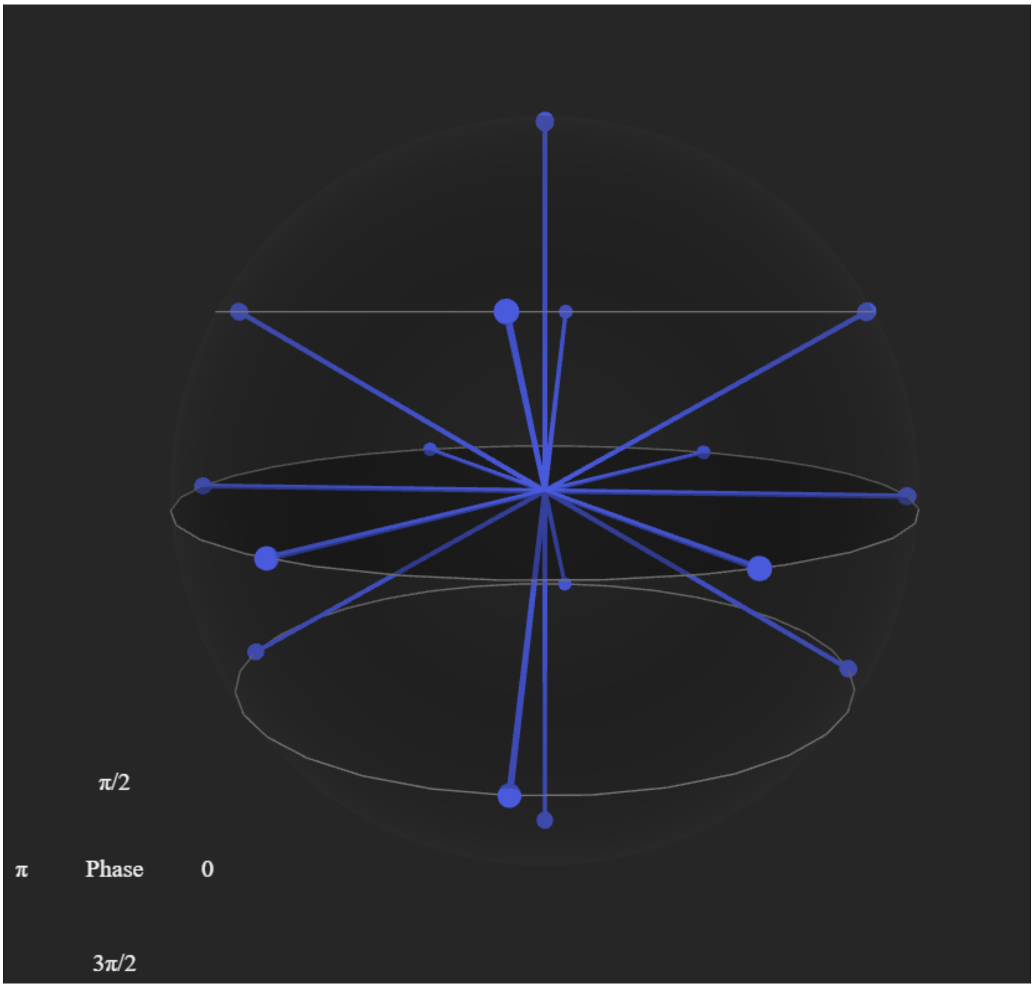
IBM Composer Circuit:



Results:


100
80
60
40
20
0
Probability (%)
0000
0001
0010
0011
0100
0101
0110
0111
1000
1001
1010
1011
1100
1101
1110
1111
Computational basis states


1.0
0.8
0.6
0.4
0.2
0.0
Amplitude
0000
0001
0010
0011
0100
0101
0110
0111
1000
1001
1010
1011
1100
1101
1110
1111
Computational basis states
π/2
π
Phase
0
3π/2



Conclusion

In conclusion, the Quantum Fourier Transform (QFT) circuit is a fundamental quantum computing circuit that plays an important role in many quantum algorithms, including Shor's algorithm for computing large numbers and the quantum phase estimation algorithm for estimating the eigenvalues of unitary operators. QFT circuit is used to transform quantum space from the time domain to the frequency domain, which can be useful for analyzing and manipulating quantum systems.

Implementing a QFT circuit in IBM Quantum Composer provides researchers and developers with a convenient and intuitive way to design, simulate, and test a QFT circuit in a variety of quantum systems, including quantum simulators and real quantum hardware. The ability to implement the QFT circuit on IBM Quantum systems is an important step towards realizing the potential of quantum computing to solve complex problems in a variety of fields, including cryptography, chemistry, and optimization.

Overall, the Quantum Fourier transform circuit is an important building block in the field of quantum computing, and the availability of tools such as IBM Quantum Composer makes it easy for researchers and programmers to study and experiment with this important circuit.