TUTORIAL WORKBOOK

FOR

UBA10

NUMERICAL METHODS

Department of Mathematics

Institute of Science and Humanities

SIMATS School of Engineering

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PERFORMANCE ANALYSIS

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<tr><td colspan="4">Department:</td></tr>
<tr><td colspan="4">Course Code & Title:</td></tr>
<tr><td>S. No.</td><td>Tutorials</td><td>Maximum Marks</td><td>Mark Scored</td></tr>
<tr><td>1</td><td>Unit 1 – Problems</td><td>150</td><td></td></tr>
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Signature of the Course Faculty

UNIT I – SOLUTION OF EQUATIONS AND EIGEN VALUE PROBLEMS

1. Solve $e^x - 3x = 0$ by the method of fixed point iteration.

Solution:

2. Using Newton-Raphson method find the real root of $f(x) = 3x + sinx - e^x = 0$ by choosing initial approximation $x_0 = 0.5$.

Solution:

3. Find the root of $4x - e^x = 0$ that lies between 2 and 3 by Newton-Raphson method.

Solution:

4. Find the approximate root of $xe^x = 3$ by the method of false position correct to three decimal places.

Solution:

5. Find a root of $xlog_{10}x - 1.2 = 0$ using Newton-Raphson method correct to three decimal places.

Solution:

6. Using Guass-Jordan method to solve the given system of equations:

$$10x + y + z = 12; 2x + 10y + z = 13; x + y + 5z = 7.$$

Solution:

7. Apply Guass Jordan method, find the solution of the following system:

$$2x - y + 3z = 8;\ -x + 2y + z = 4; 3x + y - 4z = 0.$$

Solution:

8. Solve the following system of equations, starting with the initial vector of [0,0,0] using Gauss-Seidel method.

$$6x_1 - 2x_2 + x_3 = 11;\ -2x_1 + 7x_2 + 2x_3 = 5;\ x_1 + 2x_2 - 5x_3 = -1.$$

Solution:

9. Apply Gauss-Seidel method to solve the equations:

$$28x + 4y - z = 32; x + 3y + 10z = 24; 2x + 17y + 4z = 35.$$

Solution:

10. Solve by Gauss Seidel method, the following system:

$$20x + y - 2z = 17; 3x + 20y - z = -18; 2x - 3y + 20z = 25.$$

Solution:

11. Solve the equations by Guass-Seidel method of iteration:

$$10x + 2y + z = 9; x + 10y - z = -22; \ -2x + 3y + 10z = 22.$$

Solution:

12. Determine the largest eigen value and the corresponding eigen vector of the matrix

$$A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}.$$

Solution:

13. ind the largest eigen value and the corresponding eigen vector of a matrix

$$\begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

Solution:

14. Using power method find the dominant eigen value for the given matrix.

$$A = \begin{pmatrix} 15 & -4 & -3 \\ -10 & 12 & -6 \\ -20 & 4 & -2 \end{pmatrix}.$$

Solution:

15. Find the inverse of $\begin{bmatrix} 2 & 2 & 3 \\ 2 & 1 & 1 \\ 1 & 3 & 5 \end{bmatrix}$ using Gauss Jordan method.

Solution:

UNIT II – INTERPOLATION AND APPROXIMATION

1. The table gives the distance in nautical miles of the visible horizon for the given heights in feet above the earth's surface.

x =height:	100	150	200	250	300	350	400
y =distance:	10.63	13.03	15.04	16.81	18.42	19.9	21.27

Find the values of y when x =218 ft using Newton's forward interpolation formula.

Solution:

2. Using Newton's forward interpolation formula, find the cubic polynomial which takes the following values:

x:	0	1	2	3
y:	1	2	1	10

Solution:

3. From the given table compute the value of $sin 38^{\circ}$.

x:	0	10	20	30	40
$sinx$:	0	0.17365	0.34202	0.5	0.64279

Solution:

4. Find the value of y at $x = 21$ from the data given below:

x:	20	23	26	29
y:	0.3420	0.3907	0.4384	0.4848

Solution:

5. From the following data, find θ at x =43 and x =84.

x:	40	50	60	70	80	90
θ:	184	204	226	250	276	304

Also express θ in terms of x.

Solution:

6. The following table gives the values of density of saturated water for various temperatures of saturated steam.

Temperature °C:	100	150	200	250	300
Density hg/m^3	958	917	865	799	712

Find by interpolation, the density when the temperature is 275°.

Solution:

7. Using Lagrange's interpolation find the interpolated value for x =3 of the table.

x:	3.2	2.7	1.0	4.8
$f(x)$:	22.0	17.8	14.2	38.3

Solution:

8. Using Lagrange's interpolation formula calculate the profit in the year 2000 from the following data:

Year:	1997	1999	2001	2002
Profit in Lakhs of Rs.:	43	65	159	248

Solution:

9. Using Lagrange's formula find the value of $\log_{10} 323.5$ for the given data:

x:	321.0	322.8	324.2	325.0
$\log_{10} x$:	2.50651	2.50893	2.51081	2.51188

Solution:

10. Find an approximate polynomial for $f(x)$ using Lagrange's interpolation for the following data:

x:	0	1	2	5
$y = f(x)$:	2	3	12	147

Solution:

11. Find the Lagrange polynomial $f(x)$ satisfying the following data:

x:	1	3	5	7
$y = f(x)$:	24	120	336	720

Solution:

12. Employ a third order Newton polynomial to estimate $ln2$ with the four points give in table.

x:	1	4	6	5
$f(x)$:	0	1.386294	1.791759	1.609438

Solution:

13. Find the cubic polynomial from the following table using Newton's divided difference formula and hence find $f(4)$.

x:	0	1	2	5
$y = f(x)$:	2	3	12	147

Solution:

14. Given the tables:

x:	5	7	11	13	17
$y = f(x)$:	150	392	1452	2366	5202

Evaluate $f(9)$ using Newton's divided difference formula.

Solution:

15. Using Newton's divided difference formula determine $f(3)$ from the data:

x:	0	1	2	4	5
$f(x)$:	1	14	15	5	6

Solution:

UNIT III – NUMERICAL DIFFERENTIATION AND INTEGRATION

1. Find the first and second derivatives of the function tabulated below at $x = 1.5$.

x:	1.5	2.0	2.5	3.0	3.5	4.0
y:	3.375	7.0	13.625	24.0	38.875	59.0

Solution:

2. Find $f'(10)$ from the following data:

x:	3	5	11	27	34
y:	-13	23	899	17315	35606

Solution:

3. The velocity v (km/min) of a moped which starts from rest, is given at fixed intervals of time t (min) as follows:

t:	0	2	4	6	8	10	12
v:	0	10	18	25	29	32	20

(i) Estimate approximately the distance covered in 12 minutes, by Simpson's 1/3 rule.

(ii) Estimate the acceleration at $t = 2$ seconds.

Solution:

4. Find the value of $\log 2^{\frac{1}{3}}$ from $\int_0^1 \frac{x^2}{1+x^3} dx$ using Simpson's 3/8 rule with $h = 0.25$.

Solution:

5. Evaluate $\int_0^2 \frac{x^2+2x+1}{1+(x+1)^4}\,dx$ by Gaussian two point quadrature formula.

Solution:

6. Apply three point Gaussian quadrature formula to evaluate $\int_0^1 \frac{sinx}{x} dx$.

Solution:

7. Evaluate $\int_1^2 \frac{1}{1+x^3} dx$ using Gauss 3 point formula.

Solution:

8. Use the Romberg method to get an improved estimate of the integral from $x = 1.8$ to $x = 3.4$ from the data in table with $h = 0.4$.

x:	1.6	1.8	2.0	2.2	2.4	2.6
$f(x)$:	4.953	6.050	7.389	9.025	11.023	13.464
x:	2.8	3	3.2	3.4	3.6	3.8
$f(x)$:	16.445	20.056	24.533	29.964	36.598	44.701

Solution:

9. Evaluate $\int_0^1 \frac{dx}{1+x}$ using Romberg method and hence find the value of $\log 2$.

Solution:

10. Use Romberg method to evaluate $\int_0^1 \frac{dx}{1+x^2}$ correct to 4 decimal places. Also compute the same integral using three point Gaussian quadrature formula. Comment on the obtained values by comparing with the exact values of the integral which is equal to $\frac{\pi}{4}$.

Solution:

11. Using Trapezoidal rule evaluate $\int_0^1 \int_0^1 \frac{dxdy}{x+y+1}$ with $h = 0.5$ along x-direction and $k = 0.25$ along y-direction.

Solution:

12. Evaluate $\int_{1}^{1.4}\int_{2}^{2.4}\frac{dxdy}{xy}$ using Trapezoidal rule by taking $h = k = 0.1$ and verify with actual integration.

Solution:

12. Evaluate $\int_0^{\frac{\pi}{2}} \int_{\frac{\pi}{2}}^{\pi} \cos\,(x+y)\, dxdy$ by using Trapezoidal by taking $h = k = \frac{\pi}{4}$.

Solution:

13. Evaluate $\int_1^2 \int_3^4 \frac{dxdy}{(x+y)^2}$ taking $h = k = 0.5$ by both Trapezoidal rule and Simpson's rule.

Solution:

14. Evaluate $\int_{2}^{2.4}\int_{4}^{4.4} xy\,dx\,dy$ Trapezoidal rule taking $h = k = 0.1$.

Solution:

15. Apply three point Gaussian quadrature formula to evaluate $\int_0^1 \frac{sinx}{x} dx$.

Solution:

UNIT IV-NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS

1. Using Taylor series method, compute the value of $y(0.2)$ correct to 3 decimal places from $\frac{dy}{dx} = 1 - 2xy$ given that $y(0) = 0$.

Solution:

2. Find the value of y at $x = 0.1$ from $\frac{dy}{dx} = x^2y - 1$, $y(0) = 1$ by Taylor's series method.

Solution:

3. Obtain y by Taylors series method, given that $y' = xy + 1$, $y(0) = 1$, for $x = 0.1$ and 0.2 correct to four decimal places.

Solution:

4. Using modified Euler's method, find $y(0.1)$ and $y(0.2)$ for the given equation $\frac{dy}{dx} = x^2 + y^2$, given that $y(0) = 1$.

Solution:

5. Solve $(1+x)\frac{dy}{dx} = -y^2, \quad y(0) = 1$ by modified Euler's method by choosing $h = 0.1$, find $y(0.1)$ and $y(0.2)$.

Solution:

6. Employ the classical fourth order Runge-Kutta method to integrate $y' = 4e^{0.8t} - 0.5y$ from $t = 0$ to $t = 1$ using a stepsize of 1 with $y(0) = 2$.

Solution:

6. Find the value of $y(1.1)$ using Runge-Kutta method of fourth order for the given equation $\frac{dy}{dx} = y^2 + xy;\ \ y(1) = 1.$

Solution:

8. Using Runge-Kutta method of fourth order, solve $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$, $y(0) = 1$ at $x = 0.2$.

Solution:

9. Find $y(0.8)$ given that $y' = y - x^2$, $y(0.6) = 1.7379$ by using Runge-Kutta method of order four. Take $h = 0.1$.

Solution:

10. Using Runge-Kutta method of order four, find y when $x = 1.2$ in steps of 0.1 given that $y' = x^2 + y^2$ and $y(1) = 1.5$.

Solution:

11. Given $\frac{dy}{dx} = xy + y^2$ and $y(0) = 1, y(0.1) = 1.1169, y(0.2) = 1.2773, y(0.3) = 0.2267$, evaluate $y(0.4)$ by Milne's predictor corrector method.

Solution:

12. Determine the value of $y(0.4)$ using Milne's method given $y' = xy + y^2,\ y(0) = 1$. Use Taylor's series method to get the values of $y(0.1), y(0.2)$ and $y(0.3)$.

Solution:

13. Given $\frac{dy}{dx} = x - y^2, y(0) = 0, y(0.2) = 0.02, y(0.4) = 0.0795$ and $y(0.6) = 0.1762$. Compute $y(0.8)$ using Milne's method.

Solution:

14. Using Milne's method find $y(4.4)$ given $5xy' + y^2 - 2 = 0$, given $y(4) = 1$, $y(4.1) = 1.0049$, $y(4.2) = 1.0097$ and $y(4.3) = 1.0143$.

Solution:

15. Use Milne's method to find $y(0.8)$, given $y' = \frac{1}{x+y}, y(0) = 2, y(0.2) = 2.0933$, $y(0.4) = 2.1755, y(0.6) = 2.2493$.

Solution:

UNIT V-BOUNDARY VALUE PROBLEMS IN ORDINARY AND PARTIAL DIFFERENTIAL EQUATLONS

1. Solve $y'' = x + y$ with the boundary conditions $y(0) = y(1) = 0$.

Solution:

2. Solve the equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ subject to the conditions $u(x,0) = sin\pi x$, $0 < x < 1$, $u(0,t) = u(1,t) = 0$ using Bendre Schmidt method.

Solution:

3. Using Bendre-Schmidt method solve $u_t = u_{xx}$ subject to the conditions, $u(0,t) = 0$, $u(1,t) = 0$, $u(x,0) = sin\pi x$, $0 < x < 1$ and $h = 0.2$. Find the value of u upto $t = 0.1$.

Solution:

4. Find the values of the function $u(x,t)$ satisfying the differential equation $u_t = 4u_{xx}$ and the boundary condition $u(0,t) = 0 = u(8,t)$ and $u(x,0) = 4x - \frac{x^2}{2}$ at the point $x = i$, $x = 0,1,2,3,4,5,6,7,8$, $t = \frac{1}{8}j$, $j = 0,1,2,3,4,5$.

Solution:

5. Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to the condition $u(x,0) = sin\pi x, 0 \leq x < 1;$ $u(0,t) = u(1,t) = 0$ using Crank-Nicholson method.

Solution:

6. Given that $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $u(0,t) = 0, u(4,t) = 0$ and $u(x,0) = \frac{x}{3}(16 - x^3)$. Find $u_{ij}: i = 1,2,3,4$ and $j = 1,2$ by using Crank-Nicholson method.

Solution:

7. Using Crank-Nicholson's scheme, solve $16\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, o < x < 1, t > 0$ subject to $u(x,0) = 0$, $u(0,t) = 0, u(1,t) = 100t$. Compute u for one step in t direction taking $h = \frac{1}{4}$.

Solution:

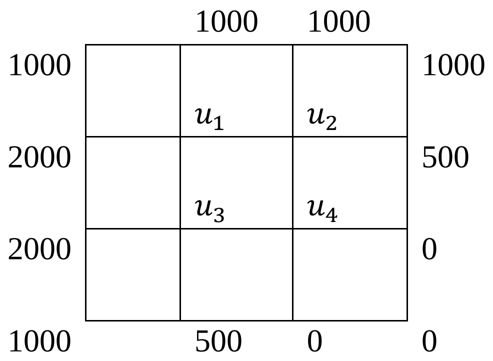
8. Evaluate the pivotal values of the equation $u_{tt} = 16u_{xx}$ taking $\Delta x = 1$ upto $t = 1.25$. The boundary conditions are $u(0,t) = u(5,t) = u_t(x,0) = 0$ and $u(x,0) = x^2(5-x)$.

Solution:

9. Solve $u_{tt} = u_{xx}, 0 < x < 2, t > 0$ subject to $u(x,0) = 0, u_t(x,0) = 100(2x - x^2)$, $u(0,t) = 0, u(2,t) = 0$, choosing $h = \frac{1}{2}$ compute u for four time steps.

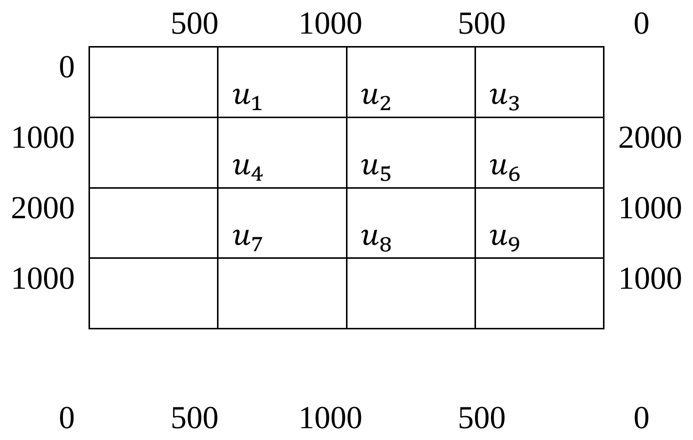
Solution:

10. Given the values of $u(x, y)$ on the boundary of the square in figure evaluate the function $u(x, y)$ satisfying the Laplace equation $\nabla^2 u = 0$ at the pivotal points of this figure by Gauss Seidel method.



Solution:

11. Solve the elliptic equation $u_{xx} + u_{yy} = 0$ for the following square mesh with boundary values as shown.



Solution:

12. Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ at the interior points of the square region given as below:

0	11.1	17.0	19.7	18.6
0	41	42	43	21.9
0	44	45	46	21.0
0	47	48	49	17.0
0	8.7	12.1	12.8	9.0

Solution:

13. By iteration method solve the elliptic equation $u_{xx} + u_{yy} = 0$ over the square region of side 4, satisfying the boundary conditions $u(0, y) = 0, 0 \leq y \leq 4,\ u(4, y) = 12 + y,\ 0 \leq y \leq 4,\ u(x, 0) = 3x, 0 \leq x \leq 4, u(x, 4) = x^2, 0 \leq x \leq 4$. By dividing the square into 16 square meshes of side 1 and always correcting the computed values to two places of decimals. Obtain the values of u at 9 interior pivotal points.

Solution:

14. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ at the nodal points of the following square grid using the boundary values indicated.

0	10	20	30
20	u_1	u_2	40
40	u_3	u_4	50
60	60	60	60

Solution:

15. Solve the Poisson's equation $\nabla^2 u = 8x^2y^2$ for the square mesh of figure with $u(x, y) = 0$ on the boundary and mesh length=1.

	u_1	u_2	u_1
	u_2	u_3	u_2
	u_1	u_2	u_1

Solution: