$y' = x^2 + y^2$; $y(0) = 1$; $y(0.1) = ?$

$y(0.2) = ?$

$f(x, y) = x^2 + y^2$

$h = 0.1$

$y(x, y) = (0, 1)$

$$y' = x^2 + y^2$$
$$y'' = 2x + 2y\frac{dy}{dx} \Rightarrow 2x + 2yy'$$
$$y''' = 2 + 2[y'y' + yy'']$$
$$y'''' = 2[2y'y'' + y'y'' + yy''']$$

$$y_0' = (0)^2 + (1)^2 = 1$$
$$y_0'' = 2(0) + 2(1)(1) \Rightarrow 2$$
$$y_0''' = 2 + 2[(y')^2 + yy'']$$
$$= 2 + 2[(1)^2 + (1)(2)] \Rightarrow 2 + 2[1+2] \Rightarrow 8$$
$$y_0'''' = 2[2(1)(2) + (1)(2) + (1)(8)]$$
$$= 2[2 + 2 + 2 + 8] \Rightarrow 28$$

$$y(0.1) \Rightarrow y_1$$

$$y_1 = y_0 + \frac{h}{1!}y_0' + \frac{h^2}{2!}y_0'' + \frac{h^3}{3!}y_0''' + \frac{h^4}{4!}y_0''''$$
$$= 1 + \frac{0.1}{1}(1) + \frac{(0.1)^2}{2!}(2) + \frac{(0.1)^3}{3!}(8) + \frac{(0.1)^4}{4!}(28)$$
$$= 1 + 0.1 + \frac{0.02}{2} + \frac{0.008}{6} + \frac{0.0028}{24}$$
$$= 1 + 0.1 + 0.01 + 0.0013 + 0.000116 \Rightarrow 1.111416$$

$y(0.2) = y_2$

$$y_2 = y_1 + \frac{h}{1!} y_1' + \frac{h^2}{2!} y_1'' + \frac{h^3}{3!} y_1''' + \frac{h^4}{4!} y_1''''$$

$h = 0.1, \quad x = 0.1, \quad y = 1.1308$

$y_1' = (0.1)^2 + (1.1308)^2$

$= 0.01 + 1.278 \Rightarrow 1.288$

$y_1'' = 2(0.1) + 2(1.1308)(1.288) \Rightarrow 0.2 + 2.912$

$\Rightarrow 3.112$

$y_1''' = 2 + 2[(1.288)^2 + (1.1308)(3.112)]$

$= 2 + 2[1.658 + 3.519]$

$= 12.354$

$y_1'''' = 2[2(1.288)(3.112) + (1.288)(3.112) + (1.1308)(12.354)]$

$= 2[8.016 + 4.008 + 13.969]$

$= 51.986$

$$y_2 = y_1 + \frac{h}{1!} y_1' + \frac{h^2}{2!} y_1'' + \frac{h^3}{3!} y_1''' + \frac{h^4}{4!} y_1''''$$

$= 1.1308 + \frac{0.1}{1!}(1.288) + \frac{0.1}{2!}(3.112) + \frac{0.1}{3!}(12.354) + \frac{0.1}{4!}(51.986)$

$=$

$y' = x^2 + y^2 \quad , \; y(0) = 1 \quad , \; y(0.1) = 1 \quad y(0.2) = ?$

$f(x, y) = x^2 + y^2 \qquad f(0, 1)$

$y(0.1) = y_1 = y_0 + h\, f(x_0, y_0)$

$= 1 + (0.1)(1)$

$= \underline{1.1}$

$y(0.2) = y_2 = y_1 + h\, f(x_1, y_1)$

$= 1 + (0.1)(1.22) \rightarrow 1.122$

$(0.1)^2 + (1.1)^2 \rightarrow 0.01 + 1.21 \rightarrow \underline{1.22}$

Modified Euler's

$y' = x^2 y - 1 \quad y(0) = 0.1, \quad y(0.2) = ?$

$f(x, y) = x^2 y - 1 \quad (0, 0.1), \quad h = 0.2$

$$y_1 = y_0 + h f\left(x_0 + \frac{h}{2}, y_0 + \frac{h}{2} y_0'\right)$$

$$= 0.1 + (0.2) f\big(0 + 0.1, 0.1 + (0.1)(-1)\big)$$

$$= 0.1 + 0.2 f(0.1, 0)$$

$$= 0.1 - 0.2$$

$$= -0.1$$

	x	y	
x_0	0	0.1	y_0
	0.2	?	

$y_0' = 0(0.1)^2 - 1 = -1$

$(0, 0.1)$

$f(0.1, 0) = 0 - 1 = -1$

$$\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2} \qquad y(0) = 1, \quad y(0.2) = ? \quad ; h = 0.2$$

$y(x)$ = value for y
$y(0) = 1$
meaning, $x = 0$ & $y = 1$
or $f(x, y) = f(0, 1)$

Using RK method.

$$y_1 = y_0 + \Delta y_0$$

$$\Delta y = \frac{1K_1 + 2K_2 + 2K_3 + 1K_4}{6}$$

$$K_1 = h\, f(x_0, y_0)$$
$$K_2 = h\, f\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right)$$
$$K_3 = h\, f\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right)$$
$$K_4 = h\, f(x_0 + h, y_0 + K_3)$$

$$f(x, y) = \frac{y^2 - x^2}{y^2 + x^2}$$

$$f(0, 1) = \frac{(1)^2 - (0)^2}{(1)^2 + (0)^2} \Rightarrow \underline{\underline{1}}.$$

$$K_1 = (0.2)\, f(0, 1)$$
$$= \underline{\underline{0.2}}.$$

$$K_2 = (0.2)\, f\left(0 + \frac{(0.2)}{2},\ 1 + \frac{(0.2)}{2}\right)$$

$$= (0.2)\, f(0.1,\ 1.1) \Rightarrow 0.2 * 0.983 = \underline{\underline{0.1966}}$$

$$K_3 = (0.2)\, f\left(0 + \frac{(0.2)}{2},\ 1 + \frac{(0.1966)}{2}\right)$$

$$= (0.2)\, f(0.1,\ 1.0983) \Rightarrow (0.2)(0.983) = \underline{\underline{0.1966}}$$

$$K_4 = (0.2)\, f(0 + 0.2,\ 1 + 0.1966)$$

$$= (0.2)\, f(0.2,\ 1.1966) \Rightarrow 0.2 * 0.945 \Rightarrow \underline{\underline{0.189}}$$

$$\Delta y = \frac{1(0.2) + 2(0.1966) + 2(0.1966) + 1(0.189)}{6}$$

$$= \frac{0.2 + 0.3932 + 0.3932 + 0.189}{6} \Rightarrow \underline{\underline{0.1959}}$$

$$f(0.1,\ 1.1) = \frac{(1.1)^2 - (0.1)^2}{(1.1)^2 + (0.1)^2}$$

$$= \frac{1.21 - 0.01}{1.21 + 0.01}$$

$$= \frac{1.2}{1.22} \Rightarrow \underline{\underline{0.983}}$$

$$f(0.1,\ 1.0983) = \frac{(1.0983)^2 - (0.1)^2}{(1.0983)^2 + (0.1)^2}$$

$$= 0.983$$

$$f(0.2,\ 1.1966) = \frac{(1.1966)^2 - (0.2)^2}{(1.1966)^2 + (0.2)^2}$$

$$= 0.945$$

$y_1 = y_0 + \Delta y_0$

$= 1 + 0.1959 \Rightarrow \underline{1.1959}$

Modified Eulers

$y' = y - \frac{2x}{y}$ $\quad y(0) = 1, \quad y(0.1) = ?$

$h = 0.1$

x	y
0	1
0.1	?

$h = 0.1 - 0$
$\Rightarrow \underline{0.1}$

$y' = f(x, y) = y - \frac{2x}{y}$

$y_1 = y_0 + h f\left(x_0 + \frac{h}{2}, y_0 + \frac{h}{2} y_0'\right)$

$= 1 + (0.1) f\left(0 + \frac{0.1}{2}, 1 + \frac{0.1}{2}(1)\right)$

$= 1 + (0.1) f(0.05, 1.05)$

$= 1 + (0.1)(0.954) \Rightarrow \underline{1.0954}$

$f(0, 1) = y - \frac{2x}{y}$

$= 1 - \frac{2(0)}{1} \Rightarrow \underline{1}$

$f(0.05, 1.05) = 1.05 - \frac{2(0.05)}{(1.05)}$

$= 1.05 - \frac{0.1}{1.05}$

$= \underline{0.954}$

RK method

$y' = \frac{xy}{1+x^2}$, $y(0)=1$, $h=0.1$, $y(0.1)=?$

x	y
0	1
0.1	?

$f(x,y) = \frac{xy}{1+x^2}$

$f(0,1) = \frac{(0)(1)}{1+(0)^2} \Rightarrow \underline{0}$

$y_1 = y_0 + \Delta y_0$

$\Delta y_0 = \frac{1k_1 + 2k_2 + 2k_3 + 1k_4}{6}$

$k_1 = hf(x_0, y_0)$

$= (0.1)\,f(0,1)$

$= (0.1)(0) \Rightarrow \underline{0}$

$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$

$$= (0.1)\, f\left(0 + \frac{0.1}{2},\ 1 + \frac{0}{2}\right)$$

$$= (0.1)\, f(0.05,\ 1) \Rightarrow (0.1)(0.049)$$

$$\Rightarrow \underline{0.0049}$$

$$k_3 = h f\left(x_0 + \frac{h}{2},\ y_0 + \frac{k_2}{2}\right)$$

$$= (0.1)\, f\left(0 + \frac{0.1}{2},\ 1 + \frac{0.0049}{2}\right)$$

$$= (0.1)\, f(0.05,\ 1.00245) \Rightarrow (0.1)(0.0498)$$

$$= \underline{0.00498}$$

$$k_4 = h f(x_0 + h,\ y_0 + k_3)$$

$$= (0.1)\, f(0 + (0.1);\ 1 + (0.0049))$$

$$= (0.1)\, f(0.1,\ 1.0049) \Rightarrow (0.1)(0.099) \Rightarrow \underline{0.0099}$$

$$\Delta y_0 = \frac{1(0) + 2(0.0049) + 2(0.0049) + 1(0.0099)}{6}$$

$$= \frac{0.0098 + 0.0098 + 0.0099}{6} \Rightarrow \underline{0.00491}$$

$$y_1 = y_0 + \Delta y_0 \Rightarrow 1 + 0.00491 \Rightarrow \underline{1.00491}$$

$$f(0.05,\ 1) = \frac{(0.05)(1)}{(1) + (0.05)^2}$$

$$= \frac{0.05}{1.0025} \Rightarrow 0.049$$

$$f(0.05,\ 1.00245) = \frac{(0.05)(1.00245)}{1 + (0.05)^2}$$

$$= \frac{0.0501}{1.0025}$$

$$= \underline{0.049}$$

$$f(0.1,\ 1.0049) = \frac{(1.0049)(0.1)}{1 + (0.1)^2}$$

$$= \frac{0.10049}{1.01} \Rightarrow 0.099$$