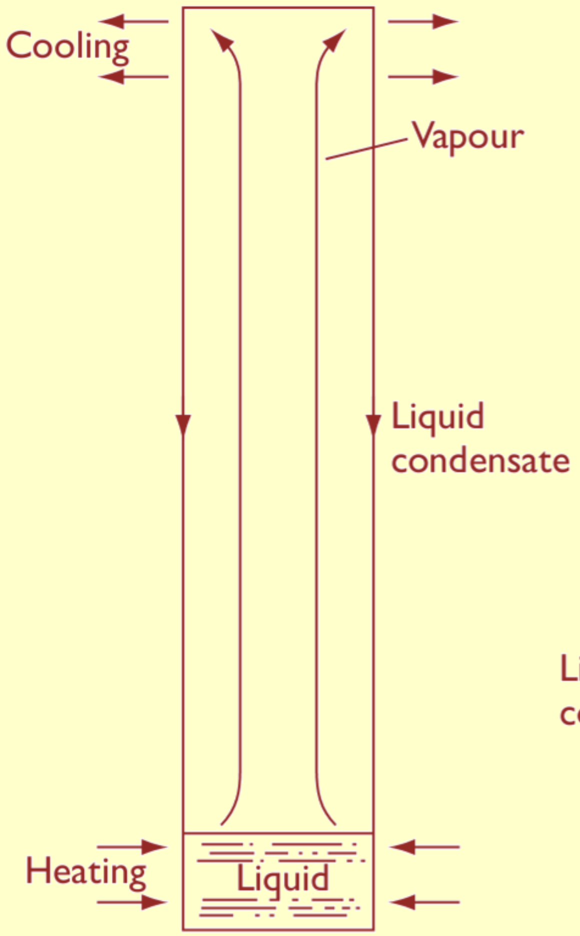
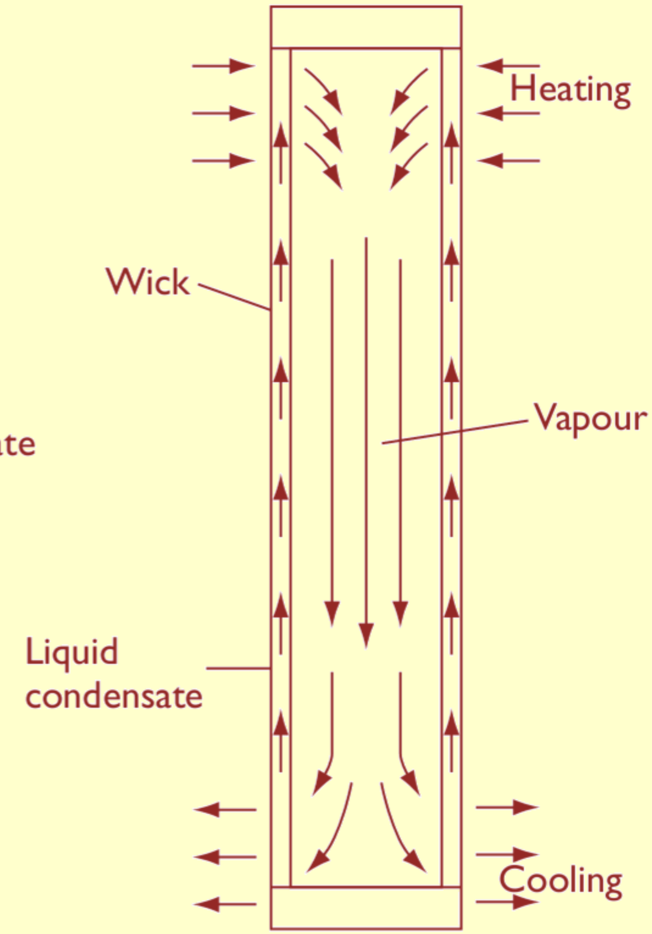


Thermo syphon



Heat Pipe

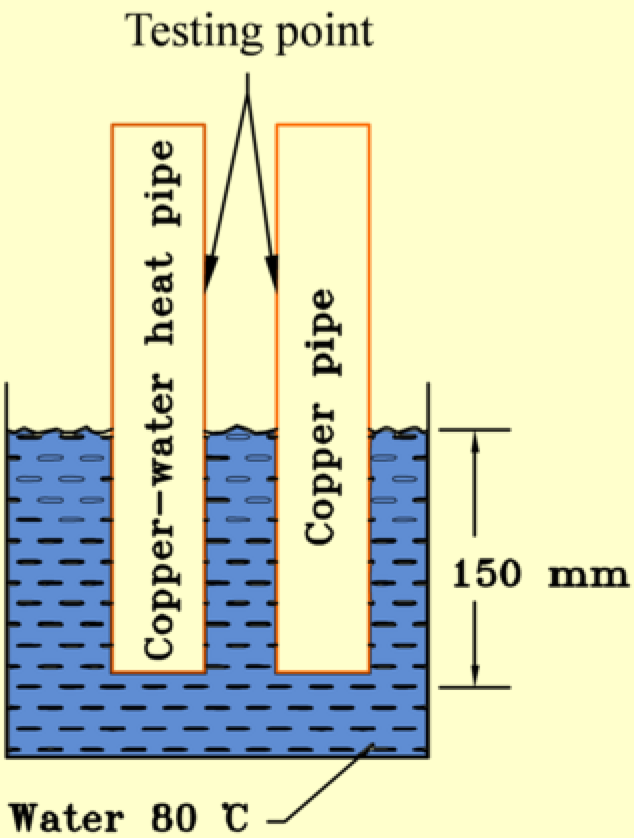
Types of Heat pipe :

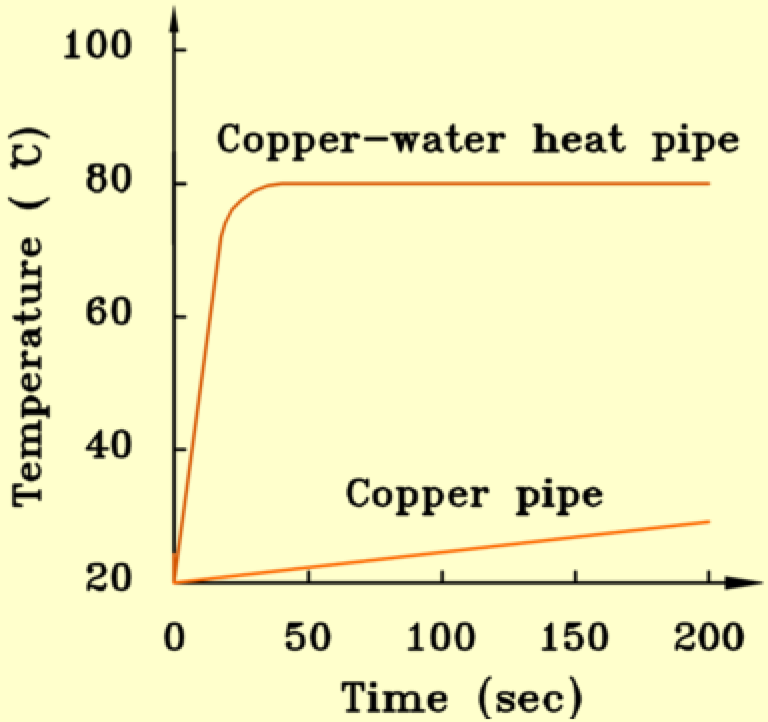
- Two-Phase Closed Thermosyphon
- Capillary-Driven Heat Pipe
- Annular Heat Pipe
- Vapor Chamber
- Rotating Heat Pipe
- Gas-Loaded Heat Pipe
- Loop Heat Pipe
- Capillary Pumped Loop Heat Pipe
- Pulsating Heat Pipe
- Monogroove Heat Pipe
- Micro and Miniature Heat Pipes
- Inverted Meniscus Heat Pipe
- Nonconventional Heat Pipes

Effective thermal conductivity = 100000 W/mK

15 cm and 0.6 cm dia can transport 300 W of heat

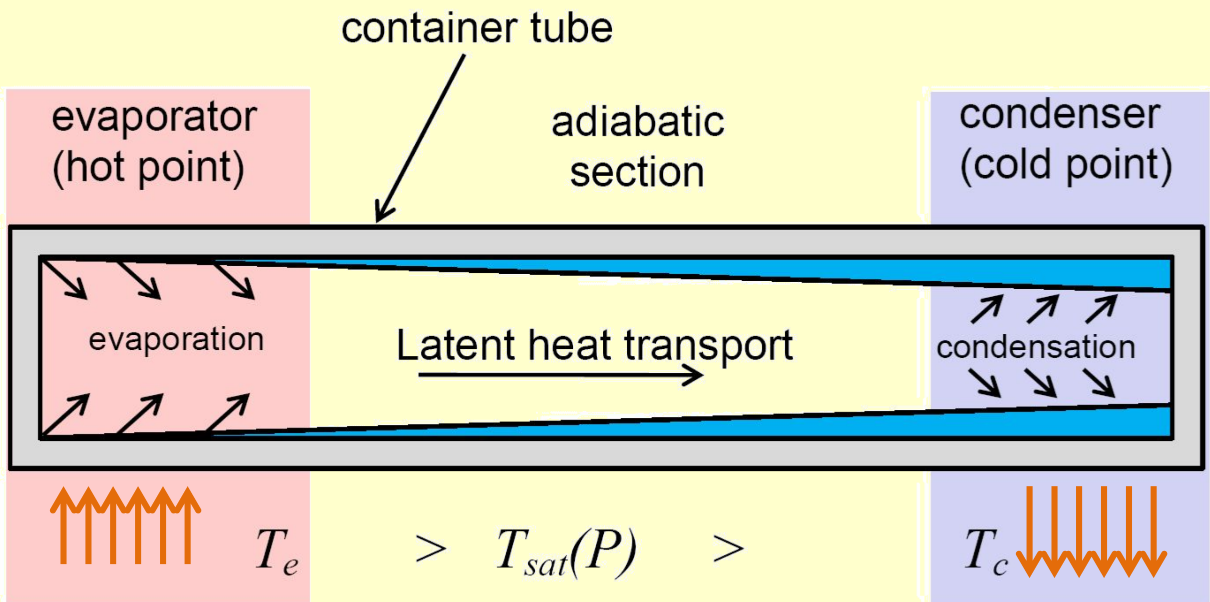
Can operate in micro gravity environment

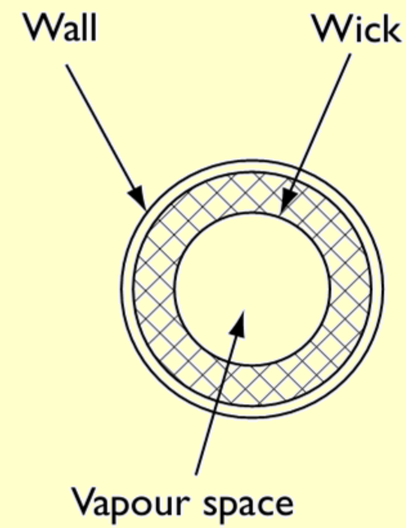


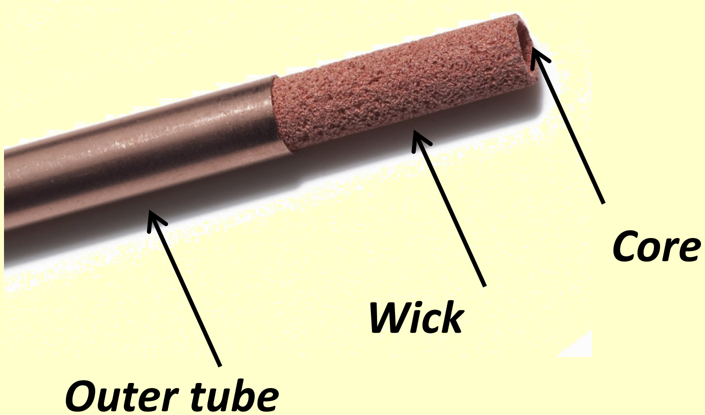


copper-water heat pipe with 6.35 mm OD and an inner diameter of 5.85 mm in comparison

STANDARD HEAT PIPES

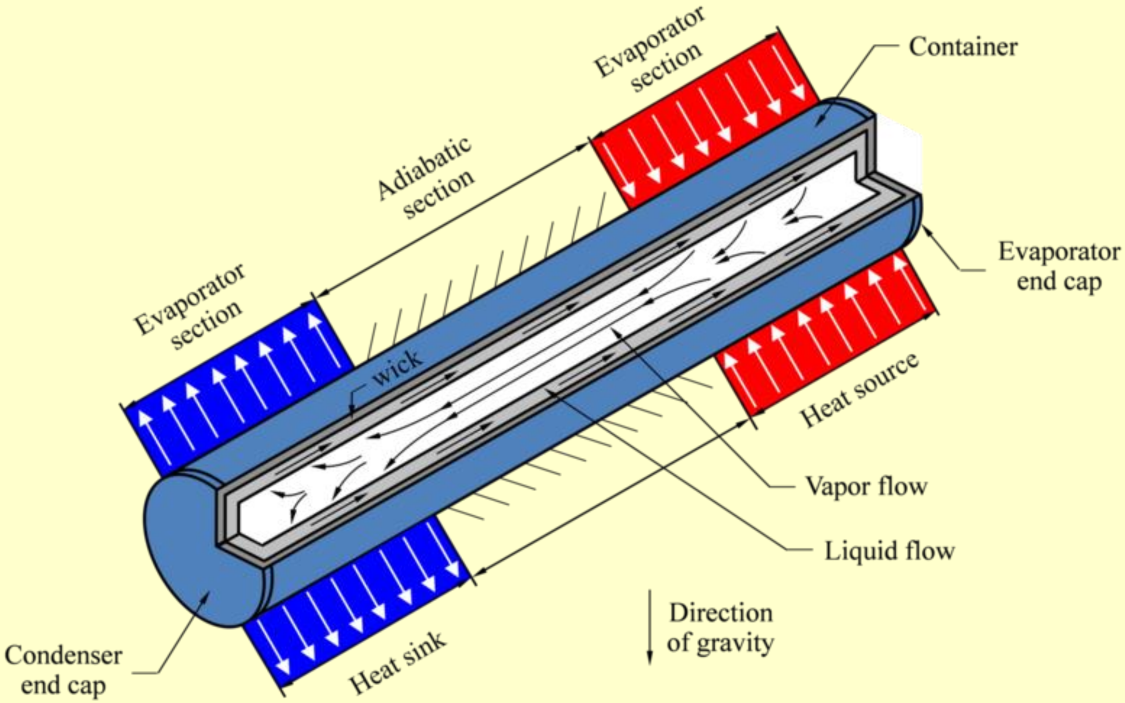


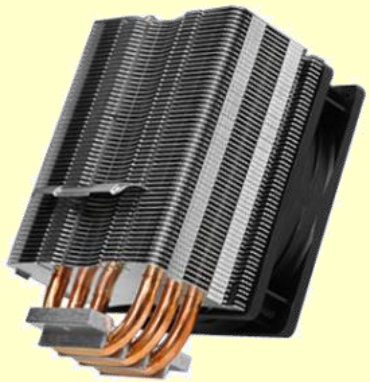


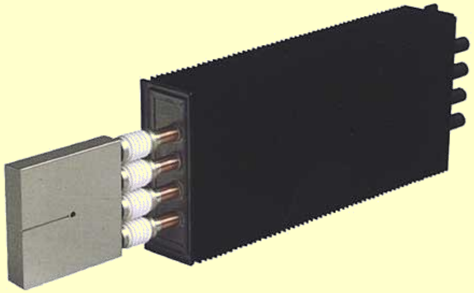


Issues :

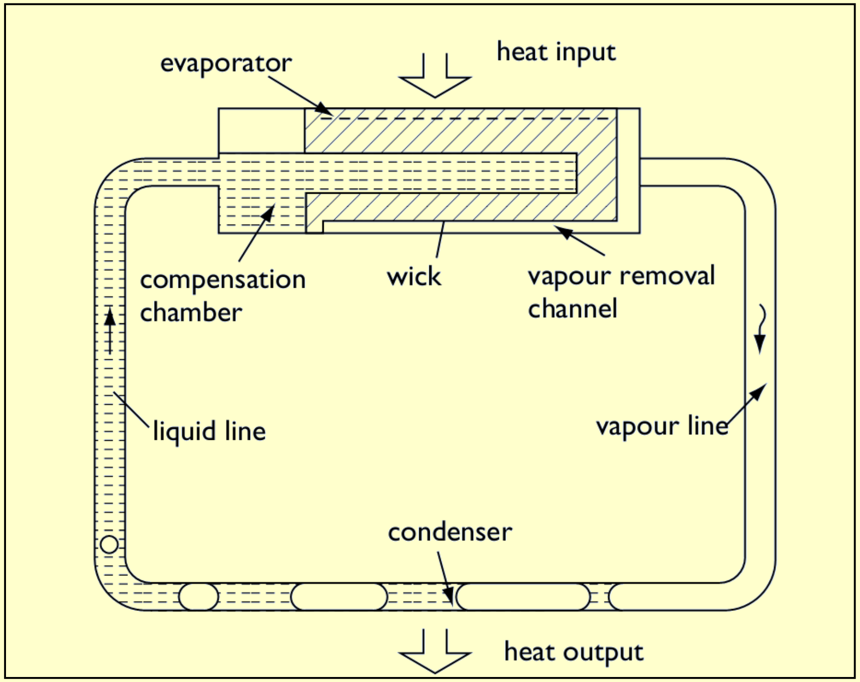
- Capillary Limit
- Liquid Entrainment

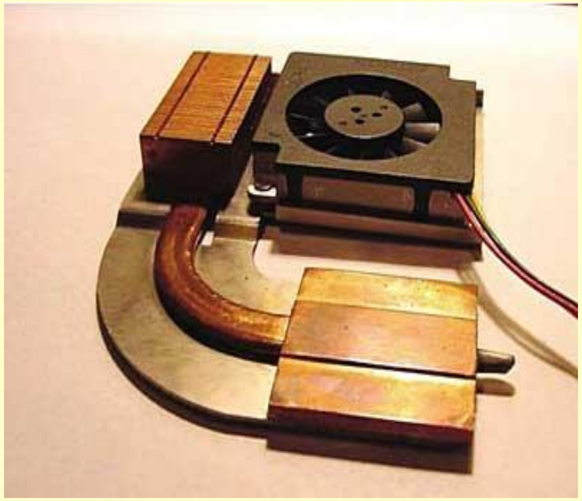

Container
Evaporator section
Adiabatic section
Evaporator section
Evaporator end cap
wick
Heat source
Vapor flow
Liquid flow
Direction of gravity
Condenser end cap
Heat sink

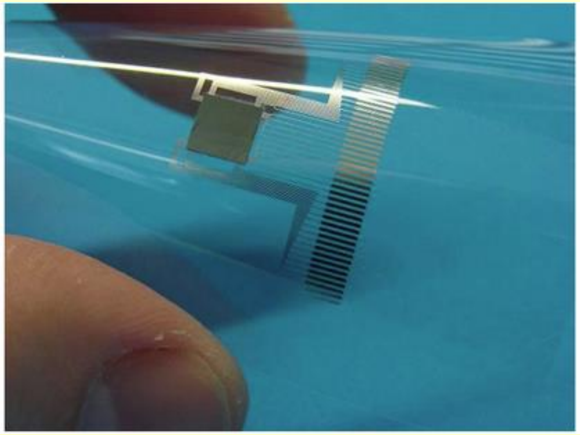




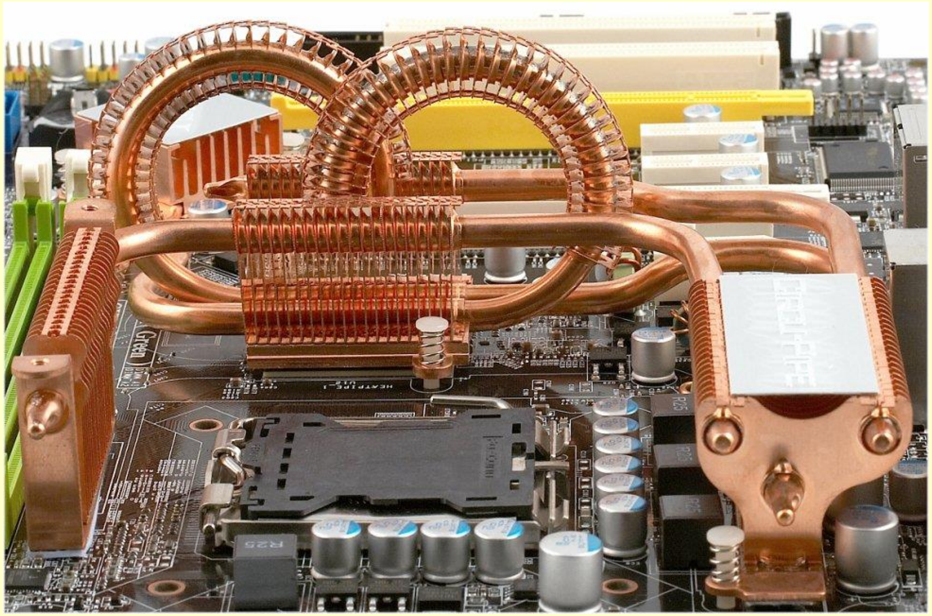
LOOP HEAT PIPES

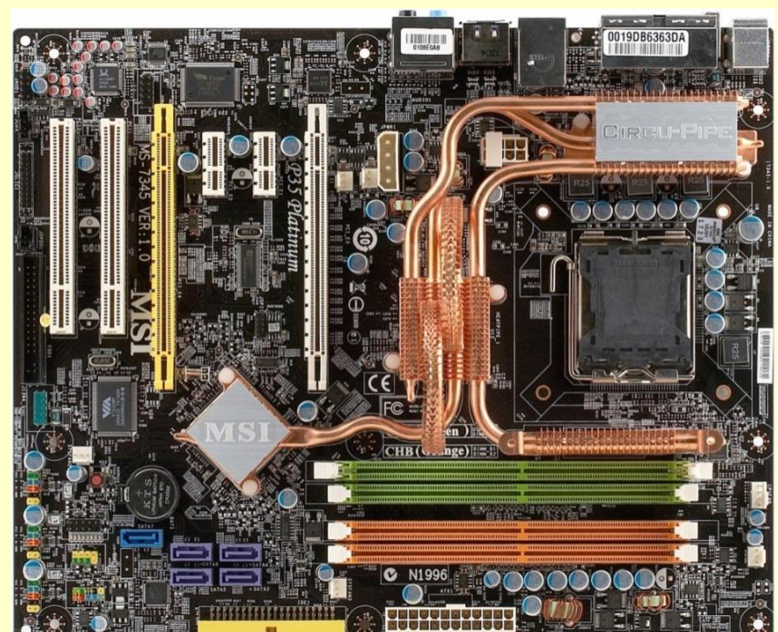




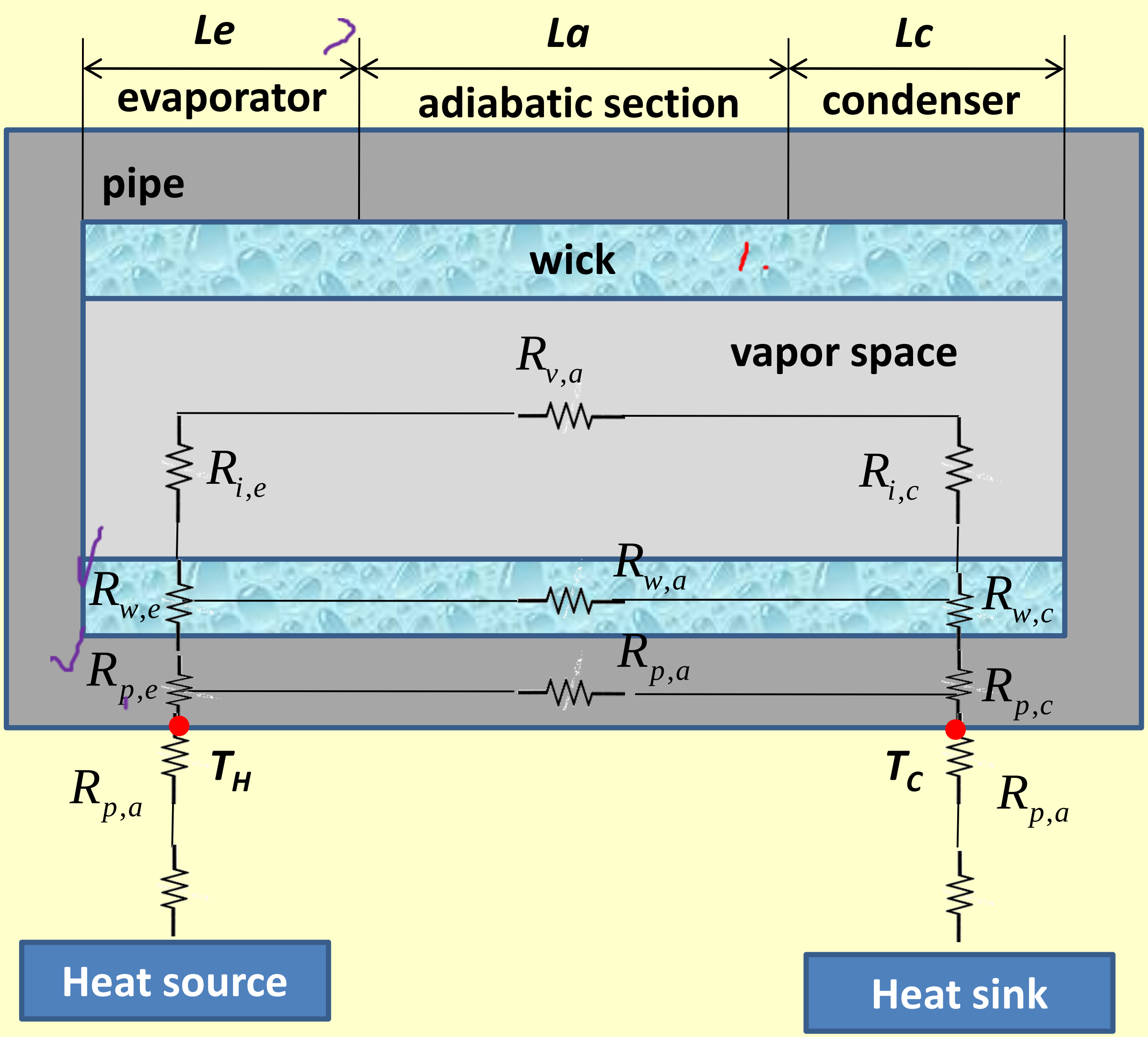


STAGED HEAT PIPES



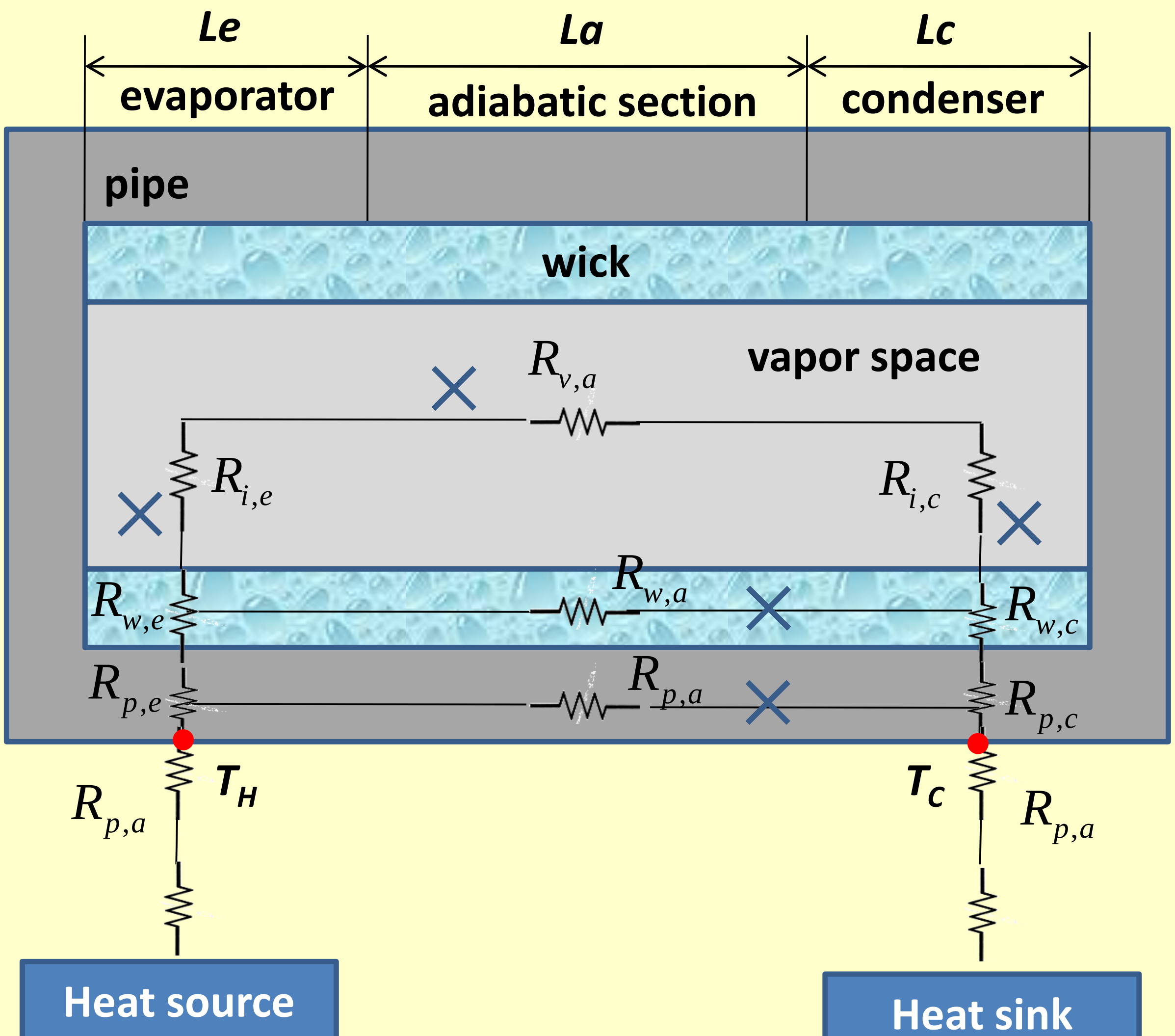


Thermal resistance and thermal conductivity of heat pipe



$$R_{p,e} = \frac{\ln\left(\dfrac{d_o}{d_i}\right)}{2\pi L_e k_p} \qquad R_{w,e} = \frac{\ln\left(\dfrac{d_o}{d_i}\right)}{2\pi L_e k_{eff}}$$

Thermal resistance and thermal conductivity of heat pipe



How to simplify the overall thermal resistance

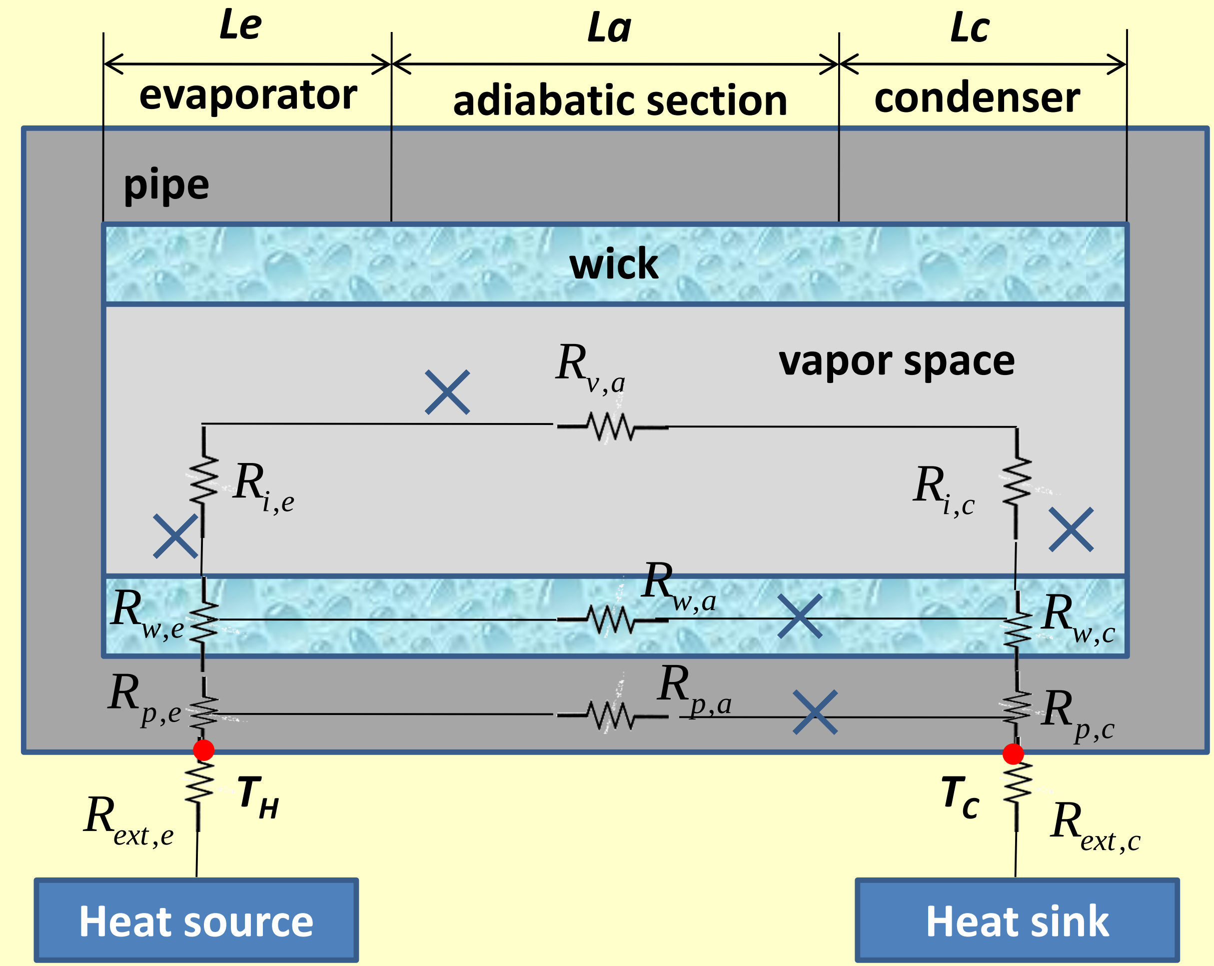
- When there is large resistance we shall consider the line to be open circuited w.r.t heat flow
- When the resistance is too large we shall consider that the line is short circuited w.r.t heat flow

$R_{p,a}$ $R_{w,a}$ } ***Very large open circuited***

$R_{i,e}$ $R_{i,c}$ $R_{v,a}$ } ***Very small Short circuited***

Res	C/W
R_{pe}, R_{pc}	10^{-1}
R_{we}, R_{wc}	10^{+1}
R_{ie}, R_{ic}	10^{-5}
R_{va}	10^{-8}
R_{pa}	10^{+2}
R_{wa}	10^{+4}

Thermal resistance and thermal conductivity of heat pipe



$$R_{p,e}=\frac{\ln\left(\frac{d_o}{d_i}\right)}{2\pi L_e k_p}\qquad R_{w,e}=\frac{\ln\left(\frac{d_o}{d_i}\right)}{2\pi L_e k_{eff}}\qquad R_{p,c}=\frac{\ln\left(\frac{d_o}{d_i}\right)}{2\pi L_e k_p}\qquad R_{w,c}=\frac{\ln\left(\frac{d_o}{d_i}\right)}{2\pi L_e k_{eff}}$$

Total thermal resistance

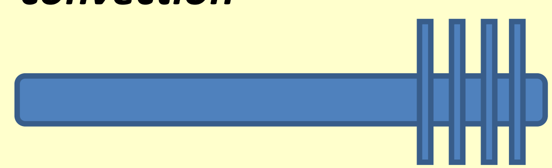
$$R_{tot}=R_{p,e}+R_{w,e}+R_{w,c}+R_{p,c}$$

$$q=\frac{T_h-T_c}{R_{tot}}$$

Contact resistance with the sink and source if both are attached by constant resistance

$$R_{ext,e\,and\,c}=\frac{R''_{t,c}}{\pi d_o L}$$

If any side heat is dissipated by convection

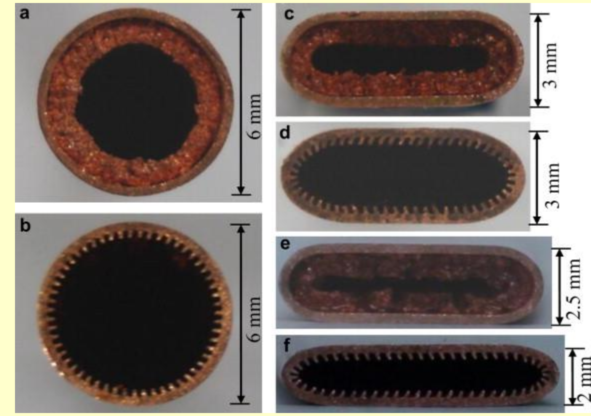


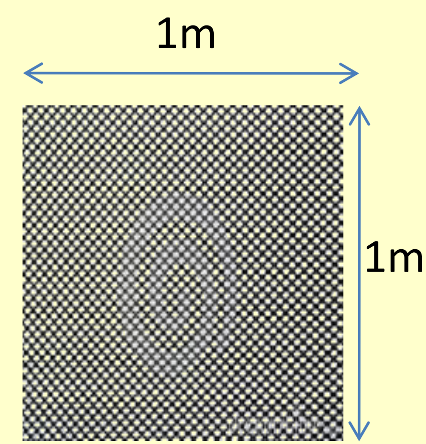
$$R_{ext,conv}=\frac{1}{h\pi d_o L}$$

fins

Effective thermal conductivity of the wick

Wick structure	k_{eff}
Wick and liquid in series	$\dfrac{k_l k_w}{\varepsilon k_w + k_l(1-\varepsilon)}$
Wick and liquid in parallel	$\varepsilon k_l + k_w(1-\varepsilon)$
Wrapped screen	$\dfrac{k_l[(k_l+k_w)-(1-\varepsilon)(k_l-k_w)]}{(k_l+k_w)+(1-\varepsilon)(k_l-k_w)}$
Packed sphere	$\dfrac{k_l[(2k_l+k_w)-2(1-\varepsilon)(k_l-k_w)]}{(2k_l+k_w)+(1-\varepsilon)(k_l-k_w)}$
Rectangular grooves	$\dfrac{(w_f k_l k_w \delta)+w k_l(0.185 w_f k_w+\delta k_l)}{(w+w_f)(0.185 w_f k_w+\delta k_l)}$
Sintered metal fibers*	$\varepsilon^2 k_l + (1-\varepsilon)^2 k_w + \dfrac{4\varepsilon(1-\varepsilon)k_l k_w}{k_l+k_s}$



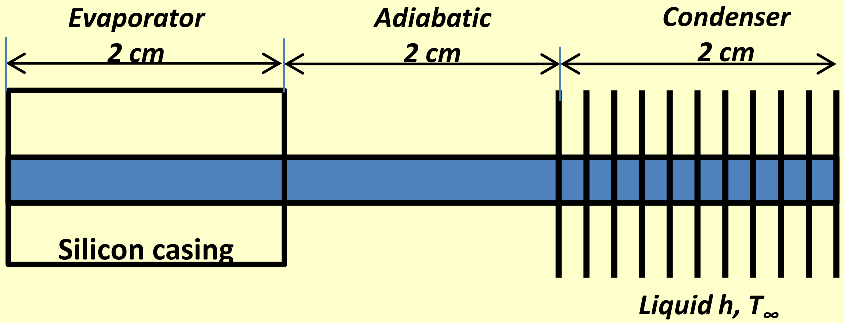


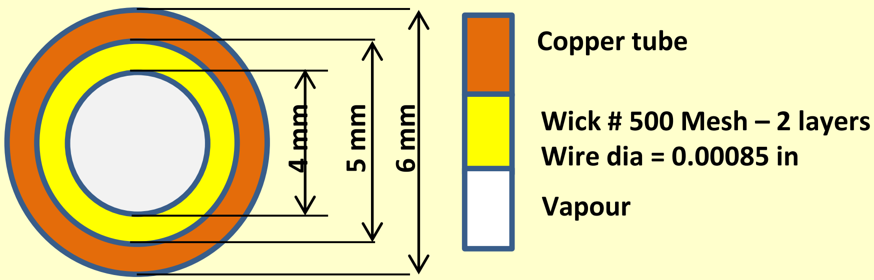
<u>**ε – Porosity → Vol of void / total volume**</u>

Example : Wrapped Screen (wire mesh) – 400 Mesh / inch
Wire diameter (D) = 2x10^{-5}m → Thickness of the screen = 2D
Length is 1 m

- **Volume of one wires(m^3/m) = 3.142 / 4 * D^2 * L = 3.14*10^{-10}**
- **Total number of wires in x and y /m^2 (take 400 / inch product) = 10160+10160 = 20320 wires/m^2**
- **Total wire volume =(20320)*3.14*10^{-10} = 6.38048 *10^{-6}**
- **Total screen volume (thick = 2 dia)= L*L*t=4x10^{-5} m3**
- **Void volume = Total screen volume - Total wire volume =3.36103 X 10^{-5}**
- **Porosity (ε) = Void volume/ Total screen volume = 0.84**

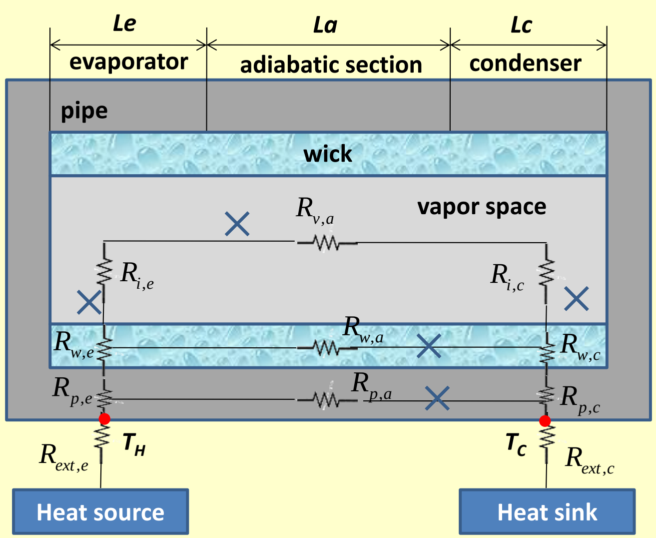
$$\varepsilon = 1 - \frac{\pi N_{PER-INCH} d_{WIRE}}{4}$$





$$k_l = 0.603\ W/mK$$
$$k_p = 401\ W/mK$$

$$R_{tot} = R_{p,e} + R_{\omega,e} + R_{\omega,c} + R_{p,c}$$



Evaporator and condenser are typically identical – Heat pipe 'R'

$$R_{tot} = 2\left(R_{p,e} + R_{w,e}\right)$$

$$k_{eff} = \frac{k_l\left[\left(k_l + k_\omega\right) - \left(1-\varepsilon\right)\left(k_l - k_\omega\right)\right]}{\left(k_l + k_\omega\right) + \left(1-\varepsilon\right)\left(k_l + k_\omega\right)}$$

$$\varepsilon = 1 - \frac{\pi N d_w}{4} = 1 - \frac{\pi\left(500/in.\right)\left(0.0085\ \ in.\right)}{4} = 0.666$$

$$k_{eff} = \frac{(0.603)\left[(0.603+401)-(1-0.666)(0.603-401)\right]}{(0.603+401)+(1-0.666)(0.603-401)}$$

$$= 1.205 W/m^\circ C$$

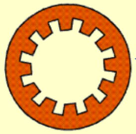
$$R_{p,e} = \frac{1n\left(\frac{d_o}{d_i}\right)}{2\pi L_e k_p} = \frac{1n\left(\frac{0.006}{0.005}\right)}{2\pi(0.02)(401)} = 3.618\times10^{-3}\ {}^\circ C/W$$

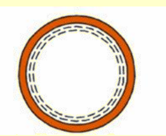
$$R_{w,e} = \frac{1n\left(\frac{d_i}{d_v}\right)}{2\pi L_e k_{eff}} = \frac{1n\left(\frac{0.005}{0.004}\right)}{2\pi(0.02)(1.205)} = 1.474^\circ C/W$$

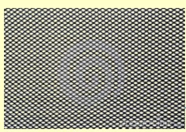
Contact Thermal resistance of solid - solid interface

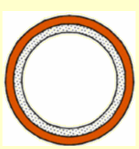
Interface	**Pressure (kN/m^2)**	$R_c \times 10^{-4}$ (m^2K/W)
Silicon chip / lapped aluminium in air	27 - 500	0.3 - 0.6
Aluminium/Aluminium with indium foil filler	100	0.07
SS/SS with indium foil	3500	0.04
Aluminium/Aluminium with Lead coating		0.01 - 0.1
Aluminium/Aluminium with thermal grease	100	0.07
SS/SS with thermal grease	3500	0.04
Silicon chip/aluminium with 0.02 mm exposy		0.2 - 0.9
Brass/Brass with 15micrometer tin solder		0.025 - 0.14

PERMEABILITY





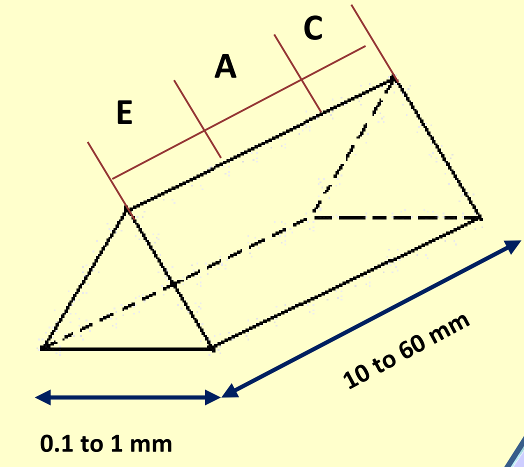


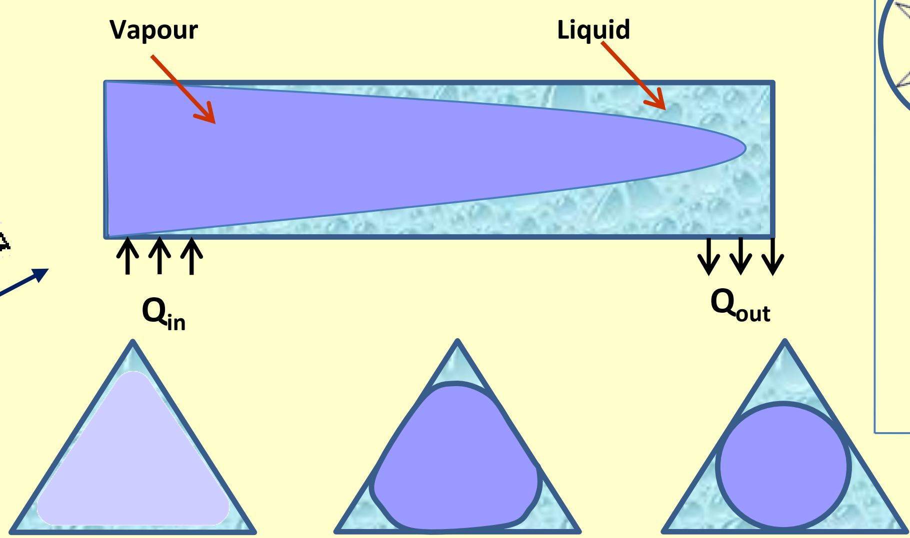


Structure	K	Data
Circular cylinder (artery or tunnel wick)	$r^2/8$	r = radius of liquid flow passage
Open rectangular grooves	$2\varepsilon\left(r_{h,l}\right)^2/(f_l\,\mathrm{Re}_l)$	ε = wick porosity ω = groove width s = groove pitch δ = groove depth $r_{h,l} = 2\omega\delta/(\omega + 2\delta)$
Circular annular wick	$2\left(r_{h,l}\right)^2/(f_l\,\mathrm{Re}_l)$	$r_{h,l} = r_1 - r_2$
Wrapped screen wick	$\dfrac{d_w^2\varepsilon^3}{122(1-\varepsilon)^2}$	d_w = wire diameter in inch $\varepsilon = 1 - (\pi N d_w/4)$ N = mesh number per inch ε = porosity (ratio of pore volume to total volume)
Packed sphere	$\dfrac{r_s^2\varepsilon^3}{37.5(1-\varepsilon)^2}$	r_s = sphere radius ε = porosity (dependent on packing mode)
Sintered metal fibers	$C_1\dfrac{y^2-1}{y^2+1}$ $y = 1 + \dfrac{C_2 d^2\varepsilon^3}{(1-\varepsilon)^2}$ $C_1 = 6.0 \times 10^{10}\,\mathrm{m}^2$ $C_2 = 3.3 \times 10^3\,1/\mathrm{m}^2$	d = fiber diameter ε = porosity (ratio of pore volume to total volume; use manufacturers data)

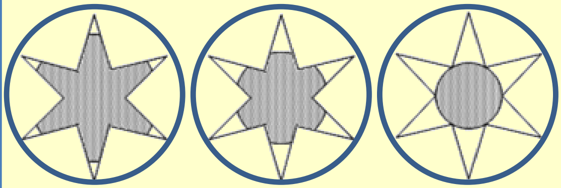
MICRO HEAT PIPES

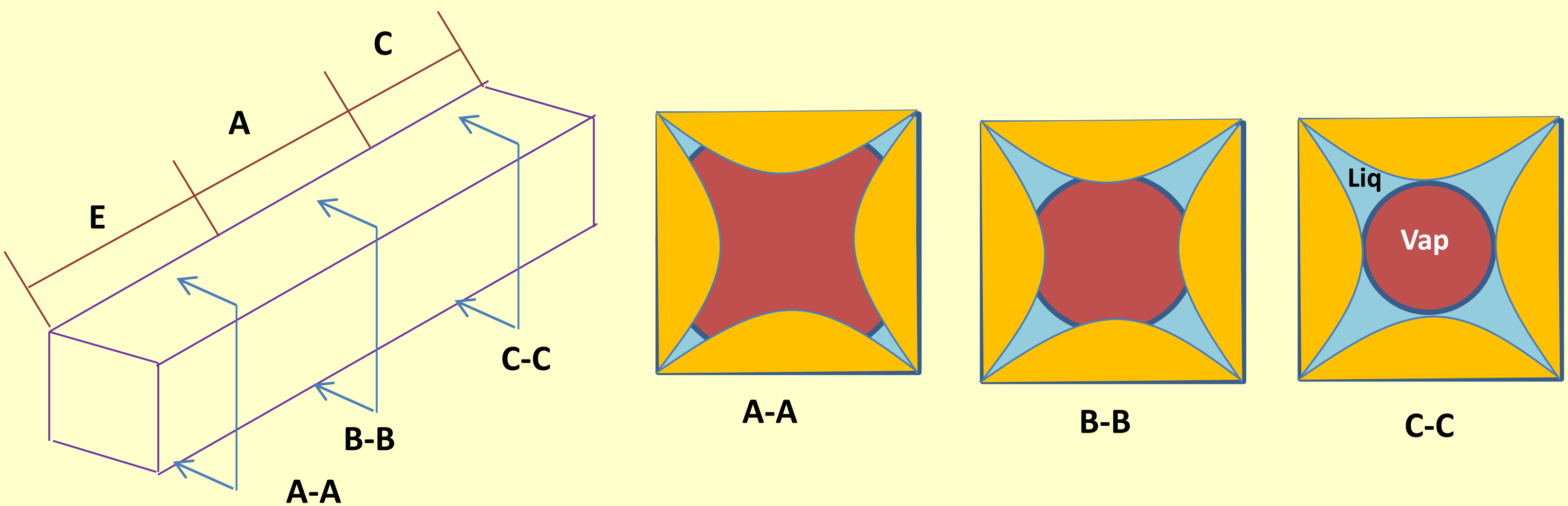
Non-Circular - These are wickless heat pipes - Sharp corner regions
D_h : 0.1 to 1 mm – L :10 to 60 mm





Other shapes





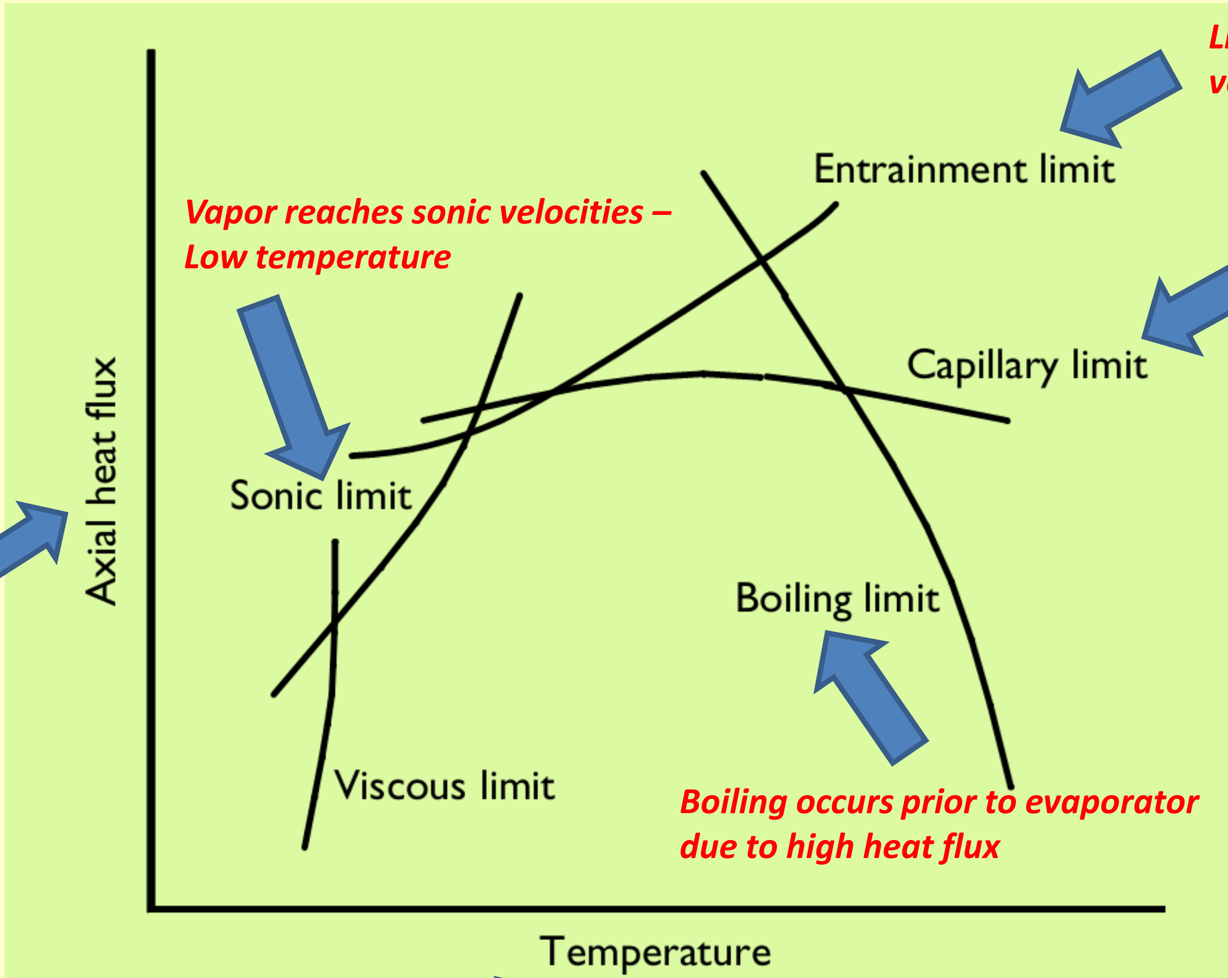
A-A B-B C-C

Heat Transfer Limitations in a Heat pipe

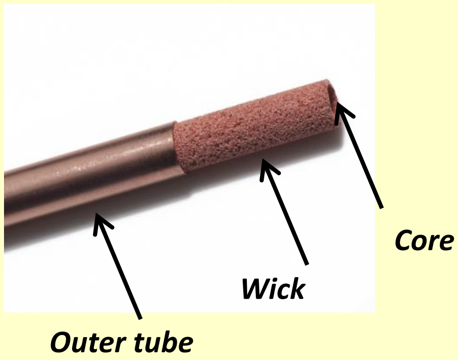
Liquid droplet entering the vapor core

Vapor reaches sonic velocities – Low temperature

Insufficient capillary pressure to pump the working fluid



Boiling occurs prior to evaporator due to high heat flux



DARCY FRICTION FACTOR

Hagen-Poiseuille Theoretical Pressure Drop

$$\Delta P = \frac{32\mu L u_m}{D^2} \qquad \frac{\Delta P}{L} = \frac{32\mu L u_m}{D^2}$$

We represent Darcy friction factor

$$f_D = \frac{\frac{\Delta P}{L} D}{\frac{1}{2}\rho u_m^2}$$

'D' is added to make the equation dimensionless

Replace for ΔP

$$f_D = \frac{\frac{\Delta P}{L} D}{\frac{1}{2}\rho u_m^2} = \frac{\frac{32\mu u_m}{D^2} D}{\frac{1}{2}\rho u_m^2} = \frac{64\mu}{\rho u_m D} = \frac{64}{\mathrm{Re}}$$

f - It should not be confused with the friction coefficient C_f

FANNING FRICTION FACTOR

$$\Delta P\, \pi \frac{D^2}{4} = \tau_W \pi D L \Rightarrow \frac{\Delta P}{L} = \frac{4\tau_W}{D}$$

We represent Fanning friction factor

$$f_F = \frac{\tau_W}{\frac{1}{2}\rho u_m^2}$$

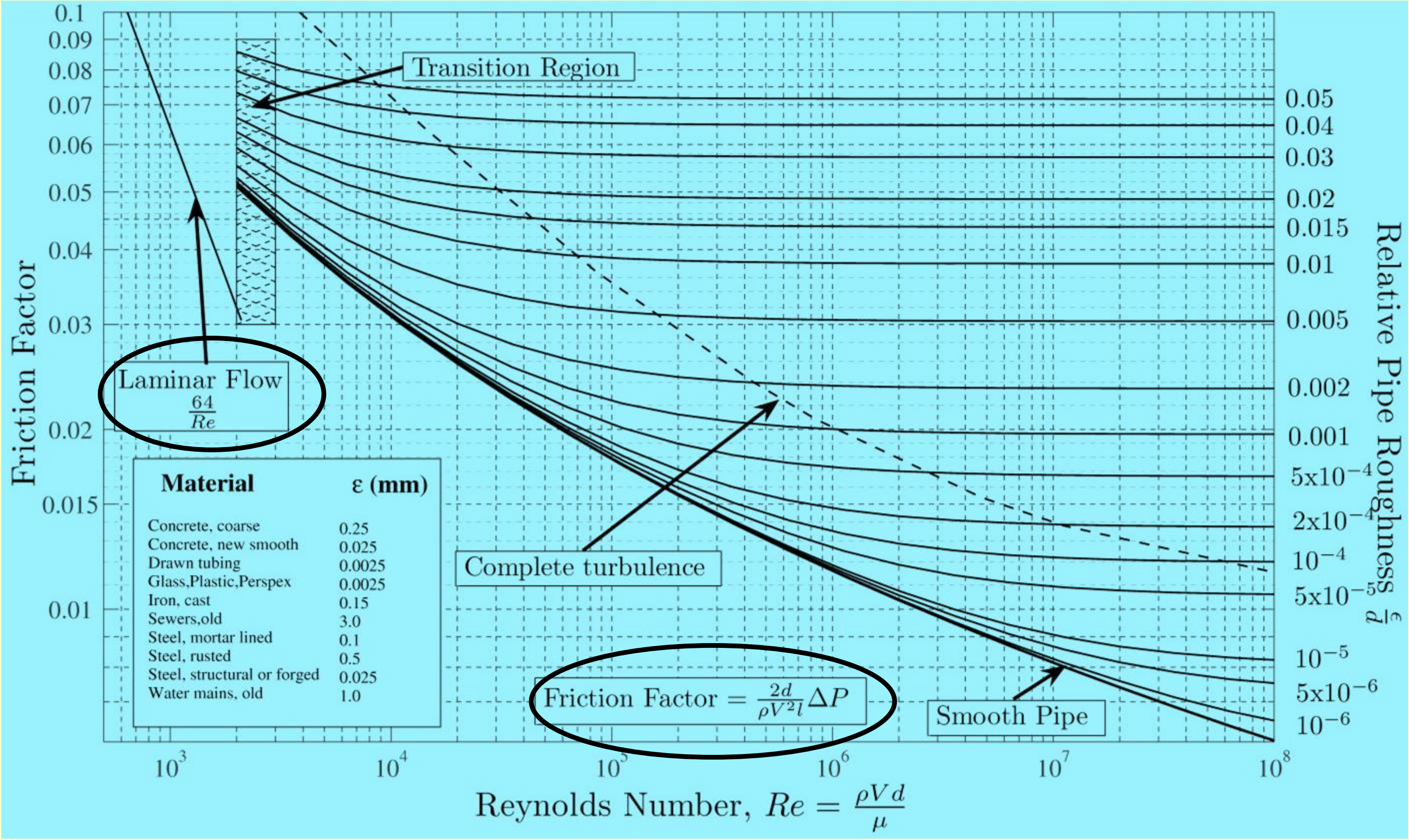
Replace in terms of ΔP

$$f_F = \frac{\tau_W}{\frac{1}{2}\rho u_m^2} = \frac{\frac{\Delta P}{L}\frac{D}{4}}{\frac{1}{2}\rho u_m^2} = \frac{f_D}{4}$$

$$f_F = \frac{f_D}{4} = \frac{16}{\mathrm{Re}}$$

MOODY'S CHART

$$f_D = \frac{\frac{\Delta P}{L} D}{\frac{1}{2}\rho u_m^2}$$



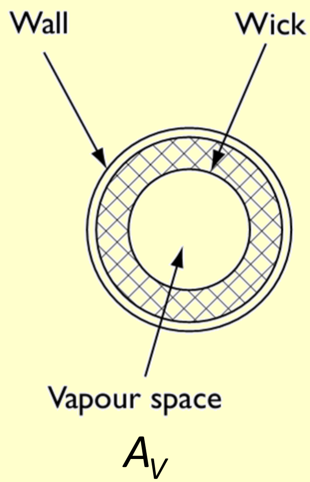
For wick we need to multiply by a correction got either empirically or experimentally

CONVERT VAPOUR PRESSURE DROP IN TERMS OF HEAT FLUX

$$f_D = \frac{\frac{\Delta P_V}{L} d_V}{\frac{1}{2}\rho u_m^2}$$

$$\Delta P_V = f_D \frac{L}{d_V}\frac{1}{2}\rho u_m^2$$

$$\Delta P_V = f_D \frac{L}{d_V}\frac{1}{2}\rho (u_m)(u_m)$$



$$u_m = \frac{q}{\rho A_V h_{fg}}$$

$$q = (\rho A_V u_m) h_{fg}$$

$$q = \dot{m} h_{fg}$$

$$u_m = \frac{\mathrm{Re}\,\mu}{\rho d_V}$$

$$\mathrm{Re} = \frac{\rho u_m d_V}{\mu}$$

$$\Delta P_V = f_D \frac{L}{D}\frac{1}{2}\rho\left(\frac{q}{\rho A_V h_{fg}}\right)\left(\frac{\mathrm{Re}\,\mu}{\rho d_V}\right)$$

$$\Delta P_V = f_D\,\mathrm{Re}\frac{L}{d_V}\frac{1}{2}\left(\frac{q}{\rho A_V h_{fg}}\right)\left(\frac{\mu}{d_V}\right)$$

$$\Delta P_V = f_D\,\mathrm{Re}\frac{L}{4r_V}\left(\frac{q}{\rho A_V h_{fg}}\right)\left(\frac{\mu}{d_V}\right)$$

$$\Delta P_V = f_D\,\mathrm{Re}\frac{L}{4r_V}\left(\frac{q}{\rho A_V h_{fg}}\right)\left(\frac{\mu}{d_V}\right)$$

$$\Delta P_V = \frac{(f_D\,\mathrm{Re})\mu_V}{8r_V{}^2\rho_V A_V h_{fg}} L_{eff}\, q$$

In terms of Fanning f $f_F = \frac{f_D}{4} = \frac{16}{\mathrm{Re}}$

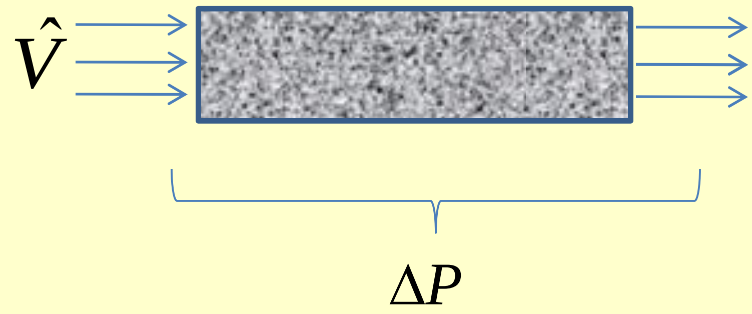
$$\Delta P_V = \frac{(f_F\,\mathrm{Re})\mu_V}{2r_V{}^2\rho_V A_V h_{fg}} L_{eff}\, q$$

$$\boxed{\Delta P_V = \frac{C(f_F\,\mathrm{Re})\mu_V}{2r_V{}^2\rho_V A_V h_{fg}} L_{eff}\, q}$$

For wick we need to multiply by a correction got either empirically or experimentally

LIQUID PRESSURE DROP AND PERMEABILITY (κ)

LIQUID VOLUME FLOW RATE (m^3/s)



Similar to Heat conduction equation

$$\hat{V} \alpha \frac{A}{\mu_l}\frac{\Delta P}{L} \Rightarrow \hat{V} = -\kappa \frac{A}{\mu_l}\frac{\Delta P}{L}$$

Permeability

$$\frac{\hat{V}}{A_W} = u_m = -\kappa \frac{\Delta P_l}{\mu_l L}$$

$$\frac{\Delta P_l}{L} = -u_m \frac{\mu_l}{\kappa}$$

$$\boxed{u_m = \frac{q}{\rho_l A_W h_{fg}}}$$

$$\Delta P_l = \left(\frac{\mu_l}{\kappa A_W h_{fg} \rho_l}\right) L_{eff}\, q$$

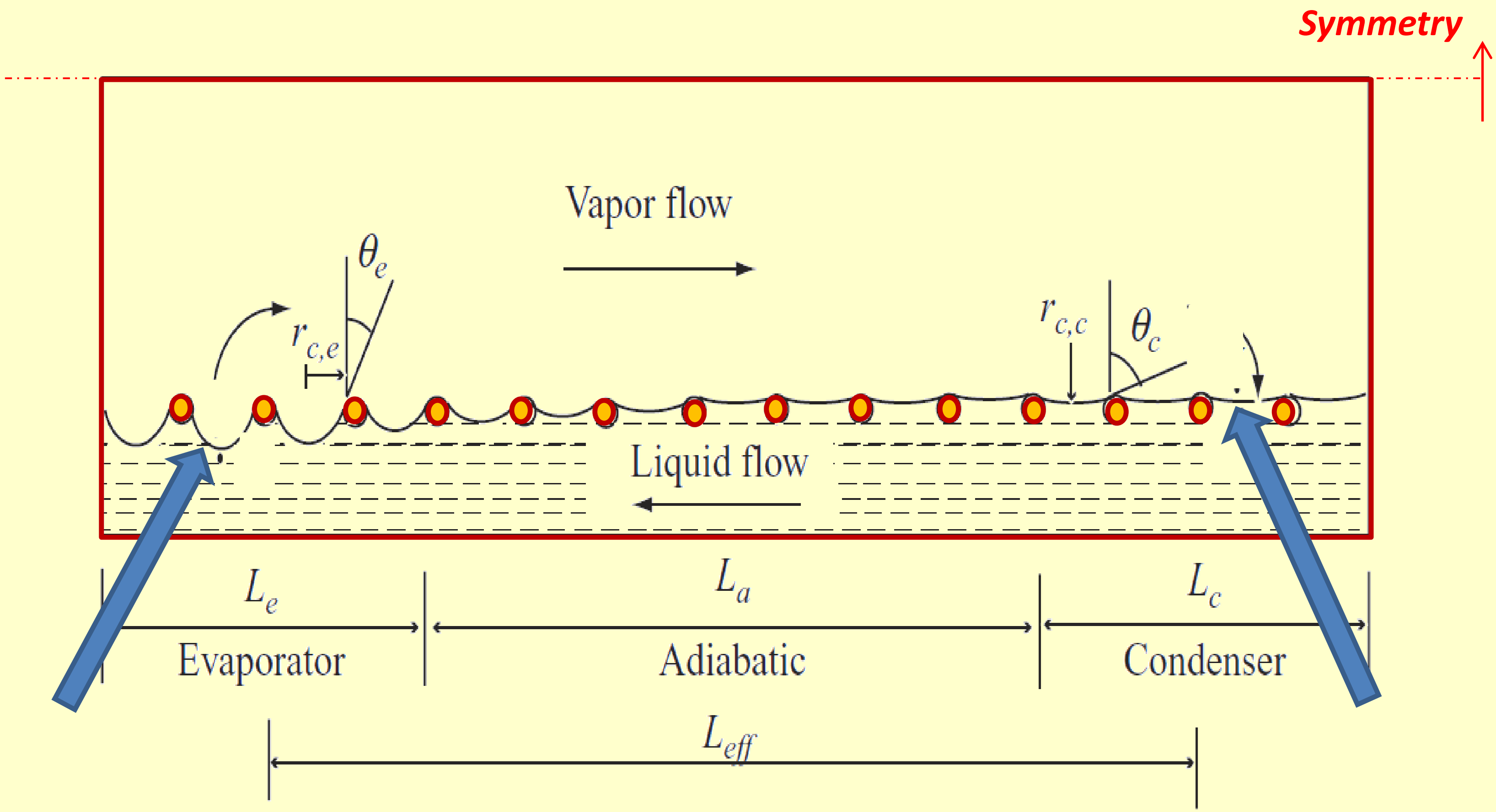
$$\frac{m^3/s}{m^2} = m/s$$

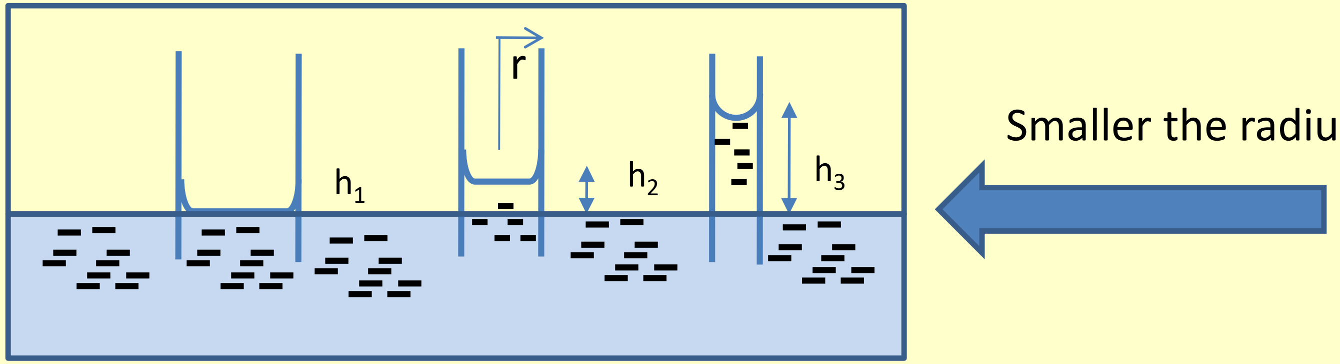
Volumetric flux
Or Darcian Velocity

Continuity

$$\hat{V} = A_W u_m$$

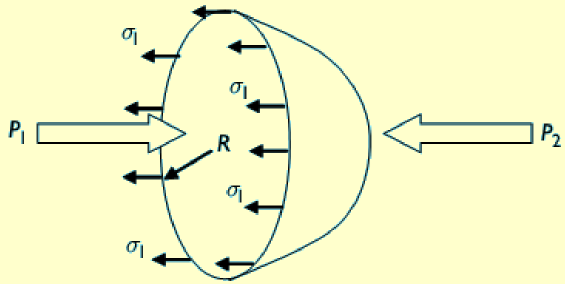
Capillary Limitation

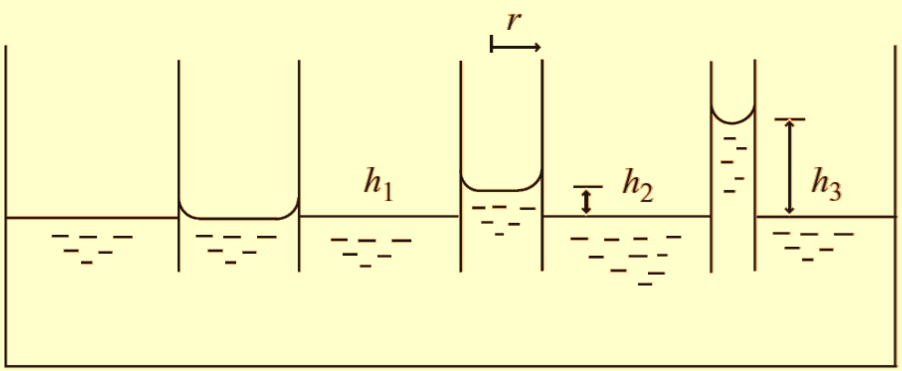




Smaller the radius larger the lift

SURFACE TENSION AND CAPILLARITY





The hemispherical surface tension force acting around the circumference

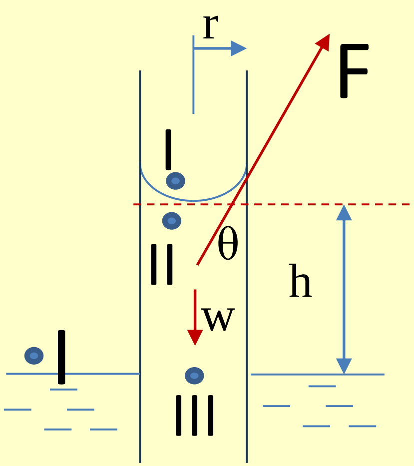
$$2\pi R\sigma_l$$

The net force on the surface due to the pressures

$$(P_1 - P_2)\pi R^2$$

Equating the above two

$$\Delta P = \frac{2\sigma_l}{R}$$



In Capillary rise

$$P_I = P_{III} \qquad F_Y = \sigma 2\pi r \cos(\theta)$$

Weight of the liquid in the tube above the bulk liquid level

$$W = mg = \rho V g = \rho\left(\pi r^2 h\right) g$$

From above balance of two equation

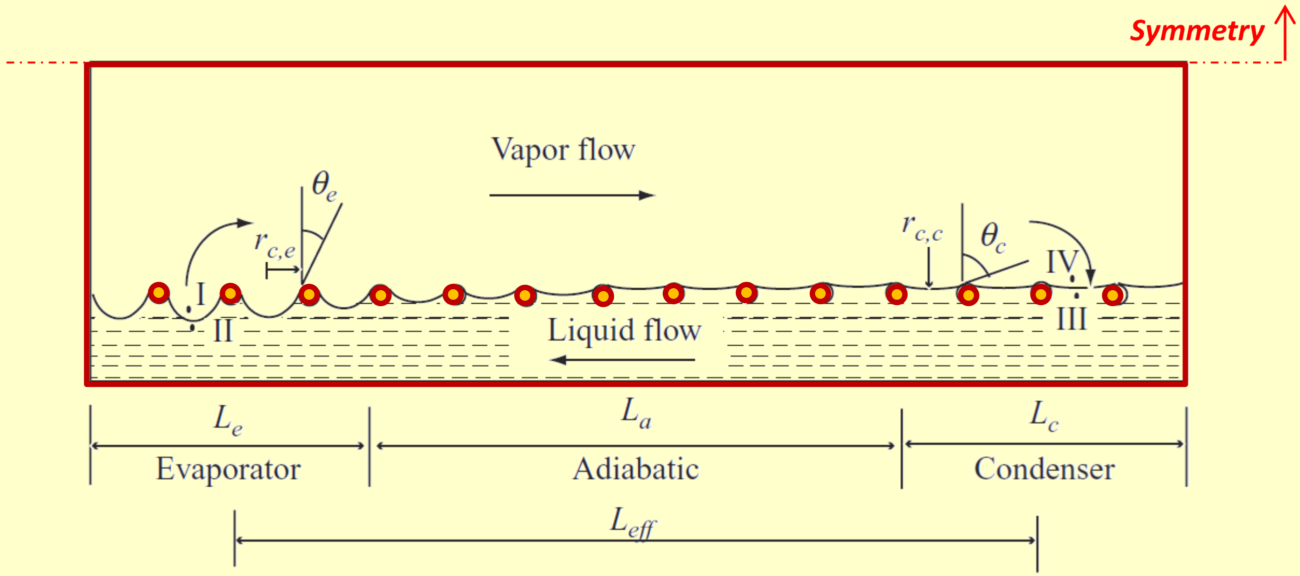
$$\rho g h = \frac{2\sigma \cos(\theta)}{r}$$

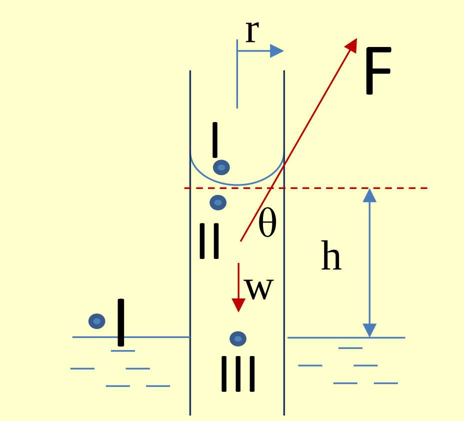
In Terms of Pressure difference

$$P_{III} - P_{II} = \rho g h$$

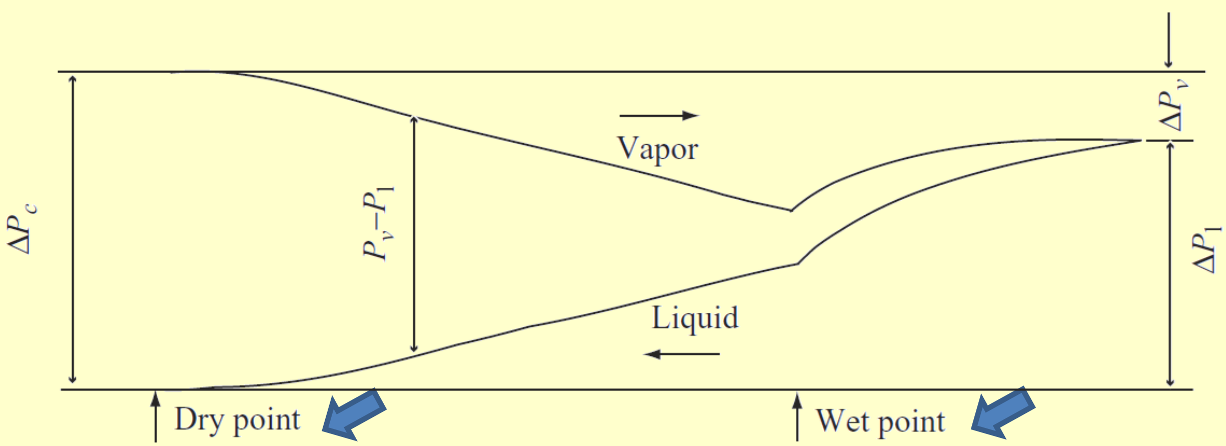
$$P_I - P_{II} = \frac{2\sigma \cos(\theta)}{r}$$

Capillary rise





$$P_I - P_{II} = \frac{2\sigma\cos(\theta)}{r}$$



Point at which the meniscus has minimum radius

Vapor and Liquid Pressure are equal or maximum radius of curvature

Capillary pressure difference / head = $P_{III} - P_{II}$

$$P_I - P_{II} = \frac{2\sigma\cos(\theta_e)}{r_{c,e}}$$

$$P_{IV} - P_{III} = \frac{2\sigma\cos(\theta_c)}{r_{c,c}}$$

For a heat pipe the net capillary force between evaporator and condenser should overcome all the pressure drops.

Capillary Limitation

Maximum capillary pressure difference

$P_{III} - P_{II}$

$$\boxed{L_{eff} = \frac{L_e}{2} + L_a + \frac{L_c}{2}}$$

$$\Delta P_{c,m} = \int_{Leff} \frac{\partial P_v}{\partial x}\,dx + \int_{Leff} \frac{\partial P_l}{\partial x}\,dx + \Delta P_{ph,e} + \Delta P_{ph,c} + \Delta P_{norm} + \Delta P_{axial}$$

Pressure drop in vapor phase (Inertial and Viscous)

Pressure drop in liquid phase (Inertial and Viscous)

Pressure drop in liquid to vapor phase Negligible

Pressure drop in vapor to liquid phase Negligible

Hydrostatic normal

Hydrostatic axial

$$\Delta P_{c,m} = \Delta P_v + \Delta P_l + \Delta P_{norm} + \Delta P_{axial}$$

Maximum Capillary pressure occur

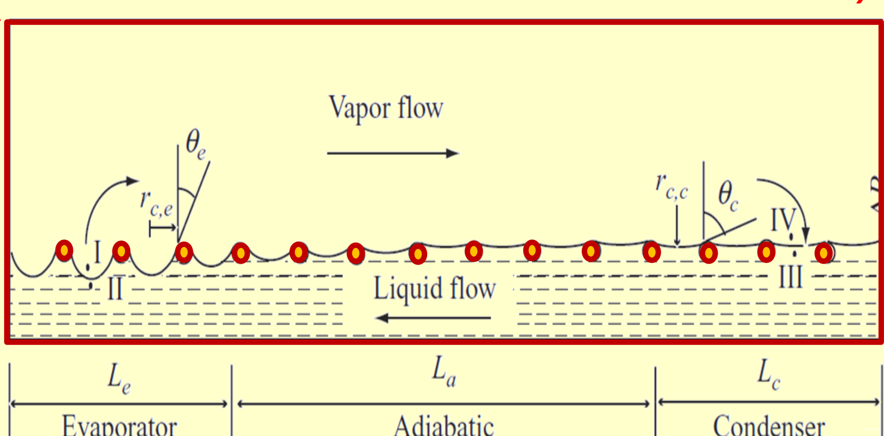
$$\theta_e = 0 \quad ; \; \theta_c = \frac{\pi}{2} \Rightarrow r_{c,c} = \infty$$

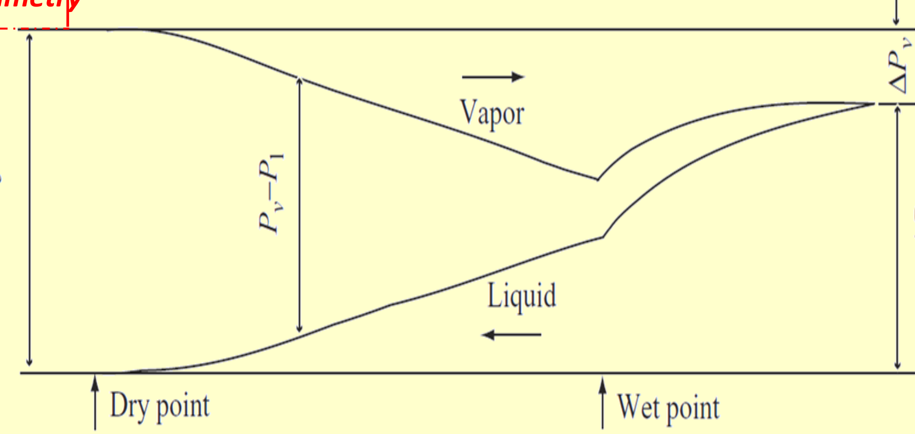
$$\Delta P_{c,m} = P_{III} - P_{II} = \frac{2\sigma}{r_{c,e}} - \frac{2\sigma}{r_{e,c}} \cong \frac{2\sigma}{r_{c,e}}$$

$$P_I - P_{II} = \frac{2\sigma \cos(\theta_e)}{r_{c,e}}$$

$$P_{IV} - P_{III} = \frac{2\sigma \cos(\theta_c)}{r_{c,c}}$$

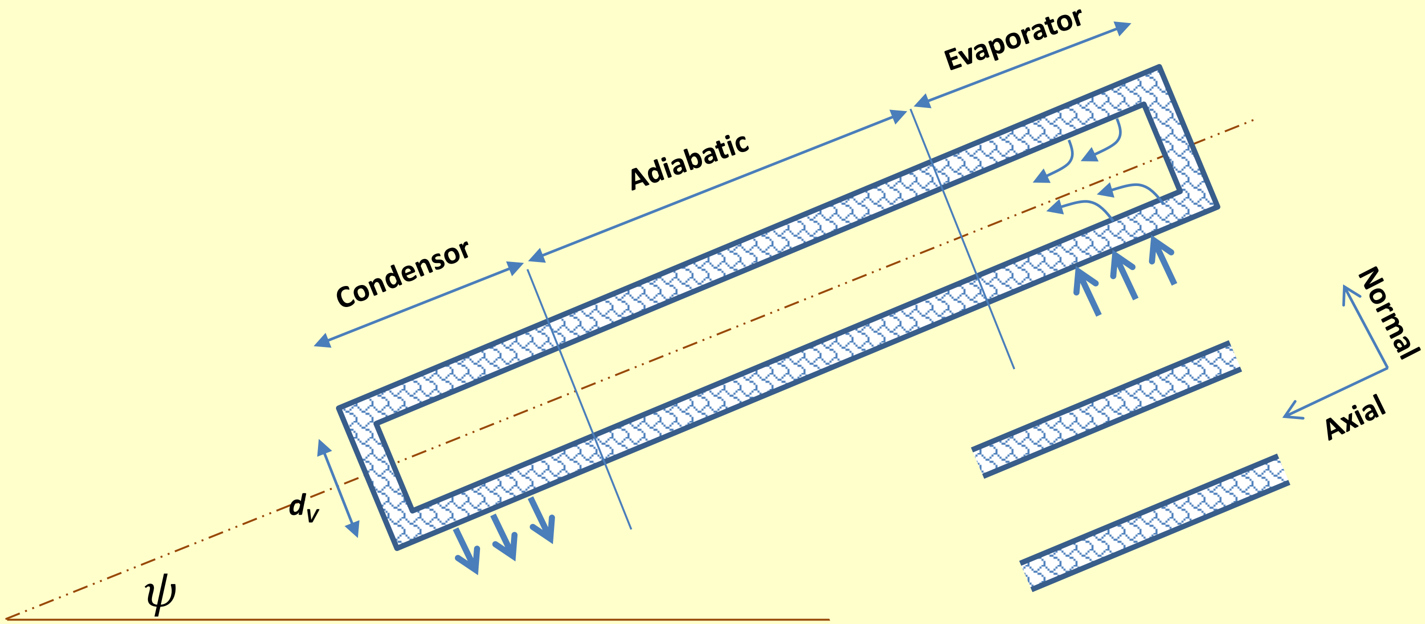
Symmetry





Critical capillary radius (r_c)

Structure	r_c	Data
Circular cylinder (artery or tunnel wick)	r	r = radius of liquid flow passage
Rectangular groove	ω	ω = groove width
Triangular groove	$\omega/\cos\beta$	ω = groove width β = half-included angle
Parallel wires	ω	ω = wire spacing
Wire screens	$(\omega + d_\omega)/2 = 1/2N$	d = wire diameter N = screen mesh number per inch ω = wire spacing
Packed spheres	$0.41r_s$	r_s = sphere radius
Sintered metal fibers*	$d/2\,(1-\varepsilon)$	d = fiber diameter ε = porosity (ratio of pore volume to total volume; use manufacturer's data)
Trapezoidal microheat pipe*	$\omega/(\cos\alpha\cos\theta)$	ω = grooove width α = half included angle θ = liquid wetting angle



$$\Delta P_{c,m} = \Delta P_v + \Delta P_l + \Delta P_{norm} + \Delta P_{axial}$$

$$\Delta P_{c,m} = \frac{2\sigma}{r_{c,e}}$$

$$\Delta P_v = \frac{C\left(f_v \operatorname{Re}_v\right)\mu_v}{2r_v^2 A_v \rho_v h_{fg}} L_{eff}\, q$$

$$\Delta P_l = \left(\frac{\mu_1}{K A_\omega h_{fg} P_l}\right) L_{eff}\, q$$

$$\Delta P_{norm} = \rho_l g d_v \cos\psi$$

$$\Delta P_{axial} = \rho_l g L \sin\psi$$

The equation can be simplified based on the orientation. In many cases the Liquid Pressure drop is negligible

$$\Delta P_v = \frac{C\left(f_v \mathrm{Re}_v\right)\mu_v}{2r_v^2 A_v \rho_v h_{fg}} L_{eff}\, q$$

"To find C"

When $\mathrm{Re}_v < 2300$ and $\boxed{Ma_v < 0.2}$

$$f_v \mathrm{Re}_v = 16$$
$$\mathrm{C} = 1.0$$
$$Ma_v = \text{Mach number of vapor}$$

When $\mathrm{Re}_v < 2300$ and $\boxed{Ma_v > 0.2}$

$$f_v \mathrm{Re}_v = 16$$
$$C = \left[1 + \left(\frac{\gamma_v - 1}{2}\right) Ma_v^2\right]^{-1/2}$$

When $\mathrm{Re}_v > 2300$ and $Ma_v < 0.2$

$$f_v \mathrm{Re}_v = 0.038 \left[\frac{2r_v q}{A_v \mu_v h_{fg}}\right]^{3/4}$$
$$\mathrm{C} = 1.0$$

When $\mathrm{Re}_v > 2300$ and $Ma_v > 0.2$ [26]

$$f_v \mathrm{Re}_v = 0.038$$
$$C = \left[1 + \left(\frac{\gamma_v - 1}{2}\right) Ma_v^2\right]^{-1/2} \left(\frac{2r_v q}{A_v \mu_v h_{fg}}\right)^{3/4}$$

$$Ma_v = \frac{V}{c} = \frac{\dot{m}}{\rho_v A_v \sqrt{\gamma_v R_v T_v}} = \frac{q}{\rho_v A_v h_{fg} \sqrt{\gamma_v R_v T_v}}$$

Structure	K	Data
Circular cylinder (artery or tunnel wick)	$r^2/8$	r = radius of liquid flow passage
Open rectangular grooves	$2\varepsilon\left(r_{h,l}\right)^2/(f_l \mathrm{Re}_l)$	ε = wick porosity ω = groove width s = groove pitch δ = groove depth $r_{h,l} = 2\omega\delta/(\omega + 2\delta)$
Circular annular wick	$2\left(r_{h,l}\right)^2/(f_l \mathrm{Re}_l)$	$r_{h,l} = r_1 - r_2$
Wrapped screen wick	$\dfrac{d_w^2\varepsilon^3}{122(1-\varepsilon)^2}$	d_w = wire diameter in inch $\varepsilon = 1 - (\pi N d_w/4)$ N = mesh number per inch ε = porosity (ratio of pore volume to total volume)
Packed sphere	$\dfrac{r_s^2\varepsilon^3}{37.5(1-\varepsilon)^2}$	r_s = sphere radius ε = porosity (dependent on packing mode)
Sintered metal fibers	$C_1\dfrac{y^2-1}{y^2+1}$ $y = 1 + \dfrac{C_2 d^2\varepsilon^3}{(1-\varepsilon)^2}$ $C_1 = 6.0 \times 10^{10}\,\mathrm{m}^2$ $C_2 = 3.3 \times 10^3\,1/\mathrm{m}^2$	d = fiber diameter ε = porosity (ratio of pore volume to total volume; use manufacturers data)
Trapezoidal microheat pipe*	$\dfrac{2\varepsilon\left(r_{h,l}\right)^2}{f_l \mathrm{Re}_l}$	$r_{h,l} = r_1 - r_2$
Rectangular artery***	$\dfrac{2r_{h,l}^2}{f_l \mathrm{Re}_l}$	ω = arterial width δ = arterial depth $r_{h,l} = \dfrac{\delta\omega}{\delta+\omega}$ $A_w = \delta\omega N$ N = number of arteries $f_l\mathrm{Re}_l = 16$ for laminar $f_l\mathrm{Re}_l = \dfrac{\mathrm{Re}_l}{4(0.79\ln(\mathrm{Re}_l) - 1.64)^2}$ for turbulent [32]

Entrainment Limitation

Weber number = $F_{INERTIA}$ / $F_{SURFACE\ TENSION}$

$$\text{Inertia force} = m.a = \rho_V.(\text{volume}).\frac{V}{t} = \rho_V.(L^3)\frac{\frac{L}{t}}{t} = \rho_V.(L^2)L\frac{\frac{L}{t}}{t} = \rho_V.(L^2)\frac{L}{t}\frac{L}{t} \propto \rho_V.V^2.L^2$$

$$\text{Surface Tension force} = \sigma L$$

$$\text{Weber number} = \frac{\rho_V.V^2.L^2}{\sigma L} = \frac{\rho_V.V^2 L}{\sigma} \quad (L \ \ is \ \ Hyd \ \ diameter \ \ of \ \ wick = d_w = 2r_w)$$

$$We = \frac{2r_w \rho_v V_v^2}{\sigma} = 1 \rightarrow V = \left(\frac{\sigma}{2r_w \rho_V}\right)^{1/2} \rightarrow q = mh_{fg} = V_V = \rho A_V h_{fg}\left(\frac{\sigma}{2r_w \rho_V}\right)^{1/2}$$

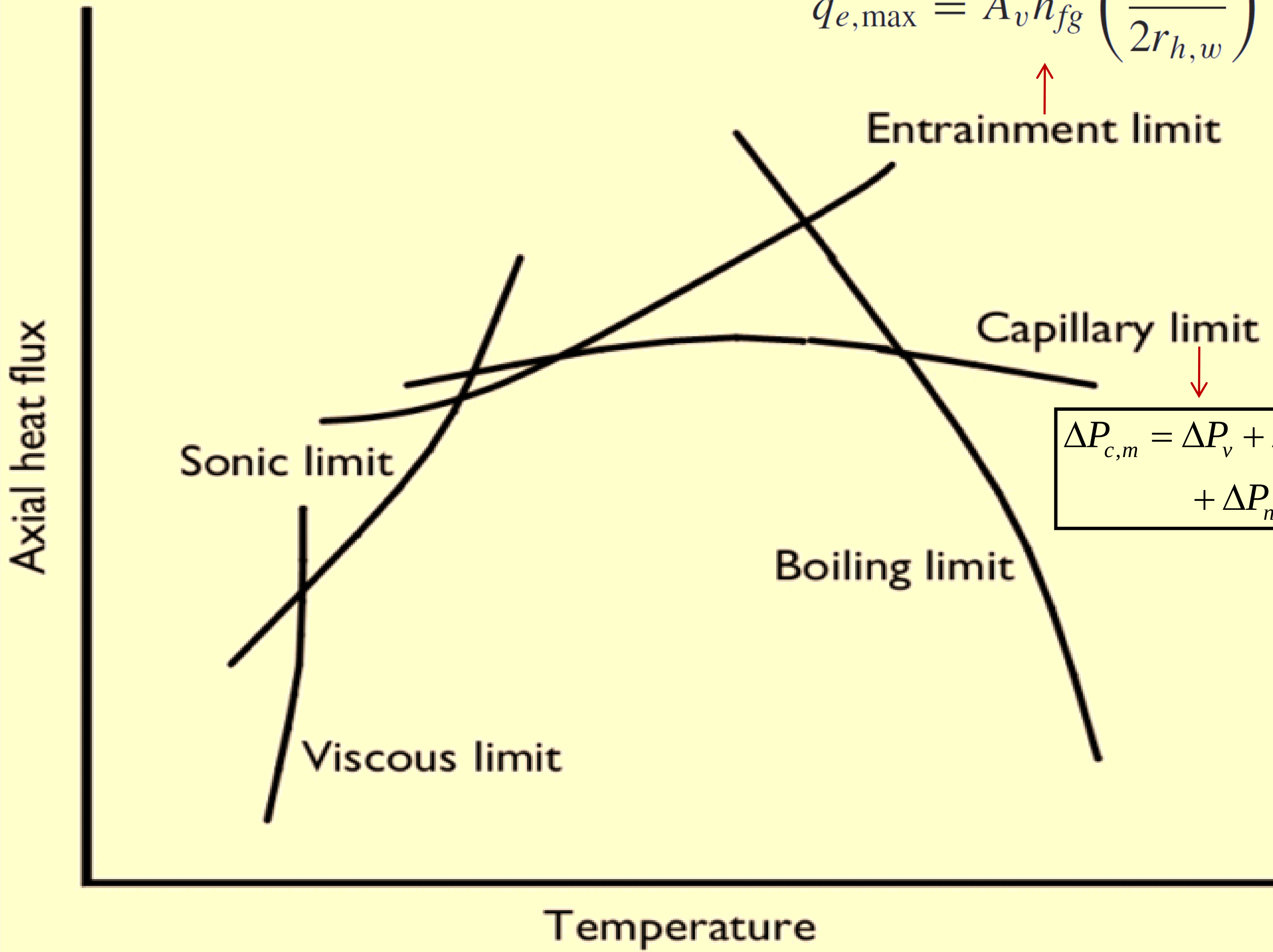
Replace velocity in terms of mass flow rate and with h_{fg} and q

$$q_{e,\max} = A_v h_{fg}\left(\frac{\sigma \rho_v}{2r_{h,w}}\right)^{\frac{1}{2}}$$

For non circular shapes

$$r_{h,w} = D_h/2 \qquad D_h = \frac{4A}{P}$$

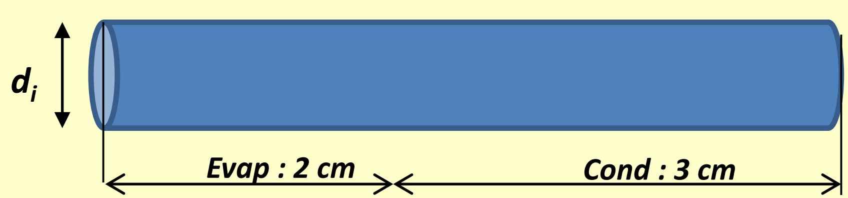
$$q_{e,\max} = A_v h_{fg} \left(\frac{\sigma \rho_v}{2 r_{h,w}} \right)^{\frac{1}{2}}$$

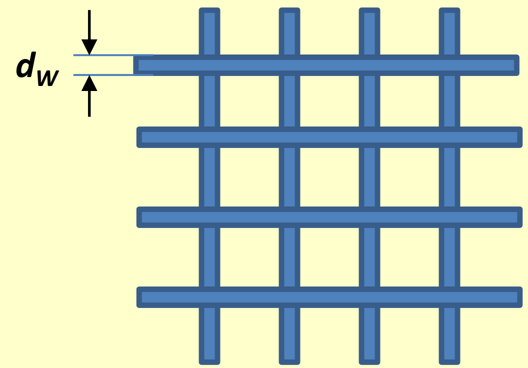


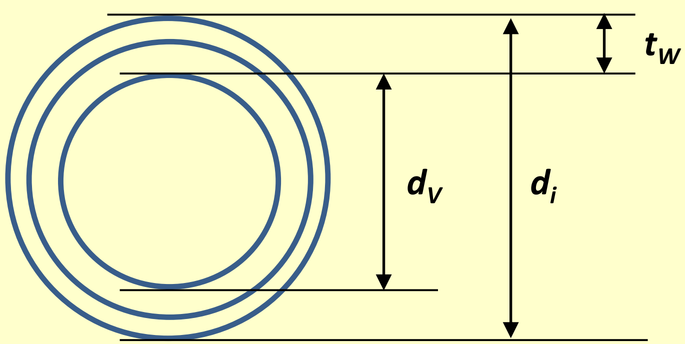
$$\boxed{\begin{aligned}\Delta P_{c,m} &= \Delta P_v + \Delta P_l \\ &\quad + \Delta P_{norm} + \Delta P_{axial}\end{aligned}}$$

Design Problem

A Heat pipe for an electronics uses ethanol as the working fluid. The evaporator is 2 cm long, the condenser is 3 cm long, and the heat pipe has no adiabatic section. The diameter of the vapour space is 3 mm and the wicking structure consists of **3 layers of of # 500 mesh 304 SS screen** (wire diameter = 0.00085 in). If the heat pipe operates at 30^{o}C in a horizontal position, determine the capillary, sonic and boiling limits.







$$\Delta P_{c,m} = \Delta P_v + \Delta P_l + \Delta P_{norm} + \Delta P_{axial}$$

Thermo physical properties of ethanol

$P_v = 10,000 Pa$

$h_{fg} = 888,600 J/kg$

$\rho_l = 781 kg/m^3,$

$\rho_v = 0.38 kg/m^3$

$k_l = 0.168 \quad W/m.K$

$\mu_l = 1.02 \times 10^{-3} kg/m/s$

$\mu_v = 0.91 \times 10^{-5} kg/m.s,$

$\sigma = 2.44 \times 10^{-2} N/m,$

$k_{\omega ire} = 14.9 W/m.k$

Dimensions

$L_e = 0.02m \quad L_c = 0.03m$

$$L_{eff} = \frac{L_e}{2} + L_a + \frac{L_c}{2} = 0.01 + 0.015 = 0.025m$$

$d_V = 0.003m, r_V = d_V/2 = 0.0015m$

$A_V = \pi r_V^2 = 7.609 \times 10^{-6} m^2$

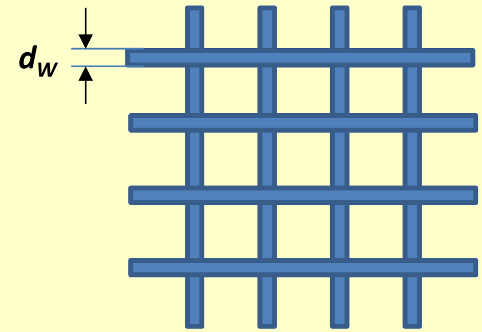
$d_W = 0.00085 \; in. = 2.159 \times 10^{-5} (m)$

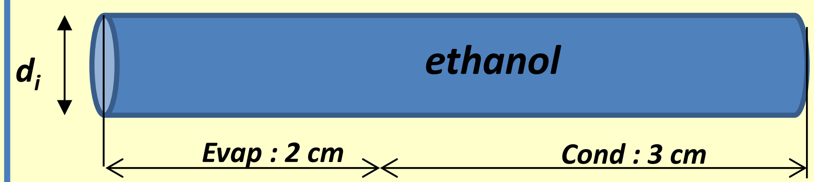
$t_W = 3(2d_W) = 12.954 \times 10^{-5} m \quad (to \quad be \quad assumed)$

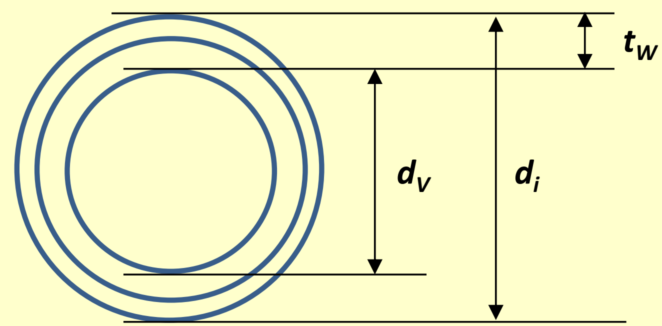
$d_i = d_v + 2t_W = 0.003259 \quad m$

$N = 500 \quad in.^{-1} = 500/in. \quad \times 1 \, inch / \; 2.54 \times 10^{-2} m$

$= 19,685 m^{-1}$







Structure	r_c	Data
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Capillary Limits $\Delta P_{C,M} = \Delta P_V + \Delta P_L + \Delta P_{norm} + \cancel{\Delta P_{axial}}$

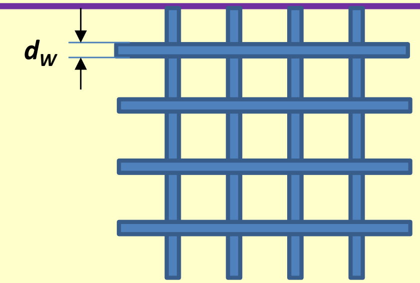
Since horizontal

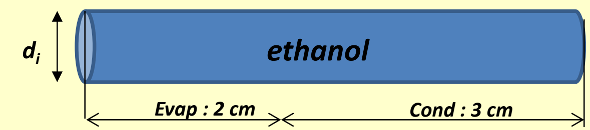
$$\Delta P_{C,M} = \frac{2\sigma}{r_{c,e}}$$

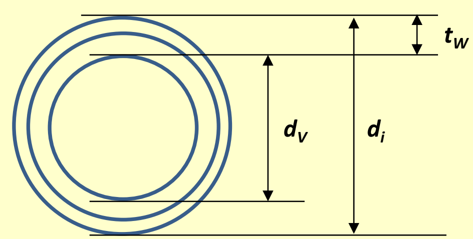
Effective capillary radius

$$r_{c,e} = \frac{1}{2N} = \frac{1}{2(500 / in.)} = 0.001 \quad in. = 2.54\times10^{-5}\, m$$

$$\Delta P_{c,m} = \frac{2\sigma}{r_{c,e}} = \frac{2\left(2.44\times10^{-2}\right)}{2.54\times10^{-5}} = 1{,}921\, Pa$$







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$$\rho_l = 781 kg / m^3,$$
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$$k_l = 0.168 \quad W / m.K$$
$$\mu_l = 1.02\times10^{-3} kg / m / s$$
$$\mu_v = 0.91\times10^{-5} kg / m.s,$$
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$$k_\omega = 14.9 W / m.k$$

Structure	K	Data
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$$\Delta P_v = \frac{C\left(f_v \mathrm{Re}_v\right)\mu_v}{2r_v^2 A_v \rho_v h_{fg}} L_{eff}\, q$$

"To find C"

When $\mathrm{Re}_v < 2300$ and $\boxed{Ma_v < 0.2}$

$$f_v \mathrm{Re}_v = 16$$
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When $\mathrm{Re}_v > 2300$ and $Ma_v < 0.2$

$$f_v \mathrm{Re}_v = 0.038 \left[\frac{2r_v q}{A_v \mu_v h_{fg}}\right]^{3/4}$$
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When $\mathrm{Re}_v > 2300$ and $Ma_v > 0.2$ [26]

$$f_v \mathrm{Re}_v = 0.038$$
$$C = \left[1 + \left(\frac{\gamma_v - 1}{2}\right) Ma_v^2\right]^{-1/2} \left(\frac{2r_v q}{A_v \mu_v h_{fg}}\right)^{3/4}$$

$$Ma_v = \frac{V}{c} = \frac{\dot{m}}{\rho_v A_v \sqrt{\gamma_v R_v T_v}} = \frac{q}{\rho_v A_v h_{fg} \sqrt{\gamma_v R_v T_v}}$$

Thermo physical properties of ethanol

$P_v = 10{,}000 Pa \qquad k_l = 0.168 \quad W/m.K$

$h_{fg} = 888{,}600 J/kg \qquad \mu_l = 1.02\times10^{-3} kg/m/s$

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$\rho_v = 0.38 kg/m^3 \qquad \sigma = 2.44\times10^{-2} N/m, k_\omega = 14.9 W/m.k$

$$L_e = 0.02m \quad L_c = 0.03m$$

$$L_{eff} = \frac{L_e}{2} + L_a + \frac{L_c}{2} = 0.01 + 0.015 = 0.025m$$

$$d_V = 0.003m, r_V = d_V/2 = 0.0015m$$

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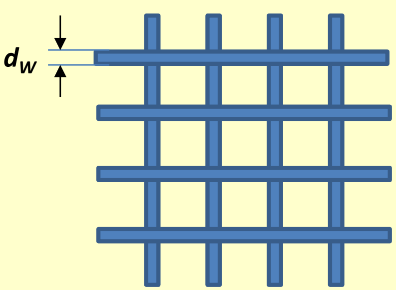
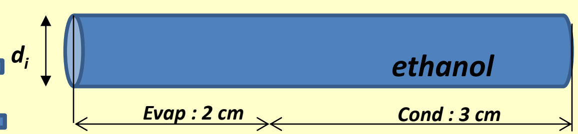
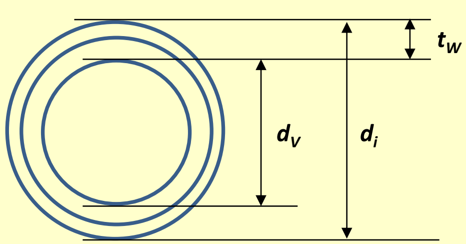
$$N = 19{,}685 m^{-1}$$







Vapour Pressure drop

$$\Delta P_V = \frac{C(f_V \mathrm{Re}_V)\mu_V}{2r_V^2 A_V \rho_V h_{fg}} L_{eff}\, q \qquad f_V \;\; is \;\; f_{FANNING}$$

No Info related to Mach number or vapor velocity give →
Therefore 1st Iteration →assume laminar Re_V < 2300 and proceed

$$\mathrm{Re}_V < 2300 \;\; and \;\; Ma_V < 0.2$$

$$C = 1\,;\; f_V R_{ev} = 16$$

$$\Delta P_V = \frac{(1)(16)(0.91\times10^{-5})}{2(0.0015)^2(7.069\times10^{-6})(0.38)(888600)} 0.025q = 0.561q$$

Liquid Pressure drop

$$\Delta P_l = \left(\frac{\mu_1}{K A_W h_{fg} \rho_l}\right) L_{eff}\, q$$

$$A_\omega = \frac{\pi(d_t^2 - d_v^2)}{4} = \frac{\pi(0.003259^2 - 0.003^2)}{4} \quad = 1.273\times10^{-6} m^2$$

$$K = \frac{d_\omega^2 \varepsilon^3 \pi}{122(1-\varepsilon)^2} = \frac{(2.159\times10^{-5})^2}{122(1-0.6495)^2} \quad = 8.52\times10^{-12}$$

$$\varepsilon = 1 - \frac{\pi N d_w}{4} = 0.6495 \qquad \Delta P_v = 3387q$$

Thermo physical properties of ethanol

$P_v = 10,000 Pa$ $\quad k_l = 0.168 \quad W/m.K$

$h_{fg} = 888,600 J/kg$ $\quad \mu_l = 1.02 \times 10^{-3} kg/m/s$

$\rho_l = 781 kg/m^3,$ $\quad \mu_v = 0.91 \times 10^{-5} kg/m.s,$

$\rho_v = 0.38 kg/m^3$ $\quad \sigma = 2.44 \times 10^{-2} N/m, k_\omega = 14.9 W/m.k$

$$L_e = 0.02m \quad L_c = 0.03m$$

$$L_{eff} = \frac{L_e}{2} + L_a + \frac{L_c}{2} = 0.01 + 0.015 = 0.025m$$

$$d_V = 0.003m, r_V = d_V/2 = 0.0015m$$

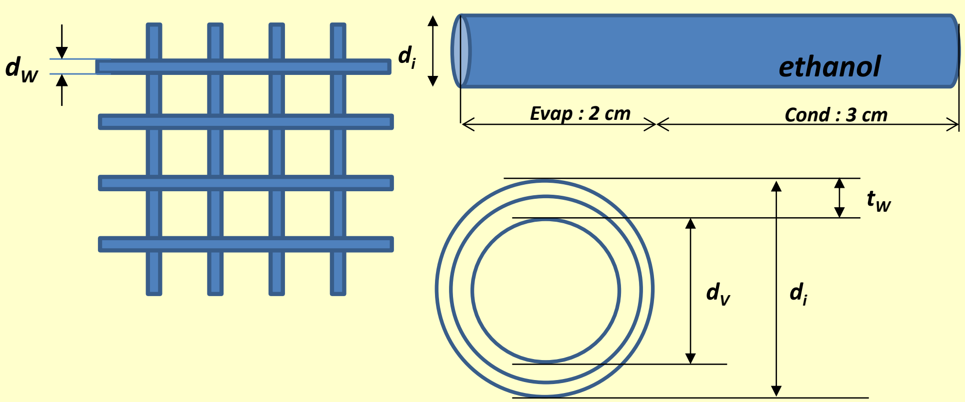
$$A_V = \pi r_V^2 = 7.609 \times 10^{-6} m^2$$

$$d_W = 0.00085\, in. = 2.159 \times 10^{-5} m$$

$$t_W = 3(2d_W) = 12.954 \times 10^{-5} m$$

$$d_i = d_v + 2t_W = 0.003259 \quad m$$

$$N = 19,685 m^{-1}$$



Hydrostatic Pressure drop

$$(\Delta P)_{norm} = \rho_l\, g\, dv \cos\psi = 781 * 9.81 * (0.003) \cos 0$$
$$= 22.98 Pa$$

$$\Delta P_{C,M} = \Delta P_V + \Delta P_L + \Delta P_{norm} + \Delta P_{axial}$$

$$1921 = 13.55q + 3387q + 22.98$$

$$Solve\ for\ q = 0.56W$$

Capillary limits

Check for Re for laminar assumption

$$\mathrm{Re}_v = \frac{\rho_v V d_v}{\mu_v} = \frac{4\dot{m}}{\pi d_v \mu_v} = \frac{4q}{\pi d_v \mu_v h_{fg}} < 2300$$

Entrainment limit

$$We = \frac{2 r_w \rho_v V_v^2}{\sigma} \qquad V_v = \frac{q}{A_v \rho_v h_{fg}}$$

$$q_{e,\max} = A_v h_{fg} \left(\frac{\sigma \rho_v}{2 r_{h,\omega}} \right)^{\frac{1}{2}} \qquad q_{e,\max} = 37.58W$$

Sonic limit

$$q_{s,\max} = 0.487 h_{fg} A_v \left(\rho_v P_v \right)^{\frac{1}{2}} \qquad q_{s,\max} = 183.5W$$

Thermo physical properties of ethanol

$P_v = 10,000 Pa \qquad k_l = 0.168 \quad W/m.K$

$h_{fg} = 888,600 J/kg \qquad \mu_l = 1.02\times10^{-3} kg/m/s$

$\rho_l = 781 kg/m^3, \qquad \mu_v = 0.91\times10^{-5} kg/m.s,$

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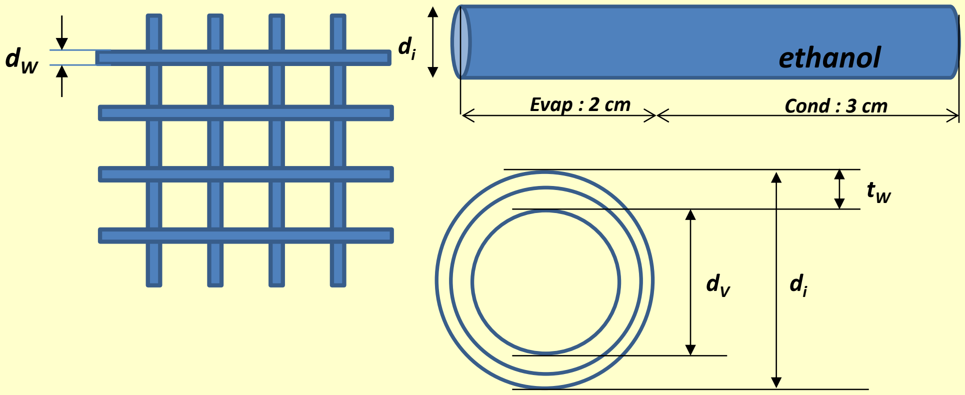

Boiling limit

$$q_{b,\max} = \frac{4\pi L_e k_{eff} T_v \sigma}{h_{fg}\rho_v \ln\left(\frac{r_i}{r_v}\right)}\left(\frac{1}{r_n} - \frac{1}{r_{c,e}}\right) \qquad q_{b,\max} = 88.8W$$