Reduce to Row Echelon Form (REF):

1)

$$\begin{aligned} x_1 + 4x_2 + 2x_3 + 8x_4 &= 12 \\ x_2 + 7x_3 + 2x_4 &= 4 \\ 5x_3 + x_4 &= 7 \\ x_3 + 3x_4 &= 5 \end{aligned}$$

Code:

```
A = [1 4 2 8; 0 1 7 2; 0 0 5 1; 0 0 1 3];

b = [12; 4; 7; 5];


A(3, :) = A(3, :) / A(3,3);

b(3) = b(3) / A(3,3);


A(4, :) = A(4, :) - A(3, :) * A(4,3);

b(4) = b(4) - b(3) * A(4,3);

disp('Row Echelon Form (System ):');

disp([A b]);
```

Output:

Row Echelon Form (System):

1.0000	4.0000	2.0000	8.0000	12.0000
0	1.0000	7.0000	2.0000	4.0000
0	0	1.0000	0.2000	7.0000
0	0	0	2.8000	5.0000

Pivot Positions:

- Row reduce the matrix A below to echelon form ,and locate the pivot columns of A.

$$A = \begin{bmatrix} 0 & -3 & -6 & 4 & 9 \\ -1 & -2 & -1 & 3 & 1 \\ -2 & -3 & 0 & 3 & -1 \\ 1 & 4 & 5 & -9 & -7 \end{bmatrix}$$

Code:

```
A = [2 6 6 4 3;
    0 3 6 4 9;
    1 2 1 3 1;
    2 3 0 3 1;
    1 4 5 9 7];
[R, pivot_cols] = rref(A);
disp('Row Echelon Form of A:');
disp(R);
disp('Pivot Columns:');
disp(pivot_cols);
```

Output:

```
Row Echelon Form of A:

        1   0  -3   0   0
        0   1   2   0   0
        0   0   0   1   0
        0   0   0   0   1
        0   0   0   0   0

Pivot Columns:

    1   2   4   5
```

The Row Reduction Algorithm:

- Apply elementary row operations to transform the following matrix first into echelon form and then into reduced echelon form:

$$\begin{bmatrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{bmatrix}$$

Code:

```
A = [0 3 -6 6 4 -5;
    3 -7 8 -5 8 9;
    3 -9 12 -9 6 15];
A([1,2], :) = A([2,1], :);
A(3, :) = A(3, :) - A(1, :);
A(2, :) = A(2, :) / 3;
disp('Row Echelon Form of A:');
disp(A);
R = rref(A);
disp('Reduced Row Echelon Form of A:');
disp(R);
```

Output:

```
Row Echelon Form of A:
  3.0000  -7.0000   8.0000  -5.0000   8.0000   9.0000
       0   1.0000  -2.0000   2.0000   1.3333  -1.6667
       0  -2.0000   4.0000  -4.0000  -2.0000   6.0000
```

Reduced Row Echelon Form of A:

```
1   0  -2   3   0  -24
0   1  -2   2   0  -7
0   0   0   0   1   4
```

Solutions of Linear Systems:

- Find the general solution of the linear system whose augmented ma trix has been reduced to

$$\begin{bmatrix} 1 & 6 & 2 & -5 & -2 & -4 \\ 0 & 0 & 2 & -8 & -1 & 3 \\ 0 & 0 & 0 & 0 & 1 & 7 \end{bmatrix}$$

Code:

```
A = [1  6  2 -5 -2 -4;
   0    0 2 -8 -1  3;
   0  0  0  0  1  7];
coeff_matrix = A(:, 1:end-1);
rhs_vector = A(:, end);
R = rref(A);
disp('Reduced Row Echelon Form of A:');
disp(R);
syms x2 x4;
x5 = 7;
x3 = 4*x4 + 5;
x1 = -6*x2 - 3*x4;
disp('General Solution:');
solution = [
  x1;
  x2;
  x3;
  x4;
  x5];
disp(solution);
```

Output:

Reduced Row Echelon Form of A:

```
   1    6    0    3    0    0

   0    0    1   -4    0    5

   0    0    0    0    1    7
```

General Solution:

```
- 6*x2 - 3*x4

             x2

     4*x4 + 5

             x4

              7
```

Null Space:

- Find the **null space** (kernel) of the matrix:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Code:

```
A = [1 2 3;

   4 5 6;

   7 8 9];
N = null(A, 'r');

disp('Basis for the Null Space of A:');

disp(N);
```

Output:

Basis for the Null Space of A:

1

-2

1

A Geometric Description of Span {v} and Span {u,v}:

For what value(s) of h will $\mathbf{y}$ be in Span $\{v1;v2;v3\}$ if

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 5 \\ -4 \\ -7 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}, \quad \text{and} \quad \mathbf{y} = \begin{bmatrix} -4 \\ 3 \\ h \end{bmatrix}$$

Code:

```
syms h;
A = [1 5 -3 -4;
  -1 -4 1 3;
  -2 -7 0 h];
A(2,:) = A(2,:) + A(1,:);
A(3,:) = A(3,:) + 2*A(1,:);
A(3,:) = A(3,:) - 3*A(2,:);
disp('Row Echelon Form of the augmented matrix:');
disp(A);
h_value = solve(A(3,4) == 0, h);
disp('The value of h for which the system is consistent:');
disp(h_value);
```

Output:

Row Echelon Form of the augmented matrix:

[1, 5, -3,   -4]

[0, 1, -2,   -1]

[0, 0, 0, h - 5]

The value of h for which the system is consistent:

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