TF502: Numerical Analysis
Homework 2

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Deadline: Oct 23, 2017

1. What is the bit representations of the floating point number 5.25 using the IEEE standard for double precision numbers? Solve this problem with pen and paper first. Use the JULIA command `bits(5.25)` to verify your result. **(5 point)**

2. We want to evaluate the function
$$f(x) = \frac{\sin(10^4 x)}{x}$$
at $x = \pi$. What is the order of magnitude of the evaluation error that you would expect when evaluating this expression using double precision numbers (eps $\approx 2 * 10^{-16}$)? **(5 points)**

3. Numeric differentiation based on central differences:

   (a) Implement a function (preferably in Julia) that uses numeric differentiation based on central differences,
   $$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h} \; .$$
   Here, the inputs of the differentiation routine are the scalar function f that we want to differentiate, the point x at which the derivative should be evaluated, and the finite perturbation $h > 0$. Use the syntax

   `diff(f,x,h) = ...` **(10 points)**

   (b) Evaluate the derivative of the function $f(x) = \exp(x)$ at $x = 0$ using the above routine `diff`. Plot the numerical differentiation error in dependence on $h \in [10^{-15}, 10^{-1}]$ and interpret the result. **(20 points)**

4. In order to evaluate the factorable function $f(x) = \sin(\cos(x)) * \cos(x)^2$ we write an evaluation algorithm of the form
$$\begin{aligned} a_0 &= x \\ a_1 &= \cos(x) \\ a_2 &= a_1 * a_1 \\ a_3 &= \sin(a_1) \\ a_4 &= a_2 * a_3 \\ f(x) &= a_4 \; . \end{aligned}$$
What is the corresponding algorithm for evaluating the derivative of $f(x)$ using the forward mode of algorithmic differentiation (AD)? What is the order of magnitude of the numerical error that is associated with evaluating the derivative of f at $x = 0$ using this AD code? **(10 points)**