Week 3 3/17 H.W.

Cobb-Douglas 生產函數

$$Q = f(L,K) = L^{\alpha}K^{\beta}\ ,\ \alpha,\beta > 0$$

① 產出彈性

- 勞動平均產量 $AP_L = \frac{Q}{L} = \frac{L^{\alpha}K^{\beta}}{L} = L^{\alpha-1}K^{\beta}$
- 勞動邊際產量 $MP_L = \frac{\partial Q}{\partial L} = \alpha L^{\alpha-1}K^{\beta}$
- 資本平均產量 $AP_K = \frac{Q}{K} = \frac{L^{\alpha}K^{\beta}}{K} = L^{\alpha}K^{\beta-1}$
- 資本邊際產量 $MP_K = \frac{\partial Q}{\partial K} = \beta L^{\alpha}K^{\beta-1}$
- 勞動產出彈性 $\varepsilon^L = \frac{MP_L}{AP_L} = \frac{\alpha L^{\alpha-1}K^{\beta}}{L^{\alpha-1}K^{\beta}} = \alpha$
- 資本產出彈性 $\varepsilon^K = \frac{MP_K}{AP_K} = \frac{\beta L^{\alpha}K^{\beta-1}}{L^{\alpha}K^{\beta-1}} = \beta$

② 生產力彈性

勞動&資本要素同時等比增加 ϕ 倍對生產函數的影響：

$$Q = f(\phi L, \phi K) = \phi^{\alpha+\beta}L^{\alpha}K^{\beta}$$

- 生產力彈性為

$$\varepsilon^{\phi} = \frac{\frac{dQ}{Q}}{\frac{d\phi}{\phi}} = \frac{\frac{dQ}{d\phi}}{\frac{Q}{\phi}} = \frac{(\alpha+\beta)\phi^{\alpha+\beta-1}L^{\alpha}K^{\beta}}{\frac{\phi^{\alpha+\beta}L^{\alpha}K^{\beta}}{\phi}} = \alpha+\beta$$

$$\varepsilon^{\phi} = \varepsilon^L + \varepsilon^K = \alpha+\beta$$

③ 替代彈性

- 邊際技術替代率 $MRTS = \frac{MP_L}{MP_K} = \frac{\alpha L^{\alpha-1}K^{\beta}}{\beta L^{\alpha}K^{\beta-1}} = \frac{\alpha}{\beta}\cdot\frac{K}{L}$
- 替代彈性 $\varepsilon^{LK} = \frac{d\ln(\frac{K}{L})}{d\ln(MRTS)} = \frac{d\ln(\frac{K}{L})}{d\ln(\frac{\alpha}{\beta})+d\ln(\frac{K}{L})} = 1$

隨堂

$Q = 3K+2L$ ，K 資本、L 勞動、Q 產出。

(1) 函數呈現固定規模報酬 (O)

(2) 函數呈現資本與勞動的邊際生產力遞減 (X)

(3) 函數呈現固定的技術替代率 (O)

(1) K、L 同增 λ 時

$\rightarrow F(\lambda K, \lambda L) = 3(\lambda K)+2(\lambda L)$

$= \lambda(3K+2L)$

(Q 也增加 λ 倍) $= \lambda Q$

(2) $MP_L = \frac{\Delta Q}{\Delta L} = 2$ $MP_K = \frac{\Delta Q}{\Delta K} = 3$

皆為固定

(3) 邊際替代率為一固定值