FEDERAL STATE AUTONOMOUS EDUCATIONAL INSTITUTION
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Report
on the practical task No.2
“Algorithms for unconstrained nonlinear optimization. Direct methods”

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Goal

This work aims for the tasks of unconstrained nonlinear optimization using such direct methods as one-dimensional methods of exhaustive search, dichotomy and golden section search and multidimensional methods of exhaustive search, coordinate descent and Nelder-Mead.

Formulation of the problem

Comparing the results for used numerical methods and finding approximate solutions for functions are the main goals of this paper. Also each part of work has extra tasks.

In first part such one-dimantional methods as exhaustive search, dichotomy and golden section search needed to implement in program language, as an example, in python, and to approximate solutions *x: f(x)→min* for the following functions and domains:

1) $f(x) = x^3,\ x \in [0, 1]$;
2) $f(x) = |x - 0.2|,\ x \in [0, 1]$;
3) $f(x) = x\,sin\frac{1}{x},\ x \in [0.01, 1]$.

In addition, there will be analysis of the number of f-calculations and the number of iterations performed in each method.

Second part is required to solve the minimization problem of generated the noisy data by the methods of exhaustive search, Gauss and Nelder-Mead. Data should be approximated by linear function $F(x,\ a,\ b) = ax + b$ and rational function $F(x,\ a,\ b) = \frac{a}{1+bx}$.

Brief theoretical part

An algorithm is any well-defined computational procedure that takes some value, or set of values, as input and produces some value, or set of values, as output. An algorithm is thus a sequence of computational steps that transform the input into the output. Also an algorithm can appear as a tool for solving a well-specified computational problem.

Usually computer professionals use the method which is the easiest to implement. But at the same time bounded resources of computers should be taken into account because computers are not yet infinitely fast and memory is not free. Optimization methods which find optimal in some sense solutions for mathematical models will help us use these resouces wisely.

There are direct, or zero-order, first-order and second-order methods. This work includes only direct methods which used only values of function. Direct methods can be one-dimensional and multidimensional.

One-dimentional direct methods that use in this work are brute-force search, dichotomy method and golden section method.

Multidimentional direct methods - exhaustive search, Gauss method and Nedler-Mead method.

Results

Part I. The one-dimentional methods.

The approximate solutions finds correctly. The number of f-calculations and the number of iterations can be found on images 1, 2 and 3 below.

```
EXHAUSTIVE SEARCH for f(x)=x^3
F-calculations:  0
The number of iterations:  999
DICHOTOMY for f(x)=x^3
F-calculations:  26
The number of iterations:  13
GOLDEN SECTION SEARCH for f(x)=x^3
F-calculations:  30
The number of iterations:  15
```

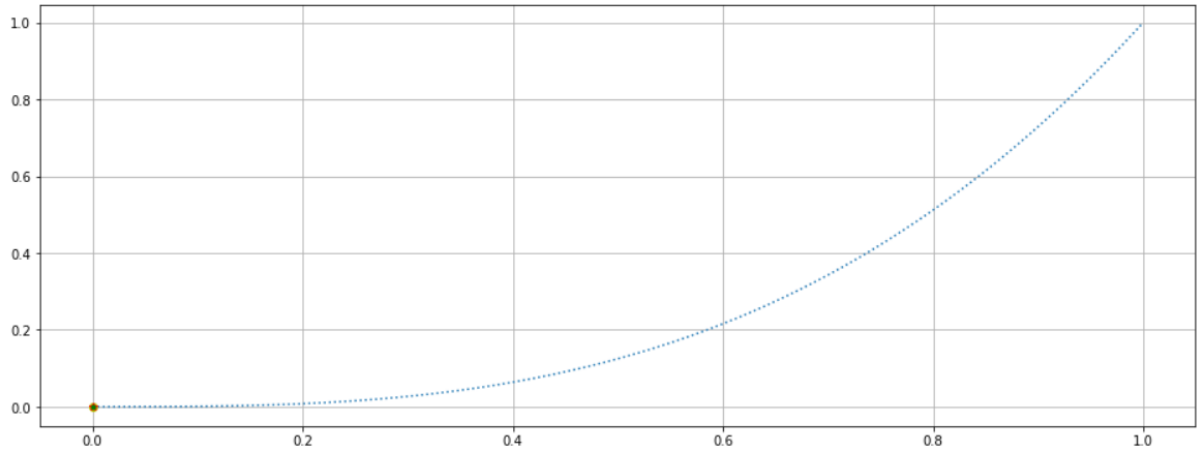


Image 1 - The results for function $f(x) = x^3$

```
EXHAUSTIVE SEARCH for f(x)=|x-0.2|
F-calculations:  400
The number of iterations:  999
DICHOTOMY for f(x)=|x-0.2|
F-calculations:  14
The number of iterations:  12
GOLDEN SECTION SEARCH for f(x)=|x-0.2|
F-calculations:  16
The number of iterations:  15
```

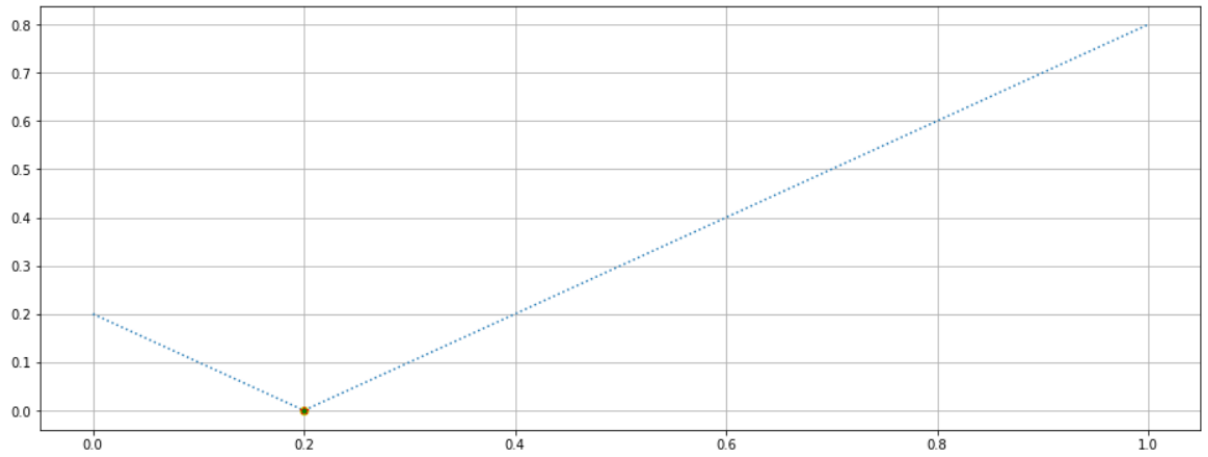


Image 2 - The results for function $f(x) = |x - 0.2|$

```
EXHAUSTIVE SEARCH for f(x)=x*sin(1/x)
F-calculations:  136
The number of iterations:  999
DICHOTOMY for f(x)=x*sin(1/x)
F-calculations:  12
The number of iterations:  13
GOLDEN SECTION SEARCH for f(x)=x*sin(1/x)
F-calculations:  16
The number of iterations:  15
```

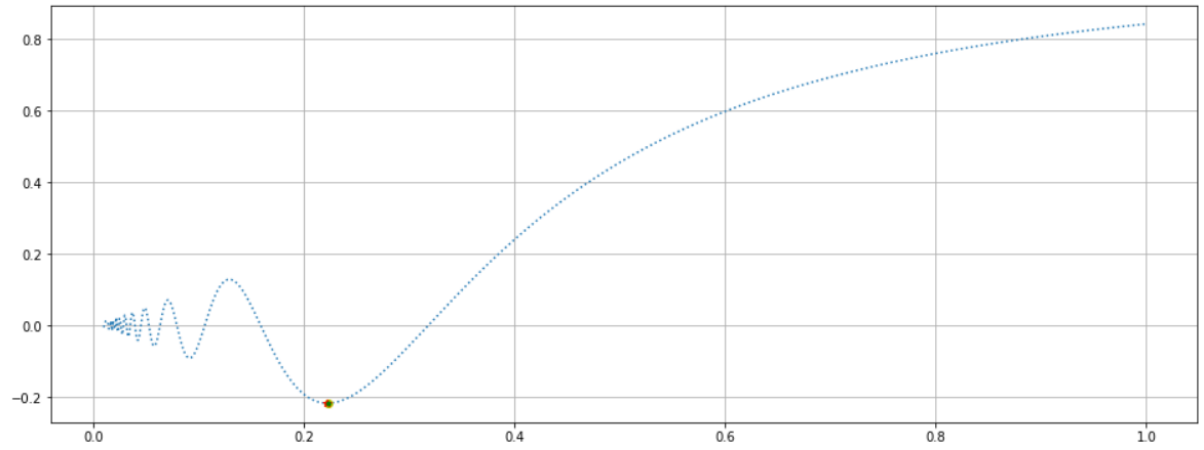
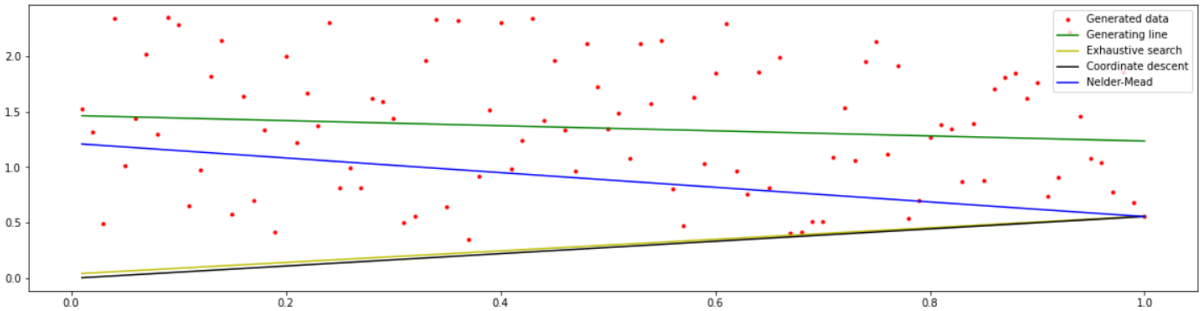


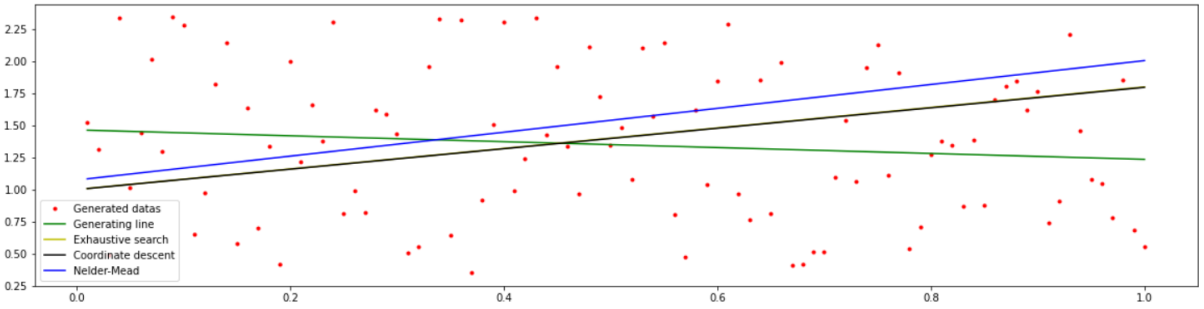
Image 3 - The results for function $f(x) = x\,sin\frac{1}{x}$

Part II. Multidimentional direct methods.

1. Linear approximant.



2. Rational approximant.



Conclusions

In conclusion, in this work was considered varied algorithms of one-dimensional and multidimentional methods and was visualized the results by matplotlib, Python library. The results show that golden section search has 15 iterations on input data and dichotomy has the least number of f-calculations among one-dimentional methods. On input data linear and rational approximants were minimized more effectively by Gauss coordinate descent.

Appendix

https://github.com/JaneKKTTme/analysis-and-development-of-algorithms