Statistics for Computer Science

Assignment 2

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Exercise 3

1. Write down the formula for likelihood function of Poisson distribution.

The formula for the Poisson probability mass function is:

$P(x, \lambda) = \frac{e^{-\lambda}\lambda^x}{x!}$

In Poisson distribution, the parameter of interest is λ. Having sequence of X_n, the probability of observing the sequence X_n will be the product of probabilities of each of them.

Therefore, the kernel of likelihood function of Poisson distribution is:

$$L(\lambda|X) = \prod_{i=1}^{N} \frac{\lambda^{X_i} e^{-\lambda}}{X_i!}$$

2. Write down the formula for log-likelihood function of Poisson distribution.

The formula for log-likelihood function of Poisson distribution is obtained by using natural logarithm on the likelihood function of Poisson distribution.

Therefore, the kernel of log-likelihood function of Poisson distribution is:

$$l(\lambda|X) = ln\left(\prod_{i=1}^{N} \frac{\lambda^{X_i} e^{-\lambda}}{X_i!}\right)$$

$$l(\lambda|X) = \sum_{i=1}^{N} X_i ln\lambda - N\lambda$$

3. Write down the likelihood equation and work out the exact formula for $\hat{\lambda}$.

$$L(\lambda) = \prod_{i=1}^{N} \frac{e^{-\lambda}\lambda^{x_i}}{x_i!} = e^{-N\lambda}\frac{\lambda^{\sum_1^N x_i}}{\prod_{i=1}^{N} x_i}$$

$$lnL(\lambda) = -N\lambda + \sum_{1}^{N} x_i ln(\lambda) - ln\left(\prod_{i=1}^{N} x_i\right)$$

$$\frac{dlnL(\lambda)}{dp} = -N + \sum_{1}^{N} x_i \frac{1}{\lambda}$$

$$\hat{\lambda} = \frac{\sum_{i=1}^{N} x_i}{N}$$

4. Create your own R-function for calculating the value of log-likelihood function of Poisson distribution.

```
x <-c(117, 109, 109, 89, 120, 88, 99, 103, 109, 91, 107, 101, 109, 117,
   96, 95, 129, 96, 105, 98)
n <- 20

#the function will take 3 parameters lambda, n and x
```

```
#n is number of obervations
#lambda is mean
#x is the sequence of observed values
#finally it will return the value of log-likelihood of Poisson distribution
poi.log.likelihood <- function(lambda, n, x){
  log.like <- sum(x) * log(lambda) - n * lambda
  return(-log.like)
}

ans.poi.log.like <- poi.log.likelihood(mean(x), n, x)
```

The value of log-likelihood function of Poisson distribution is -7612.856.

5. Using function optimize() find $\hat{\lambda}$. Compare it to the estimate you get from the exact formula.

```
#this function will take 2 parameters x and n
#x is the sequence of observed values
#n is number of obervation
#finally it will return lambda hat which is the mean
lambda.hat <- function(x, n){
  return(sum(x)/n)
}
ans.lambda.hat <- lambda.hat(x, n)

#using optimize function to obtain lambda hat
#the optimize function take the log-likelihood of Poisson distribution to optimize with the given interval
#optimize function will return maximum and objective value
#in this case we interest in the value of maximum
lambda.hat.est <- optimize(f = poi.log.likelihood, interval = c(88, 129), maximum = T, x = x, n=n)$maximum
```

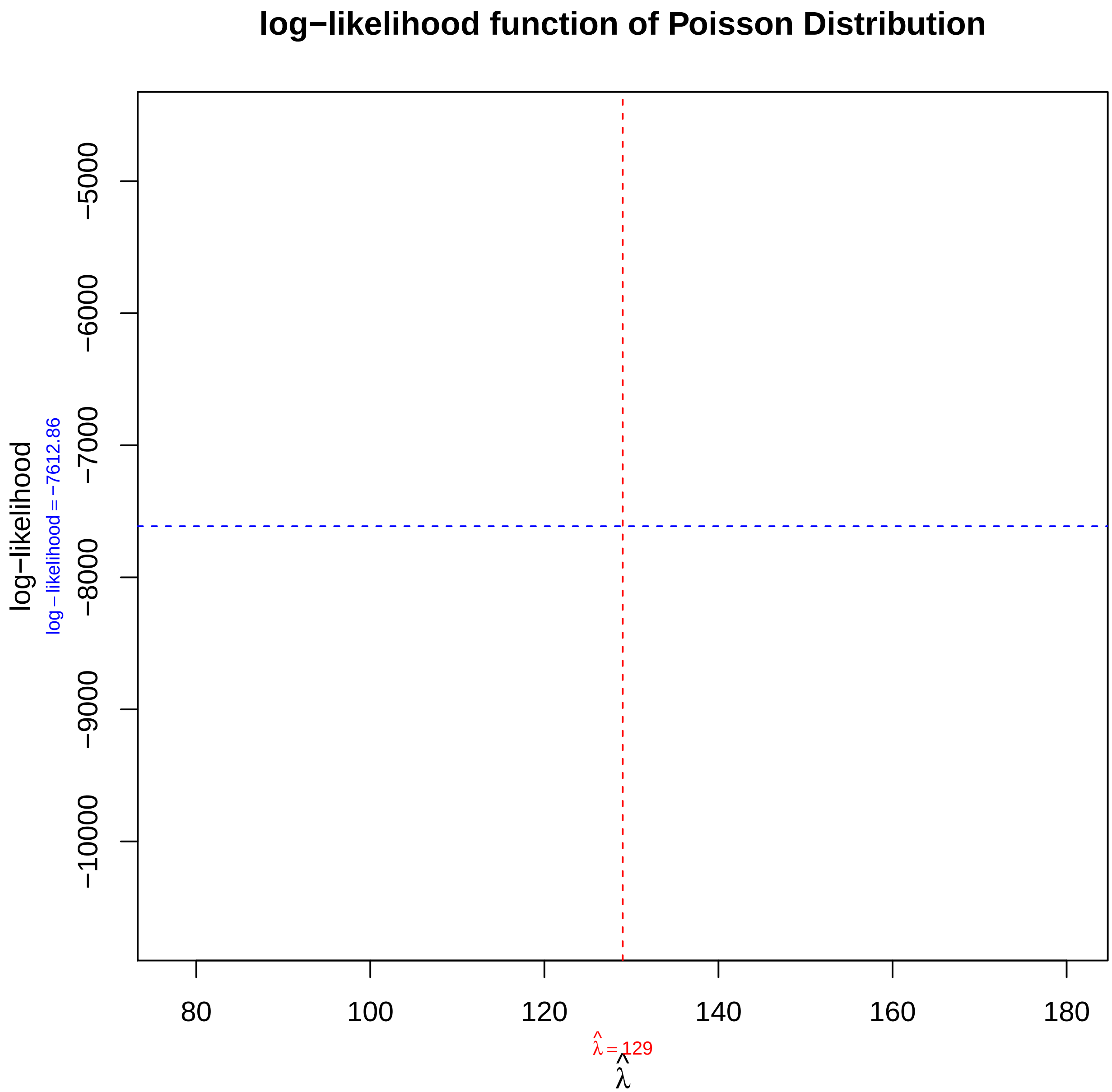
The exact value of $\hat{\lambda}$ is 104.35 and the estimate value of $\hat{\lambda}$ is 128.9999.

6. Plot the log-likelihood function, highlight the maximum and denote the maximum likelihood estimate in plot margin.

```
lambda.hat.seq <- seq(from=88, to=129, length=20)
l.lambda <- apply(X = as.matrix(lambda.hat.seq), MARGIN = 1, FUN = poi.log.likelihood, lambda=ans.lambda.hat, n=n)

plot(lambda.hat.est, ans.poi.log.like, type = 'l', main="log-likelihood function of Poisson Distribution", xlab = bquote(hat(lambda)), ylab = "log-likelihood")
abline(v = lambda.hat.est, col = 'red', lty =2)
abline(h = ans.poi.log.like, col = 'blue', lty = 2)
mtext(bquote(hat(lambda) == .(round(lambda.hat.est, 2))), side = 1, line = 2,
    at = lambda.hat.est, cex = 0.7, col = 'red')
```

```
mtext(bquote(log-likelihood == .(round(ans.poi.log.like, 2))), side = 2,
    line = 2,
      at = ans.poi.log.like, cex = 0.7, col = 'blue')
```



Exercise 4

1. Write down the null and the alternative hypotheses in mathematical form.

   Null hypothesis— $H_0 : \rho = 0$

Alternative hypothesis— $H_1 : \rho \neq 0$

2. Calculate the value of test statistic.

```
setwd(getwd())
body <- read.table(file = 'body-measurements.txt', header = T)
body <- na.omit(body)

body.f.height <- body[body$sex == 'f', 'body.H']
body.f.neck <- body[body$sex == 'f', 'neck.C']

n.x <- length(body.f.height)
n.y <- length(body.f.neck)
mu.x <- mean(body.f.height)
mu.y <- mean(body.f.neck)
sigma2.x <- sd(body.f.height)
sigma2.y <- sd(body.f.neck)
  correlation <- cor(body.f.height, body.f.neck, method = c("pearson", "kendall", "spearman"))

# Test Statistic Comparing Two Population Means: 978.969544271

z <- (1/2)*log((1+r)/(1-r))
```

```
Error in eval(expr, envir, enclos): object 'r' not found
```

Exercise 5

1. Write down the null and the alternative hypotheses in mathematical form.

Null hypothesis— $H_0 : p_1 - p_2 = 0$

Alternative hypothesis— $H_1 : p_1 - p_2 \neq 0$

2. Calculate the value of test statistic.

```
n1 <- 200
x1 <- 32
p1 <- x1 / n1

n2 <- 230
x2 <- 21
p2 <- x2 /n2

p0 <- 0.5

alpha <- 0.05
```