Expected Value

1. Let's play roulette. A standard roulette wheel has 38 numbered slots for a small ball to land in: 36 are marked from 1 to 36, with half of those black and half red; tow green slots are numbered 0 and 00.
   a. An allowable bet is to bet on either red or black. This bet is an even-money bet, which means if you win you receive twice what you bet. What is the expected value of betting $1000 on red? What should the payoff be if the game is to be a fair game?

   b. If you bet $100 on a particular number and the ball lands on that number, you are paid $3600 (this includes the $100 you bet, so your gain is $3500). What is the expected value of betting $100 on red 9? What payoff should you receive to make the game fair?

2. What is the expected value of the outcome (1, 2, 3, 4, 5, or 6) of rolling a fair die? What is the expected value of the outcome (2, 3, 4, … , 12)of rolling two fair dice?

3. You roll a fair die. If you roll 1, the house pays you $25. If you roll 2, the house pays you $5. If you roll 3, you win nothing. If you roll a 4 or 5, you must pay the house $10, and if you roll a 6, you must pay the house $15. What is the expected value of this game? How much would you have to pay, or be paid, to make the game a fair game?

4. You place a bet and then roll two fair dice. If you roll a sum of 7 or 11, you receive your bet back (you break even). If you roll a sum of 2, 3, or 12, then you lose your bet. If you roll anything else, you receive half of the sum you rolled in dollars. How much should you bet to make this a fair game?

5. Someone has a weighted coin that lands heads up with probability 2/3 and tails up with probability 1/3.
   a. If the coin comes up heads, you pay $1; if the coin comes up tails, you receive $1.50. What is the expected value of this game? Would you play? Why or why not?

   b. You pay $5 to flip the coin. If the coin comes up heads, you lose and receive nothing. For this game to be a fair game, how much would you have to receive if you flip tails?

6. You live in an area where the probability that one's bicycle is stolen is 0.2. You care deeply for your $700 road bike. What is a fair price to pay to insure your bike against theft over the life of your bike?

7. You own a $9000 car and a $850 mountain bike. The probability that your car will be stolen next year is 0.02 (2%), but the probability that your bike will be snatched is 0.1 (10%). An insurance company offers you theft insurance for your car for $200 and insurance for your bike for $75. What is the

expected value of the car insurance? What is the expected value of the bike insurance?

8. You have three options for the evening. (1) You could watch CSI on TV that you are certain to enjoy and that will provide you with a relative pleasure rating of 4. (2) You could go to a movie that is supposed to be good. You would enjoy the movie with probability 0.5 (50%); if you enjoy it, it will provide you with a relative pleasure rating of 11, but if you don't enjoy it, it will provide a negative rating of -2. (3) You could go a wine tasting dinner at a new restaurant. With probability 0.3 you will experience a pleasure rating of 15, and with probability 0.7 you will hate the food and wine chosen and the evening will provide a negative rating of -2. What are the expected values of pleasure for these individual activities? List the activities in order of expected pleasure. What would you do?

9. Coins in a fountain. You pay $1 for tow coins to toss in a fountain and see how they land. If you see two tails, then you receive $3, otherwise you lose. What is the expected value of this game? Is there one single possible outcome whereby you would actually gain or lose the exact amount computed for the expected value? If not, then why is it called *expected* value?

10. You pay $5 for three coins to toss in a fountain and see how they land. If you see no heads, then you receive $20. If you see exactly one head, then you receive $5 (the game is a draw), and if you see at least two heads, then you lose. What is the expected value of this game?

11. You grandson is looking for a job. Three companies are interested in him and he will receive at most one offer. The first company has a job open with a salary of $42,000, and the probability of getting an offer is 0.5. The second company has a job open paying $64,000 and the probability of getting an offer is 0.3. The last company has a job paying $90,000 and the probability of getting an offer is 0.1. There is a 0.1 probability of not landing any job. What is his expected income for next year?

12. There is a 50% chance that the price of gold will go up by $25 an ounce; a 20% chance that it will remain the same; and a 30% chance that it will drop by $40 an ounce. What is the expected value of a purchase of gold? Given your answer, would you invest in gold at this time if gold cost $375 per ounce?

13. You wish to invest $1000 and you have two choices. One is a sure thing and you will make a 5% profit. The other is a riskier venture. If the venture pays off, you will make a 25% profit; otherwise, you lose your $1000. What is the minimum required probability of this riskier venture paying off in order for the expected value to exceed the value of the first investment?

14. Assume that the insurance value of a life is $1,200,000. Suppose Pap smear tests will save one life in 3000. A Pap smear costs about $30. What is the expected value of a Pap smear?

15. You pay a certain amount of money to play a coin game. If you see tails, you flip again, and the game continues until you see a head, which ends the game. If you see heads on the first flip, you receive $2. If you see heads on the second flip, you receive $4. If you see heads on the third flip, you get $8 and so forth – the payoff is doubled every time. What is the expected payoff of this game? How much would you pay to play this game? Suppose you pay $1000 to play. What is the probability that you would make money? Why is this gam a paradoxical situation given the expected value?

16. Martingales. A game is played with a fair coin. You bet some amount of money on heads. If you flip heads, then you receive even money. (You get your bet back, plus an extra amount equal to the bet.) IF you flip tails, then you lose your bet. The doubling strategy is one where you continue to double your bet after each loss until you win. So, for example, if you first bet $1 and lose, then the next time you would bet $2. If you lose again, you would then bet $4, and then $8, and so on. How much would you earn if you used this strategy and lost seven times in a row before finally winning? How much money would you need to play eight times?