Playing with Pinecones and Flowers

1. Take a look at the pinecone and examine its scales. Often you can find they are clumped in spirals, emanating from the bottom. How many spirals are there? Is it different if you consider spirals that go the other way (clockwise versus counterclockwise)? Compare this with your neighbors. Do you get the same numbers?

2. Do the same analysis for a picture of a daisy, shown below. How many spirals do you get this time? Are there two different numbers if you consider spirals circulating in a different direction?

Fibonacci Numbers and Wabbit season

We've seen that sunflowers, pinecones, and pineapples all have certain special numbers of spirals on them. These are called Fibonacci numbers. They are named after Leonardo Fibonacci (actually known as Leonardo of Pisa). To determine these numbers, we create a sequence where the first two terms are 1. The next term is the sum of the previous two. In particular, $F_1 = 1$ and $F_2 = 1$. Therefore, $F_3 = F_2 + F_1 = 2$, $F_4 = F_3 + F_2 = 3$, and $F_5 = F_4 + F_3 = 5$.

1. Fill in the following chart:

F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}
1	1	2	3	5										

2. Fill in the following chart, rounding to three digits after the decimal place.

F_2/F_1	F_3/F_2	F_4/F_3	F_5/F_4	F_6/F_5	F_7/F_6	F_8/F_7	F_9/F_8	F_{10}/F_9	F_{11}/F_{10}	F_{12}/F_{11}

What do you notice?

3. Fibonacci actually never mentioned his "numbers." Instead, he wrote a textbook which had the following problem that you should solve: Suppose a newly-born pair of rabbits, one male, one female, are put in a field. Rabbits are able to mate at the age of one month so that at the end of its second month a female can produce another pair of rabbits. Suppose that our rabbits never die and that the female always produces one new pair (one male, one female) every month from the second month on. How many pairs will there be in one year?

4. Here's a strange phenomenon: Check out a Fibonacci number like $F_6 = 8$. The next Fibonacci number is $F_7 = 13$. If you convert 8 miles to kilometers, you find it is about 13 km. Try this with a few other Fibonacci numbers. Why is this working? Is there something mystical about British and metric conversions?

5. Experiment with the first few Fibonacci numbers to see if you can find a simple formula for $(F_{n+1})^2 + (F_n)^2$.

6. Let's do the rabbit experiment again, except this time suppose it takes two months before a pair of bunnies is mature enough to reproduce. Make a table for the first 10 months showing how many pairs there would be at the end of each month. Is there a pattern? Give a formula for generating the sequence.

7. Yet another rabbit experiment … this time, as before, the rabbits reproduce after they are one month old and produce a pair of bunnies each month. Now, however, each pair dies after 3 months, immediately after giving birth. Create a chart showing how many pairs we have after each month from the start through month nine.

8. Lucas sequence: Build a sequence of numbers starting with 2, 1 and adding the previous two numbers to get the next number. Write down the first 15 terms. Compute the quotients of consecutive Lucas numbers. What number do these quotients approach? Try it again with two other starting numbers (you pick them) and see what happens.

9. Using the square root key on a calculator, evaluate each number and record the answer:

$$\sqrt{\frac{F_3}{F_1}}$$

$$\sqrt{\frac{F_4}{F_2}}$$

$$\sqrt{\frac{F_5}{F_3}}$$

$$\sqrt{\frac{F_6}{F_4}}$$

$$\sqrt{\frac{F_7}{F_5}}$$

$$\sqrt{\frac{F_8}{F_6}}$$

$$\sqrt{\frac{F_9}{F_7}}$$

$Make\ a\ guess\ as\ to\ what\ number$ $\sqrt{\frac{F_{n+2}}{F_n}}$ approaches as n gets larger and larger.

10. Tribonacci. Start with the numbers 0,0, 1 and generate future numbers in the sequence by adding up the previous three numbers. Write out the first 15 terms in this sequence starting with the first 1. Evaluate with a calculator the quotients of consecutive terms (dividing the smaller term into the larger one). Do the quotients appear to be approaching a fixed number?

11. Create a new sequence of numbers starting with 0 and 1. This time, generate the next term by adding 2 times the previous term to the term before it. So, $G_n = 2G_n + G_{n-1}$. This is called a generalized Fibonacci sequence. Write out the first 15 terms. Can you use the methods we used in class to determine the exact number that $\frac{G_{n+1}}{G_n}$ approaches as n gets large?