

Week One

1. Suppose you have a certain number of chips on a table. Each player can take 1, 2, or 3 chips at a time. The two players take turns taking away chips. The player who takes the last chip loses. Begin playing with 23 chips. Determine if player 1 or player 2 has a foolproof strategy to win and why it works. Then play with 25 chips. Do the same analysis again.
2. Suppose Joe and Jane have the same amount of money in their pockets. How much would Jane have to give to Joe so that he had \$20 more than Jane?
3. There are 100 politicians at a certain convention. Each politician is either crooked or honest. We are given the following two facts:
 - a. At least one of the politicians is honest.
 - b. Given any two of the politicians, at least one of the two was crooked.Can you determine from these facts how many of the politicians are honest and how many are crooked? If so, how many? If not, why not?
4. Sue bought a book for \$5, sold it for \$6, bought it back for \$7, and sold it for \$8. How much profit did she make?
5. There are two boxes - one marked A and one marked B. Each box contains either \$1 million or a deadly snake that will kill you instantly. You must open one box. On box A there is a sign that reads: "At least one of these boxes contains \$1 million." On box B there is a sign that reads: "A deadly snake that will kill you instantly is in box A." You are told (truthfully) that either both signs are true or both are false. Which box do you open?
6. Congresswoman Smith opened the Times and saw that a bean-counting scandal had been leaked to the press. Outraged, Smith immediately called an emergency meeting with the five other members of the Special Congressional Scandal Committee, the busiest committee on Capitol Hill. Once they were all assembled, Smith declared, "As incredible as it sounds, I know that three of you always tell the truth. So now I'm asking all of you, who spilled the beans to the press?" Congressman Scholck spoke up, "It was either Wind or Pocket." Congressman Wind, outraged, shouted, "Neither Slie nor I leaked the scandal." Congressman Pocket then chimed in, "Well, both of you are lying!" This provoked Congressman Greede to say, "Actually, I know that one of them is lying and the other is telling the truth." Finally, Congressman Slie, with steadfast eyes, stated, "No, Greed, that is not true." Assuming that Congresswoman Smith's first declaration is true, can you determine who spilled the beans?
7. Three strangers, Tom, Joe, and Jane, meet at a taxi stand and decide to share a cab to cut down the cost. Each has a different destination, but all the destinations are on the same route, so no circuitous driving is required. Tom's destination is 10 miles away, Joe's is 20 miles, and Jane's is 30 miles. The taxi costs \$2 per mile including the tip, regardless of the number of passengers. How much should each person pay?
8. Jane and Tom threw a party which was attended by four other couples. Although we have no idea who shook hands with whom at the party, we do know that no one shook hands with themselves and no one shook hands with his or her partner. Given these facts, a guest might not shake anyone's hand or might shake as many as eight other people's hands. At the end of the party, Tom gathered the crowd and asked the nine other people how many hand each of them had shaken. Each person gave a different answer. How many hands did Jane shake?