

## Worksheet 6 – Rational or Irrational?

1. Reduce these "fat" fractions to lowest terms:

$$\frac{6}{24} \quad \frac{15}{9} \quad -\frac{14}{42} \quad \frac{125}{10} \quad -\frac{121}{11}$$

2. Show that the numbers below are rational by expressing each as a ratio of two integers.

$$\frac{1}{2} + \frac{5}{2}, \quad \frac{1}{2} - \frac{2}{3}, \quad \left(\frac{1}{2}\right) \times \left(\frac{6}{5}\right), \quad \frac{\frac{1}{2}}{\frac{2}{3}}, \quad \frac{\left(\frac{5}{2}\right) \times \left(\frac{6}{5}\right)}{\frac{2}{3}}$$

3. Show that each of the decimal numbers below is actually a rational number by expressing it as a ratio of two integers.

$$0.02, \quad 6.23, \quad 2.71828, \quad -168.5, \quad -0.00005$$

4. Express each fraction in its decimal expansion.

$$\frac{6}{7}, \quad \frac{17}{20}, \quad \frac{21.5}{15}$$

5. Suppose  $M = 0.499999\dots$ . Then what does  $10M$  equal? Find two expressions for the quantity  $10M - M$  and set those two expressions equal to each other. (Hint: One expression is simply  $9M$ .) Can you solve your equation to discover something marvelous about  $M$ ?

6. Express each number as a fraction.

a. 1.28901

b. 20.4545

c. 12.999

d. 2.222222...

e. 43.12121212...

f. 5.6312121212...

g. 0.0101010101...

h. 71.23999999...

7. Find a rational number that is bigger than 12.0345678 and smaller than 12.0345679.

8. Find a rational number that is bigger than 3.14159 and smaller than 3.14159001.
9. Describe an irrational number that is bigger than 5.7 but smaller than 5.72.
10. Is it possible to build an irrational number whose decimal digits are just 1's and 2's? If so, describe such a number and show why it is irrational. If not, explain why.
11. Is it possible to build an irrational number whose decimal digits are just 1's and 2's and only finitely many 2's appear? If so, describe such a number and show why it is irrational. If not, explain why.
12. We know that  $\sqrt{2}$  is irrational. Therefore  $\frac{3\sqrt{2}}{5\sqrt{2}}$  must also be irrational. Is this conclusion correct? Why or why not?
13. We know that  $2/5$  and  $7/3$  are rational. Therefore  $(2/5)/(7/3)$  is also rational. Is this conclusion correct? Why or why not?
14. For each of the following numbers, determine if the number is rational or irrational. Why?

$$\frac{4}{9},$$

$$1.75,$$

$$\frac{\sqrt{20}}{3\sqrt{5}},$$

$$\frac{\sqrt{2}}{14},$$

$$3.14159$$

$$\sqrt{\frac{16}{20}}$$

$$\sqrt{\frac{12}{7.5}}$$

$$-147$$

$$0$$

$$\frac{\sqrt{3}}{3}$$

15. Prove that  $\sqrt{5}$  is irrational.

16. Show that  $\sqrt{10}$  is irrational.

17. Show that for any prime number  $p$ ,  $\sqrt{p}$  is an irrational number.

18. The  $\sqrt{4} = 2$  which is a rational number. Try using the argument above to prove  $\sqrt{4}$  is irrational. Where does the argument break down?