Ping Pong Balls in a Room

Here's the question we're going to ponder: Approximately how many ping pong balls can fill our classroom?

1. How could you even go about getting an estimation that's better than "a lot?" What strategy should you employ?

2. What would information would you need to know to effectively estimate the answer to our question? Do you need to have exact values?

3. With that information, what is your estimate of the number? Explain how you did it.

Identical hair-follicle twins?!

Here's the next question we're going to ponder: Are there two (non-bald) people in the world who have exactly the same number hairs on their body? This is a really hard problem that may seem overwhelming at first. Before getting too frustrated before you've even begun, let's use an important problem solving strategy:

Try and solve a similar problem that is smaller and/or easier.

Here are some suggestions that we can begin with before we tackle the larger problem. As you're answering these questions, think about what larger ideas could be applied towards solving the identical hair-follicle twins.

1. Without asking everyone, do you *think* that two people in this class have the same birthday? Do you *know* that two people in this class have the same birthday? Explain your reasoning.

2. Do you *think* or *know* that two students at OLLI have the same birthday? Some of you may say "Yes" because you happen to know two people at the school with the same birthday. If that's the case, do you think or know that two students at the Montreat College (current enrollment is 1082 students) have the same birthday? Explain your reasoning.

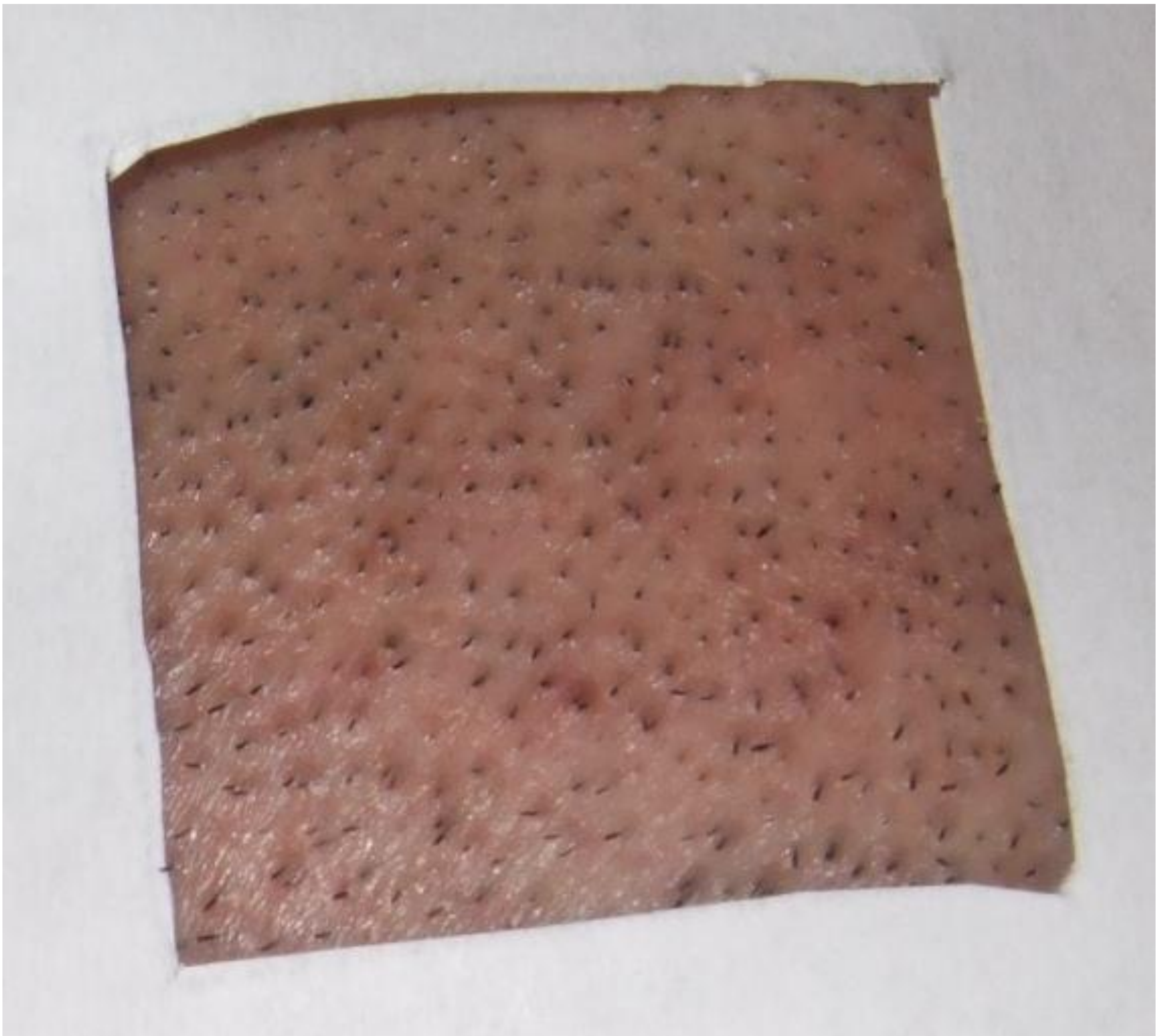
3. What's the fundamental principle that allowed you to answer questions 1 and 2? How can it help you to answer the identical hair follicle twin question? What would you have to do next?

4. There are currently about 7 billion people on earth. Suppose we could estimate the number of body hairs on any individual. How could this "gift" and knowledge of the earth's population answer whether we *know* if there are two people who are identical hair-follicle twins? Would we want to consider a very hairy person, a not-so-hairy person, or an average person to help us answer this question? Explain your reasoning.

5. Now comes the hard part – how would you *estimate* the number of body hairs on a person. Be explicit! Describe what kinds of information you would need to get started. What assumptions will you make in order to get a handle on this problem? How would you proceed to get an actual number in the end?

Sampling of a square inch hair follicles in a beard

Although this is a little gross, here's a close-up photo of a square inch of Prof. Keynes' beard. Estimate how many hair follicles there are in this region. The number doesn't have to be precise, but rather around the right amount. How can this help us to figure out the maximal number of hairs a human could have?



Additional questions about the Pigeonhole Principle

1. The pigeonhole principle states that if there are more items than pigeonholes, than at least one pigeonhole must contain at least two items. Identify in each of the two previous problems (identical hair-follicle twins and two people at Montreat College having the same birthday) what were the items and what were the pigeonholes. Explain your reasoning.

2. If there are more items than pigeonholes, do we *know* that each pigeonhole contains at least one item? Is this the same statement as the pigeonhole principle? Why or why not?

3. If there are 350 people in an auditorium, it is *likely* that two of them will share a birthday? Do we *know* that two of them will share a birthday? Is the distinction ever important?

4. How many people in an auditorium would you need in order to guarantee that at least *three* people share a birthday? What about four people? Explain how you figured this out. *Important:* Don't be afraid to use logic and think!!! The pigeonhole principle doesn't explicitly cover this. How can you modify it?

5. Someone offers to give you a million dollars in one-dollar bills. To receive the money, you must lie down; the million one-dollar bills will be placed on your stomach. If you keep them on your stomach for 10 minutes, the money is yours! Do you accept the offer? Why or why not?

6. You have 10 pairs of socks, five black and five blue, but they are not paired up. Instead, they are all mixed up in a drawer. It's early in the morning, and you don't want to turn on the lights in your dark room. How many socks must you pull out to guarantee that you have a pair of one color? How many must you pull out to have two good pairs (each pair is of the same color)? How many must you pull out to be certain you have a pair of black socks?

7. What proportion of the first 1000 natural numbers have a 3 somewhere in them? For example, 135, 403, and 339 all contain a 3, whereas 402, 677, and 8 do not.

8. What proportion of the first 10,000 natural numbers contain a 3?

9. Explain why almost all million-digit numbers contain a 3. Do nearly all million-digit numbers contain a 4?