Principles of Cyber-Physical Systems
Project: Modeling of Multiple Heaters

In this project, we will model a room heating system. Consider a house with 4 rooms that are heated by 2 heaters. The temperature in each room is controlled by a heater, if there is one in it, and also depends on the temperature of the adjacent rooms and the outside temperature. Each room can have at most one heater. Clearly, at any given time, only two of the rooms have a heater.

Let x_i be the temperature in room i $(i = 1, 2, 3, 4)$, u the outside temperature. The temperature of a room changes linearly with the difference of the temperature with the other rooms, the difference with the outside temperature, and the power of the heater if there is one in it. Specifically, the dynamics of the system is given by

$$\dot{x}_i = c_i h_i + b_i(u - x_i) + \sum_{j \neq i} a_{i,j}(x_j - x_i)$$

with constants $a_{i,j}$, b_i, c_i. In this equation, $h_i \in \{0, 1\}$ is the power status of the heater in the room: h_i is 0 if there is no heater or the heater is off, $h_i = 1$ if there is a heater and it is on. We assume that all the heaters are identical. If $a_{i,j} > 0$ then rooms i and j are adjacent.

1. Design a Simulink model (in a subsystem block) of the above room heating system. Describe briefly your design in the report. The system should have h_i and u as its inputs. It should be parameterized by the constants $a_{i,j}$ in a matrix $A = [a_{i,j}] \in \mathbb{R}^{4\times4}$, the constants b_i in a vector $b = [b_i] \in \mathbb{R}^4$, the constants c_i in a vector $c = [c_i] \in \mathbb{R}^4$, and the initial temperature of each room in a vector $x_0 \in \mathbb{R}^4$.

2. We will control the heaters by a typical thermostat, i.e. the heater is switched on at its maximum power ($h_i = 1$) if the temperature is below a certain threshold, and off ($h_i = 0$) if it is beyond another (higher) threshold. Specifically, for each room i we define thresholds on_i and off_i, and the heater in room i (if there is one) is on if $x_i \leq on_i$ and off if $x_i \geq \mathit{off}_i$.
   We will also control the placement of the heaters in the rooms (remember that there are fewer heaters than the rooms) by the following rule: a heater is moved from room j to room i ($i \neq j$) if all the followings hold:

   - room i has no heater (each room can have at most one heater)
   - room j has a heater
   - temperature $x_i \leq get_i$
   - the difference $x_j - x_i \geq \mathit{dif}_i$.

   The constants get_i and dif_i may differ for each room. When a heater can be moved to two different rooms or two heaters can be moved to the same room, you can make any choice you like (for example, always favor the room with higher index, or always favor the room with lower temperature).
   Design this controller in Stateflow (as a subsystem block), parameterized by the constants on_i in a vector $on = [on_i] \in \mathbb{R}^4$, the constants off_i in a vector $\mathit{off} = [\mathit{off}_i] \in \mathbb{R}^4$, the constants get_i in a vector $get = [get_i] \in \mathbb{R}^4$, the constants dif_i in a vector $\mathit{dif} = [\mathit{dif}_i] \in \mathbb{R}^4$. The inputs to the controller should be the temperatures x_i. The outputs should be h_i for each room i, with the constraints that $h_i \in \{0, 1\}$ and at any time, at most two of these outputs can be 1 (because there are only two heaters). Describe rigorously and succinctly your design in the report.

3. Connect the controller model to the rooms' model and simulate your system with the following data. The system should be simulated for a reasonable amount of time, i.e. when interesting events, such as moving of heaters, happen. Plot the temperature x_i of each room, the control action h_i, and the

placement of the heaters.

$$A = \begin{bmatrix} 0.00 & 0.30 & 0.40 & 0.30 \\ 0.30 & 0.00 & 0.50 & 0.00 \\ 0.40 & 0.50 & 0.00 & 0.30 \\ 0.30 & 0.00 & 0.30 & 0.00 \end{bmatrix}, \quad b = \begin{bmatrix} 0.30 \\ 0.20 \\ 0.50 \\ 0.40 \end{bmatrix}, \quad c = \begin{bmatrix} 9.00 \\ 7.00 \\ 11.00 \\ 7.00 \end{bmatrix}$$

$u = 6, \quad x_0 = \begin{bmatrix} 16.5 & 16.5 & 16.5 & 16.5 \end{bmatrix}^T$

$\mathit{off} = \begin{bmatrix} 20 & 20 & 20 & 20 \end{bmatrix}^T, \quad \mathit{on} = \begin{bmatrix} 19 & 19 & 19 & 19 \end{bmatrix}^T$

$\mathit{get} = \begin{bmatrix} 17 & 16 & 16 & 17 \end{bmatrix}^T, \quad \mathit{dif} = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}^T$

Initially, the two heaters are in room 2 and room 3.

We would like to keep the temperature of each room between 15 and 20 degree. In your simulation, was this requirement satisfied?

4. Change the parameters of the controller: on_i, get_i, dif_i while off_i remain unchanged. Simulate your system and discuss the effects of these changes. Is the temperature requirement satisfied in each case?