A multi-state Markov model to separate latent deterioration process on navy ship failure engineering

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Abstract

Maintenance optimization for naval ship equipment is essential in terms of national defense. However, the pure deterioration effect should be inferred in advance to compare different maintenance policies could be compared. In other words, exact inference of the effect of deterioration and maintenance process which are combined in observed data, annual failure count for example, is needed based on which policy selection could be performed. This paper proposes the framework for retrieving the deterioration effect by separating it from current maintenance based on the multi-state Markov model using the annual engine failure count of 99 ships. The framework consists of multiple steps including imputation, transition matrix design, optimization, and validation. Hierarchical Gaussian process model is used for imputation and three-state Markov model is constructed to estimate parameters for deterioration and maintenance. Nonlinear optimization is performed to estimate the natural (deterioration) and artificial (maintenance) effect separately. Experiments show the robust estimation of model parameters as well as accurate recovery of observed data under multiple settings thereby confirming the credibility of our model. The results of this study could be employed to make reliable system as well as reduce maintenance cost.

Keywords: Multi-state Markov model, equipment reliability and maintenance, optimization, imputation

1 Introduction

In navy ship equipment management, maintenance policy is crucial both in terms of safety and efficiency. Maintenance policy includes several controllable variables to be determined such as inspection frequency or acceptable maintenance standard. Too long inspection interval or overly lenient standard for repair would result in an unstable system, and the resulting failure costs will be very high. On the other hand, too strict maintenance with frequent inspections and excessively conservative standards would yield a great budget waste.

For an effective policy development, precise diagnosis of current system should be first addressed based on the past failure data. For example, an annual engine failure count of a specific ship is significant source for planning an optimal maintaining policy which gives information on two main processes, deterioration and maintenance. However, the failure count data only represent the combined effect; thus identifying a hidden deteriorating process is needed based on which deterioration state with different maintenance policy could be predicted.

Motivated by the fact that engine deterioration is a gradual process and its future state depends crucially on the current state, we apply multi-state Markov model to design engine management policy. A multi-state model is defined as a stochastic process, which experiences one of a few possible states (Hougaard, 1999). Much uncertainty could be reflected in this stochastic model and the future state could be predicted with added randomness. Also, multi-state approach can provide more interpretability and effective management policy in the real world since it perceives the whole system as a gradual process between the perfect state (normal) and complete failure (failure) (Qiu et al., 2015; Kołowrocki and Soszyńska-Budny, 2011). Moreover, computational complexity is greatly reduced due to its discrete state modeling. Markovian multi-state models have been widely used in infrastructure management for the prediction of future deterioration states, such as bridge (Kallen, 2007), healthcare systems (González-Domínguez et al., 2020), and pavement (Butt et al., 1987) systems. However, few has been conducted in marine domain and the existing literature mainly focus on risk assessment rather than failure prediction (Faghih-Roohi et al., 2014; Wang et al., 2020).

Many of time-based multi-state Markov approach assume that the lifetime distribution is known, which does not hold (de Jonge and Scarf, 2020). Since regular investigation and maintenance have been implemented on each system, most available failure data are the result of current management policy, and therefore the true deterioration process is unobserved, but this has been ignored on past studies. Reliable construction of transition probability matrix requires consecutive deterioration data without any maintenance intervention (Morcous, 2006). To deal with this problem, we suggest separating the effect of the latent deterioration process from the current maintenance policy.

The objective of this paper is to suggest the overall framework for ship reliability engineering when the data is incomplete and underlying deterioration process is not directly observed due to the effect of current maintenance policy. Using the collected annual engine failure count data for recent 10 years from Korean navy, we estimate the true deterioration process without maintenance intervention by taking a nonlinear optimization approach. Navy ship equipment failures are assumed as stochastic degradation processes and they are modeled as three multi-state continuous-time Markov models. The Transition probabilities between each deterioration state are represented as a 3×3 transition probability matrix (TPM), and we aim to estimate TPM. Li et al. (2014) computed the TPM by constructing the deterioration distribution, however, they did not consider the age factor. In this article, we estimated the age dependent transition probabilities with transition rate matrix obtained by Kolmogorov equation, an inhomogeneous Markov model. The results have shown that there are different deterioration patterns for each age era. Figure 1 shows the overall process of our proposed model.

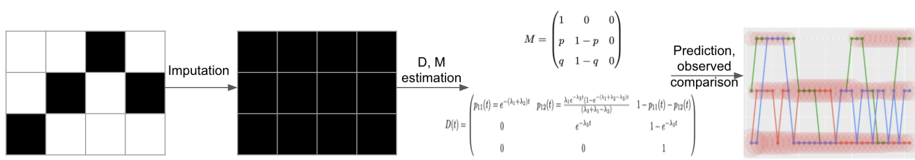

Figure 1: Process of the proposed model.

First, the missing data is imputed with Gaussian process model so as to aid the next step, parameter estimation of transition and maintenance matrix. Without this step, estimation was highly inconsistent. Second, based on the observed data, best parameter representation of deterioration and maintenance process was designed. Then these parameters are estimated for given train sets. Lastly, with the resulting parameter values, state series are predicted and compared with real observed states. The last step is necessary in that it shows the validity of our model. Following sections describe the above in detail.

The outline of this paper is as follows. In Section 2, we introduce our overall modeling framework and multi-state Markov model. We formally define the deterioration and maintenance matrix in Section3, and also describe our optimization process to separate the pure effect of deterioration process and estimate the hidden deterioration rate. Section 4 shows the result of our approach, estimated deterioration parameters and the prediction accuracy. Lastly, limitations and future developments are discussed in Section 5.

2 Problem setting

Annual propulsion engine failure count data for recent ten years (2010-2019) was collected from the equipment maintenance information system. 99 ship engines are categorized into five engine types. Figure 2 presents an overview of our data. The data has 99 vectors with its length of 31, each number representing the number of ships and their lifetime. Each cell represents the failure count; brighter color represents higher failure counts, and non-colored regions are where data are missing. As can be seen from the figure, the amount of data for each category is highly imbalanced. We arranged the failure data of 99 propulsion ship engines according to their engine type (1 to 5). As we can see, the amount of data for each engine type is highly unbalanced. Considering that the data was recorded in the recent 10 years, the recorded lifetime region differs according to its introduction time. For new models introduced after 2010 (engine type 5), failure counts were recorded on younger age (0-9 yrs), whereas old model's data introduced in 1985 were recorded on older age (25-31 yrs). Moreover, the similarity between data under the same category could be inferred; for example, same engine types tend to share existing data range and the scale of failure count. Due to the security problem of military data, only its scaled version is reported throughout the paper.

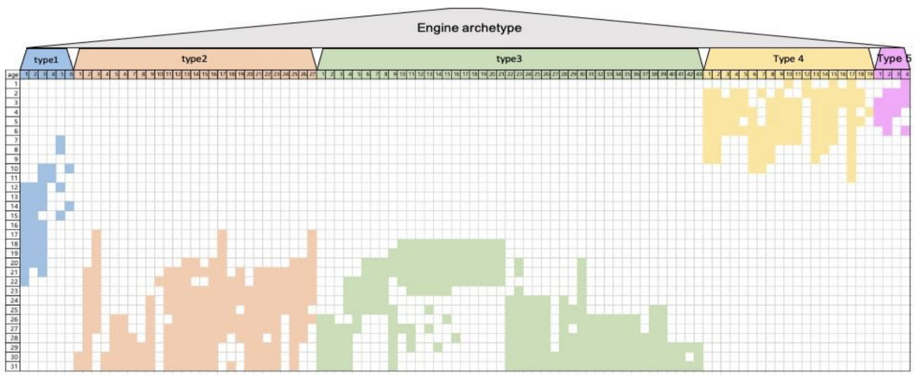

Figure 2: Overview of failure count data collected from the Korea navy ship.

3 Imputation using Gaussian Process

As the majority of original data was missing and unevenly spaced, imputation was needed to obtain a robust Markov model. After comparing with several models including hierarchical spline and Poisson process, hierarchical Gaussian process model was chosen for imputation. It is notable that due to the hierarchical structure of a system, such as shared engine types, using hierarchical model is effective (Gelman et al., 2013).

We assumed that for jth ship, failure counts for time t follow normal distribution with mean $\mu_{t,j}$. For each lifetime, standard deviation for each engine looks quite different. For example, engine 4 and 5 show large variance while engine 3 shows small variance of given data. So we attempted to give different standard deviation to each engine. Consequently, each failure counts follows normal distribution with mean $\mu_{t,j}$ and standard deviation $\sigma_{k[j]}$ where $k[j]$ is engine index for jth ship.

$$y_{t,j} \sim \mathcal{N}(\mu_{t,j}, \sigma_{k[j]}) \tag{1}$$

Mean $\mu_{t,j}$ is defined as additive Gaussian process which is sum of variables with normal distribution and Gaussian processes. μ is an overall average value of $\mu_{t,j}$; $\theta_t^{age}, \theta_j^{ship}, \theta_{k[j]}^{engine}$ follow normal distribution with zero mean and variance $\sigma^2_{age}, \sigma^2_{ship}, \sigma^2_{engine}$, respectively. Variances are defined as parameters.; γ_j and δ_k are Gaussian processes whose tth elements are $\gamma_{t,j}$ and $\delta_{t,k[j]}$, respectively.

$$\begin{aligned} \mu_{t,j} &= \mu + {\theta_t}^{age} + {\theta_j}^{ship} + \theta_{k[j]}^{engine} + \gamma_{t,j} + \delta_{t,\ k[j]} \\ t &= 1,2\dots T,\ \ j = 1,2,\dots N \end{aligned} \tag{2}$$

$$\begin{aligned} {\theta_t}^{age} &\sim \mathcal{N}(0, \sigma^2_{age}\ ) \\ {\theta_j}^{ship} &\sim \mathcal{N}(0, \sigma^2_{ship}\ ) \\ {\theta_k}^{engine} &\sim \mathcal{N}(0, \sigma^2_{engine}\ ) \end{aligned} \tag{3}$$

$$\begin{aligned} \gamma_j &\sim \mathcal{N}(\mathbf{0}, \boldsymbol{K}_{l^\gamma,\alpha^\gamma}) \\ \delta_k &\sim \mathcal{N}(\mathbf{0}, \boldsymbol{K}_{l^\delta,\alpha^\delta}) \end{aligned} \tag{4}$$

Covariance kernel for Gaussian processes γ_j and δk are defined as the exponentiated quadratic kernel. The exponentiated quadratic kernel defines the covariance of each Gaussian process between $f(x_i)$ and $f(x_j)$ where $f : \mathbb{R}^D \to \mathbb{R}$ as a function of the squared Euclidian distance between $x_i \in \mathbb{R}^D$ and $x_j \in \mathbb{R}^D$:

$$\begin{aligned} \mathrm{Cov}(f(x_i), f(x_j)) &= k(x_i, x_j) \\ &= \alpha^2 \exp\left(-\frac{1}{2l^2}\sum_{d=1}^{D}(x_{i,d} - x_{j,d})^2\right) \end{aligned} \tag{5}$$

with α and l constrained to be positive.

Since covariance function $\boldsymbol{K}_{l^\gamma,\alpha^\gamma}$ and $\boldsymbol{K}_{l^\delta,\alpha^\delta}$ follow exponentiated quadratic kernel, ijth element of $\boldsymbol{K}_{l^\gamma,\alpha^\gamma}$ and $\boldsymbol{K}_{l^\delta,\alpha^\delta}$ are defined as 6

$$\begin{aligned} (\boldsymbol{K}_{l^\gamma,\alpha^\gamma})_{ij} &= {\alpha^\gamma}^2 \exp\left(-\frac{1}{2{l^\gamma}^2}\sum_{d=1}^{D}(x_i, d - x_j, d)^2\right) \\ (\boldsymbol{K}_{l^\delta,\alpha^\delta})_{ij} &= {\alpha^\delta}^2 \exp\left(-\frac{1}{2{l^\delta}^2}\sum_{d=1}^{D}(x_i, d - x_j, d)^2\right) \end{aligned} \tag{6}$$

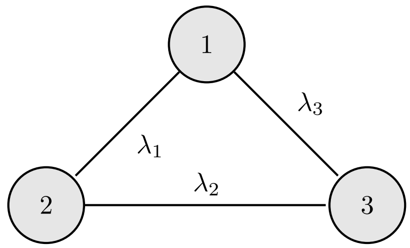

Figure 3: Deterioration rate between three states.

Prior distribution for l^γ and l^δ are Weibull distribution of shape parameter k and scale parameter λ. Since their mean largely depends on scale parameter, we reparametrized l^γ and l^δ so that scale parameter λ is fixed to 1.

$$\begin{aligned} l_\gamma &\sim \text{Weibull}(k_\gamma, \lambda_\gamma) \\ l_\delta &\sim \text{Weibull}(k_\delta, \lambda_\delta) \end{aligned} \tag{7}$$

are reparametrized into

$$\begin{aligned} l_\gamma^s &\sim \text{Weibull}(k_\gamma, 1), \quad l_\gamma = l_\gamma^s * \lambda_\gamma \\ l_\delta^s &\sim \text{Weibull}(k_\delta, 1), \quad l_\delta = l_\delta^s * \lambda_\delta \end{aligned} \tag{8}$$

Using this model and in terms of prediction accuracy, we gained appropriate posterior distribution of each parameter of Gaussian model and imputed using the best results.

4 Separating the Deterioration and Maintenencnce effect using CTMC

4.1 Continuous time multi-state Markov model

In this paper, we proposed a three-state continuous-time Markov model to describe the deterioration states over the navy ship's life cycle as Figure 3. The ship's deterioration stage can be classified into three states, which is divided by a quantile classification method with the historical failure data as shown in Table 1. The corresponding failure counts are standardized, we apply a standardized prodcedure substracting the mean and dividing by the standard error. The deterioration state of engine has a range from 1 to 3, where 1 corresponds to an normal condition, 2 corresponds to near failure and 3 to failure of engine. The state of each ship at time t is defined by standardized annual failure counts as shown in Table 1. For this three-state deterioration model, the deterioration process can only move in a forward direction.

State	Status Description	Annual failure counts
1	Normal	[-1.8304, -0.3340)
2	Near Failure	[-0.3340, -0.0703)
3	Failure	[-0.0703, 2.1273]

Table 1: Deterioration state of ship equipment according to the standardized annual failure counts.

A continuous-time multi-state model is for a continuous-time stochastic process where individuals can occupy a finite number of states. Let $\{Y(t)|t \in T\}$ represents the deterioration state of the process at time t. The transition between states follows a continuous-time Markov process, and transition probabilities only depend on a present state. For a time-homogeneous Markov chain, we can write the transition probability function from state i at time s to state j at time t as $p_{ij}(s,t) = Pr\{Y(t) = j|Y(s) = i\}$. The transition probabilities between all possible pairs (i, j) are represented by a $(n \times n)$ matrix called the transition probabilities matrix $P(s,t)$, where n is the number of possible condition states as shown in 9. In a continuous-time Markov process, transition rate matrix Q is also introduced to explain instantaneous transition rates between states.

$$P(s,t) = \begin{pmatrix} p_{11}(s,t) & p_{12}(s,t) & p_{13}(s,t) \\ p_{21}(s,t) & p_{22}(s,t) & p_{23}(s,t) \\ p_{31}(s,t) & p_{32}(s,t) & p_{33}(s,t) \end{pmatrix} \tag{9}$$

The transition rate matrix $Q(t)$, also known as an intensity matrix, has an element q_{ij} (for $i \neq j$) denoting the rate departing from i and arriving in state j. Diagonal elements q_{ii} are defined such that $q_{ii} = -\sum_{j \neq i} q_{ij}$., and therefore the rows of the matrix sum to zero. In other words, the off-diagonal elements of Q represent the rates governing the exponentially distributed variables that are used to describe the amount of time that elapses before a particular type of base substitution occurs. Transition matrix $P(t)$ can be calculated using Kolmogorov's forward and backward equations, $P'(t) = P(t)Q$ and $P'(t) = QP(t)$. The solution is uniquely derived in terms of transition rate matrix Q as **??** by the matrix exponential of Qt.

$$\begin{aligned} P(t) &= \exp(Qt) \\ &= \sum_{r=0}^{\infty} Q^r \frac{t^r}{r!} \end{aligned} \tag{10}$$

4.2 Deterioration and maintenance matrix

To separate the effect of deterioration and maintenance, we first define a deterioration matrix $D(t)$, which is a function of age t, and a maintenance matrix M, which reflects the effect of maintenance. Deterioration matrix $D(t)$ should be an upper-diagonal matrix. Since the effect of deterioration varies between different age era, unlike maintenance matrix, it is constructed in a time-inhomogeneous manner. This is a direct reflection of the fact that older engines are prone to deteriorate faster.

(11) shows the deterioration rate matrix for three-state Markov model. This transition rate matrix requires three distinct parameters for each time era. Also, the deterioration rate matrix is upper-diagonal, considering that deterioration process can only occur in degrading direction.

$$Q(t) = \begin{pmatrix} -(\lambda_{t,1} + \lambda_{t,2}) & \lambda_{t,1} & \lambda_{t,2} \\ 0 & -\lambda_{t,3} & \lambda_{t,3} \\ 0 & 0 & 0 \end{pmatrix} \tag{11}$$

Based on the constructed rate matrix, we define 3×3 *deterioration matrix* as $D(t)$; which it is a function of time t. Each elements in $D(t)$ is the deterioration probability and $D_{ij}(t)$ is the probability of transitioning from state i to j. As we mentioned in Section 2, the transition probability of the deterioration matrix can be calculated uniquely with the given deterioration rate matrix Q, which comes from the solution of Kolmogorov's forward and backward equation as below.

$$\begin{aligned} D(t) &= \exp(Qt) \\ &= \sum_{r=0}^{\infty} Q^r \frac{t^r}{r!} \\ &= I + \sum_{r=1}^{\infty} Q^r \frac{t^r}{r!} \end{aligned} \tag{12}$$

The resulting matrix exponential term can be expressed in a closed form, because the deterioration matrix $D(t)$ is positive-definite, which makes possible for Q to be always factorized as $Q = UDU^{-1}$ using eigen-decomposition, where D is the diagonal matrix of eigenvalues and U is the matrix whose columns are the corresponding eigenvectors. Using the diagonal matrix D, the deterioration matrix $D(t)$ for three-state Markov model is computed as (Jones et al., 2017). $p_{ij}(t)$ is equal to the occurence probability from state i to j.

$$D(t) = \begin{pmatrix} p_{11}(t) = e^{-(\lambda_1+\lambda_2)t} & p_{12}(t) = \frac{\lambda_1 e^{-\lambda_3 t}(1-e^{-(\lambda_1+\lambda_2-\lambda_3)t}}{(\lambda_3+\lambda_1-\lambda_2)} & 1 - p_{11}(t) - p_{12}(t) \\ 0 & e^{-\lambda_3 t} & 1 - e^{-\lambda_3 t} \\ 0 & 0 & 1 \end{pmatrix} \tag{13}$$

In the same way, we define the *maintenance matrix* M as 14, which is a transition probability matrix with two parameters. The maintenance matrix M is multiplied to the state probability vector every time the maintenance is performed. The construction of this maintenance matrix is based on several circumstantial assumptions as follows. First of all, maintenance interval is once a year. Also, some maintenance are imperfect leading to transition from state 2 to be divided to state 1 and 2 with certain probability, p and $1-p$ for each. Likewise, transition from state 3 to 1, 2, and 3 should be modeled with q, r, and $1-q-r$ probability. However, the experiment results showed that $1-q-r$ term mostly converged to 0. Thus we set $1-q-r$ as 0 and changed the probability q, r to q and $1-q$.

$$M = \begin{pmatrix} 1 & 0 & 0 \\ p & 1-p & 0 \\ q & 1-q & 0 \end{pmatrix} \tag{14}$$

4.3 Optimization on estimating the deterioration matrix

The main objective our paper is to estimate the pure deterioration effect from the data with maintenance effect intervened. This can be achieved by estimating the underlying deterioration probabilities on the deterioration matrix $D(t)$, which is parametrized by deterioration rate parameters. Thus, the estimation of deterioration rate parameters λ is the main challenge.

We propose a nonlinear least square optimization method. As deterioration process is age dependent, we divide the ship's lifetime (31 years) into four age era using time series structural change detection method (Killick et al., 2020). Consequently, the total of twelve parameters, three for each era are estimated.

We estimate the rate parameter in the transition rate matrix Q via nonlinear optimization using Stan, a probabilistic computer language for flexible statistical analysis (Carpenter et al., 2017). Stan finds optimal parameter that minimize the target function, difference between the prediction and observed failure count in our setting. $D(t)$ is a deterioration probability matrix in time t where $D(0)$ is an initial deterioration probability matrix which is an identity matrix. $y_{i,t}$ denotes the observed probability of being in state i at time

t, while $\hat{y}_{i,t}$ is its predicted value.

$$
\begin{aligned}
D(t) &= D(0)\exp(Qt), \quad \text{where } D(0) = (1,0,0)^\top \\
\text{minimize} &\sum_{i=1}^{99}\sum_{t=1}^{31} \| \, y_{i,t} - \hat{y}_{i,t} \, \|^2 \\
\hat{y}_{i,t} &= D(0) \times \{ \prod_{i=1}^{t-1} D(i)M \} \, D(t)
\end{aligned}
\tag{15}
$$

5 Result

5.1 Estimated parameters

As we parameterized the deterioration probability and the maintenance probability. Deterioration matrix $D(t)$ is parameterized by rate matrix Q including 3 rate parameters for each 4 eras: total 12 deterioration rates. Maintenance matrix is parameterized by 2 parameters, p and q. With the observed deterioration data, We split the total data into train and test set by 94:5 ratio for 1000 different combinations. We estimated the parameters of deterioration and maintenance matrix by minimizing the least-square error of the observed train set. We estimated parameters for each 1000 different train/test set and checked the distribution of each parameter including average value and standard deviation.

Table 2 shows summary of estimated values of every component of deterioration matrix $D(t)$ in each era. Since $D(t)$ regulates deterioration probability, $(D)_{21}(t)$, $(D)_{31}(t)$ and $(D)_{32}(t)$ are set to zero, and $(D)_{33}(t)$ are set to 1 for every era. Since the first row of $D(t)$ for each era shows a relatively even distribution, we can expect that they tend to deteriorate from state 1 to each state by even probability. Also, the second row shows that as time goes on, ships in state 2 tend to deteriorate to state 3 in higher probability. Each element of $D(t)$ converges while $(D)_{23}(t)$ of Era 1 and 4 shows somewhat irregular and deviated distribution because observed states on Era 1 and 4 are inconsistent for 99 ships.

Figure 4,5,6, and 7 show the distribution of estimated value of every component of deterioration matrix $D(t)$ in first, second, third and fourth era, respectively. Those distributions are results of parameter optimization of 1000 different train/test sets. Since $D_{(t)}$ regulates deterioration probability, $(D)_{21}(t)$, $(D)_{31}(t)$ and $(D)_{32}(t)$ are set to zero, and $(D)_{33}(t)$ are set to 1 for every era. Each element of $D(t)$ converges while $(D)_{23}(t)$ and $(D)_{31}(t)$ of Era 1 and 4 shows somewhat irregular and deviated distribution because observed states on Era 1 and 4 are inconsistent for 99 ships.

Figure 8 and Table 3 show distribution and summary of estimated value of $lambda_1$, $lambda_2$, and $lambda_3$, which are elements of transition rate matrix Q, for each era. Since the average value of λ_1 and λ_3

Era	Average Value			Standard Deviation		
1	0.356	0.309	0.334	0.0112	0.0144	0.0125
	0	0.662	0.338	0	0.165	0.165
	0	0	1	0	0	0
2	0.350	0.337	0.313	0.012	0.00924	0.0136
	0	0.395	0.605	0	0.0639	0.0639
	0	0	1	0	0	0
3	0.348	0.323	0.329	0.0128	0.0141	0.0137
	0	0.403	0.597	0	0.0918	0.0918
	0	0	1	0	0	0
4	0.347	0.328	0.325	0.013	0.0204	0.0204
	0	0.461	0.539	0	0.207	0.207
	0	0	1	0	0	0

Table 2: Average value and standard deviation of $D(t)$.

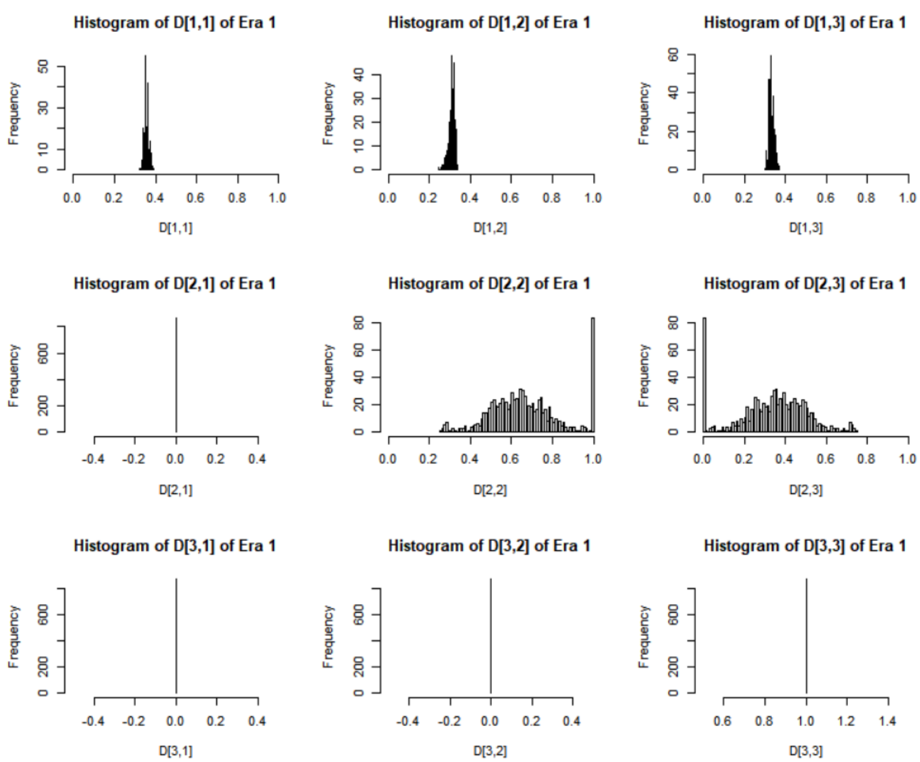

Figure 4: Estimated value of $D(t)$ in the first era from different train/test set.

are larger than λ_2 for every era, we can conclude that the deterioration rate from state 1 is larger than from state 2. Most of the parameters showed consistent values. However, few parameters including the rate at the

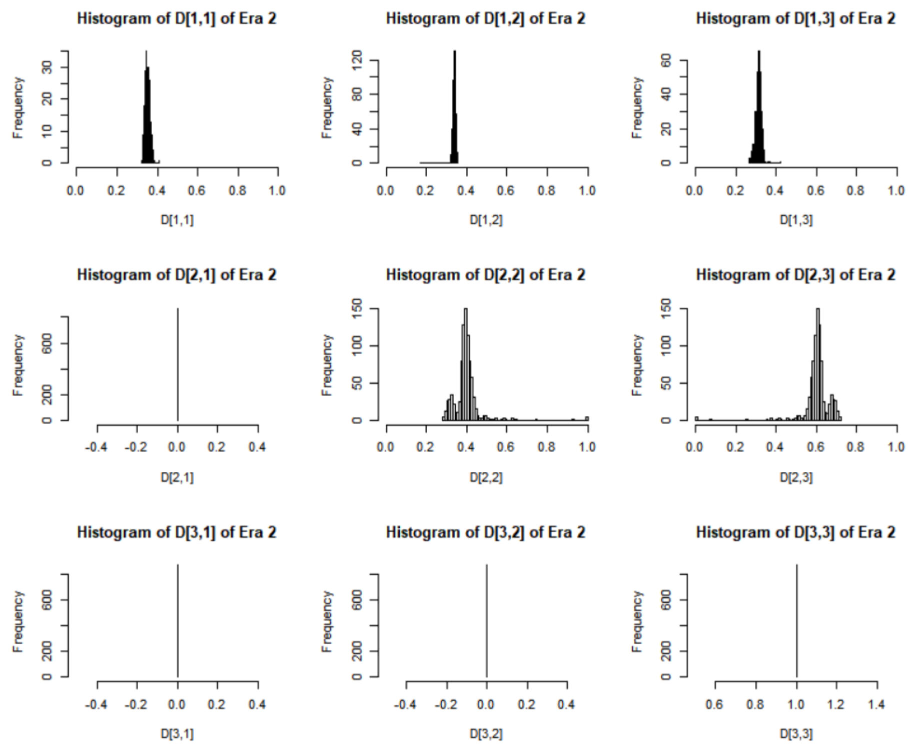

Figure 5: Estimated value of $D(t)$ in the second era from different train/test set.

first era were unstable from time to time as can be seen from Figure 8. We think the unstable system at the initial period might be the main cause. Moreover, the warranty policy could have lowered the accuracy of the failure count in the first few years.

Figure 9 shows distribution of estimated value of every component of maintenance matrix M. Since M is transition probability matrix during the maintenance of engine, $(M)_{12}$, $(M)_{13}$, $(M)_{23}$ and $(M)_{33}$ are set to zero, and $(M)_{11}$ are set to 1. We can observe that $(M)_{21}$ and $(M)_{31}$ concentrate near probability 1, which means after the maintenance, engine's states tend to go to state 1 with high probability for both state 2 or 3 engines.

Table 2 shows the estimated value of deterioration matrix $D_{(t)}$ with computed mean and standard deviation obtained from 1000 different train/test sets.

Table 4 shows summary of estimated maintenance matrix M. Since M is the transition probability matrix during the maintenance of engines, $(M)_{12}$, $(M)_{13}$, $(M)_{23}$ and $(M)_{33}$ are set to zero, and $(M)_{11}$ are set to 1. We can observe that $(M)_{21}$ and $(M)_{31}$ concentrate near probability. This means that after the maintenance, engine states tend to move from both state 2 or 3 to state 1 with high probability.

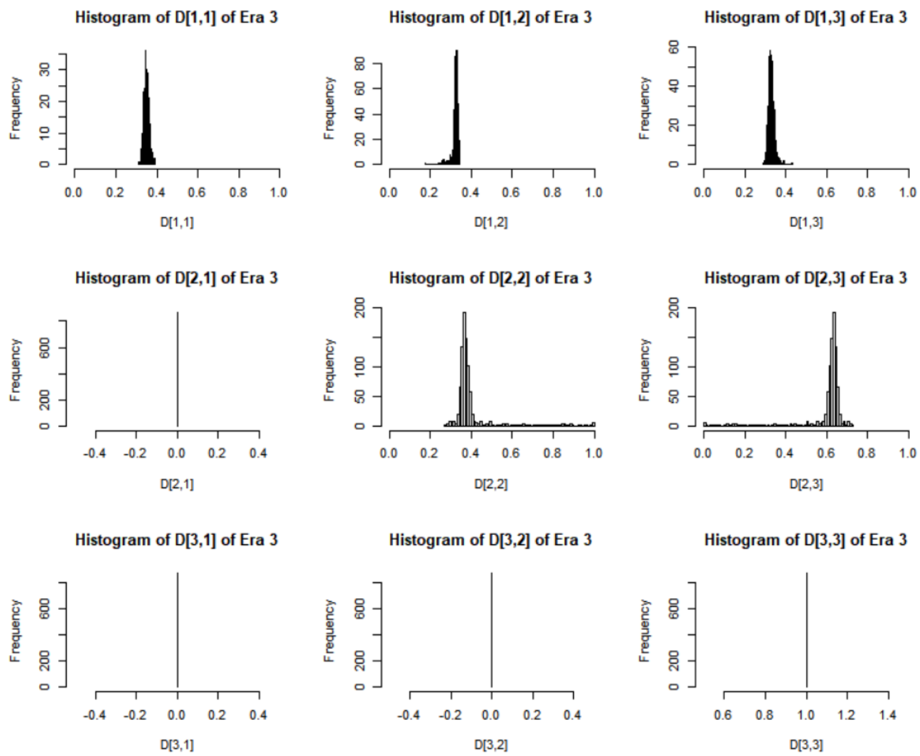

Figure 6: Estimated value of $D(t)$ in the third era from different train/test set.

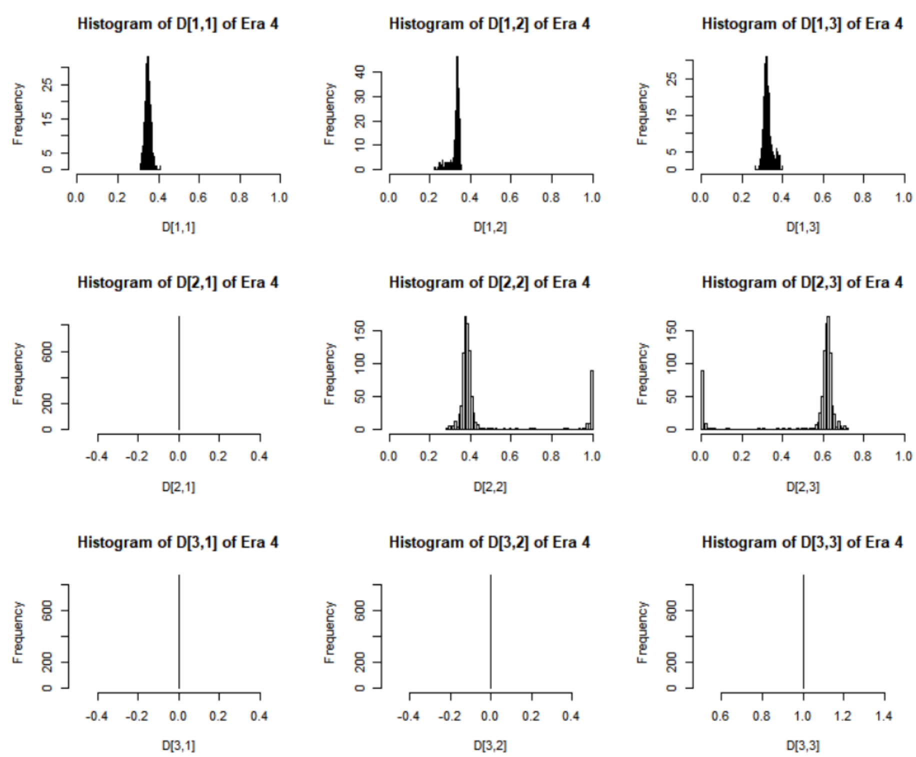

Figure 7: Estimated value of $D(t)$ in the 4th era from different train/test set.

Era	rate	Average Value	Standard Deviation
1	λ_1	0.645	0.111
	λ_2	0.387	0.111
	λ_3	0.445	0.259
2	λ_1	0.914	0.0705
	λ_2	0.137	0.0717
	λ_3	0.940	0.134
3	λ_1	0.880	0.109
	λ_2	0.176	0.0979
	λ_3	0.936	0.212
4	λ_1	0.863	0.161
	λ_2	0.197	0.150
	λ_3	0.840	0.328

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Table 3: Average value and standard deviation of λ.

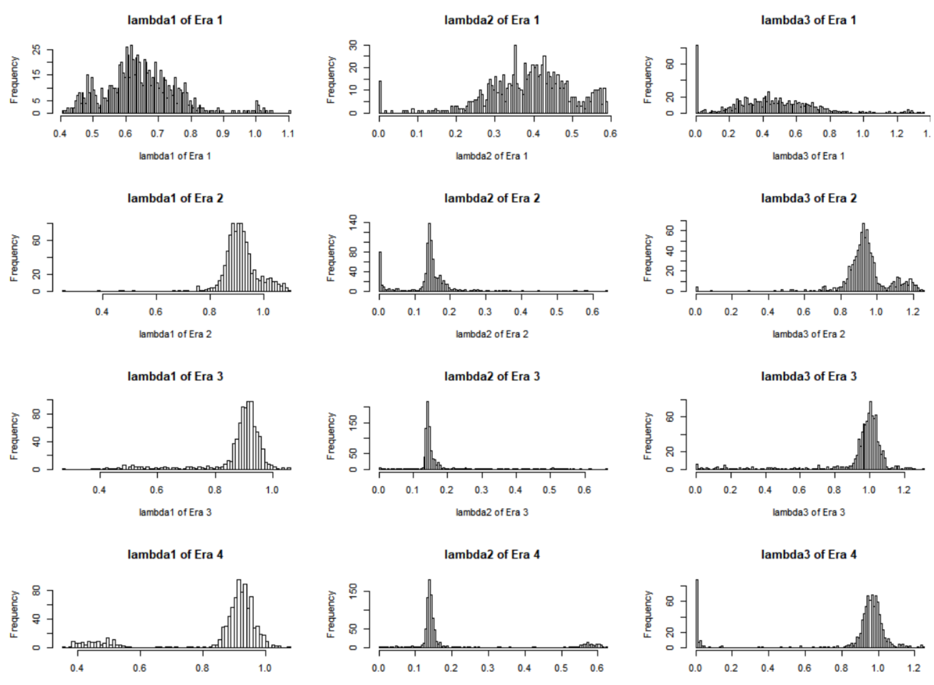

Figure 8: Estimated value of λ from different train/test set.

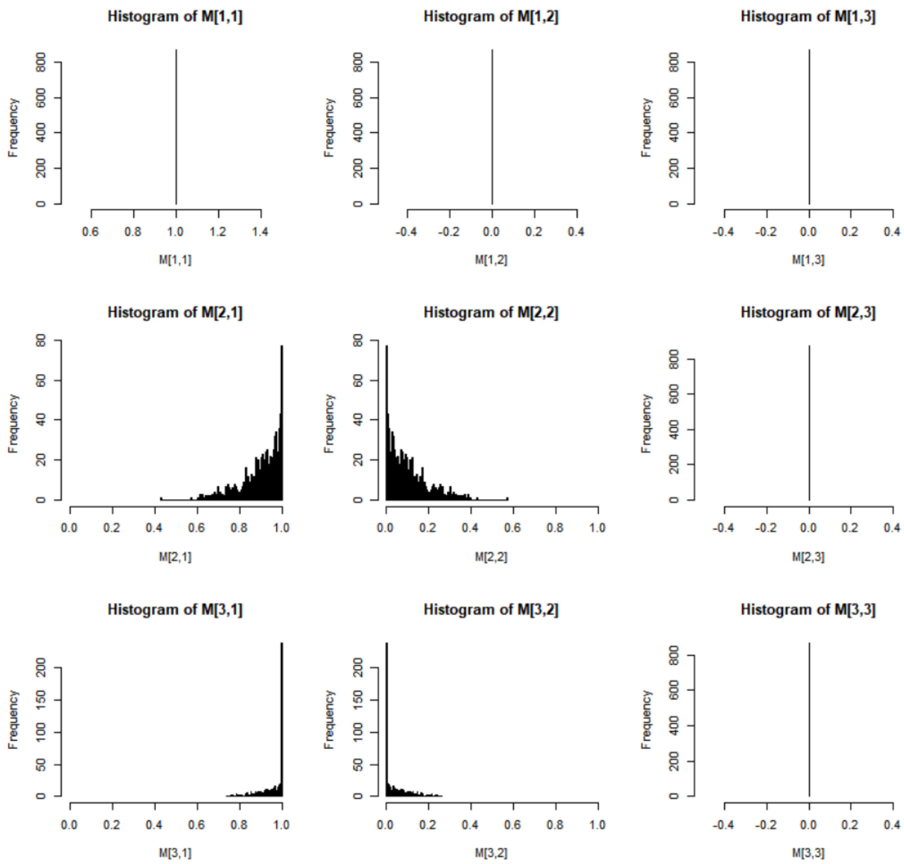

Figure 9: Estimated value of M from different train/test set.

Average Value			Standard Deviation		
1	0	0	0	0	0
0.900	0.100	0	0.0927	0.0927	0
0.951	0.049	0	0.0575	0.0575	0

Table 4: Average value and standard deviation of M.

5.2 Predicted states

The main purpose of the model is to predict the state of each engine so that management plans could be planned based on the predictions. Figure 10 shows the predicted probabilities for each state with the size of the red circles. In most cases, the predictions fit well with the observed states from three out of the sample ship's engines. For example, between year 10 and 20, the flat area which corresponds to the flat area of a bathtub in previous studies, the observed states are concentrated in the first two states and so are the predictions.

One of the best ways to summarize the accuracy of prediction is by measuring the distance between the observed and predicted states. Equation 11 shows the error measure between two state series, where N denotes the number of predicted ships, $y_i(t)$ is the observed deterioration state of ith ship at time t, and $\hat{y}_i(t)$ is the prediction of $y_i(t)$. Note that the value of N is 5 for the test set and 94 for the train set. Figure 11 shows the mean square error between the observed and predicted states for each train and test set. A total of 1000 tests have been repeated and test sets are constructed by randomly selecting five engines. Though the training error (average 20.6) is smaller than the test error (average 20.9), their difference is not big enough which supports the credibility of our model along with the relatively consistent error results for each iteration.

The credibility of the model in each set is assessed by using the mean square error (MSE) of the predictions on train and test sets. The MSE for each optimized data set is calculated as 16 where $d_i(t)$ and $\hat{d}_i(t)$ are observed and predicted states of time t, respectively.

$$MSE = \frac{1}{N}\frac{1}{31}\sum_{i=1}^{N}\sum_{t=1}^{31}(d_i(t) - \hat{d}_i(t))^2 \tag{16}$$

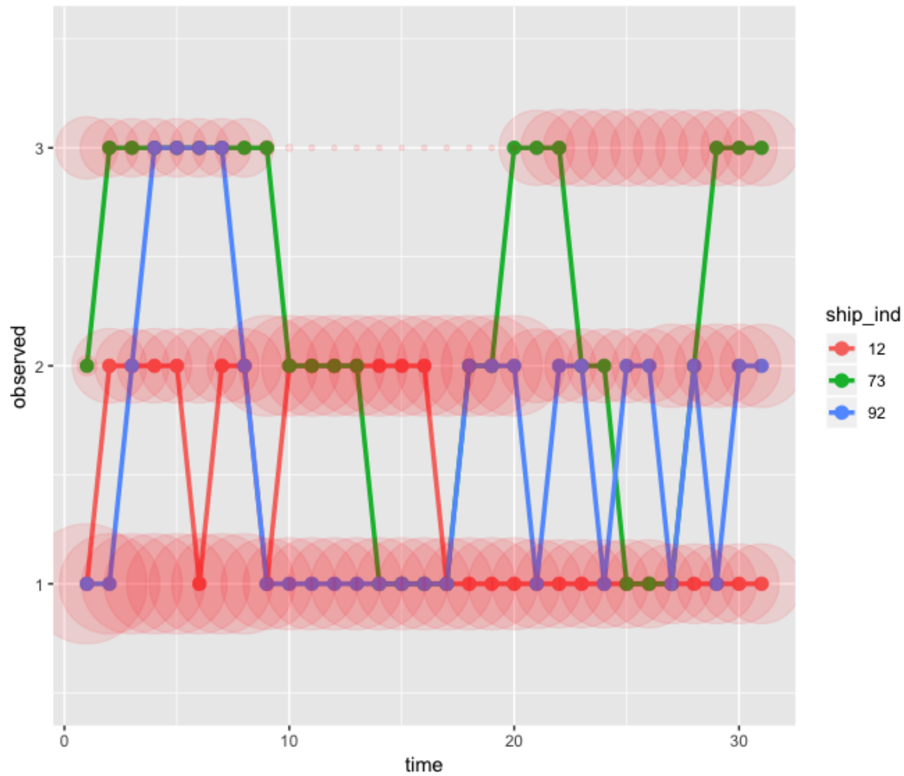

Figure 10: Predicted states and observed states for five test sets.

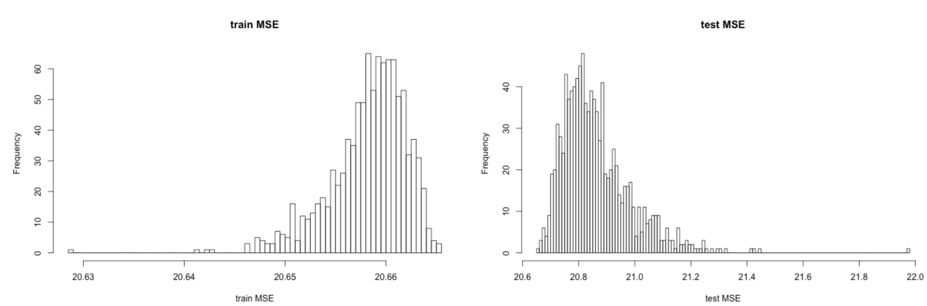

Figure 11: Train/Test MSE from different train/test set.

6 Conclusion

The multi-state model has a strong potential in reliability analysis. We have applied this framework to Korean Navy engine ship failure data and established a three-state Markov model on the engine deterioration process. The difficulty in maintenance policy suggestion is the construction of a pure deterioration matrix since the current implemented policy is reflected in the given data. We separately estimated the deterioration and maintenance effect using nonlinear optimization based on the hierarchical Gaussian process imputed data. Our approach which is validated with existing data provides a basis for reasonable policy comparison. We expect maintenance policy suggestion based on our methodology to be a main direction for further research.

The limitation or further development of this study is as follows. First, the deterioration matrix (D) could be learned in a hierarchical way. Currently, D of each engine type (1 to 5) is learned in a no pooling way but it could be updated to partial pooling, in other words, hierarchical way. Moreover, the rate between four separated time regions in each ship could be partially pooled as well. Though parameters would be added from this additional structure, as has been introduced in McElreath (2020) and Gelman et al. (2013), a hierarchical model with its advanced flexibility could increase the model accuracy. Second, prior knowledge of the engines (deterioration and operation patterns) could be reflected. Prior could greatly affect the performance of the model if properly understood in the context of the entire Bayesian analysis, from inference to prediction to model evaluation (Gelman et al., 2017).

References

Butt, A. A., Shahin, M. Y., Feighan, K. J., and Carpenter, S. H. (1987). *Pavement performance prediction model using the Markov process*. Number 1123.

Carpenter, B., Gelman, A., Hoffman, M. D., Lee, D., Goodrich, B., Betancourt, M., Brubaker, M. A., Guo, J., Li, P., and Riddell, A. (2017). Stan: a probabilistic programming language. *Grantee Submission*, 76(1):1–32.

de Jonge, B. and Scarf, P. A. (2020). A review on maintenance optimization. *European Journal of Operational Research*, 285(3):805 – 824.

Faghih-Roohi, S., Xie, M., and Ng, K. M. (2014). Accident risk assessment in marine transportation via markov modelling and markov chain monte carlo simulation. *Ocean engineering*, 91:363–370.

Gelman, A., Carlin, J. B., Stern, H. S., Dunson, D. B., Vehtari, A., and Rubin, D. B. (2013). *Bayesian data analysis*. CRC press.

Gelman, A., Simpson, D., and Betancourt, M. (2017). The prior can often only be understood in the context of the likelihood. *Entropy*, 19(10):555.

González-Domínguez, J., Sánchez-Barroso, G., and García-Sanz-Calcedo, J. (2020). Scheduling of preventive maintenance in healthcare buildings using markov chain. *Applied Sciences*, 10(15):5263.

Hougaard, P. (1999). Multi-state models: a review. *Lifetime data analysis*, 5(3):239–264.

Jones, E., Epstein, D., and García-Mochón, L. (2017). A procedure for deriving formulas to convert transition rates to probabilities for multistate markov models. *Medical Decision Making*, 37(7):779–789.

Kallen, M.-J. (2007). Markov processes for maintenance optimization of civil infrastructure in the netherlands.

Killick, R., Beaulieu, C., Taylor, S., and Hullait, H. (2020). Package 'envcpt'.

Kołowrocki, K. and Soszyńska-Budny, J. (2011). *Reliability and Safety of Complex Technical Systems and Processes: Modeling–Identification–Prediction-Optimization*. Springer Science & Business Media.

Li, L., Sun, L., and Ning, G. (2014). Deterioration prediction of urban bridges on network level using markov-chain model. *Mathematical Problems in Engineering*, 2014.

McElreath, R. (2020). *Statistical rethinking: A Bayesian course with examples in R and Stan*. CRC press.

Morcous, G. (2006). Performance prediction of bridge deck systems using markov chains. *Journal of performance of Constructed Facilities*, 20(2):146–155.

Qiu, G., Huang, S., Zhu, L., Su, X., and Chen, Y. (2015). Risk assessment of multi-state bayesian network in an oil gathering and transferring system. *Procedia Engineering*, 130:1514–1523.

Wang, S., Yin, J., and Khan, R. U. (2020). The multi-state maritime transportation system risk assessment and safety analysis. *Sustainability*, 12(14):5728.