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Invited Review

A survey of maintenance policies of deteriorating systems

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Abstract

In the past several decades, maintenance and replacement problems of deteriorating systems have been extensively studied in the literature. Thousands of maintenance and replacement models have been created. However, all these models can fall into some categories of maintenance policies: age replacement policy, random age replacement policy, block replacement policy, periodic preventive maintenance policy, failure limit policy, sequential preventive maintenance policy, repair cost limit policy, repair time limit policy, repair number counting policy, reference time policy, mixed age policy, preparedness maintenance policy, group maintenance policy, opportunistic maintenance policy, etc. Each kind of policy has different characteristics, advantages and disadvantages. This survey summarizes, classifies, and compares various existing maintenance policies for both single-unit and multi-unit systems. The emphasis is on single-unit systems. Relationships among different maintenance policies are also addressed.

Keywords: Maintenance policy; Maintenance; Reliability; Replacement; Optimisation

1. Introduction

Systems used in the production of goods and delivery of services constitute the vast majority of most industry's capital. These systems are subject to deterioration with usage and age (Valdez-Flores and Feldman, 1989). Most of them are maintained or repairable systems. For some systems, such as aircrafts, submarines, military systems, and nuclear systems, it is extremely important to avoid failure during actual operation because it can be dangerous or disastrous. Therefore, maintenance on them is necessary since it can improve reliability. The growing importance of maintenance has generated an increasing interest in the development and implementation of optimal maintenance strategies for improving system reliability, preventing the occurrence of system failures, and reducing maintenance costs of deteriorating systems.

In the past several decades, maintenance and replacement problems have been extensively investigated in the literature. McCall (1963), Barlow and Proshan (1965, 1975), Pierskalla and Voelker (1976), Osaki and Nakagawa (1976), Sherif and Smith (1981), Jardine and Buzacott (1985), Valdez-Flores and Feldman (1989), Cho and Parlar

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(1991), Jensen (1995), Dekker (1996), Pham and Wang (1996), Van Der Duyn Schouten (1996), and Dekker et al. (1997) survey and summarize the research and practice in this area in different ways. In this survey, a classification scheme of maintenance models that is amenable to current theoretical development is presented. This classification is intended to serve as guidance to both practitioners and researchers. The idea is to classify maintenance models such that a decision maker can recognize the model that best fits his maintenance problem. Although thousands of maintenance models have been published, there is a limited number of maintenance policies which all maintenance models can be based on. For example, hundreds of maintenance models fall into the age replacement policy, and many fall into the failure limit policy. Therefore, this review, unlike the previous ones, surveys existing maintenance models in terms of maintenance policies that they belong to. This survey is organized into two sections reflecting the classification scheme: maintenance policies of single-unit systems and multi-unit systems. Since maintenance policies for single-unit systems are more established, and are the basis for maintenance policies of multi-unit systems, this work is focused on single-unit systems.

Maintenance can also be categorized into two major classes: corrective and preventive ones. Corrective maintenance (CM) is the maintenance that occurs when a system fails. Some researchers refer to CM as repair. According to MIL-STD-721B, CM means all actions performed as a result of failure, to restore an item to a specified condition. Preventive maintenance (PM) is the maintenance that occurs when a system is operating. According to MIL-STD-721B, PM means all actions performed in an attempt to retain an item in specified condition by providing systematic inspection, detection, and prevention of incipient failures. In this review, maintenance will be a general term and may represent either PM or CM. Replacement is a perfect maintenance. Repair is an action made upon component or system failure and has the same meaning as CM. Repair and CM will be used alternatively throughout this review.

2. Maintenance policies of one-unit systems

As mentioned earlier, although thousands of maintenance models have been developed they can be classified into certain kinds of maintenance policies. This section summarizes, classifies, and compares maintenance policies of one-unit systems. The characteristics, advantages, and drawbacks for each kind of policy will be addressed. Maintenance models with different maintenance cost structures and/or different maintenance restoration degrees (minimal, imperfect, perfect) under the same maintenance policy will be classified into the same policy. The first five subsections in this section discuss maintenance policies with PMs and another subsection contemplates those without PMs. The last subsection provides a summary of them. The basic assumptions for single-unit systems under all maintenance polices are that the unit lifetime has increasing failure rate (IFR); there are virtually infinitely many disposable identical units with i.i.d. lifetimes; salvage value of the unit is negligible.

2.1. Age-dependent PM policy

The most common and popular maintenance policy might be the age-dependent PM policy. In some early work, the *age replacement policy* was extensively studied. Under this policy, a unit is always replaced at its age T or failure, whichever occurs first, where T is a constant (Barlow and Hunter, 1960). Later, as the concepts of minimal repair and especially imperfect maintenance (Pham and Wang, 1996) became more and more established, various extensions and modifications of the age replacement policy were proposed. This class of policies, i.e., the age replacement policy and its extensions, is referred to as the *age-dependent PM policy* in this survey since their PM times are based on the age of the unit. Under this type of policy, a unit is preventively maintained at some predetermined age T, or repaired at failure, until a perfect maintenance, preventive or corrective, is received. Note that PM at T and CM at failure might be either minimal, imperfect, or perfect. Thus, for this class of policy various maintenance models can be constructed according to different types of PMs (minimal, imperfect, perfect), CMs (minimal, imperfect, perfect), cost structures, etc. For example, PM at T might be a replacement or imperfect, CM at failure might be minimal or imperfect, maintenance cost may be a constant or a function of unit age or number of repairs, etc. Details can be found in Pham and Wang (1996), and Valdez-Flores and Feldman (1989). If T is a random variable, the policy is referred to as the random age-dependent maintenance policy that is in force when it is impractical to maintain a unit in a strictly periodic fashion. For example, a given unit may have a variable work cycle so that maintenance in midcycle is impossible or impractical. In this eventuality, the maintenance policy would have to be a random one, taking advantage of any free time available to perform maintenance. In the age replacement policy, items are replaced if they reach a certain age. This age is measured from the time of the last replacement. If only minimal repair is undertaken upon failure, the age replacement policy amounts to the "periodic replacement with minimal repair at failure" policy (see Section 2.2).

Some researchers have produced many interesting and significant results for variations of the age replacement model. Tahara and Nishida (1975) introduce the maintenance policy "replace the unit when the first failure after t_0 hours of operation or when the total operating time reaches T $(0 \leqslant t_0 \leqslant T)$, whichever occurs first; Failures in $[0, t_0]$ are removed by minimal repair". Note that if $t_0 \equiv 0$, it becomes the age replacement policy, and if $t_0 \equiv T$ it reduces to the "periodic replacement with minimal repair at failure" policy. Observe that t_0 is a *reference time* and maintenance actions are not performed exactly at that moment t_0 (unlike PM time).

Nakagawa (1984) extends the age replacement policy to replacing a unit at time T or at number N of failures, whichever occurs first, and undergoes minimal repair at failure between replacements. The decision variables for this policy are T and N. Note that this policy combines the fixed age and the repair number counting ideas. Clearly, if $N \equiv 1$, this policy reduces to the age replacement policy. Herein this policy is called T–N policy. A more general policy is that a subsystem is subject to imperfect PM at T, or CM at the Nth failure $(N = 1, 2, 3, \ldots)$, whichever occurs first, and undergoes *imperfect repair* at failure between replacements.

Two other expansions of the age replacement policy are provided by Sheu et al. (1993, 1995). Sheu et al. (1993) examine a generalized age replacement policy by using the idea similar to Tahara and Nishida (1975). In this policy if a unit fails at age $y < t$, it is subject to a perfect repair with $p(y)$, or undergoes a minimal repair with probability $q(y) = 1 - p(y)$. Otherwise, the unit is replaced when the first failure after t occurs or the total operating time reaches age T $(0 \leqslant t \leqslant T)$, whichever occurs first. The policy decision variables are t and T. Obviously, if $t \equiv 0$ then this policy becomes the age replacement policy. If $t \equiv T$ and $q(y) \equiv 1$, it becomes the "periodic replacement with minimal repair at failure" policy (see Section 2.2). Therefore, this policy is also general since it includes both age replacement policy and "periodic replacement with minimal repair at failure", which are in two different categories of this survey. Sheu et al. (1995) make another extension to the age replacement policy. They assume that a unit has two types of failures at age z, and is replaced at either the nth type 1 failure or first type 2 failure, or at age T, whichever occurs first. Type 1 failure occurs with probability $p(z)$ and is corrected by minimal repair. Type 2 failure occurs with probability $q(z) = 1 - p(z)$ and is corrected by perfect repair. Clearly, if $p(z) = 0$ this policy becomes the age replacement policy. If $p(z) \equiv 1$ and $n \equiv \infty$, it becomes the "periodic replacement with minimal repair at failure" policy (see Section 2.2). The policy decision variables are n and T. Again, this policy is quite general since it includes both the age replacement policy and the "periodic replacement with minimal repair at failure" policy.

Block et al. (1993) introduce another generalized age replacement policy, *repair replacement policy*, where units are preventively maintained when a certain time has elapsed since their last repair. That is, items are repaired if they fail and are replaced only if they survive beyond a certain fixed time from the last repair or replacement. Units are either minimally or perfectly repaired at failure or they are replaced if they survive a certain fixed time from the last repair without suffering a CM. If at failure only perfect repair is allowed, then the repair replacement policy reduces to the age replacement policy. Consequently, the concept of a repair replacement policy is a more general type of replacement policy than the age replacement policy. This policy seems convenient, since, at repair, a schedule to maintain the item is in place and so the bookkeeping to start the maintenance policy can also be undertaken at this time. Furthermore, it seems reasonable, especially for an item which is aging and has undergone minimal repairs, to have some replacement policy rather than to do nothing.

Wang and Pham (1999) make another extension of age replacement policy, called "mixed age PM policy". In this policy, After nth imperfect repair, there are two types of failures. A type 1 failure might be total breakdowns, while a type 2 failure can be interpreted as a slight and easily fixed problem. When a failure occurs, it is a type 1 failure with probability $p(t)$ and a type 2 failure with probability $q(t) = 1 - p(t)$. Type 1 failures are subject to perfect repairs and type 2 failures are subject to minimal repairs. Therefore, each repair is a perfect repair with probability $p(t)$ and a minimal one with probability $q(t) = 1 - p(t)$. After the first n imperfect repairs, the unit will be subject to a perfect maintenance at age T or at the first type 1 failure, whichever occurs first. This process continues along an infinite time horizon. The policy decision variables are T and n. Obviously, if $p(t) \equiv 0$ and $n \equiv 0$, it becomes the "periodic replacement with minimal repair at failures" policy. If $p(t) \equiv 1$ and $n \equiv 0$, it becomes the age replacement policy.

The age-dependent PM policy has received most of the attention in the literature. Studies on this type of policy went back to as early as Morse (1958). Various age-dependent PM policies, summarized from numerous existing maintenance models, are listed in Table 1. Table 1 shows that most extended policies are general and can include the age replacement policy and/or the "periodic replacement with minimal repair at failure" policy as special cases. Note also that most of them are proposed based on imperfect maintenance concepts.

2.2. Periodic PM policy

In the *periodic PM policy*, a unit is preventively maintained at fixed time intervals kT $(k = 1, 2, \ldots)$ independent of the failure history of the unit, and repaired at intervening failures where T is a constant. In some early research, the *block replacement policy* was examined in which a unit is replaced at prearranged times kT $(k = 1, 2, \ldots)$ and at its failures. The block replacement policy derives its name from the commonly employed practice of replacing a block or group of units in a system at prescribed times kT $(k = 1, 2, \ldots)$ independent of the failure history of the system and is

Table 1
Summary of age-dependent PM policies

Policy	Typical reference	PM time points	Decision variables	Special cases
Age replacement	Barlow and Hunter (1960)	Fixed age T	T	
Repair replacement	Block et al. (1993)	Time since last maintenance	Fixed time	Age replacement
T–N	Nakagawa (1984)	Fixed age T or time	T, N	Age replacement periodic PM
(T, t)	Sheu et al. (1993)	Fixed age T or time	T, t	Age replacement periodic PM
(t_0, T)	Tahara and Nishida (1975)	Fixed age T	t_0, T	Age replacement periodic PM
Mixed age	Wang and Pham (1999)	Fixed age T or time	k, T	Age replacement periodic PM
(T, n)	Sheu et al. (1995)	Fixed age T	T, n	Age replacement periodic PM

often used for multi-unit systems. Another basic PM periodic policy in this class is "periodic replacement with minimal repair at failures" policy under which a unit is replaced at predetermined times kT $(k = 1, 2, \ldots)$ and failures are removed by minimal repair (Barlow and Hunter, 1960, Policy II).

As the concepts of minimal repair and especially imperfect maintenance (Pham and Wang, 1996) became more and more established, various extensions and variations of these two policies were proposed. One expansion of the "periodic replacement with minimal repair at failure" policy is the one where a unit receives imperfect PM every T time unit, intervening failures are subject to minimal repairs, and it is replaced after its age has reached $(O+1)T$ time units, where O is the number of imperfect PMs which have been done (Liu et al., 1995). $O = 0$ is allowed in this policy, which means the unit will be replaced whenever it has operated for T time units and there will be no imperfect PM for it. The policy decision variables are O and T. Obviously, if $O = 0$, this policy becomes the "periodic replacement with minimal repair at failure" policy.

Berg and Epstein (1976) have modified the block replacement policy by setting an age limit. Under this modified policy, a failed unit is replaced by a new one; however, units whose ages are less than or equal to t_0 $(0 \leqslant t_0 \leqslant T)$ at the scheduled replacement times kT $(k = 1, 2, \ldots)$ are not replaced, but remain working until failure or the next scheduled replacement time point. Obviously, if $t_0 = T$, it reduces to the block replacement policy. This modified block replacement policy was shown to be superior to the block replacement policy in terms of the long-run maintenance cost rate.

Tango (1978) suggests that some failed units be replaced by used ones, which have been collected before the scheduled replacement times. Under this extended block replacement policy, units are replaced by new ones at periodic times kT $(k = 1, 2, \ldots)$. The failed units are, however, replaced by either new ones or used ones based on their individual ages at the times of failures. A time limit r is set in this policy, similar to t_0 in Berg and Epstein (1976). Under this policy, if a failed unit' age is less than or equal to a predetermined time limit r, it is replaced by a new one; otherwise, it is replaced by a used one. This policy is different from Berg and Epstein's because it modifies the ordinary block replacement policy by considering rules on the failed units rather than on the working ones (cf. Berg and Epstein, 1976). Obviously, if $r = T$, this policy becomes the block replacement policy.

Nakagawa (1981a,b) presents three modifications to the "periodic replacement with minimal repair at failure" policy. The modifications give alternatives that emphasize practical considerations. The three policies all establish a reference time T_0 and periodic time T^*. If failure occurs before T_0, then minimal repair occurs. If the unit is operating at time T^*, then replacement occurs at time T^*. If failure occurs between T_0 and T^*, then: (*Policy I*) the unit is not repaired and remains failed until T^*; (*Policy II*) the failed unit is replaced by a spare unit as many times as needed until T^*; (*Policy III*) the failed unit is replaced by a new one. In all these three policies, the policy decision variables are T_0 and T^*. Clearly, if $T_0 \equiv T^*$, Policies I, II, and II all become the "periodic replacement with minimal repair at failure" policy. If $T_0 \equiv 0$, Policy III becomes the block replacement policy.

Nakagawa (1980) also makes an expansion to the block replacement policy. In his policy, a unit is replaced at times kT $(k = 1, 2, \ldots)$ independent of the age of the unit. A failed unit remains failed until the next planned replacement. Another variant of the "periodic replacement policy with minimal repair" policy is also due to Nakagawa (1986), in which the replacement is scheduled at periodic times kT $(k = 1, 2, \ldots)$ and failure is removed by minimal repair. If the total number of failures is equal to or greater than a specified number n, the replacement should be done at the next scheduled time; otherwise, no maintenance should be done. The decision variable is n and T. In this policy, if $n = \infty$, this policy becomes the "periodic replacement with minimal repair at failure" policy.

Chun (1992) studies determination of the optimal number of periodic PM's under a finite planning horizon. Dagpunar and Jack (1994) determine the optimal number of imperfect PMs for a finite horizon given that the minimal repair is made at any failure between PM's.

Wang and Pham (1999) extend the block replacement policy to a general case. In their policy, a unit is imperfectly repaired at failure if the number of repairs is less than N (a positive integer). The repair is imperfect in the sense that the unit has shorter and shorter lifetime upon each repair. Upon the Nth imperfect repair at failure, the unit is preventively maintained at kT ($k = 1, 2, \ldots$) where the constant $T > 0$. The PM is imperfect in the sense that after PM the unit is "as good as new" with probability p and "as bad as old" with $(1 - p)$. Upon a perfect PM, the maintenance process repeats. The decision variables are N and T. The justification of this policy is that when a new unit is put into operation, the first N repairs at failure will be performed at a low cost. This is because the unit is young at those times and these repairs turn out to be imperfect. Usually, these repairs are just minor repairs because it is in good operating condition. After the Nth imperfect repair at failure, the unit may be in worse operating condition due to usage, aging and imperfectness of repairs, and then a major maintenance is necessary at a higher cost. If the repair at failure and PM are perfect and $N \equiv \infty$, this policy reduces to the block replacement policy. If the repair at failure is minimal and PM is perfect and $N \equiv \infty$, this policy amounts to the "periodic replacement with minimal repair at failure" policy.

Maintenance schedules under the periodic PM policy, summarized from numerous existing maintenance models, are listed in Table 2.

2.3. Failure limit policy

Under the *failure limit policy*, PM is performed only when the failure rate or other reliability indices of a unit reach a predetermined level and intervening failures are corrected by repairs. This PM policy makes a unit work at or above the minimum acceptable level of reliability. For example, Lie and Chun (1986) formulate a maintenance cost policy where PM is performed whenever a unit reaches the predetermined maximum failure rate, and failures are corrected by minimal repair. Bergman (1978) investigates a failure limit policy in which replacement policies are based on measurements of some increasing

Table 2
Summary of periodic PM policies

Policy	Typical reference	PM time points	Decision variables	Special cases
Block replacement	Barlow and Hunter (1960)	Periodic time	Periodic time	
Periodic replacement with minimal repair	Barlow and Hunter (1960)	Periodic time	Periodic time	
Overhaul and minimal repair	Liu et al. (1995)	Periodic time and its multiples	Fixed number of PMs/periodic time	Periodic replacement with minimal repair
(T_0, T^*) Policy I	Nakagawa (1981a,b)	Periodic time	Periodic time/ reference time	Periodic replacement with minimal repair
(T_0, T^*) Policy II	Nakagawa (1981a,b)	Periodic time	Periodic time/ reference time	Periodic replacement with minimal repair
(T_0, T^*) Policy III	Nakagawa (1981a,b)	Periodic time	Periodic time/ reference time	Periodic replacement with minimal repair/ block replacement
(n, T)	Nakagawa (1986)	Periodic time	Periodic time/ number of failures	Periodic replacement with minimal repair
(r, T)	Tango (1978)	Periodic time	Periodic time/ reference age	Block replacement
(N, T)	Wang and Pham (1999)	Periodic time and its multiples	Periodic time/ number of repairs	Block replacement/ periodic replacement with minimal repair
(t_0, T)	Berg and Epstein (1976)	Periodic time	Periodic time/ reference age	Block replacement

state variable, e.g., wear, accumulated damage or accumulated stress, and the proneness to failure of an active unit is described by an increasing state-dependent failure rate function. The optimal replacement rule in terms of average long-run maintenance cost rate is shown to be a failure limit rule, i.e., it is optimal to replace either at failure or when the state variable has reached some threshold value, whichever occurs first. Bergman's model includes the age replacement policy as a special case. Other research on the failure limit policy can be found in Malik (1979), Canfield (1986), Jayabalan and Chaudhuri (1992a), Jayabalan and Chaudhuri (1992c), Jayabalan and Chaudhuri (1995), Chan and Shaw (1993), Suresh and Chaudhuri (1994), Monga et al. (1997), Pham and Wang (1996). In addition, Love and Guo (1996) study failure limit policy for PM decisions under Weibull failure rates. Generally, the problem of this class of policy is that it requires much computing efforts. The failure limit policy and its extensions are summarized in Table 3.

2.4. Sequential PM policy

Unlike the periodic PM policy, a unit is preventively maintained at unequal time intervals under the *sequential PM policy*. Usually, the time intervals become shorter and shorter as time passes, considering that most units need more frequent maintenance with increased ages. An early sequential PM policy is designed for a finite span (Barlow and Proshan, 1965). Under this sequential policy, the age for which PM is scheduled is no longer the same following successive PMs, but depends on the time still remaining. Clearly the added flexibility permits the achievement of an optimum sequential policy having lower cost than that of the corresponding optimum age replacement policy. Under sequential PM, the next PM interval is selected to minimize the expected expenditure during the remaining time. Thus, this policy does not specify at the beginning of the original time span each future PM interval; rather, after each PM, it specifies only the next PM interval. This gain in flexibility leads to reduction in expected cost.

Nguyen and Murthy (1981b) introduce a sequential policy which calls for a PM if a failure has not occurred by some reference time t_i, where t_i is the maximum time that a unit should be left without maintenance after the $(i-1)$th repair (time from the last repair or replacement). In this policy, a unit is replaced after $(k-1)$ repairs. It is repaired (or replaced at the kth repair) at the time of failure or at age t_i, whichever occurs first. The decision variables are k and t_i for $i = 1, \ldots, k$, given that each PM increases the failure rate of the unit. If $k = 1$, this sequential policy reduces to the age replacement policy.

Nakagawa (1986, 1988) discusses a sequential PM policy where PM is done at fixed intervals x_k

Table 3
Summary of failure limit policies

Typical reference	Reliability index monitored	Optimality criterion	Planning horizon
Bergman (1978)	Failure rate through wear, accumulated damage or stress	Cost rate	Infinite
Malik (1979)	Reliability	Reliability	Infinite
Canfield (1986)	Failure rate	Cost rate	Infinite
Zheng and Fard (1991)	Failure rates	Cost rate	Infinite
Lie and Chun (1986)	Failure rate	Cost rate	Infinite
Jayabalan and Chaudhuri (1992a)	Failure rate	Total cost	Finite
Jayabalan and Chaudhuri (1992c)	Age others	Cost rate	Infinite
Jayabalan and Chaudhuri (1992d)	Age	Total cost	Finite
Chan and Shaw (1993)	Failure rate	Availability	Infinite
Suresh and Chaudhuri (1994)	Reliability and failure rate	Total cost	Finite
Jayabalan and Chaudhuri (1995)	Age	Total cost	Finite
Monga et al. (1997)	Reduction (age and failure rate)	Cost rate	Infinite
Love and Guo (1996)	Weibull failure rate	Cost rate	Infinite

for $k = 1, 2, , \dots, N$. The unit is replaced at the Nth PM and failures between PMs are corrected by minimal repairs, given the unit has different failure distributions between PMs (the failure rate of the unit increases with the number of PMs, or its age is reduced (1988), i.e., the first $(N - 1)$ PMs are imperfect). The policy decision variables are N and $x_k (k = 1, 2, , \dots, N)$. Nakagawa (1986, 1988) also presents two numerical examples indicating that the optimal policy satisfies $x_k \leqslant x_{k-1}$ for $k = 2, \dots, N$. Nguyen and Murthy (1981b) study this policy (Policy II in their paper). If $N = 1$, this sequential policy reduces to the "periodic replacement with minimal repair at failure" policy.

These sequential policies are practical because most units need more frequent maintenance with increased age. They are different from the failure limit policy in that it controls x_k lengths directly but the failure limit policy controls failure rate, reliability, etc., directly. Moreover, Kijima and Nakagawa (1992) develop a sequential PM policy using an accumulated damage concept.

2.5. Repair limit policy

When a unit fails, the repair cost is estimated and repair is undertaken if the estimated cost is less than a predetermined limit; otherwise, the unit is replaced. This is called the *repair cost limit policy* in the literature, as introduced by Gardent and Nonant (1963), and Drinkwater and Hastings (1967). A disadvantage of the repair cost limit policy is that the replacement or repair decision depends only on the cost of a single repair. Long-lasting situations characterized by frequent repairs whose costs are below the corresponding limit do not directly influence the time of replacement, although the repair cost rate might justify a replacement. Thus, further financial savings seem possible if the replacement decision depends on the whole history of the repair process. Considering this drawback, Beichelt (1982) examines repair cost limit policy and uses the repair cost rate (repair cost per unit time) as a criterion of replacement or repair: a unit is replaced as soon as the repair cost rate reaches or exceeds a fixed level, otherwise, it is repaired. In this policy (Beichelt, 1982), the replacement intervals are independently and identically distributed random variables. Yun and Bai (1987) propose a repair cost limit policy in which when a unit fails, the repair cost is estimated and repair is undertaken if the estimated cost is less than a predetermined limit L, where the repair is imperfect. Otherwise, the unit is replaced. This policy by Yun and Bai (1987) is generalized from the one by Drinkwater and Hastings (1967). In addition, Kapur et al. (1989), extend the repair cost limit policy to incorporate the number of repairs as a policy decision variable.

The *repair time limit policy* is proposed by Nakagawa and Osaki (1974) in which a unit is repaired at failure: if the repair is not completed within a specified time T, it is replaced by a new one; otherwise the repaired unit is put into operation again, where T is called *repair time limit*. Nguyen and Murthy (1980) study a repair time limit replacement policy with imperfect repair in which there are two types of repair – local and central repair. The local repair is imperfect while the central repair is perfect, which may take a longer time. Dohi et al., 1997 consider a generalized repair time limit replacement problem with lead time and imperfect repair, which is subject to a time constraint, and propose a nonparametric solution procedure to estimate the optimal repair time limit. Koshimae et al. (1996) consider another repair time limit policy. Under this policy, when the original unit fails, the repair is started immediately. If the repair is completed in a time limit t_0, then the repaired unit is installed as soon as the repair is finished. On the other hand, if the repair time is greater than the time limit t_0, the failed unit is scrapped and a spare is ordered immediately. It is delivered and installed after a lead time. The policy decision variable is the repair time limit t_0.

The repair limit policy and its extensions are summarized in Table 4. Note that in the existing literature, there are two types of repair limit policies: repair cost limit policy and repair time limit policy.

2.6. Repair number counting and reference time policy

Morimura and Makabe (1963a) introduce a policy where a unit is replaced at the kth failure.

Table 4
Summary of repair limit policies

Reference	CM before limit	CM after limit	Limit	Optimality criterion	Planning horizon
Hastings (1969)	Minimal	Perfect	Cost	Cost rate	Infinite
Kapur et al. (1989)	Minimal	Perfect	Cost	Cost rate	Infinite
Beichelt (1982)	Perfect	Perfect	Cost rate	Cost rate	Infinite
Beichelt (1981a,b)	Minimal	Perfect	Cost rate	Cost rate	Infinite
Nguyen and Murthy (1980)	Imperfect	Perfect	Time	Cost rate	Infinite
Yun and Bai (1988)	Minimal	Perfect	Cost	Cost rate	Infinite
Koshimae et al. (1996)	Perfect	Perfect	Time	Cost rate	Infinite
Nguyen and Murthy (1980)	Minimal	Perfect	Time	Cost rate	Infinite
Dohi et al. (1997)	Minimal	Imperfect	Time	Cost rate	Infinite
Park (1979)	Minimal	Perfect	Cost	Cost rate	Infinite
Nakagawa and Osaki (1974)	Minimal	Perfect	Time	Cost rate	Infinite
Yun and Bai (1987)	Imperfect	Perfect	Cost	Cost rate	Infinite
Wang and Pham (1996d)	Imperfect	Imperfect	Cost	Availability/cost rate	Infinite

The first $(k-1)$ failures are removed by minimal repair. Upon replacement, the process repeats. This policy is called *repair number counting policy* in this survey. The policy decision variable is k. Later, Morimura (1970) extends this policy by introducing another policy variable T critical *reference time* which is a positive number. Under this extended policy, all failures before the kth failure are corrected only with minimal repair. If the kth failure occurs before an accumulated operating time T, it is corrected by minimal repair and the next failure induces replacement. But if the kth failure occurs after T, it induces replacement of the unit. Obviously, this policy combines the ideas of counting the number of repairs and recording the elapsed time. The policy decision variables are k and T. If the policy decision variable T is zero, this policy reduces to the repair number counting policy. An imperfect repair version of the repair number counting policy is examined by Jack (1991): performing imperfect repair on failure, and replacement upon the kth failure. A policy similar to the repair number counting policy is also investigated by Park (1979) in which a unit is replaced at the kth failure and minimal repairs are performed for the first $(k-1)$ failures. Later, Lam (1988), and Stadje and Zuckerman (1990) investigate the repair number counting policy, given that the lengths of the operating intervals decrease whereas the durations of the repair increase in different ways.

Muth (1977) examines a replacement policy, similar to the reference time idea of the extended policy by Morimura (1970), in which a unit is minimally repaired up to time T and replaced at the first failure after T. This policy is referred to as *reference time policy* later in this review. Note that in this policy the maintenance action is not undertaken exactly at the reference time point T (unlike PM time). Makis and Jardine (1991, 1993) discuss an optimal replacement policy with imperfect repair at failure: a unit is replaced at the first failure after some fixed time. Makis and Jardine (1992) also introduce a general policy in which a unit can be replaced at any time and at the nth failure the unit can be either replaced or can undergo an imperfect repair. Under different conditions, this policy can reduce to the repair number counting policy, reference time policy, and "periodic replacement with minimal repair at failure" policy, respectively. Therefore, it is a quite general policy.

In general, the repair number counting policy is effective when the total operating time of a unit is not recorded or it is time consuming and costly to replace a unit in operation. It has been proven (Muth, 1977) that the reference time policy yields a lower long-run expected cost per unit time than the periodic PM policy given that the mean residual life function of the unit is strictly decreasing after some age t_0. With this condition, called positive aging, the unit deteriorates and eventually reaches a condition where it is no longer economically justifiable to perform minimal repair after repair. It is shown that the repair number counting policy yields lower asymptotic expected cost rate than the age replacement policy. Also the number of failures before replacement in the repair number counting policy is less than that in the age replacement policy. However, all these results are proven numerically for the Weibull distribution (i.e., for some specific Weibull distribution parameter values).

Phelps (1981) compares the "periodic replacement with minimal repair at failure" policy (Barlow and Hunter, 1960), the repair number counting policy (Morimura and Makabe, 1963a,b; Park, 1979), and the reference time policy (Muth, 1977), given an increasing failure rate. Phelps (1981) shows that the reference time policy, replacing after the first failure that occurs after reference time T, is the optimal of the three policies in terms of the long-run cost rate; The repair number counting policy is more economical than the "periodic replacement with minimal repair at failure" policy.

Note that generally there are no PMs scheduled for this type of policy. These policies are mainly based on counting the number of repairs and/or reference time, but the age-dependent PM policy and periodic PM policy rely on PM times, at which maintenance actions are performed. In the repair number counting and reference time policy, maintenance actions are not undertaken precisely at the reference time point T. In the repair number counting and reference time policy, number of repairs and/or reference time are policy decision variable(s). In the age-dependent PM policy and periodic PM policy, PM time is one of the policy decision variables.

2.7. On the maintenance policies for single-unit systems

The age-dependent PM policy and periodic PM policy have received much more attention in the literature. Hundreds of papers and models have been published (McCall (1963), Barlow and Proshan (1965, 1975), Pierskalla and Voelker (1976), Osaki and Nakagawa (1976), Sherif and Smith (1981), Jardine and Buzacott (1985), Valdez-Flores and Feldman (1989), Pham and Wang (1996)) under these two kinds of maintenance policies. Detailed comparisons on the age and block replacement policies can be found in Barlow and Proshan (1965, 1975) in which the general conclusion is that the age replacement policy is an economical way to the block replacement policy. Berg and Epstein (1978) compare three types of replacement policies: age, block, failure replacement policies and provided a heuristic rule for choosing the best one. In Block et al. (1990), comparisons are made between the block replacement policy and "periodic replacement with minimal repair at failure" policy. In Block et al. (1993), comparisons are made among the age replacement policy, block replacement policy, and repair replacement policy.

The failure limit policy, repair limit policy, and sequential policy are more practical, but there has been much less research done on it. The failure limit policy is also directly consistent with the maintenance objectives: improving reliability and reducing failure frequency. One of the disadvantages of the failure limit policy and sequential policy is that their PM intervals are not equal and thus it is wasteful to implement them. The periodic PM policy is perhaps more practical than the age-dependent PM policy since it does not require keeping records on unit usage. The block replacement policy is more wasteful than the age replacement policy since a unit of "young" age might be replaced at periodic times. Generally, the same argument may hold for the age-dependent PM and periodic PM policy.

Note that maintenance policies have become more and more general because they include some previous policies as special cases. This is reflected in Tables 1 and 2, and described in Sections 2.1–2.6. In general, optimal maintenance plans obtained from these general policies may result in some cost savings since the optimal maintenance schedules under them might be "globally" optimal (optimal in a larger range). However, as they become more and more complicated, these general policies may also cause inconvenience in implementation in practice. Similarly, the maintenance cost is no longer a constant and becomes more and more general. For example, it may be a function of unit age and number of repairs already performed on the unit (It is noted that Frenk et al. (1997) establish a general method for modeling complicated maintenance costs, which is also convenient for this case).

Basically, each maintenance policy for one-unit systems either depends on counting/recording of the number of repairs, PM time, or reference time. In practice, counting number of repairs and recording PM time, and reference time are all possible ways. The current research seems to intend to use two or more of them as policy decision variables in a single policy.

3. Maintenance policies of multi-unit systems

The six kinds of maintenance policies in Section 2 are designed for a system composed of a single stochastically deteriorating subsystem. A natural generalization of these maintenance policies is to consider a system with a number of subsystems. Optimal maintenance policies for such systems reduce to those for systems with a single subsystem only if there exists neither economic dependence, failure dependence nor structural dependence. In this case, maintenance decisions are independent, and the "optimal" maintenance policy is to employ one of the six classes of maintenance policies in Section 2 for each separate subsystem. However, if there exists dependence, for example, economic dependence, then the optimal maintenance policy is not one of considering each subsystem separately and maintenance decisions will not be independent. Obviously, the optimal maintenance action for a given subsystem at any time point depends on the states of all subsystems in the system: the failure of one subsystem results in the possible opportunity to undertake maintenance on other subsystems (opportunistic maintenance). In this paper, economic dependence means that performing maintenance on several subsystems jointly costs less money and/or time than on each subsystem separately. Failure dependence means that failure distributions of several subsystems are stochastically dependent.

Economic dependency is common in most continuous operating systems. Examples of such systems include aircrafts, ships, power plants, telecommunication systems, chemical processing facilities, and mass production lines. For this type of system, the cost of system unavailability (one-time shut-down) may be much higher than maintenance costs. Therefore, there is often a great potential for cost savings by implementing an opportunistic maintenance policy.

Currently, there is an increasing interest in multicomponent maintenance policies and models. As pointed out in Van Der Duyn Schouten (1996), one of the reasons that is often put forward to explain the lack of success in applications of maintenance and replacement models is the simplicity of the models compared to the complex environment where the applications occur. In particular, the fact that up to 10 years ago the vast majority of the maintenance models were concerned with one single piece of equipment operating in a fixed environment was considered as an intrinsic barrier for applications. Next we summarize maintenance policies for multi-unit systems. Cho and Parlar (1991) survey the multi-unit system maintenance models created before 1991, and Dekker et al.'s review is focused on economic dependence models published after 1991 (cf. Dekker et al., 1997). This survey is emphasized on classifications and characteristics of maintenance policies though sometimes it cites the same existing maintenance models as the previous surveys. The basic assumptions for multi-unit systems under all maintenance polices are that there are virtually infinitely many disposable identical units with i.i.d. lifetimes for all items; salvage values of all units are negligible.

3.1. Group maintenance policy

The problem of establishing group maintenance policies, which are best from the point of view of the system's reliability or operational cost, has received significant attention in the maintenance literature. One class of problem for group maintenance policies has been to establish categories of units that should be replaced when a failure occurs. This is particularly important when there are varying access costs associated with disassembly and reassembly, and simultaneous PM of categories of parts may be appropriate. A second class of group replacement studies has been concerned with reducing costs by including redundant parts into systems design. A third class of papers has been concerned with establishing group maintenance policies for systems of independently operating machines, all of which are subject to stochastic failures from the same distribution (Ritchken and Wilson, 1990). For this class of problems, there are three existing group maintenance policies. The first policy, referred to as a T-age group replacement policy, calls for a group replacement when the system is of age T. A second policy, referred to as an m-failure group replacement policy, calls for a system inspection after m failures have occurred. The third policy combines the advantages of the m-failure and T-age policies. This policy, referred to as an (m, T) group replacement policy, calls for a group replacement when the system is of age T, or when m failures have occurred, whichever comes first. The (m, T) group replacement policy requires inspection at either the fixed age T or the time when m machines have failed, whichever comes first. At an inspection, all failed units are replaced with new ones and all functioning units are serviced so that they become as good as new. The policy decision variables are m and T.

Gertsbakh (1984) introduces a policy in which a system has n identical units with exponential lifetimes, and it is repaired when the number of failed units reaches some prescribed number k, the policy decision variable. Vergin and Scriabin (1977) propose a (n, N) policy. Under this group maintenance policy, a unit undergoes preventive replacement if it has operated for N periods, and undergoes a group replacement if it has operated n periods and if either another unit fails or another unit reaches its preventive replacement age (where $n < N$). Love et al. (1982) establish another group replacement policy for a fleet of vehicles. Under this group maintenance policy a vehicle is replaced when repair cost for the vehicle exceeds a pre-set repair limit; otherwise, it is repaired. Sheu and Jhang (1997) propose a 2-phase group maintenance policy for a group of identical repairable items. The time interval $(0, T]$ is defined as the first phase, and the timer interval $(T, T + W]$ is defined as the second phase. As individual units fail, individual units have two types of failures. Type I failures are removed by minimal repairs, whereas Type II failures are removed by replacements or are left idle. A group of maintenance is conducted at time $T + W$ or upon the kth idle, whichever comes first. The policy decision variables are T, W, and k.

Wildeman et al. (1997) discuss a group maintenance policy considering that a maintenance activity carried out on a technical system involves a system-dependent set-up cost that is the same for all maintenance activities carried out on that system, and grouping activities thus saves costs since execution of a group of activities requires only one set-up. Under this policy, a rolling-horizon approach is proposed that takes a long-term tentative plan as a basis for a subsequent adaptation according to information that becomes available in the short term. This policy makes it easy to incorporate short-term circumstances such as opportunities or a varying use of components because these are either not known beforehand or make the problem intractable.

Assaf and Shanthikumar (1987) propose a group preparedness policy for a set of N machines having exponential lifetimes with constant rate. A failed machine can be repaired at any time, and the repair is perfect. The number of failed machines in the system is unknown unless an inspection is carried out. Upon an inspection, a decision will be made on whether to repair the failed machines or not, based on the number of failed machines in the system, a policy decision variable.

3.2. Opportunistic maintenance policies

As pointed out earlier, maintenance of a multicomponent system differs from that of a single-unit system because there exists dependence in multicomponent systems. One of the dependencies is economic dependence. For example, it is possible to do PM to non-failed subsystems at a reduced additional cost while failed subsystems are being repaired. Another dependence is failure dependence, or correlated failures. For example, the failure of one subsystem may affect one or more of the other functioning subsystems, and times to failures of different units are then statistically dependent (Nakagawa and Murthy, 1993). Berg (1976, 1978), suggests a preventive replacement policy for a machine with two identical components which are subject to exponential failure. Under this policy, upon a component failure the other component as well as the failed one is also replaced by a new one if its age exceeds a pre-determined control limit L. Later, Berg (1978) extends it to such an policy: both units are replaced either when one of them fails and the age of the other unit exceeds the critical control limit L, or when any of them reaches a predetermined critical age S. A unit is replaced at age T or at failure, whichever occurs first. Note that this policy will become two independent age replacement policies if $L \equiv \infty$.

Zheng and Fard (1991) examine an opportunistic maintenance policy based on failure rate tolerance for a system with k different types of units. A unit is replaced (active replacement) either when the hazard rate reaches L or at failure with the failure rate in a predetermined interval $(L - u, L)$. When a unit is replaced due to the hazard rate reaching L, all of the operating units with their hazard rate falling in the interval $(L - u, L)$ are replaced (passive replacement) at that time. A unit is subject to minimal repair at failure when the hazard rate is in interval $(0, L - u)$. The policy decision variables are L and u.

Kulshrestha (1968) investigates an opportunistic maintenance policy in which there are two classes of units, 1 and 2. Class 1 contains M standby redundant units so that upon the failure of the currently operating class-1 units, a standby takes over. When all the class-1 standbys have failed, the system suffers catastrophic failure. The class-2 units, on the other hand, form a series system; if one of them should fail, the system suffers a minor breakdown. When a minor breakdown occurs, there is a possible chance for opportunistic repair of those class-1 units which have failed.

Pham and Wang (2000) propose two new (τ, T) opportunistic maintenance policies for a k-out-of-n system. In these two policies, minimal repairs are performed on failed components before time τ- a policy decision variable, and CM of all failed components is combined with PM of all functioning ones after τ. At time T, another policy decision variable, PM is performed if the system has not been subject to a perfect maintenance before $T > \tau$. The policy decision variables are τ and T. Pham and Wang (2000) also extend these two policies to the one including the third decision variable the number of failed components to start CM, considering the k-out-of-n system may still operate even if some of its components have failed.

Dagpunar (1996) introduces a general maintenance policy where replacement of a component within a system is available at an opportunity. An opportunity arises if the failure of some other part of the system allows the component in question to be replaced. It is assumed that the opportunity process is Poisson, which is reasonable if the system consists of a large number of components which are regularly maintained. In this policy the component will be replaced if its age at an opportunity exceeds a specified control limit.

Rander and Jorgenson (1963), and Wang et al. (2001) investigate an opportunistic preparedness maintenance of multi-unit systems with $(n + 1)$ subsystems. Wang et al. (2001) examine such a preparedness policy:

(i) If subsystem i fails when the age of subsystem 0 is in the time interval $[0, t_i)$ replace subsystem i alone at a cost of C_i and at a time of w_i $(i = 1, 2, \dots, n)$.

(ii) If subsystem i fails when the age of subsystem 0 is in the time interval $[t_i, T)$ replace subsystem i and do perfect PM on subsystem 0 $(i = 1, 2, \dots, n)$. The total maintenance cost is C_{0i} and total maintenance time is w_{0i}.

(iii) If subsystem 0 survives until its age $x = T$ perform PM on subsystem 0 alone at a and at a maintenance time of w_0 at $x = T$ PM is imperfect.

(iv) If subsystem 0 has not received a perfect PM at T, perform PM on it alone at time jT $(j = 2, 3, \dots)$ until it gets a perfect PM; If subsystem 0 has not experienced a perfect maintenance and subsystem i fails after some PM, replace subsystem i and do perfect PM on subsystem 0 ($i = 1, 2, \ldots, n$) The total maintenance cost is still C_{0i} and total maintenance time is w_{0i} This process continues until subsystem 0 gets a perfect maintenance.

4. On optimal maintenance policies

Maintenance aims to improve system availability and MTBF, to reduce failure frequency and downtime. However, since maintenance incurs cost, to reduce maintenance cost is also necessary. Most research in maintenance is to study the stochastic behavior of systems under various maintenance policies, and to determine optimal system maintenance policies. The stochastic behavior of systems is mainly represented by system maintenance cost measures: maintenance cost rate, discounted cost rate, and the system reliability measures: availability, MTBF and failure frequency, etc. Generally, an optimal system maintenance policy may be the one which either

(a) minimizes system maintenance cost rate,
(b) maximizes the system reliability measures,
(c) minimizes system maintenance cost rate while the system reliability requirements are satisfied, or
(d) maximizes the system reliability measures when the requirements for the system maintenance cost are satisfied.

Fig. 1 shows various factors which may affect an optimal maintenance policy. An optimal maintenance schedule should properly consider/incorporate various maintenance policies, system configurations, shut-off rules, maintenance restoration degrees, correlated failures and repairs, failure dependence, economic dependence, non-negligible maintenance time, etc. It is noted that for a series system there exist some shut-off rules. For example, while a failed component in a series system is in repair, all other components remain in "suspended animation" (they do not age and do not fail). After the repair is completed, the system is returned to operation. At that instant, the components in "suspended animation" are as good as they were when the system stopped operating. This shut-off rule is used in Barlow and Proshan (1975). Obviously, it is practical and can be applicable in other system configurations.

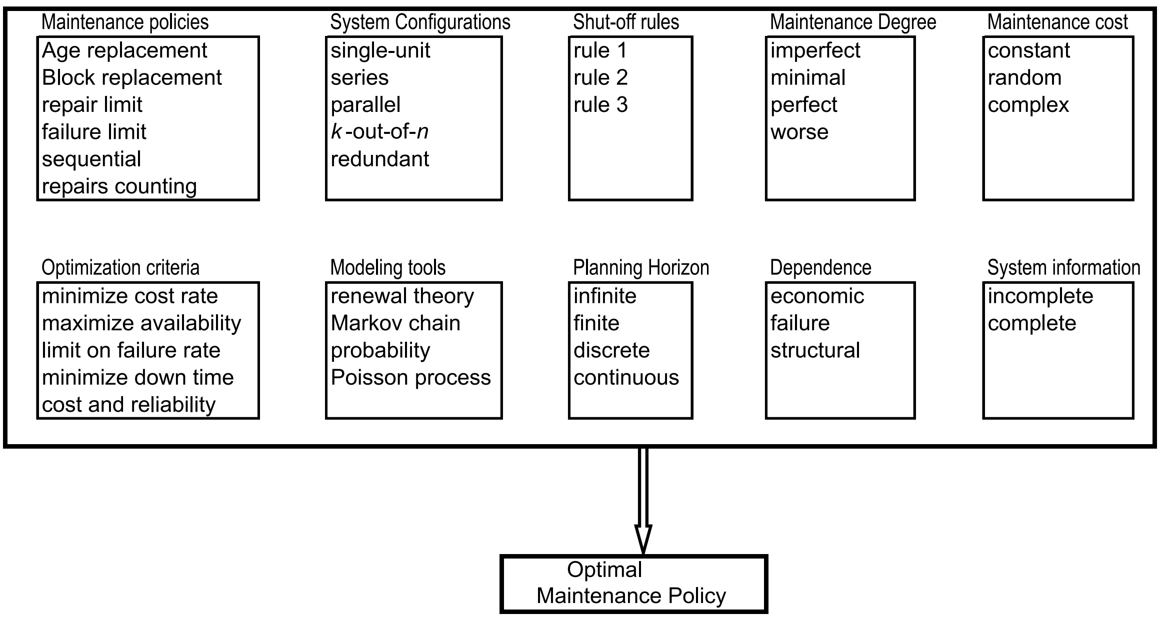

Fig. 1. Maintenance policy and its influence factors.

Hudes (1979) and Khalil (1985) discuss various shut-off rules. Besides, it is worthwhile to mention the following points:

1. Because units are building blocks for a multi-unit system it is necessary to develop effective and efficient methods for modeling reliability measures and cost rates of a single-unit system other than the existing methods. All these methods for a single-unit system will be the basis for the analysis of a multicomponent system.
2. Most optimal maintenance models in the literature use the optimization criterion: minimizing system maintenance cost rate but ignoring reliability performance. However, maintenance aims to improve system reliability performance. Therefore, the optimal maintenance policy must be based on not only cost rate but also reliability measures. It is important to note that for multicomponent systems minimizing system maintenance cost rate may not imply maximizing the system reliability measures. Sometimes when the maintenance cost rate is minimized the system reliability measures are so low that they are not acceptable in practice. This is because various components in the system may have different maintenance costs and different reliability importance in the system (Wang and Pham, 1997). Therefore, to achieve the best operating performance, an optimal maintenance policy needs to consider both maintenance cost and reliability measures simultaneously.
3. In most existing literature on maintenance theory, the maintenance time is assumed to be negligible. This assumption makes availability, MTBSF and MTBSR modeling impossible or unrealistic. Considering maintenance time will result in realistic system reliability measures.
4. The structure of a system must be considered to obtain optimal system reliability performance and optimal maintenance policy. For example, once a subsystem of a series system fails it is necessary to repair it at once. Otherwise, the system will have longer downtime and worse reliability measures. However, when one subsystem of a parallel system fails, the system will still function even if this subsystem is not repaired immediately. In fact, its repair can be delayed until it is the time to do PM on the system considering economic dependence; or repair can begin at such a time that only one subsystem operates and the other subsystems have failed and are awaiting repairs; or at the time that all subsystems fail and thus the system fails, if the system failure during actual operation is not critical.

5. Remarks

This review has tried to be reasonably complete; however, those papers which are not included were either considered not to bear directly on the topic of this survey or were inadvertently overlooked. My apologies are extended to both the researchers and readers if any relevant papers have been omitted.

6. For further reading

The following works are also of interest to the reader: Abdel-Hameed (1987a), Abdel-Hameed (1987b), Abdel-Hameed (1995), Archibald and Dekker (1996), Asher and Feingold (1984), Aven (1983), Aven and Jensen (1999), Balaban and Singpurwalla (1984), Beichelt (1976), Beichelt (1978), Beichelt and Fischer (1980), Berg (1976), Block et al. (1985), Block et al. (1988), Boland (1982), Boland and Proschan (1982), Boland et al. (1991), Boland and El-Neweihi (1998), Chan and Downs (1978), Chen and Feldman (1997), Chaudhuri and Sahu (1977), Dagpunar (1999), Dekker and Smeitink (1991), Dekker and Roelvink (1995), Dohi et al. (1998), Ebrahimi (1986), Finkelstein (1992), Finkelstein (1997), Flynn (1988), Gertsbakh (1977), Gertsbakh (1989), Goel et al. (1992), Goel et al. (1993), Goel and Taiga (1993), Gupta and Kirmani (1989), Gupta (1984), Hopp and Wu (1988), Huang and Okogbaa (1996), Ingle and Siewiorek (1977), Iyer (1992), Jack and Dagpunar (1994), Jacobsen and Arunkumar (1973), Jayabalan and Chaudhuri (1992b), Jiang et al. (1998), Karpinski (1986), Kijima et al. (1988), Laprie et al.

(1981), Lie et al. (1977), McCall (1965), Murthy and Nguyen (1981), Murthy and Nguyen (1985), Murthy and Wilson (1994), Nakagawa (1979), Nakagawa (1985), Nguyen and Murthy (1981a), Nguyen and Murthy (1989), Okumoto and Elsayed (1983), Osaki et al. (1989), Ozekici (1988), Ozekici (1996), Pignal (1987), Pijnenburg et al. (1993), Prasad and Rattihalli (1987), Popova and Wilson (1999), Rander et al. (1993), Rangan and Grace (1989), Righter (1996), Roberts and Mann (1993), Scarf (1997), Scarsini and Shaked (2000), Sengupta (1980), Sheu (1992), Sheu (1999), Smith and Dekker (1997), Stadje and Zuckerman (1996), Stadje and Zuckerman (1999), Tatsuno et al. (1983), Tilquin and Cleroux (1975a), Tilquin and Cleroux (1975b), Wang and Pham (1996a), Wang and Pham (1996b), Wang and Pham (1996c), Wijnmalen and Hontelez (1997), Yasui et al. (1988), Zheng (1995).

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