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O.R. Applications

Inventory control of spare parts using a Bayesian approach: A case study

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Abstract

This paper presents a case study of applying a Bayesian approach to forecast demand and subsequently determine the appropriate parameter S of an $(S-1, S)$ inventory system for controlling spare parts of electronic equipment. First, the problem and the current policy are described. Then, the basic elements of the Bayesian approach are introduced and the procedure for calculating the appropriate parameter S is illustrated. We apply the Bayesian approach in an innovative way to specify the initial prior distributions of the failure rates of three types of circuit packs, using the initial estimates and the failure history of similar items. Based on these priors, we determine the distributions of demand for spare parts and finally we calculate the required stock levels for each type at several locations. According to the proposed method, a lower base stock than the one currently used is sufficient to achieve the desired service level.

Keywords: Inventory; Base stock systems; Spare parts; Bayesian analysis; Case study

1. Introduction

Inventory control of spare parts plays an increasingly important role in modern operations management. The trade-off is clear: on one hand a large number of spare parts ties up a large amount of capital, while on the other hand too little inventory may result in poor customer service or extremely costly emergency actions. This paper studies a specific case, where a company producing circuit packs as spare parts for telephone switching systems has a policy of keeping its customers content by maintaining a sufficiently high service level.

The company, which for confidentiality purposes will be called by the fictitious name "Katharo Technik", KT for brevity, designs, develops and manufactures communication systems. One class of its products is circuit packs with specialised software downloaded in them. These circuit packs are parts of electronic equipment, installed in telephone switching systems at large communication firms. Continuous operation of these systems is essential for KT customers. When such a system ceases to operate because of a failure in a circuit pack, it has to be immediately restored by replacing the failed circuit pack with a readily available

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spare part. Thus, the demand for spare parts originates by the random failures of the installed circuit packs.

Since the availability of a suitable spare part at the moment it is required is crucial, the customers buy a priori the spare parts they need and keep them in KT's warehouses. That means all circuit packs in the warehouses are the property of the customers. There is a central KT warehouse and local warehouses in several locations, in close proximity to the main customers. Circuit packs are distinguished as service affecting and non-service affecting. Failure of the former has serious consequences on the availability of a switch to the end user and consequently these parts are stocked locally to reduce downtime. Non-service affecting circuit packs are stocked centrally. When a failure of a circuit pack in a switch is observed, a spare part is retrieved from a KT warehouse and serves as a replacement. The replacement is performed by maintenance personnel. The entire procedure takes four hours when the spare part comes from central stock and only two hours if it comes from a local warehouse. The defective circuit pack is transported to the KT repair centre. After repair, the circuit pack is usually restored to as good as new condition and it is put back in the central or local warehouse. If it is beyond repair to a satisfactory condition, it is replaced in stock by a new one.

KT makes an agreement with its customers on the service level it has to provide them and then tries to determine the minimum stock level that satisfies the customer requirements at each stocking location separately, taking into account the circuit packs in use. KT sees the spare parts stock as an insurance against unexpected events. In this regard there is uncertainty both with respect to the average failure rate and with respect to the usual fluctuations given an average failure rate. KT's present method tackles both types of uncertainty *separately*. The Bayesian method we propose tackles them in an *integral way* and gives a better indication of which service level one may finally expect. Although the Bayesian method is not new in inventory control (see, e.g., Kaplan, 1988; Sherbrooke, 1992; Hill, 1999, for a recent lite-rature review), the application is.

Section 2 describes current practice for stock level calculation at KT. The challenge and motivation behind this case study was to improve the current method (CM) without altering the type of the inventory control policy. The vehicle to this end is a more accurate forecast of the demand for spare parts, based not only on using all available past information, but also on being "sensitive" to the new data observed each time a failure occurs. Section 3 presents the main elements of a Bayesian approach, which is appealing because of its updating capability. As the choice of initial prior distribution parameters is a key but subjective element of the Bayesian approach, this section also contains a discussion and sensitivity analysis on the values of these parameters. In Section 4 we present the actual application of a suitably adjusted variation of the Bayesian method to the inventory control of specific circuit packs and compare the results with those under the CM of stock level determination. The main findings and conclusions are summarised in Section 5.

2. Current practice

KT has established that the $(S-1, S)$ inventory control model shall be used for managing the inventory of spare parts. Consequently, the specification of the inventory control policy entails the calculation of the base stock level S that fulfils customer requirements at each particular case. Currently, the calculation is based on the assumption that the time to failure of a circuit pack follows an exponential distribution (a valid assumption given the random nature of electronic equipment failures) with parameter λ, which is constant but different from one type of circuit pack to another. Therefore, the number of failures (demand for spare parts) of a circuit pack type during the replenishment order lead-time L follows a Poisson distribution with mean λnL, where n is the number of installed circuit packs of that type. The company calculates S for a given service level p (probability that a demand for a spare part is immediately met from stock) by determining the lowest value of S that satisfies the inequality

$$\sum_{k=0}^{S-1} \frac{(\lambda nL)^k \mathrm{e}^{-\lambda nL}}{k!} \geqslant p, \tag{1}$$

where the left-hand side expresses the probability that the demand during the replenishment lead time will not exceed $S-1$, the available stock just after one part has been retrieved from stock and a replenishment order has been issued.

The problem with the CM, as described above, is that it takes the parameter λ as known and constant. This is not always justifiable. Especially when the company wants to introduce a new type of circuit pack to the market, it is obvious that it has no real data about the failure rate. There exists only an initial estimate of the failure rate, which is used as the parameter λ and is here denoted λ_0 (KT uses its own terms for the estimates of λ, which are deliberately disguised). This estimate is obtained from a reliability prediction method used during the design of the equipment, in this case the parts count method described in MIL-HDBK-217F, which counts the number of parts in the equipment or module and adds their failure rates up (assuming every part is in a reliability series configuration).

As soon as the company receives the first real data for an item, after a 12-month installation period, it proceeds with the computation of λ_1, which is the observed real failure rate of installed circuit packs, computed from

$$\lambda_1 = r/nt. \tag{2}$$

r is the number of observed failures during the operational time t, which is usually equal to one year (8760 hours). λ_1 is calculated separately for each location and the λ_1/λ_0 ratio is used as an indication of prediction error. However, although λ_1 changes with time as new data is collected, it is not used directly as the Poisson parameter λ in (1), because of the fear that the real failure rate may thus be underestimated. Instead, λ_2 is eventually calculated as the upper 95% confidence limit of λ_1, using the fact that the confidence limits on the mean of a Poisson distribution are derived by means of the chi-square distribution (Kapur and Lamberson, 1977) with $2r+2$ degrees of freedom. The exact formula for calculating λ_2 is

$$\lambda_2 = \frac{X^2(0.95, 2r+2)}{2nt}. \tag{3}$$

Example. $n = 4010$ circuit packs of a certain type were installed at a particular location. After one year of operation ($t = 8760$ hours) $r = 171$ failures had occurred. The replenishment lead time was $L = 1428$ hours and the target service level 95% ($p = 0.95$). It turns out that $\lambda_1 = 4.868 \times 10^{-6}$ failures per hour and $\lambda_2 = 5.526 \times 10^{-6}$ failures per hour. Using (1) with $\lambda = \lambda_2$, the minimum base stock that is required is $S = 42$ spare parts, providing a service level of 95.5%.

3. General framework of the Bayesian approach

Although the CM is certainly reasonable, two important issues arise:

(a) By assuming that the parameter λ is constant, the uncertainty that characterises the average failure rate is not taken explicitly into account.
(b) By using the upper 95% confidence limit of the observed failure rate as the estimate of λ, it is possible that the method, in an effort to account for the fluctuation of the real failure rate, may be unduly conservative and may thus result in higher stock than necessary.

These concerns may be addressed using a Bayesian approach for the estimation of the demand for spare parts. Specifically, the uncertainty about λ may be treated by assigning to it a prior probability distribution, to be updated as a posterior distribution on the basis of new observations (failures). The appropriate and convenient distribution of λ is the conjugate prior for the Poisson distribution, namely the Gamma distribution with parameters α, β and density function

$$G(\lambda|\alpha,\beta) = \frac{\beta^\alpha \lambda^{\alpha-1} \mathrm{e}^{-\lambda\beta}}{\Gamma(\alpha)}, \tag{4}$$

where $\alpha > 0$, $\beta > 0$ and $\Gamma(\alpha)$ is the Gamma function. The Gamma distribution has mean α/β, variance α/β^2 and it is very flexible, as it may have a shape that is either one-tailed ($\alpha \leqslant 1$) or two-tailed ($\alpha > 1$). It is well known (e.g., Carlin and Louis, 1996) that if r demands are observed in a time period of length t, then the posterior density function of λ is $G(\lambda|\alpha', \beta')$, with $\alpha' = \alpha + r$ and $\beta' = \beta + t$. The compound Gamma–Poisson probability function for the number of failures k (demand for spare parts) during the replenishment lead-time L is

$$p(k|\alpha, \beta) = \int_0^\infty \frac{(\lambda nL)^k \, \mathrm{e}^{-\lambda nL}}{k!} \frac{\beta^\alpha \lambda^{\alpha-1} \mathrm{e}^{-\lambda\beta}}{\Gamma(\alpha)} \, \mathrm{d}\lambda. \tag{5}$$

For convenience and consistency with the measures used by the company, in our application the failure rate, λ, is defined as the number of failures occurred in 1000 installed circuit packs per year. Then, the base stock that provides at least the required service level p, which will be termed "critical stock level", is the lowest value of S that satisfies the inequality

$$\sum_{k=0}^{S-1} \int_0^\infty \frac{(\lambda nL/1000)^k \, \mathrm{e}^{-\lambda nL/1000}}{k!} \frac{\beta^\alpha \lambda^{\alpha-1} \mathrm{e}^{-\lambda\beta}}{\Gamma(\alpha)} \, \mathrm{d}\lambda \geqslant p. \tag{6}$$

To apply the method, initial values of the unknown parameters α, β of the prior Gamma distribution have to be specified. This is a critical and subjective part of the Bayesian approach, because two equations are needed to define α and β and these equations can be specified in several alternative ways. A typical approach is to estimate

(a) the mean or mode of the distribution of λ (first equation) and
(b) a percentile of the distribution of λ (second equation)

by some means, either expert elicitation or some formal, statistical method. For example, the first equation may be obtained by setting the mean of the prior Gamma equal to the original estimate of the failure rate

$$\alpha/\beta = \lambda_0 \tag{7}$$

while the second equation may come from experts' experience, e.g., in 95% of the cases the actual failure rate does not exceed twice the originally estimated failure rate λ_0:

$$P(\lambda \leqslant 2\lambda_0) = 0.95 \Rightarrow \int_0^{2\lambda_0} \frac{\beta^\alpha \lambda^{\alpha-1} \mathrm{e}^{-\lambda\beta}}{\Gamma(\alpha)} \, \mathrm{d}\lambda = 0.95. \tag{8}$$

After the observation of r failures in t years, the updating of the parameters is done through

conj prior, lik

$$\alpha' = \alpha + r, \tag{9}$$

$$\beta' = \beta + tn/1000. \tag{10}$$

Since λ is expressed per 1000 units per year, the interpretation of the updating relationships after r failures in n installed circuit packs in t years is that equivalently r failures are observed in 1000 circuit packs in $tn/1000$ years.

It is clear that different initial assumptions may be made and consequently expressions different from (7) and (8) may result, leading in turn to different α, β values. To explore the effect of these assumptions we examine below, through an example, four alternatives, which result from combinations of two decisions. The first concerns the choice to set either the mean, α/β, or the mode, $(\alpha-1)/\beta$, of the Gamma distribution equal to λ_0. The dilemma is due to the usual difficulty to characterise the single-valued estimate of the failure rate as an average or a most probable value. The second decision concerns the upper limit of the integral in (8). In the case of KT's circuit packs, some of the company experts expressed the opinion that it should be set equal to $1.5\lambda_0$ rather than $2\lambda_0$. To summarise, the four alternatives use the following expressions for determining α, β of the initial prior:

First alternative:

$$\alpha/\beta = \lambda_0 \quad \text{and} \quad \int_0^{2\lambda_0} \frac{\beta^\alpha \lambda^{\alpha-1} \mathrm{e}^{-\lambda\beta}}{\Gamma(\alpha)} \, \mathrm{d}\lambda = 0.95.$$

Second alternative:

$$\alpha/\beta = \lambda_0 \quad \text{and} \quad \int_0^{1.5\lambda_0} \frac{\beta^\alpha \lambda^{\alpha-1} \mathrm{e}^{-\lambda\beta}}{\Gamma(\alpha)} \, \mathrm{d}\lambda = 0.95.$$

Third alternative:

$$(\alpha-1)/\beta = \lambda_0 \quad \text{and} \quad \int_0^{2\lambda_0} \frac{\beta^\alpha \lambda^{\alpha-1} \mathrm{e}^{-\lambda\beta}}{\Gamma(\alpha)} \, \mathrm{d}\lambda = 0.95.$$

Fourth alternative:

$$(\alpha-1)/\beta=\lambda_0 \text{ and } \int_0^{1.5\lambda_0}\frac{\beta^\alpha\lambda^{\alpha-1}e^{-\lambda\beta}}{\Gamma(\alpha)}\,d\lambda=0.95.$$

Example. Let us reconsider the example of the previous section ($n=4010$ installed circuit packs, $r=171$ failures in $t=1$ year, $L=0.163$ years, $p=0.95$). The original estimate of the failure rate was $\lambda_0=81.5$ failures per 1000 units per year and the observed failure rate after one year is $\lambda_1=42.64$ failures per 1000 units per year.

For each of the four alternative rules, Table 1 shows the resulting α, β, the mean μ_L and the standard deviation σ_L of the compound Gamma–Poisson distribution for the demand (failures) during L, as well as the critical stock level S. The system of the two equations for α and β was solved approximately with the requirement that α is always a positive integer, so that the Gamma function is easily evaluated: $\Gamma(\alpha)=(\alpha-1)!$ The values of μ_L and σ_L refer to the number $n=4010$ of installed units. It is clear from Table 1 that the choice of the mean or the mode does not significantly affect the outcome, in contrast to the choice of the upper 95% limit on λ. The much larger variance of the demand during the replenishment lead-time L is apparently responsible for the noticeably larger critical stock level with the first and third rules.

When the initial priors are updated using the data on failures after one year ($r=171$), the new parameters α', β' are also shown in Table 1, together with the new μ'_L, σ'_L and S'. It is easily seen that almost independently of the rule used for determining the initial α, β, the updated critical stock level S' is more or less the same and much lower than S. In other words, the end result is not sensitive to the choice of the initial prior. This is a very encouraging finding, because it suggests that the effect of the only arbitrary part of the Bayesian approach diminishes quickly. However, if the prior has a very small variance, the posterior has a strong "memory", i.e., it is affected more by the prior. This would be the case for a circuit pack that has been in operation for a relatively long time and the prior would then be formed based on a large number of observed failures. Since that prior is no more an initial prior but a comprehensive summary of available information, this type of insensitivity in updating is definitely an additional appealing feature of the Bayesian approach.

It is also worth noting that the minimum base stock that is required according to the Bayesian method is $S'=39$ (rules 1, 2, 3) or $S'=40$ spare parts (rule 4), while the appropriate critical stock level according to the CM is $S'=42$. The explanation for the difference between the two methods is that the demand forecasted using the Bayesian approach is lower than the one forecasted by the current KT method. Fig. 1 shows the compound Gamma–Poisson demand distributions as they come out from the four Bayesian alternatives, along with the Poisson distribution for the demand during L, as it is evaluated by the company's CM (with $\lambda=\lambda_2=48.4$ failures per 1000 units per year). The four Gamma–Poisson distributions almost coincide, while the Poisson distribution has almost the same variance but a considerably larger mean ($\mu'_L=31.6$).

4. Application and results

The preceding section exhibited and explained the general framework of the Bayesian approach,

Table 1
Sensitivity analysis on the choice of initial prior distribution for the failure rate

Rule	α	β	μ_L	σ_L	S	α'	β'	μ'_L	σ'_L	S'
1	4	0.049	53.2	27.6	106	175	4.059	28.2	5.7	39
2	13	0.159	53.2	16.5	84	184	4.169	28.8	5.8	39
3	7	0.074	62.1	24.8	108	178	4.084	28.5	5.8	39
4	18	0.209	56.4	15.3	84	189	4.219	29.3	5.8	40

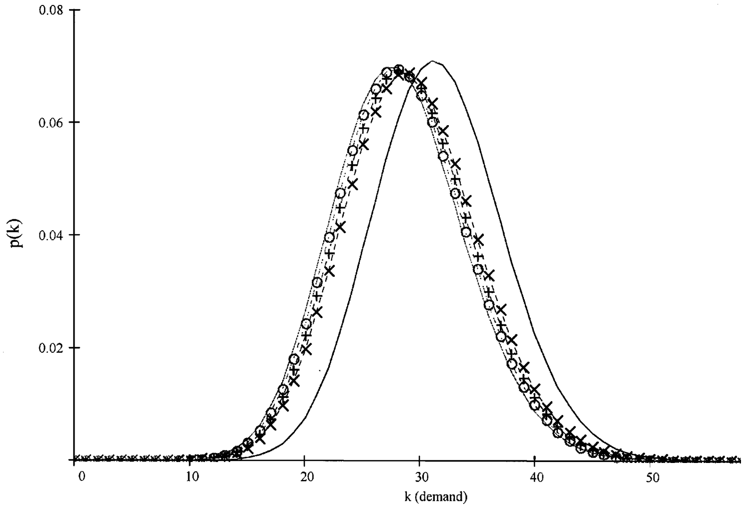

Fig. 1. Comparison between Gamma–Poisson demand distribution according to the four alternative Bayesian rules and Poisson demand distribution according to the company's CM.

making frequent reference to a specific example. The exposition reveals some problematic issues with respect to practical application of the method in the case under consideration:

(a) How accurate is the original estimate of the failure rate λ_0? Although the effect of the initial prior is diminishing quickly, a very inaccurate estimate λ_0 may lead to extremely erroneous decisions for new circuit packs, with no failure data available.
(b) How reliable are the subjective estimates of upper percentiles? The experts tend to provide such estimates without indicating or being able to specify that they refer to the 90 or 95 or some other percentile.

To resolve these issues we proceeded by first considering a generalisation of the four alternative rules examined in the previous section. Specifically, for a circuit type with initial estimate of the failure rate λ_0 (obtained by the parts count method described in Section 2), the following system of equations is formulated:

$$\alpha/\beta = \omega\lambda_0, \tag{11}$$

$$P(\lambda \leqslant \delta\lambda_0) = 0.95 \Rightarrow \int_0^{\delta\lambda_0} \frac{\beta^\alpha \lambda^{\alpha-1} e^{-\lambda\beta}}{\Gamma(\alpha)} \, d\lambda = 0.95, \tag{12}$$

where the values ω and δ are to be evaluated. In order to specify these values for our case, we made a comparison between the initial estimates of λ_0 and the actually observed failure rates λ_1 for all circuit packs for which real failure data was available. The idea is to obtain an indication of how far λ_0 may be from the actual failure rate λ_1 and then use this spread to estimate a general prior which is scaled to the value of λ_0 using Eq. (11) and (12), over all circuit packs and locations. More specifically, ω and δ are first evaluated for all circuit packs with a history. Then, their averages are computed and finally these average values are used in (11) and (12) to estimate α and β for new circuit packs, in conjunction with the estimates of their respective λ_0.

The procedure for evaluating ω and δ is illustrated below for circuit pack A, which at the end of

1997 had already been installed for four years (1994–1997) in 12 locations. Table 2 contains the failure data for that circuit pack at the 12 locations, including the data for 1998, which was used later for updating α, β.

The observed failure rate λ_1 and then the ratio λ_1/λ_0 is computed for all 12 locations and four years. The λ_1/λ_0 ratios are given in Table 3. The λ_1/λ_0 values of locations 8, 9 and 11, where less than 100 units are installed, are considered unreliable and are therefore excluded from the calculations that follow. (It is seen from Table 3 that some of these values at locations 8 and 11, where only 32 and 24 units are installed, are equal to zero, therefore misleading). We now assume that all remaining $9 \times 4 = 36$ λ_1/λ_0 ratios are independent observations of the natural spread of the actual failure rate around the predicted failure rate, caused by differences in location, product and use aspects. The average of those ratios is 0.5 and ω is set equal to this value. It is clear from (11) that this procedure amounts to set the mean α/β of the Gamma prior equal to the average observed failure rate λ_1.

Since 95% of 36 (total number of usable λ_1/λ_0 ratios) is 34.2, the value λ_1/λ_0 under which 95% of the observations lie is the 34th (or third largest) λ_1/λ_0 ratio. In this case, this value is 0.7 (after excluding the two largest values, namely 0.83 and 0.74) and consequently $\delta = 0.7$. Using $\omega = 0.5$ and $\delta = 0.7$ in (11) and (12) yields $\alpha = 25.5$ and $\beta = 0.61$. Then, the critical stock levels S for circuit packs A in 1998 result from (6) with the appropriate n and $L = 0.163$ years at each location. The leftmost columns of Table 4 contain the critical S in 1998 and the predicted service levels p at the 12 locations. The table also shows the resulting critical S in 1999 and the corresponding predicted service levels p at the 12 locations, after obtaining new data (r failures in 1998) and updating the parameters α, β according to (9) and (10). At location 1, for example,

$$\alpha' = \alpha + r = 25.5 + 95 = 120.5$$

and

$$\beta' = \beta + tn/1000 = 0.61 + 1871/1000 = 2.48.$$

Table 2
Observed failures of circuit pack A and installed basis at the 12 locations

Location	1994	1995	1996	1997	1998	Units (n)
1	70	100	92	106	95	1871
2	146	176	180	161	188	4008
3	55	65	53	61	55	1071
4	106	21	132	171	112	3784
5	15	23	13	23	16	1016
6	29	48	64	88	41	1616
7	51	37	28	36	43	752
8	0	0	1	1	0	32
9	2	3	6	5	1	64
10	163	202	143	177	140	3792
11	0	0	2	0	0	24
12	74	78	101	79	90	1840

Table 3
Values of λ_1/λ_0 ratios for circuit pack A at the 12 locations

Location	1994	1995	1996	1997	1998
1	0.46	0.66	0.60	0.70	0.62
2	0.45	0.54	0.55	0.49	0.58
3	0.63	0.74	0.61	0.70	0.63
4	0.34	0.07	0.43	0.55	0.36
5	0.18	0.28	0.16	0.28	0.19
6	0.22	0.36	0.49	0.67	0.31
7	0.83	0.60	0.46	0.59	0.70
8	0.00	0.00	0.38	0.38	0.00
9	0.38	0.58	1.15	0.96	0.19
10	0.53	0.65	0.46	0.57	0.45
11	0.00	0.00	1.02	0.00	0.00
12	0.49	0.52	0.67	0.53	0.60

Table 4
Critical stock S and predicted service level p for circuit pack A under the proposed method

Location	1998		1999	
	S	p (%)	S	p (%)
1	21	95.0	23	96.3
2	41	95.0	41	95.4
3	14	96.7	15	96.9
4	39	95.0	28	95.3
5	13	95.8	9	96.4
6	19	95.9	14	96.1
7	10	95.0	12	97.0
8	2	97.9	2	98.1
9	3	98.9	3	99.1
10	39	95.0	33	95.7
11	2	98.8	2	98.8
12	21	95.6	22	96.0
Total	224	96.2	204	96.8

The above procedure for calculating ω and δ is repeated for all circuit packs with a failure history. The average value of ω is 0.49 (meaning that λ_0 is generally a rather pessimistic estimate of the real failure rate, possibly because of the conservatism of the series configuration assumption in estimating λ_0) and the average value of δ is 1.12. For new circuit packs with no available observations, these averages are used as approximate values of ω and δ in (11) and (12) respectively to obtain the parameters α, β of the initial prior distribution.

This method to determine the prior seems to be new; at least we have not seen similar applications of Bayesian techniques to forecast demand and determine the appropriate parameters of inventory systems like the one under consideration. The method should be applied with caution in cases like the one at hand, where it recommends that the initial estimate λ_0 of the failure rate be significantly deflated, as the values of ω and δ may be valid in aggregate but they are not necessarily appropriate for *all* new items. This means that for some items the proposed method may prove somewhat aggressive in its attempt to avoid excess inventory.

In order to evaluate the difference between the proposed Bayesian (PB) method and the CM and to understand how much of this difference is due to the choice of the initial prior, we compare the following three alternative approaches for setting stock levels of new items in the first two years:

(a) The current KT approach, using λ_0 in the first year and λ_2 in the second year. For brevity and ease of presentation, this approach will be denoted CM.
(b) The original Bayesian (OB) approach using the first alternative from Section 3 to specify the initial prior for the first year, which amounts to using $\omega = 1$ and $\delta = 2$ in (11) and (12), i.e., using an unadjusted λ_0 for the mean and $2\lambda_0$ to estimate the 95th percentile. This approach is denoted OB.
(c) The Bayesian approach with the new method for specifying the initial prior for the first year, using the initial estimate λ_0 with $\omega = 0.49$ and $\delta = 1.12$ in (11) and (12). This approach is denoted PB.

We use each of the above approaches to compute the critical stock levels to achieve at least a 95% service level for selected circuit packs (including type A), differing in the initially estimated λ_0, at various locations. The results for years 1998 and 1999 are summarized in Table 5. The table shows the installed circuit packs at the locations, n, the observed failures in 1998, r, and the critical stock levels S_{CM}, S_{OB} and S_{PB} by the alternative method CM, OB and PB respectively, for 1998 and 1999. All calculations for 1998 are done as for the case of new circuit packs, namely, without using any specific failure information other than the initial estimate λ_0; the two Bayesian methods use λ_0 with their respective ω and δ to specify the initial prior. Then, the critical stock levels for 1999 are computed by taking into account the r failures observed in 1998; the CM uses λ_2, while the Bayesian methods update the parameters α and β of the initial priors.

The most important general conclusion from Table 5 is that the proposed method (PB) recommends significantly lower stock levels of spare parts for the installed circuit packs than the CM. The reduction in total stock for all three circuit pack types is 11.4% for year 1998 (initial estimates and computations without any failure observations) and 14.5% for year 1999 (after accumulation and consideration of real failure data). For 1998 the reduction is certainly expected because the CM uses λ_0 as the failure rate, while the proposed method uses $0.49\lambda_0$ as the average failure rate and $1.12\lambda_0$ as the upper 95% limit on λ. For 1999, the stock reduction may again be attributed partly to the different initial estimates of the failure rate, but it is even higher than in 1998, thus proving that the estimate λ_2 is rather conservative. Comparing the stock reduction for different types of circuit packs, one observes that for type A, which has the highest initial estimate of the failure rate ($\lambda_0 = 81.5$ failures per 1000 units per year), the reduction is 9.5% (1998) to 11.4% (1999); for type B with $\lambda_0 = 68.3$ and low usage (n), the reduction is 20.3% (1998) to 24.3% (1999) and for type C with $\lambda_0 = 6.1$ and high usage the stock reduction is 11.3% (1998) to 14.6% (1999). Thus, no systematic relationship between either λ_0 or n and stock reduction is identified.

Table 5
Critical stock levels for circuit pack types A ($\lambda_0 = 81.5$), B ($\lambda_0 = 68.3$), C ($\lambda_0 = 6.1$) under the CM (S_{CM}), the OB method (S_{OB}) and the PB method (S_{PB}) for new items

Location	Circuit pack type A								Circuit pack type B								Circuit pack type C							
	n	r	1998			1999			n	r	1998			1999			n	r	1998			1999		
			S_{CM}	S_{OB}	S_{PB}	S_{CM}	S_{OB}	S_{PB}			S_{CM}	S_{OB}	S_{PB}	S_{CM}	S_{OB}	S_{PB}			S_{CM}	S_{OB}	S_{PB}	S_{CM}	S_{OB}	S_{PB}
1	1871	95	34	52	30	26	24	24	184	3	6	6	4	6	4	3	3584	15	8	10	6	8	7	6
2	4008	188	66	109	62	45	42	42	825	22	15	21	13	10	9	8	1920	0	5	6	4	5	3	2
3	1071	55	22	31	18	18	16	15	6	0	2	2	1	1	1	1	5887	30	11	14	9	12	10	10
4	3784	112	63	103	59	30	28	27	779	25	15	20	12	11	9	9	7678	19	13	18	11	9	8	8
5	1016	16	21	29	18	8	7	7	–	–	–	–	–	–	–	–	20992	186	30	44	26	45	41	41
6	1616	41	30	45	27	15	13	13	81	1	4	5	3	5	3	2	13695	42	21	30	18	15	13	13
7	752	43	16	22	14	15	13	13	464	28	10	13	8	12	10	9	31037	206	41	64	38	49	45	45
8	32	0	3	3	2	3	2	2	4	0	1	1	1	1	1	1	289	0	2	2	2	2	2	2
9	64	1	4	4	3	4	3	2	4	0	1	1	1	1	1	1	3218	3	7	9	6	7	4	3
10	3792	140	63	103	59	36	33	33	738	29	14	19	12	12	10	10	30134	55	40	62	37	18	16	16
11	24	0	2	2	2	3	2	2	–	–	–	–	–	–	–	–	1024	1	4	4	3	4	3	2
12	1840	90	34	51	30	26	23	23	453	25	11	13	8	11	9	9	29210	114	39	60	36	31	28	27
Total	19870	781	358	554	324	229	206	203	3538	133	79	101	63	70	57	53	148668	671	221	323	196	205	180	175

Although the number of spare parts that should be kept in inventory to provide the required customer service is significantly lower under the proposed method, it must be noted that the service levels used for the determination of S_{CM}, S_{OB} and S_{PB} are *predicted* service levels under the analysis at hand and consequently the service levels from different analyses are not directly comparable. This essentially means that, according to the proposed method, the suggested values S_{PB} suffice to provide the required 95% service level, while the higher base stocks S_{CM} result in service levels that are higher than necessary and higher than those predicted by the CM.

The comparison between the respective S_{OB} and S_{PB} in Table 5 reveals the effect of the choice of the initial prior. The S_{OB} values in 1998 are expectedly higher, as the OB method uses much larger estimates of the mean and 95th percentile of the failure rate. In fact, the S_{OB} values for 1998 are even larger than the respective S_{CM} values, due to the high value of $\delta = 2$ (recall that $\omega = 1$ for OB, i.e., the average failure rate for 1998 under OB coincides with the initial estimate λ_0, which is also used by the CM). Despite the large differences between S_{OB} and S_{PB} in 1998, the S_{OB} and S_{PB} values become very similar in 1999, after updating the parameters of the initial priors using the failure data of 1998, signifying that the effect of the initial prior distribution of the failure rate diminishes quickly.

A final comment concerns the calculation of stock for items with a failure history, under the PB approach. When the critical stock levels of type A at the 12 locations were computed for 1998 taking into account the available data from 1994 to 1997, the resulting total stock (Table 4) was only 224 spare parts. When these stock levels were computed for 1998 by the proposed method but ignoring the previous failure data, the total stock (Table 5) was much higher, namely 324 spare parts. The difference is explained solely by the larger δ in the second case (1.12 as opposed to 0.7 in the first case), since the ω values are practically identical in the two cases ($\omega \approx 0.5$). The unnecessary 45% stock increase in the second case underscores the importance of exploiting the specific failure history, when it exists. It is worth noting, though, that in both cases the total stock for the following year (using the observations from 1998) is almost the same, confirming once again the robustness of the Bayesian updating to the parameters of the initial prior Gamma distribution of λ.

5. Conclusion

In this paper we have presented a case study of applying a Bayesian approach to forecasting the demand for spare parts of electronic equipment, with the objective of a more accurate determination of stock levels required to provide a negotiated service level to the users of the equipment. The proposed method is not much more complicated than the one currently in use by the company manufacturing and providing the spare parts. Using the same form of inventory control policy, i.e., the $(S-1,S)$ system, the Bayesian method results in lower total stock for achieving the required level of service. The most pronounced effect of the PB procedure is during the initial usage of new items for which no failure data is available. We reiterate, though, that the initial prior distribution must be used with caution, because it is based on aggregate data from similar items and consequently it may not be very accurate for all new items.

The method presented in this paper works under the assumption that the failure data originate from a stationary process (apart from the number of units installed in the field, for which an easy correction can be made). One should always verify this assumption. In one case we had data indicating a tenfold lower failure rate than the prior. In the Bayesian updating the posterior mean lay between the mean of the prior and the average of the data and the variance of the posterior was lower than that of the prior, because of the additivity in the updating formulas (9) and (10). This result, however, was surprising and counterintuitive (as we expected the variance to increase), yet a consequence of the stationarity assumption. Later on, the data appeared to contain errors and the correct data was in line with the prior. A similar issue is a possible ageing of the units. If that occurs one might regard old data as less important than new data and do not add the number of observations, but consider weighting the old values with a factor that decreases with time.

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References

Carlin, B.P., Louis, T.A., 1996. Bayes and Empirical Bayes Methods for Data Analysis. Chapman and Hall, London.

Hill, R., 1999. Bayesian decision-making in inventory modeling. IMA Journal of Maths Applied in Business and Industry 10, 147–163.

Kaplan, A.J., 1988. Bayesian approach to inventory control of new parts. IIE Transactions 20, 151–156.

Kapur, K.C., Lamberson, L.R., 1977. Reliability Engineering Design. John Wiley and Sons, New York.

MIL-HDBK-217F, Military Handbook, Reliability prediction of electronic equipment, Department of Defense, Washington DC, 20301.

Sherbrooke, C.G., 1992. Optimal Inventory Modelling of Systems. John Wiley and Sons, New York.