Problem 1

freq (A) = 0.875
freq (a) = 0.125

4 birds → 2·4 = 8 alleles

P of having 0 allele a → 0.344

$P = \frac{2 \cdot 4!}{8!(2 \cdot 4 - 8)!} \cdot 0.875^8 \cdot 0.125^0 = 0.344$

P of having 0 allele A → $5.9 \cdot 10^{-8}$

$P = \frac{2 \cdot 4!}{8!(2 \cdot 4 - 8)!} \cdot 0.125^8 \cdot 0.875^0 = 5.9 \cdot 10^{-8}$

P of having both alleles = 1 - P(no A) - P(no a) =
$= 1 - 5.9 \cdot 10^{-8} - 0.344 = 0.655$

Problem 2

N = 12
lose 90 %.

$H_0 = 1$
$H_t = 1 - 0.9 = 0.1$

$$\frac{H_t}{H_0} = \left(1 - \frac{1}{2N}\right)^t ; \quad \ln\left(\frac{H_t}{H_0}\right) = t \cdot \ln\left(1 - \frac{1}{2N}\right) ;$$

$$\ln\left(\frac{0.1}{1}\right) = t \cdot \ln\left(1 - \frac{1}{2 \cdot 12}\right) ; \quad t = 54.1$$

It will take 54 generations. with a pop of 12 indiv.

N = 240

$$\ln\left(\frac{0.1}{1}\right) = t \cdot \ln\left(1 - \frac{1}{2 \cdot 240}\right) ; \quad t = 1104.1$$

It will take 1104 generations with a pop of 240 indiv

Problem 3

$N = 95$ $\quad P_{fix} = P_0 = \frac{1}{2N} = \frac{1}{2 \cdot 95} = 0.00526$

$P_{loss} = 1 - P_{fix} = 1 - 0.00526 = 0.9947$

$T_{fix} = 4N = 4 \cdot 95 = 380$ generations

$T_{loss} = 2 \ln (2 \cdot 95) = 10.5$ generations ≈ 10

Problem 4

$p = \text{freq}(A) = 0.75$

$Ne = 40$

$P = \frac{2N!}{K!(2N-K)!} p^K q^{2N-K} = \frac{(2 \cdot 40)!}{60!(2 \cdot 40 - 60)!} 0.75^{60} 0.25^{2 \cdot 40 - 60} = 0.1025$

$K = 2 \cdot 40 \cdot 0.75 = 60$

Problem 5

$N = 12 \quad p_0 = 0.5 \quad q_0 = 0.5$

Generation 1 $\quad p_1 = 0.458 \quad q_1 = 1 - 0.458 = 0.542$

$P = \frac{2N!}{K!(2N-K)!} p^K q^{2N-K} = \frac{(2 \cdot 12)!}{12!(2 \cdot 12 - 12)!} 0.458^{12} 0.542^{2 \cdot 12 - 12} = 0.148$

$K = 12 \cdot 2 \cdot 0.5 = 12$

$p_2 = 0.583 \quad q_2 = 1 - 0.583 = 0.417$

$P = \frac{(2 \cdot 12)!}{12!(2 \cdot 12 - 12)!} 0.583^{12} \cdot 0.417^{2 \cdot 12 - 12} = 0.115$

$p = 0.375$

$K = 2 \cdot 12 \cdot 0.375 = 9$

$$P = \frac{24!}{9!\,15!}\, 0.375^{9} \cdot (1-0.375)^{15} = 0.166$$

$P(p = 0.375)$ in the next generation is 0.166

Problem 6

$2N = 4$ $\quad (0.25 \cdot 0.4219) + (0.25 \cdot 0.375) + (0.25 \cdot 0.469) =$

$= 0.21095$

Problem 7

Total = 55000

$$Ne = \frac{4 N_m N_f}{N_m + N_f} = \frac{4 \cdot 1100 \cdot 53900}{55000} = 4312$$

$$N_m = \frac{55000}{50} = 1100$$

$N_f = 55000 - 1100 = 53900$

$$N_m = \frac{20}{50} = 0.4 \qquad N_f = 20 - 0.4 = 19.6$$

$$Ne = \frac{4 \cdot 0.4 \cdot 19.6}{20} = 1.568$$

$H_0 = 0.028$ $\quad$ 5 generations

$$H_t = H_0 \left(1 - \frac{1}{2N}\right)^{t} = 0.028 \left(1 - \frac{1}{2 \cdot 20}\right)^{5} = 0.025$$

Problem 8

N: 104 → 62 → 10 3 generations

N = 110 in the 4th generation

$$\frac{1}{Ne} = \frac{1}{t}\left(\frac{1}{N_0} + \frac{1}{N_1} + \frac{1}{N_2} + \frac{1}{N_3}\right); \quad \frac{1}{Ne} = \frac{1}{4}\left(\frac{1}{104} + \frac{1}{62} + \frac{1}{10} + \frac{1}{100}\right);$$

Ne = 29.5 ≈ 29

Problem 1

A → a $\mu = 0.00002$ $P(A) = 0.5 = P_0$

$P_{10} = 0.5(1-0.00002)^{10} = 0.4999$
$P_{100} = 0.5(1-0.00002)^{100} = 0.499$
$P_{1000} = 0.5(1-0.00002)^{1000} = 0.49$
$P_{10000} = 0.5(1-0.00002)^{10000} = 0.4$

The freq of A decreases slowly over time.

$$P_t = P_0(1-\mu)^t$$

Problem 2

$\mu = 0.005$ $p = 0.8$
$v = 0.0015$ $q = 0.2$

$$P_t = \frac{v}{\mu+v} + \left(P_0 - \frac{v}{\mu+v}\right)(1-\mu-v)^t$$

$$q_1 = \frac{0.0015}{0.005+0.0015} + \left(0.2 - \frac{0.0015}{0.005+0.0015}\right)(1-0.005-0.0015)^1 = 0.2002$$

$q_2 = 0.2004$ $q_3 = 0.2006$

$$\hat{p} = \frac{v}{q+v} \qquad \hat{p} = \frac{0.0015}{0.005+0.0015} = 0.2307 \qquad \hat{q} = 1-\hat{p} = 0.7692$$