Problem 1

Wild → 58 a
Stubble → 42 A (lethal AA)

$freq(a) = \frac{2 \cdot 58 + 42}{100 \cdot 2} = 0.79 = p$

$freq(A) = 1 - p = 1 - 0.79 = 0.21 = q$

- Before the selection (zygotes)

$aa \rightarrow p^2 = 0.79^2 = 0.6241$
$Aa \rightarrow 2pq = 2 \cdot 0.79 \cdot 0.21 = 0.3318$
$AA \rightarrow q^2 = 0.21^2 = 0.411$

- After the selection (adults)

	aa	Aa	AA
Zygotes	58	42	x
Adult	58	42	0
Absolute fitness	1	1	0
Relative fitness	1	1	0

$$\left.\begin{aligned} p^2 W_{aa} &= 0.6241 \cdot 1 = 0.6241 \\ 2pq W_{Aa} &= 0.3318 \cdot 1 = 0.3318 \\ q^2 W_{AA} &= 0.411 \cdot 0 = 0 \end{aligned}\right\} \quad \bar{W} = 0.6241 + 0.3318 = 0.956$$

$aa \rightarrow \frac{0.6241}{0.956} = 0.653$ $\qquad p' = 0.653 + \frac{0.347}{2} = 0.827$

$Aa \rightarrow \frac{0.3318}{0.956} = 0.347$ $\qquad q' = 0 + \frac{0.347}{2} = 0.174$

$AA \rightarrow 0$

The frequency of the recessive allele will increase in each generation. Every time there will be more wild flies.

Problem 2

$p(A) = 0.5$ $AA = p^2 = 0.25$ $Aa = 0.5$

$q(a) = 0.5$ $aa = q^2 = 0.25$

- 70%

	aa	Aa	AA
zygotes	50	100	50
adults	15	100	50
W	0.3	1	1
w	0.3	1	1

assume N = 200

→ Before selection

AA → 0.25 aa → 0.25 Aa → 0.5

→ After selection (gen 1)

$$\left.\begin{aligned} p^2 W_{AA} &= 0.25 \cdot 1 = 0.25 \\ 2pq\, W_{Aa} &= 0.5 \cdot 1 = 0.5 \\ q^2 W_{aa} &= 0.25 \cdot 0.3 = 0.075 \end{aligned}\right\} \bar{w} = 0.825$$

$AA \rightarrow \frac{0.25}{0.825} = 0.3$ $p' = 0.3 + \frac{0.6}{2} = 0.6$

$Aa \rightarrow \frac{0.5}{0.825} = 0.6$

$q' = 0.1 + \frac{0.6}{2} = 0.4$

$aa \rightarrow \frac{0.075}{0.825} = 0.1$

→ After selection (gen 2)

$$\left.\begin{aligned} p^2 W_{AA} &= 0.6^2 \cdot 1 = 0.36 \\ 2pq\, W_{Aa} &= 2 \cdot 0.6 \cdot 0.4 \cdot 1 = 0.48 \\ q^2 W_{aa} &= 0.4^2 \cdot 0.3 = 0.048 \end{aligned}\right\} \bar{w} = 0.888$$

AA → 0.405 $p'' = 0.675$

Aa → 0.54 $q'' = 0.324$

aa → 0.054

$W_{AA} = 1 - S_1$; $1 = 1 - S_1$; $S_1 = 0$

$W_{Aa} = 1$

$W_{aa} = 1 - S_2$; $0.3 = 1 - S_2$; $S_2 = 0.7$

Equilibnum → $\hat{p} = \frac{S_2}{S_1 + S_2} = \frac{0.7}{0 + 0.7} = 1$

In eq all alleles will be A, there will be no more a alleles.

- 30% (gen 1)

$w_{aa} = 0.7$ $\quad p^2 w_{aa} = 0.5^2 \cdot 0.7 = 0.175$ $\quad \bar{w} = 0.925$

$w_{Aa} = 1$ $\quad 2pq\, w_{Aa} = 2 \cdot 0.5 \cdot 0.5 \cdot 1 = 0.5$

$w_{AA} = 1$ $\quad q^2 w_{AA} = 0.5^2 \cdot 1 = 0.25$

aa → $0.175 / 0.925 = 0.189$ $\quad p' = 0.54$

Aa → 0.540 $\quad q' = 0.459$

AA → 0.270

(gen 2)

$$\left.\begin{array}{l} p^2 w_{AA} = 0.29 \\ 2pq\, w_{Aa} = 0.496 \\ q^2 w_{aa} = 0.147 \end{array}\right\} \bar{w} = 0.933$$

AA → 0.31 $\quad p'' = 0.575$

Aa → 0.531 $\quad q'' = 0.4225$

aa → 0.157

The equilibnum will be reached faster in 70% killed. we can see that the frequencies of the alleles change more when killing 70%.

Problem 3

$s \rightarrow 0.12 = q$
$A \rightarrow 1 - 0.12 = 0.88 = p$

$w_{AA} = 0.88$ $\quad s_1 = 1 - w_{AA}$; $s_1 = 0.12$
$w_{As} = 1$
$w_{ss} = 0.14$ $\quad s_2 = 1 - w_{ss}$; $s_2 = 0.86$

$$\hat{p} = \frac{s_2}{s_1 + s_2} = \frac{0.86}{0.12 + 0.86} = 0.877 \simeq 0.88$$

$$\hat{q} = 1 - \hat{p} = 0.122 \simeq 0.12$$

The pop seems to be in equilibrium

$$2\hat{p}q = 2 \cdot 0.877 \cdot 0.12 = 0.21$$

$$\bar{w} = (0.877^2 \cdot 0.88) + (0.21 \cdot 1) + (0.12^2 \cdot 0.14) = 0.903$$

Problem 4

- 1, 1, 0.75 $\quad$ Selection against recessive phenotype

$A_1A_1 \rightarrow 1$
$A_1A_2 \rightarrow 1 = 1 - hs$; $1 = 1 - h \cdot 0.25$; $h = 0$
$A_2A_2 \rightarrow 0.75 = 1 - s_1$; $s_1 = 0.25$

- 1, 0.6, 0.6 $\quad$ Selection against dominant phenotype

$A_1A_1 \rightarrow 1$
$A_1A_2 \rightarrow 0.6 = 1 - hs$; $0.6 = 1 - h \cdot 0.4$; $h = 1$
$A_2A_2 \rightarrow 0.6 = 1 - s$; $s = 0.4$

- 0.4, 0.8, 0.1 $\quad$ Intermediate dominance

$A_1A_1 \rightarrow 0.4 = 1 - s_1$; $s_1 = 0.6$
$A_1A_2 \rightarrow 0.8 = 1 - s_2$; $s_2 = 0.2$ $\quad 0.8 = 1 - 0.2h$; $h = 1$
$A_2A_2 \rightarrow 1$

• 0.8, 1, 0.55   heterozygote advantage

$A_1A_1 \rightarrow 0.8 = 1 - S_1$; $S_1 = 0.2$
$A_1A_2 \rightarrow 1$
$A_2A_2 \rightarrow 0.55 = 1 - S_2$; $S_2 = 0.45$

Problem 5

	TT	Tt	tt	
N	142	84	14	$\rightarrow p = (2 \cdot 142) + 84 / 480 = 0.76$ $q = 0.24$
A	128	66	2	$\rightarrow p = (2 \cdot 128) + 66 / 392 = 0.82$ $q = 0.18$
W	0.9	0.79	0.14	
w	1	0.87	0.15	(generation 0)

$p = freq(T)$ $q = freq(t)$

$$\left.\begin{array}{l} p^2 \cdot w_{TT} = 0.672 \\ 2pq\, w_{Tt} = 0.257 \\ q^2 w_{tt} = 4.86 \cdot 10^{-3} \end{array}\right\} \bar{w} = 0.934$$

$$\left.\begin{array}{l} TT \rightarrow 0.719 \\ Tt \rightarrow 0.275 \\ tt \rightarrow 5.2 \cdot 10^{-3} \end{array}\right\}$$ adults of the next generation 1

selection

$$\left.\begin{array}{l} TT \rightarrow p^2 = 0.672 \\ Tt \rightarrow 2pq = 0.295 \\ tt \rightarrow q^2 = 0.0324 \end{array}\right\}$$ newborns of the next generation 1

Problem 6

• 1: favored dominant A allele → selection against recessive phenotype.
Both pop same fitness. Pop 1 has higher freq A (favored allele). we expect freq (A) to be 1. will be achieved faster in pop 1.

• 2: Favored dominant A allele

p is the same in both pop. Pop 1 has a higher fitness (aa), so A will be fixed faster in pop 2.

• 3: Favored recessive a allele → selection against dominant phenotype.

Pop 2 will reach faster the eq bc it has a higher freq of a (which will be fixed)

• 4: Favored recessive a allele

we can see that pop 2 has a lower W_{AA}, which means that has a higher fitness for aa. This will lead to a higher fixation.

Problem 7

$p = A_1 = 0.4$

$q = A_2 = 0.6$ → 80% die → $s = 0.8$

	A_1A_1	A_1A_2	A_2A_2	Additive effect
w	1	$1 - \frac{0.8}{2} = 0.6$	0.2	$1 - s/2$

$$\left.\begin{aligned} A_1A_1 &\rightarrow 0.4^2 = 0.16 \\ A_1A_2 &\rightarrow 2 \cdot 0.4 \cdot 0.6 = 0.48 \\ A_2A_2 &\rightarrow 0.6^2 = 0.36 \end{aligned}\right\} \text{before select.}$$

$p' = 0.4$
$q' = 0.6$

$$\left.\begin{aligned} A_1A_1 &\rightarrow W_{11} = 0.16 \cdot 1 = 0.16 \rightarrow 0.307 \\ A_1A_2 &\rightarrow W_{12} = 0.48 \cdot 0.6 = 0.288 \rightarrow 0.554 \\ A_2A_2 &\rightarrow W_{22} = 0.36 \cdot 0.2 = 0.072 \rightarrow 0.138 \end{aligned}\right\} \text{after select.}$$

$\bar{w} = 0.52$

$$p' = 0.307 + \frac{0.554}{2} = 0.58$$

$$q' = 1 - p' = 0.42$$

Problem 8

	SS	SR	RR	
without	1	0.77	0.46	→ directional ↓ purifying (against R)
with	0.68	1	0.37	→ balanced (favoring SR)

• Without warfarin

$p = freq(S) = 0.8$
$q = freq(R) = 0.2$

$$\left.\begin{aligned} p^2 w_{SS} &= 0.64 \\ 2pq\, w_{SR} &= 0.246 \\ q^2 w_{RR} &= 0.0184 \end{aligned}\right\} \bar{w} = 0.9 \quad \xrightarrow{/\bar{w}} \quad \begin{aligned} &0.71 \\ &0.273 \\ &0.0204 \end{aligned}$$

$p' = 0.71 + \frac{0.273}{2} = 0.85 \qquad q' = 0.15$

• With warfarin

$$\left.\begin{aligned} p^2 w_{SS} &= 0.435 \\ 2pq\, w_{SR} &= 0.32 \\ q^2 w_{RR} &= 0.0148 \end{aligned}\right\} \bar{w} = 0.78 \quad \xrightarrow{/\bar{w}} \quad \begin{aligned} &0.56 \\ &0.41 \\ &0.019 \end{aligned}$$

$p' = 0.765 \qquad q' = 0.235$

without warfarin

1	0.77	0.46
1	$1-s$	$1-s$
	$s_1 = 0.23$	$s_2 = 0.54$

$\hat{p} = 0.701$
$\hat{q} = 0.3$

with warfarin

0.68	1	0.37
$1-s_1$	1	$1-s_2$
$s_1 = 0.32$		$s_2 = 0.63$

$\hat{p} = 0.66$
$\hat{q} = 0.34$

Problem 9

$\mu = 1 \cdot 10^{-5}$ $A \xrightarrow{\mu} a$

$w(PKU) = \frac{w(healthy)}{2}$

AA	Aa	aa
	1	0.5

$1-s$; $s = 0.5$

$\hat{q} = \sqrt{\frac{\mu}{s}} = \sqrt{\frac{10^{-5}}{0.5}} = 0.00447$

Frequency of a at equilibrium → 0.00447
Affected ind. (at eq) → $q^2 = 2 \cdot 10^{-5}$

AA	Aa	aa
	1	$0.8 = 1-s$; $s = 0.2$

$\hat{q} = \sqrt{\frac{10^{-5}}{0.2}} = 7.1 \cdot 10^{-3}$

$q^2 = 5 \cdot 10^{-5}$

The freq of the disease has increased bc the increase of fitness → less selection pressure against a alleles. Less people die bc of the allele

Problem 10

3/10000 mice are blind → $q^2 = 3 \cdot 10^{-4}$; $q = 0.0173$
$p = 0.983$

$\mu = \frac{q}{1-q} = 0.0176$

$p^2 + 2pq \cdot 0.8 + q^2 = 1$; $q^2 = 0.842$ $\mu = 0.0919$

Second calculation takes into account reduced hetero.
→ ↓ heterozyg. and ↑ recessive allele →
↑ in μ for the a allele.