Population Genetics problems

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Genetic drift

Problem 1

$freq(A) = 0.875 \quad Freq(a) = 0.125 \quad n=4$

- P that the new colony does not have allele a?

$$\frac{8!}{8!(8-8)!} \cdot 0.875^8 \cdot 0.125^0 = 0.344$$

- P that the new colony does not have allele A?

$$\frac{8!}{8!(8-8)!} \cdot 0.125^8 \cdot 0.875^0 = 5.9 \cdot 10^{-8}$$

- P that the new colony has both alleles?

$$1 - 0,344 - 5,9 \cdot 10^{-8} = 0.655$$

Problem 2

diploid population $n=12 \quad H_0 = 1 \quad H_t = 1-0.9 = 0.1$

$$\frac{H_t}{H_0} = \left(1-\frac{1}{2N}\right)^t \rightarrow \ln\left(\frac{H_t}{H_0}\right) = t \cdot \ln\left(1-\frac{1}{2N}\right) \rightarrow t = \frac{\ln\left(\frac{H_t}{H_0}\right)}{\ln\left(1-\frac{1}{2N}\right)} \rightarrow t = \frac{\ln\left(\frac{0.1}{1}\right)}{\ln\left(1-\frac{1}{24}\right)} = 54,1$$

It will take $\simeq 54$ generations

if $n=240$:

$$t = \frac{\ln\left(\frac{0.1}{1}\right)}{\ln\left(1-\frac{1}{480}\right)} = 1.104,1$$ It will take $\simeq 1.104$ generations

Problem 3

$n=95 \quad P_{fix} = \frac{1}{190} = 0.00526 \quad P_{loss} = 1 - 0.00526 = 0.9947$

Average time of Fixation: $4 \cdot N = 95 \cdot 4 = 380$ generations

Average time to loss $= 2\ln(190) = 10.5 \simeq 10$ generations

Problem 4

$p = freq(A) = 0.75 \quad P = \frac{2N!}{k!(2N-k)!} p^k \cdot q^{2N-k} \rightarrow P = \frac{80!}{60!(20)!} \cdot 0.75^{60} \cdot 0.25^{20} = 0.1025$

$Ne = 40$

$k = 40 \cdot 2 \cdot 0,75 = 60$

Problem 5

$N=12$ — 6 males, 6 females

Generation 1. $p_1 = 0.458$ $q_1 = 1-0.458 = 0.542$

$p = freq(a) = 0.5$
$q = freq(A) = 1-0.5$

$P = \frac{2N!}{k!(2N-k)!} p^k q^{2N-k} = \frac{2\cdot 12!}{12!(12!)} \cdot 0.458^{12} \cdot 0.542^{12} = 0.148$

$k = 12\cdot 2\cdot 0.5 = 12$

$p_2 = 0.583$ $q_2 = 1-0.583 = 0.417$

$P = \frac{(2\cdot 12)!}{12!(12!)} \cdot 0.583^{12} \cdot 0.417^{12} = 0.115$

$k = 12\cdot 2\cdot 0.375 = 9$ $P = \frac{24!}{9!15!} \cdot 0.375^{9} \cdot 0.625^{15} = 0.166$

The Probability of allelle frequency is $p=0,375$ in the next generation is 0.166

Problem 6

$N=2$
$2N=4$

$(0.25\cdot 0.429) + (0.25\cdot 0.375) + (0.25\cdot 0.469) = 0,21095$

Problem 7

Total = 55.000

$Ne = \frac{4 Nm NF}{Nm + NF} = \frac{4\cdot 1100\cdot 53900}{55000} = 4312$

$Nm = \frac{55000}{50} = 1100$

$NF = 55000 - 1100 = 53900$

$Nm = \frac{20}{50} = 0,4$

$NF = 20 - 0.4 = 19.6$

$Ne = 4\cdot 0,4\cdot 19.6 = 1,568$

$H_0 = 0.028$ 5 generations

$H_r = H_0\left(1-\frac{1}{2N}\right)^r \rightarrow 0.028\left(1-\frac{1}{2\cdot 20}\right)^5 = 0.025$

Problem 8

4 generations

Bottleneck $N: 104 \to 62 \to 10 \to 110$

$$\frac{1}{Ne} = \frac{1}{t}\left(\frac{1}{N_0}+\frac{1}{N_1}+\frac{1}{N_2}+\frac{1}{N_3}\right) : \frac{1}{NE} = \frac{1}{4}\left(\frac{1}{104}+\frac{1}{62}+\frac{1}{10}+\frac{1}{100}\right)$$

$Ne = 29.5 \simeq 29$

Mutation

Problem 1

$A \to a \quad \mu = 0.00002 \quad P(A_0) = 0.5$

$P_{10} = 0.5(1-0.00002) = 0.4999 \quad P_{100} = 0.5(1-0.00002)^{100} = 0.499 \quad P_{1000} = 0.5(1-0.00002)^{1000} = 0.49$

$P_{10000} = 0.5(1-0.00002)^{10000} = 0.4$

The freq of A will decrease over time.

Problem 2

$\mu = 0.005$
$v = 0.0015$

$p = 0.8$
$q = 0.2$

$$P_t = \frac{v}{\mu+v} + \left(P_0 - \frac{v}{\mu+v}\right)\cdot(1-\mu-v)^t$$

$$q_1 = \frac{0.0015}{0.0020} + \left(0.2 - \frac{0.0015}{0.005+0.0015}\right)(1-0.005-0.0015)^1 = 0.2002$$

$q_2 = 0.2004 \quad q_3 = 0.2000$

$\hat{p} = \frac{v}{q+v} \quad \hat{p} = \frac{0.0015}{0.005+0.0015} = 0.2307 \quad q = 1-\hat{p} = 0.7692$