Biological Networks

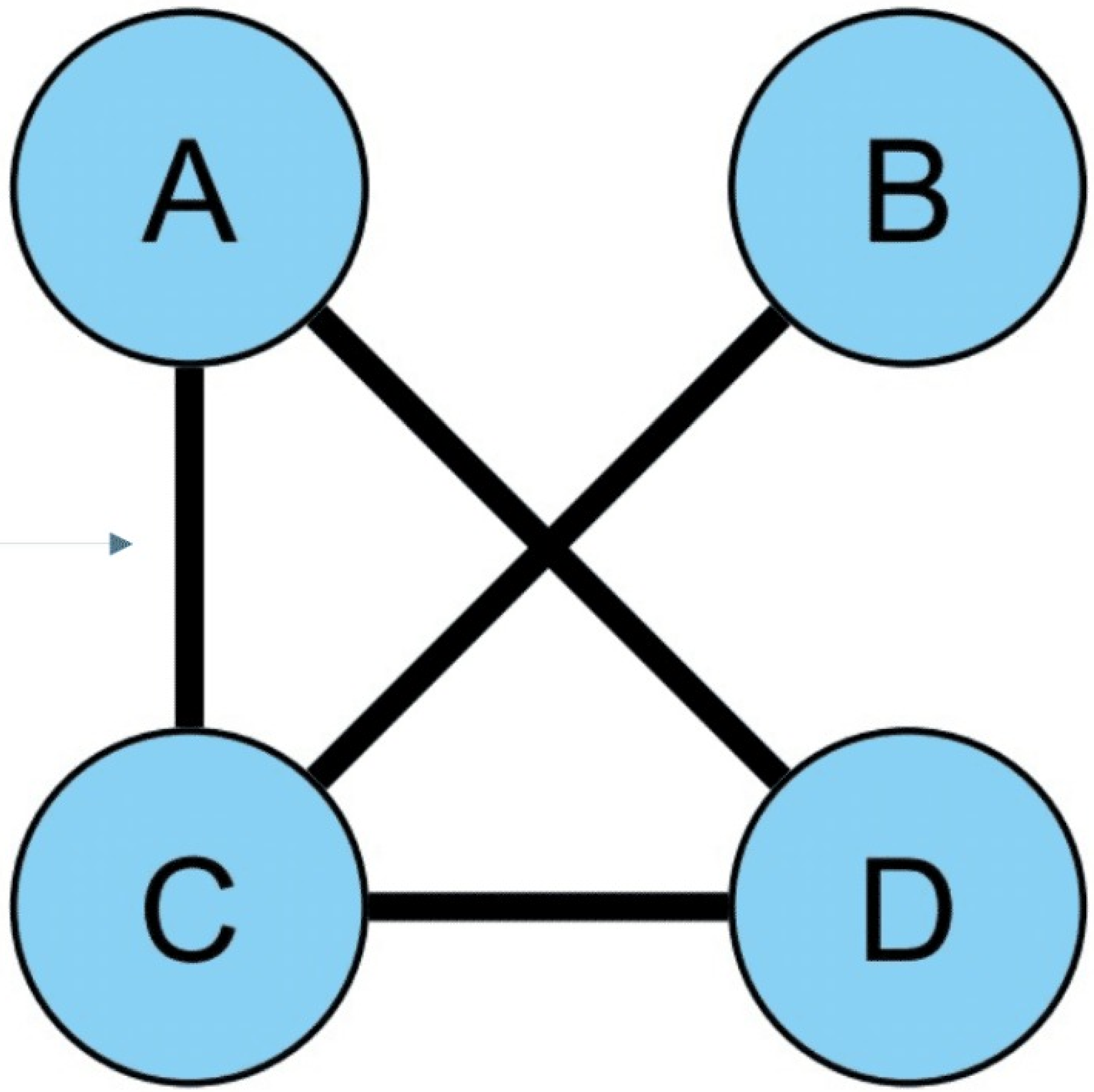
Systems & Network Biology

17 Sep 2024

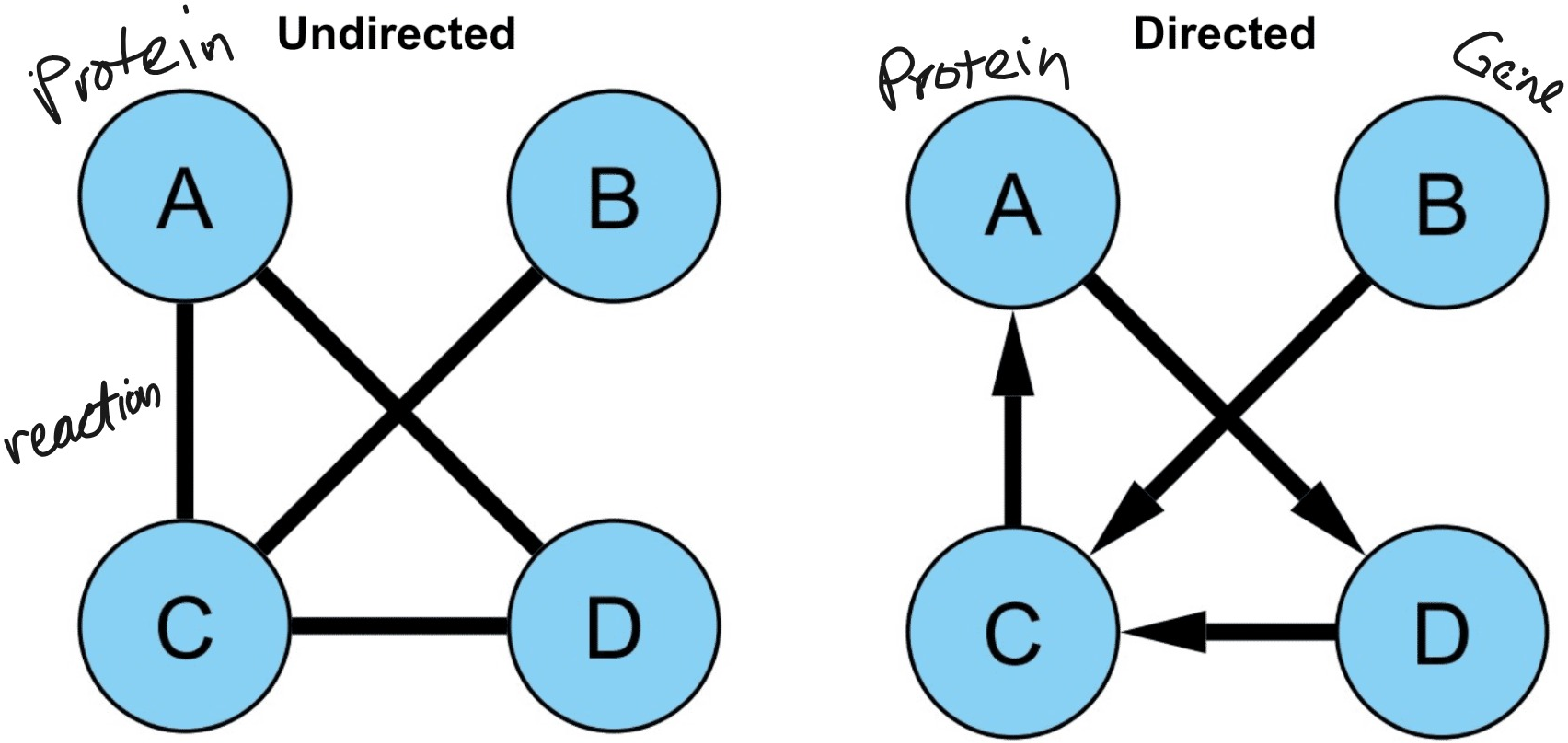
"Graph"

Network Theory

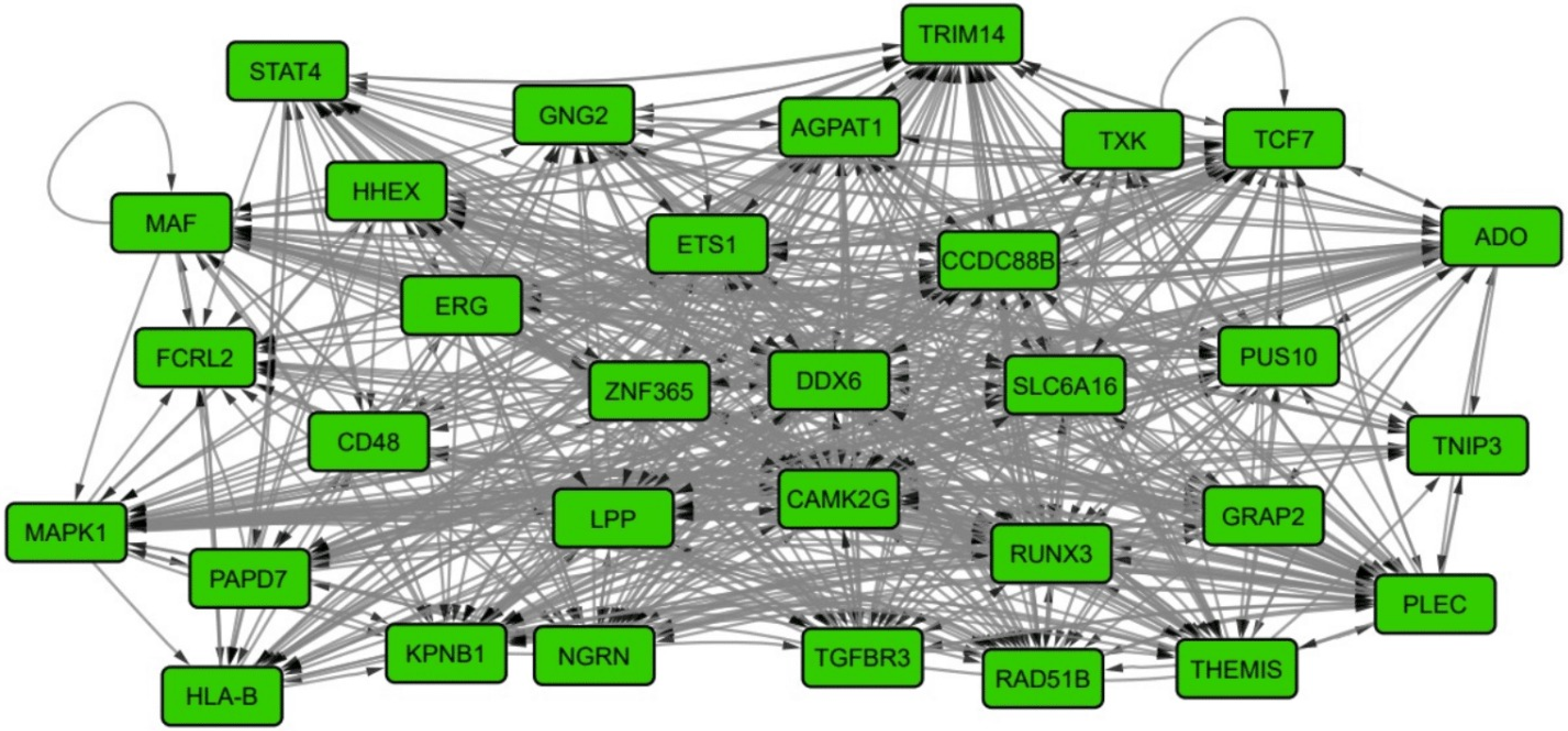
Node
vertex
site
actor
Edge
links
bonds
ties



4 nodes
4 edges
6 possible
$\rho = \frac{2}{3} = 66\%$

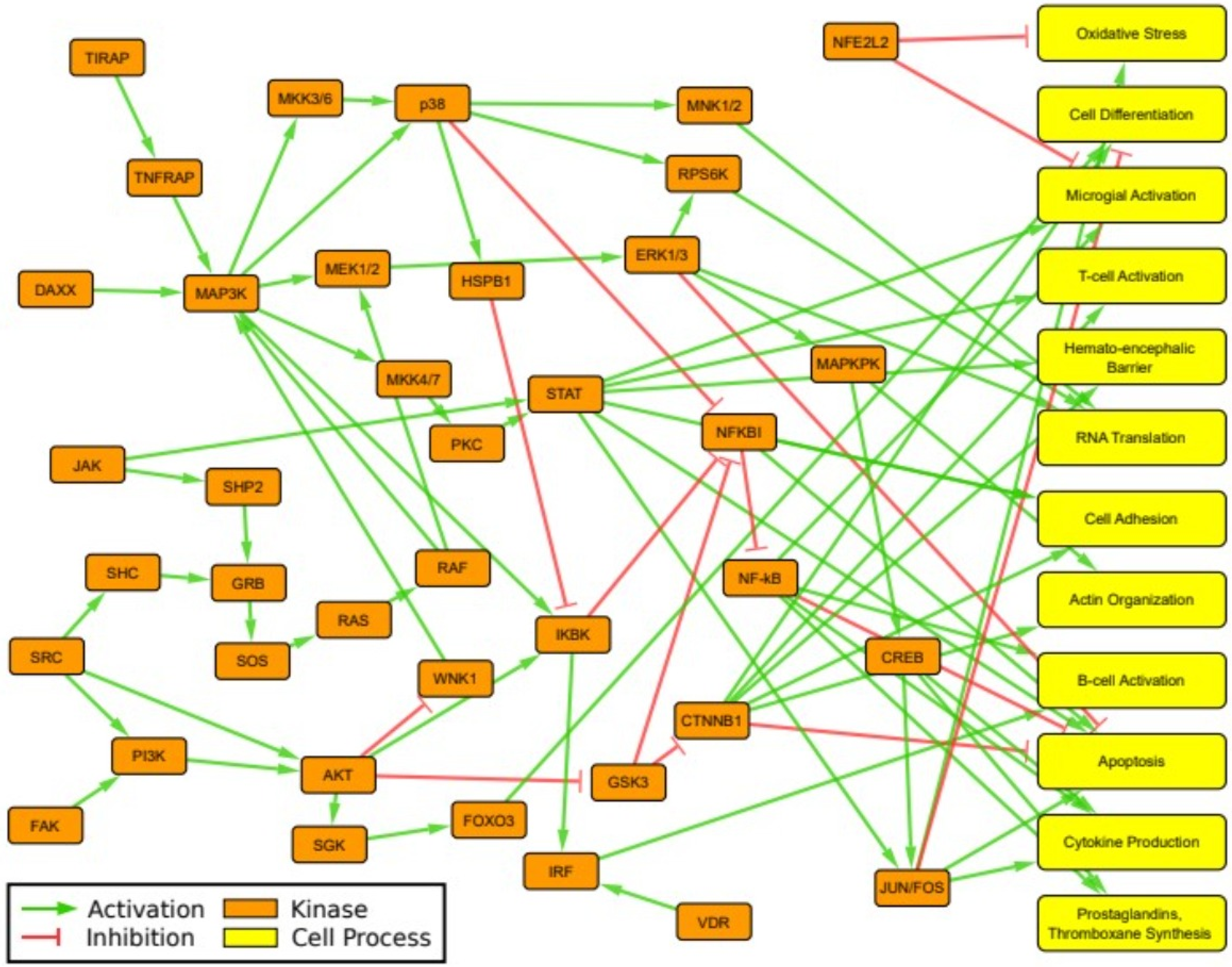

Undirected
Protein
A
B
reaction
C
D
Directed
Protein
Gene
A
B
C
D

Gene Regulatory Network



Fang et al. Nucleic Acids Research, vol. 49, no. D1, pp. D97–D103, 2021.

Protein Signaling Network

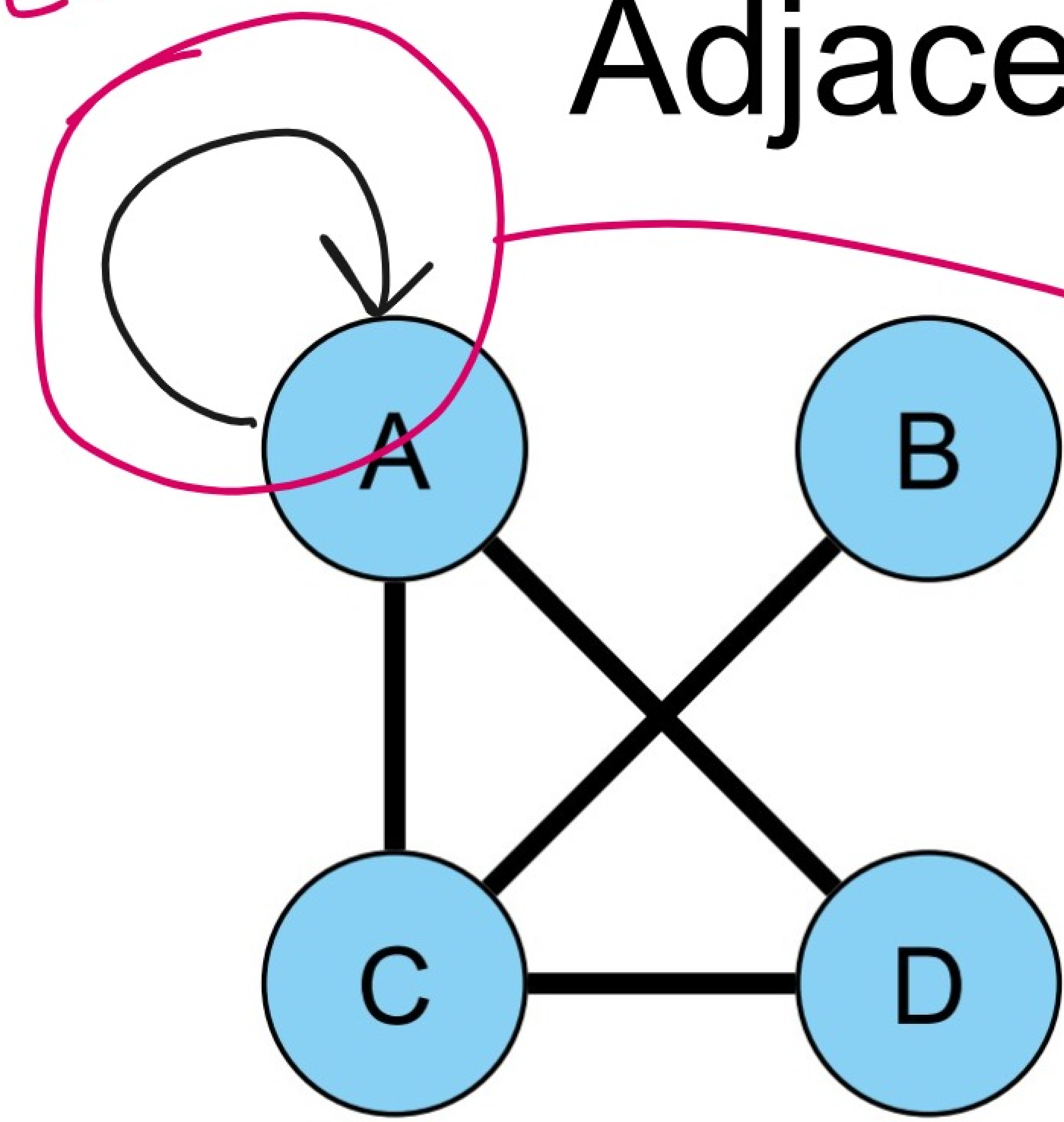


Kotelnikova et al. Proc Natl Acad Sci U S A. 2019 May 7; 116(19):9671-9676.

Adjacency Matrix

self link

undirected

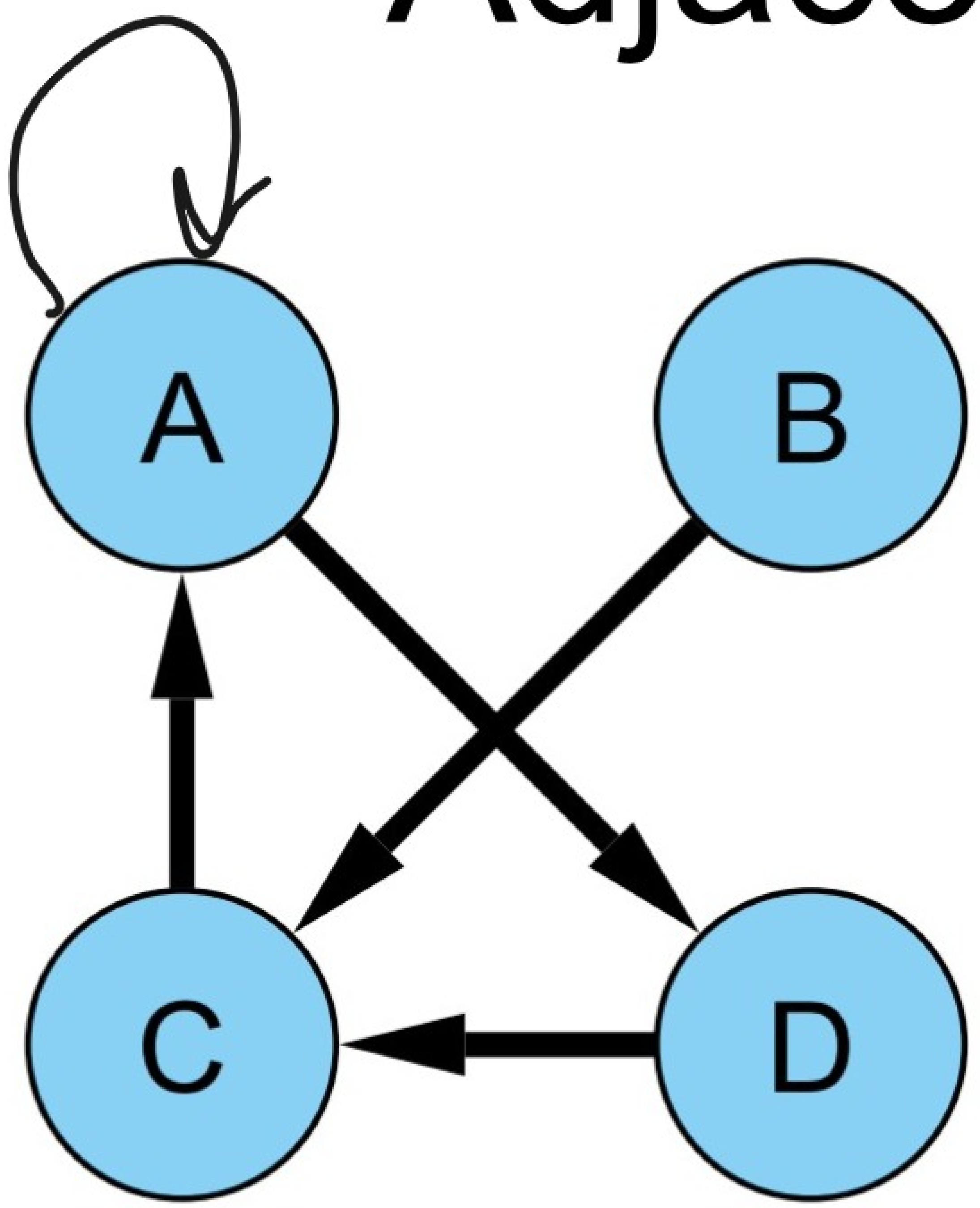


A-C, A-D, C-B, C-D

	A	B	C	D
A	2	0	1	1
B	0	0	1	0
C	1	1	0	1
D	1	0	1	0

Adjacency Matrix

directed

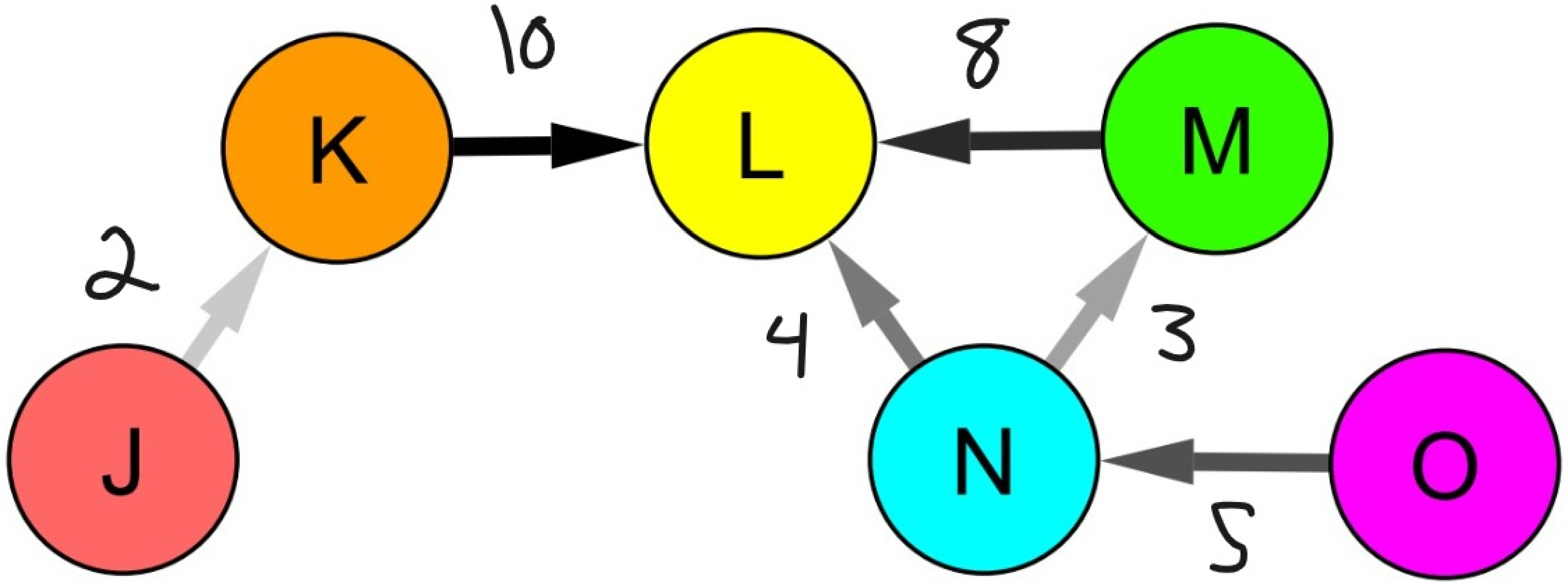


Targets

Sources

	A	B	C	D
A	1	0	0	1
B	0	0	1	0
C	1	0	0	0
D	0	0	1	0

Weighted Directed Network



Network Properties - Undirected

Degree - number of links that a node has

K = degree

m = edges

c = avg deg

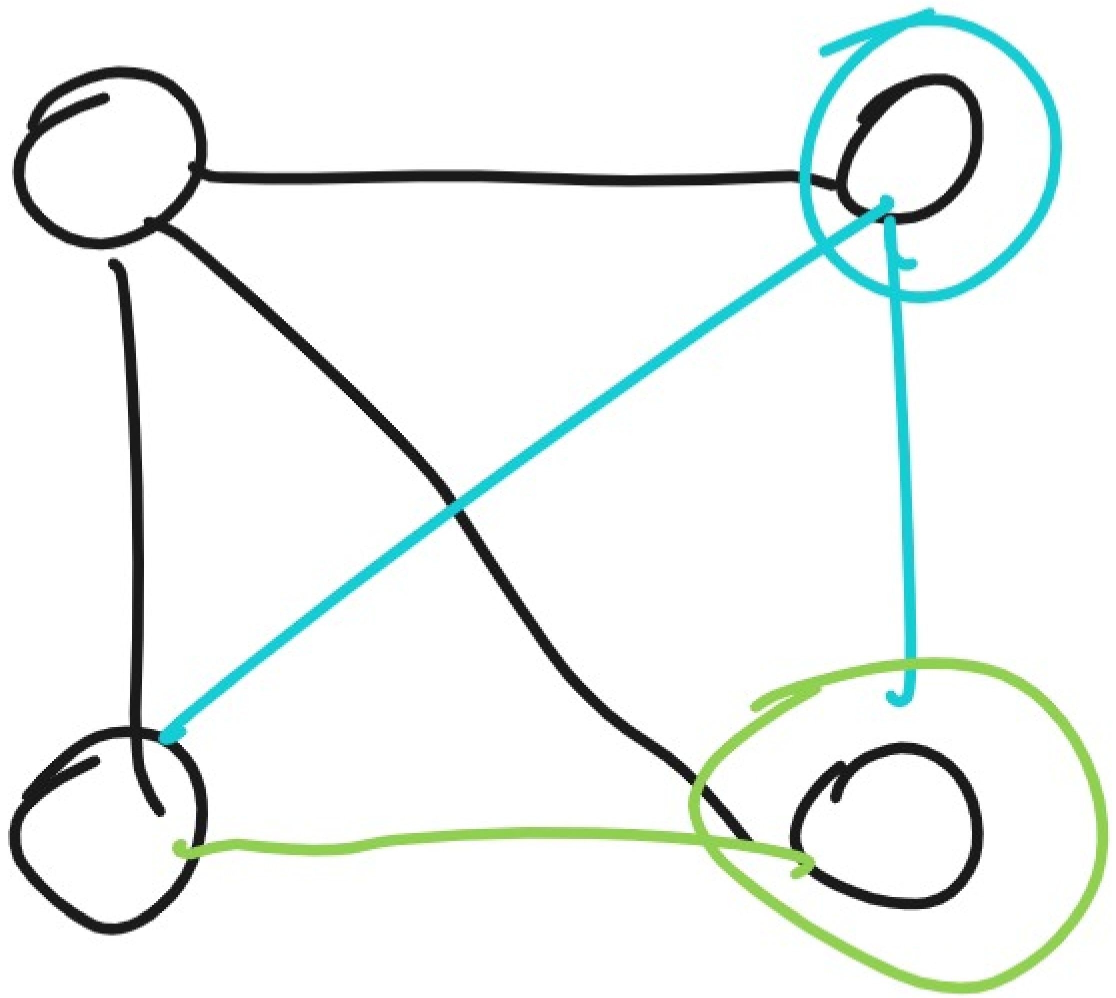
n = # nodes

$$K_i = \sum_{j=1}^{n} A_{ij}$$

$$2m = \sum_{i=1}^{n} K_i$$

$$m = \frac{1}{2} \sum_{i=1}^{n} K_i$$

$$c = \frac{1}{n} \sum_{i=1}^{n} K_i = \frac{2m}{n}$$



4 nodes

$3+2+1$

M = max # of edges

$$M = \binom{n}{2} = \frac{1}{2}n(n-1)$$

ρ = density

$$\rho = \frac{m}{M} = \frac{m}{\frac{1}{2}n(n-1)} = \frac{2m}{n(n-1)} = \frac{c}{n-1}$$

$$\rho \simeq \frac{c}{n}$$

large n

as n increase

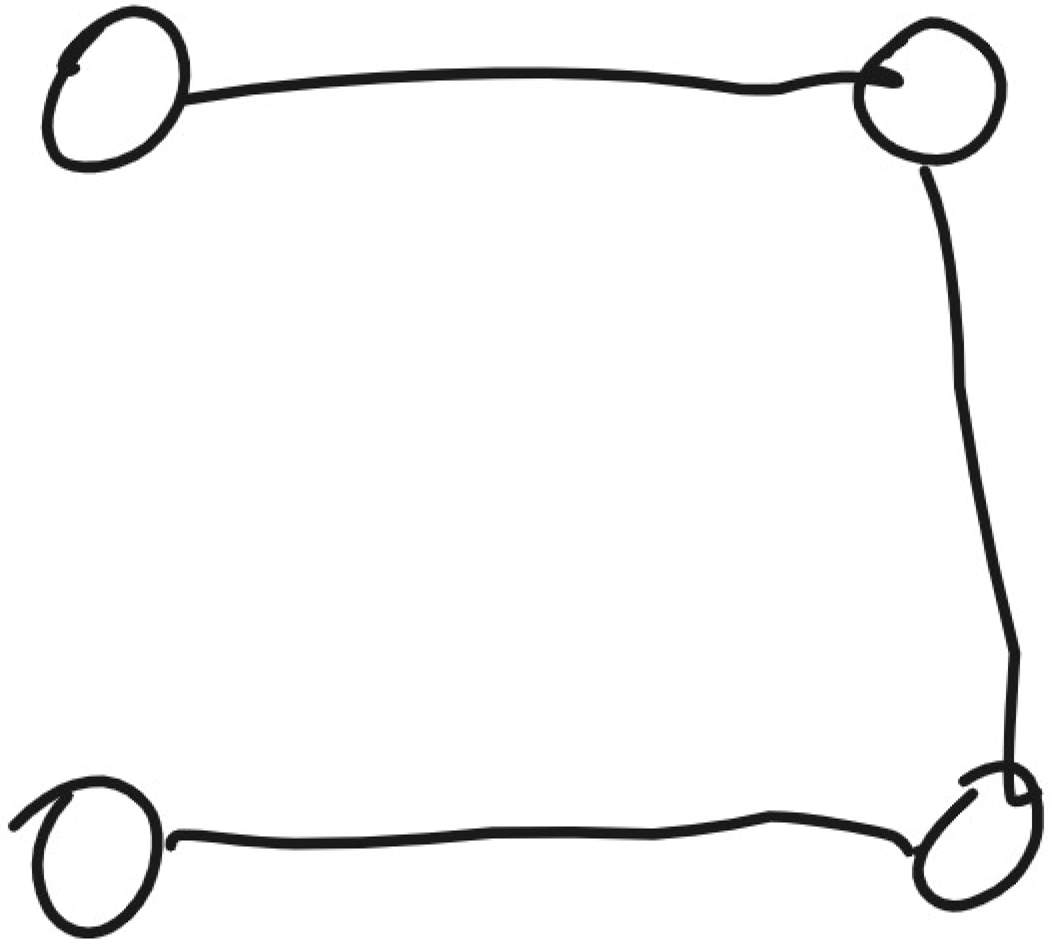
dense - p tends to a constant

sparse - p tends to 0

Erdos-Renyi random graphs

n = number of nodes

p = probability two nodes connected



$n = 4$

$p = 50\%$

$M = 6$

$$P(m) = \binom{M}{m} p^m (1-p)^{M-m}$$

\# of edges

number of combos

prob of m pairs connected

prob of rest NOT connected

$$\langle m \rangle = \sum_{m=0}^{M} m P(m) = Mp$$

expected m

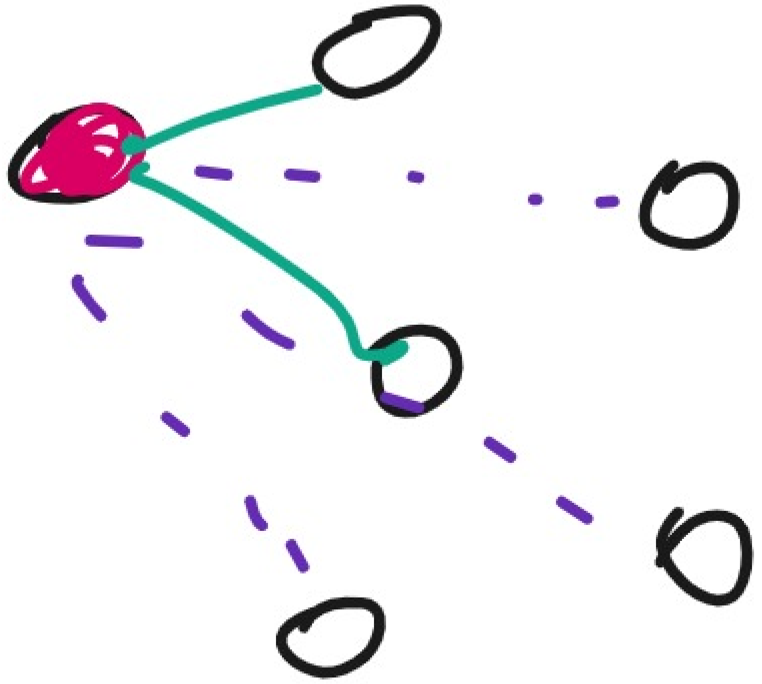
$n = 100$ $M = \frac{1}{2}(99)(100) = 4950$

$p = 0.2$ $\langle m \rangle = 990$

$$C \equiv \langle k \rangle = \sum_{m=0}^{M} \frac{2m}{n} P(m) = \frac{2}{n} \underbrace{M}_{\frac{1}{2}n(n-1)} p = (n-1)p$$

$$P(k) = \underbrace{\binom{n-1}{k}}_{\text{\# of combos}} \underbrace{p^{k}}_{\text{prob of } k \text{ neighbors}} \underbrace{(1-p)^{(n-1)-k}}_{\text{prob of NOT connected to others}}$$

prob of degree k



c = constant

$n \to \infty$

$c = (n-1)p$

$p = \frac{c}{n-1}$

$$\ln\left[(1-p)^{n-1-k}\right] = (n-1-k)\ln\left(1-\frac{c}{n-1}\right)$$

$\ln(1-x) \simeq -x$

$$\simeq -(n-1-k)\left(\frac{c}{n-1}\right) = -\left(\frac{n-1-k}{n-1}\right)c = -c$$

1

$$P(K) = \frac{(n-1)^K}{K!}\left(\frac{c}{n-1}\right)^K e^{-c} = \frac{c^K}{K!}e^{-c}$$

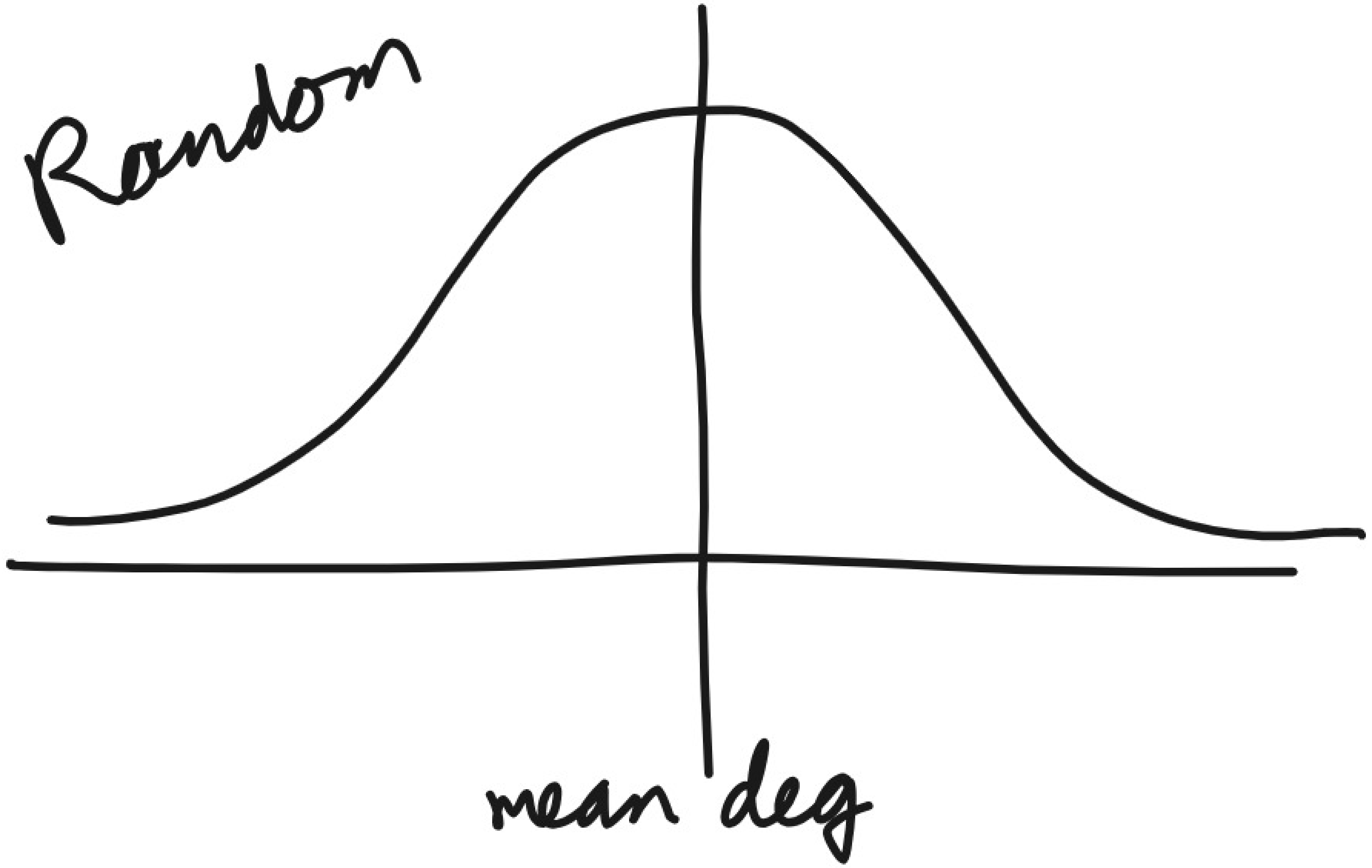
Random Graphs

↳ Degree Distribution

Binomial Distribution (all n)

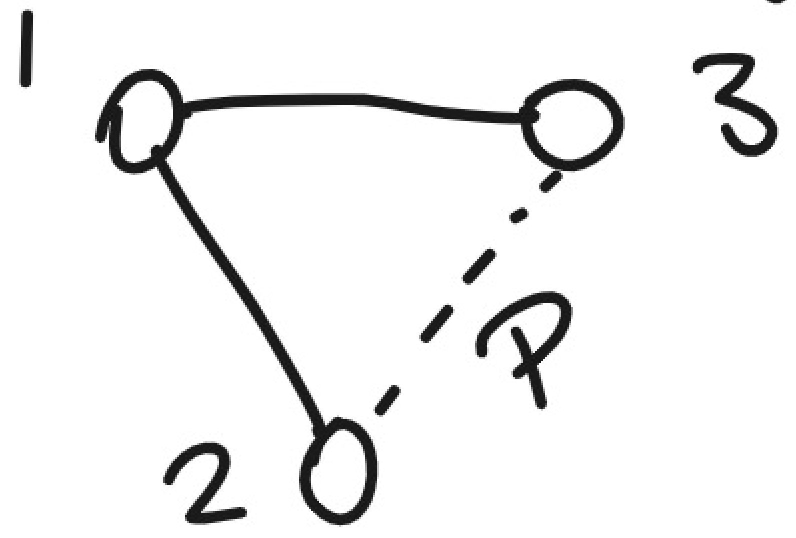
Poisson Distribution (high n, low p)

Random



Scale Free

clustering coefficient = C_i



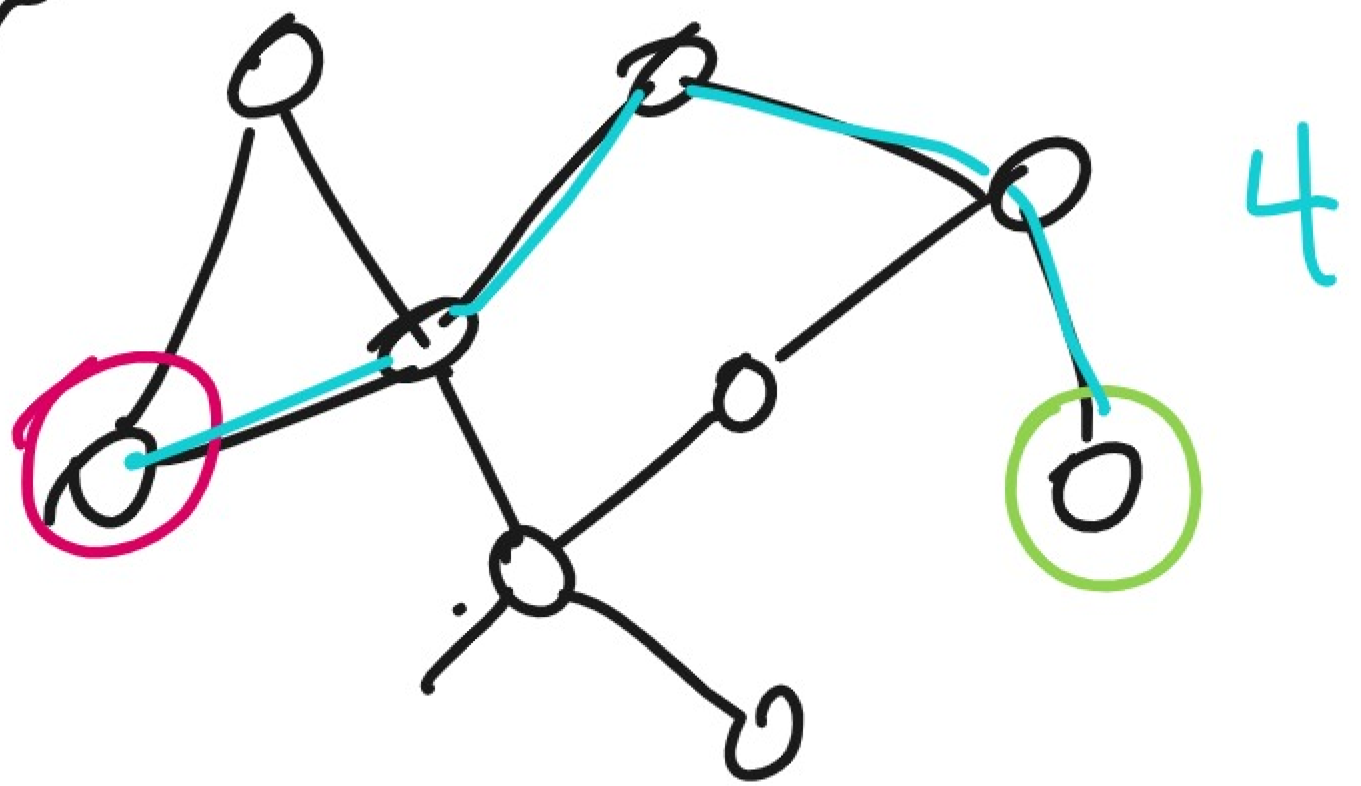
$$p = \frac{c}{n-1} = C_i$$

$$n \to \infty$$

$$C_i \to 0$$

~~Bio Nets~~

Path Length



How does avg path grow with n?

# of nodes separated by s steps $\to c^s$

$$n \sim c^s$$

$$\downarrow$$

$$s \sim \frac{\ln n}{\ln c}$$ (Random Network)

Assume

$c = 2$