# practical\_3\_positive\_feedback\_solutions

October 27, 2019

Name: Write your name here

#### 1 SYSTEMS AND NETWORK BIOLOGY - PRACTICAL 3

## 2 Positive feedback

To submit your report, answer the questions below and save the *notebook* clicking on File > Download as > iPython Notebook in the menu at the top of the page. **Rename the notebook file** to "practicalN\_name1\_name2.ipynb", where N is the number of the practical, and name1 and name2 are the first surnames of the two team members (only one name if the report is sent individually). Finally, **submit the resulting file through the** *Aula ESCI*.

Remember to label the axes in all the plots.

IMPORTANT REMINDER: Before the final submission, remember to **reset the kernel** and re-run the whole notebook again to check that it works.

The objective of this practical is to explore the behaviour of a system with a positive feedback motif. In particular, we will use numerical integration of an ordinary differential equation model to analyze the behavior of a phosphorylation/dephosphorylation cycle as the kinase concentration (acting as input signal) is cycled from a low to a high level and back.

Consider the phosphorylation/dephosphorylation cycle shown in the figure below, in which phosphorylation is induced by both a stimulus signal *S* and by the phosphorylated protein itself, in the form of a positive feedback loop. Assuming that the latter process is cooperative, the model of this system can be written as:

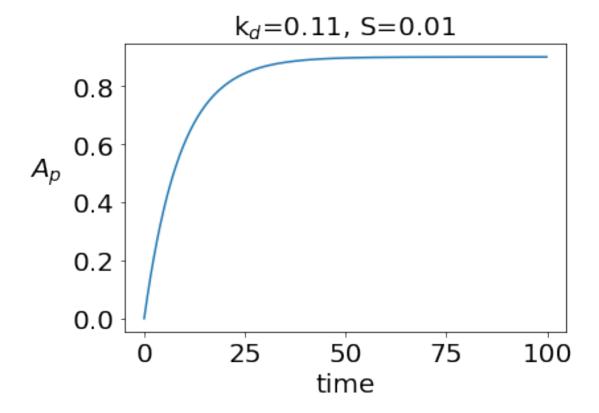
$$\frac{dA_p}{dt} = k_p SA + \beta \frac{A_p^n}{K^n + A_p^n} - k_d A_p$$

where  $k_p$  is the activation (phosphorylation) rate,  $k_d$  is the inactivation (dephosphorylation) rate, A is the concentration of the unphosphorylated protein, and  $A_p$  is the concentration of the phosphorylated protein. We will assume that the total protein concentration,  $A_T = A + A_p$ , is constant. We will consider the following parameter values: K = 50,  $A_T = 100$ ,  $k_d = 0.11$ ,  $\beta = 10$ ,  $k_p = 0.1$ , and n = 4.

In [1]:

First, integrate numerically, up to t=100, the differential equation above for a low (basal) stimulus level, such as S=0.01, assuming that initially the protein is completely unphosphory-lated.

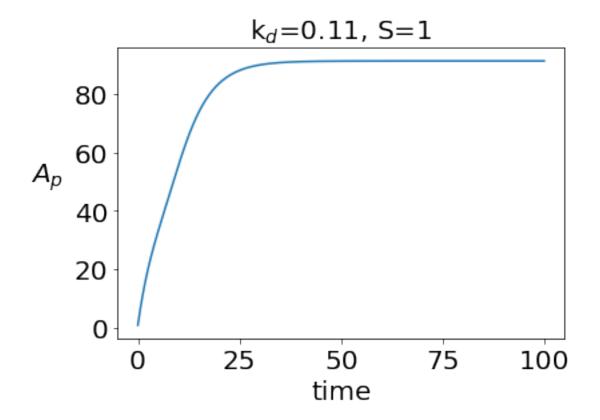
## In [2]:



Comment your result here

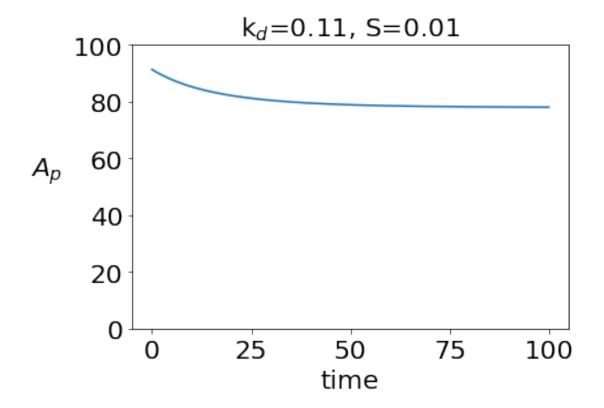
Nest, starting from the stationary situation obtained in the previous case, study the effect of suddenly applying a large stimulus signal, so that S suddenly jumps to a high level, such as S = 1.

## In [3]:



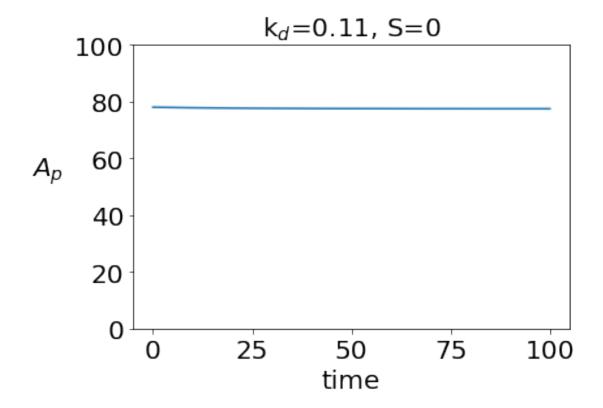
Next, starting from the latest stationary situation, integrate the equations again for S = 0.01. Does the system go back to its original state?

In [4]:



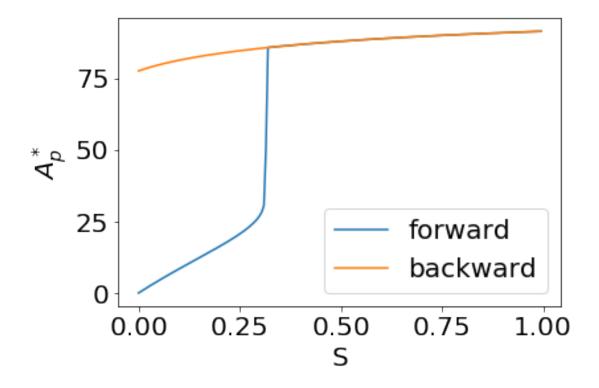
Finally, continue further to S=0, thus eliminating the stimulus completely. Does the system now go back to its original state?

In [5]:



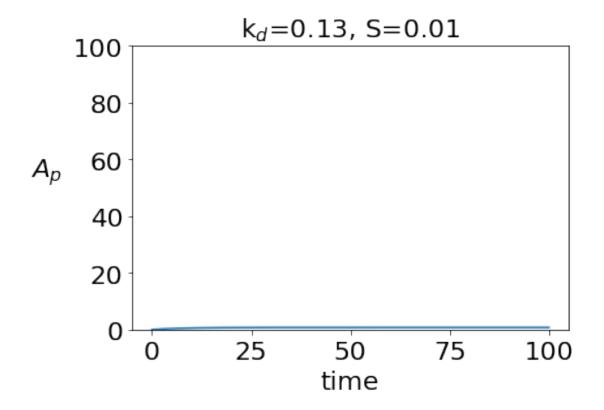
Now we will manually compute a bifurcation diagram of the system. In order to do this, choose a range of S values from 0 to 1. Start for S=0 considering that the protein is unphosphorylated at the beginning. Then, for each S value do a simulation starting each integration at the previous steady state, and save the new steady state value. Plot the results in a graph where the x axis is S, and the y axis is the steady state reached (from each simulation). Then do the same but in reverse order (starting from high S) and plot the results in a different color.

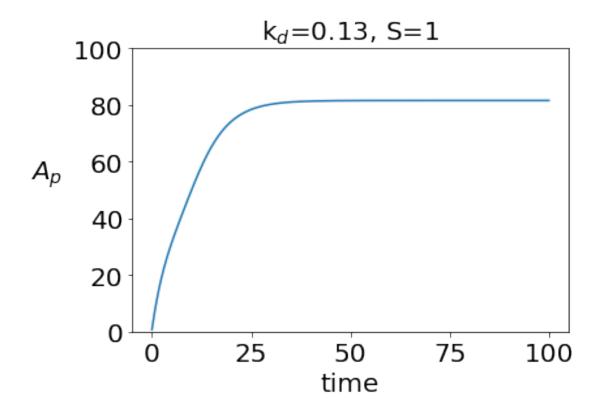
In [7]:

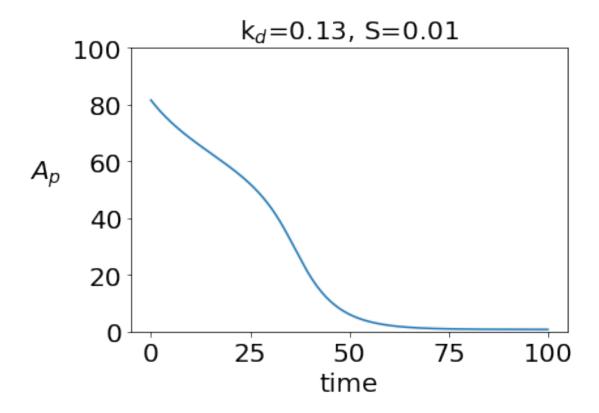


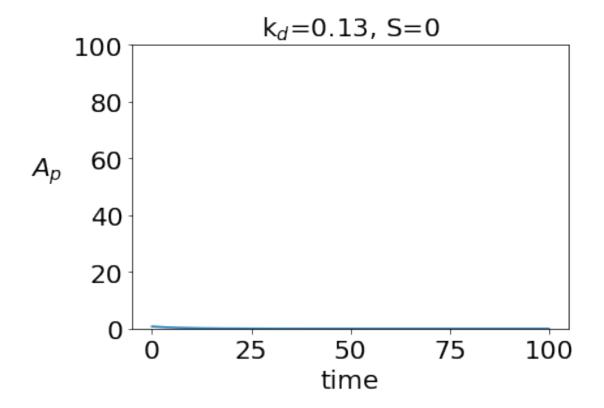
Finally, repeat the analysis above for  $k_d = 0.13$ . First, integrate for fixed values of S (increasing first from S = 0.01 to S = 1, and then decreasing back to S = 0.01 and S = 0), starting in each case from the previous steady state. Comment on the differences with respect to the case  $k_d = 0.11$ .

In [8]:



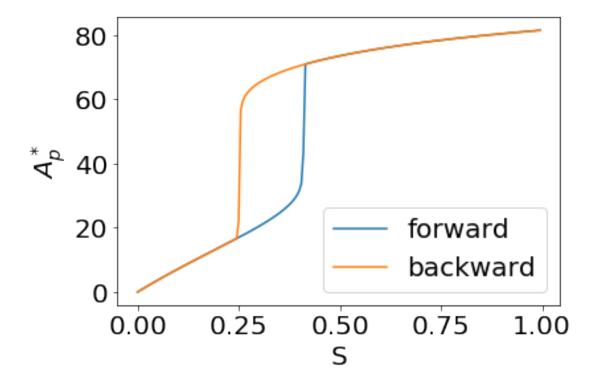






Finally, calculate numerically the bifurcation diagram for  $k_d=0.13$  and compare it again with the case  $k_d=0.11$ .

In [11]:



In []: