# practical_5_master_equation_solutions

November 5, 2019

**Name:** Write your name here

## 1 SYSTEMS AND NETWORK BIOLOGY - PRACTICAL 5

## 2 Solving the master equation

---

To submit your report, answer the questions below and save the *notebook* clicking on `File > Download as > iPython Notebook` in the menu at the top of the page. **Rename the notebook file** to ''`practicalN_name1_name2.ipynb`'', where `N` is the number of the practical, and `name1` and `name2` are the first surnames of the two team members (only one name if the report is sent individually). Finally, **submit the resulting file through the *Aula Global***.

Remember to label the axes in all the plots.

*IMPORTANT REMINDER: Before the final submission, remember to* ***reset the kernel*** *and re-run the whole notebook again to check that it works.*

---

In [9]:

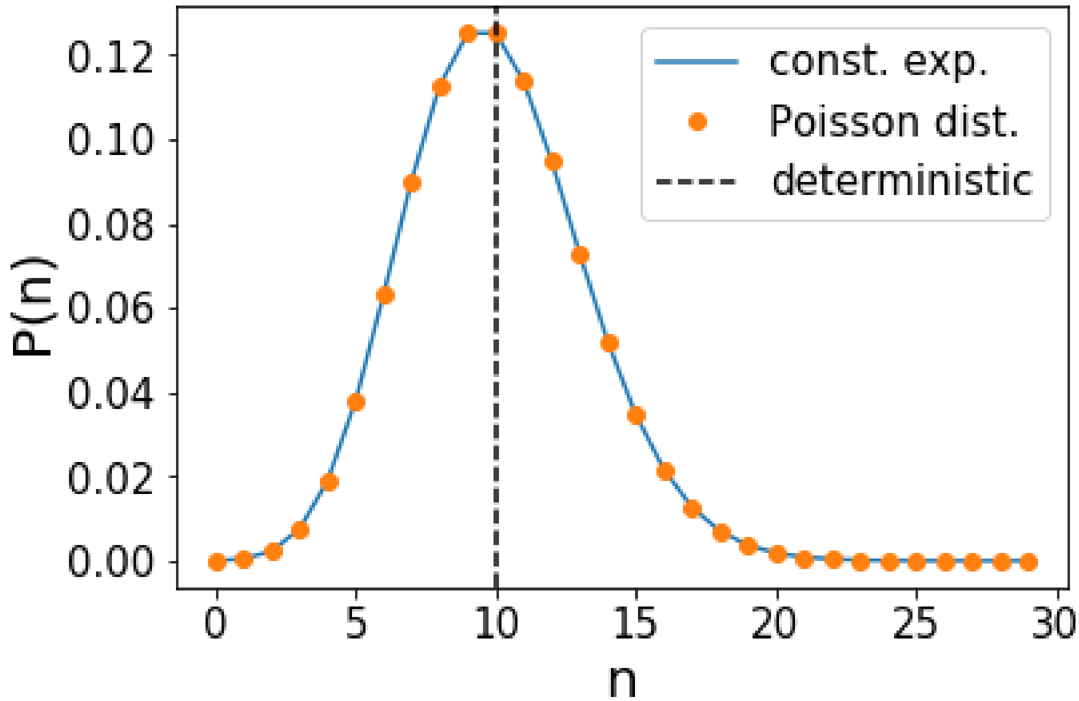
The aim of this practical is to solve the master equation of a birth-death processes in different situations. We will begin with the case of ***constitutive expression***. To that end, we will first write a computer code that calculates, following the steps described in class, the stationary probability density $P(n)$ of a birth-death process with constitutive expression ($C(n) = \alpha$, $D(n) = \delta n$), which we will use as a control in what follows, and plot the resulting distribution $P(n)$. Use the following parameter values: $\alpha = 10$, $\delta = 1$.

The solution of the master equation for this constitutive expression case can be obtained analytically, and shown to correspond to the Poisson distribution:

$$p(n) = \frac{\lambda^n}{n!} \exp(-\lambda),$$

where $\lambda = \alpha/\delta$ is the deterministic solution. Compare the numerical solution that you have obtained with the analytical expression of the Poisson distribution, and with the deterministic equilibrium of the system

In [13]:



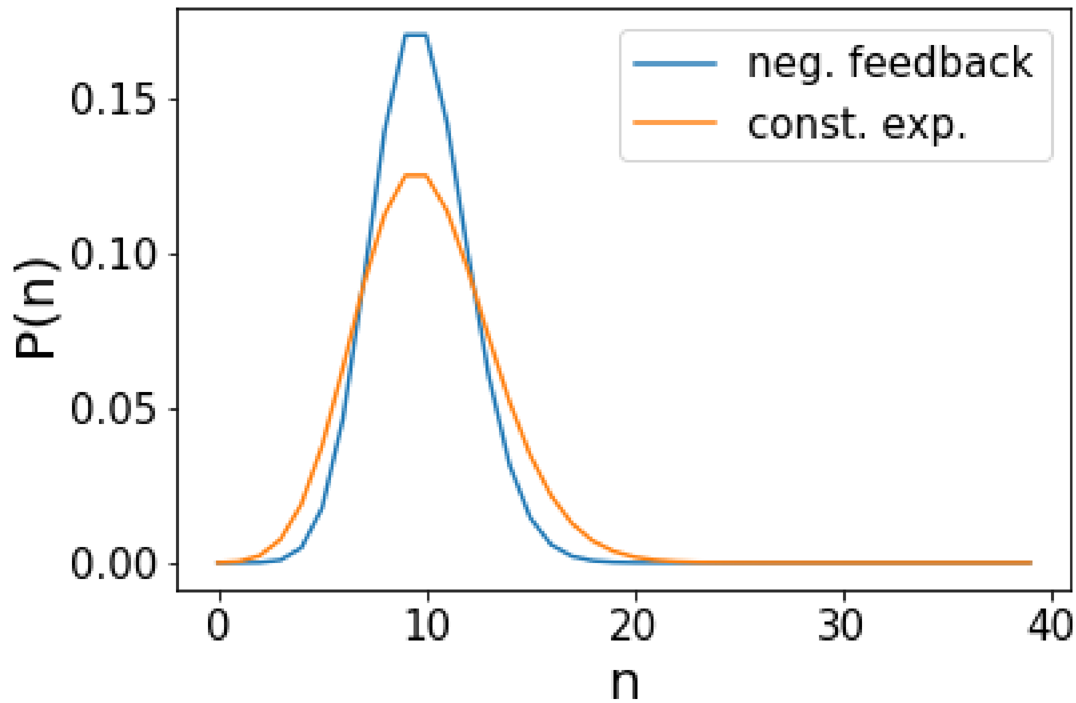

---

Comment your result here

---

Next, write another computer code that calculates the stationary probability density $P(n)$ of a birth-death process with negative feedback (defined by $C(n) = \frac{\alpha_n}{1+n/k}$), with $\alpha_n = 55$ and $k = 2$.

In [15]:



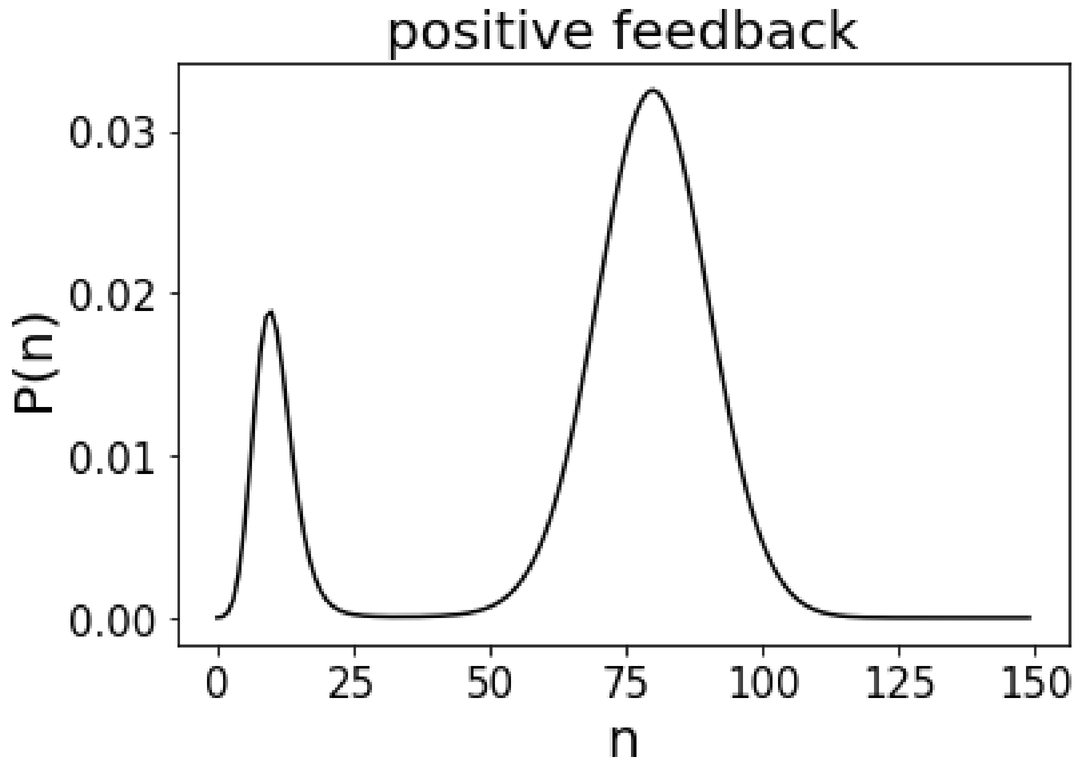

---

Compare **here** the results of the constitutive expression and the negative feedback cases. Which one is more variable? Relate this result with the properties of the negative feedback discussed in class.

---

Finally, calculate the stationary probability density $P(n)$ of a birth-death process with cooperative positive feedback with leakiness (defined by $C(n) = \alpha_0 + \frac{\alpha_p n^p}{k^p + n^p}$), with $\alpha_0 = 10$, $\alpha_p = 75$, $p = 4$, and $k = 40$.

```
In [20]:
```




---

Comment your result here. In particular, relate this result with the properties of the positive feedback discussed in class.

---

```
In [ ]:
```